Capital Investment as Real Options: A Note on Dixit-Pindyck Model.

Termos, Ali

American University of Sharjah

29 January 2008

Bank Investment as Real Option: A Note on Dixit-Pindyck Model

Ali Termos[*†]

March 1, 2008

Preliminary and Incomplete

Abstract

This paper applies Contingent Claims model a la Dixit and Pindyck (1994), on bank investment. Banks are indifferent between investing their assets on their own and extending loans to investors. The critical decision faced by the banker is the timing of the investment decision and its uncertainty. When banks make an irreversible investment decision they exercise the option to invest and give up the opportunity of waiting for new information to arrive. This lost option value is incorporated in the investment cost. Therefore, the value of the project must exceed the investment cost by the value of keeping the investment option alive. Using a third-moment mean-reversion process of the investment's volatility, the model shows that a higher mean-reversion parameter reduces both the value of the option to invest and the critical value at which the project deems feasible.

[*]Assistant professor, Department of Economics, American university of Sharjah, United Arab Emirates. [Email: atermos@aus.edu] P.O.BOX 26666 Sharjah, UAE.

[†]Acknowledgment: I thank Paul Fackler for his valuable insights on the numerical solution. All errors are mine.

1 Introduction to the model

The neo-classical theory of investment postulates that a firm should invest in a project as long as the present value of the expected stream of profits that this project will generate should exceed (or at least equal) the present value of the expenditures stream required to build the project. Thus, the net present value (NPV) of the investment project is greater than zero. This classical theory fails to capture three main characteristics in most investment decisions. First is the issue of *irreversibility.* A classical NPV calculation would implicitly assume that if economic conditions turn to be sluggish, the firm shuts down the project and recovers the money that is invested. However, this is not a realistic outcome. Second, is the issue of *uncertainty.* Again, neoclassical NPV calculation does not assess probabilities of alternative outcomes over the future of the project life. Third is the *timing* of investment. Firms, or banks in this case, can postpone their investments to obtain more information about the future, albeit never complete information. The ability to delay an irreversible investment project can affect the decision to invest. This is the main theme of the real option approach, first developed by McDonald and Siegel (1986). Firms or banks with an opportunity to invest are holding an "option" analogous to a financial call option. When a firm makes an irreversible investment, it exercises its option to invest. Doing so, the firm gives up the possibility of waiting for new information to arrive. That might affect the decision to invest now. This lost option value is an opportunity cost that must be included as part of the cost of the investment. Therefore, the value of the project must exceed the investment cost, by an amount equals to the value of keeping the investment option alive. Besides its feasibility, the real options model reflects the variations in value for any investment project in a continuous setting that is reflective of real economic variations, for example, fluctuations in interest rates.

1.1 An illustrative example

[1] Suppose that the project generates a stream of cash flow equals to the price of the output. Assume for now that price at time zero is $P_0 = 200$ and that the firm produces one output per year. This price is subject to go up to 300 with 50% chance or go down to 100 with 50% chance. So the expected value of the price is 200. Trying to avoid risk one can hedge against this price fluctuation. Let the initial investment be 1,600, the value of the project today is

$$-1600 + \sum_{t=0}^{\infty} \frac{200}{(1.10)^t}$$

$$-1600 + 2200 = 600$$

If P drops to 100 then the investor needs to sell short $\frac{2200}{200} = 11$ units. Suppose P turns down to be 100 then the investor collects $100 \times 11 = 1100$ from the hedging contract and the value of the project turns to be

$$\sum_{t=0}^{\infty} \frac{100}{(1.10)^t} = 1100$$

so the net outcome is $1100 + 1100 = 2200$.

Suppose now that the price went up to 300. The investor has sold short 11 units for $11 \times 200 = 2200$ and should pay $11 \times 300 = 3,300$ thus incur a net loss of $2200 - 3300 = -1100$ However, the value of the project is now

$$\sum_{t=0}^{\infty} \frac{300}{(1.10)^t} = 3300$$

and the net outcome is $3300 - 1100 = 2200$ same as in the previous case. Translating this example into a real option case let F_1 be the value of the investment opportunity next year. If $P = 300$ then

$$F_1 = -1600 + \sum_{t=0}^{\infty} \frac{300}{(1.10)^t} = 1700$$

If $P = 100$ then the option is unexercised so $F_1 = 0$. Now we need to find the value of the option today F_0. For that we create a portfolio

[1] The example here is borrowed from Dixit and Pindyck (1994)

with two components: first, the investment opportunity itself. Second, a certain amount of output. The portfolio is risk-free (no arbitrage). The value of the portfolio today is

$$\phi_0 = F_0 - nP_0$$

If $P_0 = 200$ then $\phi_0 = F_0 - 200n$. The value of the portfolio next year is : $\phi_1 = F_1 - nP_1$ which depends on P_1. If $P_1 = 300$ then

$$F_1 = -1600 + \sum_{t=0}^{\infty} \frac{300}{(1.10)^t} = 1700$$

Thus $\phi_{1a} = 1700 - 300n$. If $P_1 = 100$, then $F_1 = 0$, the option goes unexercised. Therefore $\phi_{1b} = 0 - 100n$. Now we need to chose n so that the portfolio ϕ_1 is risk-free. Set $\phi_{1a} = \phi_{1b}$:

$$1700 - 300n = -100n$$

so $n = 8.5$.

$$\phi_1 = 1700 - 300(8.5) = -850$$

whether P rises to 300 or falls to 100. To calculate the capital gain of this portfolio, first we should figure the payment that must be paid to the holder of the short position (the option premium).

$$Capital\ gain = \phi_1 - \phi_0 - option\ premium$$

No investor would be willing to hold a long position if capital gain is zero. The holder of a long position will require the risk free rate which is assumed 10%. Since expected price is 200 per unit, the required return is $0.1P_0 = 0.1 \times 200 = 20$ per unit. Previously, the number of units in the portfolio was found to be 8.5, therefore the option premium is $20 \times 8.5 = 170$. Then the capital gain of holding this portfolio over the year is given by:

$$\phi_1 - \phi_0 - 170$$

$$\phi_1 - (F_0 - nP_0) - 170$$

$$-850 - [F_0 - (8.5 \times 200)] - 170$$

$$= 680 - F_0$$

Since there is no arbitrage, any capital gain must equal to 10% of the initial portfolio, that is

$$680 - F_0 = 0.1\phi_0$$

$$680 - F_0 = 0.1(F_0 - 1700)$$

$$F_0 = 773$$

that is the opportunity cost of investing today. The full cost of investing today is $1600 + 773 = 2373$ which is greater than 2200.

2 Basic real options model

Assume that the bank invests directly in a portfolio of projects in order to diversify risk. The bank's investments are partly irreversible (sunk cost). These investments can be delayed, so the bank has the opportunity to wait for new information to arrive in order to reduce risk.

Let V be the value of the investment project of the bank. We can think of V as the discounted bank's cash flow from this project. Following McDonald and Siegel (1986), assume that in a given period of time V follows the following geometric Brownian motion:

$$dV = \alpha V dt + \sigma V dz \tag{1}$$

where α is the growth rate of V or the expected percentage rate of change in V. σ is the standard deviation (uncertainty), and dz is the increment of a Wiener process. The bank's investment opportunity is equivalent to a call option. An example would be if the bank is contemplating a decision to invest in either project A or project B. Therefore, the investment decision is considered as a problem of option valuation. The bank will want to maximize the value of its investment opportunity, $F(V)$.

Let S denote the cost of the investment project. The payoff from investing at time t is $V_t - S$. In a classical setting the bank's maximization problem would be represented by:

$$F(V) = max\ E[(V_T - S)e^{-rT}] \tag{2}$$

T is the future time where the investment will be made and r is the discount rate. But since we assume that V evolves stochastically, we will not be able to determine a time T. Instead, the investment condition will take the form of a critical value V^* such that it is optimal to invest as long as $V \geq V^*$. We want to determine the point at which it is optimal for the bank to invest S in return for an asset worth V. To solve for V^* we can use Contingent Claims analysis.

2.1 Contingent Claims solution

To make use of contingent claims analysis we should assume that the stochastic variations in V must be spanned by existing assets with a price that is perfectly correlated with V so that uncertainty over future values of V can be replicated by existing assets. Following the conventional procedure of the spanning technique, let P be the price of the asset that is perfectly correlated with V. Let ρ_{PM} be the correlation of P with the market portfolio M, then, $\rho_{PM} = \rho_{VM}$. Since P is perfectly correlated with V, P is assumed to evolve the same way:

$$dP = \mu P dt + \sigma P dz \tag{3}$$

μ is the risk-adjusted rate of return on this asset. By the Capital Asset Pricing Model (CAPM), μ also reflects the asset's systematic risk. The μ is given by:

$$\mu = r + \phi \rho_{PM} \sigma \tag{4}$$

where $\phi = \frac{(r_M - r)}{\sigma}$ is the aggregate market price of risk. r_M is the expected return on the market. We assume that $\alpha < \mu$ for the bank to invest in the project.

Let $\delta = \mu - \alpha$, so that $\delta > 0$. If we think of $\mu = \alpha + \delta$ as the total expected return on the project, that is , the dividend rate plus the expected rate of capital gain, then δ is an opportunity cost of delaying investing in the project and keeping the option to invest alive. If $\delta = 0$, that is if $\mu = \alpha$, then there would be no opportunity cost to keeping the option alive, and the bank would never invest in this project. Note that if δ is very large, the opportunity cost of waiting is large, thus the value of the option will be very small. α then can be expressed as:

$$\alpha = \frac{1}{dt} E[\frac{dV}{V}]$$

and δ can be expressed as a function of V:

$$\delta(V) = \mu - \frac{1}{dt} E[\frac{dV}{V}]$$

Now $F(V)$ can be determined by constructing a risk-free portfolio, determining its expected rate of return, and equating that return to the risk-free rate of interest r. To construct such a portfolio, consider holding an option to invest which is worth $F(V)$. Assume a short position of $N = F'(V)$ units of the project. The value of this portfolio is given by:

$$w = F(V) - F'(V)V \tag{5}$$

$$dw = dF(V) - dF'(V)V - F'(V)d(V)$$

where

$$F'(V) = \frac{dF}{dV}$$

This is a dynamic portfolio, so when the value V changes, $F'(V)$ may change from one short interval of time to the other. Therefore, the composition of the portfolio will be changed.

Although $N = F'(V)$ may change from one short period to another, it is fixed over each short interval of length dt. The short position in this portfolio will require a payment of $\delta V F'(V)$ dollars to the holder of the long position (the bank) every time period. An investor holding a long position in this option will demand the risk-adjusted return

$$\mu V = \alpha V + \delta V$$

where αV represents the growth of the bank's project (the capital gain) and δV represents interest earnings or the dividend stream to the depositors. The total return from holding the portfolio over a short time interval dt is given by

$$dw - \delta V F'(V)dt$$

$$dw = dF(V) - F'(V)dV - \delta V F'(V)dt \tag{6}$$

In equation (6) the term $dF'(V)V$ was omitted because we assume that $N = F'(V)$ is held constant over dt. To obtain an expression for dF we use Ito's lemma:

$$dF = F'(V)dV + \frac{1}{2}F''(V)(dV)^2 \tag{7}$$

where

$$F''(V) = \frac{d^2F}{dV^2}$$

$$(dV)^2 = (\alpha V dt)^2 + 2\alpha\sigma dt dz + (\sigma V dz)^2$$

as

$$dz^2 = dt \quad and \quad dt^2 \approx 0$$

$$(dV)^2 = \sigma^2 V^2 dz^2$$

$$(dV)^2 = \sigma^2 V^2 dt \tag{8}$$

Substituting equation (8) into (6) gives the total risk-free return on the portfolio:

$$\frac{1}{2}F''(V)(dV)^2 - \delta V F'(V)dt \tag{9}$$

again substituting equation (8) into (9) yields

$$\frac{1}{2}\sigma^2 V^2 F''(V)dt - \delta V F'(V)dt \tag{10}$$

In equilibrium equation (9) must equal equal the risk-free return in the market, that is

$$rwdt = r[F(V) - F'(V)V]dt$$

$$\frac{1}{2}\sigma^2 V^2 F''(V)dt - \delta V F'(V)dt = r[F(V) - F'(V)V]dt$$

Dividing through by dt and rearranging yields the differential equation that $F(V)$ must satisfy

$$\frac{1}{2}\sigma^2 V^2 F''(V) + (r-\delta)VF'(V) - rF(V) = 0 \tag{11}$$

$F(V)$ must also satisfy the following boundary conditions:

$$F(0) = 0 \tag{12}$$

$$F(V^*) = V^* - S \tag{13}$$

$$F(V^*) = 1 \tag{14}$$

Again V^* represents the value of the project at which it is optimal to invest.[2]

Condition (12) states that the option to invest will be of no value when $V = 0$. Equation (13) is the *value-matching* condition, that is upon investing, the bank receives a net payoff of $V^* - S$. Rewriting (13) as $V^* - F(V^*) = S$ implies that when the bank invests in the project, it gets the project value V, but gives up the opportunity to invest $F(V)$. The critical value V^* is obtained when this net gain $V^* - F(V^*)$ equals the direct cost of investment S. In other words, the value of the project V^* is set to equal the direct cost S plus the opportunity cost $F(V^*)$.

Equation (14) is the *smooth-pasting* condition. That is, if $F(V)$ were not continuous and smooth at the critical exercise point V^* , it is better for the bank to wait Δt to observe the next step of V. To solve for $F(V)$ we must solve equation (11) subject to the boundary conditions (12), (13), and (14). McDonald and Siegel (1986) suggested that the solution that satisfies condition (12) must take the form:

$$F(V) = AV^\theta \tag{15}$$

Conditions (13) and (14) can be used to solve for A which is a constant to be determined, and for the optimal value V^*. [3]

[2] Cox and Ross (1976) show that the same solution is obtained using dynamic programming analysis if we assume that all agents are risk neutral.

[3] The general solution to equation (11) as suggested by McDonald and Siegle takes the form $F(V) = A_1 V^{\theta_1} + A_2 V^{\theta_2}$ but the boundary condition (12) implies that $(r-\delta)VF'(V) =$

θ is a known constant which depends on the parameters σ, r, and δ of equation (11) where $\theta > 1$.
To obtain values for A and V^*, we substitute equation (15)into (13) and (14) so that

$$F(V^*) = AV^{*\theta} = V^* - S$$

$$A = \frac{V^* - S}{V^{*\theta}} \tag{16}$$

By equation (14), $F'(V^*) = \theta AV^{*\theta-1} = 1$. Using equation (16) to substitute for A we obtain:

$$\theta(\frac{V^* - S}{V^*}) = 1$$

$$V^* = (\frac{\theta}{\theta - 1})S \tag{17}$$

Substituting equation (17)into (16) to obtain a value for A as[4]

$$A = \frac{(\theta - 1)^{\theta-1}}{\theta^\theta S^{\theta-1}} \tag{18}$$

Equations (15),(17), and (18) give the solution to the critical values of V^* and $F(V^*)$.

3 A mean-reversion model

Geometric Brownian motion models have been criticized theoretically on the ground that these processes do not diverge over time. This implies that firms facing this sort of price or cash flow process can earn infinite profit (Metcalf and Hasset, 1995). Mean-reversion precesses in many ways provide a more plausible approximation to the stochastic nature of a project or an asset value. This section introduces the mean-reversion process based on the derivations made in the previous section.[5]

---

0 thus $A_2 = 0$, therefore the solution is confined to the form $F(V) = AV^{\theta_1}$ or just $F(V) = AV^\theta$.

[4]See appendix for solution.

[5]The analysis here follows Dixit and Pindyck (1994).

Let V now follow the mean-reversion process given below, which is known as *Geometric Ornstein-Uhlenbeck process*

$$dV = \eta(\bar{V} - V)V dt + \sigma V dz \tag{19}$$

where η is the rate of the reversion to the mean. Again, σ is the standard deviation and dz is the increment of a Wiener process. Equation (19) implies that the current value of the project is known, but future values are lognormally distributed with a variance that grows linearly with time. The future value of the project is always uncertain. Further we assume that total revenue from the project covers its average total cost, so the bank will not either shut down or abandon the project in the short-run at least. Another auxiliary assumption is needed to preserve the assumption of lognormality, is that the value of the project, V is always positive.

The expected percentage rate of change in V is

$$\frac{1}{dt}E[\frac{dV}{V}] = \frac{1}{dt}E[\frac{\eta(\bar{V} - V)V dt + \sigma V dz}{V}]$$

$$= \eta(\bar{V} - V)$$

as

$$E[dz] = 0$$

The expected absolute rate of change is given by

$$\frac{1}{dt}E[dV] = \frac{1}{dt}E[(\eta(\bar{V} - V)V dt + \sigma V dz]$$

$$\frac{1}{dt}E[dV] = \eta(\bar{V} - V)V$$

$$\frac{1}{dt}E[dV] = \eta\bar{V}V - \eta V^2 = 0$$

if

$$V = 0 \quad or \quad V = \bar{V}$$

It has a maximum at[6]

$$V = \frac{\bar{V}}{2}$$

[6]That is obtained by setting $\frac{\partial(\eta\bar{V}V - \eta V^2)}{\partial V} = 0$.

This particular process does have an analytical solution to the optimal investment problem. By substituting the value of $\delta(V) = \mu - \eta(\bar{V} - V)$ into equation (11) we obtain the following second order differential equation:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + [r - \mu + \eta(\bar{V} - V)]VF'(V) - rF(V) = 0 \qquad (20)$$

Again this equation must satisfy the boundary conditions (12), (13), and (14). Now assume that the speed of the mean-reversion process is a linear function of the value of the investment project V. A relationship of the form

$$\eta = \beta_1 + \beta_2 V \qquad (21)$$

β_2 is the "acceleration" parameter that measures the speed of adjustment by which the value of the project converges to the long-run mean. Substituting equation (21)into (20) yields

$$\frac{1}{2}\sigma^2 V^2 F''(V) + [r - \mu + (\beta_1 + \beta_2 V)(\bar{V} - V)]VF'(V) - rF(V) = 0 \quad (22)$$

One advantage of allowing for a third moment process for distribution of V is to gain insights about the skewness of this distribution around the mean.[7]

[7]One measure of skewness can be defined as $\frac{E(V-\bar{V})^3}{\sigma^3}$.

3.1 An analytical solution

The analytical solution to equation (22) is presented in detail in the appendix. As demonstrated, the solution is expressed by the following equation:

$$V^{\theta}h(V)[\frac{1}{2}\sigma^2\theta(\theta-1)+(r-\mu+\eta\bar{V})\theta-r]+$$

$$V^{\theta+1}[\frac{1}{2}\sigma^2 Vh''(V)+(\sigma^2\theta+r-\mu+\eta\bar{V}-\eta V)h'(V)-\eta\theta h(V)]=0 \quad (23)$$

Equation (23) must hold for any value of V, thus the two main bracketed terms in the equation must equal zero. The quantity in the first bracket represents a *quadratic equation* which has two solutions for θ. One is positive and the other is negative. Only the positive solution satisfies the boundary condition $F(0)=0$.

Setting the quadratic equation to zero and solving for θ as follows:

$$\frac{1}{2}\sigma^2\theta(\theta-1)+(r-\mu+\eta\bar{V})\theta-r=0$$

$$\frac{1}{2}\sigma^2\theta^2-(\frac{1}{2}\sigma^2-r+\mu-\eta\bar{V})\theta-r=0 \quad (24)$$

The positive solution to equation (24) is given by $\theta=\frac{-b+\sqrt{b^2-4ac}}{2a}$, then

$$\theta=\frac{1}{2}+\frac{(\mu-r-\eta\bar{V})}{\sigma^2}+\sqrt{[\frac{(r-\mu+\eta\bar{V})}{\sigma^2}-\frac{1}{2}]^2+\frac{2r}{\sigma^2}} \quad (25)$$

The second bracketed term of equation (23) is set to equal zero:

$$\frac{1}{2}\sigma^2 Vh''(V)+(\sigma^2\theta+r-\mu+\eta\bar{V}-\eta V)h'(V)-\eta\theta h(V)=0 \quad (26)$$

To transform equation (26) into a standard form equation we proceed as follows:

Let $x=\frac{2\eta V}{\sigma^2}$ and $h(V)=g(x)$ therefore,

$$h'(V)=g'(x)x'=g'(x)(\frac{2\eta}{\sigma^2}) \quad (27)$$

$$h''(V)=g''(x)x'(\frac{2\eta}{\sigma^2})=g''(x)(\frac{2\eta}{\sigma^2})^2 \quad (28)$$

Substituting equations (27) and (28) back into (26) yields [8]

$$xg''(x) + (y - x)g'(x) - \theta g(x) = 0 \tag{29}$$

where

$$y = 2\theta + \frac{2(r - \mu + \eta\bar{V})}{\sigma^2}$$

Equation (29) is known as Kummer's equation. Its solution is given by the confluent hypergeometric function $H[x, \theta, y(\theta)]$, which has the following series:

$$H[x, \theta, y(\theta)] = 1 + \frac{\theta}{y}x + \frac{\theta(\theta+1)x^2}{y(y+1)2!} + \frac{\theta(\theta+1)(\theta+2)x^3}{y(y+1)(y+2)3!} + \cdots \tag{30}$$

This confirmed that the solution to equation (20) is indeed of the form given by

$$F(V) = AV^\theta h(V)$$

as indicated in the appendix. Then the solution takes the form:

$$F(V) = AV^\theta H(\frac{2\eta}{\sigma^2}V; \theta, y) \tag{31}$$

Again A is a constant to be determined. A and V^* must be solved numerically using the boundary conditions (13) and (14).

To check whether there exist an analytical solution to equation (22) we substitute $\eta = \beta_1 + \beta_2 V$ into (26)

$$\frac{1}{2}\sigma^2 V h''(V) + [\sigma^2\theta + r - \mu + \beta_1\bar{V} - (\beta_1 - \beta_2\bar{V})V - \beta_2 V^2]h'(V)$$

$$-[(\beta_1 - \beta_2\bar{V})\theta + \beta_2\theta V]h(V) = 0$$

Factoring out β_1 and rearranging terms yields the following second order differential equation[9]

$$xg''(x) + [y - (1 - c_1)x - c_2x^2]g'(x) - (1 - c_1 - c_2x)\theta g(x) = 0 \tag{32}$$

where

$$y = 2\theta + \frac{2[(r - \mu + (\beta_1 + \beta_2 V)\bar{V}]}{\sigma^2}$$

[8] See appendix for solution

[9] Again, solution is provided in the appendix

$$c_1 = \frac{\beta_2}{\beta_1}\bar{V}$$

$$c_2 = \frac{\sigma^2 \beta_2}{2\beta_1^2}$$

There is no direct analytical solution to equation (32) as it was the case for equation (29) where the solution was approximated by the confluent hypergeometric expansion. Therefore, a numerical solution must be established realizing that the numerical solution of (32) is equivalent to the numerical solution of (22). The following subsection provides an elaborate discussion of the numerical solution to the bank investment problem with various scenarios as how the speed of convergence or the acceleration of the mean-reverting process affects the value of the option for the bank to invest.

3.2 A numerical solution

In solving equation (22) numerically, the parameters of the models are given the following values: $S = 1$, $r = 0.04$, $\mu = 0.08$, and $\sigma = 0.2$. Again, S is the amount of initial investment. It is normalized to one. r is the risk-free interest rate. μ is the expected rate of return on the investment, and σ is the measure of uncertainty. of These values are widely adopted in the real options literature.
Three cases are of a special interest; $\bar{V} > S$, $\bar{V} < S$, and $\bar{V} = S$. The solutions are depicted in the figures (1)-(**??**) provided in the appendix. Analyzing these figures, we can remark the following results.

- The larger is $\bar{V}$, the larger is $F(V)$ and the higher is V^*, the critical value at which it is optimal for the bank to invest (Figures (1)-(3)). Larger $\bar{V}$ implies a higher expected rate of growth of V so that the value of an option to invest V will become higher.
- For $\bar{V} < S$, a larger value of β_2, the mean reversion parameter, reduces the investment opportunity $F(V)$ (Figures (4)-(6)). This also implies that a larger value of β_2 results in a lower value of V^*.

- For a large $\bar{V}$, that is for $\bar{V} \geq S$ as β_2 gets larger, $F(V)$ becomes concave for small values of V, so β_2 rises rapidly, therefore $F(V)$ rises rapidly as well for a small value of V. However, the value of $F(V)$ diminishes with large β_2 as V gets larger (Figures (7)-(**??**)).

Assuming that small banks invest in relatively small projects and large banks invest in large projects, the implication of this real options setting to banking is that the size of the bank does not matter with respect to its financial soundness, what matters is the quality of bank's investment, whether the value of the investment exceeds its initial cost is the critical question.

4 Conclusion

Following Dixit and Pindyck (1994), a real options approach is adopted to model bank investments. A new dimension is added to the mean-reversion process by modelling the mean-reversion parameter as a linear function of the value of the project. Unlike Dixit-Pindyck model, the proposed process has no direct analytical solution so that a numerical solution was derived. As the average value of the investment project gets large so does the value of the option to invest in the project. It is also shown that as the average value of the project goes below its initial cost, a larger value of the mean-reversion parameter reduces the value of the investment option. However, as the value of the project exceeds its initial cost, then a larger mean-reversion parameter rises the value of the option only for small values of the project and reduces it as the value of the project gets larger. The implication for bank's investments are exemplified by the fact that large banks usually invest in relatively large projects compared to small banks. If we regard the rate of speed of the mean-reversion parameter as a stabilizing factor for the financial soundness of the bank, the model suggests that the size of the project should not really matter, what matters is whether the value of the project is lower or higher that the initial cost of investment.

APPENDIX

Solution to equation (18).

$$A = \frac{(\theta-1)^{\theta-1}}{\theta^{\theta} S^{\theta-1}}$$

$$V^* = (\frac{\theta}{\theta-1})S$$

$$A = \frac{(\frac{\theta-S}{\theta-1}) - S}{(\frac{\theta S}{\theta-1})^{\theta}} = \frac{(\frac{S}{\theta}) - 1}{(\frac{\theta S}{\theta-1})}$$

$$A = (\frac{S}{\theta-1})(\frac{\theta-1}{\theta S})^{\theta} = (\theta-1)^{\theta-1} S^{1-\theta} \theta^{-\theta}$$

$$A = \frac{(\theta-1)^{\theta-1}}{\theta^{\theta} S^{\theta-1}}$$

The analytical solution to equation (22).

First, we proceed by solving equation (20) so we defer the substitution of $\eta = \beta_1 + \beta_2 V$ to the end.

Cox and Ross (1976) suggested that the solution to a second order differential equation of form (20) will take the form of the following function:

$$F(V) = AV^{\theta} h(V) \tag{A-33}$$

where A and θ are constant to be determined in a way such that $h(V)$ satisfy a differential equation with known solution. We need to substitute equation (A-33) into (20), but first we derive the first and second derivatives of $F(V)$ in order to make the substitution tractable.

$$F'(V) = \theta AV^{\theta-1} h(V) + h'(V)AV^{\theta}$$

$$F'(V) = AV^{\theta}[\theta V^{-1} h(V) + h'(V)] \tag{A-34}$$

$$F''(V) = (\theta-1)\theta AV^{\theta-2} h(V) + h'(V)\theta AV^{\theta-1} + h''(V)AV^{\theta} + \theta h'(V)AV^{\theta-1}$$

$$F''(V) = AV^{\theta}[\theta(\theta-1)V^{-2} h(V) + \theta V^{-1} h'(V) + h''(V) + \theta V^{-1} h'(V)]$$

$$F''(V) = AV^{\theta}[\theta(\theta-1)V^{-2} h(V) + 2\theta V^{-1} h'(V) + h''(V)] \tag{A-35}$$

Now using equations (A-34) and (A-35) in (20) yields:

$$\frac{1}{2}\sigma^2 V^2 AV^{\theta}[\theta(\theta-1)V^{-2} h(V) + 2\theta V^{-1} h'(V) + h''(V)] +$$

$$[r-\mu+\eta(\bar{V}-V)]VAV^{\theta}[\theta V^{-1}h(V)+h'(V)]-rAV^{\theta}h(V)=0$$

Dropping A from all terms and rearranging terms result in

$$\frac{1}{2}\sigma^2 V^{\theta}[\theta(\theta-1)h(V)+2\theta Vh'(V)+h''(V)V^2]+$$

$$V^{\theta}[r-\mu+\eta(\bar{V}-V)][\theta h(V)+h'(V)V]-rV^{\theta}h(V)=0$$

$$V^{\theta}h(V)[\frac{1}{2}\sigma^2\theta(\theta-1)+(r-\mu+\eta\bar{V})\theta-r]+$$

$$V^{\theta+1}\{\frac{1}{2}\sigma^2 Vh''(V)+\sigma^2\theta h'(V)+[r-\mu+\eta(\bar{V}-V)]h'(V)-\eta\theta h(V)\}=0$$

$$V^{\theta}h(V)[\frac{1}{2}\sigma^2\theta(\theta-1)+(r-\mu+\eta\bar{V})\theta-r]+$$

$$V^{\theta+1}[\frac{1}{2}\sigma^2 Vh''(V)+(\sigma^2\theta+r-\mu+\eta\bar{V}-\eta V)h'(V)-\eta\theta h(V)]=0 \quad \text{(A-36)}$$

The solution to equation (29).

Substituting equations (27) and (28) into (26) yields

$$\frac{1}{2}\sigma^2 V[g''(x)(\frac{2\eta}{\sigma^2})^2]+(\sigma^2\theta+r-\mu+\eta\bar{V}-\eta V)[g'(x)\frac{2\eta}{\sigma^2}]-\eta\theta g(x)=0$$

$$\frac{2\eta^2 V}{\sigma^2}g''(x)+(\frac{\sigma^2\theta+r-\mu+\eta\bar{V}-\eta V}{\sigma^2})2\eta g'(x)-\eta\theta g(x)=0$$

$$\frac{2\eta^2 V}{\sigma^2}g''(x)+[2\eta\theta+\frac{2\eta(r-\mu+\eta\bar{V})}{\sigma^2}-\frac{2\eta^2 V}{\sigma^2}]g'(x)-\eta\theta g(x)=0$$

$$\eta\{\frac{2\eta V}{\sigma^2}g''(x)+[2\theta+\frac{2(r-\mu+\eta\bar{V})}{\sigma^2}-\frac{2\eta V}{\sigma^2}]g'(x)-\theta g(x)\}=0$$

$$\frac{2\eta V}{\sigma^2}g''(x)+[2\theta+\frac{2(r-\mu+\eta\bar{V})}{\sigma^2}-\frac{2\eta V}{\sigma^2}]g'(x)-\theta g(x)=0$$

The solution to equation (32).

$$\frac{1}{2}\sigma^2 Vh''(V)+[\sigma^2\theta+r-\mu+\beta_1\bar{V}-(\beta_1-\beta_2\bar{V})V-\beta_2V^2]h'(V)$$

$$-[(\beta_1-\beta_2\bar{V})\theta+\beta_2\theta V]h(V)=0$$

$$\frac{1}{2}\sigma^2 Vh''(V)+[\sigma^2\theta+r-\mu+\beta_1\bar{V}-(\beta_1-\beta_2\bar{V})V-\beta_2V^2]h'(V)$$

$$-[(\beta_1-\beta_2\bar{V})+\beta_2V]\theta h(V)=0$$

$$\frac{1}{2}\sigma^2 Vg''(x)(\frac{2\beta_1}{\sigma^2})^2+[\sigma^2\theta+r-\mu+\beta_1\bar{V}-(\beta_1-\beta_2\bar{V})V-\beta_2V^2]g'(x)(\frac{2\beta_1}{\sigma^2})$$

$$-[(\beta_1 - \beta_2\bar{V}) + \beta_2 V]\theta g(x) = 0$$

$$\frac{2\beta_1^2 V}{\sigma^2} g''(x) + [2\beta_1\theta + \frac{2(r-\mu+\beta_1\bar{V})\beta_1}{\sigma^2} - \frac{2(\beta_1 - \beta_2\bar{V})\beta_1 V}{\sigma^2} - \frac{2\beta_1\beta_2 V^2}{\sigma^2}]g'(x)$$

$$-[(\beta_1 - \beta_2\bar{V}) + \beta_2 V]\theta g(x) = 0$$

Factoring β_1 out results in:

$$\frac{2\beta_1 V}{\sigma^2} g''(x) + [2\theta + \frac{2(r-\mu+\beta_1\bar{V})}{\sigma^2} - \frac{2\beta_1 V}{\sigma^2} + \frac{2\beta_2\bar{V}V}{\sigma^2} - \frac{2\beta_2 V^2}{\sigma^2}]g'(x)$$

$$-[1 - \frac{\beta^2}{\beta_1}\bar{V} + \frac{\beta_2}{\beta_1}V]\theta g(x) = 0$$

$$xg''(x) + (y - x + \frac{\beta_2}{\beta_1}\bar{V}x - \frac{\sigma^2\beta_2}{2\beta_1^2}x^2)g'(x) - (1 - \frac{\beta_2}{\beta_1}\bar{V} + \frac{\sigma^2\beta_2}{2\beta_1^2}x)\theta g(x) = 0$$

$$xg''(x) + [y - (1-c_1)x - c_2x^2]g'(x) - (1 - c_1 - c_2x)\theta g(x) = 0$$

Graphs.

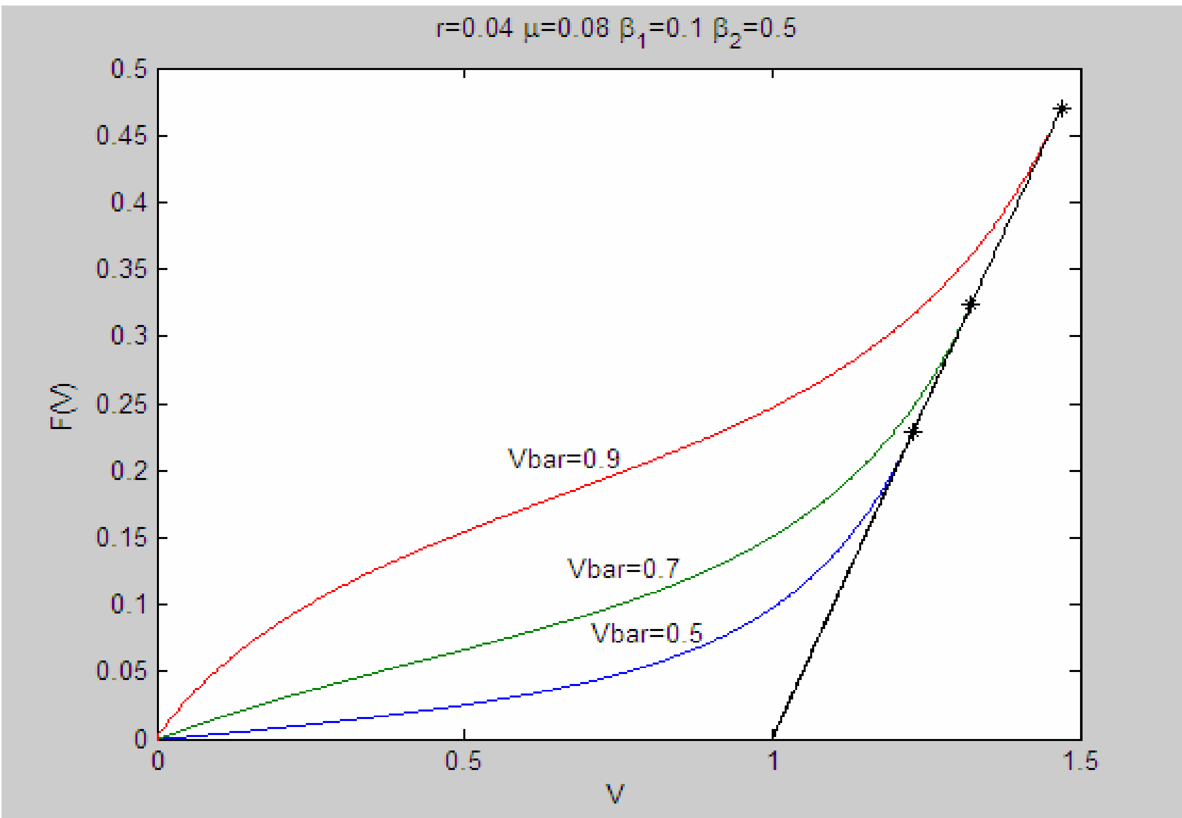

Figure 1: $F(V)$ against V for $\bar{V} < S$, $\beta_1 = 0.1$, and $\beta_2 = 0.5$.

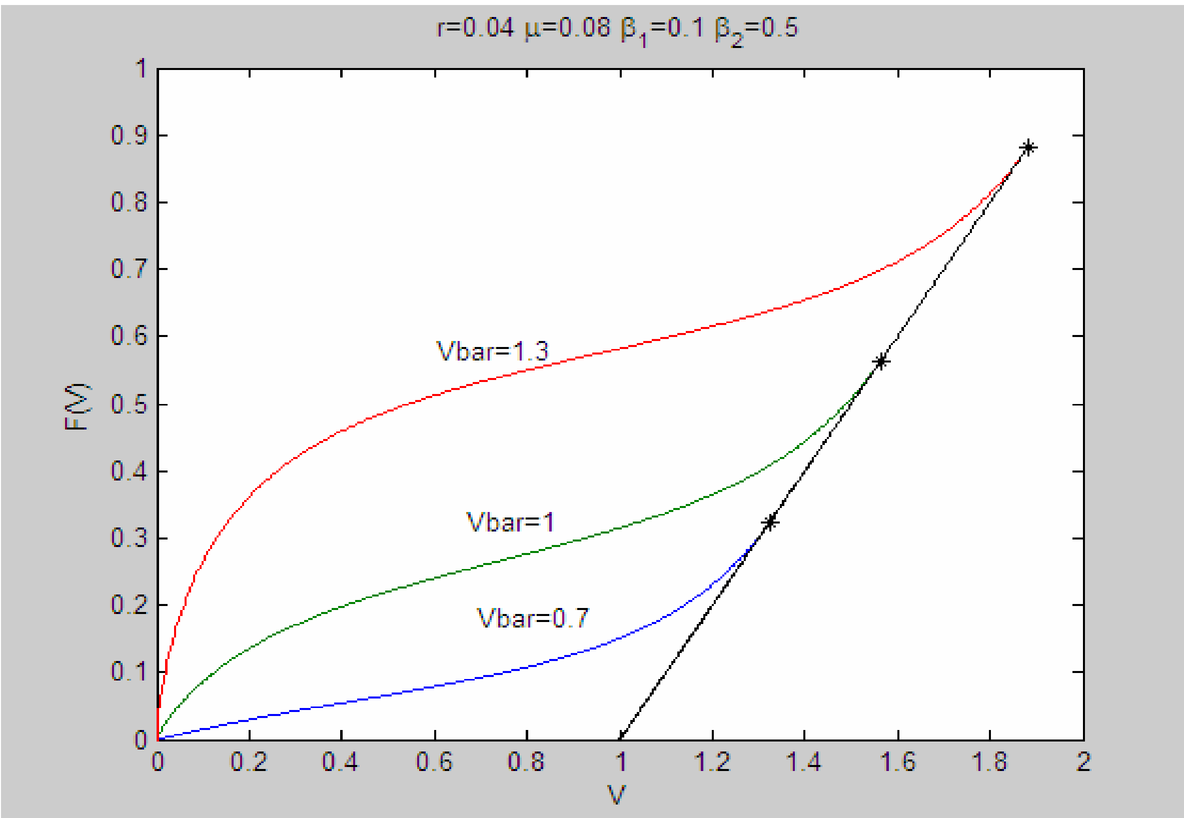

Figure 2: $F(V)$ against V for $\bar{V} < S$, $\bar{V} > S$, and $\bar{V} = S$, $\beta_1 = 0.1$, and $\beta_2 = 0.5$.

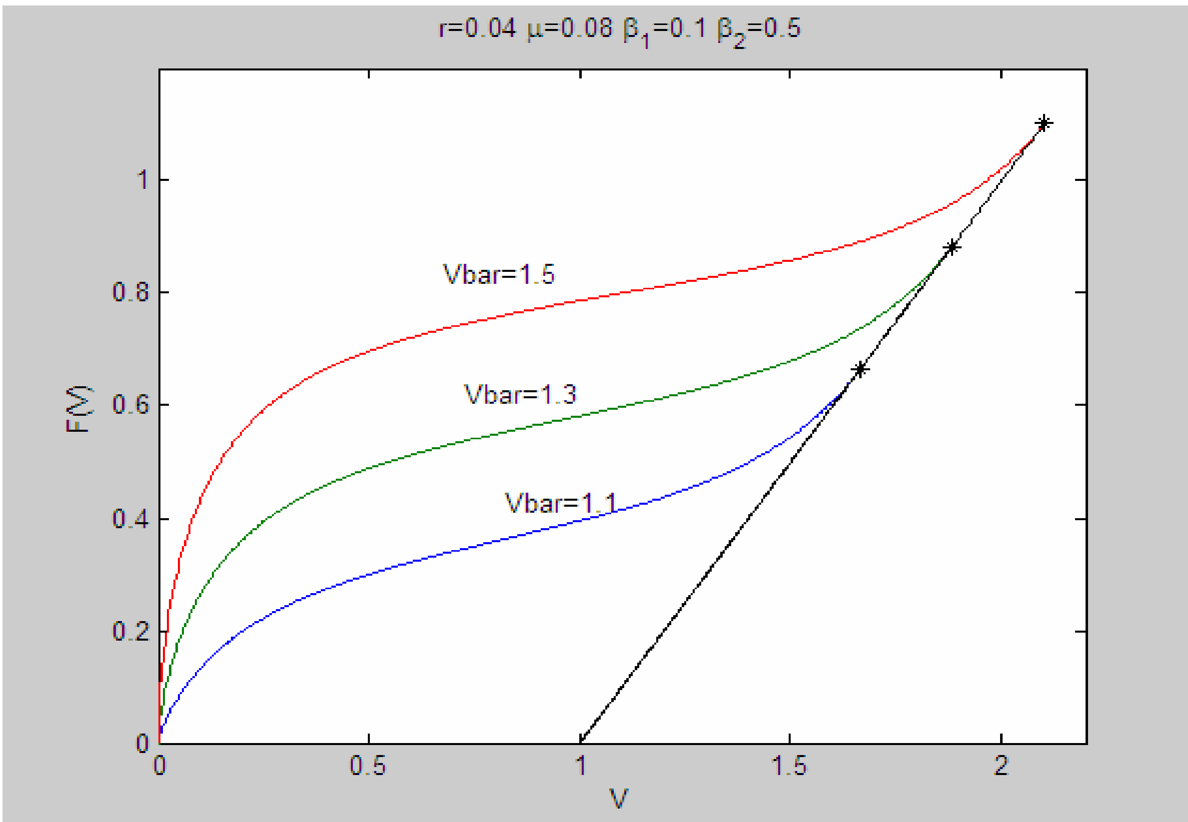

Figure 3: $F(V)$ against V for $\bar{V} > S$, $\beta_1 = 0.1$, and $\beta_2 = 0.5$.

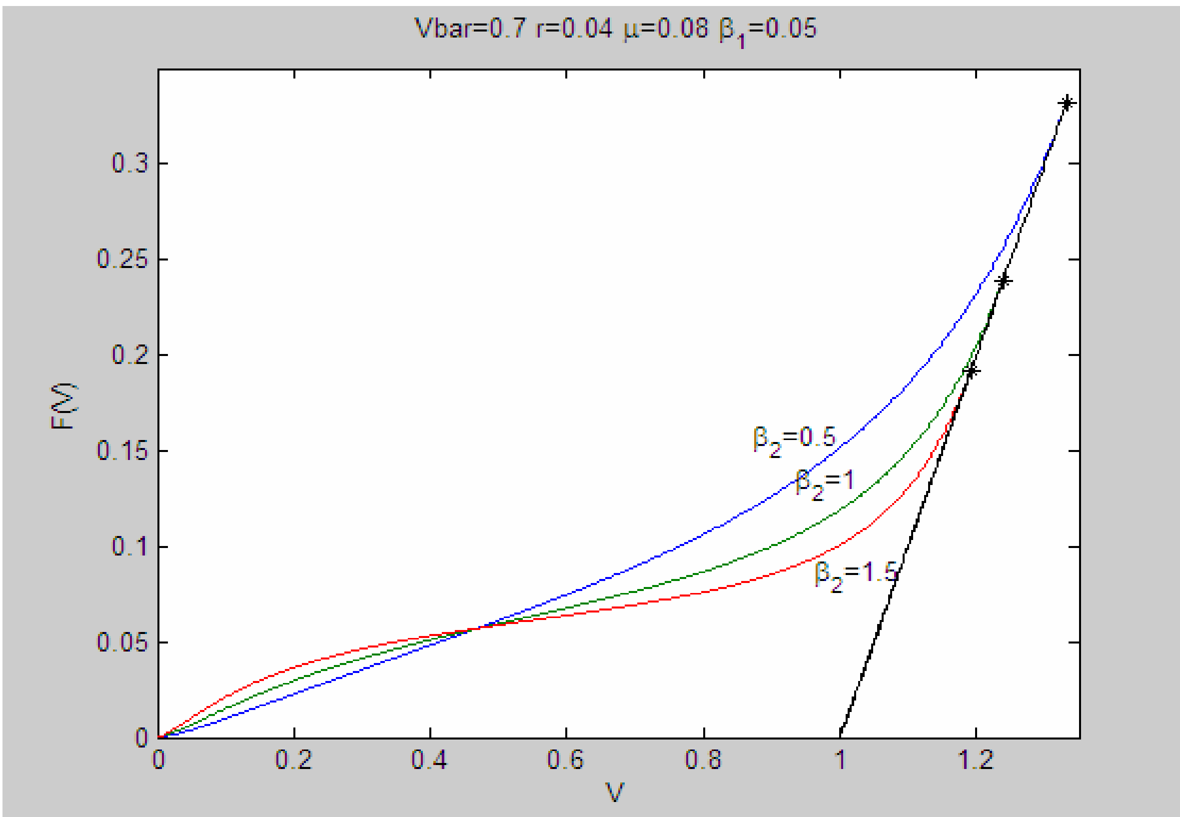

Figure 4: $F(V)$ against V for $\bar{V} < S$, $\beta_1 = 0.05$.

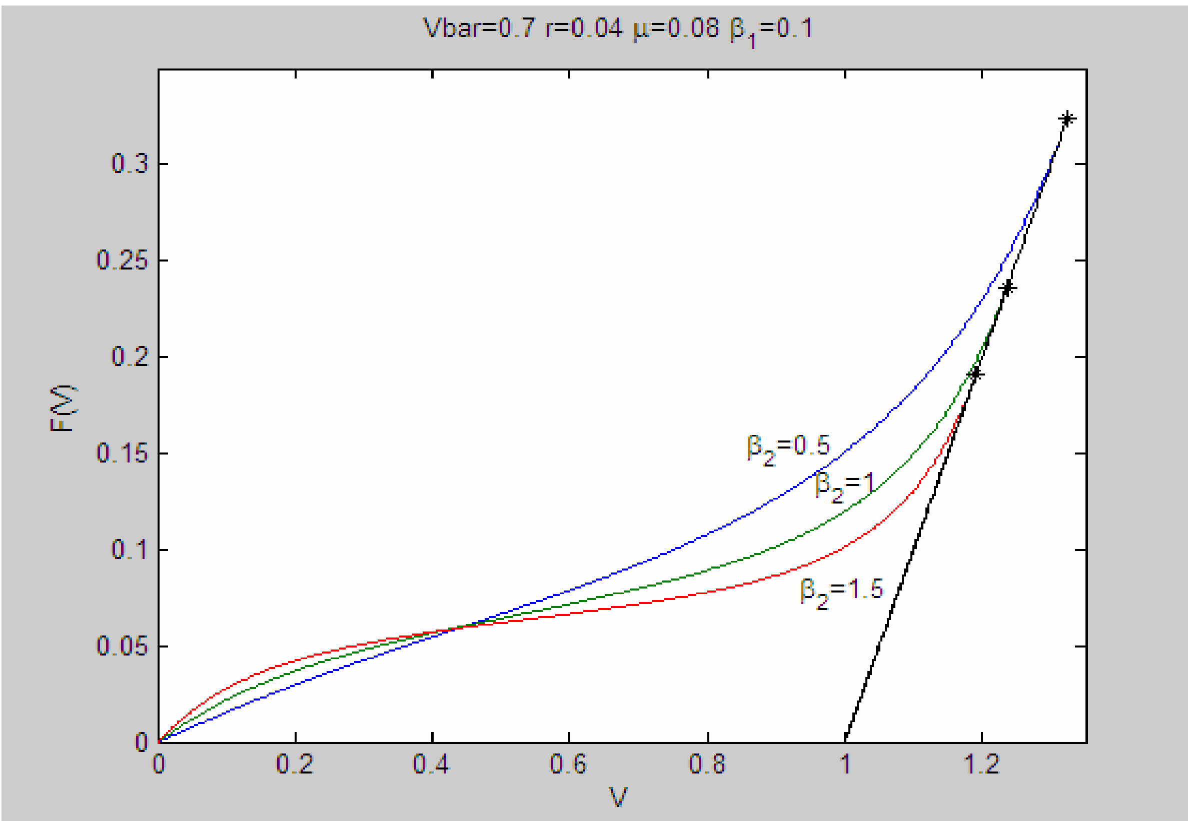

Figure 5: $F(V)$ against V for $\bar{V} < S$, $\beta_1 = 0.1$.

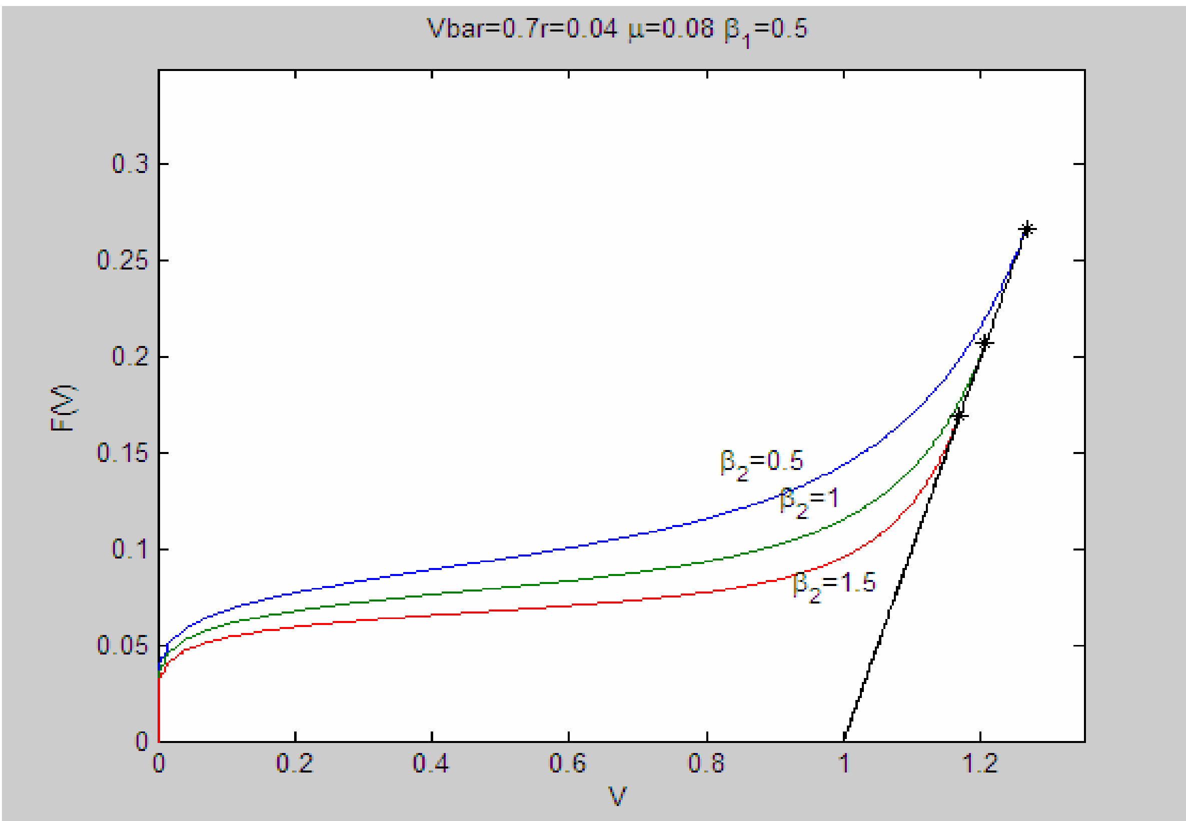

Figure 6: $F(V)$ against V for $\bar{V} < S$, $\beta_1 = 0.5$.

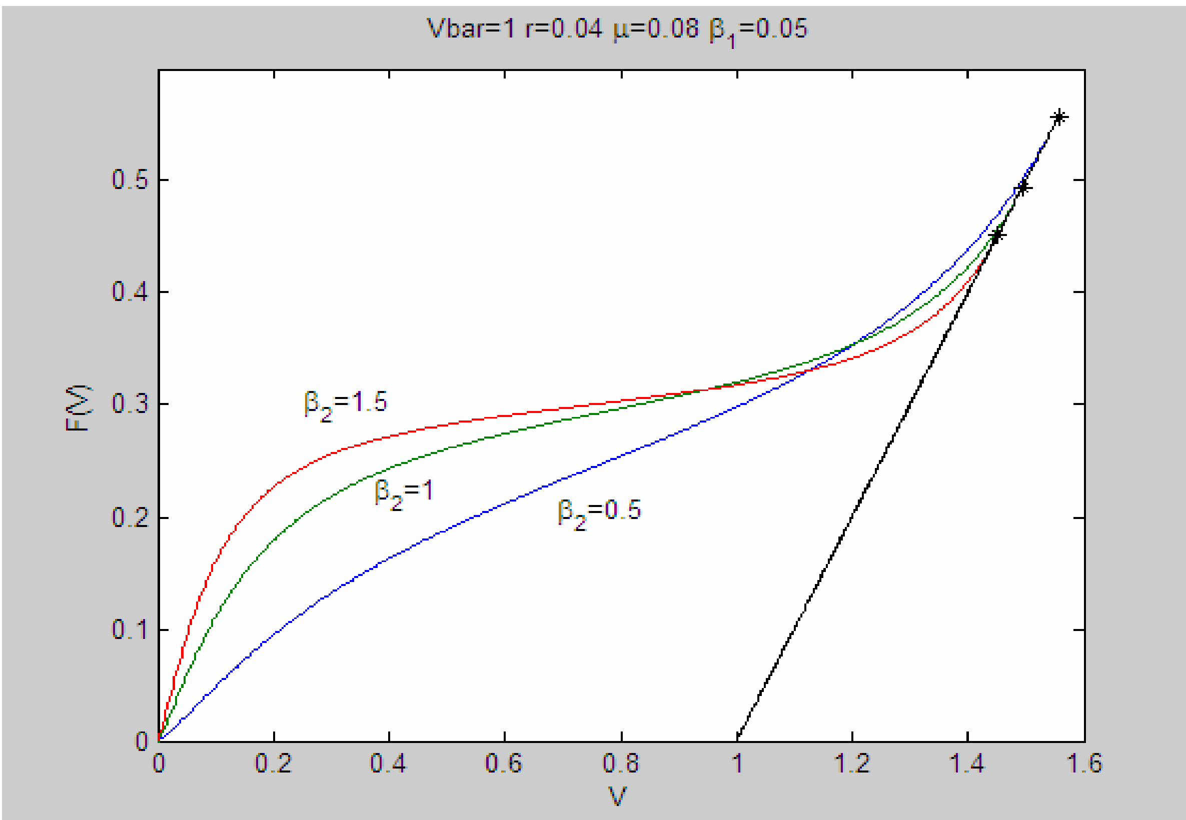

Figure 7: $F(V)$ against V for $\bar{V} = S$, $\beta_1 = 0.05$.

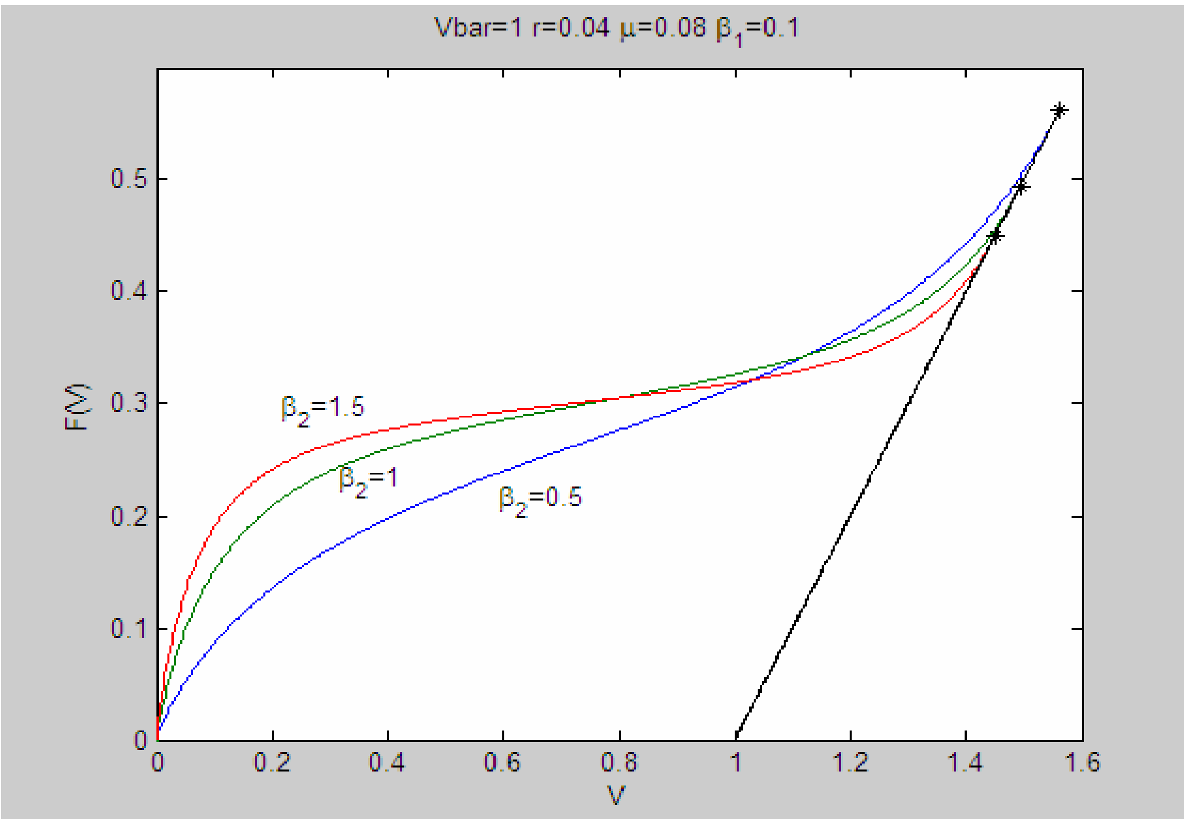

Figure 8: $F(V)$ against V for $\bar{V} = S$, $\beta_1 = 0.1$.

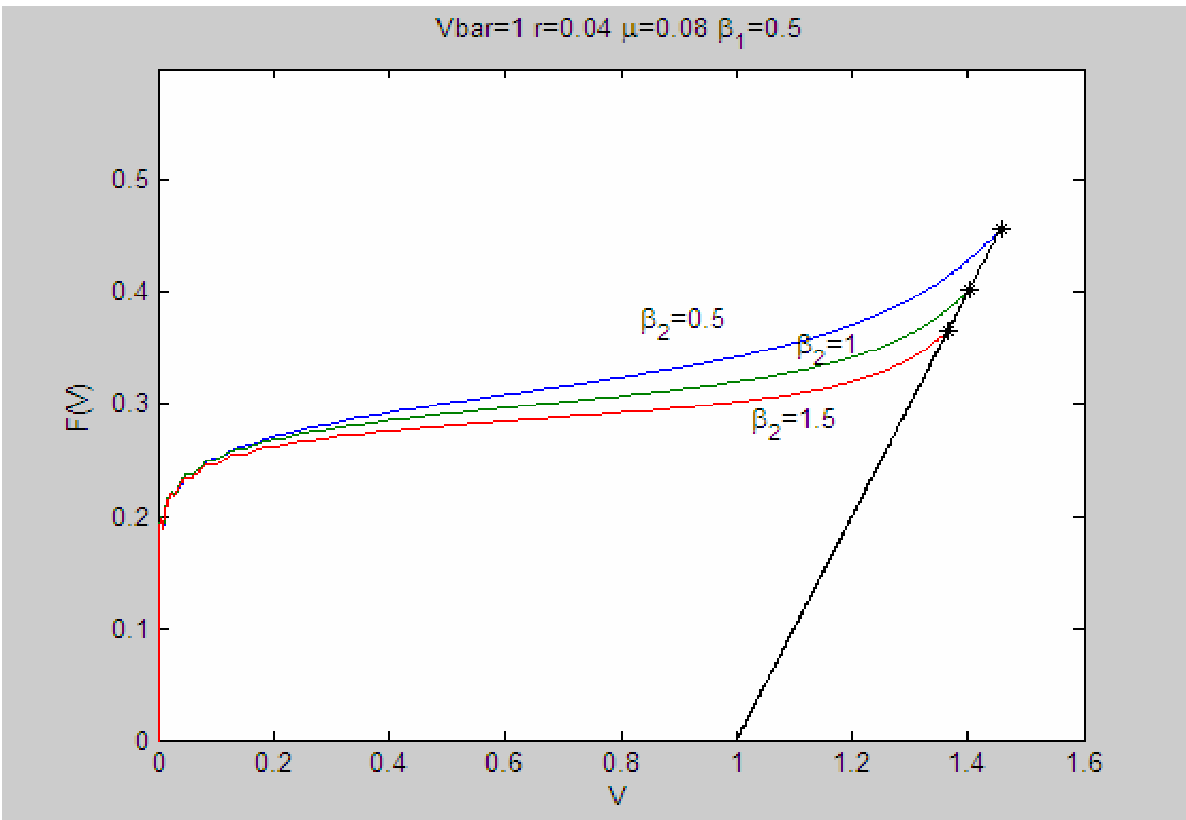

Figure 9: $F(V)$ against V for $\bar{V} = S$, $\beta_1 = 0.5$.

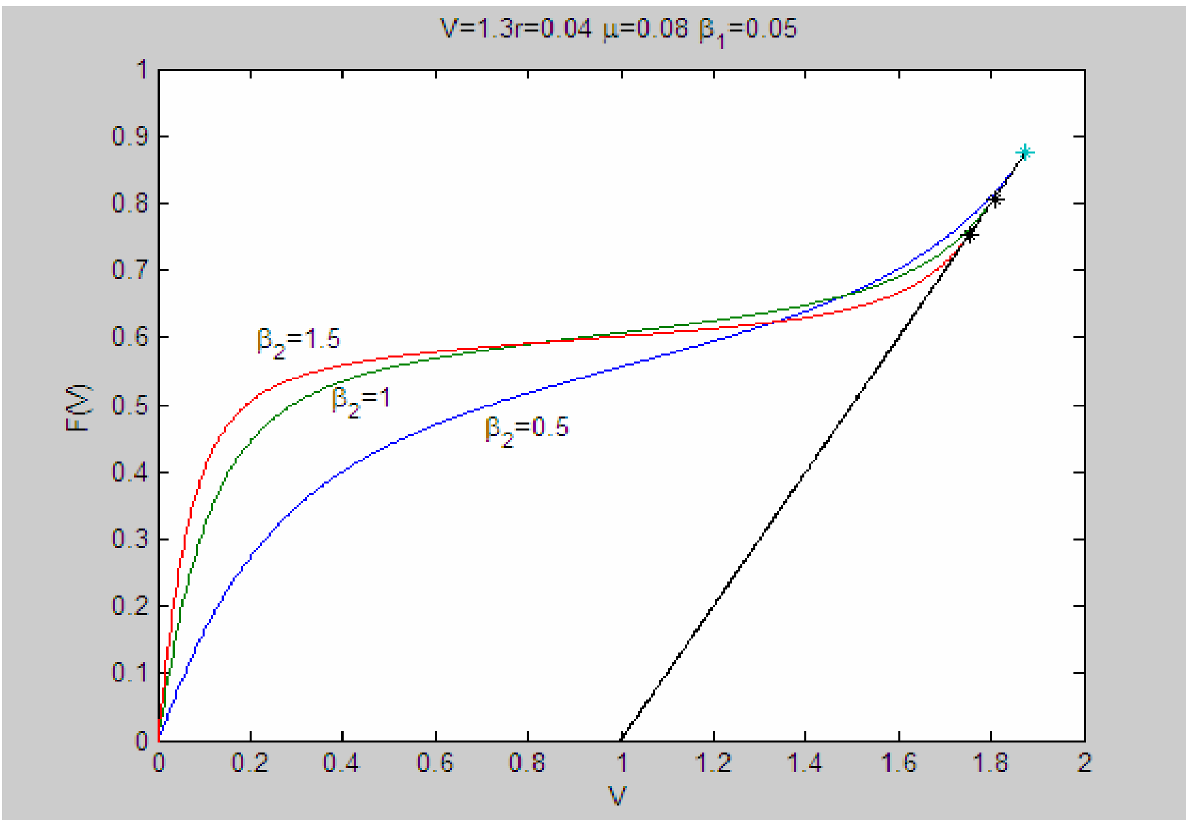

Figure 10: $F(V)$ against V for $\bar{V} > S$, $\beta_1 = 0.05$.

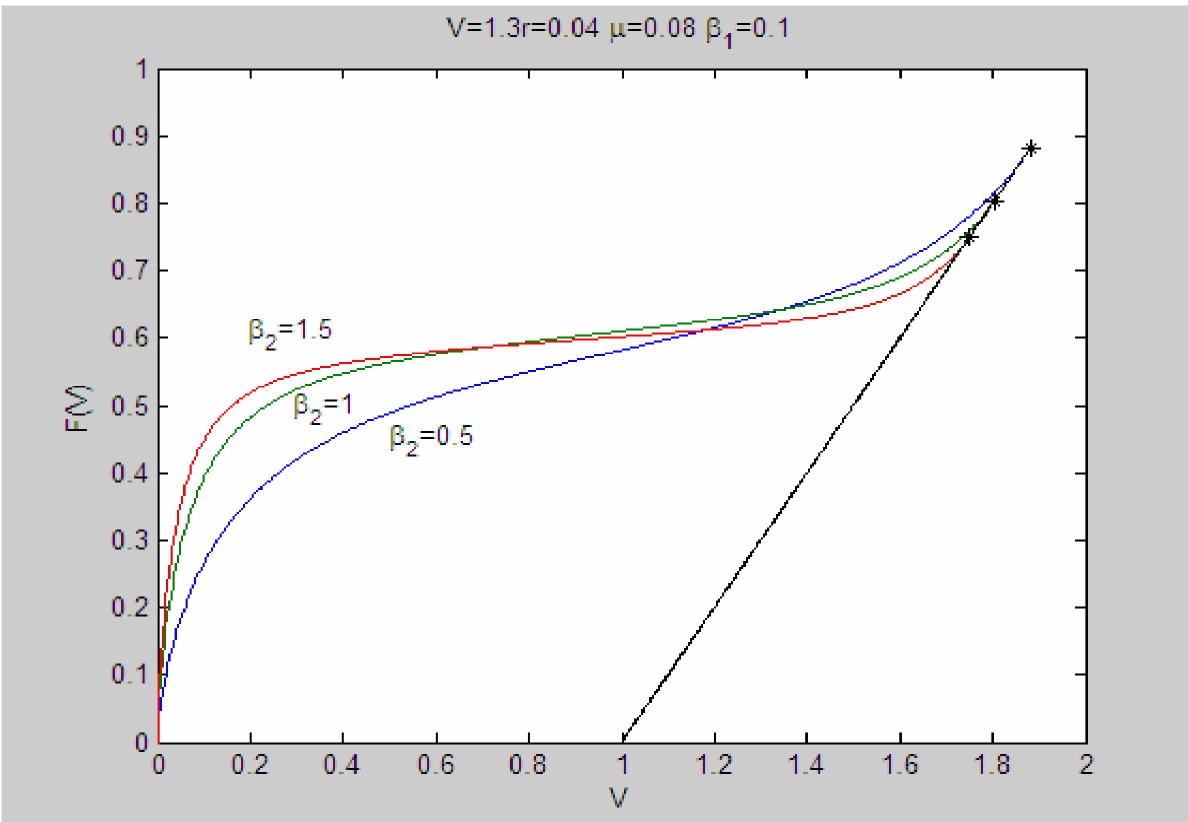

Figure 11: $F(V)$ against V for $\bar{V} > S$, $\beta_1 = 0.1$.