Bond risk premia, macroeconomic fundamentals and the exchange rate

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Abstract

We introduce a two-country no-arbitrage term-structure model to analyse the joint dynamics of bond yields, macroeconomic variables and the exchange rate. The model allows to understand how exogenous shocks to the exchange rate affect the yield curves, how bond yields co-move in different countries and how the exchange rate is influenced by the interactions between macroeconomic variables and time-varying bond risk premia.

Estimating the model with US and German data, we obtain an excellent fit of the yield curves and we are able to account for up to 75 per cent of the variability of the exchange rate. We find that time-varying risk premia play a non-negligible role in exchange rate fluctuations, due to the fact that a currency tends to appreciate when risk premia on long-term bonds denominated in that currency rise. A number of other novel empirical findings emerge.

*Any views expressed in this article are the authors' and do not necessarily represent those of the Bank of Italy.

1 Introduction

After the seminal contribution of Ang and Piazzesi (2003), several recent studies have developed no-arbitrage term structure models to determine how macroeconomic variables affect bond prices and bond risk premia. While the vast majority of these studies have analysed single countries in isolation, to date very little is known about cross-country interactions between bond prices and how they relate to macroeconomic fundamentals and exchange rate dynamics.

We propose a two-country no-arbitrage term structure model that can be used to tackle a number of largely unaddressed questions about internationally integrated bond markets. For instance, the model allows to assess how exogenous shocks to the exchange rate affect the yield curves, how bond yields co-move in different countries and how time-varying bond risk premia contribute to exchange rate fluctuations, while also controlling for other macroeconomic variables like inflation and output.

On the one hand, a number of studies have investigated the joint dynamics of exchange rates, interest rates and other macroeconomic variables (e.g.: Eichenbaum and Evans - 1995, Grilli and Roubini - 1995 and 1996), without taking bond pricing factors and bond risk premia into account. On the other hand, several two-country no-arbitrage term structure models have been proposed to analyse the relation between exchange rates, yield curves and bond risk premia (e.g.: Bansal - 1997 and Backus, Foresi and Telmer - 2001), but in these models all the dynamics are driven by latent variables and macroeconomic variables do not play any role. Our paper aims to bridge these two strands of the literature, providing a unified framework to determine how bond-pricing factors in two different countries are related to each other and to macroeconomic variables and the exchange rate.

Estimating the model with US and German data, some findings emerge that are robust to various specifications of the model and to different choices of the sample period. At short horizons changes in short-term interest rates account for approximately 30 per cent of exchange rate fluctuations, while inflation and economic growth have almost no explanatory power. At medium-to-long horizons, the explanatory power of inflation and growth becomes higher (up to 25 per cent), while that of interest rates remains about the same. We find that a significant portion of the variability of the exchange rate is accounted for by time-varying bond risk premia (more than 20 per cent at longer horizons). We analyze the dynamics of the estimated model to seek explanations for this finding. Impulse response functions reveal that increases in bond risk premia trigger a reaction of the exchange rate that is similar to the well-known *delayed overshooting* phenomenon caused by increases in policy rates. Delayed overshooting, i.e. persistent currency appreciation after an increase in policy rates, is uncovered by many empirical studies (e.g.: Eichenbaum and Evans - 1995, Grilli and Roubini - 1995 and 1996) and it is considered one of the puzzles of international finance, as it contradicts the theoretical prediction (e.g. Dornbusch - 1976) of an immediate overshooting followed by a subsequent currency depreciation. We find that also increases in bond risk premia cause *delayed overshooting*: a currency tends to persistently appreciate when expected excess returns on long-term bonds denominated in that currency rise. As emphasized by Scholl and Uhlig (2006), the *delayed overshooting* puzzle is intimately related to the *forward premium* puzzle, i.e. the empirical regularity that exchange rate fluctuations tend to reinforce rather than attenuate positive return differentials between currencies. According to our estimates, such tendency of high yielding currencies to appreciate, found by many researchers with reference to the short-term (and risk-free) segment of the bond market (e.g.: Fama - 1984, Engel - 1996), seems to extend also to the long-term segment: when investors expect large capital gains on long-term bonds denominated in a certain currency (in excess of the risk-free rate), that currency tends to appreciate. We also find that, after controlling for macroeconomic variables, there are limited spillovers between bond risk premia in the two countries, with the result that there is low correlation between bond risk premia in Germany and in the US. Finally, we find that exogenous shocks to the exchange rate (those that are not explained by other macroeconomic variables) have a negligible impact on the yield curves.

Also Dong (2006) and Chabi-Yo and Yang (2007) have recently studied the behavior of internationally integrated bond markets in a no-arbitrage framework with macroeconomic variables. We adopt a modelling strategy which is substantially different from theirs. While they take the domestic and the foreign pricing kernel as exogenously given and derive implied currency depreciation endogenously, our model features an exogenous process for currency depreciation and an endogenous foreign pricing kernel. Such approach has several advantages. First, it overcomes the well-known mismatch between model-implied and actual exchange rate found when the depreciation process is derived endogenously. Such mismatch is commonly attributed to the fact that a predominant portion of exchange rate movements is independent of interest rate movements (e.g.: Constantinides - 1992, Lothian and Wu - 2002, Leippold and Wu - 2007). The inability to produce a realistic endogenous currency depreciation process is also found by Dong (2006) and Chabi-Yo and Yang (2007), notwithstanding the fact that they explicitly consider other macroeconomic factors beyond interest rates: both papers find that large variations in the exchange rate remain unexplained by a model with endogenous currency depreciation, even after accounting for inflation and output dynamics in the domestic and foreign country. Instead, in our model there is a perfect match between the data and the model-implied exchange rate. Furthermore, we allow the exchange rate to be affected by exogenous shocks, which can capture important factors not explicitly included in the model, such as current account imbalances (e.g.: Hooper and Morton - 1978). Despite having an endogenous foreign pricing kernel, our model is still able to fit very well both the domestic and the foreign yield curve. Hence, a more realistic modelling of currency depreciation does not come at the expense of pricing accuracy. Another important advantage of our modelling strategy is that it allows to measure the feedback effect of currency depreciation on the yield curves: for example, one can estimate impulse-response functions to assess how exogenous currency shocks are transmitted to domestic and foreign yield curves and to risk premia. Finally, while in a model with endogenous currency depreciation one has to resort to approximate numerical procedures to estimate impulse responses and variance decompositions for the exchange rate, our model allows for exact analytical computation of these quantities.

Although other studies previously recognized that bond returns do not necessarily span returns in the foreign exchange market and explicitly introduced exchange rate factors orthogonal to bond market factors (e.g.: Brandt and Santa-Clara - 2002, Leippold and Wu - 2007, Graveline - 2006), our study is the first to extend Ang and Piazzesi's (2003) methodology to a two-country setting with exogenous currency depreciation. Ang and Piazzesi (2003) have inaugurated a proliferous literature which uses modern no-arbitrage pricing models to analyze the relation between the yield curve and macroeconomic fundamentals: some examples are Ang, Dong and Piazzesi (2007), Ang, Piazzesi and Wei (2006), Gallmeyer, Hollifield and Zin (2005), Hördal, Tristani and Vestin (2006) and Rudebusch and Wu (2004); for a survey, we refer the reader to Diebold, Piazzesi and Rudebusch (2005). Earlier studies investigating the relation between the yield curve and macroeconomic variables, like Fama (1990), Mishkin (1990), Estrella and Mishkin (1995) and Evans and Marshall (2007) did not consider no-arbitrage relations among yields and did not model bond pricing. As a consequence, they were able to make predictions only about the yields explicitly analyzed (typically no more than three), they did not rule out theoretical inconsistencies due to the presence of arbitrage opportunities along the yield curve and they made no predictions about risk premia and their dynamics. For these reasons, the more recent studies we mentioned above have proposed to enrich macro-finance models with rigorous asset pricing relations, imposing no-arbitrage constraints on bond prices. Our methodological contributions to this literature are in the following directions: we propose a very general setting for two-country no-arbitrage macro-finance models, adapting the canonical form derived by Pericoli and Taboga (2008) for the single-country case; we show how to introduce an exogenous process for currency depreciation in a setting with both observable and unobservable variables and we derive a new set of pricing equations which extend the Riccati equations usually found in discrete-time single-country models of the term-structure; we develop an estimation method that affords considerable simplifications over commonly employed maximum likelihood methods and is better suited to tackle the high dimensionality of two-country models; finally, we propose a strategy that allows to identify unobservable bond pricing factors and to map them to easily interpretable economic variables.

The paper is organized as follows: Section 2 introduces the new class of affine two-country models; Section 3 describes the estimation method; Section 4 discusses the empirical evidence; section 5 concludes.

2 The model

Our model of the term structure is a discrete-time Gaussian affine model. Let $(\Omega, \mathsf{F}, \mathsf{F}_t, P)$ be a filtered probability space.

The dynamics of the vector of state variables X_t (a k-dimensional F_t-adapted vector, $k \in \mathbb{N}$) obeys the following stochastic difference equation:

$$X_t = \mu + \rho X_{t-1} + \Sigma \varepsilon_t \tag{1}$$

where $\varepsilon_t \sim N(0, I_k)$ under P, μ is a $k \times 1$ vector and ρ and Σ are $k \times k$ matrices. Σ can be assumed to be lower triangular without loss of generality. The probability measure P is referred to as the 'historical probability measure', in order to distinguish it from other 'risk neutral' equivalent pricing measures, to be introduced in what follows. We assume that the first entry of the vector X_t is the natural logarithm s_t of the exchange rate S_t, the number of units of local currency you can buy with one unit of foreign currency.

The one-period domestic interest rate r_t is assumed to be an affine function of the state variables:

$$r_t = a + b^\intercal X_t \tag{2}$$

where a is a scalar and b is a $k \times 1$ vector.

A sufficient condition for the absence of arbitrage opportunities on the bond market is that there exists a risk-neutral measure Q, equivalent to P, under which the process X_t follows the dynamics:

$$X_t = \overline{\mu} + \overline{\rho} X_{t-1} + \Sigma \eta_t \tag{3}$$

where $\eta_t \sim N(0, I_k)$ under Q and such that:

1. the price at time t of a domestic bond paying one unit of domestic currency at time $t+n$ (denoted by p_t^n) equals[1]:
$$p_t^n = \mathrm{E}_t^Q \left[\exp(-r_t)\, p_{t+1}^{n-1} \right] \tag{4}$$

2. the price at time t of a foreign bond paying one unit of foreign currency at time $t+n$ (denoted by q_t^n) equals:
$$q_t^n = \mathrm{E}_t^Q \left[\exp(-r_t) \frac{S_{t+1}}{S_t} q_{t+1}^{n-1} \right] \tag{5}$$

[1] E_t^Q denotes expectation under the probability measure Q, conditional upon the information available at time t. It is a shorthand for $\mathrm{E}^Q[.|\mathsf{F}_t]$

The vector $\overline{\mu}$ and the matrix $\overline{\rho}$ are in general different from μ and ρ, while Σ does not change, by equivalence of P and Q. The prices of risk, denoted by $\lambda_0 = \Sigma^{-1}(\mu - \overline{\mu})$ and $\lambda_1 = \Sigma^{-1}(\rho - \overline{\rho})$, provide the link between the risk-neutral distribution Q and the historical distribution P:

$$\begin{aligned} \left.\frac{dQ}{dP}\right|_t &= \xi_{t+1}/\mathrm{E}_t\left[\xi_{t+1}\right] \\ \xi_{t+1} &= \prod_{j=1}^{\infty} \exp\left[-\left(\lambda_0 + \lambda_1 X_{t+j-1}\right)\varepsilon_{t+j}\right] \end{aligned}$$

Within this Gaussian framework, bond yields are affine functions of the state variables. For domestic bond yields, the well-known pricing formulae (e.g.: Ang and Piazzesi - 2003) are:

$$y_t^n = -\frac{1}{n}\ln\left(p_t^n\right) = A_n + B_n^{\intercal} X_t \tag{6}$$

where y_t^n is the yield at time t of a bond maturing in n periods and A_n and B_n are coefficients obeying the following simple system of Riccati equations, derived from (4):

$$\begin{aligned} A_1 &= a \\ B_1 &= b \\ &\dots \\ A_n &= \frac{1}{n}\left[a + (n-1)\left(A_{n-1} + B_{n-1}^{\intercal}\overline{\mu} - \frac{n-1}{2}B_{n-1}^{\intercal}\Sigma\Sigma^{\intercal}B_{n-1}\right)\right] \\ B_n &= \frac{1}{n}\left[b + (n-1)\,\overline{\rho}^{\intercal}B_{n-1}\right] \end{aligned} \tag{7}$$

The pricing formulae for foreign bonds, which we derive in the Appendix, are:

$$z_t^n = -\frac{1}{n}\ln\left(q_t^n\right) = C_n + D_n^{\intercal} X_t \tag{8}$$

where z_t^n is the yield at time t of a foreign bond maturing in n periods and C_n and D_n obey the following system of recursive equations:

$$C_1 = a - f^\intercal \overline{\mu} - \frac{1}{2} f^\intercal \Sigma \Sigma^\intercal f \tag{9}$$
$$D_1 = b - \overline{\rho}^\intercal f + f$$
$$\ldots$$
$$C_n = \frac{1}{n}\left[a + (n-1)\left(C_{n-1} + E_n^\intercal \overline{\mu} - \frac{n-1}{2} E_n^\intercal \Sigma \Sigma^\intercal E_n\right)\right]$$
$$D_n = \frac{1}{n}\left[b + f + (n-1)\,\overline{\rho}^\intercal E_n\right]$$
$$E_n = D_{n-1} - \frac{1}{n-1} f \tag{10}$$

f is a $k \times 1$ vector whose first entry is a 1 and all remaining entries are 0, so that $f^\intercal X_t = s_t$.

As clarified by equations (4) and (5), the pricing measure Q can be used to price domestic as well as foreign bonds, with the proviso that gains from currency depreciation (S_{t+1}/S_t) must be taken into account to derive the latter. Recent macro-finance studies (e.g.: Chabi-Yo and Yang - 2007, Dong - 2006) adopt a different modelling strategy: the exchange rate is not included in the set of state variables X_t, the foreign short rate is specified as an affine function of X_t and there are two different pricing measures, one for domestic bonds (say Q) and one for foreign bonds (say Q'); in this case, assuming market completeness (see e.g. Backus, Foresi and Telmer - 2001), exchange rate dynamics are completely determined by Q and Q'. Provided the model is correctly specified, these two approaches are completely equivalent: one can either specify the domestic and the foreign pricing kernels and recover the implied currency depreciation process, or specify only one of the two pricing kernels together with the currency depreciation process and recover the other pricing kernel implicitly (see Graveline - 2006 for a discussion). However, the assumption of correct specification is crucial for the two approaches to be truly equivalent: in particular, we prove in the Appendix that equivalence obtains only if the currency depreciation rate (S_{t+1}/S_t) is an F-measurable random variable[2]. Intuitively, this means that the set of state variables included in X_t must be sufficiently rich to fully account for the variability of the exchange rate, i.e. the randomness in S_t must be completely explained by the randomness in X_t. This is hardly a realistic assumption when X_t does not include the exchange rate as one of its components, as shown by the empirical evidence provided by Constantinides (1992), Lothian and Wu (2002), Leippold and Wu (2007), Chabi-Yo and Yang (2007) and Dong (2006): although the theoretical exchange rate derived in a model with endogenous currency depreciation displays a fair degree of correlation with the actual exchange

[2] The filtered probability space $(\Omega, \mathsf{F}, \mathsf{F}_t, P)$ is usually defined implicitly by the law of motion of X_t: F_t is no larger than the filtration generated by X_t, $\mathsf{F} = \sigma\left(\bigcup_{n=0}^{\infty} \mathsf{F}_t\right)$, and P is the product measure derived from the transition densities of X_t.

rate, an important portion of the variability of the actual exchange rate remains unexplained.

We accommodate the presence of an exogenous currency depreciation process, by adopting a modelling strategy popularized by Ang and Piazzesi (2003). Their seminal paper proposes a no-arbitrage affine term-structure model in which state variables can be both observable and unobservable. A wealth of other papers fruitfully applies their methodology to study the interactions between the yield curve and observable macroeconomic variables, for example Rudebusch and Wu (2004), Hördal, Tristani and Vestin (2006) and Ang, Piazzesi and Wei (2006). In our model the currency depreciation process is an observable variable and we allow for the presence of other observable macroeconomic variables, as well as some unobservables.

The structure and parametrization of the models presented in this paper is an adaptation of the canonical form we propose in a companion paper (Pericoli and Taboga - 2008). Such canonical form provides the most general identified representation within the class of Gaussian homoskedastic affine term-structure models with both observable and unobservable state variables. The presentation of the details of the model closely follows Pericoli and Taboga (2008).

Suppose that the first k^o variables included in the model are observable (remember that the first one is s_t, the natural logarithm of the exchange rate) and the remaining $k^u = k - k^o$ are unobservable. Collect their values at time t into the $k^o \times 1$ vector X_t^o and the $k^u \times 1$ vector X_t^u respectively. Equations (1-3) can be written as follows:

$$
\begin{array}{ll}
\text{Short-rate process} & \left\{ r_t = a + b^{o\top} X_t^o + b^{u\top} X_t^u \right. \\[1em]
\text{Law of motion under } P & \left\{ \begin{array}{l} X_t^o = \mu^o + \rho^{oo} X_{t-1}^o + \rho^{ou} X_{t-1}^u + \Sigma^{oo} \varepsilon_t^o \\ X_t^u = \mu^u + \rho^{uo} X_{t-1}^o + \rho^{uu} X_{t-1}^u + \Sigma^{uo} \varepsilon_t^o + \Sigma^{uu} \varepsilon_t^u \end{array} \right. \\[1em]
\text{Law of motion under } Q & \left\{ \begin{array}{l} X_t^o = \overline{\mu}^o + \overline{\rho}^{oo} X_{t-1}^o + \overline{\rho}^{ou} X_{t-1}^u + \Sigma^{oo} \eta_t^o \\ X_t^u = \overline{\mu}^u + \overline{\rho}^{uo} X_{t-1}^o + \overline{\rho}^{uu} X_{t-1}^u + \Sigma^{uo} \eta_t^o + \Sigma^{uu} \eta_t^u \end{array} \right.
\end{array}
\tag{11}
$$

where all the matrices are obtained by separating into blocks the matrices in equations (1-3).

In Pericoli and Taboga (2008), we prove that the minimal set of restrictions to be imposed in order to achieve identification is:

- $\Sigma^{oo}$is lower triangular
- $\Sigma^{uo} = 0$
- $\Sigma^{uu} = I$

- $b^u \geq 0$
- $X_0^u = 0$

The above set of restrictions, defining the canonical form of the model, imposes contemporaneous independence between random shocks to the observable variables X_t^o (the exchange rate and other macroeconomic variables) and shocks to the unobservable variables X_t^u related to the shape of the two yield curves. However, as explained in Pericoli and Taboga (2008), this does not imply statistical independence of X_t^o and X_t^u, because the above set of restrictions allows for a lagged response of the exchange rate and other observables to changes in the unobservable variables (and vice versa). Also note that the contemporaneous independence between shocks to X_t^o and X_t^u does not imply contemporaneous independence between shocks to the exchange rate and shocks to the yield curve, because the exchange rate is included in X_t^o, which in turns contributes to determine bond yields via the pricing relations (6) and (8). Hence, the model allows to reproduce events which contemporaneously affect both the exchange rate and the yield curves.

The above results on identification are easily generalized to the case where the set of state variables includes also some lags of the observables; we do not report them here and refer the reader to Pericoli and Taboga (2008).

3 Estimation method

In this section we give a detailed account of the estimation procedure we adopted. The less technically inclined reader may safely skip this section.

We use a consistent and exact two-stage estimation procedure, based on ordinary least squares regressions.

Let

$$Y_t = \left[\begin{array}{cccccc} y_t^{T_1} & ... & y_t^{T_N} & z_t^{T_1} & ... & z_t^{T_N} \end{array} \right]^\intercal$$

where $T_1, ..., T_N$ are the bond maturities used to estimate the model ($2N \geq k^u$), and let

$$\begin{aligned} A &= \left[\begin{array}{cccccc} A_{T_1} & ... & A_{T_N} & C_{T_1} & ... & C_{T_N} \end{array} \right]^\intercal \\ B &= \left[\begin{array}{cccccc} B_{T_1} & ... & B_{T_N} & D_{T_1} & ... & D_{T_N} \end{array} \right]^\intercal \end{aligned}$$

Then[3],

$$Y_t = A + BX_t \tag{12}$$

[3] Note that yields are affine in the logarithm of the exchange rate. This property of yields, proved in Section 2, is crucial for the analytical tractability of the estimation method we propose.

Let Γ be a $k^u \times 1$ vector and Δ a $k^u \times 2N$ matrix (to be specified later). Define

$$X_t^* = \begin{bmatrix} X_t^{o\intercal} & (\Gamma + \Delta Y_t)^\intercal \end{bmatrix}^\intercal$$

Then, X_t^* is of the same dimension of X_t and follows a vector autoregression:

$$X_t^* = \mu_* + \rho_* X_{t-1}^* + \Sigma_* \varepsilon_t \tag{13}$$

where:

$$\begin{aligned} \mu_* &= \left(I - \Omega\rho\Omega^{-1}\right)\Xi + \Omega\mu \\ \rho_* &= \Omega\rho\Omega^{-1} \\ \Sigma_* &= \Omega\Sigma \end{aligned}$$

and Ω is a $(k^o + k^u) \times (k^o + k^u)$ matrix whose first k^o rows are the first k^o vectors of the Euclidean basis of $\mathbb{R}^{k^o+k^u}$ and the last k^u rows are equal to the k^u rows of ΔB; Ξ is a $(k^o + k^u) \times 1$ vector whose first k^o entries are equal to 0 and the last k^u equal $\Gamma + \Delta A$.

Thus, according to equation (13) a vector comprising k^u linear affine transformation of the yields and the k^o observable factors follows a vector autoregression; the parameters governing the autoregression are a function of the parameters in the canonical form of the model (μ, ρ, Σ), of the parameters in the affine transformation (Γ and Δ) and of the parameters in the pricing equations (A and B). (13) is an equivalent representation of the model, as defined by Pericoli and Taboga (2008). In the terminology of Duffie and Kan (1996), the representation (13) is a yield-factor model; however, while Duffie and Kan (1996) include only yields in the vector of state variables, the vector autoregression (13) includes also other observable variables.

In order to be able to estimate (13) one has to fix Γ and Δ and assume that the k^u linear affine transformations of yields in the vector $\Gamma + \Delta Y_t$ are not subject to mis-pricings or measurement errors. It is standard practice in the term structure literature to assume $\Gamma = 0$ and Δ composed of k^u vectors of the Euclidean basis of $\mathbb{R}^{2N}$, each selecting only one yield , i.e. k^u yields are measured without error (Chen and Scott - 1993). We generalize this approach by assuming that the first k^u principal components of Y_t are priced without error. Hence, the rows of Δ are the eigenvectors associated to the k^u largest eigenvalues of the covariance matrix of Y_t. We make this assumption to maximize the proportion of the variability of yields coming from sources that are not prone to measurement or pricing errors. Furthermore, we avoid the somewhat arbitrary choice of which yields are to be considered exactly measured and which are not. We also transform the linear combinations thus obtained so that they have zero mean, by setting $\Gamma = -\mathrm{E}[\Delta Y_t]$.

Given the above assumptions and given consistent estimates of Γ and Δ, obtained by calculating the sample principal components of Y_t, we are able to estimate (13) by ordinary least squares, obtaining consistent estimates of μ_*, ρ_* and Σ_*.

Now, we are able obtain an equivalent representation X_t^{**} of X_t^* by rotating and translating X_t^* in such a way that the observable variables are left unchanged:

$$X_t^{**} = \Xi_* + \Omega_* X_t^*$$

where Ξ_* is any $(k^o + k^u) \times 1$ vector whose first k^o entries are equal to zero and Ω_* is any invertible $(k^o + k^u) \times (k^o + k^u)$ matrix whose first k^o rows are the first k^o vectors of the Euclidean basis of $\mathbb{R}^{k^o + k^u}$, i.e.:

$$\Omega_* = \begin{bmatrix} e_1 & \dots & e_{k^o} & v_1 & \dots & v_{k^u} \end{bmatrix}^\top$$

where:

$$\begin{array}{rcl} e_1 & = & \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}^\top \\ e_2 & = & \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \end{bmatrix}^\top \\ & & \dots \end{array}$$

and $v_1, \dots, v_{k^u}$ are k^u vectors such that Ω_* is invertible.

The equivalent representation X_t^{**} has law of motion:

$$X_t^{**} = \mu_{**} + \rho_{**} X_{t-1}^{**} + \Sigma_{**} \varepsilon_t$$

where

$$\begin{array}{rcl} \mu_{**} & = & \left(I - \Omega_* \rho_* \Omega_*^{-1}\right) \Xi_* + \Omega_* \mu_* \\ \rho_{**} & = & \Omega_* \rho_* \Omega_*^{-1} \\ \Sigma_{**} & = & \Omega_* \Sigma_* \end{array} \quad (14)$$

As proved in Pericoli and Taboga (2008), Ξ_* and Ω_* can be chosen in such a way that μ_{**}, ρ_{**} and Σ_{**} satisfy the set of minimal identifying restrictions that define the canonical form of the model (as outlined in the previous section). The set of restrictions on Σ_{**} is:

$$\Sigma_{**}^{oo} \text{ is lower triangular}$$
$$\Sigma_{**}^{uo} = 0$$
$$\Sigma_{**}^{uu} = I$$

Since the first k^o rows of Ω_* are the first k^o vectors of the Euclidean basis of $\mathbb{R}^{k^o+k^u}$ and Σ_*^{oo} is already lower triangular, the requirement that Σ_{**}^{oo} be lower triangular is trivially satisfied.

The restrictions $\Sigma_{**}^{uo} = 0$ and $\Sigma_{**}^{uu} = I$ are satisfied if:

$$e_{k^o+i}^\top = v_i^\top \Sigma_* \quad i = 1, \dots, k^u$$

Since Σ_* is invertible, the restrictions are satisfied with:

$$v_i^\top = e_{k^o+i}^\top \Sigma_*^{-1}$$

Since the distribution of any component of ε_t does not change when you multiply it by -1, you can always change the sign of an unobservable component of X_t^{**} leaving Σ_{**} unchanged, in order to satisfy the restrictions $b^u \geq 0$. The restriction $X_0^{**u} = 0$ can be satisfied only by subtracting from the unobservable components of X_t^{**} their respective values at $t = 0$.

The equivalence of X_t and X_t^* and the fact that X_t^{**} is an equivalent representation of X_t^* in canonical form guarantees that $X_t^{**} = X_t$. As a consequence, we can consistently estimate μ, ρ and Σ in two steps: i) obtain from equation (13) consistent estimates of μ_*, ρ_* and Σ_* by ordinary least squares; ii) recover the coefficients $\mu_{**} = \mu$, $\rho_{**} = \rho$ and $\Sigma_{**} = \Sigma$ by continuous transformations which preserve consistency (equation (14)).

In the second step of the estimation procedure, we estimate by OLS the coefficients A and B in equation (12). If $\widehat{Y}_t$ denote observed yields and $E_t = \widehat{Y}_t - Y_t$ the measurement errors, then:

$$\widehat{Y}_t = A + BX_t + E_t \tag{15}$$

where, by construction

$$\Delta E_t = 0$$

Note that one could also estimate (15) by non-linear least squares, by taking into account the fact that A and B are non-linear functions of the parameters $\overline{\mu}$, $\overline{\rho}$ and Σ. Although the restrictions on A and B imposed by (7) and (9) probably make the non-linear estimator more efficient if the pricing kernel is correctly specified, we believe that there are several reasons to prefer the linear estimators over the non-linear ones. First of all, the procedure we propose affords a significant computational advantage, by avoiding the numerically intensive procedures needed to solve the non-linear least squares problem; this is especially important given the high dimensionality of the problem at hand. Furthermore, we avoid the difficulties caused by local minima and flat surfaces encountered by many researchers in the process of estimating term-structure models like ours (e.g.: Kim and Orphanides - 2005). Finally, our procedure is robust to mis-specification within the exponential affine class, in the sense that it produces consistent estimates of A and B also when the pricing kernel is mis-specified, but the true pricing kernel gives rise to prices that are affine exponential in the state variables.

4 Empirical evidence

In this section we apply the theoretical results outlined in the previous sections to a two-country model where Germany is the domestic country and the United States are the foreign country. This section is organized as follows: subsection 4.1 presents the data and briefly discusses some issues related to sample choice and model selection; subsection 4.2 comments on estimation results; subsection 4.3 examines the ability of the estimated model to accurately reproduce the dynamics of risk premia on bonds and currency markets; subsection 4.4 introduces an equivalent representation of the model that allows for a clear economic interpretation of all the state variables; subsections 4.5 and 4.6 describe the results of variance decompositions and impulse response analysis respectively; subsection 4.7 discusses deviations from uncovered interest rate parity; subsection 4.8 discusses robustness to sample choice; subsection 4.9 discusses other robustness checks.

4.1 Data, sample choice and model selection

We use two datasets of zero coupon rates extracted from US and German government bond yields, recorded at a monthly frequency and provided by the Federal Reserve and the Deutsche Bundesbank respectively[4]: the two yield curves consist of ten equally spaced maturities, from 1 to 10 years. The sample goes from January 1973 to September 2007 and the yields are registered on the last trading day of each month. We utilize all the ten maturities to estimate our models.

The macroeconomic variables in our dataset are: an inflation rate and a measure of the output gap for both countries. The inflation rate is the twelve-month growth rate of the consumer price index. The output gap is HP-filtered industrial production (the smoothing parameter is set to 129,600 as suggested by Ravn and Uhlig - 2002). Both the consumer price indices and the industrial production data are taken from Datastream (the codes are BDCONPRCF, USCONPRCE,

[4] Access to these datasets is granted via the Bank for International Settlements DataBank.

BDIPTOT.G and USIPTOT.G). We also follow Engel and West (2006) and replace the June 1984 outlier in German industrial production with the average of the two neighbouring months.

The USD/DEM exchange rate is the end of month closing middle rate available on Datastream (the USD/EUR exchange rate is used to lengthen the time series after December 1998, when the euro became the single currency for the European Monetary Union).

As far as sample selection is concerned, the main discussion in the following sections is based on estimates carried out with a sub-sample of the available data, starting from January 1983 and ending in December 1998. This is the same sub-sample used by Dong (2006), who provides ample motivation for this choice: according to the evidence presented, among others, by Clarida, Galì and Gertler (1998), it is recommendable to focus on a post-1983 sample in order to avoid shifts in monetary policy regimes both in Germany and in the US. The choice of December 1998 as the end date is motivated by the introduction of the euro in Germany in January 1999. In subsection 4.8, where we discuss robustness to sample choice, we summarize the results obtained by enlarging the sample to the pre-1983 and post-1998 decades.

Our preferred model has five observable variables: the logarithm of the exchange rate, German and US inflation and the output gap in both countries. The choice of inflation and output gap as the two macroeconomic variables to include is in line with the majority of the recent macro-finance literature (e.g.: Ang, Dong and Piazzesi - 2007). The number of unobservable variables is set to four. As we explain in more detail in the following sections, four unobservables allow for an excellent bond pricing accuracy and for a neat ex-post interpretation of the results in terms of relevant economic variables. No lags of any of the variables are comprised in the state vector. In subsection 4.9 (robustness checks), we consider other specifications where the state vector includes more unobservable variables and lags of the observables.

4.2 Estimation results

Table 1 contains the estimates[5] of the coefficients of the factor dynamics (1). The four unobservable variables display a fair degree of persistence (the coefficients on their own lags are between 0.90 and 0.95), in line with that of the observable macroeconomic variables; the eigenvalues of the autoregression matrix ρ lie well inside the unit circle, with the lowest modulus equal to 0.639 and the highest equal to 0.974, so that the estimated model is covariance stationary.

The off-diagonal blocks ρ^{ou} and ρ^{uo} have many entries that are statistically different from zero (as shown by the high t-statistics). As stressed by Pericoli and Taboga (2008), while a number of macro finance studies impose the restrictions $\rho^{ou} = 0$ and $\rho^{uo} = 0$, these restrictions are not required by the canonical form introduced in Section 2. The overidentifying restrictions $\rho^{ou} = 0$ and $\rho^{uo} = 0$, together with $\Sigma^{ou} = 0$ and $\Sigma^{uo} = 0$ (which are instead necessary for identification),

[5] In interpreting results, note that estimation has been carried out with yields and inflation rates expressed in percentage points on an annualised basis.

would imply that the observable variables be independent from the unobservable and that there be no interactions between macroeconomic variables and other factors related to shape of the yield curves (for a discussion of this point, see Rudebusch, Sack and Swanson - 2006). The relaxation of the aforementioned restrictions allows instead for a lagged response of macroeconomic variables to changes in the unobservable factors (and vice versa). Recent studies (e.g.: Diebold, Rudebusch and Aruoba - 2006) find that the hypothesis of independence is challenged by formal statistical tests. The evidence presented in Table 1 lends further support to these findings.

To save space we do not report the estimated factor loadings in the pricing equations (15). A qualitative inspection of the loadings does not reveal any immediately discernible pattern and the unobservable factors have no straightforward economic interpretation (as, for instance, in Ang and Piazzesi - 2003, where the unobservable factors are highly correlated with the level, slope and curvature of the term structure). For this reason, we will introduce in the following subsections an equivalent representation of the factor dynamics that allows for a clear interpretation of all the state variables of the model.

Table 2 reports the estimated standard deviations of the pricing errors of equation (15). Pricing is very accurate, with the standard deviation never exceeding 10 basis points. Although the foreign pricing kernel has been endogenised in our model, pricing accuracy for foreign bonds is similar to that found for domestic bonds.

4.3 Risk premia

In this subsection we analyze the ability of our estimated model to reproduce observed patterns of variation in risk premia: we perform statistical tests to check whether model-implied expected returns are unbiased predictors of realized returns and we assess their in-sample predictive accuracy. We analyze expected returns on domestic bonds and on foreign bonds, considering both the perspective of a domestic and of a foreign investor.

We define risk premia as model-implied expected excess holding-period returns. More precisely, when a domestic bond maturing in n periods is bought at time t and held for $h \leq n$ periods, the risk premium $e_t^{n,h}$ is:

$$e_t^{n,h} = \mathrm{E}_t^P \left[\ln \left(p_{t+h}^{n-h} \right) - \ln \left(p_t^n \right) \right] + \ln \left(p_t^h \right)$$

The risk premium $e_t^{n,h}$ is the percentage capital gain that an investor expects to obtain by holding an n-maturity zero coupon bond for h periods, net of the risk-free interest he could alternatively receive investing in a bond expiring in exactly h periods.

The risk premium $f_t^{n,h}$ on a foreign bond from the perspective of a foreign investor is:

$$f_t^{n,h} = \mathrm{E}_t^P \left[\ln \left(q_{t+h}^{n-h} \right) - \ln \left(q_t^n \right) \right] + \ln \left(q_t^h \right)$$

while the risk premium $g_t^{n,h}$ on a foreign bond from the perspective of a domestic investor is:

$$g_t^{n,h} = \mathrm{E}_t^P \left[\ln\left(q_{t+h}^{n-h}\right) - \ln\left(q_t^n\right) + s_{t+h} - s_t \right] + \ln\left(p_t^h\right)$$

This last definition takes into account the fact that a domestic investor holding a foreign bond for h periods also profits from currency appreciation between t and $t+h$.

Realized excess holding-period returns are defined accordingly, substituting expectations of random variables with their respective realizations in the above formulae. For example, the realized excess returns $re_t^{n,h}$ on domestic bonds are defined as:

$$re_t^{n,h} = \ln\left(p_{t+h}^{n-h}\right) - \ln\left(p_t^n\right) + \ln\left(p_t^h\right)$$

Table 3 reports the summary statistics of a battery of regressions of realized excess returns on their model-implied expectations:

$$re_t^{n,h} = \alpha + \beta e_t^{n,h} + u_t \tag{16}$$

We consider both domestic and foreign bonds from the perspective of a foreign and a domestic investors. The holding period is one year ($h = 12$ in our monthly model).

The R^2 is always higher than 30 per cent and in some cases it is as high as 45 per cent. The estimated βs are always significantly different from zero at all conventional levels of confidence[6]. If model-implied expectations are not biased, tests of the joint hypothesis $\alpha = 0, \beta = 1$ should not be rejected by the data. We run Wald tests of this hypothesis (χ^2 statistics and p-values are reported in Table 3). The hypothesis of unbiasedness is never rejected at 99 and 95 per cent confidence, while it is in only one case rejected at the 90 per cent level.

The fit of the above regressions is comparable to that found with popular predictive regressions: for instance, Cochrane and Piazzesi (2005) also find that their statistical model of excess returns produces R^2 as high as 45 per cent. Furthermore, the regressions provide good evidence that the model produces unbiased estimates of risk premia. As we will explain in the next subsections, estimated risk premia play a key role in the analysis and interpretation of model dynamics, hence we believe that analysing the properties of regression (16) is an essential check on the robustness of the model.

[6] We use Newey-West standard errors with 18 lags to take into account the serial correlation induced by overlapping returns, as suggested by Cochrane and Piazzesi (2005).

4.4 Equivalent representations and interpretation of the state variables

In this subsection we introduce an equivalent representation of the factor dynamics (1) that we will exploit in the following subsections to carry out variance decompositions and impulse response analyses. While the unobservable variables, as identified in the estimated canonical form, are difficult to give a clear economic interpretation, the equivalent representation we now introduce allows to map all the state variables to easily understandable economic quantities. We believe this is an important step towards a better understanding of the mechanics of a reduced-form no-arbitrage model like ours[7] and that it helps to provide a deeper insight into the economic forces driving bond prices and currency depreciation.

Note first that the yields on the one-year domestic and foreign bonds are affine functions of the state variables:

$$
\begin{aligned}
y_t^{12} &= A_{12} + B_{12}^{\top} X_t \\
z_t^{12} &= C_{12} + D_{12}^{\top} X_t
\end{aligned}
$$

As shown in the Appendix, also the expected excess holding period returns on the ten-year domestic and foreign bonds (the latter from the perspective of a foreign investor) are affine in the state variables:

$$
\begin{aligned}
e_t^{120,12} &= J_{120,12} + K_{120,12}^{\top} X_t \\
f_t^{120,12} &= Q_{120,12} + W_{120,12}^{\top} X_t
\end{aligned}
$$

where $J_{120,12}$ and $Q_{120,12}$ are scalars and $K_{120,12}$ and $W_{120,12}$ are $k \times 1$ vectors whose functional dependence on model parameters is reported in the Appendix.

Let Γ_1 be a $k^u \times 1$ vector and Δ_1 a $k^u \times (k^o + k^u)$ matrix defined as follows:

$$
\begin{aligned}
\Gamma_1 &= \begin{bmatrix} A_{12} & C_{12} & J_{120,12} & Q_{120,12} \end{bmatrix} \\
\Delta_1 &= \begin{bmatrix} B_{12} & D_{12} & K_{120,12} & W_{120,12} \end{bmatrix}^{\intercal}
\end{aligned}
$$

Define

$$
X_t^{\#} = \begin{bmatrix} X_t^{o\intercal} & y_t^{12} & z_t^{12} & e_t^{120,12} & f_t^{120,12} \end{bmatrix}^{\intercal}
$$

[7] To our knowledge, our paper is the first to utilize equivalent representations to achieve economic identification of the latent variables in a reduced-form no-arbitrage model. We thank Pietro Veronesi, who encouraged us to think deeply about economic identification of the latent variables.

Then, $X_t^{\#}$ is of the same dimension of X_t and follows a vector autoregression:

$$X_t^{\#} = \mu_{\#} + \rho_{\#} X_{t-1}^{\#} + \Sigma_{\#} \varepsilon_t \tag{17}$$

where:

$$\begin{aligned}
\mu_{\#} &= \left(I - \Omega_{\#} \rho \Omega_{\#}^{-1} \right) \Xi_{\#} + \Omega_{\#} \mu \\
\rho_{\#} &= \Omega_{\#} \rho \Omega_{\#}^{-1} \\
\Sigma_{\#} &= \Omega_{\#} \Sigma
\end{aligned}$$

and $\Omega_{\#}$ is a $(k^o + k^u) \times (k^o + k^u)$ matrix whose first k^o rows are the first k^o vectors of the Euclidean basis of $\mathbb{R}^{k^o + k^u}$ and the last k^u rows are equal to the k^u rows of Δ_1; Ξ is a $(k^o + k^u) \times 1$ vector whose first k^o entries are equal to 0 and the last k^u equal Γ_1.

(17) is an equivalent representation of the model, where the state vector $X_t^{\#}$ comprises the 5 initial observable variables, the two yields on the one-year zero coupon bonds (the domestic and the foreign one) and the two risk premia (the expected excess holding-period returns on the domestic and the foreign ten-year bonds). Hence, the equivalent representation (17) allows to express the dynamics of the model in terms of easily understandable economic quantities, without resorting to latent factors that are not economically identified: as we show in the following subsections, this results allows for a neat interpretation of variance decompositions and impulse response analyses.

4.5 Variance decompositions

In this subsection we comment on the results of variance decompositions, based on the equivalent representation (17).

Table 4 reports the decomposition of the variance of the exchange rate. The proportion of variance explained by inflation and output increases with the time horizon, from 6 per cent at a 12-month horizon to 24 per cent at a 120-months horizon. The variance explained by shocks to the one-year interest rates is 29 per cent at a 12-month horizon and only slightly decreases as the forecast horizon increases. The percentage of variance attributable to risk premia is about 9 per cent at a 12-month horizon and increases up to 24 per cent at longer horizons. The proportion of variance of the exchange rate not explained by other variables is at first high (55 per cent), but then decreases to less than 25 per cent. In all cases, the predominant role is played by US variables, while shocks to German variables seem to have a fairly limited impact on the exchange rate.

Our findings are in line with those of previous studies which found that, although a significant part of exchange rate fluctuations is not explained by macroeconomic variables and interest rates in the short run, over longer horizons the explanatory power of fundamentals significantly increases (e.g.: Bekaert and Hodrick - 1992). However, a novel finding emerging from the variance decomposition is the fact that a significant portion of exchange rate variability is driven by bond risk premia. As we more extensively discuss in the sections on robustness checks, this finding does not appear to be dependent upon any particular modelling or identification assumption: in fact, if we estimate a vector autoregression like (17) that excludes the two variables related to bond risk premia, the proportion of variance of the exchange rate that is left unexplained increases by more than 10 per cent at all time horizons. In the following subsections, where we analyse impulse response functions and deviations from uncovered interest rate parity, we describe the economic mechanism underlying the influence of risk premia on the exchange rate.

Table 5 reports the results of decomposing the variance of risk premia at different time-horizons. Results differ depending on the country under consideration. On the one hand, about 80 per cent of the variance of the risk premium on the US bond is explained by shocks to macroeconomic variables and interest rates, both at short and at medium-to-long horizons. On the other hand, more than 70 per cent of the variance of the risk premium on the German bond remains unexplained by macro-factors at short horizons. However, also for Germany at longer horizons the explanatory power of the other state variables increases up to about 50 per cent. Interestingly, after accounting for macroeconomic shocks, there seem to be very limited spillovers from US to German risk premia and vice versa: in each country, the proportion of variance of the domestic risk premium explained by shocks to the foreign risk premium is negligible. This is confirmed also by the impulse response analysis in the next subsection.

4.6 Impulse response functions

4.6.1 Responses of the exchange rate

Figure 1 plots the responses of the logarithm of the exchange rate to one standard deviation shocks[8] to output gaps, inflation rates, one-year interest rates and risk premia.

An increase in the German (US) interest rate leads to an appreciation of the Mark (dollar). The maximal response is achieved after 13 months for Germany and after 9 months for the US. This is in line with many empirical studies (e.g.: Eichenbaum and Evans - 1995, Grilli and Roubini - 1995 and 1996) that find evidence of persistent currency appreciation after an increase in policy rates. The phenomenon is known as *delayed overshooting* and it is considered one of the puzzles of international finance, as it contradicts the theoretical prediction (e.g. Dornbusch - 1976) of an immediate overshooting followed by a subsequent currency depreciation. As emphasized by Scholl

[8] We present responses to non-factorized innovations, i.e. for each variable subjected to a shock we plot the effect of a *ceteris paribus* increase in that variable on the other variables.

and Uhlig (2006), the *delayed overshooting* puzzle is intimately related to the *forward premium* puzzle we discuss in subsection 4.7.

An increase in German inflation initially leads to a small depreciation of the Mark, followed, after 6 months, by an appreciation of greater magnitude: the overall outcome is a prolonged increase in the value of the Mark with respect to the US dollar. An increase in US inflation causes a persistent appreciation of the Dollar. Hence, in both countries, the response of the exchange rate to inflation is inconsistent with long-run PPP; it is, however, consistent with the previously mentioned *delayed overshooting* phenomenon: as noted by Clarida (2004) and Dong (2006) central banks tend to increase the interest rate in response to higher inflation; if a persistent currency appreciation follows such increase, then the combined effect of these two reactions leads to a positive lead-lag relationship between inflation and currency appreciation.

The response to a positive shock to output is different in the two countries. An increase in German output is followed by an appreciation of the Mark, while an increase in US output leads to a depreciation of the dollar. The response of the exchange rate to changes in output is notoriously difficult to predict and interpret (e.g.: Harberger - 2003, Miyajima - 2005, Zhu and McFarlane - 2005) and the empirical evidence is mixed (Froot and Rogoff - 1994); the response depends on many factors: for example, it may depend on whether the output shock is demand or supply driven and whether it has been generated in the tradeables or non-tradeables sector; furthermore, it may also depend on how the central bank reacts to output shocks if it is following a Taylor (1993) rule.

An increase in expected excess holding period returns on German (US) 10-year bonds triggers an appreciation of the Mark (dollar); the response is much stronger and more persistent for the United States than for Germany. Hence, the tendency of high yielding currencies to appreciate, found by many empirical studies with reference to the short-term (and risk-free) segment of the bond market (e.g.: Fama - 1984, Engel - 1996), seems to extend also to the long-term segment: when investors expect large capital gains on Mark (dollar) denominated 10-year bonds (in excess of the risk-free rate), the Mark (dollar) tends to appreciate. As discussed also in subsection 4.7, this evidence adds to the complexity of the *delayed overshooting* and *forward premium* puzzles: exchange rate fluctuations seem to reinforce rather than attenuate positive return differentials between currencies, not only with reference to investments in short-term deposits, but also with reference to longer-term securities.

4.6.2 Responses of risk premia

Figure 2 plots the responses of the risk premia (defined as excess expected holding period returns on the 10-year bonds) to one standard deviation shocks to the other state variables.

A positive shock to German (US) output leads to a reduction of the risk premium on the German (US) bond. The effect is more pronounced for Germany than for the US. Countercyclical variation in risk premia is consistent with the predictions of intertemporal asset pricing models (Robotti -

2002): in the presence of positive risk aversion (hence decreasing marginal utility of consumption), high risk premia are required during unanticipated recessions to induce investors away from (already low) current consumption and into risky investments.

The response to inflation is positive both in Germany and in the US. The response in Germany is very weak and reverts to negative after few months, while it is stronger and lasts longer in the US. Hence, investors tend to require higher risk premia when inflation is higher; this is consistent with the estimate of a positive price of risk for inflation in Ang and Piazzesi's (2003) Macro Lag Model.

The risk premium on the German 10-year bond tends to rise when the premium on the analogous US bond increases. Also the premium on the US bond has a positive response to increases in the German premium, although to a much lesser extent. The correlation between the two is, however, quite low (4 per cent).

Finally, we do not find any significant response of risk premia to unexpected currency appreciation and to unexpected increases in the short-term interest rates.

4.7 Deviations from uncovered interest rate parity

The theory of uncovered interest rate parity states that the currency of the country with the higher interest rate tends to depreciate, so that the positive yield differential is compensated by an expected currency loss and a risk-neutral investor is indifferent between investing in the domestic currency and investing in the foreign currency. There is ample and robust empirical evidence (e.g.: Fama - 1984, Engel - 1996) that uncovered interest rate parity does not hold in practice and that higher-yielding currencies display a tendency to appreciate rather than depreciate. This phenomenon, widely known as the *forward premium puzzle*, is exploited by a popular trading strategy, known as the *carry trade*, whereby one borrows money in low-yielding currencies and invests it in high-yielding currencies, hoping to profit both from interest rate differentials and from currency appreciation. In this subsection we thoroughly investigate the phenomenon through the lens of our model.

We need to introduce a fourth definition of risk premium, which we call *foreign exchange risk premium*:

$$\overline{g}_t^{n,h} = \mathrm{E}_t^P \left[\ln\left(q_{t+h}^{n-h}\right) - \ln\left(q_t^n\right) + s_{t+h} - s_t\right] - \mathrm{E}_t^P \left[\ln\left(p_{t+h}^{n-h}\right) - \ln\left(p_t^n\right)\right] \tag{18}$$

It is the extra-return that a domestic investor expects to gain, over an holding-period of length h, when he substitutes a domestic bond maturing in n periods with a foreign bond of equal maturity. Stated differently, it is the expected return from a carry-trade strategy whereby one buys a foreign bond and contemporaneously sells shorts a domestic bond of equal maturity. While the majority of papers dealing with the forward premium puzzle have limited their analysis of the foreign exchange risk premium to the case in which $n = h$ (holding period and bond maturity coincide), some recent papers (Meredith and Chinn - 1998 and Alexius - 2000) have shown that the empirical evidence is significantly influenced by the choice of maturity. For this reason, we choose a more flexible definition of foreign exchange risk premium, where the maturity n is allowed to vary.

In the notation of our model, the most classical version of the uncovered interest rate parity can be expressed as follows:

$$\begin{aligned} \mathrm{E}_t^P \left[s_{t+h} - s_t \right] &= \ln \left(q_t^h \right) - \ln \left(p_t^h \right) && (19) \\ &= h \left(y_t^h - z_t^h \right) && (20) \end{aligned}$$

which is equivalent to:

$$\overline{g}_t^{h,h} = 0$$

Hence, the main prediction of the uncovered interest rate parity theory is that the foreign exchange risk premium (as defined above) be equal to zero, not only on average (i.e. unconditionally), but also in any time period (i.e. conditionally). The prediction is a direct consequence of assuming risk-neutral investors. It can be proved that in our model, instead, $\overline{g}_t^{n,h}$ is an affine function of the state variables and is therefore allowed to vary over time. More specifically, $\overline{g}_t^{n,h}$ is equal to a constant plus a linear combination of the following variables[9]: 1-5) the five observable variables included in X_t^o; 6) the domestic one-year interest rate; 7) the risk premium on the domestic ten-year bond; 8) the differential between the foreign and the domestic one-year interest rate (henceforth, *the interest rate differential*); 9) the differential between the foreign and the domestic risk premium on the ten-year bond (henceforth, *the risk premium differential*).

Table 6 reports the coefficients of the linear combination for various maturities (h is fixed to 12 as in the rest of the paper). Contrary to the uncovered interest rate parity hypothesis, the foreign exchange risk premium is an increasing function of the interest rate differential across all maturities, i.e. a widening of the difference between the one-year foreign interest rate and the corresponding domestic rate increases the expected return of the carry trade, irrespective of the maturity of the bonds involved in the trade. Note, however, that this is a *ceteris paribus* statement and does not imply that a positive interest rate differential is associated to a positive foreign exchange risk premium, as implied by naive formulations of the forward premium puzzle; in fact, the interest rate differential is only one of the variables which concur to determine the foreign exchange risk premium and it may well happen that a positive interest rate differential is more than compensated

[9] The proof that the foreign exchange risk premium is an affine function of the variables listed here is very similar to the proof that expected excess holding period returns are affine in the state variables. We do not report it to save space.

by the negative effect of other variables so to produce a negative foreign exchange risk premium. For example, the coefficients in Table 6 indicate that the stronger the US dollar is with respect to the Mark, the lower is the expected profit on the US carry trade (borrowing money in Germany and investing it in the US); hence, even when the dollar is the higher yielding currency, the expected return on the carry trade can be negative if the USD/DEM exchange rate is sufficiently far above its equilibrium value.

The foreign exchange risk premium is also an increasing function of the risk premium differential, i.e. the expected return on the carry trade increases when the difference between the risk premium on the foreign 10-year bond and the risk premium on the corresponding domestic bond widens. This is consistent with the finding reported in subsection 6 that high returns on foreign bonds tend to be reinforced by currency appreciation also at longer maturities.

The dependence of the foreign exchange risk premia on other variables is hard to interpret: we report it without further comments. The effect of domestic inflation on the foreign exchange risk premium depends on the maturity under consideration, being positive for short and long maturities and negative for intermediate. The effect of foreign inflation is unambiguously positive. The effect of the output gap (both domestic and foreign) is negative. The effects of the domestic interest rate and risk premium are negative and positive respectively.

4.8 Extending the sample period

In this subsection we briefly illustrate the results obtained by estimating the model over longer time spans. We consider two prolongments of the 1983-1998 sample period discussed in the previous sections. The first extended sample goes from January 1974 to December 1998. This is the same period chosen by Inci and Lu (2004), in order to cover the entire floating exchange rate period before the introduction of the euro (more precisely, the international exchange rates became floating in August 1973, but the authors recommend starting from January 1974 to allow for an adjustment period). The second extended sample goes from January 1983 to September 2007 (the month when we last updated our dataset). The exchange rate of individual currencies of the European Monetary Union vis à vis the US dollar has been extended beyond 1998 using the USD/EUR exchange rate also by Gadea, Montanés and Reyes (2004), who argue that the extension might augment the power of PPP and unit root tests and offers a number of interesting insights by reflecting the impact of the early years of the European single currency on the results obtained for the pre-Euro period.

Econometric tests of the hypothesis that January 1983 and December 1998 constitute two breakpoints provide mixed evidence. Running Chow breakpoint tests and CUSUM tests on the individual equations of the vector autoregression (17), we find that the null hypothesis of no structural break is rejected for several equations (see Table 7) at the 95 per cent confidence level. The rejections are more frequent with the Chow test than with the CUSUM test, probably indicating that, although coefficients change after the breakpoints, the predictive accuracy of the regressions does not deteriorate much by extending the sample period. However, if we run a LR test of the joint hypothesis of stability of all the equations in the VAR (we use the LR statistic adjusted for small-sample bias proposed by Sims - 1980 and reported by Hamilton - 1994), we are not able to reject the hypothesis of no structural change (the p-values are 67 and 73 per cent for the pre-1983 and post-1998 periods respectively). Furthermore, the majority of the empirical findings presented in the previous sections remain broadly unaffected by the enlargement of the sample period. We now briefly report the differences with respect to the base sample (1983-1998).

Estimating the model with the 1974-1998 sample, we observe the following features: pricing accuracy only slightly deteriorates, with the average pricing error remaining well below 10 basis points; the ability of model-implied risk premia to predict excess returns on bonds is comparable to that found in the 1983-1998 sample; the R^2 found in the regressions of realized on model-implied expected excess returns is on average higher than 30 per cent and the p-values found in the tests for unbiasedness are even greater; we still find evidence of countercyclicality in risk premia, although less pronounced; variance decompositions of the exchange rate indicate that risk premia still explain an important fraction of exchange rate variability, although the explanatory power increases at short and medium forecasting horizons and decreases at longer horizons; impulse response functions reveal that the delayed overshooting phenomenon remains, both for short term interest rates and for risk premia, although it is somewhat weaker for short-term interest rates; deviations from uncovered interest rate parity are still driven by positive return differentials both in the short and in the long segment of the bond market.

Estimating the model with the 1983-2007 sample, we find that: pricing accuracy is virtually unaffected; the ability of model-implied risk premia to predict excess returns on bonds is comparable to that found in the 1983-1998 sample for German bonds while it slightly deteriorates for US bonds (both from the perspective of a German and of a US investor); the R^2 found in the regressions of realized on model-implied expected excess returns is however higher than 25 per cent also for US bonds; the tests for unbiasedness still yield very satisfactory p-values; countercyclicality in risk premia disappears; performing variance decompositions of the exchange rate, we find that risk premia explain an even greater fraction of exchange variability, due to the fact that also German risk premia now play a non-negligible role (remember that in the 1983-1998 sample only US risk premia seemed to matter); estimates of impulse response functions reveal that the delayed overshooting phenomenon remains an important phenomenon, both for short term interest rates and for risk premia; deviations from uncovered interest rate parity are still driven by positive return differentials both in the short and in the long segment of the bond market.

Overall, extending the sample backwards and forward we do not find striking differences in the results, except for the fact that adding recent data takes the evidence of countercyclicality in risk premia away. The findings that a portion of the variability of the exchange rate is explained by risk premia and that delayed overshooting happens also in response to changes in risk premia, two of the main empirical findings of the paper, are robust across samples.

4.9 Robustness checks

Apart from experimenting with different samples, as documented in the previous subsection, we performed a number of other robustness checks both on the data set and on the specification of the model.

We replaced HP-filtered industrial production with the 12-month growth rate of industrial production and found no significant differences. We also computed a monthly series for real GDP by linearly interpolating the quarterly observations and we applied to it the same HP filter used for industrial production: although the series based on industrial production is more volatile, it is highly correlated with that based on GDP and assigns approximately the same dates to peaks and troughs of economic activity.

We added more unobservable variables to the model, but found no appreciable improvement in pricing accuracy or differences in the dynamics of the model. We added up to four lags of the macroeconomic variables and the exchange rate, and we found no significant changes in variance decompositions and impulse response functions; however, we noted that adding more lags caused a fast growth in the condition number of the data matrix and we thus chose the most parsimonious specification (only one lag) to avoid collinearity problems.

We tried different Choleski orderings of the variables for performing variance decompositions: although numerical results obviously change, none of the main features reported in subsection 4.5 are substantially altered.

We estimated a vector autoregression like (17), but excluding the two variables related to bond risk premia, and we observed that the proportion of variance of the exchange rate left unexplained increased by more than 10 per cent at all time horizons. This check should ensure the robustness of the finding that a significant portion of exchange rate variability is driven by bond risk premia.

We ran Dai and Singleton's (2002) LPY tests to assess the ability of the model to reproduce both the historical and the risk-neutral dynamics of yields and we found no evidence of mis-specification.

Finally, we checked for structural stability during the 1983-1998 sample and we did not come across any obvious structural change.

5 Conclusions

We have proposed a no-arbitrage term structure model that allows to contemporaneously price bonds in two different countries, taking into account the dynamics of the exchange rate and of observable macroeconomic variables such as inflation and the output gap. In the model, the domestic pricing kernel and the exchange rate are specified exogenously and the foreign pricing kernel is derived endogenously. Thanks to this specification strategy, the model affords a considerable analytical tractability and allows to analyse bidirectional linkages between bond prices and the exchange rate. Estimating the model with US and German data over various sample periods, we have found that a significant portion of the variability of the exchange rate is accounted for by time-varying bond risk premia (more than 20 per cent at long time horizons). We have analyzed the dynamics of the estimated model to seek explanations for this finding. Impulse response analysis reveals that a currency tends to persistently appreciate when risk premia on long-term bonds denominated in that currency rise, that is when investors expect large capital gains on long-term bonds denominated in that currency. The *delayed overshooting* phenomenon found by many previous studies with reference to short-term policy rates seems to extend also to expected returns on long-term bonds. Furthermore, differences in bond risk premia between countries drive deviations from uncovered interest rate parity: the higher is the difference between bond risk premia in two countries, the more profitable is a carry trade strategy based on such difference. After controlling for macroeconomic variables, bond risk premia in one country have fairly limited influence on those of another country. Finally, exogenous shocks to the exchange rate have a negligible impact on the yield curves.

6 Appendix

6.1 Pricing formulae for foreign bonds

We derive the pricing formulae for foreign bonds recursively. The price of a bond expiring in one period is:

$$\begin{aligned}
q_t^1 &= \mathrm{E}_t^Q\left[\exp\left(-r_t\right)\frac{S_{t+1}}{S_t}q_{t+1}^0\right] &(21)\\
&= \mathrm{E}_t^Q\left[\exp\left(-r_t\right)\frac{S_{t+1}}{S_t}\right]\\
&= \mathrm{E}_t^Q\left[\exp\left(-r_t\right)\exp\left(s_{t+1}-s_t\right)\right]\\
&= \mathrm{E}_t^Q\left[\exp\left(-a-b^\intercal X_t+f^\intercal X_{t+1}-f^\intercal X_t\right)\right]\\
&= \mathrm{E}_t^Q\left[\exp\left(-a-b^\intercal X_t+f^\intercal\overline{\mu}+f^\intercal\overline{\rho}X_t+f^\intercal\Sigma\eta_{t+1}-f^\intercal X_t\right)\right]\\
&= \exp\left(-\left(a-f^\intercal\overline{\mu}\right)-\left(b^\intercal-f^\intercal\overline{\rho}+f^\intercal\right)X_t\right)\mathrm{E}_t^Q\left[\exp\left(f^\intercal\Sigma\eta_{t+1}\right)\right]\\
&= \exp\left(-\left(a-f^\intercal\overline{\mu}-\frac{1}{2}f^\intercal\Sigma\Sigma^\intercal f\right)-\left(b^\intercal-f^\intercal\overline{\rho}+f^\intercal\right)X_t\right)
\end{aligned}$$

Hence,

$$\begin{aligned}
z_t^1 &= -\ln\left(q_t^1\right)= &(22)\\
&= \left(a-f^\intercal\overline{\mu}-\frac{1}{2}f^\intercal\Sigma\Sigma^\intercal f\right)+\left(b^\intercal-f^\intercal\overline{\rho}+f^\intercal\right)X_t
\end{aligned}$$

so that:

$$\begin{aligned}
C_1 &= a-f^\intercal\overline{\mu}-\frac{1}{2}f^\intercal\Sigma\Sigma^\intercal f &(23)\\
D_1 &= b-\overline{\rho}^\intercal f+f
\end{aligned}$$

For foreign bonds expiring in n periods, we have:

$$\begin{aligned}
q_t^n &= \mathrm{E}_t^Q\left[\exp\left(-r_t\right)\frac{S_{t+1}}{S_t}q_{t+1}^{n-1}\right] \qquad (24)\\
&= \mathrm{E}_t^Q\left[\exp\left(-r_t+s_{t+1}-s_t\right)q_{t+1}^{n-1}\right]\\
&= \mathrm{E}_t^Q\left[\exp\left(-a-b^\intercal X_t+f^\intercal X_{t+1}-f^\intercal X_t-(n-1)\left(C_{n-1}+D_{n-1}^\intercal X_{t+1}\right)\right)\right]\\
&= \mathrm{E}_t^Q\left[\exp\left(-a-b^\intercal X_t+\left(f-(n-1)D_{n-1}\right)^\intercal\left(\overline{\mu}+\overline{\rho}X_t+\Sigma\eta_{t+1}\right)-f^\intercal X_t-(n-1)C_{n-1}\right)\right]\\
&= \mathrm{E}_t^Q\left[\exp\left(-a-b^\intercal X_t-(n-1)E_n^\intercal\left(\overline{\mu}+\overline{\rho}X_t+\Sigma\eta_{t+1}\right)-f^\intercal X_t-(n-1)C_{n-1}\right)\right]\\
&= \exp\left(-\left(a+(n-1)\left(E_n^\intercal\overline{\mu}+C_{n-1}\right)\right)-\left(b+f+(n-1)\overline{\rho}^\intercal E_n\right)^\intercal X_t\right)\\
&\quad\times\mathrm{E}_t^Q\left[\exp\left(-(n-1)E_n^\intercal\Sigma\eta_{t+1}\right)\right]\\
&= \exp\left(-\left(a+(n-1)\left(E_n^\intercal\overline{\mu}+C_{n-1}-\frac{n-1}{2}E_n^\intercal\Sigma\Sigma^\intercal E_n\right)\right)-\left(b+f+(n-1)\overline{\rho}^\intercal E_n\right)^\intercal X_t\right)
\end{aligned}$$

where:

$$E_n = D_{n-1}-\frac{1}{n-1}f$$

Since

$$\begin{aligned}
z_t^n &= -\frac{1}{n}\ln\left(q_t^n\right)\\
&= \frac{1}{n}\left(a+(n-1)\left(E_n^\intercal\overline{\mu}+C_{n-1}-\frac{n-1}{2}E_n^\intercal\Sigma\Sigma^\intercal E_n\right)\right)+\frac{1}{n}\left(b+f+(n-1)\overline{\rho}^\intercal E_n\right)^\intercal X_t
\end{aligned}$$

we get:

$$\begin{aligned}
C_n &= \frac{1}{n}\left(a+(n-1)\left(E_n^\intercal\overline{\mu}+C_{n-1}-\frac{n-1}{2}E_n^\intercal\Sigma\Sigma^\intercal E_n\right)\right)\\
D_n &= \frac{1}{n}\left(b+f+(n-1)\overline{\rho}^\intercal E_n\right)
\end{aligned}$$

6.2 Measurability of the currency depreciation rate

The standard argument used to derive the currency depreciation rate endogenously, as a function of the domestic and foreign pricing kernels, proceeds as follows (e.g.: Backus, Foresi and Telmer - 2001). Let ξ be the payoff at time $t+1$ to an asset denominated in the foreign currency. Its price $p(\xi)$, can be derived either using the domestic pricing measure Q:

$$\begin{aligned}
p(\xi) &= \mathrm{E}_t^Q\left[\exp\left(-r_t\right)\frac{S_{t+1}}{S_t}\xi\right] \qquad (25)\\
&= \mathrm{E}_t^P\left[\left.\frac{dQ}{dP}\right|_t\exp\left(-r_t\right)\frac{S_{t+1}}{S_t}\xi\right]
\end{aligned}$$

or the foreign pricing measure Q':

$$\begin{aligned} p(\xi) &= \mathrm{E}_t^{Q'}\left[\exp\left(-r_t^f\right)\xi\right] \\ &= \mathrm{E}_t^{P}\left[\left.\frac{dQ'}{dP}\right|_t \exp\left(-r_t^f\right)\xi\right] \end{aligned} \tag{26}$$

where $r_t^f = z_t^1$ is the foreign short rate. Assuming that the two above pricing relations hold for any asset ξ, which is P-mesurable and square integrable with respect to P, the equality

$$\mathrm{E}_t^{P}\left[\left.\frac{dQ}{dP}\right|_t \exp(-r_t)\frac{S_{t+1}}{S_t}\xi\right] = \mathrm{E}_t^{P}\left[\left.\frac{dQ'}{dP}\right|_t \exp\left(-r_t^f\right)\xi\right] \tag{27}$$

implies:

$$\left.\frac{dQ}{dP}\right|_t \exp(-r_t)\frac{S_{t+1}}{S_t} = \left.\frac{dQ'}{dP}\right|_t \exp\left(-r_t^f\right) \tag{28}$$

and:

$$\frac{S_{t+1}}{S_t} = \frac{\left.\frac{dQ'}{dP}\right|_t}{\left.\frac{dQ}{dP}\right|_t}\exp\left(r_t - r_t^f\right) \tag{29}$$

thanks to a basic fact from Hilbert space theory (see e.g. Luenberger - 1969, page 48, lemma 2). Hence, the currency depreciation rate is equal to the ratio between the foreign and the domestic pricing kernels.

There is a subtlety involved in the above derivation. In order for the expected value in (25) to have any meaning at all, the currency depreciation rate S_{t+1}/S_t must be an F-measurable random variable. This is by no means an innocuous assumption. Recent macro-finance studies (e.g.: Chabi-Yo and Yang - 2007 and Dong - 2006) define the probability space $(\Omega, \mathsf{F}, \mathsf{F}_t\ , P)$ implicitly by defining the law of motion of X_t: F_t is the filtration generated by X_t, $\mathsf{F} = \sigma\left(\bigcup_{n=0}^{\infty}\mathsf{F}_t\right)$, and P is the product measure derived from the transition densities of X_t; hence, assuming that the currency depreciation rate be F-measurable is tantamount to saying that the randomness in S_t is completely explained by the randomness in X_t. While this is obviously true if X_t explicitly includes the currency depreciation rate, it is hardly a realistic assumption otherwise, as shown by the empirical evidence provided by, among others, Inci and Lu (2004).

6.3 Expected excess holding period returns

Remember that the expected excess holding-period excess return on a domestic bond (from the perspective of a domestic investor) is:

$$e_t^{n,h} = \mathrm{E}_t^P \left[\ln\left(p_{t+h}^{n-h}\right) - \ln\left(p_t^n\right)\right] + \ln\left(p_t^h\right)$$

Given that bond prices are exponential affine functions of the state variables, we obtain, by substituting the pricing formulae of Section 2:

$$\begin{aligned} e_t^{n,h} &= \mathrm{E}_t^P \left[-\left(n-h\right)\left(A_{n-h} + B_{n-h}^\top X_{t+h}\right) + n\left(A_n + B_n^\top X_t\right)\right] - h\left(A_h + B_h^\top X_t\right) \\ &= -\left(n-h\right)\left(A_{n-h} + B_{n-h}^\top \mathrm{E}_t^P\left[X_{t+h}\right]\right) + n\left(A_n + B_n^\top X_t\right) - h\left(A_h + B_h^\top X_t\right) \end{aligned}$$

Since

$$\mathrm{E}_t^P\left[X_{t+h}\right] = \mu + \rho X_{t+h-1}$$

and

$$\mathrm{E}_t^P\left[X_t\right] = X_t$$

by recursive substitution one obtains:

$$\mathrm{E}_t^P\left[X_{t+h}\right] = \nu_h + \rho^h X_t$$

where

$$\nu_h = \mu + \rho\mu + \ldots + \rho^{h-1}\mu$$

Hence,

$$e_t^{n,h} = J_{n,h} + K_{n,h}^\top X_t$$

where

$$\begin{aligned} J_{n,h} &= -\left(n-h\right)\left(A_{n-h} + B_{n-h}^\top \nu_h\right) + nA_n - hA_h \\ K_{n,h}^\top &= -\left(n-h\right)B_{n-h}^\top \rho^h + nB_n^\top - hB_h^\top \end{aligned}$$

Analogous algebra yields the expressions for the holding-period return on a foreign bond (from the perspective of a foreign investor):

$$f_t^{n,h} = Q_{n,h} + W_{n,h}^\top X_t$$

where

$$
\begin{aligned}
Q_{n,h} &= -(n-h)\left(C_{n-h} + D_{n-h}^{\top}\nu_h\right) + nC_n - hC_h \\
W_{n,h}^{\top} &= -(n-h)\, D_{n-h}^{\top}\rho^h + nD_n^{\top} - hD_h^{\top}
\end{aligned}
$$

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Table 1 - Coefficient estimates

Equation	μ	Coefficients on lagged variables								
		Ex. r.	Ger. I.	US I.	Ger. O.	US O.	Lt 1	Lt 2	Lt 3	Lt 4
Exch. r.	0.057	0.916	0.006	0.002	0.001	-0.006	-0.003	0.004	0.001	0.001
	(2.129)	(42.96)	(2.361)	(0.678)	(0.581)	(-2.257)	(-2.910)	(4.225)	(1.133)	(1.664)
Ger. Infl.	0.169	-0.193	0.935	0.036	0.000	-0.002	-0.006	-0.005	-0.001	0.007
	(0.260)	(-0.911)	(40.21)	(1.116)	(0.029)	(-0.077)	(-0.584)	(-0.491)	(-0.009)	(0.861)
US Infl.	0.893	-0.473	-0.033	0.907	-0.021	0.037	-0.030	0.001	0.005	0.013
	(4.115)	(-2.675)	(-1.711)	(33.87)	(-1.886)	(1.680)	(-3.616)	(0.160)	(0.780)	(1.862)
Ger. Outp.	-0.408	0.994	-0.006	-0.088	0.702	0.076	0.000	-0.103	0.041	-0.046
	(-0.407)	(1.214)	(-0.063)	(-0.712)	(13.54)	(0.746)	(0.008)	(-2.570)	(1.346)	(-1.485)
US Outp.	0.793	-0.564	0.026	-0.057	-0.012	0.904	-0.036	0.032	0.011	0.015
	(2.707)	(-2.364)	(0.978)	(-1.591)	(-0.778)	(30.59)	(-3.175)	(2.775)	(1.204)	(1.680)
Latent 1	1.317	-0.644	0.038	-0.151	0.038	0.008	0.948	0.027	0.014	0.021
	(1.568)	(-0.941)	(0.502)	(-1.453)	(0.882)	(0.098)	(29.50)	(0.819)	(0.559)	(0.813)
Latent 2	-0.784	1.231	-0.112	-0.058	-0.060	0.078	0.055	0.903	-0.060	-0.039
	(-0.934)	(1.800)	(-1.491)	(-0.557)	(-1.387)	(0.922)	(1.726)	(26.98)	(-2.382)	(-1.510)
Latent 3	0.821	-0.945	0.053	-0.025	0.022	0.224	-0.028	0.055	0.929	0.000
	(0.977)	(-1.381)	(0.712)	(-0.242)	(0.516)	(2.641)	(-0.879)	(1.655)	(36.67)	(0.002)
Latent 4	-1.741	1.659	0.071	0.012	0.084	0.052	0.063	-0.054	0.012	0.923
	(-2.074)	(2.424)	(0.951)	(0.114)	(1.928)	(0.616)	(1.972)	(-1.621)	(0.489)	(35.31)

The table reports coefficient estimates and t-statistics for the vector autoregression (13). Each row corresponds to a state variable. Each column corresponds to each of the lagged state variables on the right hand side of the equations of the autoregression. The column labelled μ reports the estimates of the vector of intercepts. The sample period is 1983:01 to 1998:12.

Table 2 - Standard deviation of the pricing errors (in basis points)

Maturity (years)	1	2	3	4	5	6	7	8	9	10
German bonds	8	4	5	5	4	3	2	2	3	5
US bonds	6	3	4	5	4	4	4	6	5	8

The table reports the standard deviations of the pricing errors of equation (15). The standard deviations (expressed in basis points) measure the average distance between observed yields and model-implied yields. Each column corresponds to a bond maturity. The sample period on which estimates are based is 1983:01 to 1998:12.

Table 3 - Regressions of realized on model-implied expected excess returns

	German bonds			
Bond maturity	1	3	5	10
R^2	_	0.456	0.428	0.359
α (t-stat)	_	-0.915 (-1.652)	-1.580 (-1.470)	-2.491 (-1.017)
β (t-stat)	_	1.550 (5.850)	1.568 (5.307)	1.591 (3.710)
χ^2 $[\alpha = 0, \beta = 1]$ (p-val)	_	4.328 (0.115)	3.723 (0.156)	1.958 (0.376)
	US Bonds			
Bond maturity	1	3	5	10
R^2	_	0.355	0.316	0.317
α (t-stat)	_	-0.3817 (-0.647)	-0.856 (-0.777)	-3.337 (-1.256)
β (t-stat)	_	1.275 (3.492)	1.350 (3.185)	1.743 (3.512)
χ^2 $[\alpha = 0, \beta = 1]$ (p-val)	_	0.609 (0.738)	0.779 (0.678)	2.360 (0.307)
	US bonds for a German investor			
Bond maturity	1	3	5	10
R^2	0.409	0.437	0.444	0.409
α (t-stat)	0.033 (0.014)	-2.099 (-0.94)	-3.755 (-1.699)	-4.830 (-2.091)
β (t-stat)	1.181 (5.259)	1.352 (5.763)	1.419 (5.860)	1.243 (5.447)
χ^2 $[\alpha = 0, \beta = 1]$ (p-val)	0.741 (0.690)	2.493 (0.288)	4.564 (0.102)	4.810 (0.090)

The table reports estimates (t-statistics in parentheses) of equation (16), together with R^2, and χ^2 statistics (p-values in parentheses) for the null of unbiasdness. The equation is a linear regression of excess bond returns observed during the sample period on a constant and expected excess returns derived with our no-arbitrage model. Newey-West standard errors with truncation at 18 lags are used to take into account the serial correlation induced by overlapping returns. The sample period is 1983:01 to 1998:12.

Table 4 - Decomposition of the variance of the exchange rate

Proportion of the variance accounted for by shocks to:	Forecast horizon 12 months	36 months	60 months	120 months
German inflation	1.48%	1.19%	2.44%	3.67%
US inflation	0.66%	4.57%	6.26%	7.38%
German output gap	0.81%	4.03%	4.61%	4.86%
US output gap	2.74%	5.28%	7.27%	7.65%
German interest rate	2.87%	4.52%	3.70%	3.26%
US interest rate	27.17%	27.96%	25.19%	24.96%
German risk premium	2.19%	1.28%	1.72%	1.90%
US risk premium	6.98%	18.93%	21.85%	22.52%
Exchange rate	55.11%	32.23%	26.97%	23.78%

The table lists the contribution of each of the state variables in the vector autoregression (17) to the forecast variance of the exchange rate, for various forecast horizons. The Choleski ordering for the variance decomposition is the same as the order of appearance of the variables in the table (from German inflation to the exchange rate). The sample period on which estimates are based is 1983:01 to 1998:12.

Table 5 - Decomposition of the variance of risk premia

	Proportion of variance of the German risk premium explained by shocks to:				
Forecast horizon	Exchange rate	Macro variables	Interest rates	German risk pr.	US risk pr.
12 months	1.08%	16.17%	7.35%	67.88%	1.54%
36 months	1.22%	30.29%	8.45%	58.20%	1.83%
60 months	1.87%	29.88%	11.50%	54.93%	1.82%
120 months	2.92%	29.46%	13.76%	52.02%	1.84%
	Proportion of variance of the US risk premium explained by shocks to:				
Forecast horizon	Exchange rate	Macro variables	Interest rates	German risk pr.	US risk pr.
12 months	0.12%	64.69%	19.12%	0.50%	15.56%
36 months	0.20%	52.93%	26.41%	0.39%	20.07%
60 months	0.38%	47.59%	31.55%	0.33%	20.16%
120 months	0.90%	43.14%	35.86%	0.36%	19.74%

The table lists the contribution of each of the state variables in the vector autoregression (17) to the forecast variance of bond risk premia, for various forecast horizons. The Choleski ordering for the variance decomposition is the same as the order of appearance of the variables in the table (from the exchange rate to US risk premia). The sample period on which estimates are based is 1983:01 to 1998:12.

Table 6 - Affine representation of foreign exchange risk premia

Maturities of the bonds used to set up the carry trade	1	3	5	10
Constant	13.77	11.03	9.66	13.77
Exchange rate	-49.37	-47.86	-47.90	-49.37
Domestic inflation	-0.30	0.56	0.83	-0.30
Foreign inflation	4.93	4.68	4.44	4.93
Domestic output gap	-1.46	-1.26	-1.26	-1.46
Foreign output gap	-6.59	-5.80	-5.72	-6.59
Domestic 1-year interest rate	-2.28	-1.94	-1.67	-2.28
Domestic 10-year risk premium	1.89	1.52	1.45	1.89
Interest rate differential	2.79	3.18	3.45	2.79
Risk premium differential	2.19	2.11	2.29	3.19

The table reports the loadings on the set of state variables in the affine representation of foreign exchange risk premia (equation 18). Each column in the table refers to a different maturity. The sample period on which estimates are based is 1983:01 to 1998:12.

Table 7 - Structural break tests for individual equations

	Break after 1982		Break after 1998	
Equation	Chow B.P.	CUSUM	Chow B.P.	CUSUM
Exchange rate	Yes	No	Yes	Yes
German inflation	No	Yes	No	No
US inflation	Yes	Yes	Yes	No
German output gap	Yes	No	Yes	No
US output gap	Yes	Yes	Yes	No
German interest rate	Yes	No	No	No
US interest rate	Yes	No	No	Yes
German risk premium	Yes	No	Yes	No
US risk premium	No	No	Yes	No

The table reports the results of test for structural change for the single equations of the vector autoregression (17). The null hypothesis is: no structural change. The confidence level is 95 per cent. Yes indicates a rejection, No a failure to reject the hypothesis of stability. The test for a break after 1982 is conducted on the 1974:1998 sample. The test for a break after 1998 is conducted on the 1983:2007 sample.

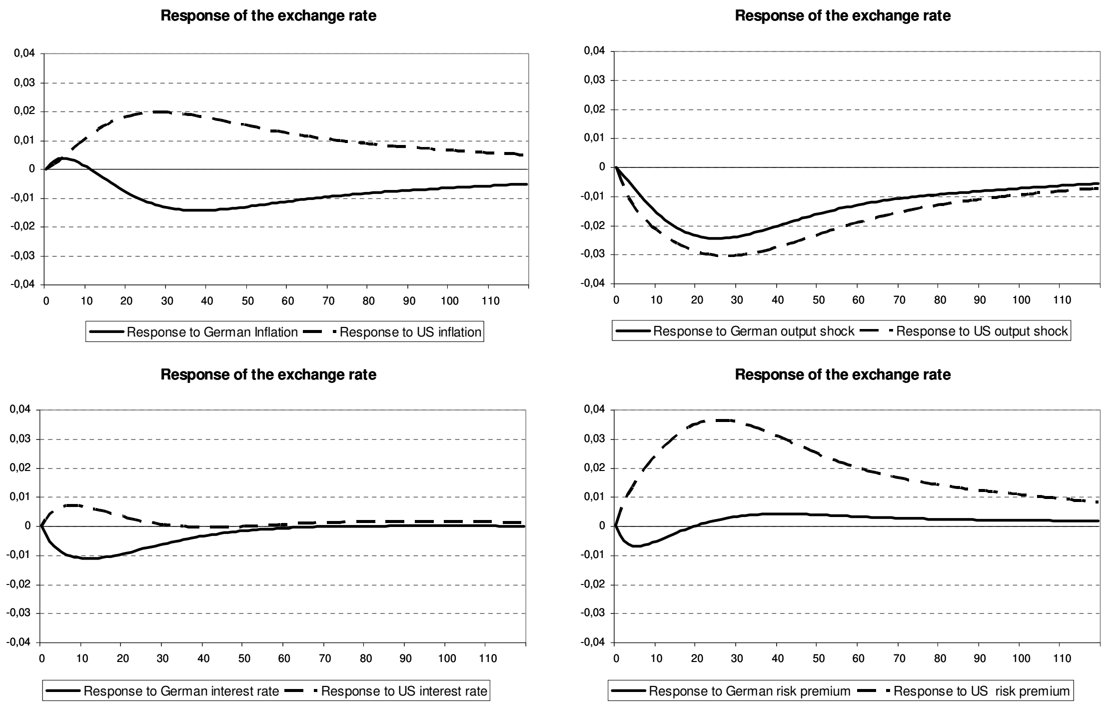

Figure 1 - Responses of the exchange rate to non-factorized one standard deviation shocks to the other state variables. Time (expressed in months) is reported on the x-axis of each plot.

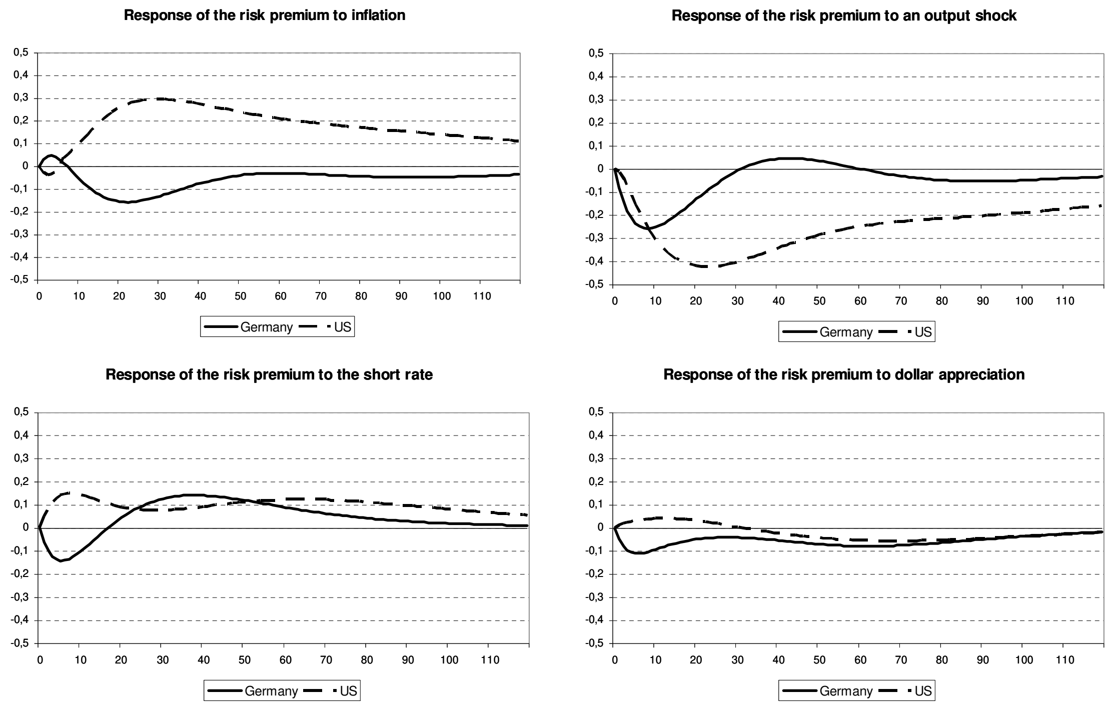

Figure 2 - Responses of risk premia to non-factorized one standard deviation shocks to the other state variables. Time (expressed in months) is reported on the x-axis of each plot.