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1. Farmer and Wendner (2004), FW in the following, consider the sensitivity of policy effects, as implied by dynamic multi-sector computable general equilibrium models, with respect to the specification of capital and investment aggregation. They argue that there is no common standard for the specification of (heterogeneous) capital in such models. They demonstrate that (small) differences in the specification of capital and investment aggregation may yield large differences in the policy effects predicted by dynamic multi-sector computable general equilibrium models.

2. There exists no framework for conducting systematic sensitivity analysis across *differing model frameworks.* Therefore, the objective of FW was to provide "...a first attempt to look deeper at the question of sensitivity of policy simulation results with regard to the specification of investment aggregation" (FW, p.470), in the hope to stimulate more academic discussion on this important issue.

The note by Aßmann and Hogrefe in this issue of *Structural Change and Economic Dynamics*, AH in the following, takes up the research question of FW. It addresses the question of whether or not FW's result carries over to a different model framework. AH conclude that FW's result indeed also holds in different model frameworks. However, AH criticize FW's model that is based on the "puzzling assumption of nominal equality between investment value in t and stock of capital in the next period" (*value capital approach* with *value shares* of investment, in the following). Consequently, AH consider FW's specification of investment aggregation in their model as "inadequate for analyzing the impact of capital specifications."

Given AH's critique, we address two issues in the following. First, we shed light on and justify the value capital approach. Second, we specify the restriction under which FW's model is equivalent to AH's model, and we demonstrate that AH's model can easily be converted into a model with value shares such as in FW.

3. Though AH, at the beginning of their comment on FW, properly refer to the value capital approach in the CGE literature of 1980s and 1990s, they dismiss this approach later on as "inadequate for analyzing the impact of capital specification." To put this claim — for which no reason is given — into appropriate perspective, we would like to remind the reader that the value capital approach had been used by most eminent economists, such as Ballard (1983, 1989) or Goulder and Summers

Table 1: VALUE SHARES (FW) VS. HOMOGENEOUS CAPITAL (AH)

FW	AH
$x_t = \frac{k_t - d^y}{d^x_t - d^y}(d^x_t)^\alpha$	$x_t = \frac{k^x_t/b_{xx} - d^y}{d^x_t - d^y}(d^x_t)^\alpha$
$c^x_{t,2} = \frac{\lambda\gamma\sigma}{\varphi}(d^x_{t-1})^\alpha(d^x_t)^{\alpha-1}$	$c^x_{t,2} = \frac{\lambda\gamma\sigma}{\varphi}(d^x_{t-1})^\alpha(d^x_t)^{\alpha-1}/[b_{xx} + b_{yy}\,p_t]$
$i^x_t = k^x_{t+1} = \eta\,\sigma(d^x_t)^\alpha$	$i^x_t = k^x_{t+1} = \sigma(d^x_t)^\alpha/[1 + b_{yy}/b_{xx}p_t]$

(1989), and FW's paper aimed at a critical evaluation of the employed value capital approach within a stylized dynamic two-sector OLG model.

Ballard et al. had to cope with the problem of setting up a dynamic input-output model without having access to empirical data on the capital coefficient matrix (numerical values for b_{ij}). The solution was to determine investment quantities per sector of origin by physical (Ballard) or value (Goulder and Summers) shares of aggregate savings, and to set next period value capital equal to the sum of sector-of-origin specific investment quantities times output prices. In this way the sector-specific nature of the capital-value aggregate could be maintained without the need of an explicit modeling of multi-sector capital stock dynamics, and without the need of empirical data of the capital coefficient matrix. Therefore, considering the purpose of the value capital approach in applied general equilibrium modeling, AH miss the main point if they are looking for a production function for the capital aggregate. Clearly, *if data* on sectoral capital coefficients *were available*, then AH's approach could be utilized — but as shown above, no new insights are generated.

4. Comparing the model of AH with that of FW, the relative price, the capital-value aggregate ($k_{t+1} = k^x_{t+1} + p_t\,k^y_{t+1}$), and consumption of young households are identical. The differences of the two models are summarized in Table 1.

Consider the following assumption:

$$b_{xx} = \eta\,, \quad b_{yy} = (1-\eta)/p_t\,. \tag{A.1}$$

Claim 1 *Suppose* (A.1) *holds. Then, the FW model is equivalent to the AH model.*

Proof. If (A.1) holds, we know: $\eta(b_{xx} + b_{yy}\,p_t) = b_{xx}$. In this case, according to Table 1, it can be easily verified that all terms in the AH framework are equivalent to those in the FW framework. $\|$

The AH model can easily be transformed into a "value-share formulation" in the tradition of the value capital approach, such as in FW. Denote savings by $s_t = \sigma\,(d^x_t)^\alpha$, and consider: $k^x_t + p_t\,k^y_t = s_t$, along with $k^x_t = b_{xx}/b_{yy}\,k^y_t$ in AH's specification.[1] Then,

[1] The relationship between the capital stocks, $k^x = b_{xx}/b_{yy}\,k^y$, follows from: $l_y = 1 - l_x$, $k^x = b_{xx}\,d^x\,l^x + b_{xx}\,d^y\,l^y$, and $k^y = b_{yy}\,d^x\,l^x + b_{yy}\,d^y\,l^y$.

$k^x_t = s_t\ [b_{xx}/(b_{xx} + p_t\, b_{yy})]$. Similarly, $k^y_t = s_t\ [b_{yy}/(b_{xx} + p_t\, b_{yy})]$. The two terms in square brackets are the corresponding *value shares* in AH's model. Clearly, under assumption (A.1), these value shares correspond to respectively η and $(1 - \eta)/p_t$.

By specifying the (physical) coefficients b_{xx} and b_{yy}, AH implicitly provide a "foundation" for the aggregate value shares of FW. It is important to emphasize, that in both model frameworks, the value shares — devoted to investments in the two sectors — depend on both the technical coefficients (b_{xx}, b_{yy}, η) and the relative price, p.

5. In their comment, AH fail to inform the reader in which ways the approach with value shares fails to be "consistent."[2] In this reply, we hope to have shown that the value share approach, amalyzed by FW, is a consistent (not "puzzling") way to model dynamic multi-sector CGE models.[3] At the same time, we understand that AH strengthened the FW result. Even with a different model framework, the main message of FW goes through. Small differences in the specification of capital and investment aggregation may yield large differences in the policy effects predicted by dynamic multi-sector computable general equilibrium models.

References

[1] Ballard, C.L., 1983. Evaluation of the Consumption Tax with Dynamic General Equilibrium Models. Ph.D. Dissertation, Stanford University, Stanford.

[2] Ballard, C.L., 1989. On the Specification of Simulation Models for Evaluating Income and Consumption Taxes. In: Rose, M. (Ed.), Heidelberg Congress on Taxing Consumption. Springer, Berlin, pp. 147 – 188.

[3] Goulder, L.H., Summers, L.H., 1989. Tax Policy, Asset Prices and Growth. A General Equilibrium Analysis. Journal of Public Economics 38, 265 – 296.

[4] Farmer K., Wendner, R., 2004. Dynamic, Multi-Sector CGE Modeling and the Specification of Capital. Structural Change and Economic Dynamics 15(4), 469 – 492.

[2] A consistent derivation of AH's equation of motion, given in their footnote 4, would require to replace the leftmost term by: $b_{xx}\,(d^x_t - d^y)$. It would also require to replace the price equation by: $p_{t+1} = (1-\alpha)\, b_0\, (d^x_{t+1})^\alpha + \alpha\, b_1\, (d^x_{t+1})^{\alpha-1}$.

[3] After all, AH's approach can readily be transformed into a value share formulation, as demonstrated above.