THE ENTROPIC ANALYSIS OF ELECTORAL RESULTS: THE CASE OF EUROPEAN COUNTRIES

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1. INTRODUCTION

Entropy is a measure of information and uncertainty which has been used recently in different areas, besides of its original utilization in physics, creating a new research area: Econophysics. Econophysics is the application of physical methods with the objective of solve economic problems and had started precisely with the study of financial markets. In the last years, the number of works using entropy in economic analysis increased, because of it interest and also for similarities between both research areas, although that relationship is not such evident, in a first view. For example, Candeal *et al.* (2001) show in their work that exists a great similarity between the theory of entropy and utility.

There are several examples that show the analogies between physics and economics, as the cases of works of Ausloos *et al.* (1999), Stanley *et al.* (2001) or Bouchaud (2002), between others. There exist also some works that show an analogy between nature and economics, which allows to use entropy to study financial markets (see, in example, Dacorogna, 1999, Ausloos, 1998 or Farmer and Lo, 1999). Bailey (1990, 1993) introduces the concept of Social Entropy Theory, applying the concept of entropy to Sociology. In other hand, Chen (2003) uses entropy in a theory of psychology which as implications in the behaviour of agents that act in financial markets. More recently the insurance market had attracted the interest of physics (see Daaroneh 2006a, 2006b, 2006c). It can be concluded that the use of entropic methods had growing in areas like finances, microeconomics, utility functions, macroeconomics or even psychology.

The concept of entropy had also been used in election studies. Kirchgässner and Schimmelpfennig (1992) use it in an indirect form to study electoral participation. According to some authors, the closeness an election is, more participated is that election, because electors think that have more decision power. In that paper, entropy is used as a measure of closeness of results and it should be positively correlated with electoral participation. Gill (2005) uses entropy as a measure of uncertainty with the objective of trying to explain voting decision of electors that effectively vote. Considering the "Contract with America", a document done by republican in 1994 USA elections, to clarify American's opinions, Gill (2005) analyses if such phenomena had changed uncertainty levels of electors, concluding

that uncertainty is important in their decisions. In other hand, Antweiler (2006) uses maximum entropy to analyse migration votes in two consecutive elections, to analyse which electors changed their vote in the first to the second election. This paper intends to explore the utilization of entropy through politics and election results, but with different objectives. It generalizes interpretation of entropy, considering it a measure of dissatisfaction and disillusion of populations in relation to politics, using data for legislative elections[1].

Some phenomena like the increase of abstention in a country, consequence of the dissatisfaction of population and of their alienation in relation to politics could be detected and analysed. This discontentment could result, for example, in the appearance of new political parties, with more division of votes and increasing entropy (result of the discontentment and uncertainty by electors[2]). Absolute majorities, while imply less dispersion of votes, are synonym of more confidence in a given party, making a reduction of entropy.

Electoral results could also be influenced by particular phenomena, like those terrorist attacks made in vespers of the elections in Spain in 2004, with consequences on the affluence to the polls by electors, and influencing levels of entropy. Elections' dates could also influence results: for example, elections on summer season suffer from more abstention. Elections' results could also be connected with aspects like safety feeling of citizens, with unpopular socioeconomic policies taken by government or even with the economic performance of a country. One of the purposes of this paper is to find these types of phenomena and try to relate them with the concept of entropy. Another objective is to analyse the reality in different European countries.

Section 2 explains the concept of entropy and how this measure could be related with electoral abstention. It is also explained how entropy is estimated. Section 3 presents the data used in this paper, while section 4 shows the results of the relationship between entropy and abstention. In section 5 we present other possible results which could be obtained with this data: the relationship between abstention and GDP growth. It is also shown how abstention could help to explain about pontual facts and also to study the relationship between entropy and opinion polls. Section 6 compares entropy with other measures of variation and concentration, namely variance and the Herfindahl concentration index. Section 7 concludes.

2. ENTROPY AND ELECTORAL ABSTENTION

Entropy is a thermodynamics' concept, linked with the disorder degree of a state, a concept created by Clausius in 1865. Later, in 1877, Ludwig Boltzmann defined a probabilistic method to measure entropy. In 20th century, von Neumann and Shannon worked this concept. In the work that is the foundation of Information Theory, Shannon (1948) defines entropy as:

$$H(X) = -\sum p_i \log p_i \tag{1}$$

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[1] **Other methods of physical statistics were applied to election results, although with other objectives (see, for example, Fortunato and Castellano, 2007, or Filho et al., 2003).**

[2] It is based on the interpretation of entropy as a measure of discontentment or disorder that Bailey (1990, 1993) use this measure in the context of Sociology.

Shannon's entropy is a measure of uncertainty which is non-negative[3]. Zero is its minimum value, and it is just verified when there exists only one event, when there is certainty. The maximum entropy value is verified when the distribution is uniform ($p_i = 1/n_i$). Entropy measures total uncertainty of a not known probability distribution. In the scope of Information Theory, Shannon (1948) uses entropy as a measure of the amount of information transmitted by a message. Entropy could be also seen as a measure of randomness of a system.

This measure could also be interpreted as a measure of surprise of a result. Simply and intuitively, the lower that is the probability of an event, the greater is the surprise of that event to occur. The value of $-\log p_i$, which exemplifies that surprise, is greater as the probability is lower. So, entropy could be identified as the average as of how much a person should be surprised for the occurrence of an event. In the other hand, when uncertainty is greater, greater is the surprise of occurrence of an event. In this sense, when uncertainty is greater, entropy could be seen as the ordinal value of information in decisor's perspective (Dionísio, 2001).

It is also possible to consider entropy, when applied to economics, as an indicator of efficiency, because entropy conservation in time is a consequence of the absence of changes in resources distribution (Hidalgo, 2006). It is the same to say that the thermal machine as no movement, or in other words, temperature between two agents is equal, so it exists efficiency[4].

Entropy for discrete distributions take its minimum when is certainty about an event. In terms of elections, it should occur if all electors vote in the same party (or candidate). The maximum entropy value will occur when we are in the presence of an uniform probability distribution. Electorally speaking, it means that all parties (or candidates) have the same number of votes, result of their uncertainty.

In this paper entropy is used for discrete distributions, to analyse the evolution of electoral results. Although its interpretation of uncertainty, entropy is interpreted in this case as a measure of dissatisfaction of electors. In election results there are no probabilities to use to estimate entropy. However, elections data are relative frequencies, which verify some of the main properties of probabilities, for the effect of calculate entropy: $p_i > 0$ and $\sum p_i = 1$. So, the estimation of entropy is done with vote proportions of each party. Suppose that in a given election there are three parties, with the following vector of vote proportions: $\{0, 5; 0, 3; 0, 2\}$. In this case, the estimation of entropy following equation 1, would take the value of $1, 0297$. Because in this study the number of parties in the elections of different countries is different, it is calculated relative entropy: entropy divided by its maximum value. Entropy takes it maximum value when we have an uniform distribution. Considering the same election, when we have three parties, vote proportions would be $\{1/3; 1/3; 1/3\}$. So, entropy would take its maximum value, using equation 1, in $1, 0986$, and relative entropy will have the value of $0, 9372$.

There exist several studies trying to explain the reason for some electors vote and others abstain. Downs (1957), in a pioneer work in this research area, identifies the decision of vote with the fact that the elector has a greater benefit, when comparing

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[3]Equation 1 refers to entropy for discrete distributions. Entropy for continuous distributions was also defined by Shannon (1948). It presents some different propoerties but it is not referred in this paper because it will not be used.

[4]Across the time were developed new concepts linked with entropy. One of those concepts is conditional entropy, the uncertainty of a variable given the knowledge of other variable. Another related concept is mutual information, a measure that intends to identify similarities between two distributions. Nowadays it is used in several tests, namely to test dependence between different distributions.

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with the cost of vote, in a similar decision when economic agents decide when they consume. One of the critics pointed to this analysis is the fact that it couldn't explain all what is behind the decision of an elector to vote. It happens because there are other kinds of motivations, like the civic duty sensation, in the act of vote. Besides this, the decision of vote could be more related with an investment decision than with a consumption one, expecting a benefit in the future (see, in example, Riker and Ordeshook, 1968, and Palfrey and Rosenthal, 1983).

Other explanations for the decision to vote are made for models that use a spatial approach to explain that decision: the grater the proximity between elector's and candidates' opinion, greater is the probability of an elector vote in that candidate. This type of analysis is done, for example, by Ferejohn and Fiorina (1974), Enelow and Hinich (1981), Paldam and Skott (1995), Kirchgässner (2003) and Gill (2005).

The purpose of this paper is not to make a model of the decision of vote or abstain, but as it was referred, to relate abstention with entropy, interpreting it as a measure of dissatisfaction. The question here is: should it be possible to use entropy as a measure of dissatisfaction of electors? In the particular case of USA elections, Gill (2005) refers that there exists a great complexity in the elections. It happens because candidates present a vague speech, letting some uncertainty in electors. Besides this, the lack of knowledge of electoral programs and the lack of time of electors to analyse them could make a contribution to create that uncertainty. The existence of asymmetric information, according to the same author, could conduct some electors to think that the best alternative is to not vote (see also Enelow and Hinich, 1981 and Feddersen and Pesendorfer, 1997). So, and according to Gill (2005), if some electors vote and others do not, this is due to different levels of uncertainty of electors.

At an aggregate level, Blais *et al.* (2003) try to identify what are the factors that have influence in the electoral participation, covering 151 elections in 61 different countries. GDP per capita is one of those variables, showing a positive impact in participation. Also the dimension of population contributes to turnout, but in a negative form: countries with more population have less relative turnout. Electoral systems that allow votation through alternative ways (mail, in advance or by proxy) and that have compulsory vote (penalizing those that do not vote) present also more relative participation in elections. Feddersen and Pesendorfer (1997) also identify the reduction of vote barriers as a potential variable which could raise turnout. In other hand, in the specific context of american elections, Abramson and Aldrich (1982) found evidence that, even with more registration facility and with the increase of education, turnout in USA, between 1966 and 1980 decreased. Other factors, like dissatisfaction and the non identification with political parties are explanations for that phenomenon.

About the personal decision of vote or abstain, the motives are obviously others. Palfam and Skott (1995) and Kirchgässner (2003) refer that political parties gain votes if, in the electors' view, they are visibly different between them. So, one possible explanation for abstention is the belief of electors that political parties have no qualitative different solutions, synonymous of dissatisfaction. Or, in other hand, that political parties present promises that they successively do not fulfill. Santo (2005) cites Georges Vedel[5] affirming that one possible reason to the increase of abstention, in the case of France, is the depolitization, which can also be interpreted as a signal of population's dissatisfaction. Citing Freire and Magalhães[6],

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[5] Vedel, G. (1962), *La Dépolitisation: Mythe ou Réalité?*, Paris, Libraire Armand Colin.

[6] Freire, A. e P. Magalhães, 2002, *A Abstenção Eleitoral em Portugal*, Lisboa, Instituto de

Santo (2005) refers that abstention could be understood as a temporary attitude of dissatisfaction in relation to the political system and inclusively as a lack of confidence[7]. The lack of confidence and depolitization, associated with the weakening of party identification, had been pointed by Abramson and Aldrich (1982) as possible explanations for the decrease of turnout, in the specific case of USA. The same authors point the declining beliefs about government responsiveness as another reason for the increase of abstention.

Note the particular case of Portugal, a country that suffered some decades with a dictatorship, that ended in 1974. In the following year, Portugal had the first free elections after dictatorship[8]. Left recently of a disturbed period of its history, it was natural that electors wanted to show their desire to decide country's way, and voted about 92% of inscribe electors[9]. In terms of electoral results, if electors believe that democracy works, it is expected that the turnout levels doesn't change dramatically. However, if for any motive people do not believe in the political class, or if exists dissatisfaction, it is expected that abstention increases.

The value of entropy decreased in Portuguese elections through 1980, signal of the decrease of uncertainty by electors. It could be concluded that exists some credibilization of politicians. The coalition of centre-right, which won 1979 and 1980 elections could helped for this. However, that coalition finished after misunderstand, and separated. That fact contributed for a more dispersion of votes, increasing entropy in the following elections, in periods marked for some political instability in the country and also economic problems (it coincides with two periods of less GDP growth).

In the following elections, entropy decreased. There were more stable years, both in political and economic terms, also related with the entrance of Portugal to European Economic Community. This phenomena helped to decrease electors' uncertainty, in two votations that resulted in to absolute majorities and with governments that finished their four years mandates. Since then, electoral results show a growing level of entropy, just broken by a slight decrease in 2002. This is a point when Portuguese population feels dissatisfacted with politicians and when political parties present some weakness and sign of division. This is well-known with the appearance of movements of independent citizens in autarquic elections[10], or with the division of a major party supporting two different candidates for presidential elections. Besides this, parties with less votation in previous elections, started to gain some weight, having inclusively seat in the Parliament.

Since the first elections in 1974 to the last in 2005, the level of abstention in Portugal presents a growing trend, worrying for political class, and could be interpreted as an uncredibility indicator of populations. The lack of confidence in the political system is also identified as a problem in the country, by Vargas (2002).

Also the Spanish case could help to understand better the relationship between entropy and abstention, namely the events happened in 2004 elections. In that year, in the days before the elections, occurred the terrorist attacks of March 11th, that could changed the course of the results. The majority of opinion polls in

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[7] Bailey (1983) refers that maximum entropy is the "system death" (page 114). The increase of entropy, in the context of elections and interpreted as social dissatisfaction shouldn't mean the "death of democracy" but sould be analysed by politicans as a problem, like the increase of abstention.

[8] All the elections considered in this paper are legislative elections.

[9] Data for portuguese elections are obtained in *Comissão Nacional de Eleições* (www.cne.pt)

[10] In 2005, a total of 7 municipalities were won by independent candidatures, all over the country.

that year indicated a victory of Popular Party (PP), even it was not an absolute majority[11]. However, after the attacks and after wrong accusations (remember that PP government blamed ETA for the attacks when the true attacks were made by extremist Islamic groups), results were different from those that opinion polls point. In this case, the dissatisfaction with a particular political party made that electors had more certains in relation to the opposition party, changing the direction of results of opinion polls, at the same time that turnout increased.

It is based on these arguments that entropy is interpreted as a measure of dissatisfaction for the political system in general. So, it is possible to analyse if entropy is related with abstention.

3. DATA

Abstention is a phenomenon that occurs in all countries of the world and has been studied for several occasions. In this paper abstention is analysed for 16 European countries: Italy, Spain, Portugal, United Kingdom, the Netherlands, Germany, Belgium, France, Norway, Sweden, Finland, Greece, Switzerland, Austria, Ireland and Denmark. The study starts with elections after II World War. First, because it is a mark on the world history. Second because is from this date that data is more frequent.

Austria, Finland, France, Norway, United Kingdom and Denmark were the countries of this analysis the first to have elections in post-II World War, in 1945; in the following year were the elections of the Netherlands and Belgium[12]; 1947 was the year of elections in Switzerland, followed by Italy, Sweden and Ireland in 1948 and Germany in 1949; Greece, Portugal and Spain have a shorter time analysis because their first free elections were, respectively, in 1974, 1975 and 1977. Table 1 indicates the first and the last year of the analysis, as well as the number of elections in each country. It is also indicated data source for each country, normally sites of official pages of elections in each country[13]. Just legislative elections were considered. Following Santo (2005), the context of the election (if it is national, regional, European, presidential, etc.) could have influence in the level of abstention. So it is necessary to choose similar elections for countries.

In terms of the evolution analysis we found that the levels of abstention increased for almost all the countries studied in our data sample. This growing trend just not occurs in Spain, Norway and Denmark. In the cases of Germany, Belgium and Sweden, until half of sample, abstention had decreased. From that time to nowadays, they have the same increasing path. The evolution of abstention can be analysed in Figure 1.

We use proportions of vote for the most voted parties to estimate discrete entropy. The number of parties used in each country, for the estimation of entropy, differs between countries but, in the same country, the number of parties is always the same. To make comparisons between different countries, and since entropy is sensitive in relation to the number of categories (parties) used for the analysis, it was calculated the relative entropy (HR): the ratio between entropy (H) and its

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[11]See, for example, www.libertaddigital.com/suplementos/elecciones2004/encuestas.html or www.lapaginadefinitiva.com/politica/elecciones2004/escopetas.htm

[12]The first election used in Belgium was just the election of 1949, because it was the first with universal suffrage.

[13]For some electios, in the absence of information about abstention, these data was recovered from www.idea.int/vt/.

Country	1st election	Last election	N. of elections	Source
Ita	1948	2008	16	elezionistorico.interno.it
Spa	1977	2008	10	www.elecciones.mir.es
Por	1975	2005	12	www.cne.pt
UK	1945	2005	17	www.election.demon.co.uk
Net	1946	2006	19	www.kiesraad.nl
Ger	1949	2005	16	www.bundeswahlleiter.de
Bel	1949	2007	19	www.ibzdgip.fgov.be
Fra	1945	2007	18	www.interieur.gouv.fr
Nor	1945	2005	16	www.regjeringen.no
Swe	1948	2006	19	www.val.se
Fin	1945	2007	18	statfin.stat.fi
Gre	1974	2007	12	www.ypes.gr
Swi	1947	2007	16	www.parties-and-elections.de
Aut	1945	2006	19	www.bmi.gv.at
Ire	1948	2007	18	electionsireland.org
Den	1945	2007	25	www.ft.dk

TABLE 1
Number of elections for each country

maximum value (HM) - $HR = H/HM$. The criterion used to choose the number of categories in each country is related with the number of parties most voted, in the first considered election. The objective was that, at the beginning of the sample, the proportion of the category "Others" shouldn't exceed 10%. It allows to see, for example, the relevance and the evolution of this category. An increase in its value means that exists more dispersion on votes, being a clear indicator of global uncertainty by electors in relation to their political opinions, which will increase entropy. While there exist countries where this category decreased (the case of Spain, that could be related with the particular event of terrorist attacks in the days before the elections in 2004 and 2008), others are where that category is, inclusively, the most important one (the case, for example, of Belgium, where in the first election it had a residual weight of 1% and, in 2007, after considered the first six parties, the value of "Others" corresponded to about 24% of the votes). Excluding the cases of Germany, Spain and Portugal, all other countries show a trend, more or less evident, of increase in that category, in spite of the cases of Italy, Switzerland and Denmark where that value had decreased in the last elections. The behaviour of the category "Others" could be seen, for the different countries, in Figure 2.

4. RESULTS

Entropy is frequently used as a measure of uncertainty. In this paper it is used to analyse electoral processes, interpreting it as a measure of uncertainty of electors for politics in general. It could be considered as a measure of dissatisfaction and disillusion in relation to politics. As referred previously, electors' dissatisfaction
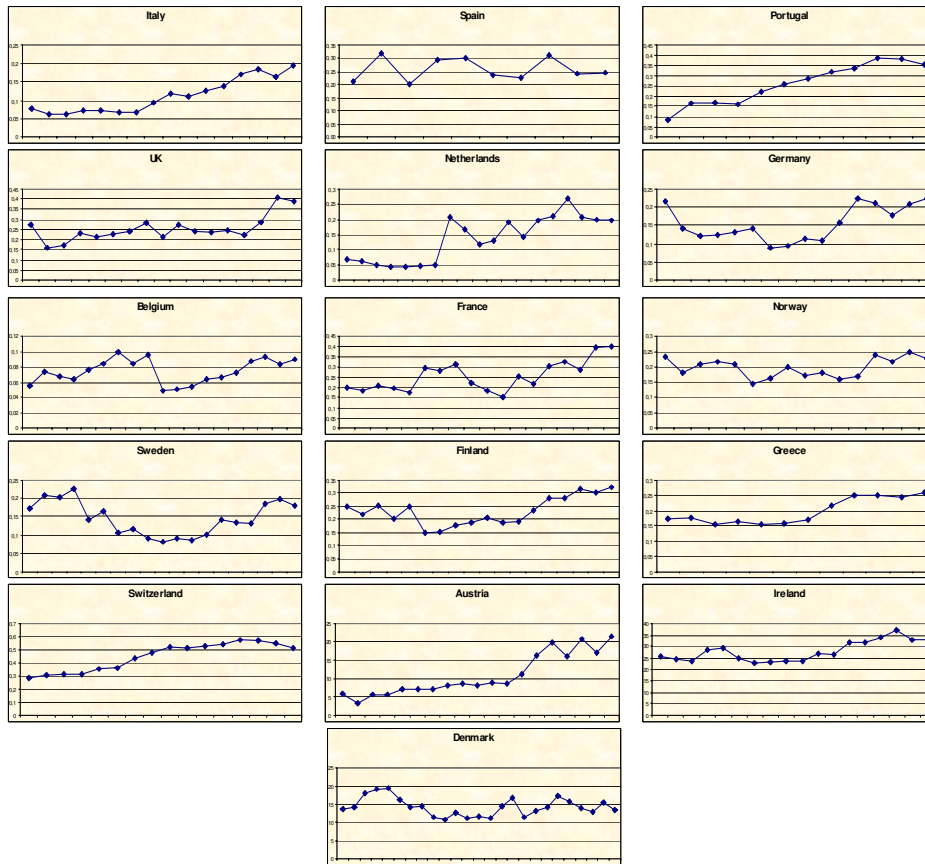
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FIG. 1 Evolution of abstention.

is caused, according to some authors, by the weakening of the interest for politics and has a consequence on the decreasing of turnout, increasing abstention. In this sense, entropy should be positively correlated with abstention which means that, the greater the dissatisfaction of electors, abstention will take higher values (see, for example, Santo, 2005 or Abramson and Aldrich, 1982). Individual results, for each of sixteen countries in analysis, could be seen in Table 2. Those values refers to the correlation between relative entropy and abstention, for country.

From the analysis of all countries in this database, just four have different correlation coefficients from expected: Portugal, France, Finland and Denmark. From these countries, just the French case is significant at 5% level. The coefficient of Portugal, besides taking a relatively high value (inclusively greater in absolute value when compared with the french coefficient), is not significant at 5%.

For the countries that present positive coefficients, more than half are significant: Italy, United Kingdom and Norway, at 5% level and Germany, Switzerland, Austria and Ireland at 1% level. Our results show a strong support for what is proposed in this paper: abstention is related with uncertainty of electors, measured by entropy. Considering that any country have a sample greater than twenty elections, results could be considered encouraging. The graphical representation of the different
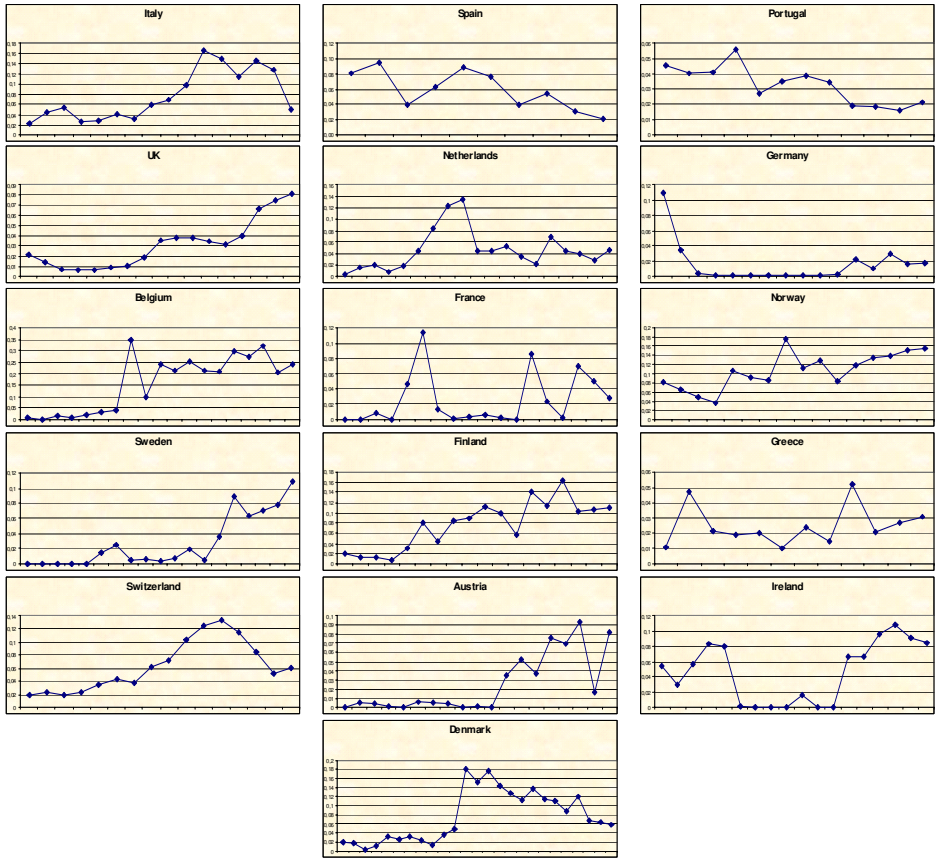
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FIG. 2 Evolution of the category "Others".

countries, considering these two variables, could be seen at Figure 3.

If one of the problems could be the small dimension of the samples in some countries, it is interesting to verify what happens we aggregate data from different countries, making just one sample. Because for the estimation of entropy were used different number of categories for countries, it was estimated a comparative measure. It was calculated the level of relative entropy ($HR = H/HM$), a measure that is greater than 0 and smaller than 1 and indicates the relationship of the value of entropy in each election and its maximum value, in the case of all categories have the same vote proportion. As the level of abstention is completely different for countries, it was done the calculus of abstention per capita, to make possible an international comparison between data[14].

The sample recovered has a total of 270 elections. Estimating the correlation coefficient between relative entropy and the respective level of abstention per capita, the observed value is $0,2971$, statistically significant. It is concluded that, besides results have some support for individual analysis, the same happens when aggregated data is considered. Figure 4 represents this relationship.

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[14] Abstention per capita represents the value of percentage of abstention for million inhabitants. A similar indicator to this is used in Strömberg (2004).

Country	Correlation between entropy and abstention
Ita	0,5927*
Spa	0,2031
Por	-0,5610
UK	0,5490*
Net	0,1884
Ger	0,8072**
Bel	0,1290
Fra	-0,5130*
Nor	0,5160*
Swe	0,0390
Fin	-0,0279
Gre	0,3092
Swi	0,6859**
Aut	0,8397**
Ire	0,7032**
Den	-0,3800

TABLE 2
Individual results of correlation between entropy and abstention. ** significant at
5% level. * significant at 1% level.

Entropy had been already used previously to analyse its relationship with turnout. Kirchgässner and Schimmelpfennig (1992) defend that, how much electors think that election's result is close, greater is their probability to decide to vote, because they consider their vote with capacity to be decisive. As elections with closer results mean higher levels of entropy, authors find evidence for United Kingdom and West Germany (elections in both countries in 1983 and 1987) for a positive relationship between entropy and turnout.

This paper presents entropy as a measure of dissatisfaction, positively correlated with abstention and, so, negatively correlated with turnout, contradicting previous results. A possible explanation could be related with the reduced number used by Kirchgässner and Schimmelpfennig (1992), besides it uses information for several regional electoral circles. Another possible explanation is that entropy, as it was estimated, is a measure calculated after elections. The factors that can have influence in the decision of vote by electors, according to the authors, is their belief that elections will have close results, and these are data supplied not by final results but by pre-electoral opinion polls.

5. OTHER POSSIBLE FACTS TO ANALYSE

Besides the relationship between abstention and entropy, the data of this paper could be used to analyse other kind of questions. In this section is analysed the possible relationship between abstention and GDP growth, the effect of specific and unexpected events during the electoral period and to analyse the relationship between opinion polls and electors uncertainty.

FIG. 3 Relative entropy and abstention.

5.1. Abstention and GDP growth

Some authors, like Blais *et al.* (2003), refer that turnout could be positively related with the level of growth in a country. In this paper it was made an analysis of this kind of relationships, first making the relationship between growth rate and abstention and that the relationship between growth rate and abstention per capita[15]. Besides of the fact that results do not show such a good verification like those between abstention and entropy, some conclusions could be retrieved. The relationship between growth rate, abstention and abstention per capita is in the Table 3. The expected relation between these variables is a negative one: higher levels of growth should make abstention to be lower, because countries with greater growth rates have tendency to have more informed and interested citizens in politics (Blais *et al.*, 2003).

In this case, results do not have such support as expected. Using abstention, and besides the majority of countries show the expected sign of correlation (eleven in sixteen countries), just four have significant results: Italy, Austria, Denmark (all with 5% of significance) and Switzerland (1% of significance). This could be caused by the few number of observations in samples. Six countries present positive

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[15]The growth rate used is the average growth between two consecutive elections.
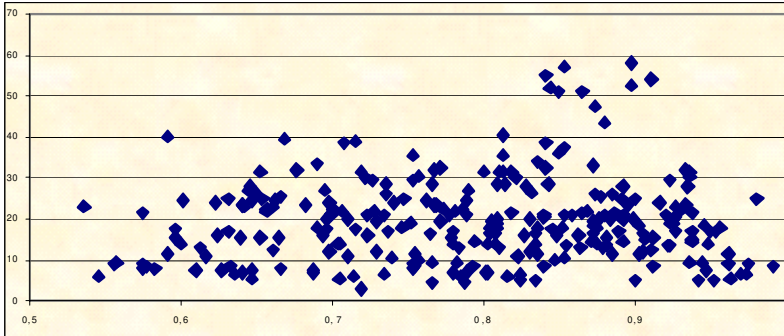
FIG. 4 Relative entropy and abstention per capita.

correlation levels (Spain, United Kingdom, Belgium, Sweden, Greece and Ireland), but none is statistically significant, using the usual levels of significance.

When it is made the same analysis using abstention per capita, results are somehow different. Just five countries show positive correlation (Spain, United Kingdom, France Sweden and Ireland), but none is significant. The remaining countries show the expected correlation signal and five of them are significant: Denmark, Austria, Belgium and Italy with 5% of significance, and Switzerland, with 2% (Greece shows a level of correlation close to be significant at a level of 5%). The relationship between abstention per capita and GDP growth rate, for each country, could be seen in Figure 5.

While analysing countries individually show some support for the expected relationship between these two variables, when data is aggregated that verification vanish. Using the results of growth rate and abstention per capita for all countries, we obtain a correlation coefficient of $0,1174$, in a total of 214 observations. The differences between this result and the correlation between entropy and abstention could be related with two different motives. First, the number of observations is smaller, once data used to calculate growth rates, sourced by IFM, are less numerous. Figure 6 shows the relationship between abstention per capita and GDP growth rate, using aggregated data.

Second, and maybe more important, is the fact that in this study are just used countries with degrees of economic development relatively high. The study of Blais *et al.* (2003) uses an heterogeneous group of 61 countries and from different continents. For those heterogeneous group GDP could be relevant. In this case, probably this is not the most important variable.

5.2. Specific events

It is usual to say that some events have the ability to change the elections' results. Is this true? At least in the case of Spain it seem to be. The electoral campaign in 2004 was coming to end, with opinion polls indicating a victory for Popular Party (PP), when happened the terrorist attacks of 11th March in Madrid. The PP government accused ETA to have the responsability for the attacks, when they were made by Islamic groups. After this, the electoral results were completely different from opinion polls, giving a clear victory for the Socialist Party (PSOE).

Country	Correlation between GDP growth and abstention	Correlation between GDP growth and abstention per capita	N
Ita	-0,7604*	-0,7449*	10
Spa	0,3158	0,1312	9
Por	-0,4018	-0,3682	11
UK	0,1290	0,1524	15
Net	-0,1445	-0,0782	15
Ger	-0,0645	-0,0238	12
Bel	0,4716	-0,5303*	16
Fra	-0,2683	0,0362	14
Nor	-0,3453	-0,2117	10
Swe	0,3120	0,36976	17
Fin	-0,4250	-0,4248	13
Gre	0,5089	-0,5936	11
Swi	-0,7429**	-0,7316**	15
Aut	-0,5604	-0,5544*	13
Ire	0,4455	0,0448	17
Den	-0,5555*	-0,5729*	16

TABLE 3

Individual results for correlations between growth and abstention and between growth and abstention per capita. ** significant at 5% level. * significant at 1% level.

This particular phenomena was the base of the work of Gassebner *et al.* (2008). Based on theretical approaches of Barro (1973) and Ferejohn (1986), and with the hypothesis that "Terrorism increases the probability that the incumbent government is replaced an the next election", they conclude that the existence of terrorist attacks and, in particular, their severity, have an effective impact in the decision of electors of change the government. This happens because national security is regarded has an extremely important public good and, when occure severe terrorist attacks, they are seen as a failure of governments.

In the particurar case of the Spanish election of 2004, electors showed his certainty to punish the PP, at the same time that turnout increased. How can be entropy used to analyse this particular event? As referred previously, entropy is a measure of uncertainty and could be interpreted as a measure of dissatisfaction facing politics in general. As data show, the relationship between abstention and entropy is high. So, how much electors are dissatisfacted with political parties, there is a less turnout. Before the attacks, there was no reason to think that elections would not have a normal behaviour. At the last day that opinion polls were published in Spain, four different organizations presented their results (IPSOS-ECO, SIGMA-DOS, Instituto Opina and Demoscopia). Just in the first one PSOE was the winning party. The results of the polls, in terms of entropy, were $\{1, 2136; 1, 1871; 1, 2182; 1, 1993\}$[16]. The entropy of the electoral results is $1, 1787$.

[16]In this case, entropy was estimated transforming the proportion of the number of deputies indicated by polls. As the proportion of deputies is not equal to the proportion of votes, to compare the results of polls with the electoral result, the entropy of this was also estimating with
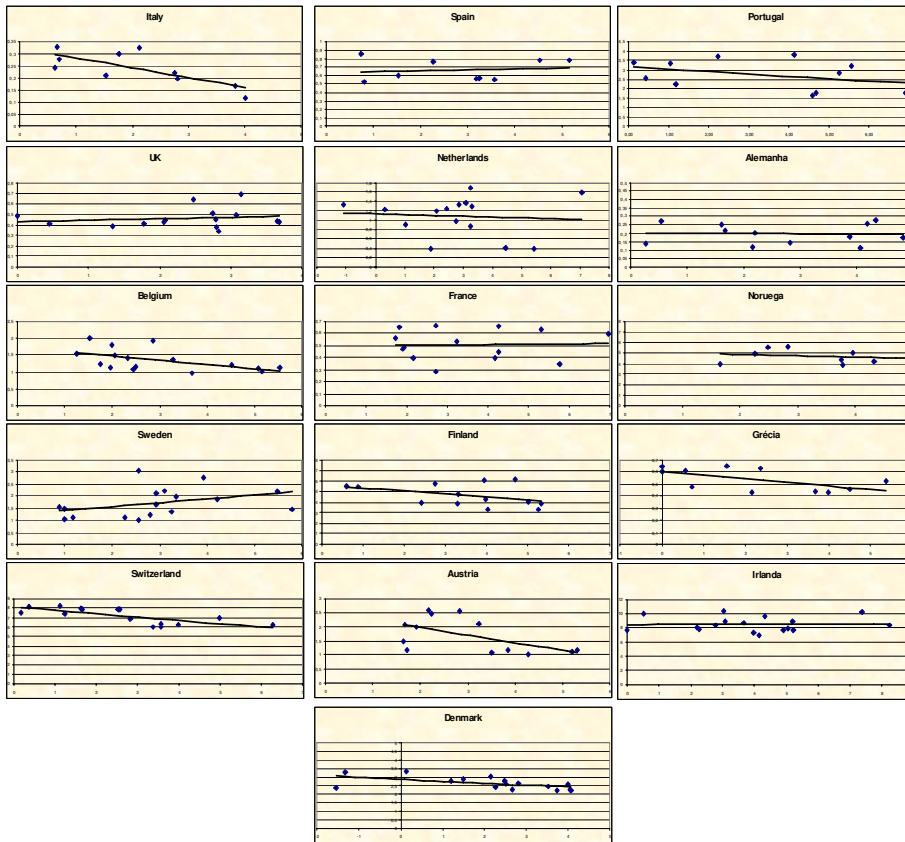
13

FIG. 5 Relation between abstention per capita and GDP growth rate.

It is confirmed that entropy decreased, in relation to all parties. Besides this, it was made a test to the means of the four polls. Under the null hypothesis that the mean is equal to $1,1787$ (the result of elections), and even with just four observations, the null hypothesis is rejected at the level of 5%, meaning that mean of polls is different from the result effectively done by the elections. Even doing a bilateral test, the evidence is that the average of the opinion polls is greater than the average of the elections, concluding that entropy decreased.

What is possible to conclude? That electors showed unhappy with what happened with the government party and mobilized to vote in PSOE. Could be made the following question: isn't entropy being interpreted as a dissatisfaction indicator? Apparently yes, but an indicator of dissatisfaction facing to politics in general. In this case, the reducion of entropy is due to the dissatisfaction with a particular party.

5.3. Opinion polls.

Other question could be made now: are the results of opinion polls misleading? Or, in average, they give the true result of intentions of electors? Unfortunately, we
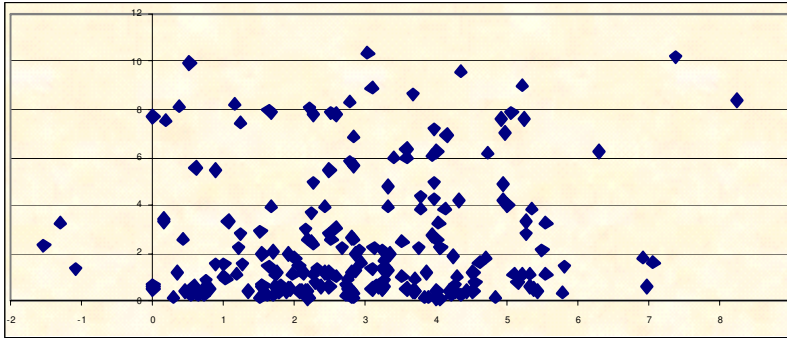
those data.

FIG. 6 Relation between abstention per capita and GDP growth for aggregated data.

have not available data for opinion polls for all countries. However, it was possible to find 147 different opinion polls[17]. Estimating the relative entropy of poll's results, and calculating its average, we made making the usual test of equality of means and we did not reject the hypothesis of equality. So, with a level of 5% of significance, there is evidence than the average of the relative entropy of polls opinion $(0, 8308)$ is not statistically different from the relative entropy of the results $(0, 8246)$. It can be concluded that, normally, surveys do not present statistically different results, in terms of entropy, from that there are shown by elections. This demonstrates that entropy could be used to detect what happens caused by unexpected events that change in the uncertainty level of citizens, as seen in the previous section.

6.   COMPARING DIFFERENT MEASURES

Besides entropy, there are other measures traditionally used to measure uncertainty, variability and concentration. The measure of variability most used is variance, which could be also interpreted as a measure of uncertainty.

Supposing, once again, that we have an election with three political parties with the following vector of vote proportions $\{0, 5; 0, 3; 0, 2\}$. In this case, the estimated variance is given by $0, 03$. For the same motive that was used the relative entropy, it was also used the relative variance: the ration between variance and maximum variance. Once again, maximum variance is estimating using an uniform

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[17]6 polls for italian elections in 2008 (http://alessandroingegno.wordpress.com) and one for the 2006 election (http://www.repubblica.it); 3 polls for spanish elections in 2008 and 4 for 2004 (http://es.wikipedia.org); 7 polls for portuguese elections of 2005 (http://www.marktest.com), 1 for 2002 (http://dossiers.publico.clix.pt/noticia.aspx?idCanal=317&id=73771), 6 for elections of 1999, 7 for 1995 and 6 for 1991 (http://margensdeerro.blogspot.com); for Sweden, in 2006 were obtained data for 25 polls of 5 different organisms(Synovate Temo, Sifo, Skop, Ruab, Demoskop), since the beggining of September, with the elections taking place on 17th Setembro (http://www.temo.se/upload/326/valjbsamtliga.htm) and for 2002 were taken data for 47 polls from 1st September to 14th September, the day before the election (http://www.temo.se/upload/326/valjbsamtliga2000_sep2002.htm); for Greece, in 2007, have reults of 13 polls (http://en.wikipedia.org/wiki/Greek_legislative_election%2C_2007); for the Netherlands it was collected one poll (http://www.politiekebarometer.nl/); for Austria were collected data from 12 polls in the month before elections (http://en.wikipedia.org/wiki/Austrian_legislative_election%2C_2006); in France were collected 9 polls (http://en.wikipedia.org/wiki/French_legislative_election%2C_2007).

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	HR	VAR_R	HHR	$Abstpc$
HR	1	0,6726**	-0,978**	0,2907**
VAR_R		1	-0,6175**	0,1535*
HHR			1	-0,2661**
$Abstpc$				1

TABLE 4

Comparison of different measures. ** significant at 5% level. * significant at 1% level.

vote vector, given by $\{1/3; 1/3; 1/3\}$. So, maximum variance should be $0,0370$, and relative variance would be $0,82$.

Another index broadly used to measure concentration drifts from industrial economics: Herfindahl concentration index, given by 2.

$$HH = \sum p_i^2 \tag{2}$$

With a vote proportion vector equal to $\{0, 5; 0, 3; 0, 2\}$, Herfindahl index will take the value of $0,38$. Once again, because different countries use different numbers of categories (parties), it was calculated the relative Herfindahl index, given by equation 3. In this case, the value would be $0,07$.

$$HHR = \frac{\left(HH - \frac{1}{N}\right)}{1 - \frac{1}{N}} \tag{3}$$

It were calculated correlations between the following variables: relative entropy (HR), relative variance (VAR_R), relative Herfindahl index (HHR) and abstention per capita ($Abstpc$). Results could be seen at Table 4.

It could be seen a strong significant correlation between those three measures. The relationship between relative entropy and relative variance takes a positive value of $0,6726$. Effectively, these are measures that have the same intention: both could be considered uncertainty and variability measures. Comparing the performance of both measures with abstention (measured per capita), relative entropy has stronger results: in spite of both variables present a significant correlation, significance is stronger for relative entropy.

When compared with Herfindahl index (HHR), entropy presents a significant correlation. As it was expected, that correlation is negative: while entropy is a measure of uncertainty and dispersion, Herfindahl index is a measure of concentration. As expected, the correlation between HHR and abstention per capita is negative. In absolute terms, relative entropy shows stronger correlation with abstention per capita, when compared with relative variance.

7. CONCLUSION

This paper has the objective of extend the use of entropy in elections. Being entropy a measure of uncertainty, it can be seen as a measure of dissatisfaction of electors in relation to politics in general. We expect that when the dissatisfaction of electors is higher, the turnout will be lower and, consequently, abstention will be higher.

The relation ship between abstention and entropy was obtained and, for 12 of the 16 countries of the sample, results present the expected signal of the coefficient, and 7 from these are significant. Just Portugal, Finland, Denmark and France have negative coefficients, and just the last one is significant. When this coefficient is calculated with grouped data, the result is statistical significant.

Then, it were identified another facts. According to some authors, it is expected that abstention should be negatively correlated with GDP. Calculating the correlation between abstention and growth rate, grouped results present a positive correlation, contrary to the expected, but statistically non-significant. However, when individual countries are considered, the unique results that are significant point to the expected negative relationship.

Entropy could also be used, in this context, to try to explain some events. It is what happens with Spain, where the terrorist attacks of 11th March 2004 changed the vote intention of electors and that it is significantly different from surveys. As opinion polls do not differ from the average of election results, it is concluded that entropy could also explain some specific events occurred during the electoral period.

Finally, it were compared three different measures normally used to measure dispersion and concentration: entropy, variance and Herfindahl index, all in relative terms. The first two measures presented, between them, a close relation. When compared with abstention, both in absolute and per capita terms, relative entropy has stronger results. Relative entropy shows an inverse relation, as expected, with the Herfindahl index. However, both measures present similar qualitative results when compared with abstention: how greater is abstention, it could be seen as dissatisfaction of populations but also has a positive correlation with electoral results.

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