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Abstract

The shift from hunting and gathering to agriculture, some 10,000 years ago, triggered the first demographic explosion in history. Along with population, working time increased, while food consumption remained at the subsistence level. For that reason, most anthropologists regard the adoption of agriculture as an economical puzzle.

I show, using a neoclassical economic model, that there is nothing puzzling about the adoption of agriculture. Agriculture brings four technological changes: an increase in total factor productivity, a stabilization of total factor productivity, less interference of children on production, and the possibility of food storage. In my model, each of those changes induces free, rational and self-interested hunter-gatherers to adopt agriculture. As a result, working time increases while consumption remains at the subsistence level, and population begins to grow until diminishing returns to labor bring it to a halt. Welfare, which depends on consumption, leisure, and fertility, rises at first; but after a few generations it falls below its initial level. Still, the adoption of agriculture is irreversible. The latter generations choose to remain farmers because, at their current levels of population, reverting to hunting and gathering would reduce their welfare.

Key words: Paleoeconomics; economic anthropology; Neolithic Revolution; hunter-gatherers; agriculture; original affluent society.

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1 Introduction

The shift from hunting and gathering to agriculture, usually termed the Neolithic Revolution (10,000 to 5,000 B.P.), triggered the first demographic explosion in the history of humankind (Bocquet-Appel 2002). In the course of few centuries, typical communities grew from about 30 individuals to 300 or more, and population densities increased from less than one hunter-gatherer per square mile, to 20 or more farmers on the same surface (Johnson and Earle 2000, 43, 125, 246).

Population was not the only thing that expanded during the Neolithic Revolution. Working time expanded as well. Ethnographical studies indicate that hunter-gatherers worked less that six hours per day, whereas primitive horticulturists worked seven hours on average, and intensive agriculturalists worked nine (Sackett 1996, 338–42). The increase in working time was, however, not accompanied by an increase in food consumption. If anything, food consumption fell a bit (Armelagos et al. 1991; Cohen and Armelagos 1984), though certainly not much, as hunter-gatherers were already chronically undernourished and constantly threatened by famine (Kaplan 2000). The lost of leisure without an increase in food consumption has convinced most anthropologists that the Neolithic Revolution reduced welfare. For that reason, they regard our ancestors' decision to farm as a puzzle in need of explanation.

I will show, using a neoclassical economic model, that there is nothing puzzling about the facts of the Neolithic Revolution. In my model, rational and self-interested hunter-gatherers freely adopt agriculture. The adoption of agriculture increases working time while consumption remains at the subsistence level, and the initially stable population begins to grow until diminishing returns to labor bring it to a halt. Welfare, which depends on consumption, leisure, and fertility, rises at first; but after few generations it falls below its initial level. Still, the shift from hunting and gathering to agriculture is irreversible. The latter generations choose to remain farmers because, at their current levels of population, reverting to hunting and gathering would reduce their welfare. Many hands make hard work, but there is nothing the hands can do about it.

The adoption of agriculture brings four technological changes: an increase in total factor productivity, a stabilization of total factor productivity, less interference of children on productive activities, and the possibility of food storage. In my model, each technological change reproduces, by itself, the facts of the Neolithic Revolution. Hence, not only are the facts of the Neolithic Revolution not puzzling: from an economist's perspective, they were inevitable.

Most models of the Neolithic Revolution assume that the total factor productivity of agriculture is larger than that of hunting and gathering, at least when the revolution takes place (Weisdorf 2005). Since that assumption is common, I will not discuss it here. The other three technological changes, on the other hand, have been (to my knowledge) disregarded by modelers, and thus merit some attention.

The instability of total factor productivity is probably the main problem of contemporary hunter-gatherers (Kaplan 2000; Johnson and Earle 2000, 57). Their resources increase and decrease periodically (daily for hunters, yearly for gatherers), and every once in a while they fail altogether. Domestication of plants and animals alleviates the problem, by smoothing (though not completely) the yield of the land (Johnson and Earle 2000, 127).

Instability is further alleviated by the possibility of storing food. Most hunter-gatherers are nomads, and carrying food around is too costly a burden for them. The alternative would be to settle down; but as they quickly deplete local resources, the trade-off is solved in favor of moving (Sahlins 1998). Early farmers, on the contrary, led sedentary lives, and produced starchy crops suitable for storing (Johnson and Earle 2000, 33).

Sedentism also reduces the cost of children, mainly because caring for them interferes with food gathering tasks requiring a high degree of mobility (Kramer and Boone 2002).

Related literature

The theories of agriculture adoption have been extensively surveyed elsewhere (Weisdorf 2005). Hence, I will limit the discussion to the two models that share with mine the inclusion of leisure in the utility function; an essential feature, if one is to assess the welfare effects of expanding working time. Those models are Marceau and Myers' (2006) and Weisdorf's (2004).

Marceau and Myers model the adoption of agriculture as a common resource problem. At low levels of technology, the whole population forms a unique band of hunter-gatherers. The members of this band coordinate to prevent the overexploitation of a common resource. As technology improves, the prospect of leaving the band to be a farmer gets more and more attractive. When technology surpasses a certain threshold, the lure of agriculture becomes irresistible and the band breaks apart into a myriad of small communities of farmers. The farmers don't cooperate to preserve the common resource and, as a result, consumption falls while working time increases.

I sustain Marceau and Myers' model fails to provide a good account of the Neolithic Revolution, for two reasons. First, the model predicts that farmers will live in smaller groups than hunter-gatherers, while the opposite is true. Second, the model builds on the unsound assumption that hunter-gatherers coordinate to prevent overexploitation, whereas farmers do not. There is mounting evidence that contemporary hunter-gatherers use individually optimal foraging strategies. They are perfectly willing to exhaust their resources, and when they fail to do so, it is due to their low population densities and inefficient technologies (Penn 2003). Farmers, on the other hand, organize themselves hierarchically, and their leaders often take measures that mitigate the tragedy of the commons. For example, they may regulate the fallow cycle to maximize the yield of the land, or manage the use of pastures to prevent overgrazing (Johnson and Earle 2000, 271, 299, 310, 311, 318, 327, 328, 388).

In *Weisdorf's* model, early farmers give away leisure in exchange for other goods produced by an emerging class of non-food specialists (e.g., craftsmen, chiefs, bureaucrats, and priests). Weisdorf's hypothesis is compelling because non-food specialists were needed to develop the innovations that followed agriculture (e.g., writing, metallurgy), and that characterize civilization. Although I will show that the demand for non-food specialists is not necessary to explain agriculture, the relevance of Weisdorf's explanation relative to my neoclassical account will have to be settled on empirical grounds.

Marceau and Myers, and also Weisdorf, assume population is constant during the transition to agriculture. That is a serious limitation, as the possibility of raising more children probably played a crucial role in our ancestors' decision to become farmers. The population explosion that took place during the Neolithic Revolution clearly points in that direction. My model addresses the issue by assuming reproduction to be a personal decision. A realistic assumption, as it is known that contemporary hunter-gatherers do control population, using such mechanisms as abortion, infanticide, prolonged lactation, and postpartum sex taboos (Cashdan 1985).

Finally, my model is also linked to the family of endogenous fertility models, pioneered by Razin and Ben-Zion (1975). In particular, it is closely related to those models in which the diminishing returns to labor operate as a Malthusian population check; for example, Boldrin and Jones (2002), Eckstein et al. (1988), and Nerlove et al. (1986).

2 A model of agriculture adoption

2.1 Model setup

Time is divided in $t = 1, 2, 3, \ldots$ periods. Each period has two seasons, indexed by $j \in \{1, 2\}$. During period t, a tribe has $N_t > 0$ identical adult members or *tribesmen*. Their lives last exactly one period. Generations do not overlap.

At the beginning of the first season, each tribesman decides how many children to have. Denote by $n_t > 0$ the number of children of a typical tribesman. In the next period, the size of the tribe will thus be $N_{t+1} = n_t N_t$.

To survive, a tribesman must eat at least $\bar{c} > 0$ units of food during each season. Denote by $c_{tj} \geq \bar{c}$ his food consumption during season j. He must also provide $\bar{c}$ units of food per season to each of his children.

Tribesmen work to earn their food. Let $w_{tj} \geq 0$ be a typical tribesman's working time during season j. He will produce $A_{tj} w_{tj}$ units of food during that season; $A_{tj} > 0$ being the typical tribesman's productivity, which he takes as given. A part of production will be lost due to children interference: κ units of food per child, where κ is high if the tribe is nomadic, and low if it is sedentary.

If the tribe is sedentary, a tribesman may store some food at the end of season one, for future consumption during season two. Denote by $s_t \geq 0$ a tribesman's food savings, and let $\sigma = \mathrm{N}$ if the tribe is nomadic and $\sigma = \mathrm{S}$ if it is sedentary. The tribesman is subject to the following food budget constraints:

$$
\begin{aligned}
A_{t1} w_{t1} - \kappa n_t &= c_{t1} + \bar{c} n_t \;\; + s_t, \\
\underbrace{A_{t2} w_{t2} - \kappa n_t}_{\text{Income}} &= \underbrace{c_{t2} + \bar{c} n_t}_{\text{Expenses}} \; \underbrace{- s_t \mathbf{1}_{\mathrm{S}}(\sigma)}_{\text{Savings}},
\end{aligned}
$$

where $\mathbf{1}_{\mathrm{S}}(\sigma)$ is an indicator function that takes value 1 when $\sigma = \mathrm{S}$, and otherwise takes value 0.

Eating food and having children make a tribesman happy, whereas work makes him unhappy. The utility function of a period t tribesman is given by

$$
u(c_{t1}, c_{t2}, w_{t1}, w_{t2}, n_t) = v(c_{t1}) + v(c_{t2}) - \frac{\gamma}{\rho + 1} w_{t1}^{1+\rho} - \frac{\gamma}{\rho + 1} w_{t2}^{1+\rho} + \beta n_t.
$$

Parameter $\beta > 0$ implies that children are valued, whereas $\gamma > 0$ implies tribesmen dislike work. Parameter $\rho > 0$ indicates that, everything else being equal, a tribesman will want to spread his workload evenly between the two seasons. The utility of consumption is strictly increasing and concave: $v' > 0$ and $v'' < 0$. Function u is an instance of Becker's (1992) Malthusian utility function, which doesn't include the quality of children as an argument. As Becker points out, before the Industrial Revolution there were virtually no opportunities to invest on the quality of children; medical care, education, and training were just too rudimentary. Hence, for our purposes, omitting the quality of children from the tribesman utility function is harmless.

The tribe chooses between two production technologies: hunting and gathering, and agriculture. In order to draw a clear "before and after" picture of agriculture adoption, assume all members of the tribe must use the same technology. Which of the two alternatives, they must agree by vote. The equality of all tribesmen entails the election of technology will always be unanimous.

The efficiency of hunting and gathering declines the more people engage on it (Johnson and Earle 2000, 54). Everyday, the tribesmen must venture a little farther from camp in order to obtain food. Eventually, the value of the remaining food falls short of the costs of obtaining it, plus the opportunity cost of lifting the camp and moving somewhere else. A large tribe of hunter-gatherers consumes the "cheaper" food sources near camp faster than a smaller tribe, and also has to incur in the costs of relocating more often. In that spirit, define the productivity of a hunter-gatherer during season j as follows:

$$A_{tj} = a_j \left(N_t w_{tj}\right)^{-\theta}, \tag{1}$$

where $a_j > 0$ is season j total factor productivity, and $0 < \theta < 1$. This condition guarantees that total production increases when the tribe's total work effort increases (i.e. $N_t w_{tj} A_{tj} = a_j \left(N_t w_{tj}\right)^{1-\theta}$ is increasing in $N_t w_{tj}$).

Just as hunting and gathering, agriculture is subject to diminishing returns. Early farmers were mostly slash-and-burners. When population increased, they were forced to speed up the fallow cycle, reducing the productivity of land (Boserup 1965). Therefore, we will also model the productivity of farmers using the formulation in (1), changing the values of a_1 and a_2.

For future use, define average working time ($\bar{w}$), average total factor productivity (μ), and the instability of total factor productivity (δ), as follows:

$$\begin{aligned} \bar{w} &= \frac{w_{t1} + w_{t2}}{2}, \\ \mu &= \frac{a_1 + a_2}{2}, \\ \delta &= \left|\frac{a_1 - a_2}{2}\right|. \end{aligned}$$

Table 1 (page 15) summarizes the notation.

2.2 The tribesman problem

Before solving the tribesman problem, two assumptions are in order. First, assume $a_1 > a_2$, so an abundant season precedes a scarce season. As a result, A_{t1} will always be larger than A_{t2} *in equilibrium*, and tribesmen will want to store some food at the end of season one, even if storing turns out to be impossible (we will confirm that $A_{t1} > A_{t2}$ in Section 2.3). Second, assume

$$\frac{\beta}{\bar{c}+\kappa} > \left\{1 + \left[\frac{a_1}{a_2}\right]^{\frac{1+\rho}{1-\theta}}\right\} v'(\bar{c}), \tag{2}$$

which implies that a tribesman will use any income over $\bar{c}$ to have children. Before the Industrial Revolution, any raise in income induced an increase in population, while consumption remained close to the subsistence level. Inequality 2 guarantees the model will produce a reasonable approximation to the dynamics of consumption before the Industrial Revolution, while letting us focus our attention on the interaction between work and fertility. The inequality will hold if children are cheap enough ($\bar{c}+\kappa$ is sufficiently low).

A tribesman solves

$$\begin{aligned}
\max_{\{c_1,c_2,w_1,w_2,n,s\}} \quad & u = v(c_1) + v(c_2) - \tfrac{\gamma}{\rho+1} w_1^{\rho+1} - \tfrac{\gamma}{\rho+1} w_2^{\rho+1} + \beta n, \\
\text{s.t.} \quad & A_1 w_1 = c_1 + (\bar{c}+\kappa) n + s, \\
& A_2 w_2 = c_2 + (\bar{c}+\kappa) n - s\mathbf{1}_{\mathrm{S}}(\sigma), \\
& c_1, c_2 \geq \bar{c}, \\
& w_1, w_2, n, s \geq 0,
\end{aligned}$$

where the t subscripts have been dropped to simplify the expressions. Table 2 (page 16) displays the solution to the tribesman problem, for the cases without and with storage (i.e. for $\sigma = \mathrm{N}$ and $\sigma = \mathrm{S}$).

2.3 Short-run equilibrium

In the short-run, population is fixed at N° (the empty dot indicates the short-run value of a variable). Equilibrium requires labor productivity to satisfy equation (1). Using that equation, together with the tribesman optimal choices (table 2), we can solve for the short-run equilibrium values of all variables. Table 3 (page 17) displays the short-round results, for the cases without and with storage.

Inspecting table 2, and recalling that $a_1 > a_2$ and $0 < \theta < 1$, we confirm that labor productivity is always larger during the abundant season ($A_1^{\circ} > A_2^{\circ}$). Also, when storing is unfeasible, the tribesmen work more during the scarce season ($w_1^\circ < w_2^\circ$), while if storing is feasible, they work more during the abundant season ($w_1^\circ > w_2^\circ$).

2.4 Long-run equilibrium

In the long-run, diminishing returns to labor operate as a Malthusian check. As population grows, labor productivity declines, until the optimal tribesman's choice is to bear exactly one child: $n^\bullet = 1$ (the full dot indicates the long-run value of a variable). From then onwards, population will remain constant.

Imposing the one child condition on the short-run results (table 3), we can compute the long-run equilibrium values of all variables. Table 4 (page 18) displays the long-run results, for the cases without and with storage.

Proposition 1 (Stability of the long-run equilibrium.) *The long-run equilibrium is stable, meaning that a small deviation from the equilibrium population level ($N^\bullet$) will always be reversed.*

2.5 The adoption of agriculture

Consider a tribe of hunter-gatherers that has reached the long-run equilibrium: each tribesman bears one child ($n^\circ = 1$) and population is at its long-run equilibrium level ($N^\circ = N^\bullet$). One good day, the tribe stumbles upon a new technology: agriculture. Suppose the tribe decides to adopt this new technology (later we will prove that was the rational decision). Agriculture brings four technological changes: an increase in average total factor productivity ($\triangle^+\mu$), a stabilization of total factor productivity ($\triangle^-\delta$), less interference of children on production ($\triangle^-\kappa$), and the possibility of food storage (a change from $\sigma = \mathrm{N}$ to $\sigma = \mathrm{S}$).

Proposition 2 (Short-run effects of agriculture.) *Each technological change of agriculture produces a short-run increase in fertility ($\triangle^+ n^\circ$), average working time ($\triangle^+\bar{w}^\circ$), and utility ($\triangle^+ u^\circ$). In sum:*

$$
\begin{array}{rclrclrcl}
\dfrac{\partial n^\circ}{\partial \mu} & > & 0, & \dfrac{\partial \bar{w}^\circ}{\partial \mu} & > & 0, & \dfrac{\partial u^\circ}{\partial \mu} & > & 0, \\
\dfrac{\partial n^\circ}{\partial \delta} & < & 0, & \dfrac{\partial \bar{w}^\circ}{\partial \delta} & < & 0, & \dfrac{\partial u^\circ}{\partial \delta} & < & 0, \\
\dfrac{\partial n^\circ}{\partial \kappa} & < & 0, & \dfrac{\partial \bar{w}^\circ}{\partial \kappa} & < & 0, & \dfrac{\partial u^\circ}{\partial \kappa} & < & 0, \\
n^\circ[\mathrm{N}] & < & n^\circ[\mathrm{S}]\,, & \bar{w}^\circ[\mathrm{N}] & < & \bar{w}^\circ[\mathrm{S}]\,, & u^\circ[\mathrm{N}] & < & u^\circ[\mathrm{S}]\,.
\end{array}
$$

The generation that adopts agriculture suddenly finds children to be more affordable: feeding one child requires less work when productivity is higher ($\triangle^+\mu,\ \triangle^-\kappa$); a more stable productivity ($\triangle^-\delta$) implies the required work will be a bit more tiring during the abundant season, but much less strenuous during the scarce one; the possibility of storing food ($\sigma = \mathrm{S}$) allows tribesmen to use first season abundance to provide for the times of scarcity. As one would expect, cheaper children translate into increased fertility ($\triangle^{+} n^{\circ}$). The effect of cheaper children on working time, on the other hand, is not as clear cut. Each tribesman could work less hours and still afford more than one children. In our case, the substitution of children for leisure dominates the income effect, so working time increases ($\triangle^{+} \bar{w}^{\circ}$). Finally, as working time increases, labor productivity falls, reducing the efficiency gains of agriculture. The loss in efficiency attenuates the surge in fertility and work, but does not change the direction of the effects.

From proposition 2 we learn that the generation that adopts agriculture is be happy with the changes. In other words, a tribe of selfish, utility-maximizing people will freely abandon hunting and gathering to become farmers. Working time will expand, but the additional toil will be more than compensated by the larger families the tribesmen will be able to afford.

As a consequence of increased fertility, population will start to grow. Eventually, it will stabilize at a new equilibrium with higher population.

Proposition 3 (Long-run effects of agriculture.) *In the long-run, fertility converges to $n^{\bullet} = 1$. The four changes of agriculture produce a long-run increase in population ($\triangle^{+} N^{\bullet}$). Working time will be longer ($\triangle^{+} \bar{w}^{\bullet}$) as a result of the increase in average total factor productivity ($\triangle^{+} \mu$), the stabilization of total factor productivity ($\triangle^{-} \delta$), and the reduction of the interference of children on production ($\triangle^{-} \kappa$). The possibility of food storage ($\sigma = \mathrm{S}$) has an ambiguous effect on working time, which may increase or decrease. Only a reduction of the interference of children on production will have a long-run effect on utility, which will fall below its pre-agriculture level ($\triangle^{-} u^{\bullet}$). In sum:*

$$
\begin{array}{rclrclrcl}
\dfrac{\partial N^{\bullet}}{\partial \mu} & > & 0, & \dfrac{\partial \bar{w}^{\bullet}}{\partial \mu} & > & 0, & \dfrac{\partial u^{\bullet}}{\partial \mu} & = & 0, \\
\dfrac{\partial N^{\bullet}}{\partial \delta} & < & 0, & \dfrac{\partial \bar{w}^{\bullet}}{\partial \delta} & < & 0, & \dfrac{\partial u^{\bullet}}{\partial \delta} & = & 0, \\
\dfrac{\partial N^{\bullet}}{\partial \kappa} & < & 0, & \dfrac{\partial \bar{w}^{\bullet}}{\partial \kappa} & < & 0, & \dfrac{\partial u^{\bullet}}{\partial \kappa} & > & 0, \\
N^{\bullet}[\mathrm{N}] & < & N^{\bullet}[\mathrm{S}]\,, & \bar{w}^{\bullet}[\mathrm{N}] & \gtrless & \bar{w}^{\bullet}[\mathrm{S}]\,, & u^{\bullet}[\mathrm{N}] & = & u^{\bullet}[\mathrm{S}]\,.
\end{array}
$$

Proposition 3 tells us that the descendants from the original farmers will be worse off than their hunter-gatherer ancestors. In spite of that, the transition to agriculture is irreversible. From proposition 2 we infer that, once the new long-run equilibrium has been reached, reverting to hunting and gathering will reduce the utility of the current generation. Hence, they will choose to remain farmers.

Proposition 4 (Long-run effect of food storage on working time.) *In the long-run, the possibility of food storage ($\sigma = \mathrm{S}$) will increase working time ($\triangle^{+} \bar{w}^{\bullet}$) if $2\theta + \rho > 1$.*

In other words, if the returns to labor fall quickly enough (θ is high), or if the tribesmen are sufficiently averse to workload instability (ρ is high), then the possibility of food storage will end up increasing working time. When storage is feasible, ρ^{-1} is the uncompensated labor supply elasticity. The overwhelming majority of estimations locate that elasticity between 0 and 1 (Blundell and MaCurdy 1999). Hence, reasonable values of ρ should be larger than 1. That dispels the ambiguity from proposition 4. If food storage becomes possible, working time will eventually increase ($\triangle^{+}\bar{w}^{\bullet}$).

In the long-run, all tribesmen eat the minimum amount and can only afford to have one child. But in the long-run the tribe is larger and, everything else being equal, that means labor productivity is lower than before. As a result, each tribesmen must work more than his ancestors just to feed himself and his child... unless the tribesman has the chance to store some food. Storing allows the tribesman to substitute a large amount of effort in the scarce season by a smaller amount in the abundant season, when he is more efficient. But even with storage things can get nasty if the returns to labor fall too fast (θ is high): all the additional work during the abundant season could reduce the yield of the land so much that everybody ends up working more than before the adoption of agriculture. Also, if the tribesmen are too inclined to smooth their labor supply through time (ρ is high), they will refuse to work much harder during the abundant season than during the scarce season. If that is the case, working time will increase even if storage is feasible.

In sum, each of the four technological changes is enough to explain the consequences of shifting from hunting and gathering to agriculture: the increased population and working time, while consumption remains at subsistence level. Thus, from an economist's perspective, not only do the facts of the Neolithic Revolution make perfect sense: they were inevitable.

Figure 1 (page 19) illustrates the result of the four changes of agriculture happening together. The figure summarizes 20 periods in the (simulated) history of a tribe. During the first ten periods, the tribesmen make a living out of hunting and gathering. Population stays at its long-run equilibrium level; working time and utility are also constant. At the beginning of period 11, the tribe discovers agriculture. Population increases at first, but after a few generations it stabilizes at a new, higher equilibrium. Working time and utility both soar in period 11. After that, they decline over time. Working time stabilizes above its pre-agriculture level; the utility of the last generations falls below the utility of their hunter-gatherer ancestors. All the while, consumption remains at the subsistence level. Yet the tribesmen of periods 12 and after will not revert to hunting and gathering, as figure 1B evidences. The "shadow" utility of hunting and gathering runs beneath the utility of agriculture. Things get bad for farmers, but their alternative gets even worse.

3 Concluding remarks

> "What needs explanation is why in contemporary contexts hunter-gatherers often demonstrate unlimited, rather than limited, material wants. Why is it that at Momega and, according to the literature, elsewhere modern hunter-gatherers have apparently insatiable demands for shotguns, rifles, motor vehicles, cassette recorders, CD players, televisions, and VCRs?" Jon Altman (1992)

Economics studies how people allocate scarce means to their unlimited wants. As essential as the principle of scarcity is to the economist's way of thinking, it is strongly rejected by other social scientists. Émile Durkheim, a founding father of both sociology and anthropology, believed people learn from their social world what and how much to desire (1953, 95). To Durkheim, the unlimitedness of wants is not part of human nature, but a product of modern Western society: an evil product that fuels the war of all against all (Durkheim 1961, 45; 1969). Max Weber, the famous sociologist and "political economist," also deemed unlimited wants extrinsic, a capitalistic creation. He provided as evidence the behavior of traditional peasants. According to Weber, peasants do not crave for more and more, but are content to live the way they are accustomed. As soon as they satisfy their very limited wants, they stop working. It follows, Weber reasons, that an employer who wants to extract more effort from peasants should lower their wages instead of raising them (Weber 1958, 59–62). Although Weber's characterization of peasant mentality has been debunked countless times (see, for example, James Scott 1985), many of those who reject his evidence as false still embrace his ideas about the cultural origin of our greediness.

When in the 1960s it was established that hunter-gatherers' work very little compared to modern standards, anthropologists thought they had found indisputable proof for Durkheim and Weber's most radical ideas. Professor Emeritus Marshall Sahlins, the dominant voice of contemporary economic anthropology, declared hunter-gatherers the "original affluent society." They are affluent, he argued, not because their means are abundant, but because their wants are few. If the behavior of hunter-gatherers obeys any laws at all, it is the laws of Zen economics (Sahlins 1968, 1998). The principles of neoclassical economics, and in particular the idea of unlimited wants, are nothing but the "origin myth of capitalist society." Economic theory, Sahlins denounced, is merely the rhetoric used by capitalism to justify and perpetuate itself (Sahlins 1976, 53, 205–207).

The affluence of hunter-gatherers turns the adoption of agriculture into a conundrum (if a parent, forced to kill the children he can't feed, can be seriously called affluent). As Hardy (1992) famously put it: "Why farm? Why give up the 20-hour work week and the fun of hunting in order to toil in the sun?" The decision of our ancestors supplied non-economists with ammunition to attack another favorite of economic principles: rationality.

In these pages I have argued that, if read properly, the facts of the Neolithic Revolution bear no evidence against the principles of unlimited wants and rationality. At least from that trench, nothing emerges that obliges us to delete the word *max* from our microeconomic textbooks, or demote nonsatiation from its rank of axiom. Neoclassical economics is perfectly able to explain the behavior of hunter-gatherers: why they work so few hours, why given the chance they become farmers, and why, when exposed to modern life, they demand DVD players, televisions, and iPods.

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TABLE 1

NOTATION SUMMARY

Variable	Sym.	Parameter	Sym.
Population size	N	Subsistence consumption	$\bar{c}$
Season j labor productivity	A_j	Cost of children interference	κ
Season j consumption per tribesman	c_j	Aversion to workload instability	ρ
Season j working time per tribesman	w_j	Weight of work in utility	$-\gamma$
Average working time	$\bar{w}$	Weight of children in utility	β
Number of children per tribesman	n	Season j total factor productivity	a_j
Utility function	u	Average total factor productivity	μ
		Instability of total factor productivity	δ
		Degree of decreasing returns to labor	θ
		Food storage is unfeasible (the tribe is nomadic)	$\sigma = \mathrm{N}$
		Food storage is feasible (the tribe is sedentary)	$\sigma = \mathrm{S}$

TABLE 2

Solution to the tribesman problem

Var.	Without storage	With storage
c_j	$\bar{c}$	$\bar{c}$
w_j	$\left[\frac{\beta/\gamma}{\bar{c}+\kappa}\frac{1}{A_1^{-(\rho+1)}+A_2^{-(\rho+1)}}\right]^{\frac{1}{\rho}} A_j^{-1}$	$\left[\frac{\beta/\gamma}{\bar{c}+\kappa}\frac{A_j}{2}\right]^{\frac{1}{\rho}}$
n	$\left[\frac{\beta/\gamma}{(\bar{c}+\kappa)^{\rho+1}}\frac{1}{A_1^{-(\rho+1)}+A_2^{-(\rho+1)}}\right]^{\frac{1}{\rho}} - \frac{\bar{c}}{\bar{c}+\kappa}$	$\left[\frac{\beta/\gamma}{(\bar{c}+\kappa)^{\rho+1}}\right]^{\frac{1}{\rho}}\left\{\left[\frac{A_1}{2}\right]^{\frac{\rho+1}{\rho}}+\left[\frac{A_2}{2}\right]^{\frac{\rho+1}{\rho}}\right\} - \frac{\bar{c}}{\bar{c}+\kappa}$
s	0	$\left[\frac{\beta/\gamma}{\bar{c}+\kappa}\right]^{\frac{1}{\rho}}\left\{\left[\frac{A_1}{2}\right]^{\frac{\rho+1}{\rho}}+\left[\frac{A_2}{2}\right]^{\frac{\rho+1}{\rho}}\right\}$

TABLE 3

Short-run equilibrium

Var.	Without storage	With storage
A_j°	$\left\{\frac{\bar{c}+\kappa}{(N^\circ)^\rho\beta/\gamma}\left[a_1^{-\frac{\rho+1}{1-\theta}}+a_2^{-\frac{\rho+1}{1-\theta}}\right]\right\}^{\frac{\theta}{\theta+\rho}} a_j^{\frac{1}{1-\theta}}$	$\left[2\frac{\bar{c}+\kappa}{(N^\circ)^\rho\beta/\gamma}\right]^{\frac{\theta}{\theta+\rho}} a_j^{\frac{\rho}{\theta+\rho}}$
c_j°	$\bar{c}$	$\bar{c}$
w_j°	$\left\{\frac{\beta/\gamma}{(N^\circ)^\theta(\bar{c}+\kappa)}\left[a_1^{-\frac{\rho+1}{1-\theta}}+a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-1}\right\}^{\frac{1}{\theta+\rho}} a_j^{-\frac{1}{1-\theta}}$	$\left[\frac{\beta/\gamma}{(N^\circ)^\theta(\bar{c}+\kappa)}\frac{a_j}{2}\right]^{\frac{1}{\theta+\rho}}$
n°	$\left\{\frac{(\beta/\gamma)^{1-\theta}}{\left[(N^\circ)^\theta(\bar{c}+\kappa)\right]^{\rho+1}}\right\}^{\frac{1}{\theta+\rho}}\left[a_1^{-\frac{\rho+1}{1-\theta}}+a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{1-\theta}{\theta+\rho}}-\frac{\bar{c}}{\bar{c}+\kappa}$	$\left\{\frac{(\beta/\gamma)^{1-\theta}}{\left[(N^\circ)^\theta(\bar{c}+\kappa)\right]^{\rho+1}}\right\}^{\frac{1}{\theta+\rho}}\left\{\left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}}+\left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}}\right\}-\frac{\bar{c}}{\bar{c}+\kappa}$
s°	0	$\left\{\frac{1}{(N^\circ)^{\theta(\rho+1)}}\left[\frac{\beta/\gamma}{\bar{c}+\kappa}\right]^{1-\theta}\right\}^{\frac{1}{\theta+\rho}}\left\{\left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}}-\left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}}\right\}$
U°	$2v(\bar{c})-\frac{\bar{c}\beta/\gamma}{\bar{c}+\kappa}+\frac{\rho}{\rho+1}\left[\frac{\beta/\gamma}{(N^\circ)^\theta(\bar{c}+\kappa)}\right]^{\frac{\rho+1}{\theta+\rho}}\left[a_1^{-\frac{\rho+1}{1-\theta}}+a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{1-\theta}{\theta+\rho}}$	$2v(\bar{c})-\frac{\bar{c}\beta/\gamma}{(\bar{c}+\kappa)}+\frac{\rho}{1+\rho}\left[\frac{\beta/\gamma}{(N^\circ)^\theta(\bar{c}+\kappa)}\right]^{\frac{\rho+1}{\theta+\rho}}\left\{\left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}}+\left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}}\right\}$

TABLE 4

Long-run equilibrium

Var.	Without storage	With storage
$N^\bullet$	$\left\{ \frac{1}{(2\bar{c}+\kappa)^{\theta+\rho}} \left[\frac{\beta/\gamma}{\bar{c}+\kappa}\right]^{1-\theta} \right\}^{\frac{1}{\theta(\rho+1)}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-\frac{1-\theta}{\theta(\rho+1)}}$	$\left\{ \frac{1}{(2\bar{c}+\kappa)^{\theta+\rho}} \left[\frac{\beta/\gamma}{\bar{c}+\kappa}\right]^{1-\theta} \right\}^{\frac{1}{\theta(\rho+1)}} \left\{ \left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}} + \left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}} \right\}^{\frac{\theta+\rho}{\theta(\rho+1)}}$
$A_j^\bullet$	$\left[\frac{(\bar{c}+\kappa)(2\bar{c}+\kappa)^{\rho}}{\beta/\gamma}\right]^{\frac{1}{\rho+1}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{\frac{1}{\rho+1}} a_j^{\frac{1}{1-\theta}}$	$\left[\frac{(\bar{c}+\kappa)(2\bar{c}+\kappa)^{\rho}}{\beta/\gamma}\right]^{\frac{1}{\rho+1}} \left\{ \left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}} + \left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}} \right\}^{-\frac{\rho}{\rho+1}} \left(2^{\theta} a_j^{\rho}\right)^{\frac{1}{\theta+\rho}}$
$c_j^\bullet$	$\bar{c}$	$\bar{c}$
$w_j^\bullet$	$\left[\frac{(2\bar{c}+\kappa)\beta/\gamma}{\bar{c}+\kappa}\right]^{\frac{1}{\rho+1}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-\frac{1}{\rho+1}} a_j^{-\frac{1}{1-\theta}}$	$\left[\frac{(2\bar{c}+\kappa)\beta/\gamma}{\bar{c}+\kappa}\right]^{\frac{1}{\rho+1}} \left\{ \left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}} + \left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}} \right\}^{-\frac{1}{\rho+1}} \left[\frac{a_j}{2}\right]^{\frac{1}{\theta+\rho}}$
$n^\bullet$	1	1
$s^\bullet$	0	$(2\bar{c}+\kappa) \left\{ \left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}} + \left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}} \right\}^{-1} \left\{ \left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}} - \left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}} \right\}$
$U^\bullet$	$2v\left(\bar{c}\right) + \frac{\beta/\gamma}{(\rho+1)} \left[\rho - \frac{\bar{c}}{\bar{c}+\kappa}\right]$	$2v\left(\bar{c}\right) + \frac{\beta/\gamma}{(\rho+1)} \left[\rho - \frac{\bar{c}}{\bar{c}+\kappa}\right]$

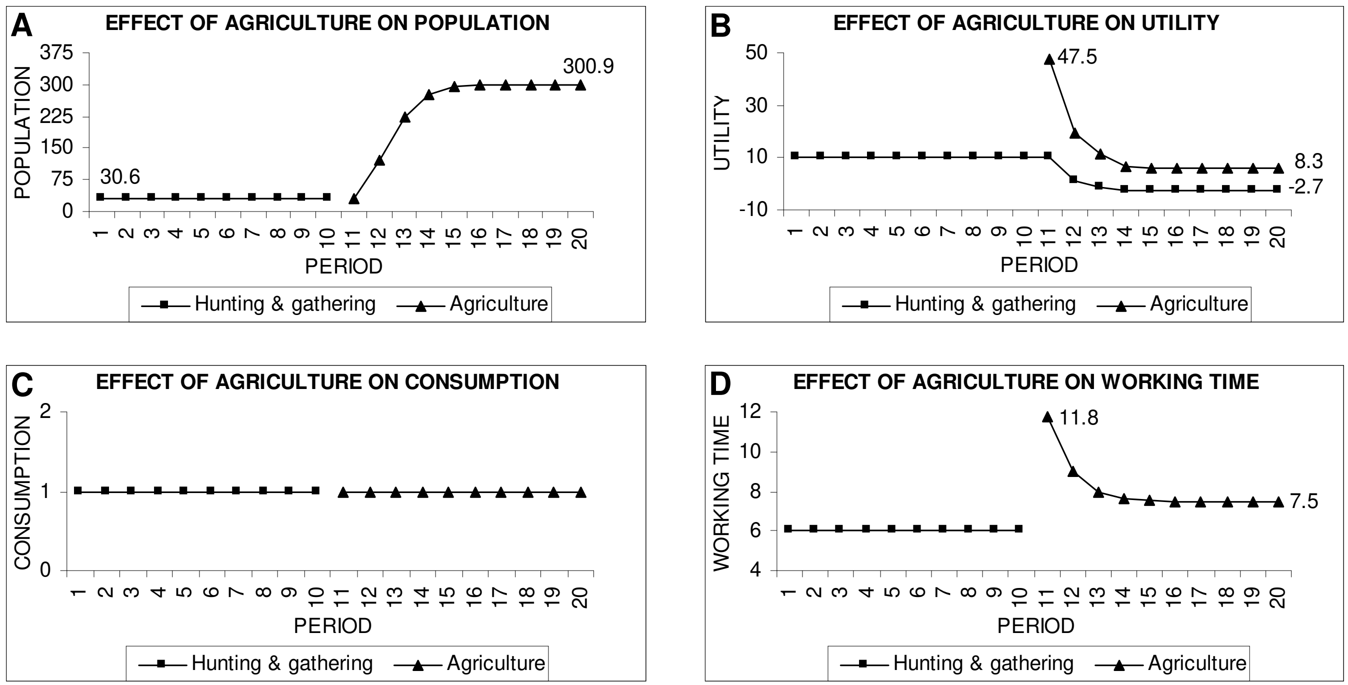

Figure 1: From periods 1 to 10 the tribesmen hunt and gather ($\mu = 5.2$, $\delta = 2.25$, $\kappa = 1$, $\sigma = \mathrm{N}$). At the beginning of period 11 they adopt agriculture ($\mu = 5.89$, $\delta = 0$, $\kappa = 0.5$, $\sigma = \mathrm{S}$). Simulation parameters: $\bar{c} = 1$, $\beta = 10$, $\gamma = 0.42$, $\rho = 1.26$, $\theta = 0.31$, $N_1 = 30.6$.

A Appendix

A.1 Preliminary results

Three functions that will be useful later:

$$\begin{aligned}
\mathrm{K}_p(x_1,x_2) &= \frac{x_1^p + x_2^p}{x_1^{p-1} - x_2^{p-1}}, \\
\mathrm{L}_p(x_1,x_2) &= \frac{x_1^p + x_2^p}{x_1^{p-1} + x_2^{p-1}}, && \text{(Lehmer mean)} \\
\mathrm{M}_p(x_1,x_2) &= \left[\frac{1}{2}x_1^p + \frac{1}{2}x_2^p\right]^{\frac{1}{p}}. && \text{(Generalized mean)}
\end{aligned}$$

Lemma 5 *If $p < q$ and $x_1 > x_2$, then $\mathrm{K}_p(x_1,x_2) > \mathrm{K}_q(x_1,x_2)$.*

Proof. From $x_1 > x_2$, it follows that

$$\frac{\partial K_p(x_1,x_2)}{\partial p} = -(\ln x_1 - \ln x_2)\frac{x_1^p x_2^{p-1} + x_1^{p-1} x_2^p}{\left[x_1^{p-1} - x_2^{p-1}\right]^2} < 0.$$

■

Lemma 6 (Lehmer mean inequality) *If $p < q$ and $x_1 \neq x_2$, then $L_p(x_1,x_2) < L_q(x_1,x_2)$.*

Lemma 7 (Generalized mean inequality) *If $p < q$ and $x_1 \neq x_2$, then $M_p(x_1,x_2) < M_q(x_1,x_2)$.*

A.2 Proof of proposition 1

For the dynamic system $N_{t+1} = n_t N_t$ to be stable, the following condition is sufficient:

$$-2 < N^{\bullet} \left.\frac{\partial n^{\circ}}{\partial N^{\circ}}\right|_{N^{\circ}=N^{\bullet}} < 0. \tag{3}$$

Condition 3 guarantees that if N_t is close to $N^{\bullet}$, then N_{t+1} will be even closer.

A.2.1 Case 1: Storage is unfeasible

If storage is unfeasible, we have that

$$N^{\circ} \frac{\partial n^{\circ}}{\partial N^{\circ}} = -\frac{\theta(\rho+1)}{\theta+\rho} (N^{\circ})^{-\frac{\theta(\rho+1)}{\theta+\rho}} \left[\frac{(\beta/\gamma)^{1-\theta}}{(\bar{c}+\kappa)^{\rho+1}}\right]^{\frac{1}{\theta+\rho}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{1-\theta}{\theta+\rho}}.$$

Plugging $N^{\bullet}$ into the previous expression we get

$$N^{\bullet} \left.\frac{\partial n^{\circ}}{\partial N^{\circ}}\right|_{N^{\circ}=N^{\bullet}} = -\underbrace{\frac{\theta(\rho+1)}{\theta+\rho}}_{\tau_1} \underbrace{\left[2 - \frac{\kappa}{\bar{c}+\kappa}\right]}_{\tau_2}.$$

From $0 < \theta < 1$ and $\rho > 1$, it follows that $0 < \tau_1 < 1$. From $\bar{c}, \kappa > 0$, it follows that $0 < \tau_2 < 2$. As a result, $-2 < (N^{\bullet}\partial n^{\circ}/\partial N^{\circ})|_{N^{\circ}=N^{\bullet}} < 0$.

A.2.2 Case 2: Storage is feasible

If storage is feasible, we have that

$$N^{\circ} \frac{\partial n^{\circ}}{\partial N^{\circ}} = -\frac{\theta(\rho+1)}{\theta+\rho} (N^{\circ})^{-\frac{\theta(\rho+1)}{\theta+\rho}} \left[\frac{(\beta/\gamma)^{1-\theta}}{(\bar{c}+\kappa)^{\rho+1}}\right]^{\frac{1}{\theta+\rho}} \left\{\left[\frac{a_1}{2}\right]^{\frac{\rho+1}{\theta+\rho}} + \left[\frac{a_2}{2}\right]^{\frac{\rho+1}{\theta+\rho}}\right\}.$$

Plugging $N^{\bullet}$ into the previous expression we get

$$N^{\bullet} \left.\frac{\partial n^{\circ}}{\partial N^{\circ}}\right|_{N^{\circ}=N^{\bullet}} = -\frac{\theta(\rho+1)}{\theta+\rho} \left[2 - \frac{\kappa}{\bar{c}+\kappa}\right].$$

So again $-2 < N^{\bullet} (\partial n^{\circ}/\partial N^{\circ})|_{N^{\circ}=N^{\bullet}} < 0$. ■

A.3 Proof of proposition 2

The following proofs build on the short-run equilibrium results of table 3. Recall that $a_1 > a_2 > 0$, $0 < \theta < 1$, and $\beta, \gamma, \rho, \bar{c}, \kappa, N > 0$.

- $\partial n^\circ / \partial \mu > 0$.

  Proof.

$$\frac{\partial n^\circ}{\partial \mu} = \frac{\rho+1}{\theta+\rho} \left\{ \frac{(\beta/\gamma)^{1-\theta}}{\left[(N^\circ)^\theta (\bar{c}+\kappa) \right]^{\rho+1}} \right\}^{\frac{1}{\theta+\rho}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-\frac{\rho+1}{\theta+\rho}} \left[a_1^{-\frac{\rho+1}{1-\theta}-1} + a_2^{-\frac{\rho+1}{1-\theta}-1} \right] > 0.$$

  ■

- $\partial n^\circ / \partial \delta < 0$.

  Proof.

$$\frac{\partial n^\circ}{\partial \delta} = \frac{\rho+1}{\theta+\rho} \left\{ \frac{(\beta/\gamma)^{1-\theta}}{\left[(N^\circ)^\theta (\bar{c}+\kappa) \right]^{\rho+1}} \right\}^{\frac{1}{\theta+\rho}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-\frac{\rho+1}{\theta+\rho}} \underbrace{\left[a_1^{-\frac{\rho+1}{1-\theta}-1} - a_2^{-\frac{\rho+1}{1-\theta}-1} \right]}_{\tau}.$$

  The sign of $\partial n^\circ / \partial \delta$ depends on τ. Since $a_1 > a_2$, term τ is negative. Hence, $\partial n^\circ / \partial \delta < 0$. ■

- $\partial n^\circ / \partial \kappa < 0$.

  Proof.

$$\frac{\partial n^\circ}{\partial \kappa} = -\frac{\rho+1}{\theta+\rho} (\bar{c}+\kappa)^{-\frac{\rho+1}{\theta+\rho}-1} \left[\frac{(\beta/\gamma)^{1-\theta}}{(N^\circ)^{\theta(\rho+1)}} \right]^{\frac{1}{\theta+\rho}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right] < 0.$$

  ■

- $n^\circ[\mathrm{N}] < n^\circ[\mathrm{S}]$.

  Proof.

$$n^\circ[\mathrm{S}] - n^\circ[\mathrm{N}] = \left\{ \frac{(\beta/\gamma)^{1-\theta}}{\left[(N^\circ)^\theta (\bar{c}+\kappa) \right]^{\rho+1}} \right\}^{\frac{1}{\theta+\rho}}$$
$$2^{-\frac{1-\theta}{\theta+\rho}} \underbrace{\left[\mathrm{M}_{-\frac{1}{1-\theta}} \left(a_1^{\rho+1}, a_2^{\rho+1} \right)^{\frac{1}{\rho+\theta}} - \mathrm{M}_{\frac{1}{\theta+\rho}} \left(a_1^{\rho+1}, a_2^{\rho+1} \right)^{\frac{1}{\rho+\theta}} \right]}_{\tau}.$$

The sign of $n^\circ[\mathrm{S}] - n^\circ[\mathrm{N}]$ depends on τ, which will be positive if

$$\mathrm{M}_{-\frac{1}{1-\theta}}\left(a_1^{\rho+1}, a_2^{\rho+1}\right) < \mathrm{M}_{\frac{1}{\theta+\rho}}\left(a_1^{\rho+1}, a_2^{\rho+1}\right).$$

But

$$-\frac{1}{1-\theta} < \frac{1}{\theta+\rho}.$$

Thus, from the generalized mean inequality, we conclude $\tau > 0$, so $n^\circ[\mathrm{N}] < n^\circ[\mathrm{S}]$. ■

- $\partial \bar{w}^\circ / \partial \mu > 0$.

  Proof. Instead of $\partial \bar{w}^\circ/\partial\mu$, consider $\partial \ln \bar{w}^\circ/\partial\mu$, which has the same sign as $\partial \bar{w}^\circ/\partial\mu$.

$$\frac{\partial \ln \bar{w}^\circ}{\partial \mu} = \frac{1}{1-\theta}\underbrace{\left[\frac{\rho+1}{\theta+\rho}\mathrm{L}_{-\frac{\rho+1}{1-\theta}}\left(a_1, a_2\right)^{-1} - \mathrm{L}_{-\frac{1}{1-\theta}}\left(a_1, a_2\right)^{-1}\right]}_{\tau}.$$

The sign of $\partial \ln \bar{w}^\circ/\partial\mu$ depends on τ. But

$$\begin{aligned}
\frac{\rho+1}{\theta+\rho} &> 1, \\
\mathrm{L}_{-\frac{\rho+1}{1-\theta}}\left(a_1, a_2\right) &> 0, \\
\mathrm{L}_{-\frac{1}{1-\theta}}\left(a_1, a_2\right) &> 0.
\end{aligned}$$

Therefore,

$$\tau > \mathrm{L}_{-\frac{\rho+1}{1-\theta}}\left(a_1, a_2\right)^{-1} - \mathrm{L}_{-\frac{1}{1-\theta}}\left(a_1, a_2\right)^{-1}.$$

On the other hand,

$$-\frac{\rho+1}{1-\theta} < -\frac{1}{1-\theta}.$$

Thus, from the Lehmer mean inequality it follows that

$$\mathrm{L}_{-\frac{\rho+1}{1-\theta}}\left(a_1, a_2\right) < \mathrm{L}_{-\frac{1}{1-\theta}}\left(a_1, a_2\right),$$

or equivalently

$$\mathrm{L}_{-\frac{\rho+1}{1-\theta}}\left(a_1, a_2\right)^{-1} - \mathrm{L}_{-\frac{1}{1-\theta}}\left(a_1, a_2\right)^{-1} > 0.$$

That implies $\tau > 0$, so $\partial \ln \bar{w}^\circ/\partial\mu > 0$ and $\partial \bar{w}^\circ/\partial\mu > 0$. ■

- $\partial \bar{w}^\circ / \partial \delta < 0$.

  Proof. Instead of $\partial \bar{w}^\circ / \partial \delta$, consider $\partial \ln \bar{w}^\circ / \partial \delta$, which has the same sign as $\partial \bar{w}^\circ / \partial \delta$.

$$\frac{\partial \ln \bar{w}^\circ}{\partial \delta} = \frac{1}{1-\theta} \underbrace{\left[\frac{\rho+1}{\theta+\rho} \mathrm{K}_{-\frac{\rho+1}{1-\theta}} \left(a_1, a_2\right)^{-1} - \mathrm{K}_{-\frac{1}{1-\theta}} \left(a_1, a_2\right)^{-1} \right]}_{\tau}.$$

  The sign of $\partial \ln \bar{w}^\circ / \partial \delta$ depends on τ. But

$$\begin{aligned} \frac{\rho+1}{\theta+\rho} &> 1, \\ \mathrm{K}_{-\frac{\rho+1}{1-\theta}} \left(a_1, a_2\right) &< 0, \\ \mathrm{K}_{-\frac{1}{1-\theta}} \left(a_1, a_2\right) &< 0. \end{aligned}$$

  Therefore,

$$\tau < \mathrm{K}_{-\frac{\rho+1}{1-\theta}} \left(a_1, a_2\right)^{-1} - \mathrm{K}_{-\frac{1}{1-\theta}} \left(a_1, a_2\right)^{-1}.$$

  On the other hand,

$$-\frac{\rho+1}{1-\theta} < -\frac{1}{1-\theta}.$$

  Thus, from lemma 5 it follows that

$$\mathrm{K}_{-\frac{\rho+1}{1-\theta}} \left(a_1, a_2\right) > \mathrm{K}_{-\frac{1}{1-\theta}} \left(a_1, a_2\right),$$

  or equivalently

$$\mathrm{K}_{-\frac{\rho+1}{1-\theta}} \left(a_1, a_2\right)^{-1} - \mathrm{K}_{-\frac{1}{1-\theta}} \left(a_1, a_2\right)^{-1} < 0.$$

  That implies $\tau < 0$, so $\partial \ln \bar{w}^\circ / \partial \delta < 0$ and $\partial \bar{w}^\circ / \partial \delta < 0$. ■

- $\partial \bar{w}^\circ / \partial \kappa < 0$.

  Proof.

$$\frac{\partial \bar{w}^\circ}{\partial \kappa} = -\frac{1}{\theta+\rho} \left(\bar{c}+\kappa\right)^{-\frac{1}{\theta+\rho}-1} \left\{ \frac{\beta/\gamma}{\left(N^\circ\right)^\theta} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-1} \right\}^{\frac{1}{\theta+\rho}} \left[a_1^{-\frac{1}{1-\theta}} + a_2^{-\frac{1}{1-\theta}} \right] < 0.$$

  ■

- $\bar{w}^{\circ}[\mathrm{N}] < \bar{w}^{\circ}[\mathrm{S}]$.

  Proof.

$$\begin{aligned}\bar{w}^{\circ}[\mathrm{S}] - \bar{w}^{\circ}[\mathrm{N}] \quad &= \quad \left\{\frac{\beta/\gamma}{2\left(N^{\circ}\right)^{\theta}\left(\bar{c}+\kappa\right)}\right\}^{\frac{1}{\theta+\rho}} \\ &\quad \underbrace{\left\{\mathrm{M}_{\frac{1}{\theta+\rho}}\left(a_1,a_2\right)^{\frac{1}{\theta+\rho}} - \mathrm{M}_{-\frac{\rho+1}{1-\theta}}\left(a_1,a_2\right)^{\frac{1}{\theta+\rho}} \frac{\mathrm{M}_1\left(a_1^{-1/(1-\theta)}, a_2^{-1/(1-\theta)}\right)}{\mathrm{M}_{\rho+1}\left(a_1^{-1/(1-\theta)}, a_2^{-1/(1-\theta)}\right)}\right\}}_{\tau}.\end{aligned}$$

  The sign of $\bar{w}^{\circ}[\mathrm{S}] - \bar{w}^{\circ}[\mathrm{N}]$ depends on τ. But

$$\frac{1}{\theta+\rho} > -\frac{\rho+1}{1-\theta},$$

  and thus, from the generalized mean inequality,

$$\mathrm{M}_{\frac{1}{\theta+\rho}}\left(a_1,a_2\right)^{\frac{1}{\theta+\rho}} > \mathrm{M}_{-\frac{\rho+1}{1-\theta}}\left(a_1,a_2\right)^{\frac{1}{\theta+\rho}} > 0.$$

  Also, $1 < \rho + 1$. So again, from the generalized mean inequality,

$$0 < \frac{\mathrm{M}_1\left(a_1^{-1/(1-\theta)}, a_2^{-1/(1-\theta)}\right)}{\mathrm{M}_{\rho+1}\left(a_1^{-1/(1-\theta)}, a_2^{-1/(1-\theta)}\right)} < 1.$$

  Therefore, $\tau > 0$, and that implies $\bar{w}^{\circ}[\mathrm{S}] - \bar{w}^{\circ}[\mathrm{N}] > 0$. ■

- $\partial u^{\circ}/\partial \mu > 0$.

  Proof.

$$\frac{\partial u^{\circ}}{\partial \mu} = \frac{\rho}{\theta+\rho}\left[\frac{\beta/\gamma}{\left(N^{\circ}\right)^{\theta}\left(\bar{c}+\kappa\right)}\right]^{\frac{\rho+1}{\theta+\rho}}\left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{\rho+1}{\theta+\rho}}\left[a_1^{-\frac{\rho+1}{1-\theta}-1} + a_2^{-\frac{\rho+1}{1-\theta}-1}\right] > 0.$$

  ■

- $\partial u^{\circ}/\partial \delta < 0$.

  Proof.

$$\frac{\partial u^{\circ}}{\partial \delta} = \frac{\rho}{\rho+1}\left[\frac{\beta/\gamma}{\left(N^{\circ}\right)^{\theta}\left(\bar{c}+\kappa\right)}\right]^{\frac{\rho+1}{\theta+\rho}}\left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{1-\theta}{\theta+\rho}}\underbrace{\left[a_1^{-\frac{\rho+1}{1-\theta}-1} - a_2^{-\frac{\rho+1}{1-\theta}-1}\right]}_{\tau} < 0.$$

  The sign of $\partial u^{\circ}/\partial \delta$ depends on τ. Since $a_1 > a_2$, term τ is negative. Hence, $\partial u^{\circ}/\partial \delta < 0$. ■

- $\partial u^{\circ}/\partial\kappa < 0$.

  Proof.

$$\frac{\partial u^{\circ}}{\partial\kappa} = -\frac{\rho}{\theta+\rho}\left(\bar{c}+\kappa\right)^{-\frac{\rho+1}{\theta+\rho}-1}\left[\frac{\beta/\gamma}{\left(N^{\circ}\right)^{\theta}}\right]^{\frac{\rho+1}{\theta+\rho}}\left[a_1^{-\frac{\rho+1}{1-\theta}}+a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{1-\theta}{\theta+\rho}} < 0.$$

  ■

- *In the short run* $u^{\circ}[\mathrm{N}] < u^{\circ}[\mathrm{S}]$.

  Proof.

$$u^{\circ}[\mathrm{S}] - u^{\circ}[\mathrm{N}] = \frac{\rho}{1+\rho}\left[\frac{\beta/\gamma}{\left(N^{\circ}\right)^{\theta}\left(\bar{c}+\kappa\right)}\right]^{\frac{\rho}{\theta+\rho}}\left(n\left[\mathrm{S}\right]-n\left[\mathrm{N}\right]\right).$$

  But $n^{\circ}[\mathrm{N}] < n^{\circ}[\mathrm{S}]$, and thus, $u^{\circ}[\mathrm{N}] < u^{\circ}[\mathrm{S}]$. ■

A.4 Proof of proposition 3

The following proofs build on the long-run equilibrium results of table 4. Recall that $a_1 > a_2 > 0$, $0 < \theta < 1$, and $\beta, \gamma, \rho, \bar{c}, \kappa > 0$.

- $\partial N^\bullet / \partial \mu > 0$.

  Proof.

$$\frac{\partial N^\bullet}{\partial \mu} = \frac{1}{\theta} \left\{ \frac{1}{(2\bar{c}+\kappa)^{\theta+\rho}} \left[\frac{\beta/\gamma}{\bar{c}+\kappa} \right]^{1-\theta} \right\}^{\frac{1}{\theta(\rho+1)}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-\frac{\theta\rho+1}{\theta(\rho+1)}} \left[a_1^{-\frac{\rho+1}{1-\theta}-1} + a_2^{-\frac{\rho+1}{1-\theta}-1} \right] > 0.$$

  ■

- $\partial N^\bullet / \partial \delta < 0$.

  Proof.

$$\frac{\partial N^\bullet}{\partial \delta} = \frac{1}{\theta} \left\{ \frac{1}{(2\bar{c}+\kappa)^{\theta+\rho}} \left[\frac{\beta/\gamma}{\bar{c}+\kappa} \right]^{1-\theta} \right\}^{\frac{1}{\theta(\rho+1)}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-\frac{\theta\rho+1}{\theta(\rho+1)}} \underbrace{\left[a_1^{-\frac{\rho+1}{1-\theta}-1} - a_2^{-\frac{\rho+1}{1-\theta}-1} \right]}_{\tau}.$$

  The sign of $\partial N^\bullet / \partial \delta$ depends on τ. Since $a_1 > a_2$, term τ is negative. Hence, $\partial N^\bullet / \partial \delta < 0$. ■

- $\partial N^\bullet / \partial \kappa < 0$.

  Proof.

$$\frac{\partial N^\bullet}{\partial \kappa} = -\frac{1-\theta}{\theta(\rho+1)} (\bar{c}+\kappa)^{-\frac{1-\theta}{\theta(\rho+1)}-1} \left[\frac{(\beta/\gamma)^{1-\theta}}{(2\bar{c}+\kappa)^{\theta+\rho}} \right]^{\frac{1}{\theta(\rho+1)}} \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}} \right]^{-\frac{1-\theta}{\theta(\rho+1)}} < 0.$$

  ■

- $N^\bullet[\mathrm{N}] < N^\bullet[\mathrm{S}]$.

$$N^\bullet[\mathrm{S}] - N^\bullet[\mathrm{N}] = \left\{ \frac{1}{(2\bar{c}+\kappa)^{\theta+\rho}} \left[\frac{\beta/\gamma}{\bar{c}+\kappa} \right]^{1-\theta} \right\}^{\frac{1}{\theta(\rho+1)}} 2^{-\frac{1-\theta}{\theta(\rho+1)}} \underbrace{\left[\mathrm{M}_{\frac{\rho+1}{\theta+\rho}} (a_1, a_2)^{\frac{1}{\theta}} - \mathrm{M}_{-\frac{\rho+1}{1-\theta}} (a_1, a_2)^{\frac{1}{\theta}} \right]}_{\tau}.$$

The sign of $N^{\bullet}[\mathrm{S}] - N^{\bullet}[\mathrm{N}]$ depends on τ, and τ will be positive if

$$\mathrm{M}_{-\frac{\rho+1}{1-\theta}}(a_1, a_2) < \mathrm{M}_{\frac{\rho+1}{\theta+\rho}}(a_1, a_2).$$

But

$$-\frac{\rho+1}{1-\theta} < \frac{\rho+1}{\theta+\rho}.$$

Thus, from the generalized mean inequality, we conclude $\tau > 0$, so $N^{\bullet}[\mathrm{N}] < N^{\bullet}[\mathrm{S}]$.

- $\partial \bar{w}^{\bullet}/\partial \mu > 0$

Proof. Instead of $\partial \bar{w}^{\bullet}/\partial \mu$, consider $\partial \ln \bar{w}^{\bullet}/\partial \mu$, which has the same sign as $\partial \bar{w}^{\bullet}/\partial \mu$.

$$\frac{\partial \ln \bar{w}^{\bullet}}{\partial \mu} = \frac{1}{1-\theta} \underbrace{\left[\mathrm{L}_{-\frac{\rho+1}{1-\theta}}(a_1, a_2)^{-1} - \mathrm{L}_{-\frac{1}{1-\theta}}(a_1, a_2)^{-1} \right]}_{\tau}.$$

The sign of $\partial \ln \bar{w}^{\bullet}/\partial \mu$ depends on τ. But

$$-\frac{\rho+1}{1-\theta} < -\frac{1}{1-\theta}.$$

Thus, from the Lehmer mean inequality it follows that

$$\mathrm{L}_{-\frac{\rho+1}{1-\theta}}(a_1, a_2) < \mathrm{L}_{-\frac{1}{1-\theta}}(a_1, a_2),$$

or equivalently

$$\mathrm{L}_{-\frac{\rho+1}{1-\theta}}(a_1, a_2)^{-1} - \mathrm{L}_{-\frac{1}{1-\theta}}(a_1, a_2)^{-1} > 0.$$

That implies $\tau > 0$, so $\partial \ln \bar{w}^{\bullet}/\partial \mu > 0$ and $\partial \bar{w}^{\bullet}/\partial \mu > 0$. ■

- $\partial \bar{w}^{\bullet}/\partial \delta > 0$

Proof. Instead of $\partial \bar{w}^{\bullet}/\partial \delta$, consider $\partial \ln \bar{w}^{\bullet}/\partial \delta$, which has the same sign as $\partial \bar{w}^{\bullet}/\partial \delta$.

$$\frac{\partial \ln \bar{w}^{\bullet}}{\partial \delta} = \frac{1}{1-\theta} \underbrace{\left[\mathrm{K}_{-\frac{\rho+1}{1-\theta}}(a_1, a_2)^{-1} - \mathrm{K}_{-\frac{1}{1-\theta}}(a_1, a_2)^{-1} \right]}_{\tau}.$$

The sign of $\partial \ln \bar{w}^{\bullet}/\partial \delta$ depends on τ. But

$$-\frac{\rho+1}{1-\theta} < -\frac{1}{1-\theta}.$$

Thus, from lemma 5 it follows that

$$\mathrm{K}_{-\frac{\rho+1}{1-\theta}}\left(a_1,a_2\right) > \mathrm{K}_{-\frac{1}{1-\theta}}\left(a_1,a_2\right),$$

or equivalently

$$\mathrm{K}_{-\frac{\rho+1}{1-\theta}}\left(a_1,a_2\right)^{-1} - \mathrm{K}_{-\frac{1}{1-\theta}}\left(a_1,a_2\right)^{-1} < 0.$$

That implies $\tau < 0$, so $\partial \ln \bar{w}^{\bullet}/\partial\delta < 0$ and $\partial \bar{w}^{\bullet}/\partial\delta < 0$. ■

- $\partial \bar{w}^{\bullet}/\partial\kappa > 0$

  Proof.

$$\frac{\partial \bar{w}^{\bullet}}{\partial\kappa} = -\frac{1}{\rho+1}\frac{c}{(2c+\kappa)^{\frac{\rho}{\rho+1}}\left(c+\kappa\right)^{\frac{\rho+2}{\rho+1}}}\left(\beta/\gamma\right)^{\frac{1}{\rho+1}}\left[a_1^{-\frac{\rho+1}{1-\theta}}+a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{1}{\rho+1}}\left[a_1^{-\frac{1}{1-\theta}}+a_2^{-\frac{1}{1-\theta}}\right] < 0.$$

  ■

- $\partial u^{\bullet}/\partial\kappa > 0$

  Proof.

$$\frac{\partial \bar{w}^{\bullet}}{\partial\kappa} = \frac{c\beta/\gamma}{\left(\rho+1\right)\left(c+\kappa\right)^2} > 0.$$

  ■

A.5 Proof of proposition 4

$\bar{w}^{\bullet}[\mathrm{S}] > \bar{w}^{\bullet}[\mathrm{N}]$ if and only if

$$\Delta\bar{w}^{\bullet}(\rho,\theta) \equiv \frac{1}{2}\left[\frac{(2\bar{c}+\kappa)\,\beta/\gamma}{\bar{c}+\kappa}\right]^{\frac{1}{1+\rho}}$$

$$\left\{\left[a_1^{\frac{\rho+1}{\theta+\rho}} + a_2^{\frac{\rho+1}{\theta+\rho}}\right]^{-\frac{1}{\rho+1}}\left[a_1^{\frac{1}{\theta+\rho}} + a_2^{\frac{1}{\theta+\rho}}\right] - \left[a_1^{-\frac{\rho+1}{1-\theta}} + a_2^{-\frac{\rho+1}{1-\theta}}\right]^{-\frac{1}{\rho+1}}\left[a_1^{-\frac{1}{1-\theta}} + a_2^{-\frac{1}{1-\theta}}\right]\right\} > 0.$$

Function $\Delta\bar{w}^{\bullet}$ is continuous in $\rho > 0$ and $0 < \theta < 1$. Also, provided that $a_1 > a_2 > 0$, it is straightforward that $\Delta\bar{w}^{\bullet}$ takes value 0 if and only if $\rho + 2\theta = 1$.

From continuity it follows that $\Delta\bar{w}^{\bullet}$ will have the same sign for all ρ and θ in the set

$$\mathcal{A}_{+} \equiv \{(\rho,\theta) : \rho > 0,\, 0 < \theta < 1, \text{ and } \rho + 2\theta > 1\}.$$

One point in set $\mathcal{A}_{+}$ is $(1, 1/2)$. Evaluating $\Delta\bar{w}^{\bullet}$ at that point we get.

$$\Delta\bar{w}^{\bullet}(1,1/2) = \frac{1}{2}\left[\frac{(2\bar{c}+\kappa)\,\beta/\gamma}{\bar{c}+\kappa}\right]^{\frac{1}{2}}\underbrace{\left\{\mathrm{H}_{2/3}(a_1,a_2) - \mathrm{H}_2(a_1,a_2)\right\}}_{\tau},$$

where

$$\mathrm{H}_p(a_1,a_2) = \frac{a_1^p + a_2^p}{\left(a_1^{2p} + a_2^{2p}\right)^{1/2}}.$$

But, for all $p > 0$ and $a_1 \neq a_2$,

$$\frac{\partial\,\mathrm{H}_p(a_1,a_2)}{\partial p} = -a_1^p a_2^p\,(\ln a_1 - \ln a_2)\,\frac{a_1^p - a_2^p}{\left(a_1^{2p} + a_2^{2p}\right)^{3/2}} < 0.$$

Hence $\mathrm{H}_{2/3}(a_1,a_2) > \mathrm{H}_2(a_1,a_2)$, term $\tau > 0$, function $\Delta\bar{w}^{\bullet}(1,1/2) > 0$, and finally $\Delta\bar{w}^{\bullet}(\rho,\theta) \geq 0$ for all $(\rho,\theta) \in \mathcal{A}_{+}$.

The proof that $\Delta\bar{w}^{\bullet}(\rho,\theta) \leq 0$ for all $(\rho,\theta) \in \mathcal{A}_{-} \equiv \{(\rho,\theta) : \rho > 0,\, 0 < \theta < 1, \text{ and } \rho + 2\theta < 1\}$ is analogous, so I omit it. ■