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Mendicino, Caterina

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January 2008

Online at https://mpra.ub.uni-muenchen.de/9425/
MPRA Paper No. 9425, posted 05 Jul 2008 05:16 UTC

On the Amplification Role of Collateral Constraints*

Caterina Mendicino†

This Draft: March 2008

Abstract

Following the seminal contribution of Kiyotaki and Moore (1997), the role of collateral constraints for business cycle fluctuations has been highlighted by several authors and collateralized debt is becoming a popular feature of business cycle models. In contrast, Kocherlakota (2000) and Cordoba and Ripoll (2004) demonstrate that collateral constraints *per se* are unable to propagate and amplify exogenous shocks, unless unorthodox assumptions on preferences and production technologies are made. The aim of this paper is to examine the contribution of costly debt enforcement procedures in the amplification of business cycle fluctuations through collateral constraints. We show that for realistic degrees of inefficiency, collateral constraints can significantly amplify the effects of productivity shocks on output even under standard assumptions on preferences and technology.

*I have benefited from extensive discussions with Martin Floden, Zvi Hercowitz, Lars Ljungqvist and Nobu Kiyotaki. I am also grateful to Kosuke Aoki, Ian Christensen, Giancarlo Corsetti, Carlos De Resende, Cesaire Meh, Miguel Molico, Ron Morrow, Andrea Pescatori, Pierre St Amant and Hajime Tomura for useful feedback on this project. I thank seminar participants at the 2008 Annual Meeting of the American Economic Association, 2006 Far East Meeting of the Econometric Society, 11th conference of the Society of Computational Economics, 2005 Money Macro Finance group annual meeting, the X Vigo workshop on Dynamic Macroeconomics, Swedish Riksbank, European University Institute, ECB, and Bank of Canada for valuable suggestions. This paper is built on the second chapter of my dissertation. All remining errors are mine.

†Monetary and Financial Analysis Department, Bank of Canada, 234 Wellington St., Ottawa, K1A 0G9, Ontrario, Canada; Email: menc@bankofcanada.ca; Homepage: http://www.bankofcanada.ca/ec/cmendicino/

1 Introduction

Standard Real Business Cycle theories succeed in accounting for business cycle observations of aggregate quantities, such as output, investment and consumption, by relying mainly on large and persistent aggregate productivity shocks. Kiyotaki and Moore (1997) and Kiyotaki (1998) show that if debt is fully secured by collateral, even small and temporary productivity shocks can have large and persistent effects on economic activity. They document that the credit system may act as a powerful mechanism by which small shocks propagate into the economy. Kiyotaki and Moore's theoretical work has been very influential and an increasing number of papers have documented the contribution of collateralized debt to business cycle fluctuations. For example, on the international transmission of business cycles, see Iacoviello and Minetti (2007); on the role of the housing and collateralized debt in the transmission and amplification of shocks, see Iacoviello (2005) and Iacoviello and Neri (2008); on the macroeconomic implications of mortgage market deregulation, see Campbell and Hercowitz (2005); on the role of nominal debt in sudden stops, see Mendoza (2006) and Mendoza and Smith (2007); and on over borrowing, see Uribe (2007). A common assumption in this strand of the business cycle literature is that a certain degree of debt enforcement inefficiency in the credit market limits the agents'debt to be a fraction of the value of their collateral.

Collateralized debt is becoming a popular feature of business cycle models, despite the fact that Kocherlakota (2000) and Cordoba and Ripoll (2004) demonstrated that collateral constraints *per se* are unable to propagate and amplify exogenous shocks, unless unorthodox assumptions on preferences and production technologies are made. However, papers on the amplification role of collateral constraints have neglected the role of inefficiencies in the liquidation of the collateralized assets. As documented by Djankov, Hart, McLiesh and Shleifer (2006), debt enforcement procedures around the world are significantly inefficient. They study debt enforcement with respect to an insolvent firm, documenting the time required to resolve the insolvency and the cost of the insolvency process, to assess the degree of efficiency of debt enforcement in 88 countries. They find that all procedures are extremely time consuming, costly, and inefficient and that the degree of inefficiency varies enormously between countries. According to their findings, an average of 48 percent of the firm's value is lost in debt enforcement worldwide.[1] Table 1 summarizes their results.[2]

This paper aims to reconcile these two strands of the business cycle literature by exploring the role of costly debt enforcement procedures in the amplification of productivity shocks. We limit our analysis to the class of models with inelastic capital supply and borrowing limits *à la* Kiyotaki and Moore (1997). We document that the magnitude of amplification depends substantially on the degree of debt enforcement efficiency assumed in the credit market. As a result, collateral constraints significantly amplify the effects of productivity shocks on output even under standard assumptions on preferences and technology.

The key insight is that the degree of inefficiency in the debt enforcement procedure affects the sensitivity of output to shocks through the redistribution of capital between borrowers and lenders. The intuition is as follows. The amount of capital that borrowers can hold is limited by the existence of credit constraints. Thus, compared to an economy without colateral contraints, the allocation of capital between borrowers and lenders turns out to be inefficient and total production is distorted below the frictionless level.

Given the fact that all producers face the same technology, borrowers experience higher marginal productivity of capital. A redistribution of capital between the two groups of agents thus generates significant variations in total production. For example, a positive productivity shock, by relaxing those borrowing constraints that initially limited capital holdings among borrowers, allows for a more efficient allocation of capital between agents. An increase in the share of capital held by borrowers, who have higher marginal productivity, amplifies the effect of the shock on output. The degree of debt enforcement inefficiency determines the difference in marginal productivity between borrowers and lenders and thus the size of the endogenous amplification generated by the model. The sensitivity of output to productivity shocks varies in a non-linear way with respect to the degree of credit market inefficiency.

The model features negligible amplification in only two parameterizations: autarky and fully efficient debt enforcement procedures. In the absence of a credit market, capital is allocated in a very inefficient way, so the

[1] Efficiency is defined in relation to the present value of the terminal value of the firm after bankruptcy costs.

[2] In the left column we report the measure of efficiency by income group. The right column shows data on debt enforcement for a subsample OECD countries by legal origins.

gains from a better allocation of resources are potentially very big. However, the redistribution of capital induced by the shock itself is limited since impatient agents can finance their capital expenditure only through their own income. Thus, the share of borrowers' capital increases by little and the amplification of the shocks on total output is negligible. On the other hand, under fully efficient debt enforcement procedures, the allocation of capital between borrowers and lenders is already very close to the frictionless level. Thus, the changes in total output following on the redistribution of capital are minimal and the amplification generated by the model is negligible.

This paper improves upon previous literature by documenting the contribution of inefficiency in the debt enforcement procedure to the amplification of business cycle fluctuations which other authors have not considered. We document that Kocherlakota (2000) and Cordoba and Ripoll (2004) results are not robust to different assumptions on the degree of inefficiencies. In fact, for realistic degrees of credit market inefficiency, collateral constraints significantly amplify the effects of productivity shocks on output even under standard assumptions on preferences and technology.

The paper is organized as follows. Section 2 presents the benchmark model and section 3 studies the model's dynamics. Section 4 discusses how the model's endogenous amplification relates to measures of the efficiency of the debt enforcement procedure observed in the U.S. Section 5 presents some extensions of the model. Section 6 draws some conclusions.

2 Benchmark Model

Model 1. Following Kiyotaki and Moore (1997), we consider a discrete time economy populated by two types of agents who trade two kinds of goods: a durable asset and a non-durable commodity. The durable asset (k) does not depreciate and has a fixed supply normalized to one. The commodity good (c) is produced with the durable asset and cannot be stored. At time t there are two competitive markets in the economy: the asset market, in which one unit of the durable asset can be exchanged for q_t units of the consumption good, and the credit market. The economy is populated by a continuum of ex-ante heterogeneous agents of unit mass: n_1 *Patient Entrepreneurs* (denoted by 1) and n_2 *Impatient Entrepreneurs* (denoted by 2). In order to impose the existence of flows of credit in this economy we assume ex-ante heterogeneity based on different subjective discount factors: $\beta_2 < \beta_1 < 1$.

This assumption ensures that in equilibrium patient households lend and impatient households borrow.

Both agents produce the commodity good using the same technology:

$$y_{it} = Z_t k_{it-1}^{\alpha}, \tag{1}$$

where Z_t represents an aggregate technology shock. We assume that agents have access to the same concave production technology: $\alpha_1 = \alpha_2 < 1$. However, following previous literature, technology is specific to each producer and only the household that started the production process has the skills necessary to complete the process. Nevertheless, agents cannot precommit to produce. This means that if household i decides not to put his effort into production between t and $t+1$, there would be no output at $t+1$, but only the asset k_{it}. Agent are free to walk away from the production process and from debt contracts between t and $t+1$. This results in a default problem that makes creditors willing to protect themselves by collateralizing the borrower's asset. Creditors know that in the case where the borrower chooses not to produce and neglects his debt obligations, they can still get his asset. However, we assume that the lenders can repossess the borrower's assets only after paying a proportional transaction cost, $[(1-\gamma)E_t q_{t+1} k_{it}]$. Thus, agents cannot borrow more than a certain amount such that the next period's repayment obligation cannot exceed the expected value of next period assets:

$$b_{it} \leq \gamma E_t \left[q_{t+1} k_{it} \right], \tag{2}$$

where $(1-\gamma)$ is the cost lenders must pay to repossess the collateral asset. The lower γ, the more costly, and thus inefficient, the debt enforcement procedure.

Agents face the following problem:

$$\max_{\{c_{it}, k_{it}, b_{it}\}} E_0 \textstyle\sum_{t=0}^{\infty} (\beta_i)^t U(c_{it}) \qquad i = 1, 2$$

$$s.t.$$

$$c_{it} + q_t(k_{it} - k_{it-1}) = y_{it} + \tfrac{b_{it}}{R_t} - b_{it-1}$$

$$y_{it} = Z_t k_{it-1}^{\alpha}$$

$$b_{it} \leq \gamma E_t \left[q_{t+1} k_{it} \right],$$

where k_{it} is a durable asset, c_{it}, a consumption good, and b_{it}, the debt level. Agents' optimal choices of bonds and capital are characterized by:

$$\frac{U_{c_{i,t}}}{R_t} \geq \beta_i E_t U_{c_{i,t+1}} \tag{3}$$

and

$$q_t - \beta_i E_t \frac{U_{c_{i,t+1}}}{U_{c_{i,t}}} q_{t+1} \geq \beta_i E_t \frac{U_{c_{i,t+1}}}{U_{c_{i,t}}} \left(F_{k_i,t+1}\right), \tag{4}$$

where $F_{k_i,t} = \alpha Z_t k_{it-1}^{\alpha-1}$ is the marginal product of capital. Households feature a concave utility function $U(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}$. The first equation relates the marginal benefit of borrowing to its marginal cost. For constrained agents, the marginal benefit is always bigger than the marginal cost of borrowing. If $\mu_{i,t} \geq 0$ is the multiplier associated with the borrowing constraint, then the Euler equation becomes:

$$\frac{U_{c_{i,t}}}{R_t} - \mu_{i,t} = \beta_i E_t U_{c_{i,t+1}}. \tag{3.a}$$

The second equation states that the opportunity cost of holding one unit of capital, $\left[q_t - \beta_i E_t \frac{U_{c_{i,t+1}}}{U_{c_{i,t}}} q_{t+1}\right]$, is bigger or equal to the expected discounted marginal product of capital. For constrained agents the marginal benefit of holding one unit of capital is given not only by its marginal product but also by the marginal benefit of being allowed to borrow more:

$$q_t - \beta_2 E_t \frac{U_{c_{2,t+1}}}{U_{c_{2,t}}} q_{t+1} = \beta_2 E_t \frac{U_{c_{2,t+1}}}{U_{c_{2,t}}} \left(F_{k_2,t+1}\right) + \gamma E_t q_{t+1} \frac{\mu_t}{U_{c_{2,t}}}. \tag{4.a}$$

Collateral constraints alter the future revenue from an additional unit of capital for the borrowers. Holding an extra unit of capital relaxes the credit constraint and thus increases their shadow price of capital. This additional return encourages borrowers to accumulate capital even though they discount their revenues more heavily than lenders.

In the deterministic steady state, the group of impatient households is credit constrained.[3] Following previous literature, we analyze the properties

[3] Consider the Euler equation of the impatient household:

$$\frac{u_{c_{2,t}}}{R_t} - \mu_{2,t} = \beta_2 E_t u_{c_{2,t+1}}.$$

In the steady state it implies:

$$\mu_2 = \left(\frac{1}{R} - \beta_2\right) u_{c_2}.$$

Since the steady state interest rate is determined by the discount factor of the patient agent:

$$\mu_2 = \left(\frac{1}{R} - \beta_2\right) u_{c_2} = \left(\beta_1 - \beta_2\right) u_{c_2};$$

As long as $\beta_2 < \beta_1 < 1$, the lagrange multiplier associated with the borrowing constraint for the impatient household is strictly positive in the deterministic steady state.

of the model in a neighborhood of the steady state, in which impatient households borrow up to the maximum:

$$b_{2,t} = \gamma E_t \left[q_{t+1} k_{2t} \right]$$

and

$$k_{2t} = \frac{W_{2,t} - c_{2,t}}{\left[q_t - \gamma E_t \frac{q_{t+1}}{R_t} \right]},$$

where $W_{2,t} = y_{2,t} + q_t k_{2,t} - b_{2,t-1}$ is the impatient agent's wealth at the beginning of time t, and $d_t = \left[q_t - \gamma E_t \frac{q_{t+1}}{R_t} \right]$ represents the difference between the price of capital and the amount he can borrow against a unit of capital, i.e. the downpayment required to buy a unit of capital. The creditors' capital decision is determined at the point where the opportunity cost of holding capital equals its marginal product:

$$q_t - \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} q_{t+1} = \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} \left(F_{k_1,t+1} \right). \tag{3.b}$$

The total stock of capital k_t is given by:

$$k_t = (1-n) k_{1t} + n k_{2t}. \tag{5}$$

3 Results

Benchmark parameter values. As a benchmark case, we set the parameters' value on a quarterly base as in table 2. Patient households' discount factor equals 0.99 such that the average annual rate of return is about 4 percent. The discount factor for impatient agents, β_2, equals 0.95 This value is in line with previous estimates by Lawrence (1991), Samwick (1998) and Warner and Pleeter (2001). See also Hendricks (2007). The baseline choice for the fraction of the population that is borrowing constrained, n_2, is set to 50 percent. This value is also in the range of estimates in the literature. Campbell and Mankiw (1989) estimate around 40 percent of the population to be rule-of-thumb consumers. According to Iacoviello (2005), in the U.S. about 55 percent of the population is credit constrained. The coefficient of relative risk aversion, σ, equals 2.2, which is in the range suggested by several previous authors. Since it is difficult to pin down the share of capital in production, we use a pragmatic approach. As a benchmark value, we begin by setting α=0.4. In the next section we investigate the sensitivity of our results to different parameter values and to the introduction of labor as an input of production.

A look at the steady state. In what follows, we analyze how the deterministic steady state of the model is affected by the degree of efficiency in the debt enforcement procedure, as proxied by γ. Figure 1.a shows the marginal productivity of capital, and thus the efficiency of production, as a function of γ. Since in the deterministic steady state the group of impatient households is credit constrained, their capital holding is less than the level that maximizes total output.[4] *Ceteris paribus,* a higher γ improves the allocation of capital between the two groups of agents and reduces the difference between borrowers' and lenders' marginal productivity. Using the equations representing the households' optimal choice of capital evaluated at the steady state it is possible to show that as long as $\gamma < \frac{1}{\beta_1}$,

$$\frac{F_{k_2}}{F_{k_1}} = \frac{\beta_1 \left[1 - \beta_2 - \gamma(\beta_1 - \beta_2)\right]}{(1 - \beta_1)\, \beta_2} > 1, \tag{6}$$

where $F_{k_i} = \alpha \left(\frac{K_i}{n_i}\right)^{\alpha-1}$. The steady state allocation of capital depends on the subjective discount factors, the population weights for the two groups of agents, and the degree of credit market inefficiency, γ:

$$K_2 = \frac{1}{\left\{1 + \frac{n_1}{n_2}\left[\frac{\beta_2(1-\beta_1)}{\beta_1[1-\beta_2-\gamma(\beta_1-\beta_2)]}\right]^{\frac{1}{\alpha-1}}\right\}}. \tag{7}$$

Compared to the frictionless case, the allocation under credit constraints reduces the level of capital held by borrowers and implies a difference in the marginal productivity of capital for the two groups of producers. Thus, total production is distorted below the efficient level. More efficient debt enforcement procedures reduce the efficiency loss in terms of output. In fact, a more efficient credit market implies credit expansion and thus a rise in the level of capital held by borrowers (figure 1.b).[5] A better allocation of capital between the two groups leads to an increase in total production.

[4] The efficient allocation of capital between the two groups would be given by equality between the marginal products of the two groups:

$$F_{k_1,t} = F_{k_2,t}.$$

Thus, given the aggregate condition on capital and since the total population is normalized to be equal to the unit interval, each group of agents would get the same amount of capital in the deterministic steady state.

[5] The price of the collateral asset is also higher. In the steady state, the asset prices depend on the marginal productivity of capital. More specifically, the households' optimal

Impulse Responses. We now consider the response of the model economy to a productivity shock when γ=1.[6] Aggregate production follows an AR(1) process given by

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_{Zt},\ \varepsilon_{Zt} \backsim^{iid} N(0, \sigma_\epsilon),$$

with $\rho_Z = 0.9$. We assume that the economy is at the steady state level at time zero and then is hit by an unexpected decrease in aggregate productivity of 1 percent. The results are reported in figure 2.a. An aggregate negative shock reduces production and thus the earnings of both groups of agents. Since the shock is temporary, borrowers sell a part of their capital stock to smooth the effect of the shock on consumption. Given the assumption of inelastic supply of capital, in order for the capital market to clear, lenders have to increase their demand for capital. The user cost of holding capital decreases. Movements in the relative price of capital, altering the value of the collateral asset, affect the ability to borrow and, in turn, borrowers' expenditure decisions. Thus, constrained agents are negatively affected not only by the direct impact of the shock but also by the reduced availability of credit resulting from a reduction in the value of their collateral. Via the reduction in borrowers' current investment expenditures, the shock impacts total production over time due to the higher marginal productivity of capital. However, the response of total output displayed in figure 2.b shows that when γ=1 and standard parameter values are used to calibrate the technology and utility functions, the amplification generated by the model is negligible. This result is in accordance with Cordoba and Ripoll (2004).

Amplification and Persistence. How does the efficiency of the debt enforcement procedure affect the amplification of shocks to output? Since the first impact of the shock would always be equal to the shock itself, we look at the second-period effect of the shock. Figure 3.a plots the percentage deviation of output attributed to the model's endogenous propagation

choice of capital gives

$$q = \frac{\beta_1}{1-\beta_1} F_{k_1}.$$

[6] In order to limit concerns on the occassionally binding nature of the borrowing constraint, we base our analysis on the effects of negative productivity shocks. We condition on the initial state of the economy being the deterministic steady state and assume that the economy is hit by an unexpected reduction in productivity. Thus, the lagrange multiplier associated with the collateral constraints is always positive. As a result, the borrowing constraint always binds.

mechanism – i.e. the second-period variation in output that exceeds the exogenous impact on output directly implied by the autocorrelation of the shock. The sensitivity of output to productivity shocks varies in a non-linear way with respect to the degree of credit market inefficiency, and it is up to 30 percent stronger than that obtained with a fully efficient debt enforcement procedure (γ=1).

Under credit constraints, borrowers' capital holding is below the frictionless level. Given that all producers face the same technology, borrowers experience higher marginal productivity of capital. A redistribution of capital between the two groups of agents generates significant variations in terms of total production. In the absence of a credit market, capital is allocated in a very inefficient way, so the gains from a better allocation of resources are potentially very big (see figure 1.a). On the other hand, the redistribution of capital induced by the shock itself is limited since impatient agents cannot finance their capital expenditure through the credit market. Thus, the amplification of the shocks on total production is negligible. A more efficient the credit market allow for a larger the redistribution of capital, enhancing the endogenous amplification generated by the model. However, as γ rises the difference in marginal productivity of capital between lenders and borrowers is smaller (see figures 1.a, 1.c), which reduces the sensitivity of total output to shocks. Under fully efficient debt enforcement procedures the model feature negligible amplification. In this last case, the allocation of capital between borrowers and lenders is already very close to the frictionless level. Thus, the gains or losses in terms of aggregate production coming from the redistribution of capital are minimal and the amplification generated by the model is in fact negligible.[7]

Strictly speaking, the second-period elasticity of total output with respect to technology shocks can be written as:

$$\epsilon_{yz} = \epsilon_{yk_2}\epsilon_{k_2z} = \frac{F_{k_2} - F_{k_1}}{F_{k_2}}\alpha\frac{y_2}{y}\epsilon_{k_2z}. \tag{8}$$

[7]Kiyotaki and Moore's (1997) theoretical work shows that the amplification generated by the model can be potentially very large. They assume $\gamma = 1$. Under their assumption of a linear production function for impatient agents, the relative difference in the marginal productivity of capital is quite large, and total production is far from the efficient level even when $\gamma = 1$. Thus, the redistribution of capital between borrowers and lenders enhances the response of total production to shocks even in the presence of a fully efficient credit market. Their assumption of a linear production technology has been criticized by Cordoba and Ripoll (2004), who document that when agents face concave production functions no amplification is endogenously generated by the model.

The first term is the productivity gap between constrained and unconstrained agents (see figure 1.a), α represents the share of capital in production, while $\frac{y_2}{y}$ is the production share of constrained agents, and $\epsilon_{k_2 z}$ is the elasticity of borrowers' capital with respect to the shock (i.e. the redistribution of capital to impatient agents). In order to explain the non-linear relationship between the impact of the shock on output and the degree of credit friction, we first focus on the redistribution of capital between the two groups of agents ($\epsilon_{k_2 z}$). The right panel of figure 3.b shows the reaction of borrowers' capital expenditure when the shock hits the economy. The impact of the shock on capital expenditure displays an inverted-U relationship with the degree of inefficiency in the debt enforcement procedures.

The downpayment – that is, the difference between the price of capital and the amount agents can borrow against a unit of capital – represents the amount required to buy a unit of capital:

$$D\hat{P}_t = \frac{q_{ss}}{DP_{ss}}\hat{q}_t - \gamma \frac{q_{ss}}{R_{ss}}\left[\hat{q}_{t+1} - \hat{R}_t\right].$$

The reactions of the borrowers' capital expenditures and the downpayment are symmetrically opposite (figure 3.c). The stronger the effect of the shock on downpayment, the weaker the reaction of capital. The shape of the relationship between the degree of inefficiency in the credit market and the effect on downpayment can be explained by the existence of two opposite forces determining the intensity of the downpayment reaction. Higher γ implies a smaller productivity gap and thus a weaker the reaction of q_t and, in turn, DP_t itself to the shock. The decrease in the downpayment is thus lower than would otherwise be, leaving capital more expensive and borrowers' capital expenditure further reduced. At the same time, a more pronounced reduction in the demand for loanable funds causes a more dramatic fall in the real interest rate, R_t, which implies a more sizeable reduction in the downpayment and thus a less pronounced decline in k_{2t}. The sensitivity of borrowers' capital expenditures to a productivity shock depends on which of the two opposite effects prevails. Non-linearity in the redistribution of capital between the two groups of agents contributes to non-linearity in the sensitivity of output to productivity shocks.

Nevertheless, the elasticity of output to z_t maintains an inverted-U shape, independent of the non-linearity in $\epsilon_{k_2 z}$. Assume that lenders' utility function is linear in consumption, so that the interest rate is constant over the business cycle. Now the dynamics of the downpayment depend only on q_t.

The higher the level of γ, the weaker the effect on the downpayment and thus the larger the impact of the shock on capital. However, the relationship between γ and the second-period impact of z_t on y_t still has an inverted U-shape (figure 3.d). As shown by equation (8), the elasticity of total output to productivity shocks depends on the production share of constrained agents and the productivity gap. The fraction of total output produced by constrained agents increases with γ, since more efficient enforcement procedures induce a better allocation of capital in the economy (*production share effect*). However, for the same reason, the *productivity gap* decreases with γ. See figures 1.a and 1.c. Thus, regardless of the shape of the capital reaction to technology shocks, the second-period impact of the shock on total output has a non-linear shape. However, the non-linearity in ϵ_{yz} is enhanced by the inverted U-shape of ϵ_{k_2z}.

4 Quantitative Results

Results presented above show that for values of γ below unity the model with collateral constraints can generate amplification and persistence of productivity shocks of non-negligible magnitude. However, the relation being hump-shaped, the magnitude of amplification varies significantly. In what follows we investigate the quantitative relevance of the amplification generated by the model when the efficiency in the debt market is set to the level reported for the US (0.858). Under the benchmark calibration, the amplification on output is about 24 percent.

We now investigate the sensitivity of the results to the share of capital in production and the discount factor of borrowers. Regarding the discount factor of borrowers, we compare the results for three different values of β_2: 0.91, 0.95 and 0.97. These values are in the ballpark of previous estimates. For the share of capital in the production process, α, we follow Angeletos and Calvet (2006) and assume two different values of this parameter: α=0.4, which corresponds to the standard definition of capital, and α=0.7, which reflects a broader definition and includes both physical and intangible capital.

Table 3.a shows that when the degree of efficiency in the debt market equals that reported for the US, the amplification endogenously generated by the model is quantitatively significant. The degree of endogenous amplification generated by the model can be as low as 19 percent (β_2=0.97, α=0.4)

and as high as 39 percent (β_2=0.95, α=0.7). In any case, the magnitude of amplification is sizable and significantly higher than what we observe in the version of the model where inefficiencies in the liquidation of the collateralized asset are neglected (γ=1). As a result, for realistic degrees of efficiency in the debt enforcement procedures, collateral constraints can significantly amplify the effects of productivity shocks on output even under standard assumptions on preferences and technology.

It is not surprising that a lower discount factor for impatient agents is associated with more sizable endogenous amplification. A lower β_2 implies a higher degree of heterogeneity in the model, a wider productivity gap between borrowers and lenders (see eq.6), and thus greater amplification. The role of capital intensity in production in generating amplification is such that for $\beta_2 = 0.91$, α=0.4 amplifies the effect of the shock to a greater extent. However, when the gap in discount factors shrinks, stronger amplification is given by α=0.7 (see figure 4). The relation between γ and the intensity of the output reaction to productivity shocks is clearly non-linear with respect to α. In contrast to Kocherlakota (2000), we document that output amplification is not a strictly increasing function of the capital share. In fact, a lower α does not necessarily imply lower amplification of shocks. In the model presented here, this result holds only for a low degree of heterogeneity or a high degree of inefficiency in the debt enforcement procedures.

5 Extension of the Model: Introducing Labour Supply

According to Cordoba and Ripoll (2004), if aggregate labor is not fixed but rather optimally supplied, the amplification role of collateral constraints is dramatically reduced. To explore the robustness of the results presented above, we now consider the case where household work is also an input of production. We assume that each household works in its own firm and gets utility from leisure.

Model 2. Following Greenwood *et al.* (1988), we adopt the following utility function

$$U(c_{it}, L_{it}) = \frac{1}{1-\sigma}\left(c_{it} - \chi\frac{L_{it}^{\eta}}{\eta}\right)^{1-\sigma} \tag{9}$$

and production function

$$y_{it} = Z_t k_{it-1}^{\alpha} L_{it}^{1-\alpha}. \tag{10}$$

Endogenous amplification of the shocks is already present in the first period.[8] In response to a 1 percent decrease in productivity, total output decreases by 1.47 percent. However, it is possible to show that the first period amplification is independent of γ. Given the household's labor supply,

$$\chi L_{it}^{\eta-1} = (1-\alpha) Z_t k_{it}^{\alpha} L_{it}^{-\alpha}, \tag{11}$$

individual's production can be written in terms of the capital input:

$$y_{it} = Z_t^{\frac{\eta}{\alpha+\eta-1}} k_{it-1}^{\alpha+\frac{(1-\alpha)\alpha}{\alpha+\eta-1}} \frac{1-\alpha}{\chi}.$$

When productivity decreases by 1 percent, output decreases by $\frac{\eta}{\alpha+\eta-1} = 1.47$ percent. As figure 5.a shows, *ceteris paribus*, different degrees of inefficiency in the credit market imply different magnitudes of the endogenous amplification of output in response to the same shock. The second impact still varies with the degree of credit friction. The elasticity of total output with respect to technology shocks can be written as in equation (8), but now multiplied by $\frac{\eta}{\alpha+\eta-1}$:

$$\epsilon_{yz} = \frac{\eta}{\alpha+\eta-1} \epsilon_{yk_2} \epsilon_{k_2 z}. \tag{12}$$

Figure 5.a shows that the output response delivered by the model with collateral constraints can be much stronger and persistent than the response generated by the representative agent model. In this latter framework, the economy is populated only by patient agents and there are no limits to credit. In the model with collateralized debt, the reaction of output to a productivity shock is between 50 percent and 130 percent higher than the variation directly induced by the shock. So, despite the non-linearity featured by the model, a degree of amplification significantly higher than that generated by the representative agent model is displayed.

Model 3. In order to take into account the implications for amplification of the wealth effects on labor supply the following utility function is assumed[9]:

$$U(c_{it}, L_{it}) = \frac{1}{1-\sigma} \left(c_{it}^{(1-\varphi)} (1-L_{it})^{\varphi} \right)^{1-\sigma}. \tag{13}$$

[8] We calibrate the labor supply elasticity to 0.5 (η=2). The weight on leisure is chosen so that hours worked in the initial steady state is around 1/3 of total time depending on γ (χ=1).

[9] We calibrate $\varphi = 0.6$, so that hours worked in the initial steady state are around 1/3 of total time.

As in Cordoba and Ripoll (2004), when $\gamma = 1$, introducing labor supply according to a utility function of this type is detrimental for the amplification of shocks. However, the result only holds for a very limited range of the debt enforcement procedure parameter. Figure 5.b shows that for any value of $\gamma < 0.98$, the magnitude of the second-period amplification is bigger than that produced by the equivalent representative agent model.[10] Below this value, output responds between 5 percent and 25 percent more than the reaction directly induced by the variation in productivity.

Tables 3.b compares different versions of the model under the benchmark parameter values. First we document that, in accordance with previous literature, if we do not take into account inefficiency in the debt enforcement process (γ=1), different versions of the model predict either negligible amplification of productivity shocks to output (model 1) or even detrimental effects of collateral constraints on the output reaction (model 3). The only exception is the case where the wealth effects of labor supply are ignored (model 2). Next, we set the efficiency in the debt market to be equal to that reported for the US. All versions of the model display a significant amplification of the shock to output. Model 1 reports an amplification of 33 percent, and model 3, about 30 percent; an exceptionally high amplification is generated by model 2. Similar results are obtained if we use the average degree of inefficiency reported for groups of OECD economies differing in terms of legal origins. Setting γ equal to these averages implies a reaction of output between 15 and 48 percent higher than that obtained in the representative agent model. Thus, if realistic values of the degree of credit market inefficiency are assumed ($\gamma < 1$), the role of collateral constraints in terms of the amplification of productivity shocks is significantly enhanced, even under standard assumptions on the utility function and production process.

6 Conclusion

The aim of this paper is to quantify the amplification generated by collateral constraints in relation to the degree of frictions in the credit market. To this end, we analyze a stylized business cycle version of the Kiyotaki and Moore (1997) model. We document that the existence of costly debt enforcement procedures – which has been captured by calibrating the degree

[10]According to Djankov et al. (2006), the degree of efficiency in the debt enforcement procedures around the world does not reach such a high level.

of debt enforcement efficiency in the model as in the data – generates significant amplification of productivity shocks to output. Previous literature by ignoring potential inefficiencies in debt enforcement procedures, neglects an important source of amplification for this class of models and thus minimizes the role of collateral constraints as an endogenous amplification mechanism.

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Appendix .1 Benchmark Model: Equilibrium Conditions

The system of non-linear equations is given by four first order conditions:

$$\frac{U_{c_{1,t}}}{R_t} = \beta_1 E_t U_{c_{1,t+1}}, \tag{E.1}$$

$$\frac{U_{c_{2,t}}}{R_t} - \mu_{2,t} = \beta_2 E_t U_{c_{2,t+1}}, \tag{E.2}$$

$$q_t - \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} q_{t+1} = \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} F_{k_1,t+1}, \tag{E.3}$$

$$q_t - \beta_2 E_t \frac{U_{c_{2,t+1}}}{U_{c_{2,t}}} q_{t+1} = \beta_2 E_t \frac{U_{c_{2,t+1}}}{U_{c_{2,t}}} F_{k_2,t+1} + \gamma E_t q_{t+1} \frac{\mu_{2t}}{U_{c_{2,t}}}; \tag{E.4}$$

our aggregate conditions:

$$n_1 k_{1t} + n_2 k_{2t} = K_{1t} + K_{2t} = 1, \tag{E.5}$$

$$y_t = n_1 y_{1t} + n_2 y_{2t}, \tag{E.6}$$

$$n_1 b_{1t} + n_2 b_{2t} = 0; \tag{E.7}$$

one budget constraint:[11]

$$c_{2t} + q_t(k_{2t} - k_{2t-1}) = y_{2t} + \frac{b_{2t}}{R_t} - b_{2t-1}; \tag{E.8}$$

one borrowing constraint:

$$b_{2,t} = \gamma E_t \left[q_{t+1} k_{2t} \right]; \tag{E.9}$$

the resource constraint:

$$y_t = n_1 c_{1t} + n_2 c_{2t}; \tag{E.10}$$

the two technologies:

$$y_{1t} = Z_t k_{1t-1}^{\alpha}, \qquad y_{2t} = Z_t k_{2t-1}^{\alpha}. \tag{E.11}$$

This gives twelve equations for twelve unknowns: $\{\mu_{2t}, q_t, R_t, y_t\}$ and $\{c_{it}, k_{it}, b_{it}, y_{it}\}_{t=0}^{\infty}$ for i=1,2.

Appendix .2 Benchmark Model: Steady State

From E.1 we find the steady state interest rate:

$$\frac{1}{R} = \beta_{1.} \tag{ss.1}$$

From E.2, the lagrange multiplier:

$$\mu_2 = (\beta_1 - \beta_2)\, u_{c_2}. \tag{ss.2}$$

[11] Using Walras' Law, we can drop at each t one of the two budget constraints.

Using E.3 and E.4:

$$q = \frac{\beta_1}{1-\beta_1} F_{k_1} = \frac{\beta_2}{1-\beta_2-\gamma(\beta_1-\beta_2)} F_{k_2}. \tag{ss.3}$$

Substituting for K_1 using the aggregate condition on capital: $K_1 = 1 - K_2$, we find the steady state allocation of capital to the group of borrowers, K_2:

$$\frac{\beta_1}{1-\beta_1}\left(\frac{1-K_2}{n_1}\right)^{\alpha-1} = \frac{\beta_2}{1-\beta_2-\gamma(\beta_1-\beta_2)}\left(\frac{K_2}{n_2}\right)^{\alpha-1}.$$

Thus:

$$K_2 = \frac{1}{\left\{1 + \frac{n_1}{n_2}\left[\frac{\beta_2(1-\beta_1)}{\beta_1[1-\beta_2-\gamma(\beta_1-\beta_2)]}\right]^{\frac{1}{\alpha-1}}\right\}}.$$

Thus we find the steady state borrowing level,

$$b_2 = \gamma\,[qk_2] = -b_1, \tag{ss.4}$$

and total production,

$$y = n_1 y_1 + n_2 y_2, \tag{ss.5}$$

where

$$y_1 = k_1^\alpha, \quad y_2 = k_2^\alpha.$$

From E.8 we find the consumption of the borrowers,

$$c_2 = y_2 - b_2\left(1 - \frac{1}{R}\right),$$

and, from the resource constraint the consumption of the group of lenders,

$$n_1 c_1 = y - n_2 c_2. \tag{ss.8}$$

Table 1: Debt Enforcement around the World			
Around the World		OECD	
income level	efficiency	legal origins	efficiency
high	77.35	*english*	77.0
upper middle	46.11	*french*	69.7
lower middle	35.03	*german*	72.2
Total	51.97	*nordic*	84.9
Source: Djankov,Hart, McLiesh and Shleifer (2006)			

Table 2: Benchmark Parameter Values			
preferences		**shock process**	
discount rate	$\beta_1 = 0.99$	autocorrelation	$\rho_z = 0.9$
	$\beta_2 = 0.95$		
	$\sigma = 2.2$	**population**	$n = 0.5$
technology	$\alpha = 0.4$	**borrowing limit**	$\gamma = 1$

Table 3.a: Model Results			
efficiency	**output amplification**		
$\gamma = 85.8$	$\beta_2 = 0.91$	$\beta_2 = 0.95$	$\beta_2 = 0.97$
$\alpha = 0.4$	0.3046	0.2437	0.1716
$\alpha = 0.7$	0.3033	0.3506	0.3070
$\gamma = 1$			
$\alpha = 0.4$	0.0523	0.0263	0.0132
$\alpha = 0.7$	0.1889	0.1018	0.0529
Other parameters set as in table 2.			

Table 3.b: Model Results

efficiency		output amplification model 1	model 2	model 3
representative agent model		0	0.54	0
$\gamma = 1$	100	0.0645	0.8196	-0.0834
US	85.8	0.3046	1.0584	0.2167
OECD, efficiency by legal origins				
english	77.0	0.2504	0.8348	0.1827
french	69.7	0.2027	0.7192	0.1461
german	72.2	0.2186	0.7525	0.1584
nordic	84.9	0.2980	1.0201	0.2141

model 1: no labor supply
model 2: household labor, utility eq. (9)
model 3: household labor, utility eq. (13)
Other parameters set as in table 2.

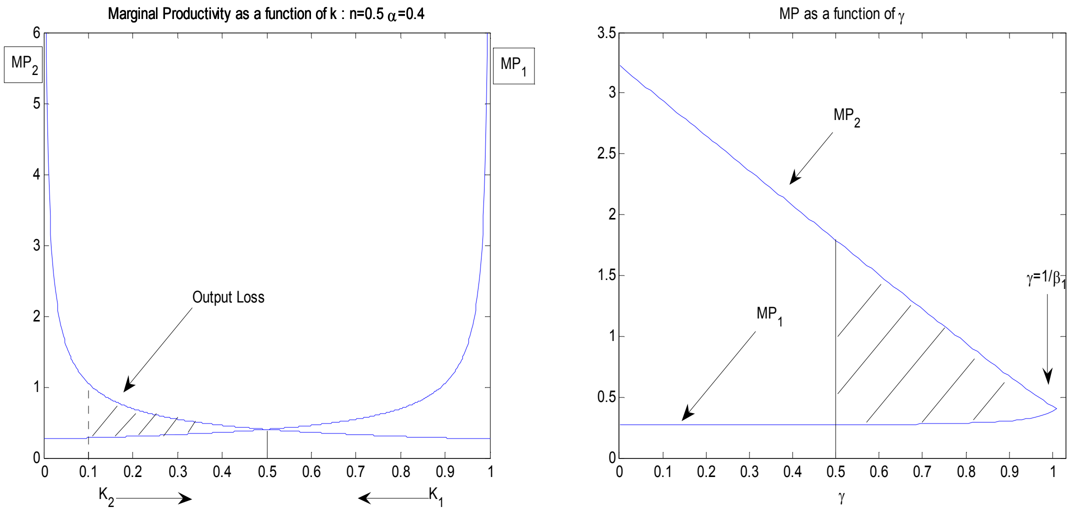

Figure 1.a shows how marginal productivity of capital for borrowers and lenders varies with respect to the capital holding and γ.

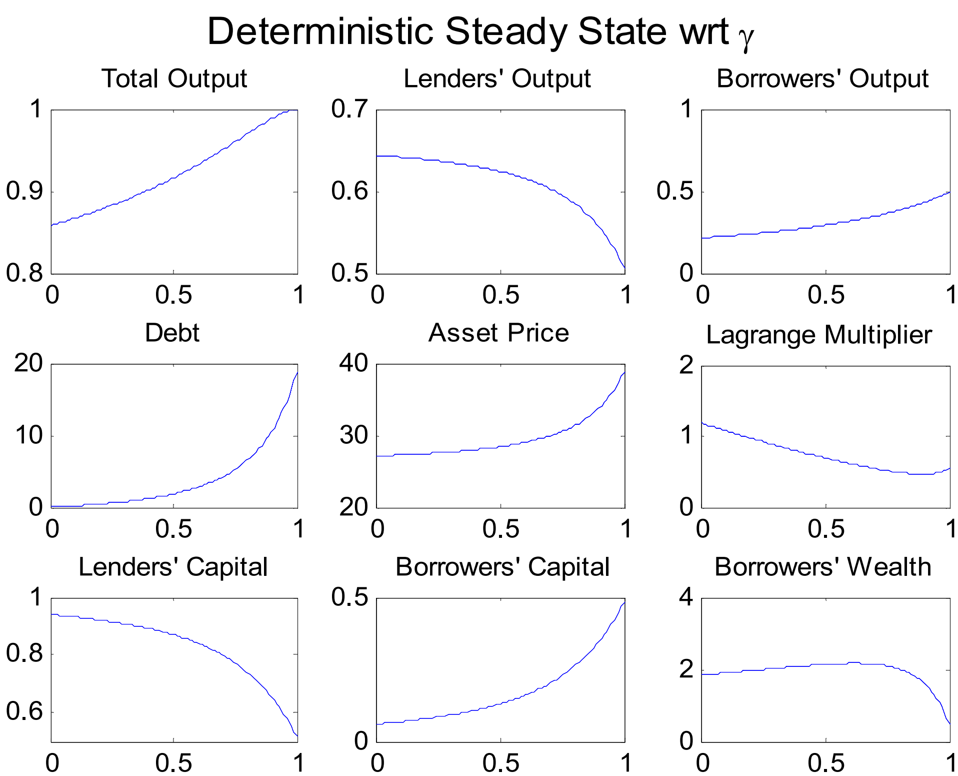

Figure 1.b shows how the steady state values of the model's variables change with respect to the degree of credit market inefficiency, γ.

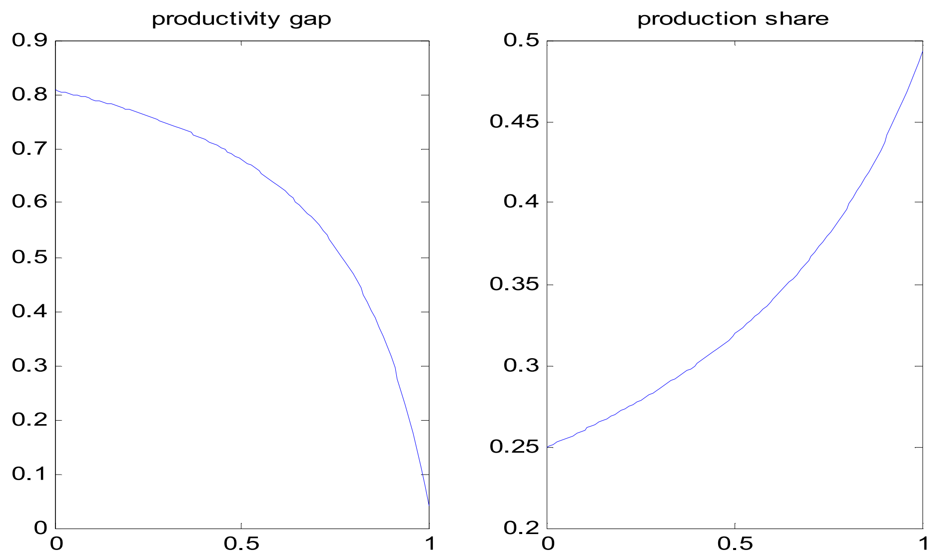

Figure 1.c shows how the steady state productivity gap and borrowers share of production varies with respect to γ.

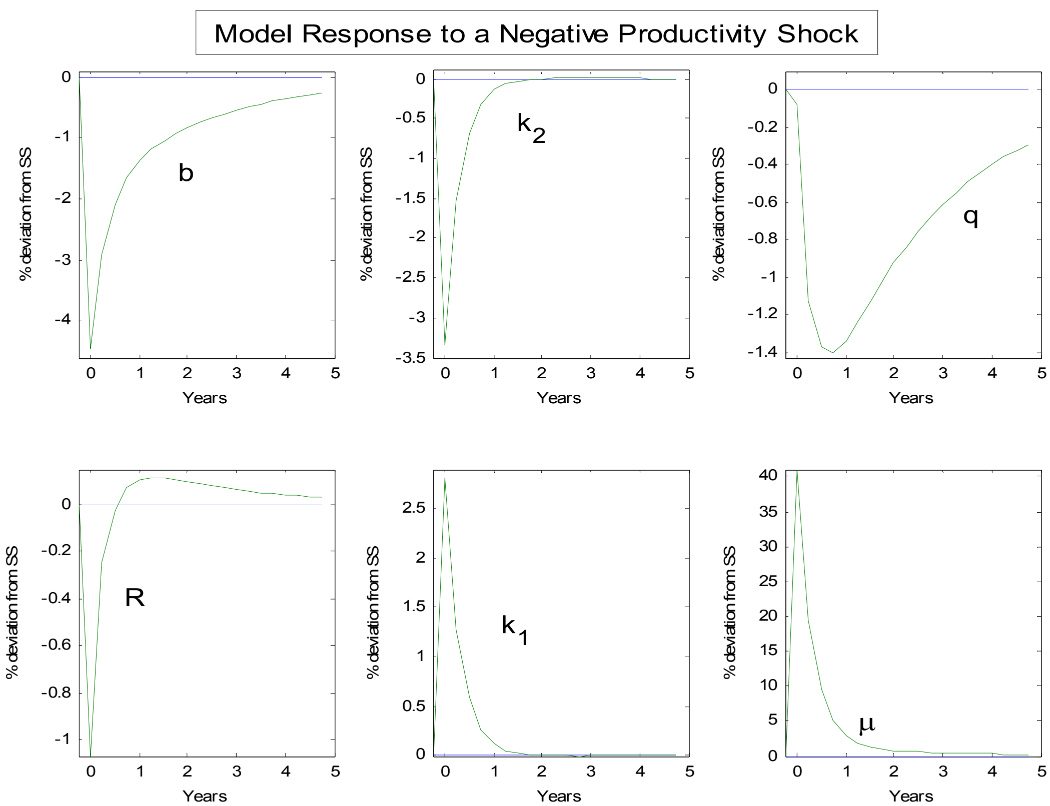

Figure 2.a shows the responses to a 1% decrease in productivity. The units on the vertical axes are percentage deviations from the steady state, while on the horizontal axes are years.

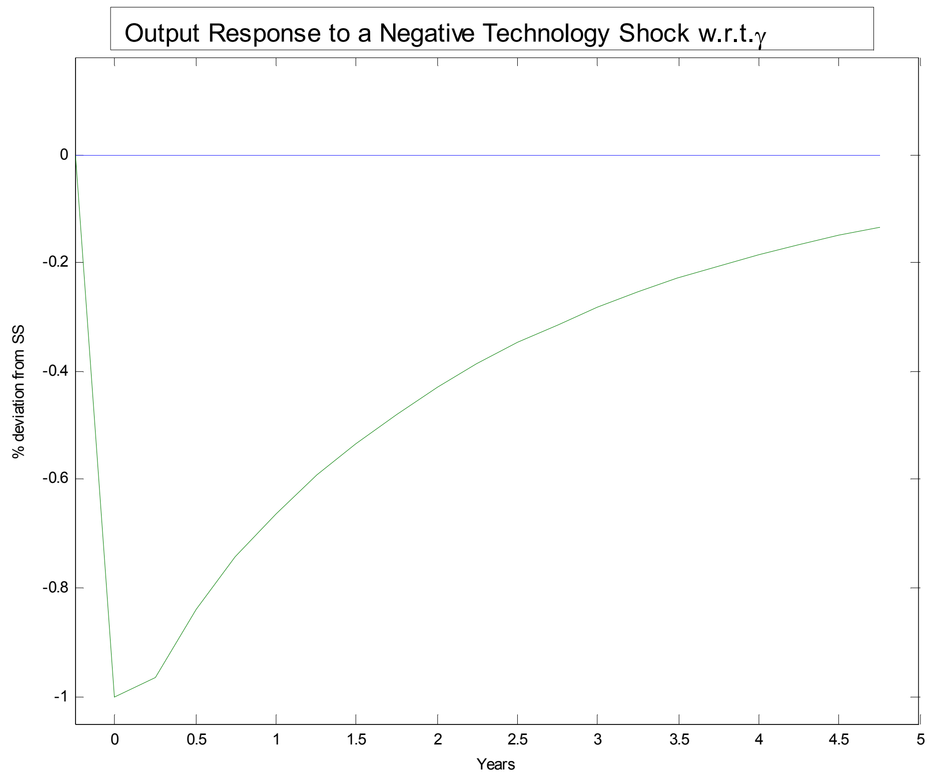

Figure 2.b shows the response of total aggregate output to a 1% decrease in productivity

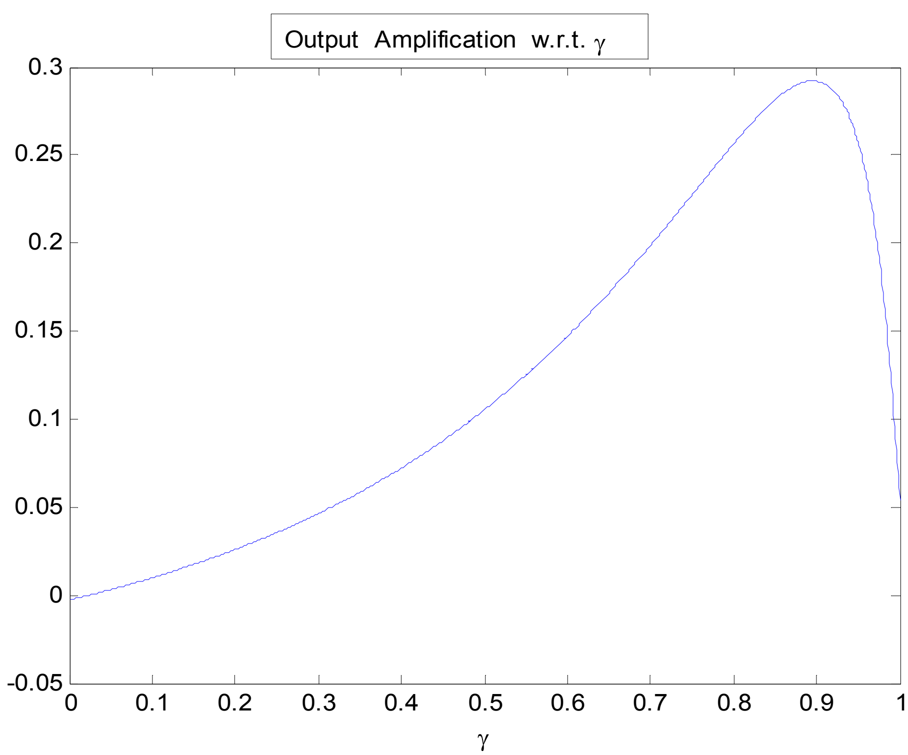

Figure 3.a second period amplification of the shock on production— endogenous reaction to shocks

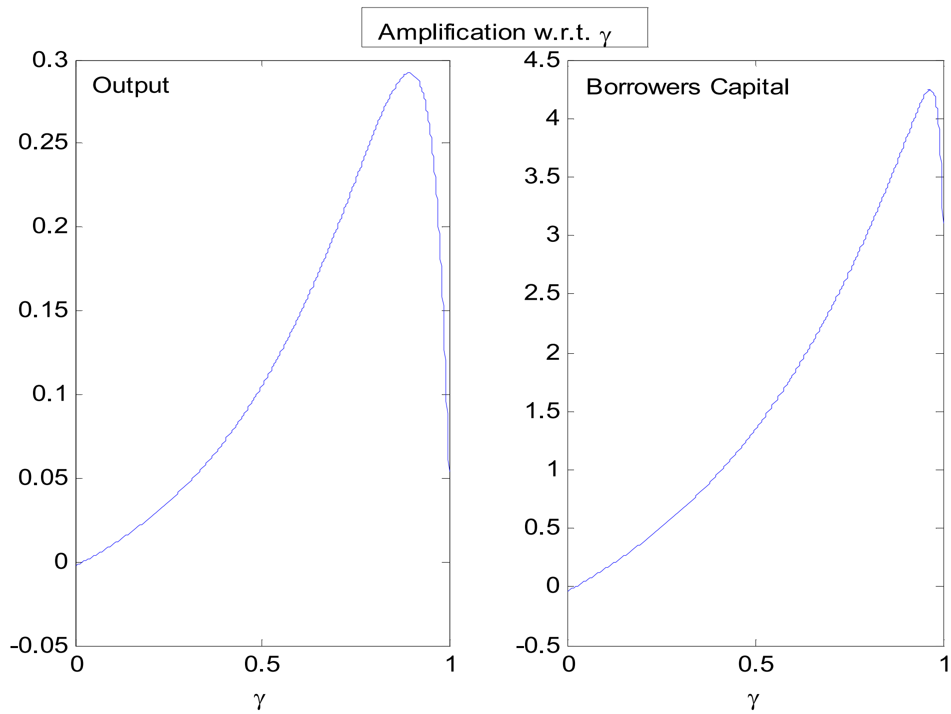

Figure 3.b second period amplification of the shock on production— endogenous reaction to shocks

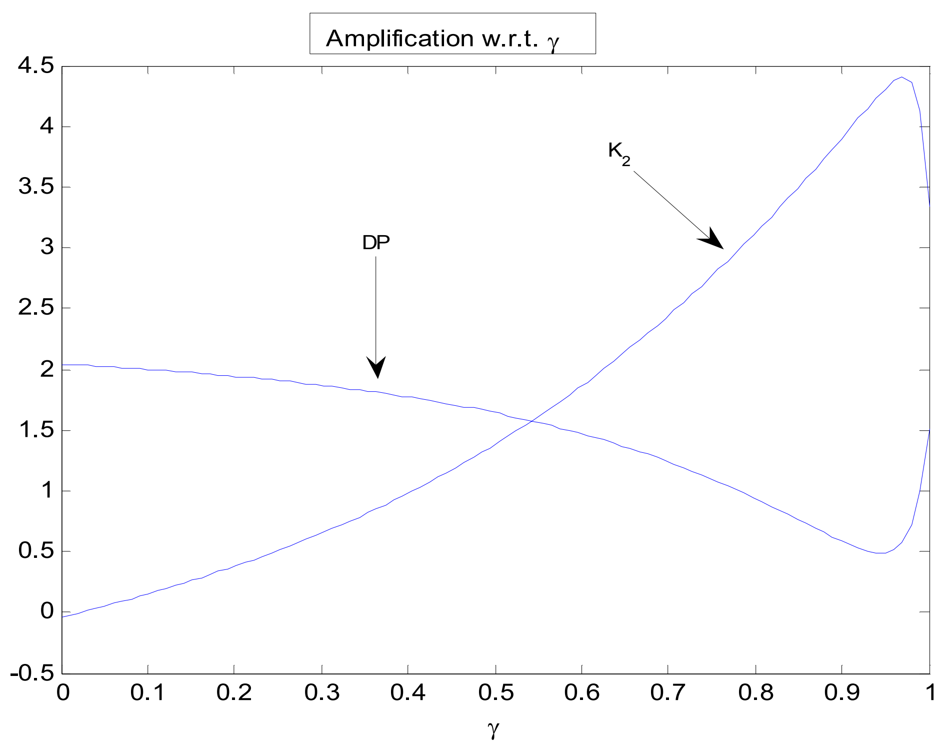

Figure 3.c second period amplification of the shock on production— endogenous reaction to shocks

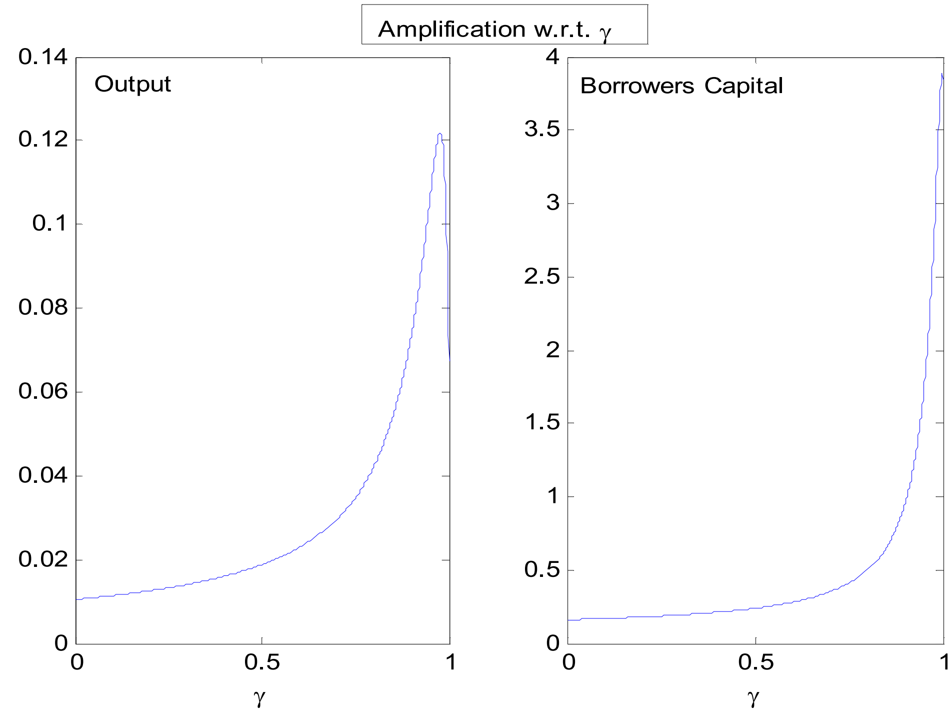

Figure 3.d second period amplification of the shock on production— endogenous reaction to shocks

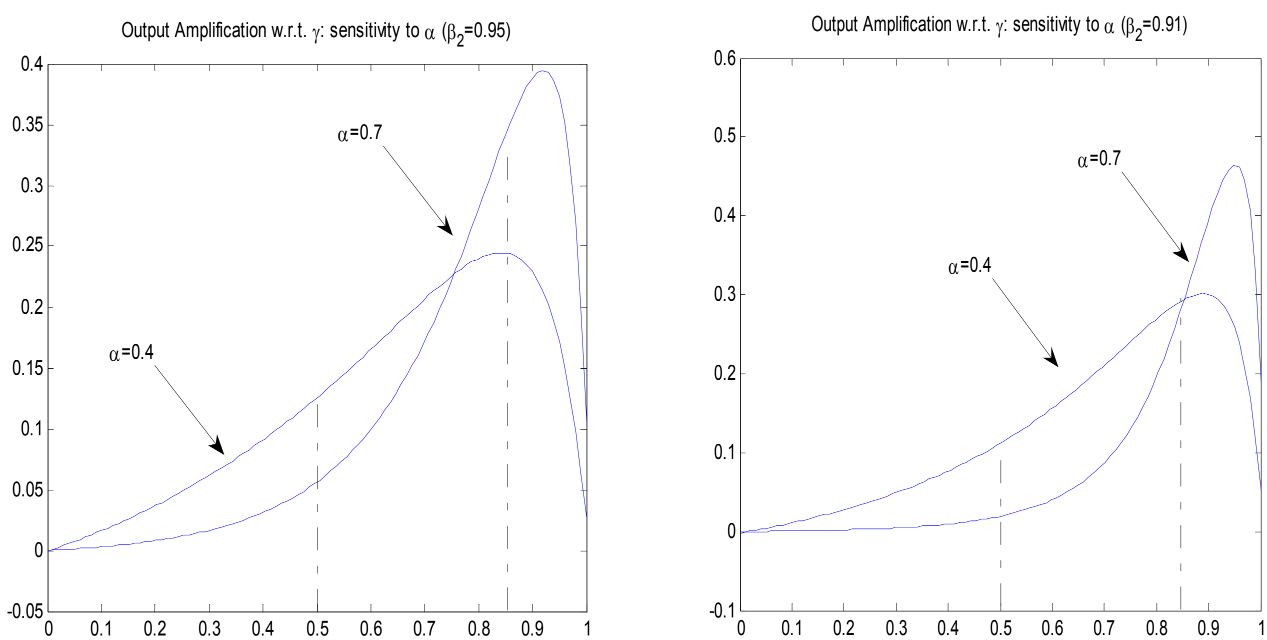

Figure 4. second period amplification of the shock on production— endogenous reaction to shocks

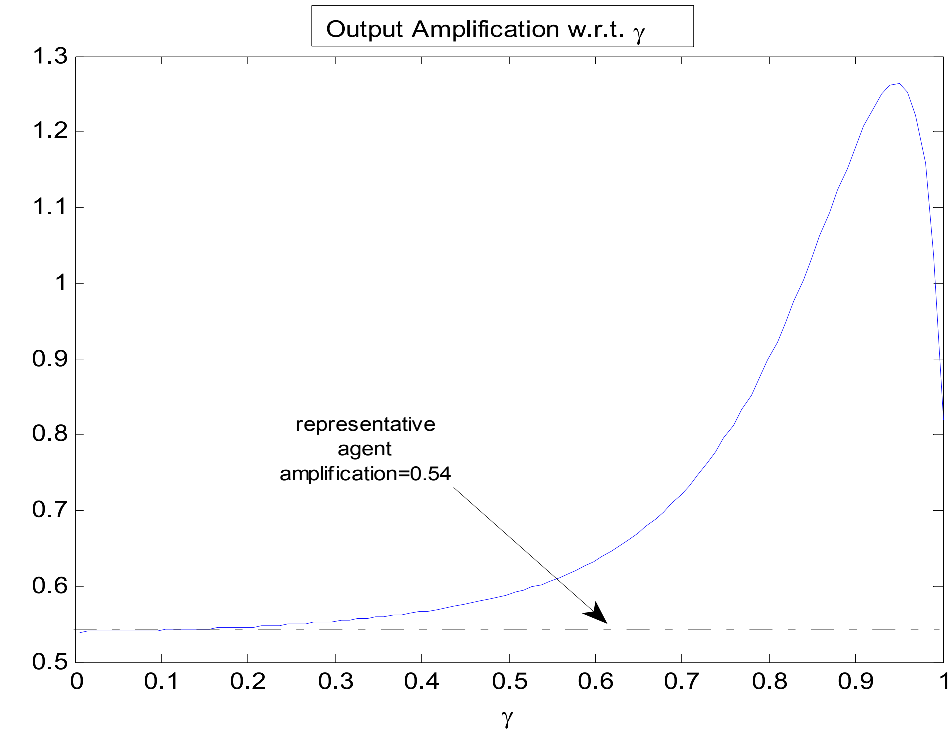

Figure 5.a second period amplification of the shock on production— endogenous reaction to shocks

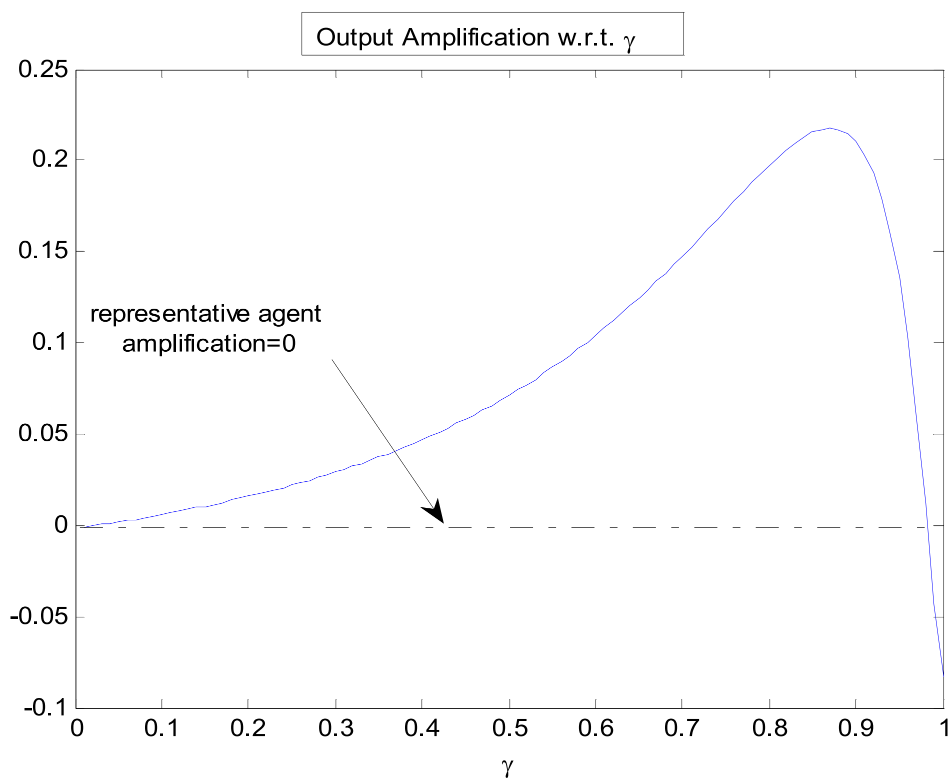

Figure 5.b second period amplification of the shock on production— endogenous reaction to shocks