Auctions design with private costs of valuation discovery

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Abstract This paper extends the pre-bid R&D and auctions design literature to an independent private value setting where each bidder incurs a private-information valuation discovery cost upon entry. The seller commits to a mechanism before the bidders' entry decisions. The main findings are as follows. Firstly, a second-price auction with no entry fee and a reserve price equal to seller's valuation is ex ante efficient. Secondly, a second price auction with the same reserve price and appropriate **ex ante** entry fees is revenue-maximizing. Every bidder's ex ante entry fee equals the hazard rate of his entry cost distribution, evaluated at the desired entry-threshold for him. Thirdly, the revenue-maximizing entry differs from the ex ante efficient entry. Fourthly, even for the symmetric setting, the ex ante efficient/revenue-maximizing entry could be asymmetric. Lastly, for the symmetric setting, when the cumulative distribution function of the entry costs changes rather slowly with respect to its argument, the efficient entry must be symmetric across bidders and it is the unique entry equilibrium of the efficient auction. If the hazard rate of the entry cost distribution is additionally increasing, then the revenue-maximizing entry must also be symmetric and it is the unique entry equilibrium of the revenue-maximizing auction. These results mean that large dispersion in the entry costs restores the symmetry in the efficient/revenue-maximizing entry.

1 Introduction

The impact of valuation discovery costs on bidders' entry decisions and auctions design, has been extensively studied in the pre-bid R&D and auctions design literature. In this paper, valuation discovery costs refer to the costs for bidders to discover their valuations of the auctioned object.[1] Milgrom (1981), French and McCormick (1984), McAfee and McMillan (1987), Engelbrecht-Wiggans (1987, 1993), Harstad (1990), Levin and Smith (1994), Ye (2004) and Lu (2006) among others study the case where bidders' valuation discovery costs are fixed. Matthews (1984), Tan (1992), Bag (1997), Persico (2000) and Bergemann and Välimäki (2002) consider the case where the information quality depends on the bidders' investment. Bergemann and Välimäki (2005) present a thorough review of the literature.

Examples abound however where the bidders' costs on the pre-bid R&D activities such as acquiring and analyzing information are their private knowledge. Piccione and Tan (1996) pointed out that several aspects of the pre-bid R&D process in the Outer Continental Shelf wildcat auctions are private knowledge of an individual bidder. In many procurements of research or construction projects, the bidders have to spend huge amount of resources to estimate their own costs of finishing the project through various pre-bid R&D activities, such as examining the specific requirements of the buyer and investigating local geological conditions, etc. The valuation discovery costs could be the bidders' business secret and these costs could vary significantly across bidders because the levels of their pre-bid R&D activities and/or their efficiency in carrying out these activities can be very different. In this paper, we study a setting where the valuation discovery costs are bidders' private information, and derive the auctions that maximize the expected total

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[1]Many other studies focus on entry costs that are incurred by bidders who know their valuations. These studies include Green and Laffont (1984), Samuelson (1985), Stegeman (1996), Menezes and Monteiro (2000), Lu (2004), Celik and Yilankaya (2005) and Tan and Yilankaya (forthcoming).

surplus and seller's expected revenue, respectively.[2] A general independent private value (IPV) framework allowing asymmetry across bidders is adopted in this study. The seller commits to a mechanism before the bidders' entry decisions.

We first establish useful connections between the first order conditions that characterizes the desired entry thresholds of entry costs and the expected payoff of these threshold types in a second-price auction with no entry fee and a reserve price equal to seller's valuation. These connections show that the above mentioned auction is ex ante efficient, but it is not revenue-maximizing. Instead, a second price auction with the same reserve price and appropriate **ex ante** entry fees is revenue-maximizing. These entry fees are positive and extract all the expected surplus of the entrants of the threshold types. Specifically, these entry fees equal the hazard rates of the bidders' entry cost distributions evaluated at the bidders' entry thresholds. Our findings confirm and generalize the insights from the existing literature. In a private value setting where bidders' information quality depends on their investment, Bag (1997) shows that a sealed-bid second-price auction with ex ante entry fees uniquely implements the first-best outcome and is optimal to the seller. In a more general setting, Bergemann and Välimäki (2002) further show that the Vickrey-Clark-Groves mechanism renders both ex ante and ex post efficiency. Levin and Smith (1994) look at the symmetric mixed-strategy (strictly) entry equilibrium in a symmetric setting with fixed entry costs. They find that the second-price auction with no entry fee and a reserve price equal to seller's valuation is both ex ante efficient and revenue-maximizing for a private value case. Lu (2006) further shows that there is no loss of generality in considering the entry patterns where every bidder participates with probability of either 0 or 1 for the revenue-maximizing (meanwhile ex ante efficient) auction. As a result, while the above mentioned second-price auction remains ex ante efficient, positive ex ante entry fees are generally necessary to extract the surplus of entrants for revenue maximization. According to our findings, the efficiency of the above mentioned

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[2]The setting of private-information discovery costs is also adopted by Rezende (2005).

second price auction and the essentiality of ex ante entry fees for revenue maximization also apply to the setting with private-information entry costs. Moreover, unlike the case with fixed entry costs, the revenue-maximizing entry diverges from the ex ante efficient entry when entry costs are private information. When bidders' entry costs are fixed, the revenue-maximizing entry coincides with the ex ante efficient entry, as the seller can use ex ante entry fees to extract all the expected surplus of the entrants. However, when the entry costs are private information of bidders, the seller can no longer do so. This explains the discrepancy between the revenue-maximizing entry and the ex ante efficient entry, and thus the revenue-maximizing auction must diverge from the ex ante efficient auction.

We find that even for symmetric setting with private-information entry costs, the ex ante efficient/revenue-maximizing entry could be asymmetric rather than symmetric. In the following example, there are 2 potential bidders. Bidders' private values follow a uniform distribution on $[0, 1]$, and bidders' entry costs follow a uniform distribution on $[0.4, 0.5]$. The seller's valuation is 0. In this setting, the expected total surplus takes the maximum of 0.05 when the participation thresholds of entry costs for the 2 bidders are 0.5 and 0.4, respectively; seller's expected revenue takes the maximum of 0.025 when the entry thresholds for the 2 bidders are 0.45 and 0.4, respectively. The intuition behind the asymmetry in the ex ante efficient/revenue-maximizing entry lies in that the marginal contribution of an additional entrant's valuation to the expected total surplus/the seller's expected revenue strictly decreases with the number of other entrants. This implies that given the sum of the ex ante participating probabilities of any two bidders, the marginal contribution of their valuations to the total surplus/the seller's revenue increases as their ex ante participating probabilities diverges. Thus increasing the distance between the entry thresholds of the bidders while maintaining the sum of their ex ante entry probabilities must increase the total surplus/the seller's revenue, provided that this adjustment

in the entry thresholds does not substantially change the expected entry costs.[3]

Two interesting issues then arise. First, can we provide sufficient conditions for the efficient/revenue-maximizing entry to be symmetric in a symmetric setting? Second, can we provide sufficient conditions for the symmetric efficient/revenue-maximizing entry to be the unique equilibrium of the proposed efficient/revenue-maximizing auction? Our findings are the following. When the cumulative distribution function of the entry cost changes rather slowly with respect to its argument, the efficient entry must be symmetric across bidders and it is the unique entry equilibrium of the proposed efficient auction. If the hazard rate of the entry cost distribution is additionally increasing, then the revenue-maximizing entry must also be symmetric and it is the unique entry equilibrium of the proposed revenue-maximizing auction. Therefore, large dispersion in the entry costs restores the symmetry in the efficient/revenue-maximizing entry. This result is in contrast to the existing findings of asymmetric efficient/revenue-maximizing entry in a setting with commonly known costs or where the quality of the bidders' information depends on their investment. Bag (1996) shows that the efficient/revenue-maximizing investment decisions can be asymmetric in a symmetric setting where the quality of the bidders' information quality depends on their investment. Lu (2006) in a setting with commonly known costs shows that when there are sufficiently many bidders, the efficient/revenue-maximizing entry must be an asymmetric one where every bidder participates with probability of 1 or 0.[4]

This paper is organized as follows. In Section 2, we consider a general IPV setting where potential bidders have different distributions on both valuations and valuation discovery costs. The ex ante efficient auction and revenue-maximizing auction are established. In Section 3, we focus on issues in the symmetric IPV setting, where potential

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[3]This condition holds when the ranges of private entry costs are rather small, especially when entry costs are fixed as in Lu (2006).

[4]Lu (2004) and Celik and Yilankaya (2005) find that asymmetric revenue-maximizing entry may also arise in a symmetric setting where fixed entry costs are incurred by bidders who know their valuations.

bidders share identical distributions on valuations and valuation discovery costs. We show that the efficient/revenue-maximizing entry can be asymmetric. We further provide sufficient conditions for the efficient/revenue-maximizing entry to be symmetric and for the symmetric entry to be the unique equilibrium of the proposed efficient/revenue-maximizing auction. Section 4 concludes.

2   Auctions Design under General IPV Setting

There are $N(\geq 2)$ potential bidders who are interested in a single item, where N is public information. Denote this group of potential bidders by $\mathcal{N} = \{1, 2, ..., N\}$. The seller's valuation is v_0, which is public information. Bidder i has to incur an entry cost of c_i in order to enter the auction.[5] After entry, he observes his private value v_i. Both c_i and v_i are assumed to be private information of bidder i. The cumulative distribution function of c_i is $G_i(c_i)$ with density function of $g_i(c_i)$, while the cumulative distribution function of v_i is $F_i(v_i)$ with density function of $f_i(v_i)$. The support of $G_i(c_i)$ is $[\underline{c}_i, \overline{c}_i]$, and the support of $F_i(v_i)$ is $[\underline{v}_i, \overline{v}_i]$. We assume $g_i(\cdot) > 0$ on its support. The distributions of c_i and v_i, $i \in \mathcal{N}$ are assumed to be public information. The entry costs can be interpreted as the bidders' efficiency in discovering their valuations. In this paper, we study a setting where the bidders' valuations do not depend on their efficiency in discovering their valuations. Specifically, we assume c_i and v_j, $\forall i, j \in \mathcal{N}$ are mutually independent. The seller and bidders are assumed to be risk neutral. The timing of the auction is as follows.

Time 0: The group of potential bidders $\mathcal{N}$, the seller's valuation v_0 and the distributions $F_i(\cdot)$, $G_i(\cdot)$, $i \in \mathcal{N}$ are revealed by Nature as public information. Every bidder i observes his private cost c_i, $i \in \mathcal{N}$.

Time 1: The seller announces the rule of the auction. We assume that the seller has

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[5]This assumption is widely adopted in the literature. However, this assumption precludes the possibility that a bidder may simply submit a bid equal to the unconditional expected valuation.

the power of committing to his announcement.

Time 2: The bidders simultaneously and confidentially make their entry decisions. If they do not enter, they simply take the outside option which gives them zero payoff. If they enter, they have to incur their private entry costs. If the seller announces an **ex ante** entry fee for bidder i, he has to also pay this entry fee to the seller upon entry. All entrants observe their private values after entry.

Time 3: All entrants bid.[6] If no one participates, the game is over.

Time 4: The payoffs of the seller and all the participating bidders are determined according to the announced rule at time 1.

We study the ex ante efficient auction rule and the revenue-maximizing auction rule announced at time 1. Here, the **ex ante efficient auction** refers to the auction maximizing the expected total surplus of seller and bidders; and the **revenue-maximizing auction** refers to the auction maximizing the expected revenue of the seller. We assume that the seller can impose **ex ante** entry fees which the entrants must pay to the seller before they observe their valuations. These **ex ante** entry fees differ from the entry fees that the seller may impose for the bidding stage. In this paper, only the **ex ante** entry fees are relevant.

Before we proceed to consider the auctions design, we first characterize all the feasible equilibrium entry patterns.

Lemma 1: *Any equilibrium entry pattern can be described through a vector of entry thresholds* $\mathbf{C^e} = (c_1^e, ..., c_N^e)$ *satisfying the following properties: (i)* $c_i^e \in [\underline{c}_i, \overline{c}_i]$, $\forall i \in \mathcal{N}$; *(ii) if* $c_i < c_i^e$, *bidder* i *participates with probability 1; if* $c_i > c_i^e$, *bidder* i *participates with probability 0.*

Proof: See Appendix.

Given entry thresholds $\mathbf{C^e}$ where $c_i^e \in [\underline{c}_i, \overline{c}_i]$, $\forall i \in \mathcal{N}$, we assume ($i$) if $c_i^e > \underline{c}_i$, bidder

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[6]Every entrant may or may not observe the other participants. The auctions designed later work in both cases.

i participates if and only if $c_i \leq c_i^e$; (ii) if $c_i^e = \underline{c}_i$, no type of bidder i participates. This simplification is reasonable, because if $c_i^e > \underline{c}_i$ bidder i with cost c_i^e at least weakly prefers participation, and if $c_i^e = \underline{c}_i$ then bidder i with cost c_i^e at least weakly prefers nonparticipation. Moreover, this simplification only further specifies the participation of the threshold type c_i^e. The expected total surplus and seller's expected revenue are not affected. For convenience, we define the following auction, which will be referred to frequently hereafter.

Definition 1: *We define $\mathcal{A}_0$ as the second-price auction with no entry fee and a reserve price equal to the seller's valuation v_0.*

Next, we establish the following results regarding the restricted ex ante efficient auction/revenue-maximizing auction that implements given entry thresholds $\mathbf{C^e} = (c_1^e, ..., c_N^e)$, where $c_i^e \in [\underline{c}_i, \overline{c}_i]$, $\forall i \in \mathcal{N}$.

Proposition 1: (i) *Among all auctions implementing any given entry thresholds $\mathbf{C^e} = (c_1^e, ..., c_N^e)$, a second-price auction with a reserve price equal to seller's valuation and appropriate* **ex ante** *entry fee (or subsidy) for every bidder provides the highest seller's expected revenue as well as the highest expected total surplus.* (ii) *The ex ante entry fees (or subsidies) are charged upon entry* **before** *the valuations are learned by the entrants, and are set at levels such that the threshold-type entrants get zero expected payoff.* (iii) *In the above auction, the expected surplus of bidder i with entry cost $c_i(< c_i^e)$ is $c_i^e - c_i$.*

Proof: See Appendix.

For given entry thresholds $\mathbf{C^e}$ where $c_i^e \in [\underline{c}_i, \overline{c}_i]$, $\forall i \in \mathcal{N}$, we denote the highest expected total surplus and the highest seller's expected revenue attainable through the Proposition 1 auction by $S(\mathbf{C^e})$ and $R(\mathbf{C^e})$, respectively. We next introduce a convenient way of writing $S(\mathbf{C^e})$ and $R(\mathbf{C^e})$.

We define set $\mathcal{K} = \{(k_1, k_2, ..., k_N)|k_i \in \{0, 1\}, \ i \in \mathcal{N}\}$, where k_i denotes bidder i's ex post entry status. Specifically, $k_i = 1$ stands for the participation of bidder i, while $k_i = 0$ represents the non-participation of bidder i. In addition, $k_0 \equiv 1$ symbolizes the

participation of the seller. For any $\mathbf{k} = (k_1, k_2, ..., k_N) \in \mathcal{K}$, use $v_{\mathbf{k}}$ to denote the highest valuation of all ex post participants including the seller. Then $v_{\mathbf{k}}$ can be written as $v_{\mathbf{k}} = \max_{\{k_j = 1, 0 \leq j \leq N\}}\{v_j\}$. We use $f_{\mathbf{k}}(v_{\mathbf{k}})$ and $F_{\mathbf{k}}(v_{\mathbf{k}})$ to denote the density and cumulative distribution function of $v_{\mathbf{k}}$, respectively. Furthermore, we use $V_{\mathbf{k}}$ to denote the expectation of $v_{\mathbf{k}}$. According to Proposition 1, $S(\mathbf{C^e})$ and $R(\mathbf{C^e})$ can be written as the following

$$S(\mathbf{C^e}) = \Big(\sum_{\{\mathbf{k} \in \mathcal{K}\}} V_{\mathbf{k}} Pr(\mathbf{k}) \Big) - \sum_{i \in \mathcal{N}} \int_{\underline{c}_i}^{c_i^e} c_i g_i(c_i) dc_i, \tag{1}$$

$$R(\mathbf{C^e}) = \Big(\sum_{\{\mathbf{k} \in \mathcal{K}\}} V_{\mathbf{k}} Pr(\mathbf{k}) \Big) - \sum_{i \in \mathcal{N}} c_i^e G_i(c_i^e), \tag{2}$$

where $Pr(\mathbf{k}) = \prod_{i \in \mathcal{N}}(G_i(c_i^e))^{k_i}(1 - G_i(c_i^e))^{1-k_i}$ is the ex ante probability that the ex post participation status denoted by $\mathbf{k}$ happens. The term $\sum_{\{\mathbf{k} \in \mathcal{K}\}} V_{\mathbf{k}} Pr(\mathbf{k})$ is the ex ante expected contribution of the valuations of all players including the seller to the expected total surplus if potential bidders participate according to threshold-vector $\mathbf{C^e}$. The term $\sum_{i \in \mathcal{N}} \int_{\underline{c}_i}^{c_i^e} c_i g_i(c_i) dc_i$ is the ex ante expected contribution (negative) of the entry costs of bidders to the expected total surplus if the potential bidders participate according to thresholds $\mathbf{C^e}$. The difference between these two terms is then the expected total surplus $S(\mathbf{C^e})$. Following Proposition 1(iii), we know that the ex ante expected information rents of bidder i is $\int_{\underline{c}_i}^{c_i^e}(c_i^e - c_i)g_i(c_i)dc_i$. This leads to the seller's expected revenue $R(\mathbf{C^e})$ in (2), which is the difference between the expected total surplus $S(\mathbf{C^e})$ and all the bidders' ex ante expected information rents $\sum_{i \in \mathcal{N}} \int_{\underline{c}_i}^{c_i^e}(c_i^e - c_i)g_i(c_i)dc_i$.

2.1 Ex Ante Efficient Auction

We derive the ex ante efficient auction through two steps. First, we characterize the first order conditions for the ex ante efficient thresholds $\mathbf{C^{e*}} = (c_1^{e*}, ..., c_N^{e*})$, which maximize $S(\mathbf{C^e})$. Second, we show that auction $\mathcal{A_0}$ implements $\mathbf{C^{e*}}$ and achieves $S(\mathbf{C^{e*}})$.

First, we characterize the first order conditions for the efficient threshold-vector $\mathbf{C^{e*}} = (c_1^{e*}, ..., c_N^{e*})$ that maximizes $S(\mathbf{C^e})$. Let us consider the entry threshold c_i^{e*} for bidder

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i, $\forall i \in \mathcal{N}$. Define $\mathcal{K}_{-i} = \{(k_1, ..., k_{i-1}, k_{i+1}, ..., k_N)|k_j \in \{0,1\}, \ j \neq i.\}$, $\forall i \in \mathcal{N}$. For any $\mathbf{k}_{-i} = (k_1, ..., k_{i-1}, k_{i+1}, ..., k_N) \in \mathcal{K}_{-i}$, we use $\mathbf{k}_1(\mathbf{k}_{-i})$ to denote the N-element vector where the i-th element is 1 and other elements are same with $\mathbf{k}_{-i}$, while we use $\mathbf{k}_0(\mathbf{k}_{-i})$ to denote the N-element vector where the i-th element is 0 and other elements are same with $\mathbf{k}_{-i}$. We then have

$$S(\mathbf{C^e}) = \sum_{\{\mathbf{k}_{-i} \in \mathcal{K}_{-i}\}} Pr(\mathbf{k}_{-i})[G_i(c_i^e)V_{\mathbf{k}_1(\mathbf{k}_{-i})} + (1 - G_i(c_i^e))V_{\mathbf{k}_0(\mathbf{k}_{-i})}] - \sum_{i \in \mathcal{N}} \int_{\underline{c}_i}^{c_i^e} c_i g_i(c_i) dc_i,$$

where $Pr(\mathbf{k}_{-i}) = \prod_{j \neq i} G_j(c_j^e)^{k_j}(1 - G_j(c_j^e))^{1-k_j}$ is the ex ante probability that the ex post participation status denoted by $\mathbf{k}_{-i}$ happens. This leads to

$$\frac{\partial S(\mathbf{C^e})}{\partial c_i^e} = g_i(c_i^e) \sum_{\{\mathbf{k}_{-i} \in \mathcal{K}_{-i}\}} [(V_{\mathbf{k}_1(\mathbf{k}_{-i})} - V_{\mathbf{k}_0(\mathbf{k}_{-i})} - c_i^e)Pr(\mathbf{k}_{-i})], \tag{3}$$

as $\sum_{\{\mathbf{k}_{-i} \in \mathcal{K}_{-i}\}} Pr(\mathbf{k}_{-i}) = 1$.

Since $\mathbf{C^{e*}}$ maximizes $S(\mathbf{C^e})$, then we must have the following characterization for $\mathbf{C^{e*}}$. For all $i \in \mathcal{N}$,

$$\frac{\partial S(\mathbf{C^{e*}})}{\partial c_i^e} = \begin{cases} 0, & \text{if} \quad c_i^{e*} \in (\underline{c}_i, \overline{c}_i), \\ \geq 0, & \text{if} \quad c_i^{e*} = \overline{c}_i, \\ \leq 0, & \text{if} \quad c_i^{e*} = \underline{c}_i. \end{cases} \tag{4}$$

Clearly, (4) are only the necessary conditions for the efficient entry. In Section 3.3, we will provide sufficient conditions characterizing the efficient entry in a symmetric setting.

Define $S_i(\mathbf{C^e}) = \sum_{\{\mathbf{k}_{-i} \in \mathcal{K}_{-i}\}} [(V_{\mathbf{k}_1(\mathbf{k}_{-i})} - V_{\mathbf{k}_0(\mathbf{k}_{-i})} - c_i^e)Pr(\mathbf{k}_{-i})]$. This term on the right hand side of (3) is the marginal contribution of bidder i with entry cost c_i^e to the expected total surplus, given that other bidders participate in auction $\mathcal{A}_0$ according to $\mathbf{C^e}$. Before we move forward, we first show the following Lemma.

Lemma 2: $V_{\mathbf{k}_1(\mathbf{k}_{-i})} - V_{\mathbf{k}_0(\mathbf{k}_{-i})}$ *is the expected payoff of bidder i with zero entry cost from participating in auction $\mathcal{A}_0$, if all other participants are those bidders with $k_j = 1$ in vector* $\mathbf{k}_{-i}$.

Proof: See Appendix.

It then follows from Lemma 2 that $S_i(\mathbf{C^e})$ is the expected payoff of bidder i with cost c_i^e when he participates in auction $\mathcal{A}_\mathbf{0}$, when all other potential bidders participate according to $\mathbf{C^e}$. This insight, together with (3) and (4), leads to the following proposition which addresses the ex ante efficient auction.

Proposition 2: *The second-price auction $\mathcal{A}_\mathbf{0}$ is ex ante efficient.*

Proof: Since $g_i(\cdot) > 0$, it is clearly a Nash equilibrium that every bidder participates in auction $\mathcal{A}_\mathbf{0}$ according to $\mathbf{C^{e*}}$, as (4) is satisfied for $\mathbf{C^{e*}}$. In addition, $\mathcal{A}_\mathbf{0}$ clearly renders an expected total surplus of $S(\mathbf{C^{e*}})$. □

Proposition 2 shows that $\mathcal{A}_\mathbf{0}$ is ex ante efficient in a more general environment than the fixed costs settings of Levin and Smith (1994) and Lu (2006). Proposition 2 accommodates the flexibility of corner solutions, as indicated by (4). An example of corner solution is provided in the following symmetric setting, where $v_0 = 0$, $N = 2$, $F_i(v_i) = v_i$, $\forall v_i \in [0, 1]$, and $G_i(c_i) = 10(c_i - 0.4)$, $\forall c_i \in [0.4, 0.5]$. In this setting, $S(\mathbf{C^e})$ takes the maximum of 0.05 when $c_1^e = 0.5$ and $c_2^e = 0.4$. This means that one bidder always participates, while the other one never participates.

In addition, in the above example there exists another symmetric entry equilibrium where $c_1^e = c_2^e = 0.4231$ for $\mathcal{A}_\mathbf{0}$. Thus, an issue of multiplicity of entry equilibria for the efficient auction $\mathcal{A}_\mathbf{0}$ arises. The multiplicity of entry equilibria means that $\mathcal{A}_\mathbf{0}$ can be efficient or not depending on the entry equilibrium the bidders play. In Section 3.3, we will address this issue in a symmetric setting.

The following Corollary further discusses the uniqueness of ex ante entry fees that implement the efficient entry $\mathbf{C^{e*}}$ in a second price auction with a reserve price equal to the seller's valuation.

Corollary 1: *Suppose $\mathbf{C^{e*}}$ is the efficient entry to be implemented. (i) A zero entry fee for every bidder implements $\mathbf{C^{e*}}$. (ii) If $c_i^{e*} \in (\underline{c}_i, \overline{c}_i)$, then the ex ante entry fee for bidder i cannot be other than zero. (iii) If $c_i^{e*} = \overline{c}_i$, the ex ante entry fee for bidder i can be set*

at any level which is smaller than or equal to $S_i(\mathbf{C^{e*}})$ which is nonnegative. No other ex ante entry fees works. (iv) If $c_i^{e*} = \underline{c}_i$, the ex ante entry fee for bidder i can be set at any level which is greater than or equal to $S_i(\mathbf{C^{e*}})$ which is nonpositive. No other ex ante entry fees works.

Proof: See Appendix.

2.2   Revenue-Maximizing Auction

We now study the revenue-maximizing auction. From (1), (2) and (3), we have

$$\frac{\partial R(\mathbf{C^e})}{\partial c_i^e} = g_i(c_i^e)[S_i(\mathbf{C^e}) - \frac{G_i(c_i^e)}{g_i(c_i^e)}], \ \forall i \in \mathcal{N}. \tag{5}$$

Suppose that $\mathbf{C^{e\dagger}} = (c_1^{e\dagger}, ..., c_N^{e\dagger})$ maximizes $R(\mathbf{C^e})$, then we have the following characterization for $\mathbf{C^{e\dagger}}$. For all $i \in \mathcal{N}$,

$$\frac{\partial R(\mathbf{C^{e\dagger}})}{\partial c_i^e} = \begin{cases} 0, & \text{if} \quad c_i^{e\dagger} \in (\underline{c}_i, \overline{c}_i), \\ \geq 0, & \text{if} \quad c_i^{e\dagger} = \overline{c}_i, \\ \leq 0, & \text{if} \quad c_i^{e\dagger} = \underline{c}_i. \end{cases} \tag{6}$$

Clearly, (6) are only the necessary conditions for the revenue-maximizing entry. In Section 3.3, we will provide sufficient conditions characterizing the revenue-maximizing entry in a symmetric setting.

Define $R_i(\mathbf{C^e}) = S_i(\mathbf{C^e}) - \frac{G_i(c_i^e)}{g_i(c_i^e)}$. Based on Lemma 2, we have that $R_i(\mathbf{C^{e\dagger}})$ is the expected payoff of bidder i with cost c_i^e, if he participates in a second-price auction with a reserve price equal to v_0 and an ex ante entry fee of $\frac{G_i(c_i^e)}{g_i(c_i^e)}$ for bidder i, provided that all other potential bidders participate according to $\mathbf{C^{e\dagger}}$.[7] Based on this insight, we obtain from (5) and (6) the following proposition that addresses the revenue-maximizing auction.

Proposition 3: *Suppose that $\mathbf{C^{e\dagger}}$ maximizes $R(\mathbf{C^e})$, then a second-price auction with reserve price equal to seller's valuation and ex ante entry fees E_i for bidder i defined*

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[7]As pointed out in Proposition 1, the ex ante entry fees are charged before the valuations are learned by the entrants.

*below leads to the seller the highest expected revenue. The entrants pay their ex ante entry
fees before their valuations are learned. The ex ante entry fees E_i, $i \in \mathcal{N}$ are defined as*

$$E_i = \begin{cases} S_i(\mathbf{C^{e\dagger}}) = \frac{G_i(c_i^{e\dagger})}{g_i(c_i^{e\dagger})}, & \text{if } c_i^{e\dagger} \in (\underline{c}_i, \overline{c}_i), \\ S_i(\mathbf{C^{e\dagger}}) \geq \frac{1}{g_i(\overline{c}_i)}, & \text{if } c_i^{e\dagger} = \overline{c}_i, \\ \text{any number} \geq S_i(\mathbf{C^{e\dagger}})(\leq 0), & \text{if } c_i^{e\dagger} = \underline{c}_i. \end{cases} \tag{7}$$

Proof: Since $g_i(\cdot) > 0$, it is clearly a Nash equilibrium that every bidder participates in
the above defined auction according to $\mathbf{C^{e\dagger}}$, while (6) and (7) hold.

From Proposition 1(ii), if $c_i^{e\dagger} > \underline{c}_i$, the entry fees should be set at the levels such that
the threshold types get zero expected payoff. Therefore, E_i should be $S_i(\mathbf{C^{e\dagger}})$ if $c_i^{e\dagger} > \underline{c}_i$.
From (5) and (6), $S_i(\mathbf{C^{e\dagger}}) = \frac{G_i(c_i^{e\dagger})}{g_i(c_i^{e\dagger})}$ if $c_i^{e\dagger} \in (\underline{c}_i, \overline{c}_i)$, and $S_i(\mathbf{C^{e\dagger}}) \geq \frac{1}{g_i(c_i^{e\dagger})}$ if $c_i^{e\dagger} = \overline{c}_i$.
If $c_i^{e\dagger} = \underline{c}_i$, any entry fee which is bigger than $S_i(\mathbf{C^{e\dagger}})(\leq 0)$ implements the threshold
participation. $\square$

From Proposition 3, if the entry cost is private information of bidders, then essentially
the revenue-maximizing auction involves positive individual ex ante entry fees unless
$c_i^e = \underline{c}_i, \forall i \in \mathcal{N}$. When $c_i^e = \underline{c}_i, \forall i \in \mathcal{N}$, we have a degenerate case where it is inefficient
for any bidder to participate in any chance, i.e., $\int_{v_0}^{\overline{v}_i}(v_i - v_0)f_i(v_i)dv_i \leq \underline{c}_i, \forall i \in \mathcal{N}$.

Similar to the case of efficient auction, there may exist multiple entry equilibria for
the revenue-efficient auction of Proposition 3. The multiplicity of entry equilibria means
that the Proposition 3 auction can be revenue-maximizing or not depending on the entry
equilibrium the bidders play. In Section 3.3, we will address this issue in a symmetric
setting.

The following Corollary further discusses the uniqueness of the revenue-maximizing
ex ante entry fees that implement the entry $\mathbf{C^{e\dagger}}$ in a second price auction with a reserve
price equal to the seller's valuation.

Corollary 2: *Suppose $\mathbf{C^{e\dagger}}$ is the revenue-maximizing entry to be implemented. (i) If
$c_i^{e\dagger} \in (\underline{c}_i, \overline{c}_i)$, then the ex ante revenue-maximizing entry fee for bidder i cannot be other*

than $\frac{G_i(c_i^{e\dagger})}{g_i(c_i^{e\dagger})}$. (ii) If $c_i^{e\dagger} = \bar{c}_i$, the revenue-maximizing ex ante entry fee for bidder i must equal $S_i(\mathbf{C^{e\dagger}})$ which is higher than $\frac{1}{g_i(c_i^{e\dagger})}$. (iii) If $c_i^{e\dagger} = \underline{c}_i$, the revenue-maximizing ex ante entry fee for bidder i can be set at any level which is greater than or equal to $S_i(\mathbf{C^{e\dagger}})$ which is nonpositive. No other ex ante entry fees works.

Proof: See Appendix.

Based on Corollary 2, we further emphasize the following properties of the ex ante entry fees in the revenue-maximizing auctions.

Corollary 3: (i) *There is no loss of generality to consider only nonnegative ex ante entry fees for the revenue-maximizing auction.* (ii) *The ex ante entry fees for any entrant in the revenue-maximizing auction must be positive.*

Conditions (4) and (6) mean that if the entry costs are private information of the bidders, then the ex ante efficient entry generally differs from the revenue-maximizing entry. In contrast, when the entry costs are fixed, the ex ante efficient entry is also revenue-maximizing, although the ex ante entry fees can differ across the ex ante efficient auction and the revenue-maximizing auction. The intuition behind this contrast is as follows. If the entry costs are fixed, the seller can always extract all the expected surplus of the participants. Thus the entry pattern maximizing expected total surplus also maximizes the seller's expected revenue. However, if the entry costs are private information of the bidders, then the seller has no way to extract all the surplus of the participants according to Proposition 1(iii). This leads to the discrepancy between the ex ante efficient entry and the revenue-maximizing entry when entry costs are private information of the bidders. It follows that the revenue-maximizing auction must generally be different from the ex ante efficient auction.

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3 Further Issues in Symmetric IPV Setting

In Section 2, we consider the unrestricted ex ante efficient and revenue-maximizing auctions in a general IPV setting. In this section, we further study some special issues for a symmetric setting where the distributions of the bidders' valuations and entry costs are the same across all potential bidders. The common cumulative distribution function of all c_i, $i \in \mathcal{N}$ is $G(\cdot)$ with density function of $g(\cdot)$. The common cumulative distribution function of all v_i, $i \in \mathcal{N}$ is $F(\cdot)$ with density function of $f(\cdot)$. The support of $G(\cdot)$ is $[\underline{c}, \overline{c}]$, and the support of $F(\cdot)$ is $[\underline{v}, \overline{v}]$. We assume $v_0 < \overline{v}$ and $g(\cdot) > 0$ on its support.

Clearly, all the findings in Section 2 apply to the above specified symmetric IPV setting. In this section, we investigate some special issues which are unique to the above symmetric setting. We first establish the efficient and revenue-maximizing auctions within the symmetric-entry class. We then show that even for the symmetric setting, the ex ante efficient/revenue-maximizing entry can be asymmetric, i.e., the desired participation thresholds are different across bidders. To address the issues of multiplicity of entry equilibria and the optimality of asymmetric entry, we further establish general sufficient conditions for the optimality and uniqueness of symmetric entry. According to our results, the more dispersed the distribution of the entry costs, the more likely we have symmetric efficient and revenue-maximizing entries. These results justify the conventional wisdom of looking at only the symmetric entries for efficient/revenue-maximizing auction.

3.1 Efficient and Revenue-Maximizing Auctions in Symmetric-Entry Class

Symmetric entry across bidders implies that the thresholds c_i^e are same across all potential bidders. Suppose $c_i^e = c^e \in [\underline{c}, \overline{c}], \forall i \in \mathcal{N}$. We define $S_s(c^e) = S(\mathbf{C_s^e})$ and $R_s(c^e) = R(\mathbf{C_s^e})$,

where $\mathbf{C_s^e} = (c^e, ..., c^e)$. With this restriction, we have

$$\frac{dS_s(c^e)}{dc^e} = \sum_{\{i \in \mathcal{N}\}} \frac{\partial S(\mathbf{C_s^e})}{\partial c_i^e} = N \frac{\partial S(\mathbf{C_s^e})}{\partial c_i^e}, \ \forall i \in \mathcal{N}, \tag{8}$$

$$\frac{dR_s(c^e)}{dc^e} = \sum_{\{i \in \mathcal{N}\}} \frac{\partial R(\mathbf{C_s^e})}{\partial c_i^e} = N \frac{\partial R(\mathbf{C_s^e})}{\partial c_i^e}, \ \forall i \in \mathcal{N}. \tag{9}$$

(8) and (9) lead to

$$\frac{\partial S(\mathbf{C_s^e})}{\partial c_i^e} = \frac{dS_s(c^e)}{dc^e}/N, \ \forall i \in \mathcal{N}, \tag{10}$$

$$\frac{\partial R(\mathbf{C_s^e})}{\partial c_i^e} = \frac{dR_s(c^e)}{dc^e}/N, \ \forall i \in \mathcal{N}. \tag{11}$$

Suppose that c^{e*} maximizes $S_s(c^e)$ and $c^{e\dagger}$ maximizes $R_s(c^e)$. Define $\mathbf{C_s^{e*}} = (c^{e*}, ..., c^{e*})$ and $\mathbf{C_s^{e\dagger}} = (c^{e\dagger}, ..., c^{e\dagger})$. Then we have the following characterizations for c^{e*} and $c^{e\dagger}$ from (10) and (11). For all $i \in \mathcal{N}$,

$$S_i(\mathbf{C_s^{e*}}) = \frac{\partial S(\mathbf{C_s^{e*}})}{\partial c_i^e}/g(c^{e*}) = \begin{cases} 0, & \text{if} \quad c^{e*} \in (\underline{c}, \overline{c}), \\ \geq 0, & \text{if} \quad c^{e*} = \overline{c}, \\ \leq 0, & \text{if} \quad c^{e*} = \underline{c}, \end{cases} \tag{12}$$

and

$$R_i(\mathbf{C_s^{e\dagger}}) = \frac{\partial R(\mathbf{C_s^{e\dagger}})}{\partial c_i^e}/g(c^{e\dagger}) = \begin{cases} 0, & \text{if} \quad c^{e\dagger} \in (\underline{c}, \overline{c}), \\ \geq 0, & \text{if} \quad c^{e\dagger} = \overline{c}, \\ \leq 0, & \text{if} \quad c^{e\dagger} = \underline{c}. \end{cases} \tag{13}$$

$S_i(\mathbf{C_s^{e*}})$ is the expected payoff of bidder i with cost c^{e*} when he participates in auction $\mathcal{A}_0$ if all other potential bidders participate according to threshold c^{e*}. $R_i(\mathbf{C_s^{e\dagger}})$ is the expected payoff of bidder i with cost $c^{e\dagger}$ when he participates in a second price auction with an ex ante entry fee of $\frac{G(c^{e\dagger})}{g(c^{e\dagger})}$ and a reserve price equal to v_0, if all other potential bidders participate according to threshold $c^{e\dagger}$. Thus, (12) and (13) lead to the following results. The proofs are similar to those of Propositions 2 and 3 and thus omitted.

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Proposition 4: (*i*) *In a symmetric IPV setting with private-information entry costs for bidders, the second-price auction $\mathcal{A}_0$ is ex ante efficient in the symmetric-entry class.* (*ii*) *Suppose $c^{e\dagger}$ maximizes $R_s(c^e)$, then a second-price auction with a reserve price equal to seller's valuation and an ex ante entry fee, E, defined below maximizes the seller's expected revenue in the symmetric-entry class. The ex ante entry fee, E, is defined as*

$$
E = \begin{cases}
S_i(\mathbf{C_s^{e\dagger}}) = \frac{G(c^{e\dagger})}{g(c^{e\dagger})}, & \text{if} \quad c^{e\dagger} \in (\underline{c}, \overline{c}), \\
S_i(\mathbf{C_s^{e\dagger}}) \geq \frac{1}{g(\overline{c})}, & \text{if} \quad c^{e\dagger} = \overline{c}, \\
\text{any number} \geq S_i(\mathbf{C_s^{e\dagger}})(\leq 0), & \text{if} \quad c^{e\dagger} = \underline{c}.
\end{cases}
\tag{14}
$$

From (12) and (13), $\mathbf{C_s^{e*}}$ and $\mathbf{C_s^{e\dagger}}$ must be at least locally efficient and revenue-maximizing, respectively. In Section 3.3, we will provide sufficient conditions for them to be globally efficient and revenue-maximizing, respectively. Furthermore, in our analysis above only symmetric entry equilibria are in the feasible set. However, there may exist multiple entry equilibria for the Proposition 4 auctions as pointed out in Sections 2.1 and 2.2. The multiplicity of entry equilibria means that the Proposition 4 auctions can be efficient/revenue-maximizing or not depending on the entry equilibrium the bidders play. This issue of multiplicity of entry equilibria will be partially sorted out later in Section 3.3.

Proposition 4 shows that restricting the entry to be symmetric across bidders does not change the main ideas of Propositions 2 and 3. Similar to Corollaries 1 and 2, the uniqueness of the ex ante entry fee schedule which implements the ex ante efficient/revenue-maximizing entry can be discussed.

3.2   Asymmetry in the Efficient/Revenue-Maximizing Entry

Consider a symmetric setting where $v_0 = 0$, $N = 2$, $F(v) = v$, $\forall v \in [0,1]$, and $G(c) = 10(c - 0.4)$, $\forall c \in [0.4, 0.5]$. Direct calculations using (1) and (2) give the following results. $S(\mathbf{C^e})$ takes the maximum of 0.05 when $c_1^e = 0.5$ and $c_2^e = 0.4$, and

$R(\mathbf{C^e})$ takes the maximum of 0.025 when $c_1^e = 0.45$ and $c_2^e = 0.4$. If we restrict $c_1^e = c_2^e$, then we have $S(\mathbf{C^e})$ takes the maximum of 0.023 when $c_1^e = c_2^e = 0.4231$, and $R(\mathbf{C^e})$ takes the maximum of 0.01875 when $c_1^e = c_2^e = 0.4187$. In other words, the ex ante efficient/revenue-maximizing entry is asymmetric. This example indicates that "symmetric" entry is generally restrictive for auctions design.

Define W_n as the expectation of the highest valuation of the seller and $n(\geq 0)$ symmetric bidders. The following Lemma provides some properties of the series W_n, $n \geq 0$.

Lemma 3: *Both* $W_n - W_{n-1}$ *and* $(W_{n+1} - W_n) - (W_{n+2} - W_{n+1})$ *decrease with* $n(\geq 0)$.

Proof: See Appendix.

The intuitions behind the asymmetry of the efficient/revenue-maximizing entry are as follows. Let us consider the case with 2 potential bidders (N=2). From (1) and (2), the first common component of $S(c_1^e, c_2^e)$ and $R(c_1^e, c_2^e)$ can be written as

$$W_2 \prod_{i=1}^{2} G(c_i^e) + W_1[G(c_1^e)(1 - G(c_2^e)) + G(c_2^e)(1 - G(c_1^e))] + W_0 \prod_{i=1}^{2}(1 - G(c_i^e))$$

$$= (W_1 - W_0) \sum_{i=1}^{2} G(c_i^e) + \frac{(W_2 - W_1) - (W_1 - W_0)}{4}[(\sum_{i=1}^{2} G(c_i^e))^2 - (G(c_1^e) - G(c_2^e))^2].$$

From Lemma 3, $W_2 - W_1 < W_1 - W_0$ as $W_{n+1} - W_n$ (the contribution of the valuation of an additional bidder if there are already n bidders) decreases with n. Thus for given $\sum_{i=1}^{2} G(c_i^e)$, we want to maximize $G(c_1^e) - G(c_2^e)$ in order to maximize the above common component of $S(c_1^e, c_2^e)$ and $R(c_1^e, c_2^e)$. Therefore, if the symmetric entry threshold maximizing the above common component is an inner solution, we must have that the unrestricted solution must be asymmetric. The above arguments can be generalized to the case where $N > 2$ by focusing on the entries of any two bidders while assuming the entry thresholds of all other bidders are fixed.

It is clear that if $\underline{c}$ and $\bar{c}$ are close enough, increasing the difference between the entry thresholds of any 2 bidders while keeping the sum of their ex ante entry probabilities unchanged will lead to higher expected total surplus/the seller's expected revenue because the second terms in (1) and (2) do not change much.

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3.3 Optimality and Uniqueness of Symmetric Entry

In a symmetric setting, for the proposed efficient auction $\mathcal{A}_0$, there may exist multiple entry equilibria. In the Section 3.2 example, the unrestricted efficient entry thresholds are $c_1^e = 0.5$ and $c_2^e = 0.4$, while the symmetric efficient thresholds are $c_1^e = c_2^e = 0.4231$. According to Propositions 2 and 4, both of these two entry patterns are implemented through the same second price auction $\mathcal{A}_0$.

In the same example, the unrestricted revenue-maximizing entry thresholds are $c_1^e = 0.45$ and $c_2^e = 0.4$, while the symmetric revenue-maximizing thresholds are $c_1^e = c_2^e = 0.4187$. In Propositions 3 and 4, two different auctions are proposed to implement these two different entry patterns. Nevertheless, for the Proposition $4(ii)$ auction, there may exist multiple entry equilibria, just like the case of the efficient auction $\mathcal{A}_0$.

To address the issues of multiplicity of entry equilibria and optimality of the symmetric entry, we present the following results.

Proposition 5: *If* $\frac{G(c_1)-G(c_2)}{c_1-c_2} < \frac{1}{(W_1-W_0)-(W_2-W_1)}$, $\forall c_1, c_2$, *then there exists a unique entry equilibrium for each of the Proposition 4 auctions. These entry equilibria are symmetric across bidders.*

Proof: See Appendix.

Since $G(\cdot)$ belongs to $[0,1]$, the condition $\frac{G(c_1)-G(c_2)}{c_1-c_2} < \frac{1}{(W_1-W_0)-(W_2-W_1)}$ can easily be satisfied if the distribution of the entry costs is quite dispersed. A sufficient condition for $\frac{G(c_1)-G(c_2)}{c_1-c_2} < \frac{1}{(W_1-W_0)-(W_2-W_1)}$ is $g(\cdot) < \frac{1}{(W_1-W_0)-(W_2-W_1)}$. Therefore, if the entry costs follow a uniform distribution, then the bigger the range of the entry costs, the less likely there exist asymmetric equilibria. In this sense, greater dispersion of entry costs decreases the likelihood of asymmetric entry equilibria. In the Section 3.2 example where asymmetric entries emerge for auction $\mathcal{A}_0$, the range of the entry costs is rather small. As a result, the condition in Proposition 5 is violated.

The efficient entry is always implemented through $\mathcal{A}_0$ according to Proposition 2. Proposition 5 further provides sufficient condition for $\mathcal{A}_0$ to implement a unique entry

equilibrium, which is symmetric. We thus have the following results.

Corollary 4: *If $\frac{G(c_1)-G(c_2)}{c_1-c_2} < \frac{1}{(W_1-W_0)-(W_2-W_1)}$, $\forall c_1, c_2$, then the following results hold. (i) The ex ante efficient entry must be symmetric. (ii) This ex ante efficient entry is the unique entry equilibrium of the efficient auction $\mathcal{A}_0$. (iii) The efficient entry threshold is fully characterized through (12).*

The following proposition further presents sufficient conditions for the revenue-maximizing entry to be symmetric.

Proposition 6: *If $\frac{G(c_1)-G(c_2)}{c_1-c_2} < \frac{1}{(W_1-W_0)-(W_2-W_1)}$, $\forall c_1, c_2$ and the hazard rate $\frac{G(\cdot)}{g(\cdot)}$ increases, then the revenue-maximizing entry must be symmetric.*

Proof: See Appendix.

Proposition 6 provides sufficient conditions for the revenue-maximizing entry to be symmetric. According to Proposition 4, this symmetric revenue-maximizing entry is implemented through the Proposition 4(ii) auction. Proposition 5 further provides sufficient condition for the Proposition 4(ii) auction to implement a unique entry equilibrium, which is symmetric. We thus have the results in Corollary 5.

Corollary 5: *If $\frac{G(c_1)-G(c_2)}{c_1-c_2} < \frac{1}{(W_1-W_0)-(W_2-W_1)}$, $\forall c_1, c_2$ and the hazard rate $\frac{G(\cdot)}{g(\cdot)}$ increases, then the following results hold. (i) The unique entry equilibrium of the Proposition 4(ii) auction is globally revenue-maximizing. (ii) The globally revenue-maximizing threshold is symmetric and fully characterized by (13).*

4   Conclusion

This paper extends the pre-bid R&D and auctions design literature to an independent private value (IPV) setting where each bidder has a valuation discovery cost that is his private information. This framework allows asymmetry across bidders in the distributions of their entry costs and private valuations. Unlike the case of fixed costs, bidders enjoy information rents when entry costs are their private information. Due to these information

rents, discrepancy appears between the ex ante efficient entry and the revenue-maximizing entry. This further leads to the divergence between the ex ante efficient auction and the revenue-maximizing auction. The ex ante efficiency is implemented through a second-price auction with no entry fee and a reserve price equal to seller's valuation. However, the revenue-maximizing auction generally involves positive individual ex ante entry fees for bidders. The revenue-maximizing ex ante entry fee for each bidder equals the hazard rate of his entry cost distribution, evaluated at the corresponding revenue-maximizing entry threshold for him.

These findings hold when we restrict the entries to be symmetric across symmetric bidders. We find that even for a symmetric setting with private-information entry costs, the ex ante efficient/revenue-maximizing entry can be asymmetric. The possibility of asymmetric optima arises due to the fact that the marginal contribution of an additional entrant's valuation to the expected total surplus/the seller's expected revenue strictly decreases with the number of other entrants. Nevertheless, when the cumulative distribution function of the entry costs changes rather slowly with respect to its argument, the efficient entry must be symmetric across bidders and it is the unique entry equilibrium of the proposed efficient auction. If the hazard rate of the entry cost distribution is additionally increasing, then the revenue-maximizing entry must also be symmetric and it is the unique entry equilibrium of the proposed revenue-maximizing auction. These results mean that the more dispersed the entry costs distribution, the more likely the efficient/revenue-maximizing entry is symmetric. In other words, large dispersion in the entry costs restores the symmetry in the efficient/revenue-maximizing entry.

Appendix

Proof of Lemma 1: Let us consider any entry equilibrium $\mathcal{E}$ implemented by an auction rule. If all bidders other than i adopt their equilibrium entry strategy described by $\mathcal{E}$, the bidder i's equilibrium entry strategy in $\mathcal{E}$ must be his best entry strategy. Given that all bidders other than i adopt the equilibrium entry strategy in $\mathcal{E}$, there must exist an entry threshold $c_i^e \in [\underline{c}_i, \overline{c}_i]$ such that bidder i's best entry strategy is described by property (ii) in Lemma 1. This is true because the expected payoff of bidder i from participating in any given auction decreases strictly and continuously with his entry cost, given that all bidders other than i adopt their equilibrium entry strategy in $\mathcal{E}$. □

Proof of Proposition 1: Let us first consider auction $\mathcal{A}_0$. Suppose all bidders other than i participate in auction $\mathcal{A}_0$ according to thresholds $\mathbf{C^e} = (c_1^e, ..., c_N^e)$. Denote bidder i's expected surplus by $S_i(c_i; \mathbf{C^e})$ if he participates in $\mathcal{A}_0$ while his entry cost is c_i. Then $S_i(c_i; \mathbf{C^e})$ decreases strictly and continuously with c_i. Set an ex ante entry fee (or subsidy) for bidder i as $E_i = S_i(c_i^e; \mathbf{C^e})$, $\forall i \in \mathcal{N}$. Clearly, for a second-price auction with ex ante entry fee (or subsidy) E_i for bidder i and a reserve price equal to seller's valuation, bidder i's expected payoff is $c_i^e - c_i$ if he participates and his entry cost is c_i. Hence, the above auction with ex ante entry fee E_i for bidder i implements entry thresholds $\mathbf{C^e}$. Note that for any auction implementing entry thresholds $\mathbf{C^e}$, the total expected entry costs are the same. Thus the above designed auction achieves the highest possible expected total surplus among the class of auctions implementing $\mathbf{C^e}$, as the auction always awards the item to the participant (including the seller) with the highest valuation.

Moreover, for any auction implementing entry thresholds $\mathbf{C^e}$, the expected surplus of bidder i with entry cost $c_i \leq c_i^e$ can not be smaller than $c_i^e - c_i$, if he participates. This is due to the fact that a type c_i can always mimic a type c_i^e, and by doing so he gets at least a payoff of $c_i^e - c_i$. Recall that in a second-price auction with ex ante entry fee E_i for bidder i and a reserve price equal to seller's valuation, bidder i's expected surplus is exactly $c_i^e - c_i$ if he participates and his entry cost is c_i. As a result, this auction achieves the highest possible seller's expected revenue among all auctions implementing any given entry threshold-vector $\mathbf{C^e}$. □

Proof of Lemma 2: This can be seen from the following arguments. Note that the economic meaning of $V_{\mathbf{k}_1(\mathbf{k}_{-i})} - V_{\mathbf{k}_0(\mathbf{k}_{-i})}$ is the marginal contribution of bidder i with zero entry cost to the expected total surplus if he participates in auction $\mathcal{A}_0$, and all other participants are those bidders with $k_j = 1$, $j \neq i$ in vector $\mathbf{k}_{-i}$. Hence, $V_{\mathbf{k}_1(\mathbf{k}_{-i})} - V_{\mathbf{k}_0(\mathbf{k}_{-i})}$ can be alternatively written as

$$V_{\mathbf{k}_1(\mathbf{k}_{-i})} - V_{\mathbf{k}_0(\mathbf{k}_{-i})} = \int_{v_0}^{\overline{v}_i} \left\{ (v_i - v_0) F_{\mathbf{k}_0(\mathbf{k}_{-i})}(v_0) + \int_{v_0}^{v_i} (v_i - v) f_{\mathbf{k}_0(\mathbf{k}_{-i})}(v) dv \right\} f_i(v_i) dv_i, \quad \text{(A.1)}$$

where $F_{\mathbf{k}_0(\mathbf{k}_{-i})}(\cdot)$ and $f_{\mathbf{k}_0(\mathbf{k}_{-i})}(\cdot)$ are the cumulative distribution function and density function of $v_{\mathbf{k}_0(\mathbf{k}_{-i})}$, respectively. In addition, note that the right hand side of (A.1) can also be interpreted as the expected payoff of bidder i with zero entry cost when he participates in auction $\mathcal{A}_0$, if all other participants are those bidders with $k_j = 1$ in vector $\mathbf{k}_{-i}$. $\square$

Proof of Corollary 1: First, from (3) and (4), the ex ante entry fees of Corollary 1 implement the efficient entry $\mathbf{C}^{e*}$ in a second price auction with a reserve price equal to the seller's valuation. Second, if the ax ante entry fees are defined differently from those specified in Corollary 1, then it is clear that the entry threshold c_i^{e*} can not be implemented. $\square$

Proof of Corollary 2: First, from the proof of Proposition 3, the ex ante entry fees of Corollary 2 implement the efficient entry $\mathbf{C}^{e\dagger}$ and extract all the expected surplus of the threshold types in a second price auction with a reserve price equal to the seller's valuation. Second, if the ex ante entry fees are defined differently from those specified in Corollary 2, then it is clear that either the entry threshold $c_i^{e\dagger}$ can not be implemented or the surplus of the threshold types can not be extracted completely. $\square$

Proof of Lemma 3: We use $H_n(\cdot)$ to denote the cumulative distribution function of the highest valuation of the seller and $n(\geq 0)$ symmetric bidders. Then $H_n(\cdot) = F^n(\cdot)$ on its support $[v_0, \overline{v}]$, $\forall n \geq 1$. Without loss of generality, we assume $v_0 \in (\underline{v}, \overline{v})$. $H_n(\cdot)$ has a mass point at v_0. It follows that $W_n = v_0 F^n(v_0) + \int_{v_0}^{\overline{v}} x dF^n(x) = \overline{v} - \int_{v_0}^{\overline{v}} F^n(x) dx$, $\forall n \geq 0$. This leads to that $W_n - W_{n-1} = \int_{v_0}^{\overline{v}} (1 - F(x)) F^{n-1}(x) dx$ and $(W_n - W_{n-1}) - (W_{n+1} - W_n) = \int_{v_0}^{\overline{v}} (1 - F(x))^2 F^{n-1}(x) dx$, $\forall n \geq 1$. Therefore, we have both $W_{n+1} - W_n$ and $(W_{n+1} - W_n) - (W_{n+2} - W_{n+1})$ decrease with $n(\geq 0)$. $\square$

Proof of Proposition 5: We prove the proposition using contradiction. Note that a symmetric entry equilibrium always exists. Suppose that there is another asymmetric entry equilibrium $\mathbf{C^e}$. Then, we can find i_1, $i_2 \in \mathcal{N}$ such that $c_{i_1}^e > c_{i_2}^e$. We use $Pr(n)$, $n = 0, 1, ..., N-2$ to denote the probabilities with which there are n bidders from the other $N-2$ bidders participating in the auction. According to Lemma 3, $W_{n+1} - W_n$ is the expected payoff of a bidder from participating in auction $\mathcal{A}_0$, if his entry cost is zero and there are other n participants.

Since entry thresholds $c_{i_1}^e$, $c_{i_2}^e$ can be corner solutions, we must have that

$$\sum_{n=0}^{N-2} Pr(n)\{(1 - G(c_{i_2}^e))(W_{n+1} - W_n) + G(c_{i_2}^e)(W_{n+2} - W_{n+1})\} \geq c_{i_1}^e + E, \quad (A.2)$$

$$\sum_{n=0}^{N-2} Pr(n)\{(1 - G(c_{i_1}^e))(W_{n+1} - W_n) + G(c_{i_1}^e)(W_{n+2} - W_{n+1})\} \leq c_{i_2}^e + E, \quad (A.3)$$

where E is the ex ante entry fee in the Proposition $4(i)$ or $4(ii)$ auction. (A.2) and (A.3) lead to

$$(G(c_{i_1}^e) - G(c_{i_2}^e)) \sum_{n=0}^{N-2} Pr(n)[(W_{n+1} - W_n) - (W_{n+2} - W_{n+1})] \geq c_{i_1}^e - c_{i_2}^e. \quad (A.4)$$

From Lemma 3, we have $\sum_{n=0}^{N-2} Pr(n)[(W_{n+1} - W_n) - (W_{n+2} - W_{n+1})] \leq (W_1 - W_0) - (W_2 - W_1)$. As $\frac{G(c_1) - G(c_2)}{c_1 - c_2} < \frac{1}{(W_1 - W_0) - (W_2 - W_1)}$, $\forall c_1, c_2$, we have the left hand side of (A.4) must be smaller than $c_{i_1}^e - c_{i_2}^e$. This contradicts with (A.4). Therefore, there exists no asymmetric entry equilibrium for the Proposition 4 auctions. □

Proof of Proposition 6: We prove the proposition using contradiction. Suppose that the revenue-maximizing entry $\mathbf{C^e}$ is asymmetric. Then, we can find i_1, $i_2 \in \mathcal{N}$ such that $c_{i_1}^e > c_{i_2}^e$. We use $Pr(n)$, $n = 0, 1, ..., N-2$ to denote the probabilities with which there are n bidders from the other $N-2$ bidders participating in the auction.

Since entry thresholds $c_{i_1}^e$, $c_{i_2}^e$ can be corner solutions, Proposition 3 gives

$$\sum_{n=0}^{N-2} Pr(n)\{(1 - G(c_{i_2}^e))(W_{n+1} - W_n) + G(c_{i_2}^e)(W_{n+2} - W_{n+1})\} \geq c_{i_1}^e + \frac{G(c_{i_1}^e)}{g(c_{i_1}^e)}, \quad (A.5)$$

$$\sum_{n=0}^{N-2} Pr(n)\{(1 - G(c_{i_2}^e))(W_{n+1} - W_n) + G(c_{i_2}^e)(W_{n+2} - W_{n+1})\} \leq c_{i_2}^e + \frac{G(c_{i_2}^e)}{g(c_{i_2}^e)}. \quad (A.6)$$

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(A.5) and (A.6) lead to

$$(G(c^e_{i_1}) - G(c^e_{i_2})) \sum_{n=0}^{N-2} Pr(n)[(W_{n+1} - W_n) - (W_{n+2} - W_{n+1})]$$

$$\geq (c^e_{i_1} - c^e_{i_2}) + (\frac{G(c^e_{i_1})}{g(c^e_{i_1})} - \frac{G(c^e_{i_2})}{g(c^e_{i_2})}) \geq c^e_{i_1} - c^e_{i_2}, \tag{A.7}$$

as hazard rate $\frac{G(\cdot)}{g(\cdot)}$ increases. From Lemma 3, we have $\sum_{n=0}^{N-2} Pr(n)[(W_{n+1} - W_n) - (W_{n+2} - W_{n+1})] \leq (W_1 - W_0) - (W_2 - W_1)$. As $\frac{G(c_1) - G(c_2)}{c_1 - c_2} < \frac{1}{(W_1 - W_0) - (W_2 - W_1)}$, $\forall c_1, c_2$, we have the left hand side of (A.7) must be smaller than $c^e_{i_1} - c^e_{i_2}$. This contradicts with (A.7). Therefore, the revenue-maximizing entry must be symmetric. □

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