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Advertising and the degree of product  
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# Loss Leader or Low Margin Leader? Advertising And The Degree of Product Differentiation\*

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## Abstract

This paper attempts to isolate the conditions that give rise to loss leader pricing. I show that for sufficiently low distance between firms, the advertised good is priced below cost irrespective of whether firms advertise the same or different products. Instead, if products are sufficiently differentiated, loss leader pricing may result only if firms advertise the low reservation value product, otherwise the advertised good is a low margin leader. Thus, whether the advertised good is a loss leader or a low margin leader is primarily a function of the extent of differentiation between competing firms.

*Keywords:* Informative advertising, loss leader, low margin leader, product differentiation

*JEL Classification:* L13; L15; M37

## 1 Introduction

Advertising (both price and nonprice) provides information to consumers and thus helps direct them to the firms where they potentially get the greatest surplus. In this way, both consumers and firms potentially benefit from advertising (Bagwell and Ramey; 1994a, 1994b). For this reason, the relationship between advertising and prices and advertising and profits has long been a

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subject of great interest to economists and thus has spawned a huge literature. Important contributions include Benham (1972), Butters (1977) and Grossman and Shapiro (1984). A common finding in this literature is the negative association between advertising and prices (Benham, 1972; Grossman and Shapiro, 1984; Bagwell and Ramey, 1994a, 1994b; Milyo and Waldfogel, 1999; among others).

In most theoretical studies however, each firm is assumed to sell a single product. This severely limits both the scope of applications of the findings as well as the scope of interactions between advertising and prices (for instance, the pervasive issue of loss leader pricing can not be studied in single product firms). In contrast to the theoretical literature, both empirical evidence (Milyo and Waldfogel, 1999; Walters and Mackenzie, 1988) and casual observation support the hypothesis that firms advertise only a subset of their products.<sup>1</sup>

The analysis of advertising becomes more fruitful (in terms of the richness of the behaviours that can be analysed) when one considers multiproduct firms. In this setting, advertising can be used both as a source of information for consumers and also as a tool to extract more consumer surplus through a careful selection of the products to advertise and astute pricing of the advertised and the non-advertised products. The advertised goods could be sold at a loss (loss-leader good) or the firm could simply reduce the margins on the advertised goods but still sell at a profit (low margin leader good).<sup>2</sup> This paper attempts to understand the conditions that lead to the advertised good

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<sup>1</sup>There are several reasons why firms may want to advertise only a handful of their products (Ellison, 2005; pages 607-611). One obvious reason is that many firms, especially retail firms, carry hundreds if not thousands of products and therefore printing an encyclopaedia (read brochure) to communicate all the prices would be very costly and even more, the intended readers may be put off by the sheer size of the brochure. Another possible rationale for advertising only a subset of the products is the desire of firms to create some ambiguity in the minds of consumers which the firms can capitalize on. The failure by firms to avail all the information to consumers generates a need for consumer search. However if search is costly, either consumer search will be limited or consumers will solely rely on the little information provided by the firms. In either case, firms can potentially exploit the ambiguity resulting from the less than full search.

<sup>2</sup>A good is termed a loss (low margin) leader if it is deliberately priced below marginal cost (low but above marginal cost) in order to attract consumers to the advertising store and hence promote sales of higher margin products. According to Investopedia, "A classic example [of a loss leader] is that of razor blades. Companies like Gillette essentially give their razor units away for free, knowing that customers will have to buy their replacement blades, which is where the company makes all of its profit. Another example is Microsoft's Xbox video game system, which was sold at a loss of more than \$100 per unit to create more potential to profit from the sale of higher-margin video games." (investopedia.com)

being priced below cost (loss leader) or above cost (low margin leader). The central question is: Why are the leader products sometimes priced below marginal cost (sold at a loss) and sometimes priced above marginal cost? In other words, when or under what conditions should we expect the leader good to be sold at a loss? In particular, what is the role of the extent of differentiation between the competing firms in the loss leader - low margin leader story? I also study the implications of leader pricing for firm profits.

I consider two firms, each selling two products but only advertising a single product. The firms are located at the end points of a linear city of unit length. Consumers (who are uniformly distributed on the unit interval) are assumed to be completely uninformed about prices and firm locations (products) unless they are reached by advertising. In other words, I am assuming that search costs are prohibitive – so that consumers do not actively engage in information acquisition activities. Following Butters (1977), advertising messages are randomly distributed over consumers.

I show that loss-leader pricing crucially depends on how strong competition between the firms is. The strength of competition between the firms is a function of both advertising and the degree of product differentiation. For sufficiently low distance between the competing firms, the advertised good is priced below cost, irrespective of whether firms advertise the same or different products. If instead products are sufficiently differentiated, the advertised good is priced above marginal cost.

It turns out that the extent of marketing is not affected by the choice of the advertised product. The advertising intensity is the same whether firms advertise similar or different products. Equally, profits are not affected by the choice of the advertised product. Hence, in equilibrium firms will randomly select the good to advertise. The intuition for this result is as follows: Since all consumers have the same reservation value for each product and the reservation value is perfectly known to all (consumers and firms), the advertised price provides a sufficient statistic for the consumer's expected surplus. As a result, it does not matter for the consumer's visitation decision whether firms advertise the same or different products.

Closely related are Lal and Matutes (1994) who study price and advertising by multiproduct firms and Ellison (2005) who studies a vertically differentiated

goods model in which firms only advertise the low quality good.<sup>3</sup> However, neither Lal and Matutes nor Ellison explicitly model the advertising decision. Generally, when firms advertise, the post advertising equilibrium is characterized by a market partitioned into segments of variably informed consumers and this has additional implications for firm pricing. Moreover, both Lal and Matutes and Ellison only consider the case where the market is fully covered. However, as Soberman (2004) shows, for some constellations of the differentiation parameter, some informed consumers find it profitable not to purchase.

Also related is Bagwell and Ramey (1994a) who study dissipative advertising. They show that "ostensibly uninformative" advertising may bring about coordination economies. These coordination economies manifest themselves in the form of greater demand to the firm and lower prices to consumers. Thus, they find a negative association between advertising and price. We differ with them in several ways: First, in our model, advertising is (directly) informative whereas in theirs, it is not. Secondly, whereas in our model firms advertise only a subset of the products they sell, in theirs, they neither advertise the prices nor the products they sell. Rather, firms advertise for instance, their size. Also, in Bagwell and Ramey, product differentiation is unimportant.

The paper is organized as follows. Section 2 sets out the model. Sections 3 and 4 examine price and advertising when firms advertise the same and respectively different products and differentiation is low. Section 5 studies price and advertising when differentiation is high and section 6 concludes.

## **2 Model and Preliminaries**

### **2.1 Model**

The model extends the Grossman and Shapiro (1984) model – as simplified by Tirole (1988) in two dimensions, viz, firms sell multiple products and I allow for the market not to be covered in equilibrium. Consider a linear city of unit length served by two firms, firm 0 and firm 1, where each firm's identity corresponds to its location. Each firm sells two products, product 1 and product 2 – which are either complements or independent but not substitutes.

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<sup>3</sup>This paper draws freely from Simbanegavi (2005).

However, across firms, the products are substitutes.<sup>4</sup> Firms advertise only one of their products. Firms randomly sent out advertisements to consumers. Let  $\phi_i$  denote the advertising (ad) reach of firm  $i$ ;  $i = 0, 1$ . The cost of reaching fraction  $\phi_i$  of consumers is  $a\phi_i^2/2$ ,  $a > t/2$ .<sup>5</sup> Each good is produced at a constant marginal cost,  $c$ , and firms simultaneously and non-cooperatively choose prices and advertising intensities to maximize profits. There is no entry or exit and there are no fixed costs. Product differentiation is exogenous.

Consumers (who are rational) are uniformly distributed on  $[0, 1]$  according to taste. That is, each consumer is identified by a point on the unit interval that corresponds to her most preferred brand. I assume that consumers are completely unaware of the existence of firms or products unless they are reached by advertising. I also assume that search costs are prohibitive – so that consumers do not actively engage in information acquisition activities. Each informed consumer buys at most one unit of each product and uninformed consumers stay out of the market. A unit of good 1(2) generates gross surplus of  $v_1(v_2)$  and consumers incur a shopping cost of  $t$  per unit distance. If an informed consumer chooses not to purchase, they get zero surplus. Transportation costs are applied to both goods (bundling) – the decision to visit a particular store is based on the expected surplus from buying at that store.

## 2.2 Timing

The timing of the actions is as follows: In stage 1, firms simultaneously decide on the advertised price and the advertising level, and send out the ads. In stage 2, consumers receive ads and make purchase plans based on the advertised price and the expected price of the non-advertised good. At the same time, firms simultaneously decide on the price of the non-advertised good. Lastly, in stage 3, purchases are made and payoffs are realized.

## 2.3 Preliminaries

Given the advertising intensities  $\phi_0$  and  $\phi_1$  : fraction  $\phi_0\phi_1$  of consumers receive advertising messages from both firms (fully informed); fraction  $\phi_i(1 - \phi_j)$ ;  $i, j =$

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<sup>4</sup>Think of, for example, two retail stores which specialize one in Philips products (TV and video camera) and the other in Sony products (TV and video camera).

<sup>5</sup>The parameter  $t$  (which is defined below) measures the transportation cost incurred by consumers when visiting a store.

$0, 1; j \neq i$  receive ads from firm  $i$  but not firm  $j$  (partially informed) and fraction  $(1 - \phi_0)(1 - \phi_1)$  receive no ads from either firm (uninformed). I assume that  $\phi_0\phi_1$  is large enough so that firms find it worthwhile to compete for the fully informed consumers.<sup>6</sup>

Suppose firm  $i$  advertises good  $k$  and firm  $j$  advertises good  $\ell$ , so that  $p_{ik}$  and  $p_{j\ell}$  are the advertised prices for firms  $i$  and  $j$  respectively. Let  $p_{i\ell}^E$  and  $p_{jk}^E$  denote the expected prices of the non-advertised products at firms  $i$  and  $j$ , respectively,  $k, \ell = 1, 2$ . If the firms advertise the same good, say good  $k$ , then there are three possible configurations of the expected prices of good  $\ell$ , the non-advertised good: Either  $p_{i\ell}^E = p_{j\ell}^E$ ,  $p_{i\ell}^E < p_{j\ell}^E$  or  $p_{i\ell}^E > p_{j\ell}^E$ . Since the non-advertised products are (physically) similar and the consumers are rational, the most reasonable beliefs that consumers can have are that  $p_{i\ell}^E = p_{j\ell}^E$ . Hence I assume that  $p_{i\ell}^E = p_{j\ell}^E$ .

The problem can be solved backwards, starting with stage 2.<sup>7</sup> In the second stage, having committed itself to the advertised price, firm  $i$ 's problem is to choose the non-advertised price,  $p_{i\ell}$ , to maximize  $(p_{i\ell} - c) D_i(p_{ik}, p_{j\ell}, p_{i\ell}^E, p_{jk}^E, \phi_i, \phi_j)$ . Observe that the demand that firm  $i$  faces is independent of  $p_{i\ell}$ , the non-advertised price.<sup>8</sup> Since demand is determined by the advertised prices and the expected prices  $p_{i\ell}^E$  and  $p_{jk}^E$ , it follows that profits are maximized by ensuring that all visiting consumers buy the non-advertised good, that is, by setting  $p_{i\ell} = p_{i\ell}^E; i = 0, 1$ .

As for consumers, the decision to visit a store is based on expected surplus. The uncertainty here derives from the fact that one of the goods they anticipate purchasing is not advertised – and hence the price is not known with certainty. A fully informed consumer located at  $x \in (0, 1)$  gets expected surplus  $v_k + v_\ell - p_{ik} - p_{i\ell}^E - tx$  buying from firm  $i$  and surplus  $v_k + v_\ell - p_{j\ell} - p_{jk}^E - t(1 - x)$  buying from firm  $j$ . Thus, firm  $i$  faces demand  $D_i^{full} = \phi_i\phi_j(p_{j\ell} + p_{jk}^E - p_{ik} - p_{i\ell}^E + t) / 2t; j \neq i$  from the fully informed consumers. Partially informed consumers purchase whenever it is individually rational to do so. Let  $y_i$  denote the location of a consumer who receives only firm  $i$ 's ad(s). Buying yields surplus  $v_k + v_\ell - p_{ik} - p_{i\ell}^E - ty_i$ . Given the consumers' outside option, firm  $i$  thus faces demand  $D_i^{partial} = \phi_i(1 - \phi_j)(v_k + v_\ell - p_{ik} - p_{i\ell}^E) / t$

<sup>6</sup>If this assumption is not satisfied, a symmetric equilibrium may not exist as at least one firm may find it profitable to charge a high price and only serve its captive consumers.

<sup>7</sup>Stage 3 is superfluous. It is only meant to close the model.

<sup>8</sup>Of course the actual quantity demanded of the non-advertised good depends on  $p_{i\ell}$ .

from the partially informed consumers.

So what is  $p_{i\ell}^E$ ? I use a standard argument to deduce this. Observe that in equilibrium consumers' expectations for the non-advertised good must coincide with the actual price charged by firm  $i$  for this good. As a matter of fact, in this model, if firms did not advertise at all, the market would unravel for the simple reason that consumers would foresee themselves being held to the reservation prices once search costs are sunk (Lal and Matutes, 1994; Bagwell, 2005; p.75). Therefore by extension, rational consumers must anticipate being held to the reservation value of the non-advertised good once they are in the store. It follows therefore that  $p_{i\ell}^E = v_\ell, i = 0, 1; \ell = 1, 2$ . Hence,  $p_{i\ell} = p_{i\ell}^E = v_\ell$ .

Firm  $i$  thus faces the demand:

$$D_i = \phi_i [(1 - \phi_j) (v_k - p_{ik}) / t + \phi_j (p_{j\ell} - p_{ik} + v_k - v_\ell + t) / 2t] ; i \neq j. \quad (1)$$

If firms advertise the same good however, the demand reduces to

$$D_i = \phi_i [(1 - \phi_j) (v_k - p_{ik}) / t + \phi_j (p_{j\ell} - p_{ik} + t) / 2t]. \quad (1')$$

For what follows, I make the following assumption:

**Assumption A1.**  $c + \sqrt{2at} < \min \{v_1, v_2\}$ .

Assumption A1 ensures that in equilibrium consumers visit the stores in the single product case. If the equilibrium price is greater than or equal to  $\min \{v_1, v_2\}$ , visiting consumers get negative surplus since they incur positive transportation costs and hence no consumer would visit a store.<sup>9</sup>

### 3 Low differentiation

In this section I assume low differentiation. That is, I assume that the market is fully covered, i.e., *the consumer who travels the furthest distance (unit interval) finds it profitable to purchase*). The market is covered when  $v_k \geq p_{ik} + t$ . This says that the gross surplus from purchasing a unit of good  $k$  at firm  $i$  exceeds the total cost of purchasing this good at firm  $i$  for the consumer who travels the

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<sup>9</sup>This assumption is implicit in the analyses of, for example, Bagwell (2005; Section. 5), Tirole (1988; Chap. 7) and Soberman (2004).



furthest distance.<sup>10</sup> It follows therefore that all consumers who receive at least one ad from firm  $i$  will make a purchase. That is,  $\tilde{y}_i \equiv \frac{v_k - p_{ik}}{t} = 1$ . The demand function (1) reduces to  $D_i = \phi_i [(1 - \phi_j) + \phi_j (v_k - v_\ell + p_{j\ell} - p_{ik} + t) / 2t]$ .

### 3.1 Price and Advertising

Assume that firm  $i$  advertises good  $k$  and firm  $j$  advertises good  $\ell$ ,  $i, j = 0, 1; i \neq j$  and  $k, \ell = 1, 2$ . Then, firm  $i$ 's profit is:

$$\pi_i = \max_{\{p_{ik}, \phi_i\}} \left\{ (p_{ik} + v_\ell - 2c) \left( \phi_i (1 - \phi_j) + \frac{\phi_i \phi_j (v_k - v_\ell + p_{j\ell} - p_{ik} + t)}{2t} \right) - \frac{a\phi_i^2}{2} \right\}. \quad (2)$$

Observe that our formulation allows for the firms to advertise the same product. In this case,  $v_k = v_\ell$  and firm  $i$ 's objective function becomes

$$\pi_i = \max_{\{p_{ik}, \phi_i\}} \left\{ (p_{ik} + v_\ell - 2c) \left( \phi_i (1 - \phi_j) + \frac{\phi_i \phi_j (p_{jk} - p_{ik} + t)}{2t} \right) - \frac{a\phi_i^2}{2} \right\}.$$

Differentiating (2) with respect to  $p_{ik}$  and  $\phi_i$  gives

$$\frac{\partial \pi_i}{\partial p_{ik}} = 2t + p_{j\ell} \phi_j + (2c - 2p_{ik} - t + v_k - 2v_\ell) \phi_j = 0, \quad (3)$$

$$\frac{\partial \pi_i}{\partial \phi_i} = 0.^{11}$$

Equation (3) gives the relation between prices and advertising. One of the features that distinguishes the present model from the models of Lal and Matutes (1994) and Ellison (2005) is that in the present model, consumers are ex-post heterogeneous – some receive ads and some don't. In other words, although all consumers are ex-ante uninformed, ex-post they are differentially informed – some are fully informed of all advertised prices, others are only partially informed and yet others are not informed at all. In contrast, in Lal and Matutes and respectively Ellison's works, all consumers are ex-post fully informed as regards to the advertised prices. Below I characterize the relationship between the advertised prices and the advertising intensity.

**Proposition 1** *Irrespective of whether firms advertise the same or different*

<sup>10</sup>In equilibrium, the non-advertised good yields zero expected surplus and hence doesn't affect the visitation decision.

<sup>11</sup> $\frac{\partial \pi_i}{\partial \phi_i} = ((p_{ik} - p_{j\ell} - 2c + t - v_k + 2v_\ell) p_{ik} + v_\ell^2 + 2p_{j\ell}c - 2ct + 2cv_k - p_{j\ell}v_\ell - 2cv_\ell + tv_\ell - v_kv_\ell) \phi_j - 2tv_\ell + 4ct - 2p_{ik}t + 2at\phi_i = 0$

products, there is a negative association between advertising and prices. That is,  $\frac{\partial p_{ik}}{\partial \phi} < 0, i = 0, 1; k = 1, 2$ .

**Proof.** From the first order conditions for firms  $i$  and  $j$ , we get that  $\frac{\partial \pi_i}{\partial p_{ik}} = 2t + p_{j\ell}\phi_j + (2c - 2p_{ik} - t + v_k - 2v_\ell)\phi_j = 0$  and  $\frac{\partial \pi_j}{\partial p_{j\ell}} = 2t + p_{ik}\phi_i + (2c - 2p_{j\ell} - t + v_\ell - 2v_k)\phi_i = 0$ . Solving firm  $j$ 's first order condition for  $p_{j\ell}$  gives  $p_{j\ell} = \frac{1}{2\phi_i}(2t - \phi_i(-2c + t + 2v_k - v_\ell) + \phi_i p_{ik})$ . Substituting this value into the first order condition for firm  $i$ , and solving for  $p_{ik}$  gives  $p_{ik} = -\frac{1}{\phi}(t\phi - 2c\phi - 2t + \phi v_\ell)$  (assuming  $\phi_i = \phi_j = \phi$ ). Finally, differentiating  $p_{ik}$  with respect to  $\phi$  yields:  $\frac{\partial p_{ik}}{\partial \phi} = -2\frac{t}{\phi^2} < 0$ . ■

This result is similar to the finding of Grossman and Shapiro (1984). A firm that advertises more expects a larger demand and hence can afford to lower prices (Bagwell and Ramey, 1994a). Intuitively, a firm that advertises more will have more consumers in the competitor's backyard aware of its product. In order to induce these consumers to ditch the nearby firm in favour of the distant firm, the distant firm must compensate these consumers for the additional search costs they incur. Ideally the firm would want to give targeted discounts to these consumers (consumers in the competitor's backyard). However, because advertising cannot be targeted (i.e., price discrimination is not feasible), the firm can only compensate these consumers by charging lower prices and thus assuring them greater expected surplus.

Evaluating the first order conditions gives:

$$p_k = 2c + \sqrt{2at} - v_\ell \quad (4)$$

$$\phi_i = \phi_j = 2 / \left(1 + \sqrt{2a/t}\right) \quad (5)$$

$$\pi = 2a / \left(1 + \sqrt{2a/t}\right)^2. \quad (6)$$

Two results follow:

**Proposition 2** *When firms advertise the same good, the advertised good is priced below cost irrespective of whether firms advertise the low or the high reservation price good. The non-advertised good is priced at its reservation price.*

**Proof.** From (4),  $p_k = 2c + \sqrt{2at} - v_\ell = c + \sqrt{2at} - (v_\ell - c)$ . Clearly,  $p_k < c$  if and only if  $\sqrt{2at} - (v_\ell - c) < 0$  and  $\sqrt{2at} - (v_\ell - c) < 0$  if and only if

$c + \sqrt{2at} < v_\ell$ . But this is nothing other than Assumption 1. Hence, we conclude that indeed  $p_k < c$ . The second part of the proposition is already shown in section 2.3. ■

Since consumers do not actively search in this model, market shares are determined solely by the advertised prices. Holding the firm's advertising reach constant, the lower the advertised price the higher the consumers' expected surplus and hence the greater the likelihood that each ad received results in a sale.

When firms advertise the low reservation value good, competition for market share is tougher as the firm that succeeds in attracting more consumers will sell more units at the higher non-advertised price. The larger the difference between the reservation values the greater the incentive to undercut. Given low differentiation, this leads to a much lower equilibrium advertised price. When firms advertise the high reservation price good instead, there are two opposing effects. On the one hand, low differentiation induces firms to compete more aggressively for market share (products are similar). On the other hand, firms realize that visiting consumers will pay a lower (reservation) price on the non-advertised good and this restrains the aggression. However, the low differentiation effect dominates and firms advertise prices below marginal cost in either case.

Examples of markets in which differentiation is generally low are the grocery retail market and the liquor retail market. Supermarkets, for instance, sell products that are almost (if not exactly) physically similar. Competition therefore is mainly on prices. Because they carry similar products, consumers do not have a strong inclination to buy from one particular store as opposed to another and, as a result, price competition is intense. Moreover, when firms advertise similar products, there is even more downward pressure on the advertised prices since advertising similar products facilitates comparison shopping. The presence of non-advertised products further exacerbates price competition since firms anticipate earning higher profits from the sale of the higher margin non-advertised products. This intense price competition results in firms offering loss leaders.

We next consider the equilibrium price and advertising configurations when firms advertise different products. The question here is whether advertising different products softens price competition. One is tempted to answer this

question in the affirmative as advertising different products ideally reduces comparison shopping. The following proposition however shows that this is not the case. In particular,

**Proposition 3** *When firms advertise different products the advertised prices are asymmetric – the firm advertising a higher reservation value good quotes a higher advertised price. However, the advertised prices are below marginal cost and the extent of marketing and profits are exactly the same as when firms advertise the same product.*

The intuition behind Proposition 3 is straightforward. The incentives to attract consumers to the store are driven by the expected profit from the sale of the non-advertised good. When firm  $i$  advertises the low reservation value good (say good 1), the incentive to attract consumers is greatest as each visiting consumer will pay  $v_2$ . Consequently, the firm advertises a much lower price. In contrast firm  $j$ , which advertises the high reservation value good (good 2), advertises a higher price. This is meant to compensate for the fact that visiting consumers will only pay  $v_1$ , which is lower. Thus the firm advertising the high reservation value good will have a higher advertised price. Consumers understand this and thus will not be fooled by firm  $i$ 's low advertised price<sup>12</sup>.

Proposition 3 is driven mainly by two factors: First, firms know consumers' reservation values and second, consumers' expectations are correct in equilibrium. Although each firm only advertises one product among its offerings, rational consumers already know the non-advertised prices. Hence, the outcome is as if both goods were advertised.

The above results allow us to rationalize the "surprising" finding of Walters and MacKenzie (1988) that, in retail markets, loss-leader pricing fails to stimulate store traffic and hence is unprofitable. Walters and MacKenzie interpret their finding as "pointing to the fact that locational convenience and overall price perceptions are more important determinants of patronage than weekly specials" (p. 60). I turn their explanation on its head. Because differentiation is typically low in the retail sector, weekly specials are crucial determinants of

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<sup>12</sup>Because firms cannot commit to not fleece consumers once search costs are sunk, they have to offer price discounts as a way to commit to leave consumers sufficient surplus to make the shopping trip worthwhile.

visitation<sup>13</sup>. As a result, a firm that offers such specials would substantially increase its market share if rivals would not follow suit. Realizing this, firms always try to match price cutting by rivals and this enables them to maintain their market shares.

This is a typical prisoner's dilemma. A firm that succeeds in undercutting its rival can greatly increase its profits since it then faces a large demand and sells the non-advertised good at its reservation price. On the other hand, if both firms undercut (symmetrically), then each firm maintains its market share. In this sense, undercutting is a (weakly) dominant strategy for each individual firm. However, this strategy does not maximize joint profits and firms settle for an equilibrium with low advertised prices but with the same level of demand.<sup>14</sup> This makes loss-leader pricing appear as if it were less important.<sup>15</sup> I find the present argument more convincing, for if price specials were unimportant, why are retailers "placing greater emphasis on hotter price specials"? (Lal and Matutes, 1994; p. 345).<sup>16</sup>

#### 4 High differentiation (Regime H)

In the previous section, I studied the case when differentiation is low; that is,  $v_k - p_{ik} - t > 0$ , where  $p_{ik}$  is the advertised price. For such parameter constellations, all consumers who receive at least one advertising message from firm  $i$  will make a purchase as they are guaranteed positive surplus from doing so. However, as Soberman (2004) notes, for some parameter constellations, some informed consumers may optimally choose not to purchase. This happens

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<sup>13</sup>When I asked him why they advertised so frequently in the newspaper (weekly basis), the manager of a large non-food retail chain in Cape Town candidly responded "If we did not advertise, instead of 8000 customers over the weekend, we would only get about 4000. So you can see why".

<sup>14</sup>That loss leader pricing does not increase demand in our model is a consequence of the unit demands assumption. With downward sloping demands, total demand can increase in equilibrium but market shares will not.

<sup>15</sup>Since competitors always match price cutting by rivals, the full effect of loss leader pricing on store traffic and profits is never realized in equilibrium. This gives a biased reading of the importance of loss leader pricing. This suggests a different empirical method to test for the effect of loss leader pricing – counterfactual analysis. What would be the effect on profits of firm  $i$  were competitors to not reciprocate when firm  $i$  lowers its price?

<sup>16</sup>I asked a manager of a major food retail chain why they advertise and the answer I got pretty much confirms the intuition above. She responded that "it's the competition thing. If we don't frequently advertise to inform our consumers of the specials we have they will go to those stores that advertise".

whenever the parameter values are such that  $t/2 < v_k - p_{ik} < t$ . This is the condition for high differentiation (Soberman, 2004) or the condition for incomplete (market) coverage (Hamilton, 2004).

In this section, I consider the case when differentiation is high. Differentiation is said to be high if, given the prices, at least one partially informed consumer does not make a purchase. In the double inequality above, the condition:  $v_k - p_{ik} < t$  tells us that the consumer located at firm  $j$  but who only receives advertising messages from firm  $i$  would get negative surplus were they to travel to firm  $i$  since the consumer surplus is less than the transportation costs. Hence, this consumer will not purchase, even though they are informed. The other condition:  $t/2 < v_k - p_{ik}$  tells us that a consumer located in between the two firms who travels distance no more than  $t/2$  to go to firm  $i$  will purchase whenever they receive at least one advertising message. These conditions give the degree of product differentiation compatible with ‘high’ differentiation. In particular, under high differentiation, the degree of product differentiation is such that  $t \in \left( \frac{v_1+v_2}{2} - c, \frac{2(v_1+v_2-2c)}{3} \right)$  and the demand that firm  $i$  faces is given by equation (1) exactly.<sup>17</sup>

The objective, as before, is to try to pin down the conditions that cause the leader good (advertised good) to be priced below cost or to be a low margin leader. To proceed, suppose firm  $i$  advertises good  $k$  and firm  $j$  advertises good  $\ell$ , so that the advertised prices are respectively  $p_{ik}$  and  $p_{j\ell}$ ;  $k, \ell = 1, 2; i \neq j$ . Firm  $i$  and firm  $j$ 's objective functions are then:

$$\pi_i^H = \max_{p_{ik}, \phi_i} (p_{ik} + v_\ell - 2c) \phi_i \left( (1 - \phi_j) \frac{v_k - p_{ik}}{t} + \phi_j \frac{p_{j\ell} + v_k - p_{ik} - v_\ell + t}{2t} \right) - a \frac{\phi_i^2}{2} \quad (7)$$

$$\pi_j^H = \max_{p_{j\ell}, \phi_j} (p_{j\ell} + v_k - 2c) \phi_j \left( (1 - \phi_i) \frac{v_\ell - p_{j\ell}}{t} + \phi_i \frac{p_{ik} + v_\ell - p_{j\ell} - v_k + t}{2t} \right) - a \frac{\phi_j^2}{2}. \quad (8)$$

Observe that our formulation allows for the case where firms advertise the

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<sup>17</sup>In regime  $H$ , we have that  $t/2 < v_k - p < t$ . The first inequality, evaluated at the full information (highest) price, gives the upper bound to regime  $H$  while the second inequality gives the lower bound. Evaluating this condition at  $p_{1F}^H = t + 2c - v_2$  gives  $t \in \left( \frac{v_1+v_2}{2} - c, \frac{2(v_1+v_2-2c)}{3} \right)$ .

same good, good  $k$ . In this case, firm  $i$ 's problem reduces to

$$\pi_i^H = \max_{p_{ik}, \phi_i} (p_{ik} + v_\ell - 2c) \phi_i \left( (1 - \phi_j) \frac{v_k - p_{ik}}{t} + \phi_j \frac{p_{jk} - p_{ik} + t}{2t} \right) - a \frac{\phi_i^2}{2}. \quad (9)$$

Consider first the case when the firms advertise the same good. Differentiating with respect to  $p_{ik}$  and then solving for  $p_{ik}$  (at the symmetric equilibrium:  $p_{ik} = p_{jk} = p_k$  and  $\phi_i = \phi_j = \phi$ ) one gets:<sup>18</sup>

$$p_k = [4c + 2(v_k - v_\ell) + (t - 2c - (2v_k - v_\ell))\phi] / (4 - 3\phi). \quad (10)$$

Equation (10) gives the equilibrium price as a function of the advertising intensity<sup>19</sup>. A careful analysis of (10) shows that loss-leader pricing is possible under high differentiation. More precisely, let  $k = \ell$  so that firms advertise the same product. Then,

**Proposition 4** *Let the firms advertise the same product. When firms advertise the low reservation value good, equilibrium may entail loss-leader pricing even when firms' products are sufficiently differentiated (high differentiation). However, when firms advertise the high reservation value good, the advertised price exceeds marginal cost (low margin leader).*

**Proof.** Let firms advertise good  $k$  so that  $p_k$  is the advertised price. Notice that (from (10))  $\partial p_k / \partial t > 0$ . Since  $t > \underline{t} = \frac{v_1 + v_2}{2} - c$ , it follows that  $p_k = \frac{4c + t\phi - 2c\phi + 2(v_i - v_j) - \phi(2v_i - v_j)}{4 - 3\phi} > p_k|_{t=\underline{t}} \equiv \underline{p}_k = c + \frac{(v_k - v_\ell)}{2}$ ;  $k, \ell = 1, 2; k \neq \ell$ . If  $v_k < v_\ell$ , advertising good  $k$  (the low reservation price good) gives  $\underline{p}_k < c$ . Thus, for  $t \rightarrow \left(\frac{v_1 + v_2}{2} - c\right)^+$  and for  $|v_k - v_\ell|$  large,  $p_k < c$ . However, if  $v_k > v_\ell$ , then advertising good  $k$  (the high reservation price good) gives  $\underline{p}_k > c$  and hence  $p_k > c$ . ■

The intuition for this result is as follows: when differentiation is high, price advertising is primarily informative. Products are less similar and therefore price differences have to be large to induce consumers to switch to the distant

<sup>18</sup>Due to complexity of the first order conditions, we cannot solve explicitly for  $p_{ik}$ ,  $p_{j\ell}$ ,  $\phi_i$  and  $\phi_j$ .

<sup>19</sup>In contrast to Proposition 2, the relation between price and advertising is positive when differentiation is high (Soberman, 2004). When products are sufficiently differentiated, demand is more elastic in the presence of informational product differentiation. Higher advertising, by increasing the share of fully informed consumers, reduces overall demand elasticity and thus allows firms to charge higher prices.

supplier. However, for  $t \rightarrow \left(\frac{v_1+v_2}{2} - c\right)^+$ , most partially informed consumers make a purchase and thus competition for market share can be intense. A higher non-advertised price adds to the incentives to compete vigorously. Thus, when firms advertise the low reservation price good, undercutting may result in prices below cost. However, when products are sufficiently differentiated ( $t$  is high), the incentive to undercut is reduced. Hence, for large  $t$ , firms advertise prices above marginal cost when they advertise the low valuation good. When firms advertise the high reservation price good however, it is never optimal to advertise prices below marginal cost. The fact that differentiation is high (products are less similar) and the fact that the non-advertised (reservation) price is lower when firms advertise the high reservation price good both induce firms to advertise higher prices.

I next consider the case when firms advertise different products. As argued earlier, advertising different products potentially has additional implications as it reduces comparison shopping. Differentiating (7) and respectively (8) with respect to  $p_{ik}$  and respectively  $p_{j\ell}$  and solving simultaneously for  $p_k$  and  $p_\ell$ ;  $k \neq \ell$  yields,<sup>20</sup>

$$p_k = [4c + 2(v_k - v_\ell) + (t - 2c - (2v_k - v_\ell))\phi] / (4 - 3\phi) \quad (11)$$

$$p_\ell = [4c + 2(v_\ell - v_k) + (t - 2c - (2v_\ell - v_k))\phi] / (4 - 3\phi) \quad (12)$$

It turns out that the equilibrium advertised prices are exactly the same as when firms advertise the same product (observe that the expressions for  $p_k$  in (10) and (11) are identical). It is therefore immediate from the proof of Proposition 4 that loss leader pricing is possible only for the firm advertising the low reservation value good. It follows therefore that if firm  $i$  advertises good  $k$  while firm  $j$  advertises good  $\ell$ , and good  $k$  happens to be the low reservation value good, then for  $t \rightarrow \left(\frac{v_k+v_\ell}{2} - c\right)^+$  and for  $|v_k - v_\ell|$  large,  $p_k < c$ . That is, loss leader pricing is possible, depending on the extent of product differentiation. However, for firm  $j$ , which advertises good  $\ell$ ,  $p_{j\ell}|_{t=\underline{t}} = c - \frac{v_k - v_\ell}{2}$ . Since  $v_k < v_\ell$ , it follows that the lowest price that can be charged by firm  $j$  exceeds the marginal cost. Thus, firm  $j$ 's advertised good is a low margin leader – independent of the degree of product differentiation. The intuition is the same as when firms advertise the same product: the firm advertising the high

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<sup>20</sup>I suppress subscripts  $i$  and  $j$ .



reservation value good realizes that visiting consumers will only pay a lower (reservation) price for the non-advertised good – hence the diminished incentive to lower the advertised price. However, because advertising is necessary for visitation, the asking price has to be below the reservation price. This explains low margin leader pricing.

In the present model advertising serves as a commitment device: it guarantees consumers sufficient surplus to make the visitation worthwhile (Lal and Matutes, 1994). Firms however, tend to overcommit. Overcommitment is more pronounced when firms’ products are similar as it implies loss leader pricing. When products are sufficiently different, the advertised good is a *low margin leader* but not a loss leader. The distinction being that for a low margin leader the price is lower than would be the case if advertising was not necessary but still price exceeds marginal cost. Thus, from Propositions 1, 3 and 4 we can pin down the conditions that give rise to loss leader pricing.

**Corollary 1** *The following conditions are sufficient for loss-leader pricing;*  
*(i) Low differentiation and (ii) Firms advertise only a subset of their products.*

Our finding that when differentiation is low it is immaterial for loss leader pricing whether firms advertise the same or different products, is somewhat surprising. Intuition suggests that advertising different products ought to soften price competition as it makes across firm price comparisons more blurred. It appears the devil is in the model assumptions. The explanation, as argued earlier, is that in the present model the advertised price is a sufficient statistic for the consumer’s expected surplus. This is so since the reservation value of the non-advertised good is the same for all consumers and is perfectly known to firms. As a result, it does not matter for the consumer whether firms advertise the same or different products. Hence firms are indifferent as to which good to advertise since the choice doesn’t affect profits. I conjecture that in a more general setting where reservation values differ among consumers or where firms do not know consumers’ reservation values for certain, this might no longer be the case.

The present model provides theoretical support to the findings of Milyo and Waldfogel (1999). In a study of the effect of price advertising on prices in the liquor retail market, Milyo and Waldfogel find that advertising stores substan-

tially cut only the prices of the products they advertise and moreover, that the effect of price advertising on prices is different depending on whether a rival in the vicinity is advertising the same good. Advertised prices are much lower if rivals close by are advertising the same good. Propositions 2 and 3 above are in agreement with these findings. Although tough competition for market share leads to advertised prices below cost, the tough price competition does not get (directly) transmitted to the non-advertised products. Instead, the competition is transmitted from the non-advertised products to the advertised products.

Our model also informs on the conditions giving rise to price dispersion. As a corollary to Propositions 2-4 we have;

**Corollary 2** *Where advertising is necessary for the proper functioning of markets, a sufficient condition for price dispersion is that firms advertise different products.*

When firms advertise similar products, the equilibrium is symmetric signifying absence of price dispersion. Price dispersion for identical products arises only if firms advertise different products<sup>21</sup>.

## 5 Concluding Remarks

I study price advertising when firms advertise only a subset of their products. I find some support for the empirical findings that price advertising affects advertised and non-advertised prices differently. I show that the degree of product differentiation is decisive for loss leader pricing. When differentiation is low, equilibrium is characterized by loss leader pricing while when products are sufficiently different, equilibrium is characterized by low margin leader pricing. Based on this analysis, I provide a game theoretic (and coherent) explanation to the finding of Walters and Mackenzie (1988).

Assuming that consumers' reservation values are known to firms, though standard in the literature, masks potentially interesting dynamics. This assumption, together with rationality, fixes the price of the non-advertised good.

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<sup>21</sup>A caveat is in order here. The result holds good provided firms find it profitable to compete for the fully informed consumers. If the proportion of fully informed consumers is small, we may have an asymmetric equilibrium in which one of the firms only sells to its captive market and the other firm sells to its captive plus the fully informed consumers. In this case price dispersion results even when firms advertise the same good.

However, in reality, this is not the case. Future research will consider relaxing this assumption.

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