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Abstract

This paper studies different types of commitment policy in an economy where the deterministic steady state is inefficient. We show how a policy suggested by the approach of policy design entails positive long-run inflation, even in the purely forward-looking canonical New Keynesian model. The long-run inflation target is robust to inflation persistence due to backward-looking rule-of-thumb behaviour by price setters. The optimal long-run inflation target is positive in all but one of the six theoretical cases studied. We evaluate policies on the basis of both the deterministic equilibrium and the stochastic equilibrium and present robustness analysis in terms of two structural parameters.

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1 Introduction

This paper studies optimal monetary policy within New Keynesian models, the canonical purely forward-looking model and its variant due to backward-looking rule-of-thumb behaviour by a fraction of price setters. The topic of what constitutes optimal monetary policy in microfounded models with nominal price rigidities, which Clarida et al. (1999) intriguingly label the science of monetary policy from a New Keynesian perspective, has been the subject of a very large body of literature.

On the one hand, the literature has emphasised that discretionary conduct of policy leads, in addition to the well-known inflation bias stressed by Kydland and Prescott (1977) and Barro and Gordon (1983), to the so-called stabilisation bias. Woodford (1999, 2003, Chapter 7) and Clarida et al. (1999) discuss how a central bank that is able to credibly commit can influence private sector expectations in a way that leads to more favorable responses to cost-push shocks. In particular, Woodford (1999, 2003, Chapter 7) shows that optimal policy under commitment is history dependent whereas discretionary policy is purely forward-looking. The logic behind the optimality of history dependence is quite intuitive. In an economy where private sector expectations are formed rationally, commitment by the central bank can influence these expectations only if the central bank's earlier commitments are sustained in later periods. Hence, successful steering of private sector expectation requires that central bank's conduct in later periods depends not only on the current state of the economy but also on the state of the economy in earlier periods.

On the other hand, within the commitment class of policy, the timeless perspective optimality concept proposed by Woodford (1999) has received a great deal of attention[1]. Policy is labelled timeless as it reflects a type of commitment that, unlike the zero-optimal policy, constraints the policy reaction function to be the same in the initial period as in all succeeding periods. By ignoring the temptation of exploiting the expectations existing in the economy, commitment policy is timeless, namely time-invariant, as the central bank abides to the same rule of conduct in all periods, including the one in which the policy is introduced. Indeed, the commitment class of policy is time-inconsistent from the perspective of Kydland and Prescott (1977), but timeless perspective commitment policy achieves time invariance whereas the zero-optimal commitment policy, although being truly optimal given the policy problem under commitment, implies that central bank's behaviour in the initial period differs from central bank's behaviour in all subsequent periods. The zero-optimal policy and the timeless perspective policy in fact differ only with respect to the central bank's posited behaviour in the initial period. Given the intertemporal nature of the aggregate-

[1] This solution has been used many times since; for example, Clarida et al. (1999), King and Wolman (1999), McCallum and Nelson (2004), Rotemberg and Woodford (1999), Svensson and Woodford (2003), and Walsh (2003), to name but a few.

supply relation, it follows that the two policies imply different transition paths for the endogenous variables to the same optimal long-run values, that is the zero-optimal policy and the timeless perspective policy share the same optimal long-run inflation target.

More recently, Blake (2001) and Jensen and McCallum (2002) show that timeless perspective policy, as usually described, is not optimal in the sense of Taylor (1979). Taylor (1979) proposes adopting a monetary policy that, given complete knowledge, in terms of both structural equations and exogenous shock processes, of the structure of an economy characterised by rational expectations, is optimal on average. Monetary policy is optimal on average if it yields the smallest unconditional expectation of the central bank's objective function, which, following Woodford (2001), is derived as an approximation to the utility of the representative household[2]. Blake (2001) and Jensen and McCallum (2002) expose how an alternative timeless perspective policy suggested by the approach of policy design delivers superior results, in terms of the unconditional expectation of the central bank's objective function, than the timeless perspective policy à la Woodford (1999), which is instead implied by the approach of optimal control[3]. Specifically, they both compare the two timeless perspective policies on the basis of the stochastic equilibrium and consider a canonical New Keynesian model, namely a model where the aggregate-supply relation, often referred to as the New Keynesian Phillips curve (NKPC)[4], is purely forward-looking and it includes a cost-push shock term.

In light of the importance of the Phillips curve for the conduct of monetary policy, it is surprising how little work has analysed these alternative commitment policies in an economy where inflation depends on a convex combination of expected future inflation and lagged inflation. This is particularly surprising given the NKPC's failure to capture the fact that inflation is persistent, and the current emphasis on the analysis of robustness of different types of commitment policy.

This paper aims to partially fill this gap by studying these different types of commitment policy in the purely forward-looking canonical New Keynesian economy and its alterations due to backward-looking rule-of-thumb behaviour by a fraction of price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003). In so doing, differently from Blake (2001) and Jensen and McCallum (2002), we consider the empirically realistic case of an economy where the deterministic steady state is inefficient

[2]Woodford (2003, Chapter 6) provides the most detailed description of the utility-based framework for the evaluation of monetary policy.

[3]The same remark in Jensen and McCallum (2002) applies here, policy design means, as in Prescott (1977), a procedure that entails choosing the optimal parameters in the policy rule after having solved the model with a policy rule that is function of all relevant state variables. In what follows, we refer to the timeless perspective commitment policy put forward by Blake (2001) and Jensen and McCallum (2002) as being the alternative timeless perspective policy whereas by standard timeless perspective policy we mean the timeless perspective commitment policy à la Woodford (1999).

[4]Aggregate-supply relation and Phillips curve are used interchangeably in this paper.

as the distortions due to monopolistic competition are not offset by subsidies to production. For every theoretical economy we describe each commitment policy in terms of target criterion and optimal long-run inflation target. With this latter respect, we derive six optimal long-run inflation targets thus confronting the policy design approach to commitment policy vis-à-vis the optimal control approach.

We show how the timeless perspective commitment policy suggested by the approach of policy design entails optimal positive long-run inflation target, even in the purely forward-looking basic New Keynesian model. The intuition for this result is quite neat, hinging on the discount factor. What drives the optimality of zero steady-state inflation under Woodford's (1999) timeless perspective commitment policy, or the associated zero-optimal commitment policy, is the fact that the monetary authority shares the private sector discount factor. The output cost resulting from the anticipation of higher inflation occurs earlier in time and it is thus weighted more strongly than the output benefit stemming from higher inflation. However, expected future inflation enters the NKPC with a coefficient that is smaller than the unitary coefficient on actual inflation, with the consequence that the two effects brought about by higher inflation exactly offset each other. Hence, despite the inefficiency of the nonstochastic steady state and the existence of a positively sloped long-run Phillips-curve trade-off, there is no long-run incentive for positive inflation and optimal steady-state inflation is zero[5]. Under the alternative timeless perspective policy, the central bank's discount factor differs from the private sector's discount factor: the private sector prefers current consumption to future consumption whereas the central bank does not discount future, namely every loss is equally costly for the central bank. On the one hand, the private sector's discount factor appears in the model's structural equations, thus resulting in the long-run Phillips curve trade-off. On the other hand, the central bank now equally weighs the increase in output caused by higher inflation and the cost of the reduction in output as a result of expected higher inflation. It follows that the stimulative effect of higher inflation on output is greater than the output cost of higher inflation. The long-run Phillips curve trade-off is then exploited and it is optimal for the central bank to commit to positive steady-state inflation.

With this respect, the paper can be read as a second answer to what causes positive inflation to be endogenously optimal in the long-run. It can be either optimality in the sense of Taylor (1979), which is achieved by the commitment policy suggested by the approach of policy design, or, as shown in Pontiggia (2007), costly disinflation. The former in fact rationalises a positive optimal long-run inflation target even in a model where the central bank is in theory capable of disinflating without incurring any loss in output. Indeed, the optimal long-run inflation target is zero in only one of the six theoretical cases studied.

[5] Steady-state and long-run are used interchangeably in this paper.

Moreover, differently from the standard timeless perspective commitment policy, the alternative timeless perspective commitment policy is robust to the introduction of backward-looking rule-of thumb behaviour à la Galì and Gertler (1999) by this implies that all price setters behave identically once shocks are eliminated from the economy. Indeed, Galì-Gertler's backward-looking rule-of-thumb behaviour does not alter the steady state that would obtain under forward-looking behaviour by all price setters, neither in terms of long-run Phillips-curve trade-off nor in terms of steady-state central bank's loss function. This arguably desirable property does not apply in the case of backward-looking rule-of-thumb behaviour à la Steinsson (2003) as indexation to past output gap by rule-of-thumb price setters changes both the long-run trade-off between output gap and inflation and the steady-state central bank's loss function.

We evaluate the alternative commitment policies both the on the basis of the deterministic equilibrium and on the basis of the stochastic equilibrium. In so doing, our main objective is, as in Jensen and McCallum (2002), to simply rank the alternative commitment policies. We present robustness analysis for ample ranges of two structural parameters rather than, as in Blake (2001) and Jensen and McCallum (2002), coefficients that are functions of structural parameters. Indeed, the coefficients in the economy's structural equations are functions of two key model's primitives: the average duration that an individual price is fixed, namely the degree of price stickiness, and the fraction of firms that reset prices in a backward-looking manner, that is the degree of rule-of-thumb behaviour.

We follow the cited papers and report average values of the central bank's objective function. Regardless of the details of price setting, zero-optimal commitment policy invariably delivers the highest level of welfare on the basis of the deterministic equilibrium and the alternative timeless perspective policy is univocally superior to the standard timeless perspective policy. Moreover, the timeless perspective commitment policy suggested by the approach of policy design always implies better welfare on the basis of the deterministic equilibrium than a policy of zero inflation at all times, whereas the same it is not always true under the timeless perspective commitment policy implied by the approach of optimal control. As for welfare on the basis of the stochastic equilibrium, in the presence of backward-looking rule-of-thumb behaviour by price setters, we also analyse welfare on a conditional basis. The ranking between the three alternative commitment policies is not strictly univocal, but the alternative timeless perspective policy is usually found to rank first, followed by the standard timeless perspective policy and both timeless perspective policies outperform the zero-optimal policy.

The remainder of the paper is organised in three sections. Section 2 formalises the policy problem and characterises the three commitment policies discussed in the literature in terms of target criterion and optimal long-run inflation target. Section 3 compares the three policies on the basis of both the deterministic equilibrium and the stochastic equilibrium and presents robustness analysis in terms of the two key model's primitives. Section 4 provides concluding remarks.

2 Model and Policy Description

We consider the *basic neo-Wicksellian model* in Woodford (2003) and its alteration due to backward-looking rule-of-thumb behaviour by a fraction of price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003)[6]. The representative household's period utility, $U_t \equiv u(C_t; \xi_t) - \int_0^1 v(h_t(i); \xi_t) di$[7], can be approximated to second order as in Woodford (2003, 2.13, p. 396)

$$U_t = -\frac{\overline{Y}\widetilde{u}_c}{2}\left[(\sigma^{-1} + \varpi)(x_t - x^*)^2 + (1 + \varpi\theta)\theta var_i \log p_t(i)\right] + t.i.p + O\left(\left\|\Phi_y, \widetilde{\xi}, \varrho\right\|^3\right) \tag{1}$$

where $\sigma^{-1} > 0$ measures the inverse of the intertemporal elasticity of substitution of aggregate expenditure, $\varpi > 0$ denotes the elasticity of real marginal cost with respect to firm's own output, and $\theta > 1$ is the elasticity of substitution between goods. The term $t.i.p$ collect terms that are independent of monetary policy, namely irrelevant to the welfare ranking of alternative equilibria. The third-order residual is a function of the parameter, Φ_y, which summarises the distortions in the natural steady-state level of output, denoted by $\overline{Y}$, that are brought about by monopolistic competition, the vector of exogenous disturbances, $\widetilde{\xi}$, and the parameter vector that indexes aspects of policy that determine steady-state values of inflation and output gap, ϱ[8]. By output gap, x_t, we mean the log-difference between the actual level of output and the natural level of output, that is the equilibrium level of output that obtains under monopolistic competition and perfectly flexible prices. x^* denotes the steady-state efficiency gap, namely the constant over time gap between the steady-state efficient level of output, which is the equilibrium level of output that obtains under perfect competition and perfectly flexible prices, and the steady-state natural level of output.

[6] A description of the model used here, apart from Woodford (2003), can also be found in Pontiggia (2007), which presents detailed derivations of the aggregate-supply relations and the central bank's loss functions under backward-looking rule-of-thumb behaviour by price setters. Indeed, the hybrid Phillips curve and the central bank's objective in the case of backward-looking rule-of-thumb behaviour a là Steinsson (2003) correct the ones reported in Steinsson (2003).

[7] The period utility is postulated to be separable in consumption and labour effort. C_t is the Dixit and Stiglitz (1977) aggregate of the household's consumption of each of the individual goods that are supplied, which are indexed by i over the unit interval. ξ_t is a vector of exogenous real shocks, namely exogenous shocks to household's impatience to consume and to the household's willingness to supply labour. $h_t(i)$ is the supply of type i labour. For any given realisation of ξ_t, the period utility function, $u(C_t; \xi_t)$, is assumed to be concave and strictly increasing in C_t whereas the period disutility of supplying labour of type i, $v(h_t(i); \xi_t)$, is assumed to be convex and increasing in $h_t(i)$.

[8] $\widetilde{u}_c$ denotes the partial derivative, evaluated at steady state, of the indirect utility function, $\widetilde{u}(Y_t; \widetilde{\xi}_t)$, with respect to the level of production.

In addition to stabilising output gap, around a level that exceeds the inefficient natural level of output by the steady-state efficiency gap, it is hence also appropriate for monetary policy to aim to curb price dispersion across goods, $var_i \log p_t(i)$. This is achieved by stabilising the aggregate price level, but how fluctuations in the general price level affect price dispersion, hence welfare, depend upon the details of the price setting.

Following the theoretical literature on optimal monetary policy, we assume that the central bank's policy instrument is the short-term nominal interest rate, which reflects the actual practice of monetary policy by central banks. The combination of cashless economy, namely there are no costs associated with varying the nominal interest rate, and central's bank control of the nominal interest rate implies that the policy problem is fully characterised by the aggregate-supply relation and the central bank's loss function. Given the central bank's optimal choices for inflation and output gap, the aggregate-demand relation simply determines the path of nominal interest rate necessary to achieve the optimal path for the output gap.

On the one hand, both the central bank's loss function, L_t, and the aggregate-supply relation depend upon the details of the price setting. On the other hand, regardless of the details of the price setting, the discounted sum of utility of the representative household equals to second order the discounted sum of central bank's loss function. The utility-based central bank's objective function at an arbitrary time $t = 0$ is in fact given by

$$W = E_0 \sum_{t=0}^{\infty} \beta^t U_t = -\Omega(1-\beta) E_0 \sum_{t=0}^{\infty} \beta^t L_t + t.i.p + O\left(\left\| \Phi_y, \widetilde{\xi}, \varrho, \Delta_{-1}^{1/2} \right\|^3 \right) \tag{2}$$

where E_0 denotes the expectations operator conditional on information up through date 0, $0 < \beta \leq 1$ is the discount factor and the constant Ω is given by $\Omega = \overline{Y}\widetilde{u}_c(\sigma^{-1} + \varpi)\theta/2\kappa$[9].

Generally, a central bank acting under commitment faces the problem of minimising welfare losses from period 0 forward subject to the constraint that the evolution of inflation and output gap represent a possible rational-expectation equilibrium. Woodford (1999), Clarida et al. (1999), and Erceg et al. (2000), among others, follow Taylor (1979) in measuring policy performance by reporting average values of the welfare criterion (2), that is values of the unconditional expectation of (2). The law of iterated expectations entails that welfare is still given by (2) where the conditional expectation operator E_0 is replaced by the unconditional expectation operator E, namely $W = -\Omega(1-\beta)E\sum_{t=0}^{\infty}\beta^t L_t$. Except for discounting, this measure corresponds to the unconditional expectation of the central bank's loss function. Indeed, Blake (2001) and Jensen and McCallum (2002) point out that the undiscounted problem, under which the central

[9] The third-order residual is also a function of the initial degree of price dispersion, Δ_{-1}.

bank minimises $W = -\Omega(1-\beta)E\sum_{t=0}^{\infty} L_t$, implies an alternative timelessly optimal rule which is globally optimal with respect to the unconditional expectation of the central bank's objective function.

In what follows, superscripts FL, GG, and S denote respectively the purely forward-looking New Keynesian model, the model with rule-of-thumb behaviour à la Galì and Gertler (1999), and the model with rule-of-thumb behaviour à la Steinsson (2003). Superscript Z denotes the zero-optimal commitment policy, superscript W designates Woodford's (1999) timeless perspective policy, and superscript BJM indicates the timeless perspective policy put forward by Blake (2001) and Jensen and McCallum (2002). We can now proceed to characterise each commitment policy in terms of target criterion and optimal long-run inflation target.

2.1 Basic New Keynesian Model

Given Calvo (1983) constraints on price setting, forward-looking behaviour by all price setters implies an aggregate-supply relation of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{3}$$

whereas the central bank's loss function is given by

$$L_t = \pi_t^2 + \lambda (x_t - x^*)^2 \tag{4}$$

where π_t is the aggregate inflation rate. The output gap coefficient, κ, in (3) is a function of the model's structural parameters, namely

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)(\sigma^{-1} + \varpi)}{(1+\varpi\theta)\,\alpha} \tag{5}$$

where $0 < \alpha < 1$ is the degree of price stickiness, that is the constant probability a firm faces in each period of not resetting her price. The coefficient on output fluctuations in (4) is given by $\lambda = \kappa/\theta$.

A central bank acting under commitment faces the problem of choosing bounded paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise the discounted sum of losses subject to the constraint that the sequences must satisfy (3) each period. The Lagrangian associated with this problem is of the form

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}\pi_t^2 + \frac{\lambda}{2}(x_t - x^*)^2 + \varphi_t \left[\pi_t - \beta\pi_{t+1} - \kappa x_t\right] \right\} \tag{6}$$

where φ_t is the Lagrangian multiplier associated with period t aggregate-supply relation[10]. Differentiating

[10] Note that in what follows, given the law of iterated expectations, we replace $E_t\pi_{t+1}$ with π_{t+1} when writing the period t

with respect to π_t and x_t, we get the two first-order conditions

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t} = 0 \Rightarrow \pi_t + \varphi_t - \varphi_{t-1} = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}_0}{\partial x_t} = 0 \Rightarrow \lambda(x_t - x^*) - \kappa\varphi_t = 0 \tag{8}$$

Under zero-optimal commitment policy, there is no fulfillment of the expectations existing at the time of the policy implementation, that is (7) in period 0 holds with $\varphi_{-1} = 0$. Hence, zero-optimal commitment policy is characterised by the optimality conditions (8) for all $t \geq 0$ and

$$\pi_t + \varphi_t = 0 \qquad t = 0 \tag{9}$$

$$\pi_t + \varphi_t - \varphi_{t-1} = 0 \qquad t \geq 1 \tag{10}$$

Combining (8) with (9) delivers the central bank's target criterion in period 0

$$\pi_0 = -\frac{\lambda}{\kappa}(x_0 - x^*) \tag{11}$$

whereas combining (8) with (10), the central bank in any period $t \geq 1$ behaves according to

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1}) \tag{12}$$

Woodford (2003) hence concludes

> *Thus it is optimal (from the point of minimizing discounted losses from date zero onward) to arrange an initial inflation, given that the decision to do so can have no effect upon expectations prior to date zero (if one is not bothered by the non-time-consistency of such a principle of action). The optimal policy involves positive inflation in subsequent periods as well, but there should be a commitment to reduce inflation to its optimal long-run value of zero asymptotically.*
> Woodford (2003, p. $414 - 5$).

Despite the inefficiency of the nonstochastic steady state, namely $x^* > 0$, and the existence of a positively sloped long-run Phillips-curve trade-off, as implied by (3) evaluated at steady state, there is in fact an advantage for having positive inflation only in period 0, whereas there is no long-run incentive for

aggregate-supply relation.

positive inflation. This is because the increase in output in any period caused by higher inflation in the same period, φ_t, is exactly offset by the cost of the reduction in output in the previous period as a result of expected higher inflation, φ_{t-1}. The steady-state efficiency gap thus enters (11), but it does not appear in (12). Hence, the optimal long-run inflation target is zero. Indeed, integrating forward the NKPC implies that, regardless of policy, the discounted sum of future output gaps can be rewritten as a function solely inflation at time 0, $\pi_0 = \kappa E_0 \sum_{t=0}^{\infty} \beta^t x_t$. Accordingly, the discounted sum of losses in (2) can be rewritten as

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\} = \lambda E_0 \left\{ \sum_{t=0}^{\infty} \beta^t x^{*2} \right\} - \frac{2x^*}{\theta} \pi_0 + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2 \right] \right\} \tag{13}$$

with the first term on the right-hand side indicating a loss that is independent of monetary policy. All the other terms, which depend on monetary policy, are minimised by choosing $\pi_t = 0$ each period, which implies $x_t = 0$ each period, except the one proportional to x^*. The presence of this term implies a welfare gain from an initial positive rate of inflation, but because it only applies to inflation in the initial period, it is optimal to commit to zero long-run inflation. Moreover, the linear term in π_0 affects the zero-optimal commitment policy for periods later than 0 as the NKPC implies an intertemporal linkage between current inflation and future inflation. The welfare gain resulting from positive π_0 can be obtained with less increase in period 0 output gap, x_0, thus resulting in less increase in λx_0^2, if it is associated with an increase in expected inflation at date one, $E_0\pi_1$. Given that the loss associated with $E_0\pi_1$ occurs later in time, and is thus weighted less strongly, the transition to zero-optimal inflation lasts for more than one period.

Woodford (1999) and Clarida et al. (1999) argue convincingly that zero-optimal commitment policy is not attractive as it implies time variance in terms of the policy reaction function. Indeed, Woodford (1999) puts forward another commitment policy, which he labels timeless perspective. The policy proposal is simple to outline. What makes the zero-optimal commitment policy time-variant is the separate treatment of initial period and all other periods. At time 0, the central bank sets inflation according to (11) and promises to set (12) at any later date. Yet, if a central bank reoptimised in any later period, it would find optimal to set inflation according to (11), updated to that period. By ignoring the conditions existing in the economy at the policy's implementation, commitment policy is in fact timeless, namely time-invariant, as it can be thought as a policy rule that was chosen in the distant past, and the current values of inflation and output gap are the values chosen form that earlier perspective to satisfy the two conditions (7) and (8). Woodford's timeless perspective commitment policy thus ignores the start-up condition (9) and the central bank's target criterion in all periods $t \geq 0$ is given by (12). Hence, despite $x^* > 0$, there is never advantage from having positive inflation.

More recently, it has been recognised that the use of (12) in all periods $t \geq 0$ is not optimal within the class of time-invariant policy rules. Specifically, Blake (2001) and Jensen and McCallum (2002) show that there is a slightly different policy rule that is globally optimal with respect to the unconditional expectation of the central bank's objective function. This alternative timeless perspective policy implies that the central bank solves the undiscounted constrained control problem, namely (6) becomes

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \left\{ \frac{1}{2}\pi_t^2 + \frac{\lambda}{2}(x_t - x^*)^2 + \varphi_t \left[\pi_t - \beta\pi_{t+1} - \kappa x_t\right] \right\} \tag{14}$$

Differentiating with respect to π_t and x_t, (8) is unaffected but instead of (7) we obtain

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t} = 0 \Rightarrow \pi_t + \varphi_t - \beta\varphi_{t-1} = 0 \tag{15}$$

From a timeless perspective, the central bank sets policy according to (8) and (15) in all periods $t \geq 0$. Combining the first order conditions, the central bank's target criterion is given by

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - \beta x_{t-1}) + \frac{(1-\beta)\lambda}{\kappa} x^* \tag{16}$$

Comparing (16) with (12), we note that this alternative time-invariant policy rule brings about an incentive for committing to positive inflation. Specifically, evaluating (16) at steady state delivers

$$\overline{\pi} + \frac{(1-\beta)\lambda}{\kappa}\overline{x} = \frac{(1-\beta)\lambda}{\kappa} x^* \tag{17}$$

Given the upward sloping relationship between steady-state output gap, $\overline{x}$, and steady-state inflation, $\overline{\pi}$, implied by (3)

$$\overline{x} = \frac{(1-\beta)}{\kappa}\overline{\pi} \tag{18}$$

the timeless perspective policy put forward by Blake (2001) and Jensen and McCallum (2002) entails positive steady-state inflation of the form

$$\overline{\pi}^{FLBJM} = \frac{(1-\beta)\kappa}{\theta\kappa + (1-\beta)^2} x^* \tag{19}$$

which collapses to zero in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$) and in the absence of steady-state distortions (i.e. $x^* = 0$).

The logic behind this result is quite intuitive, hinging on the discount factor. If the central bank shares the same discount factor of the private sector, the cost resulting from the anticipation of higher inflation occurs earlier in time and it is thus weighted more strongly (by a factor $1/\beta > 1$) than the benefit stemming from higher inflation (weighted by a factor 1). However, expected future inflation enters the NKPC with a coefficient β that is smaller than the unitary coefficient on actual inflation. Hence, as in (12), the increase in output in any period caused by higher inflation in the same period, φ_t, is offset by the cost of the reduction in output in the previous period as a result of expected higher inflation, φ_{t-1}. Accordingly, there is no long-run incentive for positive inflation and optimal steady-state inflation is zero. Under Blake (2001) and Jensen and McCallum (2002) timeless perspective policy, the central bank's discount factor differs from the private sector's discount factor. Specifically, the private sector prefers current consumption to future one ($\beta^{PS} \equiv \beta < 1$), whereas the central bank does not discount the future, namely every loss is equally costly for the central bank ($\beta^{CB} = 1$).On the one hand, the private sector's discount factor appears in the model's structural equations, thus resulting in the long-run Phillips curve trade-off. On the other hand, the central bank now equally weighs the increase in output in any period caused by higher inflation in the same period and the cost of the reduction in output in the previous period as a result of expected higher inflation. Hence, the stimulative effect of higher inflation on output is greater than the output cost of higher inflation. The long-run Phillips curve trade-off is then exploited and it is optimal for the central bank to commit to positive steady-state inflation.

2.2 Rule-of-thumb behaviour

We now proceed to compare the three alternative commitment policies when the Phillips curve becomes hybrid due to the presence of backward-looking rule-of-thumb price setters. We consider first rule-of-thumb behaviour à la Galì and Gertler (1999) and subsequently turn our attention to Steinsson's (2003) rule-of-thumb behaviour. In both cases, we formalise the policy problem and we characterise the three commitment policies in terms of target criterion and optimal long-run inflation target.

2.2.1 Rule-of-thumb behaviour à la Galì and Gertler

Given Calvo (1983) constraints on price setting, backward-looking rule-of-thumb behaviour à la Galì and Gertler (1999) by a fraction ω of firms implies an aggregate-supply relation of the form

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_1 x_t \tag{20}$$

whereas the central bank's loss function is given by

$$L_t = \pi_t^2 + \lambda_1(x_t - x^*)^2 + \lambda_2(\pi_t - \pi_{t-1})^2 \tag{21}$$

The coefficients in the hybrid Phillips curve are given by $\chi_f = \alpha\phi^{-1}$, , $\chi_b = \omega\phi^{-1}$, and $\kappa_1 = (1-\omega)\alpha\kappa\phi^{-1}$ with $\phi = [\alpha + \omega - (1-\beta)\omega\alpha]$. The coefficients in the central bank's loss function are given by $\lambda_1 \equiv \lambda$ and $\lambda_2 = \omega\,[(1-\omega)\alpha]^{-1}$.

A central bank acting under commitment faces the problem of choosing bounded paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise the discounted sum of losses subject to the constraint that the sequences must satisfy (20) each period. The Lagrangian associated with this problem is of the form

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}\left[\pi_t^2 + \lambda_1(x_t - x^*)^2 + \lambda_2(\pi_t - \pi_{t-1})^2\right] + \varphi_t\left[\pi_t - \chi_f\beta\pi_{t+1} - \chi_b\pi_{t-1} - \kappa_1 x_t\right]\right\} \tag{22}$$

Differentiating with respect to π_t and x_t, we get the two first-order conditions

$$\frac{\partial\mathcal{L}_0}{\partial\pi_t} = 0 \Rightarrow \pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \beta\lambda_2(\pi_{t+1} - \pi_t) + \varphi_t - \chi_f\varphi_{t-1} - \beta\chi_b\varphi_{t+1} = 0 \tag{23}$$

$$\frac{\partial\mathcal{L}_0}{\partial x_t} = 0 \Rightarrow \lambda_1(x_t - x^*) - \kappa_1\varphi_t = 0 \tag{24}$$

Under zero-optimal commitment policy, (23) in period 0 holds with $\varphi_{-1} = 0$. Hence, zero-optimal commitment policy is characterised by the optimality conditions (24) for all $t \geq 0$ and

$$\pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \beta\lambda_2(\pi_{t+1} - \pi_t) + \varphi_t - \beta\chi_b\varphi_{t+1} = 0 \qquad t = 0 \tag{25}$$

$$\pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \beta\lambda_2(\pi_{t+1} - \pi_t) + \varphi_t - \chi_f\varphi_{t-1} - \beta\chi_b\varphi_{t+1} = 0 \qquad t \geq 1 \tag{26}$$

Combining (24) with (25) delivers the central bank's target criterion in period 0

$$\pi_0 = \frac{1}{1 + \lambda_2(1+\beta)}\left\{\lambda_2\pi_{-1} + \beta\lambda_2\pi_1 + \frac{\lambda_1}{\kappa_1}\left[\beta\chi_b x_1 - x_0 + (1-\beta\chi_b)x^*\right]\right\} \tag{27}$$

whereas combining (24) with (26), the central bank in any period $t \geq 1$ behaves according to

$$\pi_t = \frac{1}{1 + \lambda_2(1+\beta)}\left\{\lambda_2\pi_{t-1} + \beta\lambda_2\pi_{t+1} + \frac{\lambda_1}{\kappa_1}\left[\beta\chi_b x_{t+1} + \chi_f x_{t-1} - x_t + (1-\beta\chi_b - \chi_f)x^*\right]\right\} \tag{28}$$

Under Woodford's timeless perspective commitment policy, the start-up condition (25) is ignored and the central bank's target criterion in all periods $t \geq 0$ is given by (28). Note that, as implied by (27) and (28), central bank's behaviour now depends on the economy's rate of inflation prior to the adoption of the optimal policy, π_{-1}. Moreover, given $x^* > 0$, there is an advantage for having positive inflation not only in period 0 but in any other period as well. Indeed, Pontiggia (2007) shows that the positive optimal long-run inflation target, which equally obtains under zero-optimal and the standard timeless perspective policy, is given by[11]

$$\overline{\pi}^{GGWZ} = \frac{(1-\alpha)(1-\beta)\omega\kappa}{(1-\omega)\alpha\theta\kappa + (1-\alpha)(1-\beta)^2\omega} x^* \tag{29}$$

Under Blake (2001) and Jensen and McCallum (2002) timeless perspective policy, the central bank solves the undiscounted constrained control problem, namely (22) becomes

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \left\{ \frac{1}{2} \left[\pi_t^2 + \lambda_1 (x_t - x^*)^2 + \lambda_2 (\pi_t - \pi_{t-1})^2 \right] + \varphi_t \left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_1 x_t \right] \right\} \tag{30}$$

Differentiating with respect to π_t and x_t, (24) is unaffected but instead of (23) we obtain

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t} = 0 \Rightarrow \pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \lambda_2(\pi_{t+1} - \pi_t) + \varphi_t - \chi_f \beta \varphi_{t-1} - \chi_b \varphi_{t+1} = 0 \tag{31}$$

From a timeless perspective, the central bank sets policy according to (24) and (31) in all periods $t \geq 0$. Combining the first order conditions, the central bank's target criterion is given by

$$\pi_t = \frac{1}{1+2\lambda_2} \left\{ \lambda_2 \pi_{t-1} + \lambda_2 \pi_{t+1} + \frac{\lambda_1}{\kappa_1} \left[\chi_b x_{t+1} + \chi_f \beta x_{t-1} - x_t + (1 - \chi_b - \chi_f \beta) x^* \right] \right\} \tag{32}$$

which again depends on the economy's rate of inflation prior to the adoption of the optimal policy.

Evaluating (32) at steady state delivers (17). This is because rule-of-thumb behaviour à la Galì and Gertler (1999) desirably implies that all price setters behave identically once shocks are eliminated from the economy. Indeed, Galì-Gertler's backward-looking rule-of-thumb behaviour does not alter the steady state that would obtain under forward-looking behaviour by all price setters, neither in terms of long-run Phillips-curve trade-off nor in terms of steady-state central bank's loss function. Hence, the alternative

[11] This is found by combining the target criterion with the hybrid Phillips curve, all evaluated at steady state. See Pontiggia (2007) for details of the derivation. Optimal steady-state inflation collapses to zero in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$), in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$), and in the absence of steady-state distortions (i.e. $x^* = 0$).

timeless perspective commitment policy is robust to the introduction of backward-looking rule-of thumb behaviour, when this is characterised as in Galì and Gertler (1999), with the optimal long-run inflation target being invariant to the degree of rule-of-thumb behaviour.

2.2.2 Rule-of-thumb behaviour à la Steinsson

Given Calvo (1983) constraints on price setting, backward-looking rule-of-thumb behaviour à la Steinsson (2003) by a fraction ω of firms implies an aggregate-supply relation of the form

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_2 x_t + \kappa_3 x_{t-1} \tag{33}$$

whereas the central bank's loss function is given by

$$L_t = \pi_t^2 + \lambda_1 (x_t - x^*)^2 + \lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right]^2 \tag{34}$$

with $\kappa_2 = \left[(1-\omega)\alpha\kappa - (1-\alpha)\alpha\beta\omega\delta\right]\phi^{-1}$ and $\kappa_3 = (1-\alpha)\omega\delta\phi^{-1}$. The structural parameter δ denotes the degree of indexation to lagged output gap by rule-of-thumb price setters.

A central bank acting under commitment faces the problem of choosing bounded paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise the discounted sum of losses subject to the constraint that the sequences must satisfy (33) each period. The Lagrangian associated with this problem is of the form

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\pi_t^2 + \frac{\lambda_1}{2}(x_t - x^*)^2 + \frac{\lambda_2}{2}\left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right]^2 \\ +\varphi_t \left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_2 x_t - \kappa_3 x_{t-1}\right] \end{array} \right\} \tag{35}$$

Differentiating with respect to π_t and x_t, we get the two first-order conditions

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t} = 0 \Rightarrow \left\{ \begin{array}{c} \pi_t + \varphi_t - \chi_f \varphi_{t-1} - \beta \chi_b \varphi_{t+1} \\ +\lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] - \beta\lambda_2 \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0 \tag{36}$$

$$\frac{\partial \mathcal{L}_0}{\partial x_t} = 0 \Rightarrow \lambda_1 (x_t - x^*) - \beta\lambda_2 (1-\alpha)\delta \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] - \kappa_2 \varphi_t - \beta\kappa_3 \varphi_{t+1} = 0 \tag{37}$$

Under zero-optimal commitment policy, (36) in period 0 holds with $\varphi_{-1} = 0$. Hence, zero-optimal commitment policy is characterised by the optimality conditions (37) for all $t \geq 0$ and

$$\pi_t + \lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] - \beta\lambda_2 \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] + \varphi_t - \beta\chi_b \varphi_{t+1} = 0 \qquad t = 0 \tag{38}$$

$$\left\{ \begin{array}{c} \pi_t + \varphi_t - \chi_f \varphi_{t-1} - \beta \chi_b \varphi_{t+1} \\ +\lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1}) \right] - \beta \lambda_2 \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t) \right] \end{array} \right\} = 0 \qquad t \geq 1 \tag{39}$$

Combining (37) with (38) delivers the central bank's target criterion in period 0

$$\pi_0(1+\lambda_2) = \lambda_2 \left(\pi_{-1} + (1-\alpha)\delta x_{-1} \right) + \frac{\lambda_1}{(1-\alpha)\delta}(x_0 - x^*) + \underbrace{(\chi_b - \frac{\kappa_3}{(1-\alpha)\delta})}_{0} \beta \varphi_1 - (\frac{\kappa_2}{(1-\alpha)\delta} + 1)\varphi_0 \tag{40}$$

whereas combining (37) with (39), the central bank in any period $t \geq 1$ behaves according to

$$\pi_t = \frac{1}{1+\lambda_2} \left\{ \lambda_2 \left(\pi_{t-1} + (1-\alpha)\delta x_{t-1} \right) + \frac{\lambda_1}{(1-\alpha)\delta}(x_t - x^*) - \left(\frac{\kappa_2}{(1-\alpha)\delta} + 1 \right) \varphi_t + \chi_f \varphi_{t-1} \right\} \tag{41}$$

Given the intertemporal nature of (37), the central bank's target criterion cannot in fact be expressed as a function of inflation and output gap only. The Lagrangian multiplier becomes an additional endogenous variable, but, in any period, the Lagrangian multiplier associated with the subsequent period aggregate-supply relation does not enter the target criterion. This is because, as shown in (40), the coefficient on the Lagrangian multiplier associated with the subsequent period hybrid Phillips curve is constantly equal to zero. It follows that analysing monetary policy in the presence of rule-of-thumb behaviour à la Steinsson (2003) requires considering the two first-order conditions separately.

Under Woodford's timeless perspective commitment policy, the start-up condition (38) is ignored and the central bank's target criterion in all periods $t \geq 0$ is given by (41). Note that, as implied by (41), central bank's behaviour now depends both on output gap and inflation existing prior to the adoption of the optimal policy, x_{-1} and π_{-1}. Moreover, given $x^* > 0$, there is an advantage for having positive inflation not only in period 0 but in any other period as well. Indeed, Pontiggia (2007) shows that the positive optimal long-run inflation target, which equally obtains under zero-optimal and the standard timeless perspective policy, is given by[12]

$$\overline{\pi}^{SWZ} = \frac{(1-\alpha)(1-\beta)\kappa\theta^{-1}\omega \left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta \right]}{\left\{ \begin{array}{c} (1-\omega)(1-\alpha)(\theta^{-1}-\delta)(1-\beta)^2\alpha\omega\kappa + \\ \left[(1-\omega)\alpha\kappa + (1-\alpha)^2\beta\omega\delta \right] \left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta \right] \end{array} \right\}} x^* \equiv \Psi x^* \tag{42}$$

[12] This is found by combining the first-order conditions with the hybrid Phillips curve, all evaluated at steady state. See Pontiggia (2007) for details of the derivation. Optimal steady-state inflation collapses to zero in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$), in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$), and in the absence of steady-state distortions (i.e. $x^* = 0$).

Under Blake (2001) and Jensen and McCallum (2002) timeless perspective policy, the central bank solves the undiscounted constrained control problem, namely (35) becomes

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{c} \frac{1}{2}\pi_t^2 + \frac{\lambda_1}{2}(x_t - x^*)^2 + \frac{\lambda_2}{2}\left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right]^2 \\ +\varphi_t \left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_2 x_t - \kappa_3 x_{t-1}\right] \end{array} \right\} \tag{43}$$

Differentiating with respect to π_t and x_t, both first-order conditions are affected. (36) becomes

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t} = 0 \Rightarrow \left\{ \begin{array}{c} \pi_t + \varphi_t - \chi_f \beta \varphi_{t-1} - \chi_b \varphi_{t+1} \\ +\lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] - \lambda_2 \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0 \tag{44}$$

whereas (37) becomes

$$\frac{\partial \mathcal{L}_0}{\partial x_t} = 0 \Rightarrow \lambda_1 (x_t - x^*) - \lambda_2 (1-\alpha)\delta \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] - \kappa_2 \varphi_t - \kappa_3 \varphi_{t+1} = 0 \tag{45}$$

From a timeless perspective, the central bank sets policy according to (44) and (45) in all periods $t \geq 0$. Combining the first order conditions, the central bank's target criterion is given by

$$\pi_t = \frac{1}{1+\lambda_2} \left\{ \lambda_2 \left(\pi_{t-1} + (1-\alpha)\delta x_{t-1}\right) + \frac{\lambda_1}{(1-\alpha)\delta}(x_t - x^*) - \left(\frac{\kappa_2}{(1-\alpha)\delta} + 1\right)\varphi_t + \chi_f \beta \varphi_{t-1} \right\} \tag{46}$$

which again depends on the economy's rate of inflation and output gap prior to the adoption of the optimal policy. (33) evaluated at steady state implies an upward sloping relationship between steady-state output gap and steady-state inflation of the form

$$\overline{x} = \frac{(1-\beta)(1-\omega)\alpha}{(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta}\overline{\pi} \equiv \Gamma \overline{\pi} \tag{47}$$

Combining (47) with the steady-state version of (44) and (45), the alternative timeless perspective policy implies positive steady-state inflation of the form[13]

$$\overline{\pi}^{SBJM} = \frac{\left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta\right](1-\beta)(1-\omega)\alpha\kappa}{\theta\left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta\right]^2 + \left[(1-\omega)\alpha\kappa + (1-\alpha)^2\delta^2\theta\omega\right](1-\beta)^2(1-\omega)\alpha} x^* \equiv \Theta x^* \tag{48}$$

If the rule-of thumb is characterised as in Steinsson (2003), the optimal long-run inflation target under the alternative timeless perspective commitment policy ceases to be the same as in the purely forward-

[13] Under $\delta = 0$, optimal steady-state inflation collapses to (19).

looking New Keynesian model. Steinsson's (2003) rule-of-thumb behaviour in fact alters both the long-run trade-off between output gap and inflation and the steady-state loss function that obtain in the absence of backward-looking behaviour.

Table 1 summarises the results as for the optimal long-run inflation target, evincing the relationship between the six targets derived. The optimal steady-state inflation target is indeed positive in all but one of the theoretical cases studied.

3 Quantitative Analysis

In evaluating welfare, our main objective is, as in Jensen and McCallum (2002), to simply rank the alternative commitment policies. Evaluating welfare requires specifying values for the economy's structural parameters. Four parameters (β, θ, ϖ, σ^{-1}) are chosen to equal those used in Woodford (2003, p. 431), which stem from the estimation results in Rotemberg and Woodford (1997). These values are given in Table 2. The steady-state efficiency gap, x^*, is accordingly set equal to 0.2[14]. In the absence of an empirical estimate for the degree of indexation to lagged output gap, δ, by rule-of-thumb price setters, we follow Steinsson (2003) and set it to 0.052.

The remaining two structural parameters are in fact the key model's primitives: the average duration that an individual price is fixed, namely the degree of price stickiness, α, and the fraction of firms that reset prices in a backward-looking manner, that is the degree of rule-of-thumb behaviour, ω. Galì and Gertler (1999) report estimates of ω between 0.077 and 0.552, with 3 of their 6 estimates between 0.2 and 0.3. As for the degree of price stickiness, empirically realistic values of the average price duration based on macroeconomic data vary between 2 and 5 quarters, namely $0.5 \leq \alpha \leq 0.8$. Available empirical estimates using microeconomic data, as in Bils and Klenow (2004), suggest instead a lower average price duration of around 1.5 quarters, that is a value of α of about 0.33. We thus consider $0.33 \leq \alpha \leq 0.8$ and extend the range for the degree of rule-of-thumb behaviour up to 0.7, namely $0.01 \leq \omega \leq 0.7$, as $\omega = 0.7$ implies that the hybrid Phillips curve under rule-of-thumb behaviour, regardless of its specification, puts equal weight on future expected inflation and lagged inflation. In the presence of rule-of-thumb behaviour by price setters, we pick $\alpha = 0.66$, namely an average price duration of 3 quarters, and $\omega = 0.3$ as our benchmark values because they both sit in the middle of the respective range of available empirical estimates[15].

[14] The value is implied by the definition of x^*, $x^* = \Phi_y/(\varpi + \sigma^{-1})$, with $\Phi_y = 1 - (1-\theta)/\theta$. See Pontiggia (2007) for further details.

[15] The results presented below are unaffected if one were to consider different benchmark estimates for both α and ω. Specifically, we have considered limiting values for ω (i.e. $\omega = 0.01$ and $\omega = 0.7$) when analysing robustness with respect to α and limiting values for α (i.e. $\alpha = 0.33$ and $\alpha = 0.8$) when studying robustness with respect to ω.

We compare the alternative commitment policies both on the basis of the deterministic equilibrium and on the basis of the stochastic equilibrium, with $\overline{W}$ denoting the level of welfare that obtains in the former. Specifically, we follow the cited papers and report average values of the central bank's objective function. That is, we evaluate alternative policies by taking the unconditional expectation of (2) with respect to the distribution of exogenous shocks.

We assume that inflation and output gap at time 0, as well as at time -1 under backward-looking rule-of-thumb behaviour by price setters, are at their unconditional expectation, namely at the long-run values implied by the policy under consideration. This ensures that the desirability of the chosen plan for inflation and output gap does not depend upon conditions at time 0 and at time -1. Moreover, when we compare policies on the basis of the stochastic equilibrium we also consider welfare conditional upon initial conditions. Indeed, Jensen and McCallum (2002) point out the non-optimality of Woodford's timeless perspective commitment policy on a conditional basis in the context of the purely forward-looking New Keynesian model. In the presence of backward-looking rule-of-thumb behaviour by price setters, we hence analyse welfare conditional on inflation and output gap at time 0, as well as at time -1, being at their conditional value of zero. Following Erceg et al. (2000), we throughout express welfare as a proportion of the steady-state level of output, namely Ω is now given by $\Omega = (\sigma^{-1} + \varpi)\theta/2\kappa$. Moreover, we report percent welfare levels .

We consider first welfare on the basis of the deterministic equilibrium and we later turn our attention to welfare on the basis of the stochastic equilibrium.

3.1 Welfare on the basis of the deterministic equilibrium

3.1.1 Basic New Keynesian Model

Woodford (2003, Ch. 6) shows that the unique nonexplosive solution for the Lagrange multiplier in (6), which is consistent with the zero-optimal commitment policy not fulfilling the period-minus-one Phillips curve (i.e. $\varphi_{-1} = 0$), is of the form

$$\varphi_t = -(1 - u_1^{t+1})\frac{\lambda}{\kappa}x^* \tag{49}$$

where $u_1 < 1$ is given by $u_1 = \left[\gamma - (\gamma^2 - 4\beta)^{0.5}\right]/2\beta$ with $\gamma = 1 + \beta + (\kappa^2/\lambda)$. Substituting this solution for the multiplier in (7) implies that inflation under the zero-optimal commitment policy evolves according to

$$\pi_t = (1 - u_1)\frac{\lambda}{\kappa}u_1^t x^* \tag{50}$$

Similarly, substituting (49) in (8), the path for output gap under the zero-optimal commitment policy is given by

$$x_t = u_1^{t+1} x^* \tag{51}$$

Substituting (50) and (51) into (4), yields the single-period loss function

$$L_t^{FLZ} = \left[1 + (1-u_1)^2 \frac{\lambda}{\kappa^2} u_1^{2t} + u_1^{2(t+1)} - 2u_1^{t+1}\right] \lambda x^{*2} \tag{52}$$

which implies that welfare on the basis of the deterministic equilibrium is of the form

$$\overline{W}^{FLZ} = -\Omega\lambda(1-\beta)x^{*2} E \sum_{t=0}^{\infty} \beta^t \left[1 + (1-u_1)^2 \frac{\lambda}{\kappa^2} u_1^{2t} + u_1^{2(t+1)} - 2u_1^{t+1}\right] \tag{53}$$

Given $\beta < 1$ and $u_1 < 1$, all the terms entering the sums to infinity are converging geometric series. Hence, it is possible to eliminate the infinite sums, so that (53) becomes

$$\overline{W}^{FLZ} = -\frac{(\sigma^{-1} + \varpi)}{2} x^{*2} - \frac{(1-\beta)(\sigma^{-1}+\varpi)}{2} x^{*2} \left\{ \frac{1}{1-\beta u_1^2} \left[u_1^2 + \frac{(1-u_1)^2 \lambda}{\kappa^2} \right] - \frac{2u_1}{1-\beta u_1} \right\} \tag{54}$$

Under the timeless perspective policy put forward by Blake (2001) and Jensen and McCallum (2002), welfare depends on monetary policy. Combining the steady-state loss function, $\overline{L}^{FL} = \overline{\pi}^2 + \lambda(\overline{x} - x^*)^2$, with (19) and (18), yields

$$\overline{L}^{FLBJM} = \frac{\kappa^2}{\theta\kappa + (1-\beta)^2} x^{*2} \equiv \frac{1}{[1 + (1-\beta)^2(\theta\kappa)^{-1}]} \lambda x^{*2} \tag{55}$$

which implies that welfare on the basis of the deterministic equilibrium is of the form

$$\overline{W}^{FLBJM} = -\Omega \overline{L}^{FLBJM} \equiv -\frac{(\sigma^{-1} + \varpi)}{2\,[1 + (1-\beta)^2(\theta\kappa)^{-1}]} x^{*2} \tag{56}$$

Under Woodford's timeless perspective commitment policy, welfare is independent of monetary policy, with the steady-state loss function being given by

$$\overline{L}^{FLW} = \lambda x^{*2} \tag{57}$$

which implies that welfare on the basis of the deterministic equilibrium is invariant to the degree of price stickiness, that is

$$\overline{W}^{FLW} = -\Omega \overline{L}^{FLW} \equiv -\frac{(\sigma^{-1} + \varpi)}{2} x^{*2} \tag{58}$$

Unsurprisingly, welfare on the basis of the deterministic equilibrium under the alternative timeless perspective policy is univocally better than the one under the standard timeless perspective policy as the former is globally optimal with respect to the unconditional expectation of the central bank's objective function, namely

$$\frac{\overline{W}^{FLBJM}}{\overline{W}^{FLW}} = \frac{1}{[1 + (1-\beta)^2 (\theta\kappa)^{-1}]} < 1 \tag{59}$$

Figure 1 plots $\overline{W}^{FLZ}$, $\overline{W}^{FLW}$, and $\overline{W}^{FLBJM}$ for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$, with $\overline{W}^{FLW}$ not depending on α. Zero-optimal commitment policy delivers the highest level of welfare given that, as discussed above, the transition to the steady state is always welfare-enhancing. Welfare on the basis of the deterministic equilibrium under both zero-optimal commitment policy and the alternative timeless perspective commitment policy is monotically increasing in the degree of price stickiness. This is consistent with the results presented in Blake (2001) and Jensen and McCallum (2002), which show that a greater weight on output fluctuations is welfare-worsening. In the present framework, the output gap coefficient, κ, hence the coefficient on output fluctuation λ, is monotically decreasing in the degree of price stickiness. Therefore, a higher average price duration results in better welfare on the basis of the deterministic equilibrium.

It is relevant to note that using Soderlind's (1999) method, with the time horizon set to 1000 periods[16], to derive welfare under the zero-optimal policy delivers the same level of welfare that obtains under the analytical solution, as given by (54), up to the seventh decimal figure, namely up to the fifth decimal figure when, as done here, welfare levels are expressed in percent terms.

3.1.2 Rule-of-thumb behaviour à la Galì and Gertler

We do not attempt to analytically derive the evolution of inflation and output gap under the zero-optimal commitment policy, hence welfare on the basis of the deterministic equilibrium, $\overline{W}^{GGZ}$, but we resort to Soderlind's (1999) method.

Under both timeless perspective policies, the steady-state loss function, namely $\overline{L}^{FL} \equiv \overline{L}^{GG}$, can be seen as the sum of the loss that obtains in the presence of zero long-run inflation, $\overline{L}^{FLW}$, which is independent of

[16] Whenever we employ Soderlind's method, the time horizon is set to 1000 periods.

monetary policy, and the loss attributable to positive optimal long-run inflation , $\overline{\pi}^2+\lambda_1\overline{x}^2-2\lambda_1 x^*\overline{x}$, which depends on monetary policy. Hence, we are able to address whether positive long-run inflation improves welfare relative to a policy of zero inflation at all times, which implies $\overline{W}^{FLW}$.

As described above, the alternative timeless perspective policy is robust to the introduction of backward-looking rule-of thumb behaviour à la Galì and Gertler (1999). It follows that $\overline{W}^{GGBJM} \equiv \overline{W}^{FLBJM}$, which in univocally better than $\overline{W}^{FLW}$.

Under Woodford's timeless perspective policy, combining the steady-state loss function with (29) and (18), yields

$$\overline{L}^{GGW} = \lambda_1 x^{*2} + \lambda_1 \frac{(1-\alpha)(1-\beta)^2\omega \left\{ \begin{array}{c} \theta\kappa\left[\omega+\alpha(\omega-2)\right] \\ -(1-\alpha)(1-\beta)^2\omega \end{array} \right\}}{\left[(1-\omega)\alpha\theta\kappa+(1-\alpha)(1-\beta)^2\omega\right]^2} x^{*2} \equiv \lambda_1 x^{*2}(1+\Delta) \tag{60}$$

which implies that welfare on the basis of the deterministic equilibrium is of the form

$$\overline{W}^{GGW} = -\frac{(\sigma^{-1}+\varpi)}{2}(1+\Delta)x^{*2} \tag{61}$$

The standard timeless perspective policy is hence not univocally superior to a policy of a policy of zero steady-state inflation. The condition that guarantees $\overline{W}^{GGW} > \overline{W}^{FLW}$ is easily seen to be $\Delta < 0$, where the sign of Δ is determined by the term in curly brackets in (60). Solving in terms of ω yields

$$\omega\left[(1+\alpha)\theta\kappa-(1-\alpha)(1-\beta)^2\right] \leq 2\alpha\theta\kappa \Leftrightarrow \omega \leq \frac{2\alpha\theta\kappa}{\left[(1+\alpha)\theta\kappa-(1-\alpha)(1-\beta)^2\right]} \tag{62}$$

Figure 2 plots condition (62), holding with equality, for the full range of the degree of price stickiness, $0.01 \leq \alpha \leq 0.99$. The values of ω that imply $\overline{W}^{GGW} < \overline{W}^{FLW}$ are observed to be outside the estimates of the degree of rule-of-thumb behaviour reported in Galì and Gertler (1999), especially when considering empirically realistic values of the degree of price stickiness based on macroeconomic data. However, while the alternative timeless perspective policy is univocally superior, in terms of welfare on the basis of the deterministic equilibrium, to a policy of zero inflation at all times, the same it is not always true under the standard timeless perspective policy.

Figure 3 plots $\overline{W}^{GGZ}$, $\overline{W}^{GGW}$, and $\overline{W}^{GGBJM}$ for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Given the relationship between α and κ, welfare on the basis of the deterministic equilibrium under all commitment policies is increasing in the degree of price stickiness. Zero-optimal commitment policy ranks first and the alternative timeless perspective commitment policy is confirmed to entail welfare on the basis of the deterministic equilibrium that is always better than the one implied by the standard timeless perspective policy.

As for the relation between welfare on the basis of the deterministic equilibrium and the degree of backward-looking rule-of-thumb behaviour, Figure 4 plots $\overline{W}^{GGZ}$, $\overline{W}^{GGW}$, and $\overline{W}^{GGBJM}$ for $0.01 \leq \omega \leq 0.7$. Blake (2001) and Jensen and McCallum (2002) timeless perspective policy implies the same level of welfare that obtains in the purely forward-looking model, which is invariant to the degree of rule-of-thumb behaviour. Welfare on the basis of the deterministic equilibrium under zero-optimal policy is observed to be monotonically decreasing in the degree of rule-of-thumb behaviour. Woodford's timeless perspective policy instead implies that a larger fraction of firms resetting prices in a backward-looking rule-of-thumb manner is initially welfare-enhancing, although never delivering better welfare levels than the alternative timeless perspective commitment policy, and subsequently becomes welfare-worsening. With this latter respect, Figure 5 plots, for the full range of the degree of price stickiness, the locus of combinations of the degree of price stickiness and the degree of rule-of-thumb behaviour such that $\overline{W}^{GGW}$ is at its relative maximum, with the absolute maximum obtaining under the outermost right combination of α and ω. Once again, zero-optimal policy is better than the two timeless perspective policies, with the alternative timeless perspective policy being univocally superior to the standard timeless perspective policy.

3.1.3 Rule-of-thumb behaviour à la Steinsson

We do not attempt to analytically derive the evolution of inflation and output gap under the zero-optimal commitment policy, hence welfare on the basis of the deterministic equilibrium, $\overline{W}^{SZ}$, but we resort to Soderlind's (1999) method.

Steinsson's (2003) rule-of-thumb behaviour alters both the long-run trade-off between output gap and inflation and the steady-state loss function (i.e. $\overline{L}^{S} = \overline{L}^{FL} + \lambda_2(1-\alpha)^2\delta^2)\overline{x}^2)$ that obtain in the purely forward-looking New Keynesian model. Under both timeless perspective policies, the steady-state loss function can again be seen as the sum of the loss that obtains in the basic New Keynesian model, $\overline{L}^{FLW}$, which is independent of monetary policy, and the loss due to positive long-run inflation, $\overline{\pi}^2 + (\lambda_1 + \lambda_2(1-\alpha)^2\delta^2)\overline{x}^2 - 2\lambda_1 x^* \overline{x}$, which depends on monetary policy. Under the standard timeless perspective policy, combining the steady-state loss function with (42) and (47), yields

$$\overline{L}^{SW} = \lambda_1 x^{*2} + \Psi x^{*2}\left[\Psi - 2\lambda_1\Gamma + (\lambda_1 + \lambda_2(1-\alpha)^2\delta^2)\Gamma^2\Psi\right] \tag{63}$$

which implies that welfare on the basis of the deterministic equilibrium is of the form

$$\overline{W}^{SW} = -\frac{(\sigma^{-1} + \varpi)\theta}{2\kappa}\overline{L}^{SW} \tag{64}$$

Under the alternative timeless perspective policy, combining the steady-state loss function with (48) and (47), yields

$$\overline{L}^{SBJM} = \lambda_1 x^{*2} + \Theta x^{*2}\left[\Theta - 2\lambda_1\Gamma + (\lambda_1 + \lambda_2(1-\alpha)^2\delta^2)\Gamma^2\Theta\right] \tag{65}$$

which implies that welfare on the basis of the deterministic equilibrium is of the form

$$\overline{W}^{SBJM} = -\frac{(\sigma^{-1} + \varpi)\theta}{2\kappa}\overline{L}^{SB} \tag{66}$$

As under Galì-Gertler's rule-of-thumb behaviour, we can compare welfare under positive optimal steady-state inflation vis-a-vis welfare under a policy of zero steady-state inflation. However, the derivation of the condition that guarantees an increase in welfare on the basis of the deterministic equilibrium is cumbersome, but quantitative analysis confirms that the timeless perspective in Blake (2001) and Jensen and McCallum (2002) is univocally superior, in terms of welfare on the basis of the deterministic equilibrium, to a policy of zero inflation at all times, whereas the same it is not always true under Woodford's timeless perspective policy.

The results that obtain under backward-looking rule-of thumb behaviour à la Galì and Gertler (1999) are unaffected. Figure 6 plot respectively $\overline{W}^{SZ}$, $\overline{W}^{SW}$, and $\overline{W}^{SBJM}$ for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Given that the coefficient on output fluctuation is monotically decreasing in the degree of price stickiness, welfare on the basis of the deterministic equilibrium under all commitment policies is increasing in the degree of price stickiness. Zero-optimal commitment policy ranks first and the alternative timeless perspective commitment policy is confirmed to entail welfare on the basis of the deterministic equilibrium that is always better than the one implied by the standard timeless perspective policy.

As for the relation between welfare on the basis of the deterministic equilibrium and the degree of backward-looking rule-of-thumb behaviour, Figure 7 plot respectively $\overline{W}^{SZ}$, $\overline{W}^{SW}$, and $\overline{W}^{SBJM}$ for $0.01 \leq \omega \leq 0.7$. Welfare on the basis of the deterministic equilibrium under both zero-optimal policy and Blake (2001) and Jensen and McCallum (2002) timeless perspective policy is observed to be monotonically decreasing in the degree of rule-of-thumb behaviour. Woodford's timeless perspective policy implies that a larger fraction of firms resetting prices in a backward-looking rule-of-thumb manner is associated initially with increases in welfare, although never equalling welfare levels entailed by the alternative timeless perspective commitment policy, and subsequently becomes welfare-worsening. Indeed, Figure 8 plots, for the full range of the degree of price stickiness, the locus of combinations of the degree of price stickiness and the degree of rule-of-thumb behaviour such that $\overline{W}^{SW}$ is at its relative maximum, with the absolute maximum obtaining under the outermost right combination of α and ω. Once again, zero-optimal policy is better than the the two timeless perspective policies, with the alternative timeless perspective policy being univocally superior to the standard timeless perspective policy.

3.2 Welfare on the basis of the stochastic equilibrium

Having shown how the alternative commitment policies univocally rank on the basis of the deterministic equilibrium, we now follow the theoretical literature on optimal monetary policy and proceed to evaluate policies on the basis of the stochastic equilibrium, which stems from augmenting the aggregate-supply relation with a cost-push shock. For instance, the NKPC is now given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \mu_t \tag{67}$$

where, using the terminology in Clarida et al. (1999), μ_t represents a cost-push shock, which is assumed to be autoregressive of order one with AR parameter ρ and innovation shock ϵ_t being i.i.d, namely $\mu_t = \rho\mu_{t-1} + \epsilon_t$[17]. We calibrate the standard deviation of the cost-push shock innovation to 0.016, which is the value estimated in Smets and Wouters (2003) and we set the AR parameter, ρ, to 0[18]. Augmenting the aggregate-supply relation with a cost-push shock does not alter the central bank's target criterion, but it implies that the monetary authority should react to movements in output which are caused by a cost-push shock while it should not react to movements in output which are caused by any other shocks[19].

The experiment we undertake in order to rank the alternative commitment policies on the basis of the stochastic equilibrium is simple to illustrate. We consider a draw of 100 positive cost-push shocks, which we maintain across all the theoretical cases studied[20]. For each (α, ω) pair we calculate welfare as the mean value that obtains across the 100 shocks. From this level of welfare we then subtract the

[17] Of course, (20) and (33) are also augmented with μ_t.

[18] The same remark in Jensen and McCallum (2002) applies here, changing the standard deviation of the mark-up shock innovation would only scale welfare values up or down proportionately. The results we present are robust to the possibility of a positive AR parameter. Specifically, we have considered $\rho = 0.5$ and $\rho = 0.8$.

[19] The model includes exogenous real shocks to technology, to Government purchases, to household's impatience to consume, and to the household's willingness to supply labour.

[20] The results we report are not altered when considering a larger number of shocks. Specifically, we have considered a draw of 1000 positive cost-push shocks.

corresponding welfare on the basis of the deterministic equilibrium, thus reporting welfare that is purely due to the stabilisation of the cost-push shock.

Soderlind's (1999) method solves for the evolution of endogenous variables under zero-optimal commitment policy. However, it can be used for the evolution of the endogenous variables under timeless perspective policy on the provision that a dummy control variable is introduced into the system. That is, while under zero-optimal policy the output gap is the control variable and the central bank's only constraint is the aggregate-supply relation, under timeless perspective policy the control variable equals zero at all times and the central bank is constrained by the aggregate-supply relation and the target criterion[21].

3.2.1 Basic New Keynesian Model

Figure 9 plots welfare on the basis of the stochastic equilibrium under the three commitment policies for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness. This is contrary to the relation between welfare on the basis of the deterministic equilibrium and degree of price stickiness, but the logic for this result is quite intuitive. In an economy where price setting is staggered, a higher degree of price stickiness implies a higher degree of price dispersion, which is in fact costly as it brings about dispersion of output levels across goods[22]. Hence, a higher average price duration results in larger losses associated with the stabilisation of the cost-push shock.

Figure 9 is rather inconclusive as for how the three commitment policies rank in terms of welfare on the basis of the stochastic equilibrium. Figure 10 thus reports the difference in welfare levels between any pair of commitment policies. As in Jensen and McCallum (2002), the magnitude of the differences in welfare levels is, here as well as in the presence of rule-of-thumb behaviour by price setters, not large, but, insofar as the solution method in Soderlind (1999) is accurate up to the fifth decimal figure, is distinctly non-zero[23]. The ranking between the alternative commitment policies is then univocal: the alternative timeless perspective policy ranks first, followed by the standard timeless perspective policy and both timeless perspective policies outperform the zero-optimal policy.

[21] As described above, under rule-of-thumb behaviour à la Steinsson (2003), the target criterion is replaced by the first-order conditions with respect to inflation and output gap

[22] The degree of price dispersion at time t in the purely forward-looking model is given by $\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)$, which is easily seen to be increasing in α.

[23] In what follows, a difference in welfare levels of order up to 10^{-5} is thus considered to be distinctly non-zero.

3.2.2 Rule-of-thumb behaviour à la Galì and Gertler

Figure 11 plots welfare on the basis of the stochastic equilibrium under the three commitment policies for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness. This is because, as in the purely forward-looking model, a higher degree of price stickiness implies a higher degree of price dispersion, hence larger welfare losses. The ranking between the alternative commitment policies, as depicted in Figure 12, is not univocal. On the one hand, both timeless perspective policies outperform the zero-optimal policy. On the other hand, the standard timeless perspective policy is observed to be superior to the alternative timeless perspective policy for $0.33 \leq \alpha \leq 0.54$, with the difference being distinctly non-zero for $0.38 \leq \alpha \leq 0.49$. Higher degrees of price stickiness, namely $0.55 \leq \alpha \leq 0.8$, instead imply that the alternative timeless perspective policy outperforms the standard timeless perspective policy.

Figure 13 plots welfare on the basis of the stochastic equilibrium under the three commitment policies for alternative values of the degree of rule-of-thumb behaviour, $0.01 \leq \omega \leq 0.7$. Under all policies, welfare is monotonically decreasing in the degree of rule-of-thumb behaviour. A larger fraction of firms resetting prices in a backward-looking is indeed associated with higher degree of price dispersion, hence larger losses associated with the stabilisation of the cost-push shock[24]. The ranking between the alternative commitment policies, as depicted in Figure 14, is not univocal. On the one hand, the standard timeless perspective policy is invariably superior to the zero-optimal policy. On the other hand, the alternative timeless perspective policy is seen to be inferior to the other two commitment policies for relatively high degree of rule-of-thumb behaviour. Specifically, the zero-optimal policy outperforms the alternative timeless perspective policy for $0.48 \leq \omega \leq 0.62$, with the difference being distinctly non-zero for $0.49 \leq \omega \leq 0.61$, whereas the standard timeless perspective policy is superior to its alternative version for $0.4 \leq \omega \leq 0.64$, with the difference being distinctly non-zero for $0.42 \leq \omega \leq 0.64$.

Note that backward-looking rule-of-thumb behaviour implies inferior welfare on the basis of the stochastic equilibrium than in the purely forward-looking model. Intuitively, backward-looking rule-of-thumb behaviour invariably increases the degree of price dispersion, which results in additional welfare losses associated with the stabilisation of the cost-push shock.

Finally, we consider welfare on the basis of the stochastic equilibrium on a conditional basis. Specifically, we analyse welfare conditional on inflation and output gap at time 0, as well as at time -1, being

[24] The degree of price dispersion at time t in the model with rule-of-thumb à la Galì and Gertler (1999) is given by $\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\omega)(1-\alpha)}(\pi_t - \pi_{t-1})^2 + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)$, which is easily seen to be increasing in both α and ω.

at their conditional value of zero. There is one main point to take from Figures (15)-(18), which are analogous to Figures (11)-(14). While the welfare levels on a conditional basis are practically the same as on an unconditional basis, the ranking between the alternative commitment policies is now univocal: the alternative timeless perspective policy ranks first, followed by the standard timeless perspective policy and both timeless perspective policies outperform the zero-optimal policy[25].

3.2.3 Rule-of-thumb behaviour à la Steinsson

Figure 19 plots welfare on the basis of the stochastic equilibrium under the three commitment policies for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness as a higher degree of price stickiness implies a higher degree of price dispersion, hence larger welfare losses. The ranking between the alternative commitment policies, as depicted in Figure 20, is univocal: the alternative timeless perspective policy ranks first, followed by the standard timeless perspective policy and both timeless perspective policies outperform the zero-optimal policy.

Figure 21 plots welfare on the basis of the stochastic equilibrium under the three commitment policies for alternative values of the degree of rule-of-thumb behaviour, $0.01 \leq \omega \leq 0.7$. Under all policies, welfare is monotonically decreasing in the degree of rule-of-thumb behaviour. A larger fraction of firms resetting prices in a backward-looking is indeed associated with higher degree of price dispersion, hence larger losses associated with the stabilisation of the cost-push shock[26]. The ranking between the alternative commitment policies, as depicted in Figure 22, is not univocal. On the one hand, both timeless perspective policies are invariably superior to the zero-optimal policy. On the other hand, the alternative timeless perspective policy is seen to be inferior to its standard version for relatively high degrees of rule-of-thumb behaviour. Specifically, the standard timeless perspective policy outperforms the alternative timeless perspective policy for $0.47 \leq \omega \leq 0.58$, with the difference being distinctly non-zero only for $0.51 \leq \omega \leq 0.55$.

Note that backward-looking rule-of-thumb behaviour à la Steinsson (2003) implies superior welfare on the basis of the stochastic equilibrium than under rule-of-thumb behaviour à la Galì and Gertler (1999). Intuitively, indexation to lagged output gap curbs the degree of price dispersion, which results in smaller welfare losses associated with the stabilisation of the cost-push shock.

[25] Indeed, the standard timeless perspective policy is superior to its alternative version only for extremely high degrees of rule-of-thumb behaviour, namely $\omega \geq 0.68$.

[26] The degree of price dispersion at time t in the model with rule-of-thumb à la Steinsson (2003) is given by $\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\omega)(1-\alpha)}\left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1}\right]^2 + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)$ which is easily seen to be increasing in both α and ω.

Finally, we consider welfare on the basis of the stochastic equilibrium on a conditional basis. Specifically, we analyse welfare conditional on inflation and output gap at time 0, as well as at time -1, being at their conditional value of zero. There is one main point to take from Figures (23)-(26), which are analogous to Figures (19)-(22). While the welfare levels on a conditional basis are practically the same as on an unconditional basis, the ranking between the alternative commitment policies changes. As for the degree of price stickiness, zero-optimal policy is better than the two timeless perspective policies, with the standard timeless perspective policy being univocally superior to the alternative timeless perspective policy. As for the degree of rule-of-thumb behaviour, zero-optimal policy is inferior to the two timeless perspective policies, with the standard timeless perspective policy still being univocally superior to the alternative timeless perspective policy.

4 Conclusions

This paper studies different types of commitment policy in an economy where the deterministic steady state is inefficient. We consider the canonical purely forward-looking New Keynesian model and its variant due to backward-looking rule-of-thumb behaviour by a fraction of price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003). We study the zero-optimal policy and the timeless perspective policy. Woodford (1999) labels policy timeless, namely time-invariant, as it reflects a type of commitment that, unlike the zero-optimal policy, constraints the policy reaction function to be the same in the initial period as in all succeeding periods. More recently, Blake (2001) and Jensen and McCallum (2002) show that the timeless perspective policy, as usually described, is not optimal in the sense of Taylor (1979), who proposes adopting a monetary policy that is optimal from the perspective of the average value of the central bank's loss function.

We show how this alternative timeless perspective policy entails optimal positive long-run inflation target, even in the purely forward-looking basic New Keynesian model. The intuition for this result is quite neat, hinging on the discount factor. Under Blake (2001) and Jensen and McCallum (2002) timeless perspective policy, the central bank's discount factor differs from the private sector's discount factor: the private sector prefers current consumption to future consumption whereas the central bank does not discount future, namely every loss for the central bank is equally costly. On the one hand, the private sector's discount factor appears in the model's structural equations, thus resulting in the long-run Phillips curve trade-off. On the other hand, the central bank now equally weighs the increase in output caused by higher inflation and the cost of the reduction in output as a result of expected higher inflation. It follows that the stimulative effect of higher inflation on output is greater than the output cost of higher inflation. The long-run Phillips curve trade-off is then exploited and it is optimal for the central bank to commit to positive steady-state inflation.

With this respect, the paper can be read as a second answer to what causes positive inflation to be endogenously optimal in the long-run. It can be either optimality in the sense of Taylor (1979), which is achieved by the commitment policy suggested by the approach of policy design, or, as shown in Pontiggia (2007), costly disinflation. The former in fact rationalises a positive optimal long-run inflation target even in a model where the central bank is on paper capable of disinflating without incurring any loss in output. Indeed, the optimal long-run inflation target is zero in only one of the six theoretical cases studied.

Moreover, differently from the standard timeless perspective commitment policy, the alternative timeless perspective commitment policy is robust to the introduction of backward-looking rule-of thumb behaviour à la Galì and Gertler (1999) as this does not alter the steady state that would obtain under forward-looking behaviour by all price setters. This arguably desirable property does not apply in the case of backward-looking rule-of-thumb behaviour à la Steinsson (2003) as indexation to past output gap by rule-of-thumb price setters changes both the long-run trade-off between output gap and inflation and the steady-state central bank's loss function.

We evaluate the alternative commitment policies both the on the basis of the deterministic equilibrium and on the basis of the stochastic equilibrium. In so doing, our main objective is, as in Jensen and McCallum (2002), to simply rank the alternative commitment policies. We present robustness analysis for ample ranges of two structural parameters: the degree of price stickiness and the degree of rule-of-thumb behaviour. We follow the cited papers and report average values of the central bank's objective function. Regardless of the details of price setting, zero-optimal commitment policy invariably delivers the highest level of welfare on the basis of the deterministic equilibrium and the alternative timeless perspective policy is univocally superior to the standard timeless perspective policy. Moreover, the alternative timeless perspective commitment policy always implies better welfare on the basis of the deterministic equilibrium than a policy of zero inflation at all times, whereas the same it is not always true under the standard timeless perspective commitment policy. As for welfare on the basis of the stochastic equilibrium, the ranking between the three alternative commitment policies is not strictly univocal, but the alternative timeless perspective policy is usually found to rank first, followed by the standard timeless perspective policy and both timeless perspective policies outperform the zero-optimal policy.

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6 Tables

Model \ Commitment Policy	ZW		BJM
FL	$\overline{\pi} = 0$		$\overline{\pi} > 0$
			$=$
GG	$\overline{\pi} > 0$	$\neq$	$\overline{\pi} > 0$
	$\neq$		$\neq$
S	$\overline{\pi} > 0$	$\neq$	$\overline{\pi} > 0$

Table 1. Optimal long-run inflation target

Structural parameter	β	θ	ϖ	σ^{-1}
Value	0.99	7.88	0.47	0.16

Table 2. Woodford's (2003) benchmark calibration (quarterly)

7 Figures

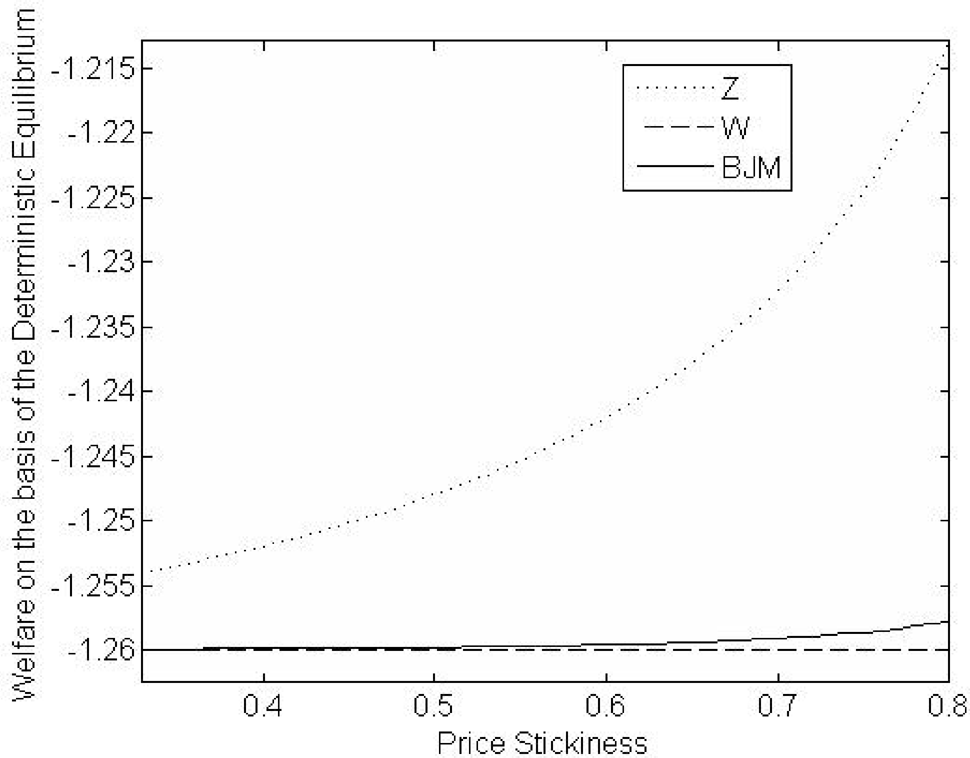

Figure 1. Welfare on the basis of the deterministic equilibrium in the purely forward-looking New Keynesian model.

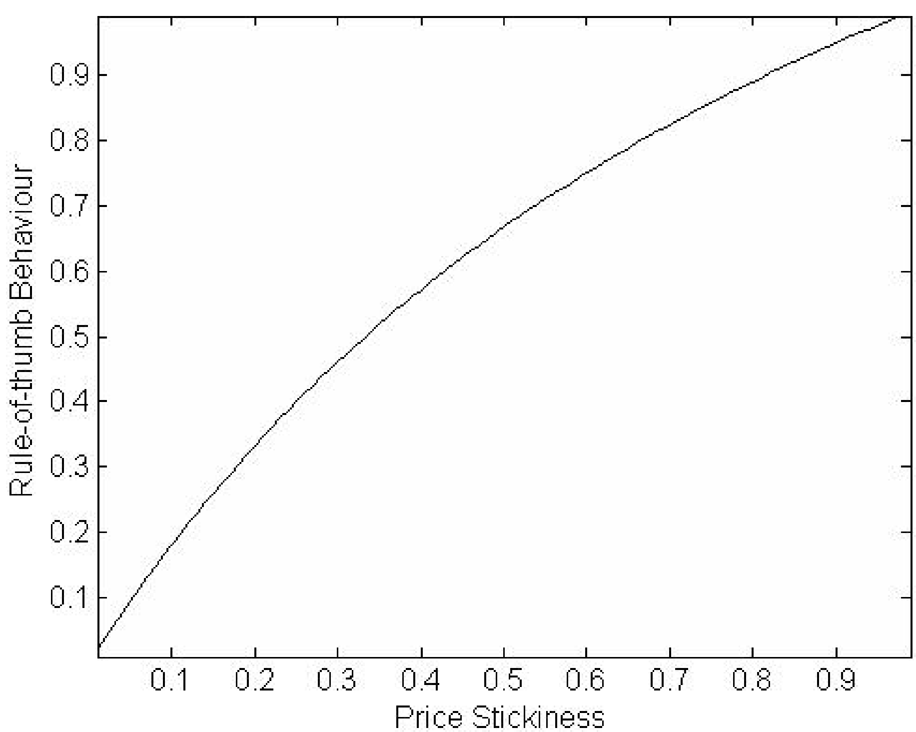

Figure 2. Condition (62). Points below the line imply $\overline{W}^{GGW} > \overline{W}^{FLW}$, points above the line imply $\overline{W}^{GGW} < \overline{W}^{FLW}$.

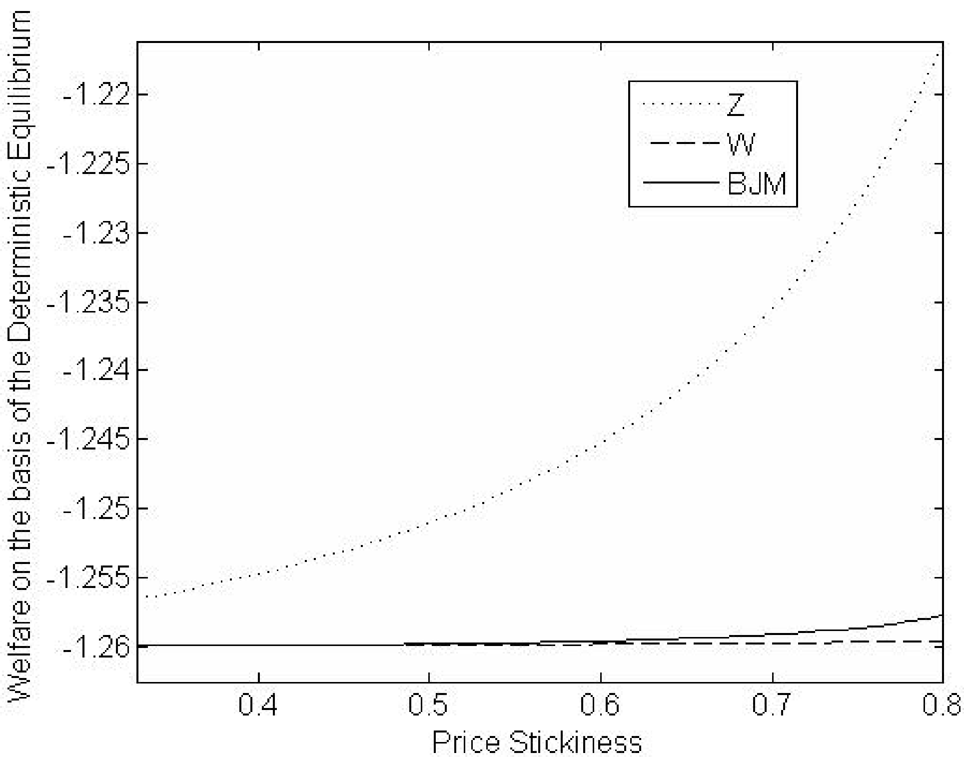

Figure 3. Welfare on the basis of the deterministic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999).

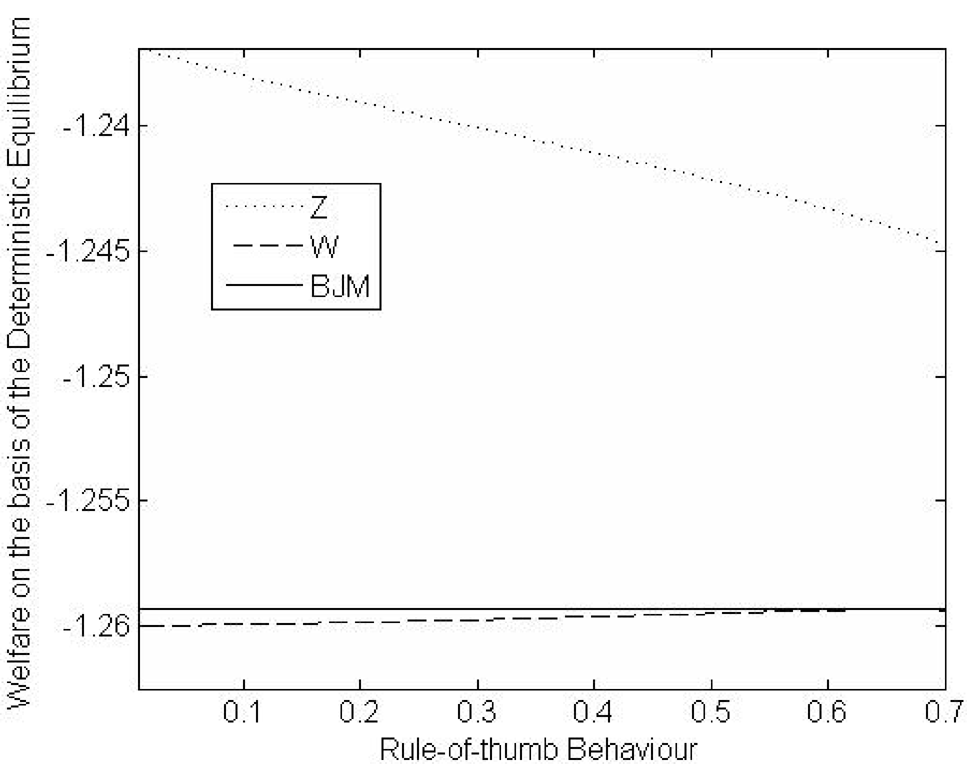

Figure 4. Welfare on the basis of the deterministic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999).

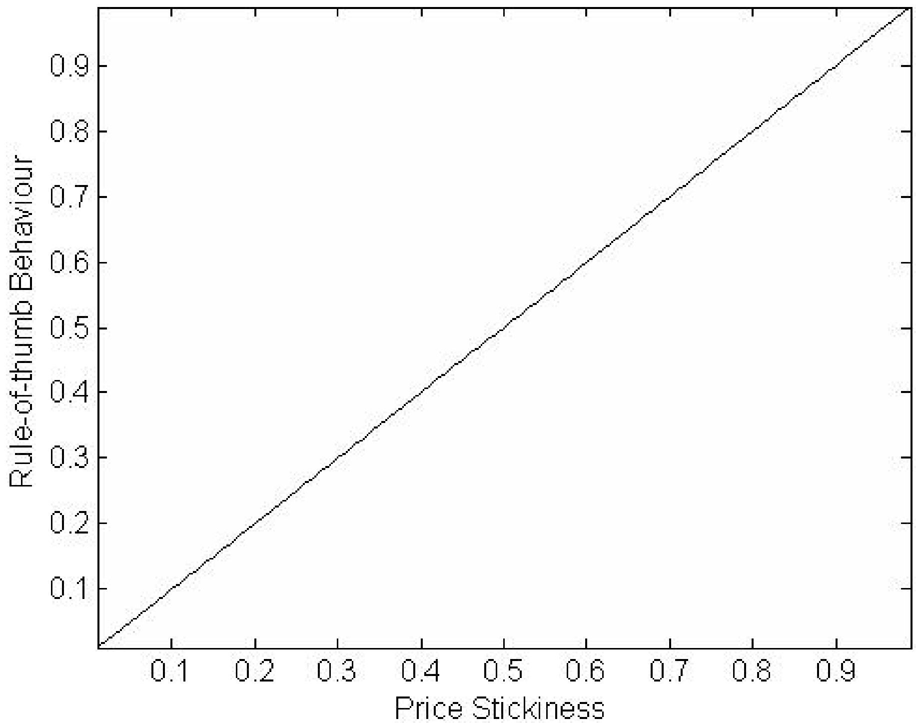

Figure 5. Locus of combinations of the degree of price stickiness and the degree of rule-of-thumb behaviour such that $\overline{W}^{GGW}$ is at its relative (absolute) maximum.

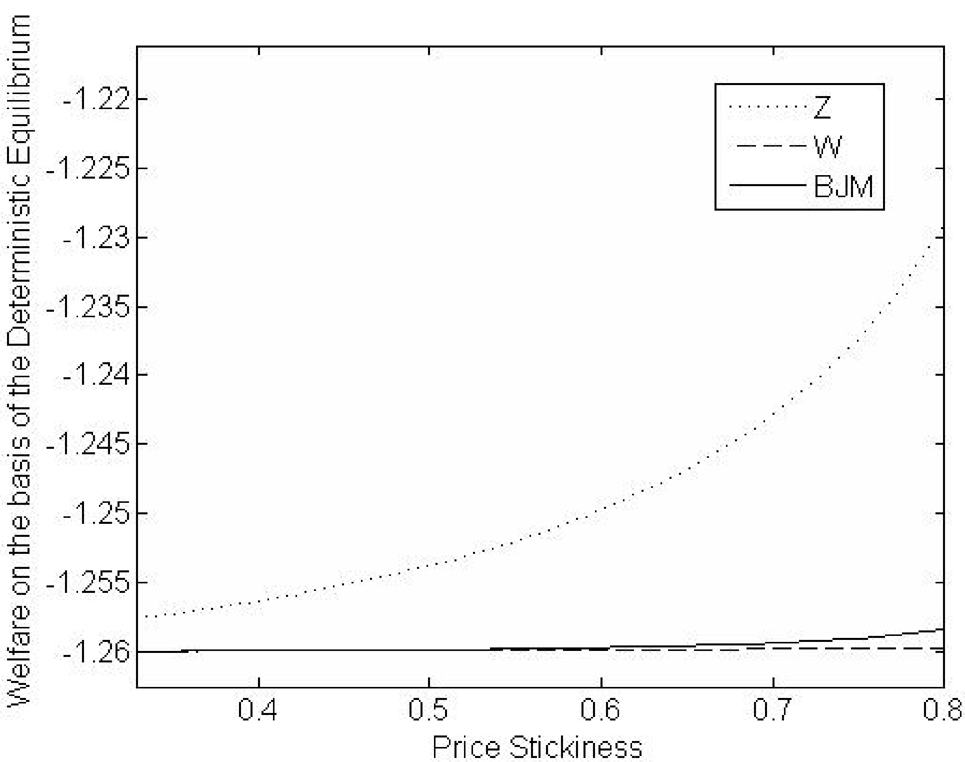

Figure 6. Welfare on the basis of the deterministic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003).

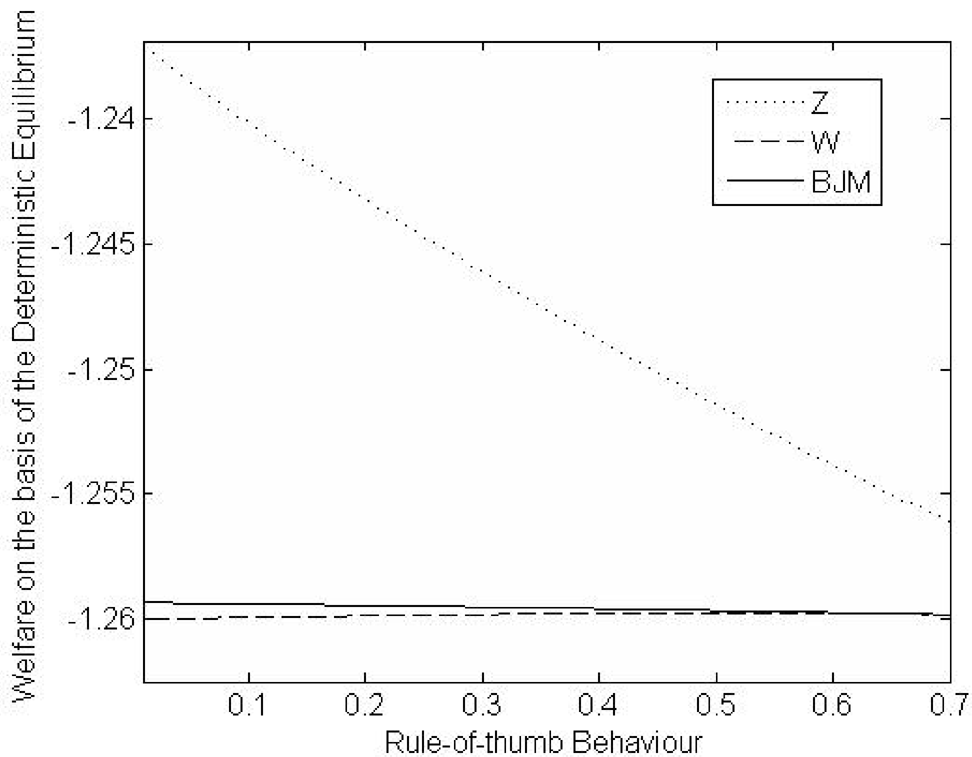

Figure 7. Welfare on the basis of the deterministic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003).

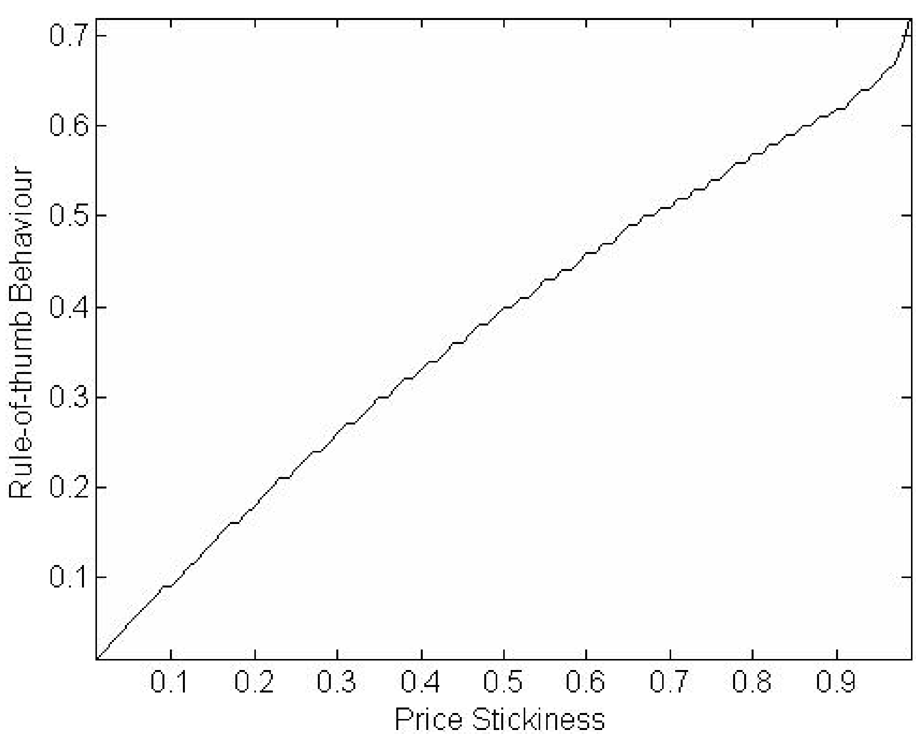

Figure 8. Locus of combinations of the degree of price stickiness and the degree of rule-of-thumb behaviour such that $\overline{W}^{SW}$ is at its relative (absolute) maximum.

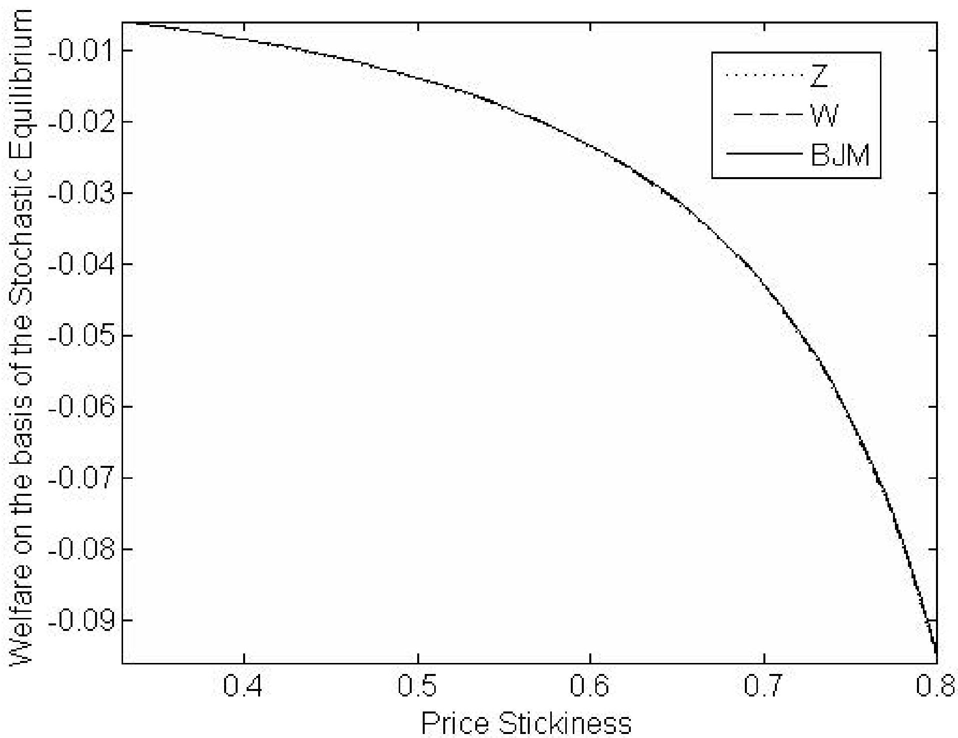

Figure 9. Welfare on the basis of the stochastic equilibrium in the purely forward-looking New Keynesian model.

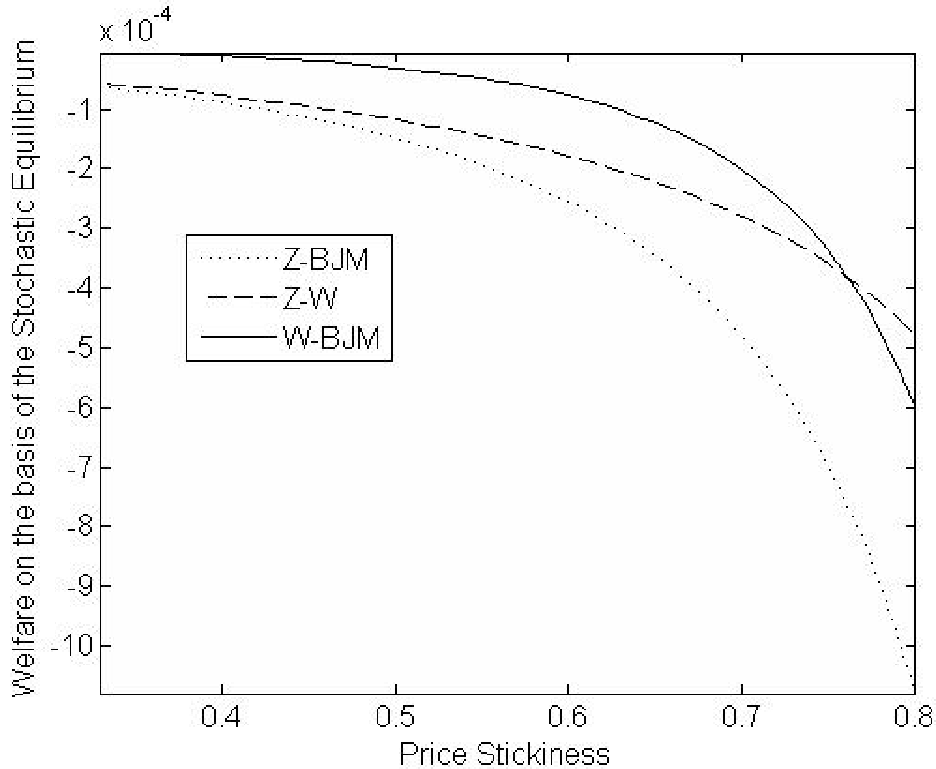

Figure 10. Welfare on the basis of the stochastic equilibrium in the purely forward-looking New Keynesian model: difference in the welfare levels between any pair of commitment policies.

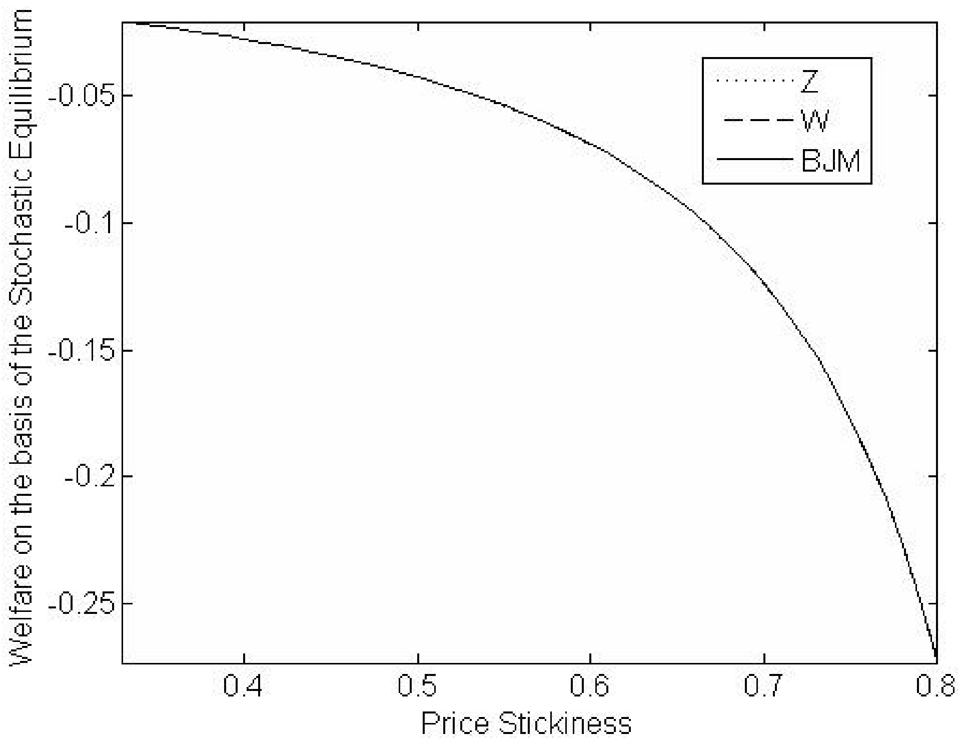

Figure 11. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999).

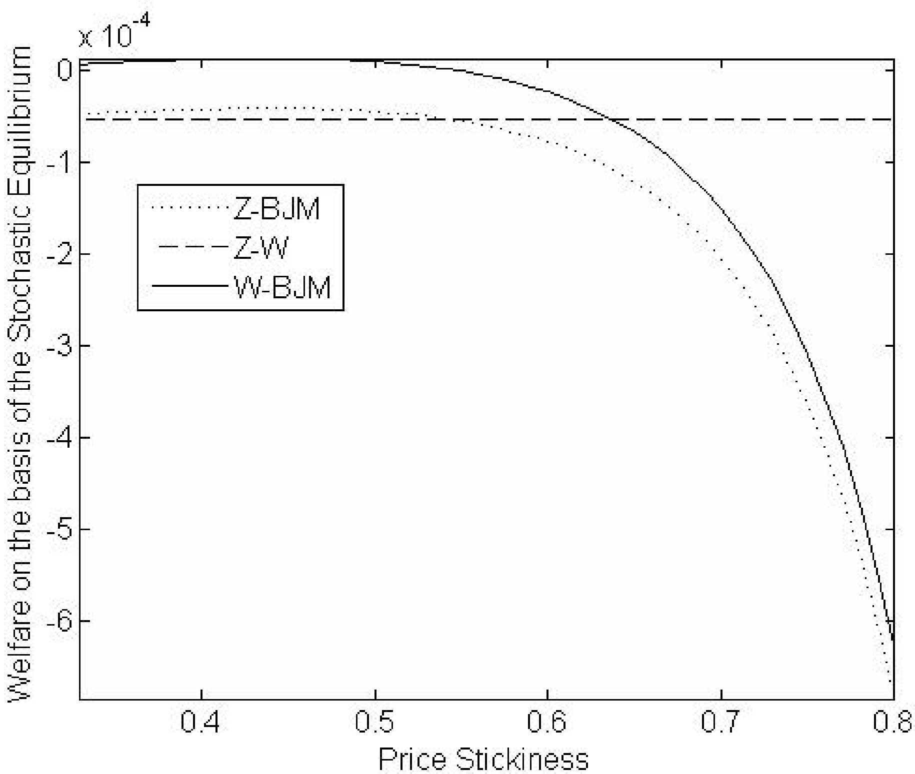

Figure 12. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999): difference in the welfare levels between any pair of commitment policies.

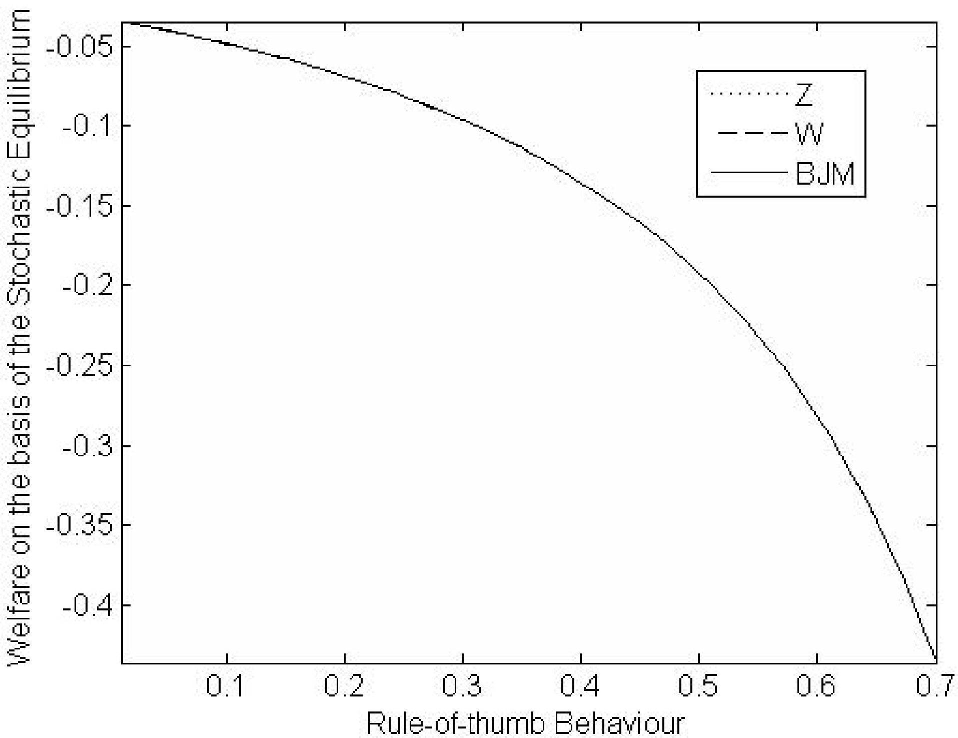

Figure 13. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999).

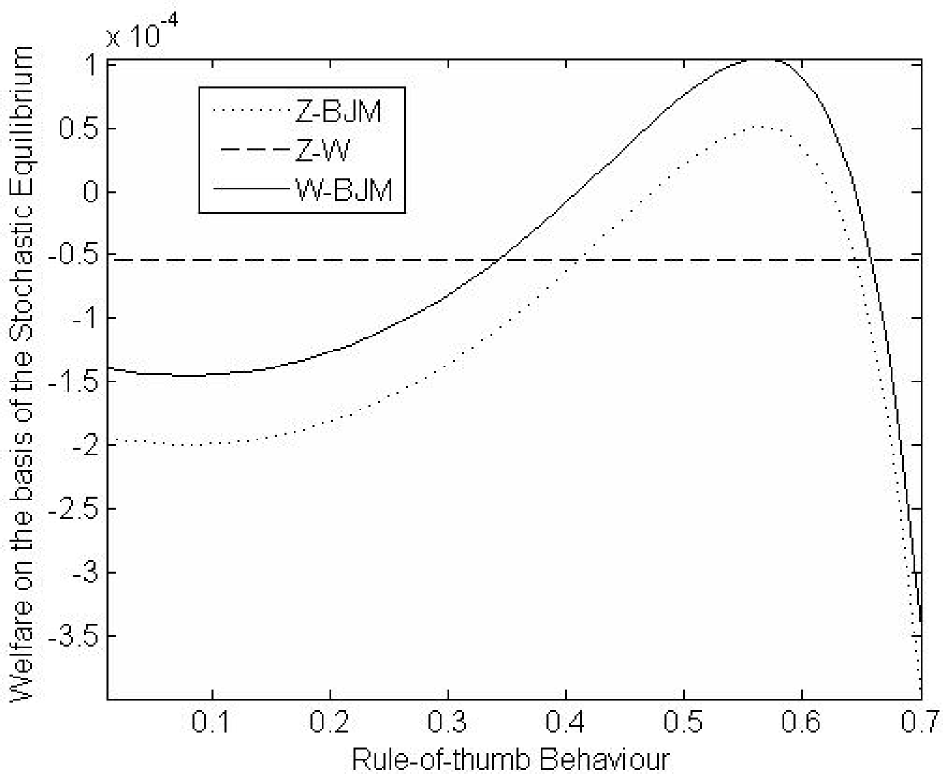

Figure 14. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999): difference in the welfare levels between any pair of commitment policies.

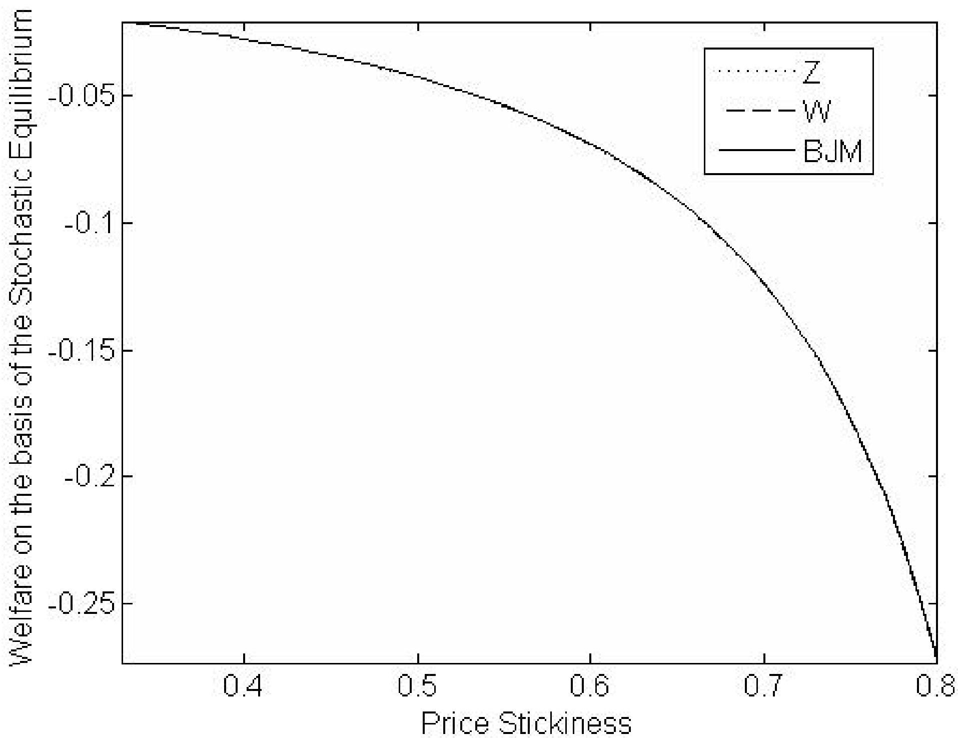

Figure 15. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999).

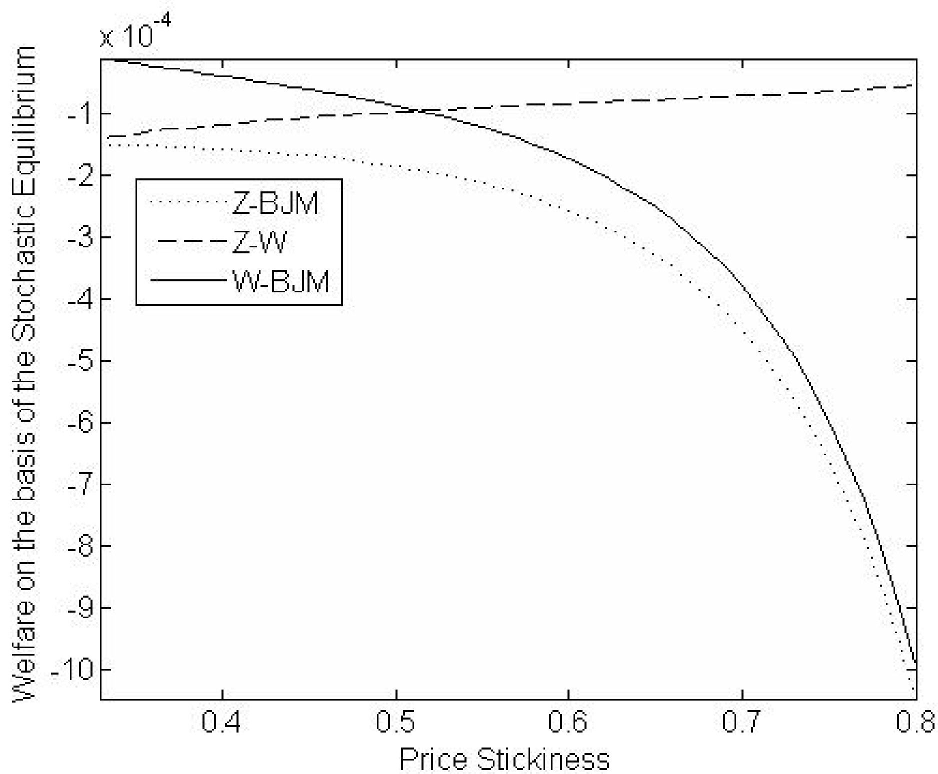

Figure 16. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999): difference in the welfare levels between any pair of commitment policies.

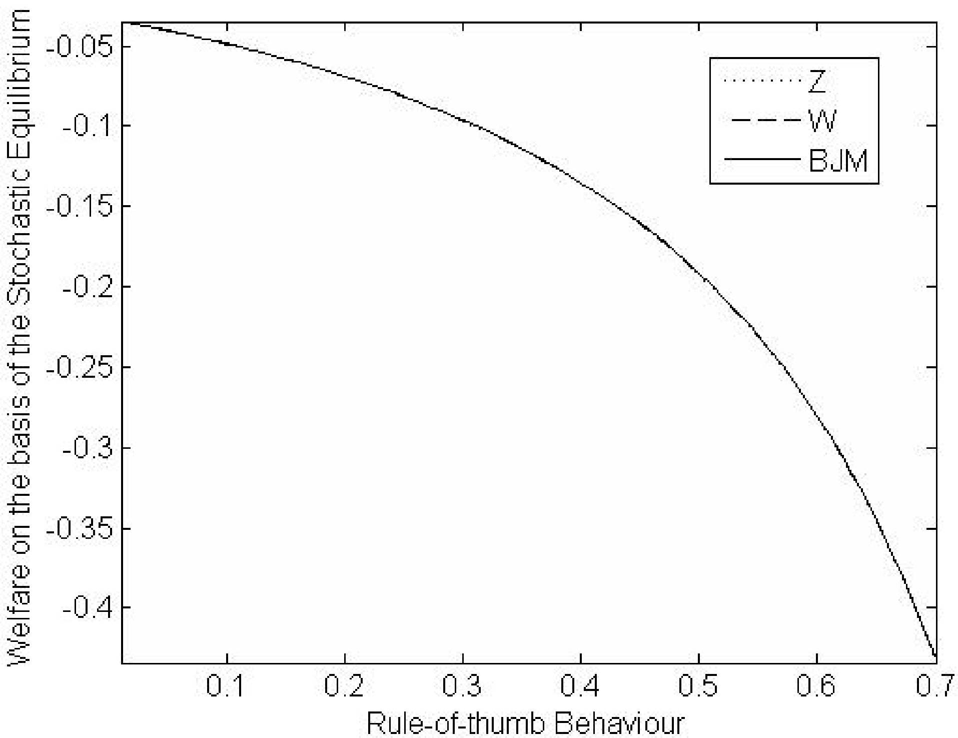

Figure 17. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999).

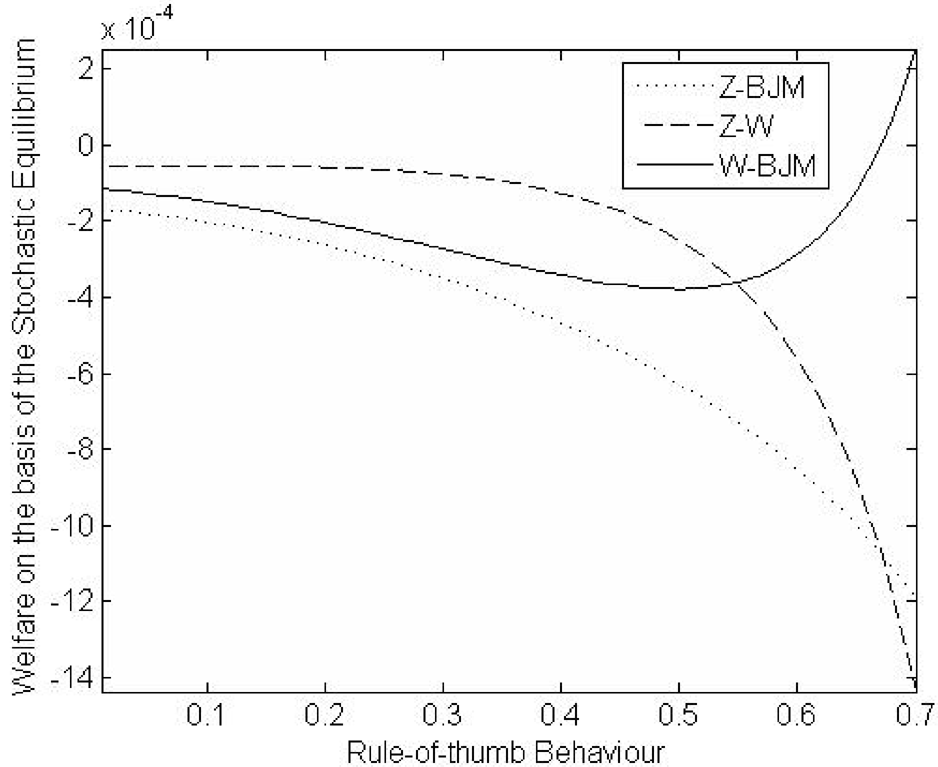

Figure 18. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Galì and Gertler (1999): difference in the welfare levels between any pair of commitment policies.

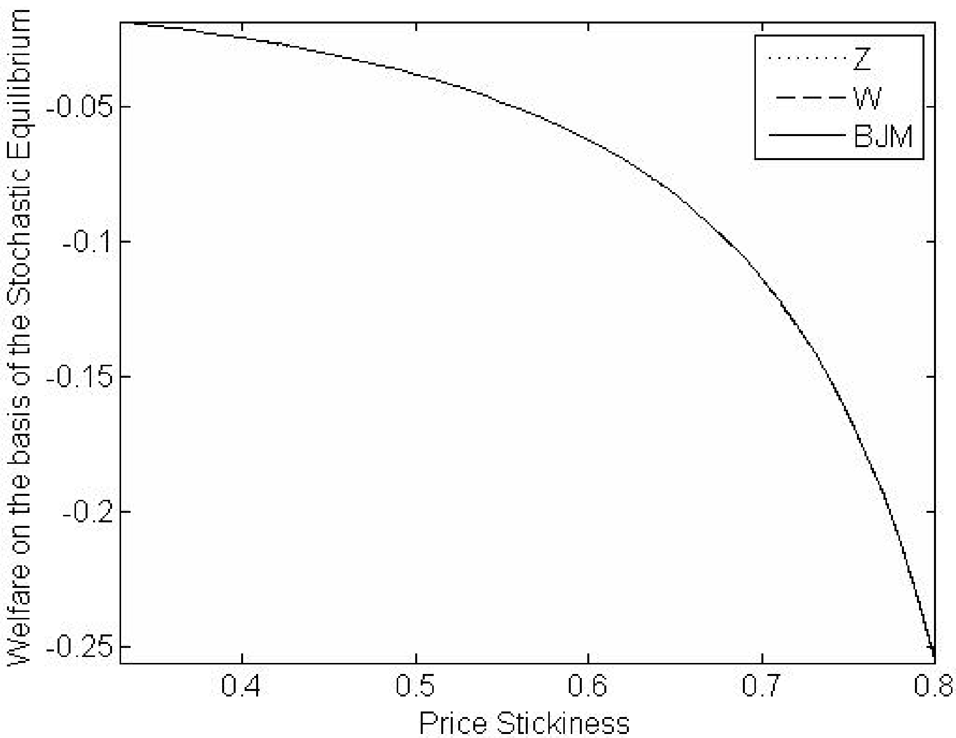

Figure 19. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003).

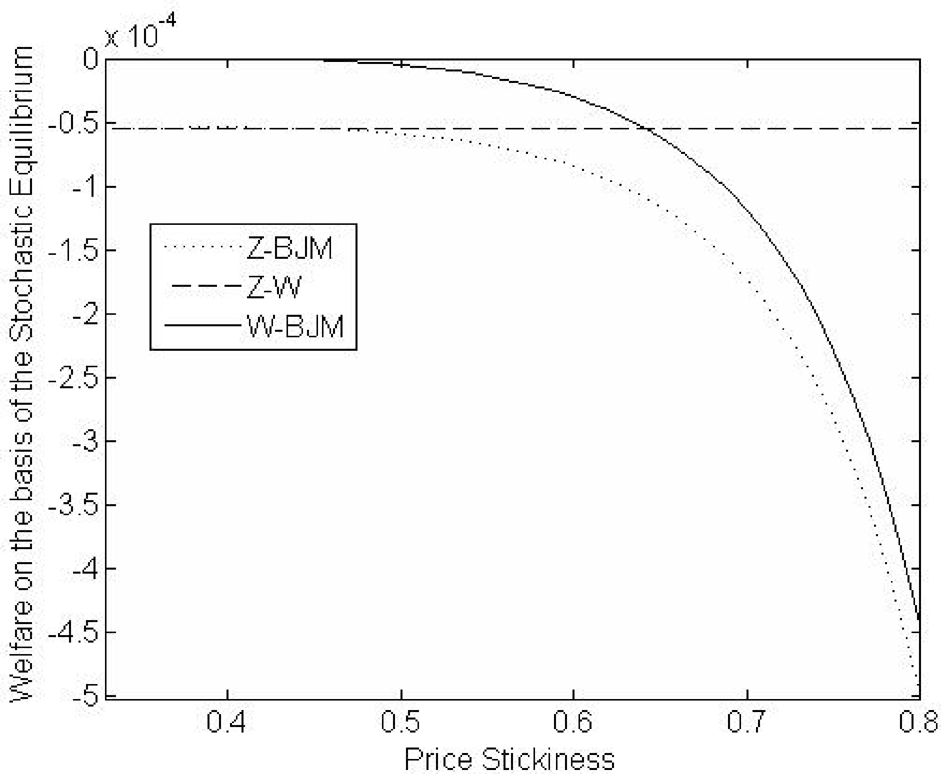

Figure 20. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003): difference in the welfare levels between any pair of commitment policies.

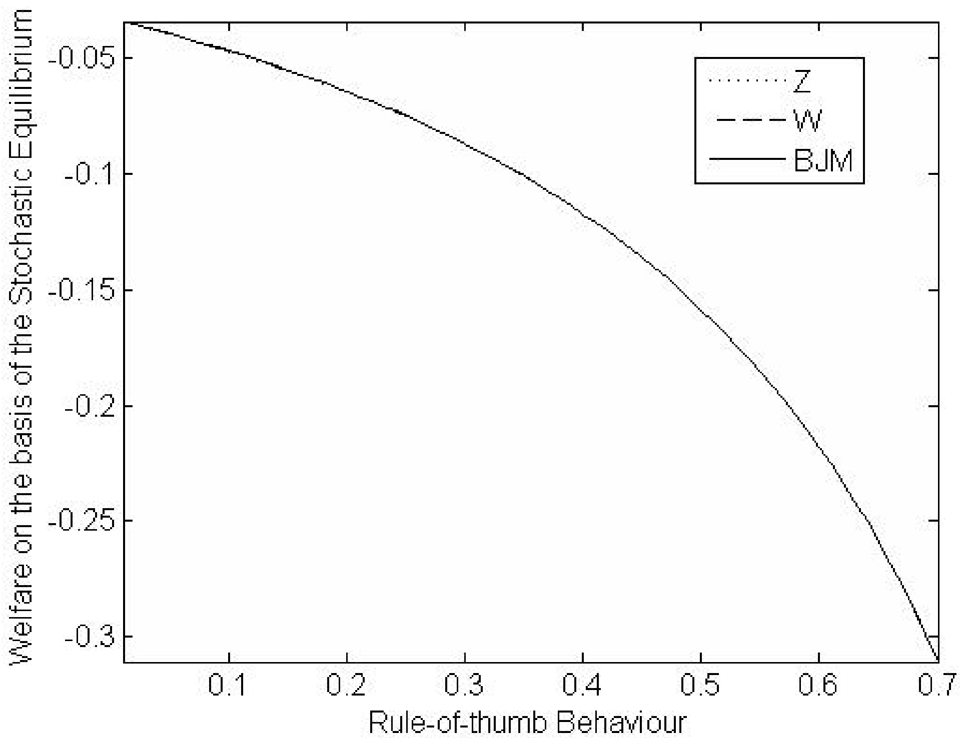

Figure 21. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003).

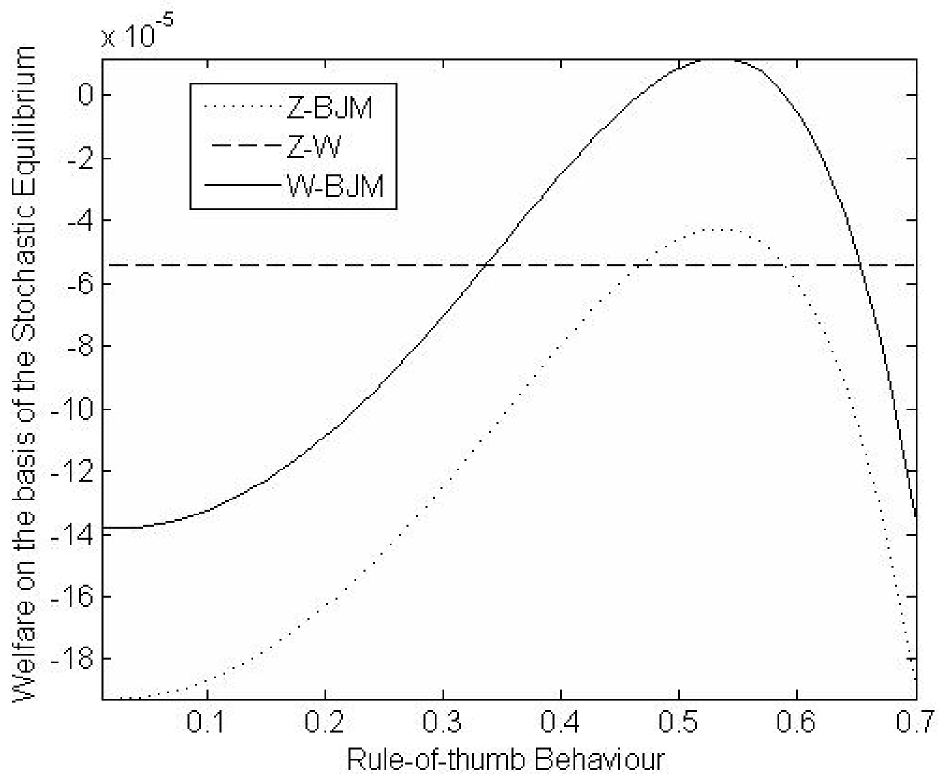

Figure 22. Unconditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003): difference in the welfare levels between any pair of commitment policies.

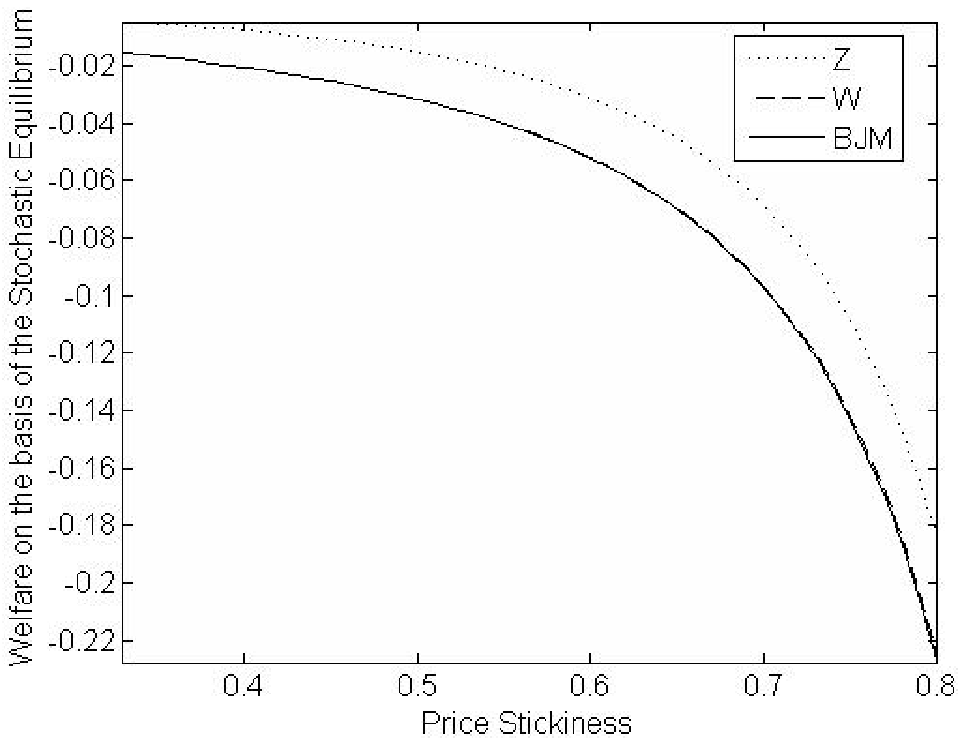

Figure 23. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003).

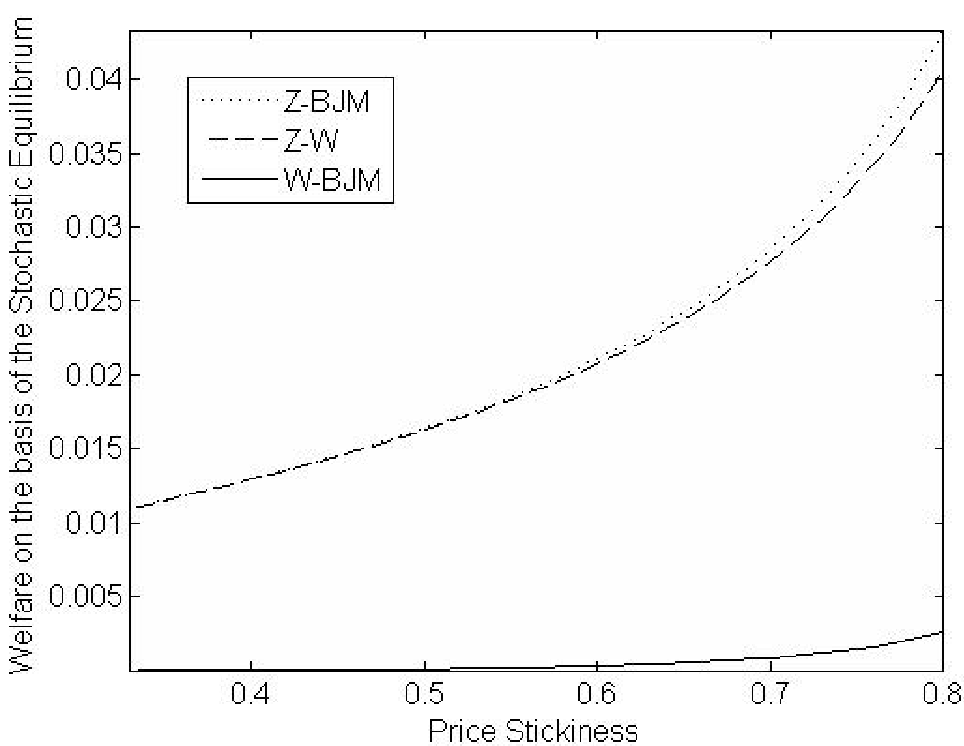

Figure 24. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of price stickiness in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003): difference in the welfare levels between any pair of commitment policies.

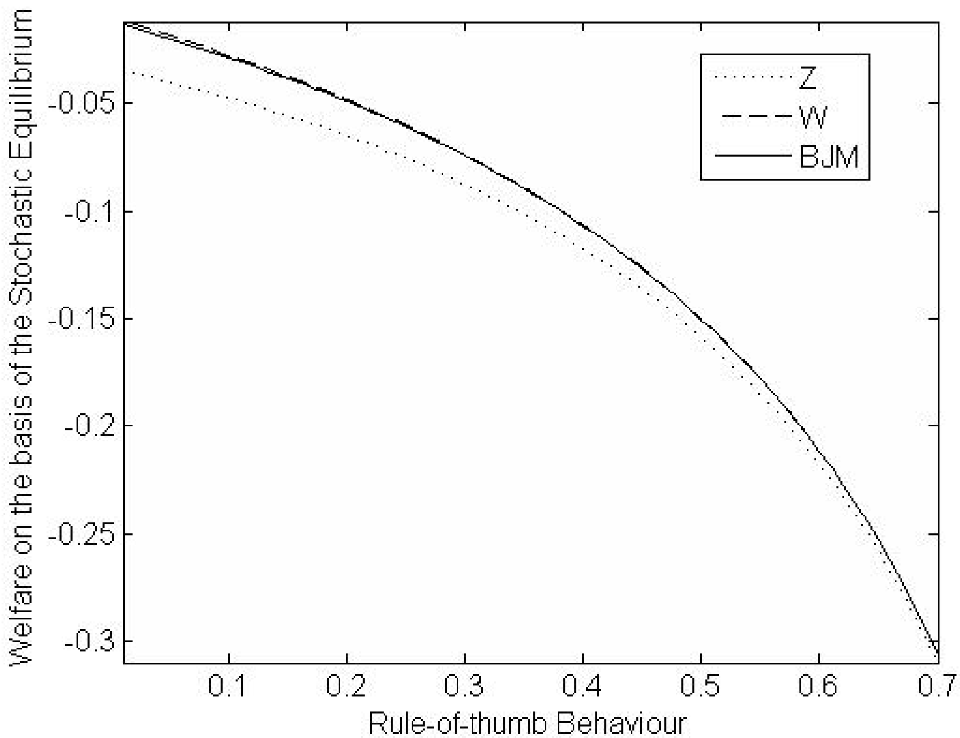

Figure 25. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003).

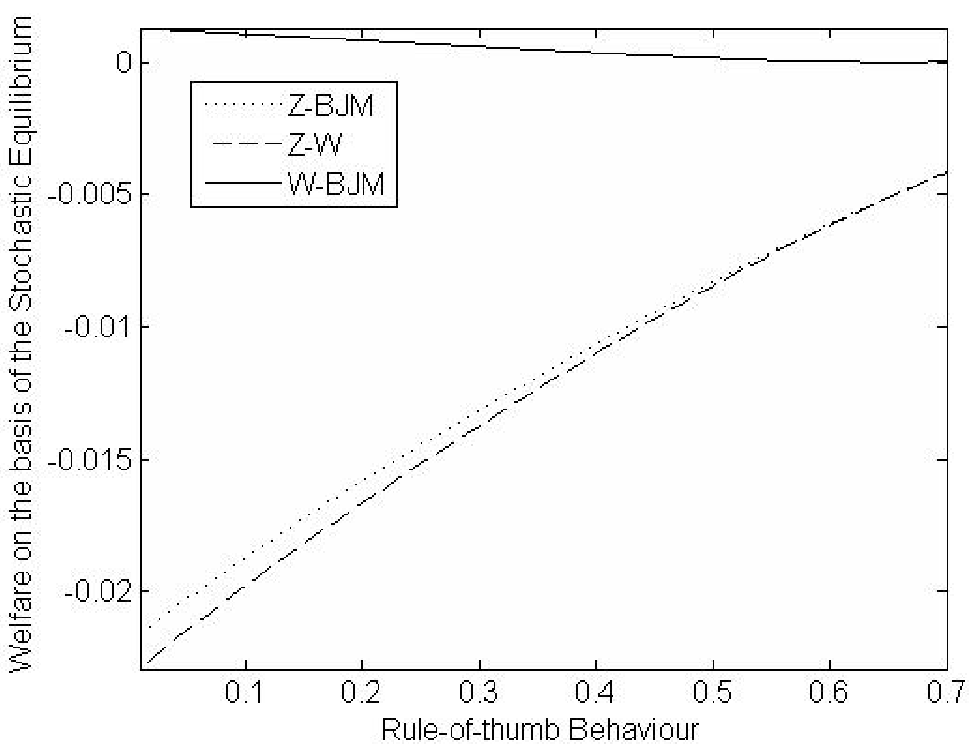

Figure 26. Conditional welfare on the basis of the stochastic equilibrium for different values of the degree of rule-of-thumb behaviour in the New Keynesian model with rule-of-thumb behaviour à la Steinsson (2003): difference in the welfare levels between any pair of commitment policies.