Review of Howard DeLong (1991), "A refutation of Arrow's theorem", with a reaction, also on its relevance in 2008 for the European Union

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Abstract

There will be many researchers who discover voting theory afresh and who will want to understand it and its interesting paradoxes. Arrow's theorem (1951, 1963) is the most celebrated result in social choice theory. It has been criticized a lot but Howard DeLong (1991), "A refutation of Arrow's theorem", is a monograph that actually succeeds. The booklet has received insufficient attention in the literature. This review also compares DeLong's approach with my own book "Voting theory for democracy" (2007) and comments on the relevance in 2008 for the European Union, with respect to the veto power of its Member States and their citizens.

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Introduction

There will be many researchers who discover voting theory afresh and who will want to understand it and its interesting paradoxes. Voting is crucial for our notion of democracy. Voting schemes are already implemented on the internet and these programs will become more complicated and more official, and it is very likely that that they will shape the environment in which we live. So it is advisable that we arrive at a good understanding of both voting theory and democracy.

The contention is: DeLong (1991) takes issue with Arrow (1951, 1963) and his Impossibility Theorem on a social welfare function generating mechanism.

The contenders are: Howard DeLong is the author of "A profile of mathematical logic", DeLong (1971), which is a beautiful textbook that combines elegant mathematics with history, wit, relevance and decency. Kenneth Arrow is the co-winner of the Nobelprize 1972, with John Hicks, and has once been characterized as the "Eric Clapton of economics".

The book is: DeLong (1991) is a hardcover booklet with 96 pages, including references and index. It is as well written as his book on logic, with again this combination of elegant mathematics with history, wit, relevance and decency. It is not a textbook however but a well organized monograph and, indeed, refutation of Arrow's Theorem. The book started originally with a lecture thirteen years earlier and has been developed into its current form.

The present author of this review is: An econometrician who worked at the Dutch Central Planning Bureau (CPB) 1982-1991 and who there discussed Arrow's theorem as well, see Colignatus (1990c). The analysis evolved into Colignatus (1992a) and eventually my book Colignatus (2001, 2007b) "Voting theory for democracy" (VTFD).

I only learned about the booklet in the mid of 2007, after completing the second edition of VTFD and when I published my own book on logic, Colignatus (2007b), "A logic of exceptions" (ALOE). Having ALOE, I ventured on contacting DeLong since I had delighted in his logic book as a student back in 1980. The academic year 2007-2008 I was too busy and only this last week I had the time to contact an internet bookshop, so Saturday July 19 2008 I received my first copy, and it is a wonderful read. It is special to see the same logic again too.

What you are reading now is both a review and a reaction. The use of both terms clarifies that I aspire at giving an unbiased review, but that I also have my own approach, so that perhaps I don't quite succeed in this goal. In other circumstances I perhaps might have hesitated with a reaction but given the continued misunderstandings about Arrow's Theorem I better do react. It seems that DeLong's refutation has not received sufficient attention and this review and reaction are in order. Note also that the book need no longer reflect DeLong's current thinking, as I actually gather from a comment from him. Nevertheless, I would like to give a general support to this book and the idea that Arrow's approach has been refuted. Yet, the devil is in the details, and it should be clarified that I do not support everything. For example, DeLong rejects the Pareto principle, which however is important for the European Union, with respect to the veto power of its Member States and their citizens.

DeLong:

> "In general, Arrow's theorem certainly can add weight to the belittlement of democracy and can thus be used to weaken commitment to democratic institutions. In my judgment this potential effect of the theorem is not too remote to be worth worrying about. It is for this reason that I have been so unrelenting in my criticisms. I hope to stimulate others to correct possible deficiencies in this monograph by, for example, publishing objections to the proof which I may have overlooked or misstated." (p52)

The following gives a general overview and then comments by page.

The denotation is: KA = Arrow (1951, 1963), AT = Arrow's Theorem, HD = DeLong (1991), TC90 = Colignatus (1990c), TC92 = Colignatus (1992a), VTFD = Colignatus (2001, 2007b), TC = TC90 & TC92 & VTFD, and SCT = social choice theory.

The AT can be seen as $a \Rightarrow falsum$ where *a* are the axioms and *falsum* is the contradiction. The conundrum is that AK claims that the axioms *a* are reasonable and morally desirable for democracy, while logic and mathematical rigour show that we have to reject them. Note that AT is a mathematical truth and that the problem arises with the other claims. From VTFD p342 I can usefully copy:

> AT = the Arrow Theorem
> ARC = the Arrow Reasonableness Claim = the properties are reasonable
> AMC = the Arrow Moral Claim = that they are to be imposed
> AGV = the Arrow General View = AT & ARC & AMC

Note that the AGV also generates the Arrow conclusion (AC) : "there is no reasonable or morally desirable constitution for democracy". Generally, people confuse the AT and AC, and overlook the AGV.

General overview of the book

AT versus AGV and AC

HD states: "The most common criticisms are that the result is correct but irrelevant, or that one or another of Arrow's assumptions is wrong. Yet the theorem has not been the object of a full-scale attack. That is what will be attempted here." (p1)

I think that this is imprecise. The theorem itself is the AT and this AT is mathematically correct. What is problematic is its interpretation and relevance. The "attack" should not be directed at the AT but at the AGV and AC and those who hold this misunderstanding.

In a way HD distinguishes these angles anyhow: "My intention is to refute not only the theorem but, in addition, to show that its philosophical base is antidemocratic and should be abandoned."

If the book had carried this in a longer title, it would have gotten more attention, appeal and convincing power.

Nevertheless, the booklet does not sharply distinguish AT from AGV and AC, which is also confusing for the reader. It is a confusion that is also present in the original AK and that took me some struggle to get rid off. Henceforth, where HD speaks about "Arrow's theorem" in the sense of the AGV then I will not write AT but AGV (and it is not necessary to state AC).

The method of refutation and the role of the lot

HD starts with the notions of democracy and then shows that AGV does not capture those notions. First of all, democratic procedures can use the lot as a tie-breaking rule, while AGV does not recognize ties but turns these into cycles, i.e. logical contradictions. The impossibility derives from a bad modelling of the possibility of aggregate indifference and the resolution of deadlocks. Secondly, HD discusses the various other axioms in the AT and also exposes their deficiency for modelling democracy. Finally, HD concludes that the AGV had a dismal effect on academic thinking on democracy while it is quite unfounded.

This method can usefully be compared to TC. That is, TC90, TC92 and VTFD also hold that democracy is more complex, yet, distinguish AT and AGV and, when discussing the AT, then they consider the AT on itself, with its internal logic and within its simplified world.

- TC90 emphasizes the difference between voting results and deciding. The axioms don't provide a decision framework. TC90 also contains another argument but does not emphasize it. This situation is turned around in TC92.
- TC92 recasts TC90 in a different format and concentrates on the AGV, i.e. AK's meta-mathematical claims on the value of the AT, i.e. that the axioms would be reasonable and morally desirable. TC92 then emphasizes, with more clarity than TC90, that if the axioms cause an impossibility then this very fact shows that they are neither reasonable nor morally desirable. I have henceforth found that this approach is the best.
- VTFD expands the discussion. It starts with a general notion of democracy and the theory of law, presents various voting routines, restates TC90 and TC92, and presents a new voting mechanism, the Borda Fixed Point method (as a development of a suggestion in TC90). Majority voting is presented as a tie-breaking rule for Pareto optimal points.

TC90 p7 states (where the random lot also occurs):

> "Black (1848) also wrote: "the committee adopts as its decision ("resolution") that motion, if any, which is able to get a simple majority over every other". From Black's elaborate description of procedures too it is evident that he was conscious of the distinction between the simple act of casting a vote, and the complex effort of arriving at a decision. But somehow this important distinction got lost in the subsequent onslaught."
>
> "Dahl (1956) calles the Condorcet situation a 'deadlock'. In other words: there indeed could be a case for social indifference; and to further resolve it: allow for a chairperson, or for power, or for dice, or a budget, or for more bargaining – or whatever gets Buridan's ass going. But while indifference indeed is the solution, it is another tragedy that Dahl prefers the solution of intransitivity of social choice."

Thus, on method, HD provides a much richer environment than TC. The booklet provides insightful references to ancient Greek and Roman writers and American practice with democracy. Each axiom is put into historical and democratic context, and it is explained what logically is required for an impossibility proof. The reader comes away from the book with the idea that the AGV has indeed been refuted.

Agreement and wonder

HD and TC thus agree that AK did not have a theorem to write home about. He only found that a misspecified social decision making situation can be inconsistent.

The booklet increases the wonder why so many intelligent writers fell for the AGV. HD has a bit the tendency to blame the "Arrow theorem" (AT or AGV of AC ?) for the subsequent impact that it had, but I would rather wonder why people let that impact occur. Perhaps the AT is a bit arcane mathematics, impressing to non-mathematicians and deluding to mathematicians who don't know much about the real world, with a bizarre wasteland of non-comprehension, also without proper distinction between AT and AGV.

I fully agree with HD p63: "In my view Arrow's separation of social choice theory from actual democratic social choices has been a fateful blunder. (footnote). (….) With only a small amount of exaggeration we can say that social choice theory is a fascinating academic game but otherwise has been unimportant in human culture." I agree, though with the proviso that the AGV has been highly misleading and has caused that many academics turned cynical about democracy, which has not been good for society.

Remaining doubts

However, the book still leaves some doubt. A main reason is, as said, that HD does not strictly distinguish AT and AGV, so that also the reader can be confused. There are two explicit reasons for doubt:

(1) The refutation is not a refutation in a normal sense, where the theorem would be shown to be false. It is not so much the internal logic that has been refuted but rather the applicability of the axioms for democracy.

(2) HD thus mentions all kinds of conditions that have to be fulfilled for proper democracy. He does not show these conditions to be consistent. This still leaves open that the AT might be adjusted in some way such that those conditions appear to suffer the same impossibility. Indeed, some readers might reject the lot as a proper democratic tool. Thus there might not be a generally acceptable way to turn voting cycles into indifference and subsequently turn a voting result into a decision.

I am sensitive to these two points since they are covered by TC. It is clear that HD has been aware of points (1) and (2). But given that TC has dealt with them in a different way, my impression is that HD itself still leaves too much room for doubt on them. This lingering doubt may be one of the reasons why the booklet apparently did not make a big splash.

Point (1) is dealt with rudimentarily by TC90 and more explicitly by TC92. When axioms result into a contradiction then they cannot be reasonable or morally desirable. The verbal and meta-mathematical reasoning is turned into a precise argument in deontic logic. AK has the choice between inconsistency or incompleteness.

Point (2) is dealt with by VTFD by suggesting the Borda FP method for Pareto points starting from some Status Quo. This procedure is called “majority voting”.

A technical comment for the following discussion

In the following I will comment per page. I will use the phrase “This is an elaboration issue” where points (1) and (2) apply. This means that the HD observation is valid but runs the possibility of objection that the AGV might still be recovered by some elaboration. Because of TC we can be sure that such recovery is not possible but having only HD it still might be feasible.

Discussion by pages

1. Introduction

p1: Contains quotes by Robert Paul Wolff and Paul Samuelson that curiously support the AGV (Arrow’s general view). Note, again, why it is useful to make the distinction between AT and AGV. If we would say that these authors support AT, then this is merely correct, since it is a mathematical theorem. But the impact comes from the other claims, that they apparently also support. It may be noted that Samuelson is a renowed economist and a strong supporter of empirical methods yet has done little econometrics himself. The distance between mathematics and the real world may be too large.

p6-13: Discuss the lot to break ties. These pages establish that this is a ancient, renowned and decent democratic tool. A social scientist might object that the use of a tie breaking rule by lot is precisely the argument that democracy is not ideally possible, i.e. that people have to resort to tricks. Another example is your breakfast: one day you put cheese and another day marmalade

on your bread, and the social scientist will complain that you are not consistent. HD does not quite deal with this objection except by stating that these are facts of life and decent democratic methods. However, the answer by TC is: Whatever all this may be, given that AT holds, we are forced for practical democracy to drop one of the axioms. In this situation, everyone has to pick his ideal. SCT may come up with a suggestion, and it is unscientific not to try. Indeed VTFD does.

HD's discussion of the practice of using the lot, since ancient times, is valuable. It really helps to see it in perspective. Not all lots are for tie breaking, also for handling information. But the tie breaking is a sufficient blow to the AGV. Hence, the AGV is not practical but desiring of something impossible. With the AGV refuted, the rest of the book is logically superfluous (but still a good read).

And the AGV is silly. Using the example of HD, its argument can be restructured in the following way. It is reasonable and morally desirable to count the age of people by their birthdays. After 21 birthdays all persons should be 21 years of age. However, there are people who then have only 5 birthdays (February 29). They are 5 years of age. This is false. Hence there is no reasonable and morally desirable way to count the age.

p9: Gives a good discussion of an irrational element in the AT.

p9, footnote 7: "In my judgment these are best treated by approval voting." However, see Colignatus (2005) for an explanation that approval voting lacks a sound moral base, especially when the Status Quo is lacking.

The discussion here causes these two questions: (1) Why did Arrow not discuss tie-breaking rules and the lot in his thesis ? He also discarded the Status Quo, that is so important in TC. Perhaps we see here a mathematician on the hunt for a theorem, with a social sauce to make it more interesting for his environment at the Cowles Commission. (2) Why didn't other authors jump on that and kill the AGV right in 1951 – or at least somewhat later ?

2. Collective rationality

p15: States that people might be forced by intimidation. This is a wrong objection. The AT presumes individual choice and non-dictatorship.

p16: States that people can be indifferent but still must make a choice, e.g. by flipping a mental coin. This is a wrong objection. In principle the generality of the proof of the AT allows both for indifference or for the interpretation of such a mental flip as the revelation of actual preference. The AT is not dynamic so it does only consider one breakfast and not different breakfasts with their variety. At best this is an elaboration issue.

p17: States: "The social choice is thus not derived from the set of preference orderings (…) but those preferences orderings *and* a random selector." This is OK in so far as we would want to make a distinction between voting and deciding, but, the logic of the situation requires that we would respect the final (random) choice as the collective result. If we would not accept the final result as the collective choice then there would be no reason to flip the coin.

The conclusion "Hence Connexity is not a reasonable restriction to place on social choice" thus is not warranted.

p17-21: These give a very curious discussion. HD: "Preference is in fact a nontransitive relation." Economists like me will not tend to accept this. In economics, preference is transitive, rather by definition of rationality. If you flip cheese and marmalade on your sandwiches, then

we presume the desire for variation, making the issue rational again. The examples provided by HD cannot convince. A cards game with a sucker and a shark (see e.g. wikipedia on nontransitive dice) is creating a different topic of discussion. The AT generally assumes that preferences are independent but HD now suggests dependence. A possible situation is that a husband prefers to differ from his wife but the wife prefers to have the same opinion as her husband: and there is no solution. For simplicity, the AT abstracts from this kind of difficulty. This is not an objection but an elaboration issue.

p19: Describes that $A > B > C > A$, with HD: "it is quite rational for society to have a definite preference with regards to any pair, but to have no definite preference with respect to all the candidates." This is gibberish. This confuses voting versus deciding.

- What HD calls a "definite preference with regards to any pair" is only a voting result, that needs to be judged in the light of the other votes to see what it means.
- In terms of decision making, this group is indifferent, and may make choice with a tie breaking rule.

Note that this HD statement is inconsistent with the earlier statement that the lot can break a tie. Note also the distinction between "preference" and "definite preference". It comes close to the distinction between voting and deciding, but is still confused on what a final decision means.

p20: Gives the *sorites*, the ancient paradox from pulling out hairs, where one hair does not make a difference, so that you can pull hairs without becoming bald. The example or case used here concerns rules for admission to college, where the rules appear to cause a paradox. Clearly, this is not a proof for intransitive or nontransitive preference, only reason to advise that college to devise a consistent procedure.

p20-21: Contains the example of a chess club that wants to buy a chess computer, with $A > B$ and $B > C$, with ">" now meaning "beats", and then $C > A$. HD does not mention that chess players know about the Elo rating, see VTFD section 7.7. Again, it may be that $A > B > C > A$ but this does not imply that this is preference. Preference is rational and transitive (otherwise there are irrational emotions).

HD: "We can certainly agree that some people feel strongly that preference must be transitive. To me this is like a strong feeling that the set of positive integers is larger than the set of even numbers." Having read the logic book it is fun to immediately understand what this is about. However, HD is not accurate. What is meant by "larger" need not only be the possibility suggested by Cantor of a bijective projection but might also be: "For any natural number $K > 0$, let $\text{pos}(K) = \{n \mid n \text{ is positive and } n < K\}$ and let $\text{even}(K) = \{n \mid n \text{ is even and } n < K\}$. Then the set of positive numbers is "larger" than the set of even numbers iff for all K, $\#(\text{pos}(K) > \#(\text{even}(K))$." I haven't checked whether this is only ad hoc, and cannot be extended to other concepts, yet for this case it captures the intuition. Cantor's suggestion is fine, and his use of the word "larger" has found wide application, but it does not capture what other people can mean when they use the word "larger". If Cantor provided the mathematically superior interpretation of "larger" then this should be associated with transitive preference, not the other way around.

p22: These pages argue that we cannot impose the condition of collective rationality but must prove it from the bottom up. This is a misunderstanding of the structure of the AT. The AT has a conditional form: if we assume such and such, then there is a contradiction.

Some of the axioms actually are not axioms but part of a definition of what a democratic society must be: we want a rational society, not an irrational one.

In overview, this chapter of the book has given no valid argument. Though the reader will come away with a grasp of the issues involved.

3. Pareto principle

p25: sets the stage on defining what the discussion is about.

p26: objects that the axiom need not be fulfilled in cases of lack of information and expectations that are not realized. This is an elaboration issue.

p26-27: gives examples of inconsistent preferences: (a) the wish to execute illegal immigrants and society's wish not to execute innocent children, (b) the wish of desperate people to sell themselves into slavery while the constitution warrants freedom, and (c) the wish of a terrorist that he dies and everyone agreeing with the idea but the law requiring a fair trial. The argument is that the collective system of democracy with its checks and balances and system of justice does not allow room for specific individual preferences.

In itself this seems correct. The AGV has already shown incorrect. This new point is another elaboration issue.

Note the following. Seen from a positive angle, AK's research develops social choice theory in a decent axiomatic approach, trying to determine the relevant axioms for democracy. In that case we start with a simple world, like Euclid starting with points and lines instead of fractals. In that case it is very enlightening to start with the Pareto principle. If all people wish to burn witches then they should be consistent and also burn witch babies. When they are shocked by that consequence then apparently their preferences are different and they should revise their earlier statement. If not, and if indeed the constitution has come about with some different properties, then Arrow's axioms need refinement. But this is an elaboration issue. See VTFD chapters 9 and 10.

p28-29: These pages invoke the principle of "self-paternalism" and quote James Madison on the "cool and deliberate sense of the community", and this is done to denounce the Pareto principle.

This is another clear moment where it appears useful that I comment on these points in more detail. It generally looks like the medieval labour of counting angles on pinheads, yet, here a real devil is exposed.

VTFD explains that the Pareto principle is the same as assigning veto powers to minorities, and it invokes James Madison in defense. But HD invokes Madison against the Pareto principle !

(Let the People decide.)

Anyway, the AT uses the "weak" Pareto principle, and only from scratch. VTFD suggests to first apply Pareto from the Status Quo, to determine what nobody vetos, and then to apply the Borda FP on what remains. The weak Pareto principle is somewhat present in the Borda FP procedure but conditioned by various other assumptions. VTFD considers the Pareto principle extremely important, yet, also arrives at a decision, which may be the Status Quo. If the government wishes to put a railway through your back garden, you have a right to compensation.

Following VTFD the Pareto principle is for example essential for the European Union for the assignment of veto powers to the Member States and their citizens. That is, France could veto issues concerning France, but France should not be able to veto issues that concern Portugal. System wide changes can only be made unanimously, causing the need for communication, deliberation, bargaining and consensual agreement.

p29. Crucially, in its rejection of the Pareto principle, HD refers to Sen 1982. I have not looked this up but presume this is the same as Sen (1970) on the "Impossibility of a Paretian liberal". However, the VTFD abstract states: "Sen's theorem on the impossibility of a Paretian liberal suffers from the same problem, i.e. that the mathematics do not fit the verbal explanations around it."

This chapter does not convince. There are valid considerations, yet, they are essentially issues of elaboration. Crucially, the Pareto principle is misrepresented, as non-Madisonian, while it is essential to protect minority rights.

4. Independence of irrelevant alternatives

As stated, we have to drop at least one of the axioms in order to get consistency, and TC suggests that IIA is the axiom to drop.

VTFD also decides to rebaptise this axiom into the "axiom of pairwise decision making", APDM, because that is what it really is. On p34, HD makes the same observation and states: "It is sometimes called Pairwise Determination. Arrow, however, uses the same name for both principles." It is good to see this point confirmed.

p31-32: Contain a strange example where betters are irrational in not observing a dependence in a betting situation. This is not AT's world. In the AT, the items to decide upon are independent. At best this is an elaboration issue, but most likely it only requires a redefinition of what the choice situation actually is.

p32-33: Contain a correct example, where the paradox arises from collective choice (and not from the items). An "irrelevant" person on ballot dies and this causes a different winner. The objection by HD is to create individual preferences that depend on the ballot. Again, this is not AT's world. In the AT, individual preferences are given, and don't change when the ballot changes. TC follows AK and AT, first in TC90 only negatively as a logical rejection, and then in VTFD positively with a suggestion for a voting routine that is robust under changes of the budget of items. Indeed, Arrow's assumption of ballot-independent preferences is a useful point to start. However, HD's objection in itself is valid for democracy and a relevant issue for elaboration. One can imagine that voters are influenced by who actually is on ballot. The example provided by HD is a sound explanation why this can be so. It is not inconceivable that the problem can be reworked again into an AT case with redefined items, yet, considerations like these are elaboration.

p33: Makes the point that democracies sometimes choose policies that likely are infeasible. The US Fifth Amendment ought to protect people from theft but it still occurs. Again, this is not the AT world. One can suppose that people know better, might rephrase that amendment, or abstain from rephrasing it since it has a nice ring to it and everyone knows that it is only intentional. I consider it hardly a valid objection, but we can see it as a point for elaboration. Does everyone understand what is on ballot ? Are these true items or only rosy promisses ?

p34: Raises the issue of cardinality, with some interesting ideas of the influence of "vote trading", in particular in a two-stage setting. I refer to the discussion in VTFD on cheating ("strategic voting") for a proper view on cardinality and ordinality in voting theory. HD's comments here are too little developed to convince as an objection. Cheating on true preferences is not a valid objection to the APDM, AT and the AGV.

p35: Repeats the idea that the random lot may cause different outcomes. This however misconstrues the argument. If an individual is indifferent on a pair and determines the actual choice by a coin, in one setting, then this is the actual preference ordening to work with, at least in the theorem, and it is incorrect to suggest that in another setting another flip can be made. There are situations where it is crucial that each different setting has its own coin flip, yet, this is not so for the AT. This is not a valid objection and at best an issue for elaboration. The use of the lot was already sufficient to destroy the AGV, but with respect to the APDM it is only an

issue of elaboration – with the question whether it is really relevant that outcomes may be random.

HD concludes on APDM: "It is not, therefore, something reasonable for a democrat to assume." We can wholly agree with that, though the arguments in this chapter are not convincing.

TC has a different setup. One of the axioms has to go. The other axioms are acceptable as a definition of what a (basic) voting routine should try to achieve. Hence APDM has to go.

PM. TC90 p24 states: "Also, it would seem that the very social process of making a constitution makes people as a whole conscious of the fact that the rules must be feasible for the constitution to exist, (…) This will create some pressure to settle on one of the feasible rules (…)" (formulas omitted). TC92 turns this into a theorem, which is restated in VTFD, Theorem A.2, page 263: "For a reasonable society, the ARC is invalid." The realm of items to be decided upon is extended with the possible rules for the constitution. Since the axioms are inconsistent, they are not feasible for a constitution, and thus are rejected (by a reasonable society).

5. Nondictatorship

p37: Gives the example of selecting a president, each month a different one, by lot. HD provides examples where this method is used, both historical and a hypothetical shipwreck island, and suggests that this still is democracy. My problem is that both kind of examples cannot convince me. Hypothetical examples are weak by themselves. For reality: in South Holland, during Carnival, towns choose a Prince who receives the keys to the town. It is very difficult to take this serious as a way to run a city. Carnival has been around since ages and it is a useful temporary compensation for the daily travail. Yet, when we consider an economy where it is carnival all the time and work only a few days per year, then we tend to know that this is too good to be true. Perhaps in a smaller organization where all involved know what is going on, it is possible to randomly switch the cerimonial leader, but as soon as leadership comes with demands, it seems better to select on qualities than by lot.

p37: "The objection against a dictator is not that his existence is intrinsically incompatible with democracy." This tends to change the meaning of a dictator. In the AT world it is an individual who apparently is allowed to exert all his or her selfish preferences. And it is not assumed that these are altruistic preferences. To draw by lot is to invoke havoc on the rights of minorities. VTFD p263 states: "The Jorgenson quote points to his preference for a benevolent and non-selfish dictatorship, but, also since such dictatorships tend to turn sour, my impression is that he would be an associate of a real dictator. Most likely, he did not understand the situation when the quote was printed."

p37-38: "The objection against the use of lot is that quite often leaders are chosen who are unwilling or incompetent to serve. Where these circumstances do not apply – say, in some clubs or unions or businesses – there is no democratic objection to the procedure of regularly choosing dictators by the lot." (a) This tends to change the meaning of "dictator". (b) Join a union and check it out. (c) But what to do with the US of A, a bit larger than a club ?

p38: Note that HD combines the notion of lot with the notion of dictatorship. In itself the notion of lot would be sufficient. It is an added feature here that this is combined with the selection of a supreme decision maker. On this page we find a more convincing example, namely the choice of dessert after dinner. Rather than a majority outvoting a minority the whole year around, everyone by turn has a chance to determine what dessert will be that day. What is a bit worrying to me is that this is only the outer appearance or the format how this kind of equality is implemented. It need not be that lot really is the fundamental phenomenon that is of interest here. In VTFD, minorities have veto power with respect to their fundamental rights, reflected in

the Status Quo. The SQ here is that there is no dessert (in doubt, a session in the Senate will clarify this). Thus if the majority in favour of ice cream wishes to get that, they have to bargain with the minority in favour of cake. Taking turns, possibly by lot, is one possible solution, but not the only one. Technically, choosing dessert is being a "dictator" only for one item on one evening, while the AT dictator really decides everything. Technically, if the whole year is considered, there is no single issue "dictator". Alltogether the argument is not quite convincing and at best it causes an elaboration. The AGV was already killed by the lot, to settle aggregate indifference, but the combination with a dictator does not convince.

p39: Is a bit curious since it supports the axiom of non-dictatorship, somewhat formulated as a criticism on the axiom of non-dictatorship.

This chapter does not convince. It provides relevant historical context and suggests ways to implement equality and to control power, but it does not focus on the AT and the AGV.

6. The invalidity of Arrow's argument

In this chapter HD intends to show that the reasoning in the AT is invalid.

This is a bit of a complex issue. For if the AT were to be rejected for logical reasons, then apparently the axioms might not be inconsistent, and we would require another explanation for the voting paradoxes. As said, TC follows another route, and, having checked and double checked, found the logic in the AT to be sound.

p42: The argument of selfreference. If one of the items on ballot is one of the axioms, and is rejected, then that axiom cannot be used for that actual rejection. This is relevant for the proof since it relies on "arbitrary items", which thus could also be the axioms of the constitution. "The proof (of AT) is invalid since the argument by which the conclusion was established depends on those principlea applying to the social choice mechanism."

For the issue of selfreference, DH refers to his book on logic and likely I can usefully do the same for my book.

It is also useful to note that we have already seen such an element of selfreference in my discussion of Theorem A.2 in VTFD. Note that this requires an extension of the domain of items. As VTFD states: AK must choose here between inconsistency or incompleteness. Apparently HD prefers inconsistency. TC tends to go for incompleteness, since, when we only consider the commodity domain or the selection of candidates on ballot, then the consistent AT nicely explains the voting paradoxes (seeming contradictions). It also shows that we have to reject the APDM. Subsequently, when we extend the domain so that the AT* (note the star) becomes inconsistent, then we must make some amends, e.g. dynamic rounds, such that "selfreference" is not killing to democracy.

It is not clear what HD tries to achieve here. By the look of it he introduces selfreference on the road to a theorem that democracy is impossible because of such selfreference. The destruction of the proof of the AT only causes that those axioms might be consistent, while he still deems them irrelevant for real democracy. Nevertheless it is right on target that such selfreference necessarily must exist, since people decide on constitutions, and that AK, AT and AGV don't really deal with that.

p43: Gives a curious statement: "(…) there are still substantial difficulties in finding language which rules out all implicit or explicit self-referential arguments which might upset Arrow's proof. (…) we ought to be skeptical that Arrow's proof is valid until we see that language." Skepticism is OK. Yet, formulated in this way, the condition would be that we already need to

know what only future generations can discover, before we are allowed to judge now what we consider acceptable. Rather, we would have to decide upon the proof given the standards of today. The statement "I have seen no error" is a sound description when you have seen no error.

p43-46:Develop a curious argument on substitution of variables. Suppose the US President has a choice between *A* and *B* as Chief of Naval Intelligence. Also *A* is the most able person and *B* is a Soviet spy. (Note that the "is" here is "element of".) However, the President does not know that and prefers *B* over *A*. DH now does some substitutions, using the "is" as pure mathematical equality, claiming that this is allowed under the AT assumptions. The argument then runs that if "the President publicly stated that he preferred the (…) spy (…) to the most talented (…) officer" then he would be impeached or declared incompetent. According to DH this shows that Arrow's proof of the impossibility is invalid.

Well, what to think of this ? First of all, the confusion of identity and being an element does not occur in the AT world. So the substitution is not valid. Secondly, this actually is an age-old question. The Morning Star and Evening Star were considered different, got different names, but actually appear to be the planet of Venus. This kind of question is of a different order. I don't think that this invalidates the theorem, under the assumptions made.

DH has a point: "One might want to get around this difficulty by assuming that all voters know all the relevant facts of identity. However, this is a quite unrealistic assumption and is almost never fulfilled in practice." This comes across as a fallacy of composition. The AT is a mathematical theorem where identity is clear. The application to reality is a different issue. It is an elaboration to extend the model with the handling of information. However, this adaptation allows more scope for impossibilities. Or there is a possibility theorem, namely when the dictator makes the choices and everybody else gives those names for the things that they themselves prefer.

p46: Contains a point where KA states that he "represents preference by a notation not customarily employed in economics, though familiar in mathematics and particularly in symbolic logic". This must be read to mean that economists at that time used utility functions while KA uses orderings. DH states: "But without restrictions on the replacement property of equality that representation is flawed." But in the mathematical world of the AT there are no problems of identity and the objection is invalid.

Again, the chapter does not convince. Personally, I found it enlightening to look at the AT from the angle of the logical problem of identity, since it is a philosophical issue, and a practical one in real decision situations. The chapter provides scope for elaboration and the examples help to put the issue across. But they don't warrant the conclusion that the AT is flawed in the sense that DH apparently thinks. It is true, however, that the issue of selfreference causes that AK must choose between inconsistency or incompleteness (and likely the latter since he did not refer to selfreference).

7. Impossibility proofs

In the field of logic, there is quite some experience with impossibility proofs, and HD now applies that experience to the AT and AGV. To see what actually happens requires a longer quote:

p47: "What would constitute a good impossibility proof of the sort Arrow intended? By focussing on this question we can see how Arrow misunderstood what is needed to produce an impossibility proof for democratic social choice. There are four necessary steps in such a proof, which can be illustrated by comparing what Arrow attempted with what Alonzo Church

achieved in his remarkable proof that it is impossible for there to be a decision procedure for arithmetic."

Note that (i) the Church result is an empirical claim, rather a thesis than a proof, (ii) the AT is a mathematical proof, (iii) the AGV contains claims on reason and morality.

The longer quote that shows some confusion between mathematics, empirics, reason and morality, and this again shows that it is sensible to distinguish the AT from the AGV (and indeed reason from morality).

p47-48: Give us Step 1. "Propose a thesis." Indeed, the AGV here turns up as: "The claim that this definition explicates the ordinary sense of a democractic decision might be called "Arrow's thesis" (footnote to Alan Taylor)". The booklet would have won in clarity if this distinction had been made from the beginning. Indeed, while before it was said that the "theorem" was false, now it is explained that Chapters 2 through 5 show that the "thesis" is false.

p48-49: Give us Step 2. "Give evidence for the thesis." Clearly, with those chapters available, DH concludes that KA failed to provide the evidence. Also, the use of the random lot (to settle ties) is counterevidence. "But if Arrow's thesis is false then his theorem, even if valid, cannot have the significance he intended and many others accept." Yes, here we see a clear distinction between the AT and the AGV. And TC agrees with DH that the AGV (or thesis) cannot hold so that the theorem has not the significance commonly attached to it. It efficiently explains many voting paradoxes, while the whole axiomatic approach provides a sound system for reference. Yet this review and DH disagree on the value of the chapters 2 – 5. AK provided adequate evidence for the AT, and is short on AGV but came a long way, leaving for us fruitful scope for elaboration.

p49-50: Give us Step 3: "Create a valid exact proof." Note that the validity of Church's proof does not convince me, see ALOE. The argument in DH Chapter 6 to show the invalidity of the proof of AT also has limited power. The notion of selfreference, also used in TC90 and elaborated in TC92, indeed causes the choice between inconsistency or incompleteness. It is not accurate to say that it shows invalidity.

p50-51: Give us Step 4: "Draw the impossibility conclusion." This is a useful suggestion. It is useful to label, next to Church's empirical "thesis" and mathematical "theorem" also the (meta) "conclusion". Namely, the thesis is "every predicate with an algorithm is general recursive", the theorem is "a particular decision predicate for arithmetic is not general recursive", the conclusion is: "there is no algorithm for arithmetic". In the same way the AGV causes the "Arrow conclusion"AC: "there is no reasonable or morally desirable constitution for democracy". Clearly, there has been a lot of confusion of AT and AC, where people overlook the AGV. (Based upon this page, I put the AC already in the introduction.)

p51: DH rightly warns that Step 4 is only warranted when the preceding steps have been completed. It is a bit arbitrary to argue whether people like Samuelson, Wolff, Dahl, Jorgenson and Sen have been too sloppy in checking those steps or honestly believed that they had checked but still were sloppy in that execution.

p51: Contains a curious argument: "Finally, it should be noted that inconsistency in a political constitution is not like inconsistency in a formal system based on classical symbolic logic. (…) It fact, it might be argued that some contradictions should be built into a political constitution so that it gains the most widespread support. Contradictions, when they arise in practice, can then be taken care of piecemeal by judicial interpretation or subsequent legislation." I suppose this is to mean that first the constitution tells you that the government cannot put a train through your backyard without compensation, and then legislation is enacted that you get a dollar, and then

the agency says that it will not actually pay since the costs of payment are larger than that dollar.

8. Philosophical presuppositions of Arrow's proof

p53-54: Gives the amusing detail how Bentham, one of the originators of economic theory, criticized the US Declaration of Independence for errors, a "hodge-pogde of confusion and absurdity".

p56: Gives criticism by Hamilton and Madison of Athenian democracy – while the book earlier used Athens as an example of a democracy using the lot. But fortunately, footnote 39 provides the likely explanation that Hamilton and Madison read Roman texts, critical of Athens too.

p57: Gives a good summary of the actual situation around the AGV:

> "If its significance is reduced to the claim that, without tie-breaking techniques, ties lead to democratic indecision, or that the order of voting can alter the outcome of elections, or that a majority can be against every choice when at least three choices are available, nothing new has been shown, although the argument might now be tighter than before. Again, such a defense would be a kind of refutation: the argument would be shown to be sound but since the result would be unimportant all the claims that Arrow's theorem is somehow profound would be false."

p57-60: Repeat arguments of Chapter 2 – 5. Perhaps some such arguments cannot be tied to a specific axiom, but they fall under the same theme: the philosophy behind the AGV is antidemocratic. The statements are informative, in particular where they oppose the Arrowian system with the ideas of the US Founding Fathers. Yet the argument remains somewhat unconvincing. Note that a major step is to introduce a Status Quo and allow individuals veto rights with respect to a deterioration for themselves. Thus the argumentation now directed to the other axioms should be directed to the lack of a SQ in AK's original system.

The discussion does remind of the exposition by McCloskey (2006) on virtue ethics. She criticizes the Bentham reduction to a utility function and suggests us to think more in terms of the virtues. Likely these discussions can help towards an elaboration of voting theory for democracy. Nevertheless, in voting, choices have to be made, and the utility / preference approach has clear advantages.

9. The influence of the theorem

p61-62: There is a seeming problem with "selfish" as "totally disregarding others" or "including some regard for others" – where it still will be selfish to be yourself even when you have some consideration for others. This is an elaboration issue.

p63-69: These pages provide a wonderful list of research topics in social choice, formulated as "what could have been" had we not been distracted by the AT, but clearly on the agenda for the future.

p70-71: Opposes the misery of the AT and its consequences with what actually should be the general goal of social choice theory. It refers to DeLong(1992?), "The court of common reason", to be written, and apparently not finished yet in 2008. HD refers here to the combination of ideal and practice, e.g. as Turing simultaneously designed both his mathematical and his practical machine. Possibly HD intends to construct a both ideal and practical social

welfare function generating mechanism, like TC did with the BordaFP method on Pareto points with respect to the SQ ?

p71-73: “Previous attempts at refuting Arrow’s theorem have failed not because relevant or cogent objections were not raised (…) but rather because the question of what constitutes excellence in social choice theory was not faced. All disciplines contain crackpot, poor, mediocre and good-but-not-first-rate work. Yet it is the quality of what the practitioners of the discipline consider achievements deserving of the highest honor that decisively influence how the discipline will be judged.” Here, HD states various objections to the logical validity of the AT. But these need not convince, as we have seen.

Subsequently: “When the social choice theorist (turned applied mathematician) makes the claim that he is modeling democracy, then freedom ends and empirical work must begin.” But this approach is a bit awkward. HD already stated: “why should those who accept Arrow’s theorem look for contradictions in actual democracies when they know from a theoretical point of view that any democracy *must* contain them ? Why try to improve logically what is logically incapable of being improved ?” (p70). Thus, HD tends to acknowledge that social choice theory has turned into a rather sterile mathematical exercise (“a fascinating academic game”) with people claiming to study democracy but actually only manipulating axioms, but he does not acknowledge that this also means that the rules of the game have changed, with different “standards of excellence”. When an academic in SCT uses the word “democracy” it does not mean what Washington, Jefferson and Madison meant.

So I do not agree that “previous attempts at refuting Arrow’s theorem failed”, I would rather hold that people on the receiving end did not listen. Mathematicians can have autistic tendencies but are still held in high esteem, and perhaps others have an interest to do so. If a research fund is only granted if the research contains math, then the Dean does not support empirical work.

p74-75: HD suggests that a deep study of history and language will reveal more of our notion of “democracy”. But when I look at the various voting routines that history has generated, and that are basically collected in VTFD, then the scope might actually be a bit simple. Arrow’s axioms come a long way, they require only a few adaptations and then there is the Borda FP rule applied to Pareto points with respect to the SQ. Naturally, we need a rich democratic setting, and in that setting the choice between say three candidates may involve bargaining but perhaps also some kind of voting routine.

p75: HD imposes the standard of mathematical logic, empirical observation, linguistic analysis and experiment. He wants AT / AGV to satisfy them all, but for refutation only a single counterargument is sufficient. In itself this might be correct. However, AK in presenting the AT did some work to defend the choice of axioms *a*. And then used mathematics to prove the AT: $a \Rightarrow falsum$. Additional conditions of linguistic analysis and experiment need not be required for people to come convinced of the relevance of the theorem. HD: “It will perhaps be objected that these standards are too exacting.” Yes, I think that people will tend to think that it a bit too much to extend the scope of analysis with linguistic analysis and experiment. The TC approach would rather be used to destroy the (meta-) mathematical value, i.e. for those readers who are swayed by the math. In the end, though, both HD and TC agree that the AT does not model democracy.

p76-77: HD explains that scientific support for social choice might require as much resources as the weather forecasts and other enterprises. He compares us with the Mayans who knew the wheel but used it only as a toy. “What is being done in social choice theory to help achieve the great moral goals of democracy: dignity, equality, freedom, and the flourishing of every human being?” Right.

Note that TC90 was blocked from publication at the CPB while an adjunct director stated a bit off-handedly “this is above the combined intelligence of the whole Bureau” - which was logically inconsistent with my presence and anyway not an adequate response. My impression was that this adjunct director was so convinced about the truth of some “Arrow theorem” (AT, AGV or AC) that it caused him to even censure science. This is a small example of the kind of antidemocratic spirit that this theorem has fed.

Conclusion

The booklet Delong (1991) is lucid, extremely well researched, with a depth of historical and practical knowledge, impressively decent, accurate, witty, well versed, to the point, and targetted at what we all should worry about: our democracy, our living conditions, the world we live in, the rules that we abide by, and how we wish to shape to our freedom and compassion for others.

The booklet convinces. The thesis by the young Kenneth Arrow (1951, 1963) on what democracy would actually mean and that it would be impossible, is refuted. The refutation succeeds. The so-called ‘impossibility of democracy’ is rejected. Democracy is possible. What remains is a distinction between the inconsistency of certain axioms and the interpretation of what this means, but that distinction now is much better clarified.

Delong (1971) taught me logic, and taught me well. It is a pleasure to see that we independently embarked on Arrow’s treatment of the paradoxes of voting. The paradoxes of logic occlude our thought and the paradoxes of voting occlude our freedom. Thus, clearly they must be dealt with. In both ranges professor DeLong provides crucial guidance.

There are some minor points that I disagree with him but those pale in comparison to what I have learnt from him.

In a way, I would say that I learnt a lot from Arrow (1951, 1963) and Sen (1970) as well, since in a way they started the whole issue and elaborated on it (that is, if we allow us to disregard all the other students of democracy). However, I have never encountered a publication by these economists that expresses the limitations as does DeLong (1991). It still leaves us to wonder why this clear criticism has never caused these Nobel prize winners to retract their statements.

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