Foreign Reserve Adequacy in Sub-Saharan Africa.

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African Department

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Abstract

This paper looks at the question of adequacy of reserves in sub-Saharan African countries in light of the shocks faced by these countries. Literature on optimal reserves so far has not paid attention to the particular shocks facing low-income countries. We use a two-good endowment economy model facing terms of trade and aid shocks to derive the optimal level of reserves by comparing the cost of holding reserves with their benefits as an insurance against a shock. We find that the optimal level of reserves depends upon the size of these shocks, their probability, and the output cost associated with them,.
16B

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I. Introduction

Policy makers in sub-Saharan Africa (SSA), as elsewhere, often need to find an operational way to assess reserve adequacy. Assessing adequacy needs to be viewed in the broader context of macroeconomic policies. A specific level of reserves may be adequate when alternative sources of financing exist or adjustment can be quickly attained. The same level of reserves, however, may not be adequate if there are no alternative sources of financing, no exchange rate instrument, and/or there is a reluctance or inability to correct a current account deficit. In addition, a large number of economic fundamentals, besides international reserves, can amplify the impact of adverse shocks and render a country crisis-prone in the event of a shock. These include risky short-term financing structures; stock imbalances due to maturity, currency, and interest rate mismatches; and high leverages in public and private sector balance sheets.

Recent studies have attempted to assess reserve adequacy by weighing the consumption smoothing benefits of holding reserves against their cost (Aizenman and Lee, 2005, Garcia and Soto, 2004, and Jeanne and Ranciere, 2006). Jeanne and Ranciere (2006) consider a small open economy with a single good consumed domestically and abroad. The economy is vulnerable to sudden stops in capital flows from abroad. Reserves allow the country to smooth domestic absorption in response to sudden stops, but yield a lower return than the interest. They come up with a closed form, analytical solution for the optimal level of reserve holdings under the above mentioned assumptions and apply it to industrial and emerging market countries. They find that under plausible calibrations the model can explain reserves of the magnitude observed in many industrial and emerging countries.

SSA countries, however, are routinely faced with substantially different shocks than industrial and emerging market economies. These shocks include abrupt changes in the terms of trade and aid flows, which can contribute to higher volatility in aggregate output and, in extreme cases, to economic crisis. Recent reserve models of consumption smoothing do not tend to take into account exogenous shocks such as changes in terms of trade and aid flows that affect most developing countries.

To address these issues, this paper extends the Jeanne and Ranciere (2006) reserve model of consumption smoothing and applies it to simulate reserve holdings in SSA in light of likely shocks. We extend the model in two ways. First, we have two goods in the economy – one traded and another non-traded, with exogenous shocks to the relative price of the traded good, or the terms of trade. Second, every period the economy receives a stochastic transfer of traded good, called aid. We use this extended model to simulate the level of reserves the economy described by our model would hold and contrast it with actual holdings of SSA countries.

The country simulations considered here take into account various aspects of vulnerability. Standard indicators consider financing needs (reserves/imports) and elements of balance sheet vulnerabilities (reserves/money and reserves/short-term external debt). Some of the LIC-specific indicators take into account many of the factors, notably the cost and risk of shocks and the interest cost of financing reserve holdings. The use of a small open economy two-goods model allows us to simulate the optimal level of reserves across a broad spectrum of shocks and output costs, but the "optimal level" of reserves is sensitive to the choice of key parameters such as the risk aversion, the term premium and the probability of shocks, and results in the paper are illustrative of model simulations for a given set of parameters. However, they leave many key considerations out. Inevitably, then, their application requires judgment.

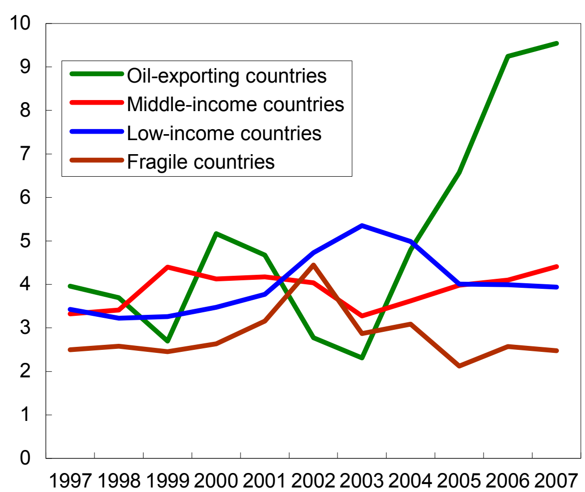
The paper is organized as follows. Section II provides some background on the currently used measures of foreign reserve adequacy in Sub-Saharan African countries. Section III focuses on the specific shocks faced by SSA countries and their impact on some key macroeconomic variables. Section IV presents the basic model used for simulations that takes in to account these shocks. Simulation results from the model are presented in Section V. Section VI presents concluding remarks.

II. Foreign Reserves in sub-Saharan Africa

A. The State of Play

Foreign reserves for all of SSA are reached an all time high of US$127 billion in 2007. Over the past 10 years fast reserve accumulation by oil exporters and steady accumulation by South Africa are notable. This reflects low initial reserve holdings, increasing openness of SSA economies, and a policy choice to build precautionary levels to insure against balance of payment risks. Other SSA countries have kept reserves roughly stable as a share of imports (see Figure 1).

Figure 1. Reserves in Months of Imports[1]



Traditional measures suggest reserve levels vary greatly across countries and country groupings (See Figures 2 and 3).

- Current account-based measures—gross official reserves in months of imports—are particularly useful for SSA countries, as an indication of how rapidly countries would need to adjust to shocks. At end-2007, reserves covered 5.8 months of imports, up from 3.7 months in 1997-2002. This reflects a wide range across countries, though, with above average cover in oil exporting countries and below average cover in fragile countries.

- Since some of the countries in the region are also subject to potential capital outflows, capital account-based measures of reserve adequacy are important too. The ratio of reserves to short term debt, especially relevant for countries that face risks related to short-term external financing, was less than 1 for only a handful of countries.

Reserve levels of other regions offer yardsticks for comparison. Reserve levels for the SSA as a whole are lower than those for the Middle East and North Africa where reserves have served as a store of value in resource rich countries, but higher than those of developing countries in general (Table 1). But with structural changes affecting balance of payments flows and the diverse macroeconomic settings and vulnerabilities of different countries, the experience of other regions provide only limited guidance for SSA countries about the adequate level of reserves in future.[1] Also, many countries may accumulate reserves as a side- effect of monetary and exchange rate policies, such as efforts to stem real exchange rate appreciation. In this case, the observed level of reserves is no benchmark for adequacy.

Table 1. Comparisons of International Reserves Across Regions, 1995-07

	In Months of Imports				Ratio to Short Term Debt			
	1995-04	2005	2006	2007	1995-04	2005	2006	2007
Sub-Saharan Africa	3.5	4.6	5.3	5.2	2.8	4.2	4.8	5.8
Middle East And North Africa	8.5	10.9	11.8	11.7	3.1	3.5	3.9	4.0
Emerging Asia	5.8	8.0	8.6	9.0	2.5	3.8	4.3	4.8
Other Developing Asia[1]	8.3	11.2	12.3	12.3	3.4	4.1	4.6	4.7
Western Hemisphere	3.4	4.6	5.0	5.5	2.1	7.1	9.2	10.9
Developing Countries	4.3	4.2	3.8	3.5	1.7	1.5	1.3	1.3
Emerging Market Economies	6.7	8.9	9.1	9.4	8.1	8.4	8.7	9.6
Least Developed Countries	6.0	7.9	8.2	8.5	3.7	5.0	5.4	5.9

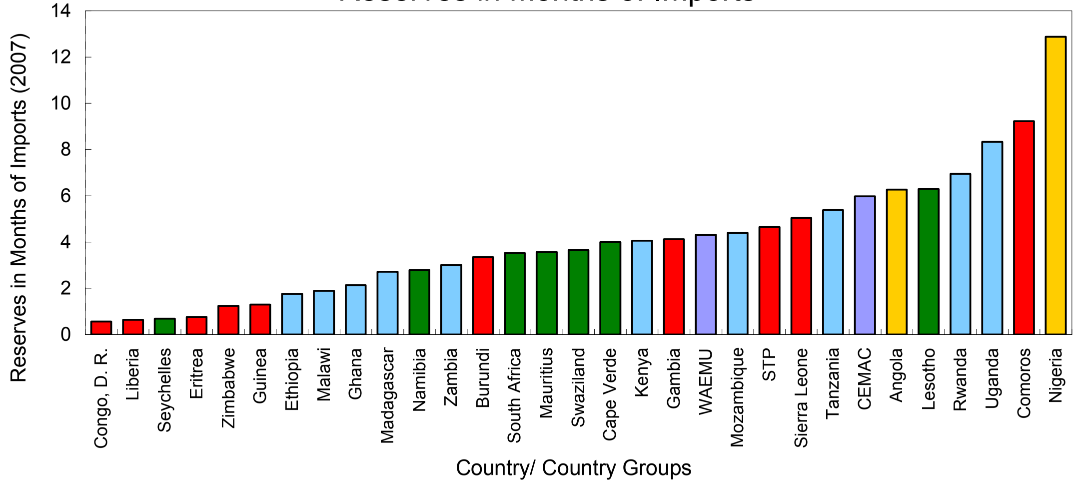
Source: IMF, *World Economic Outlook.*

[1] Excluding China & India.

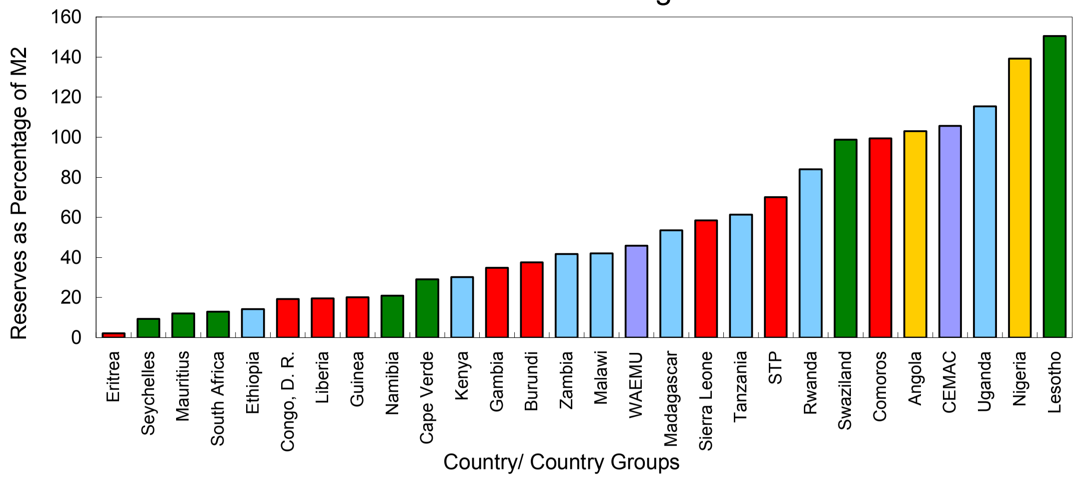
[1] For the CFA zone countries, for instance, the creation of a regional financial market with the possibility of emission of domestic-currency-denominated treasury bonds may help smoothing consumption.

Figure 2. Reserves in Sub-Saharan African Countries, 2007

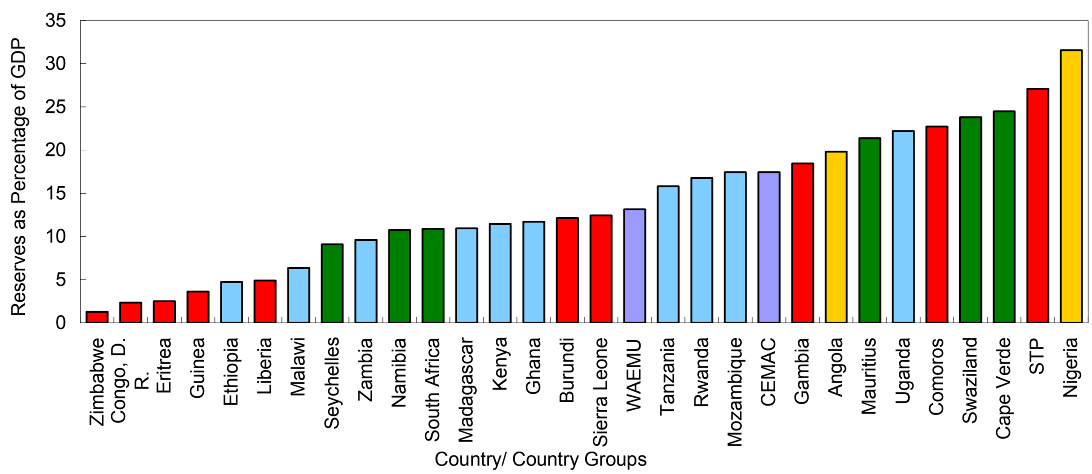
Reserves in Months of Imports[1]



Reserves as Percentage of M2



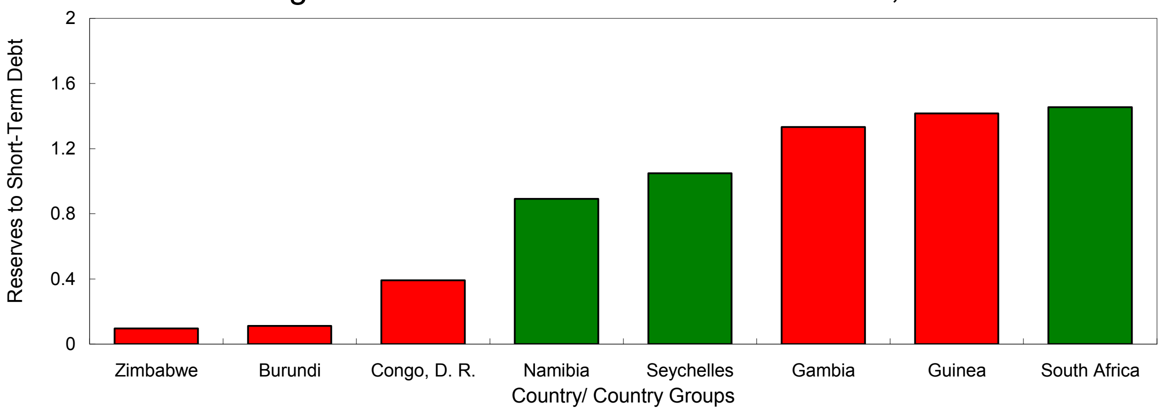
Reserves as Percentage of GDP



Source: *World Economic Outlook*

[1] Imports of Goods and Services

Figure 3. Reserves to Short-Term Debt <2, 2007[1]



Source: *World Economic Outlook*
[1]Short-Term Debt by Remaining Maturity

III. SHOCKS FACING SUB-SAHARAN AFRICA

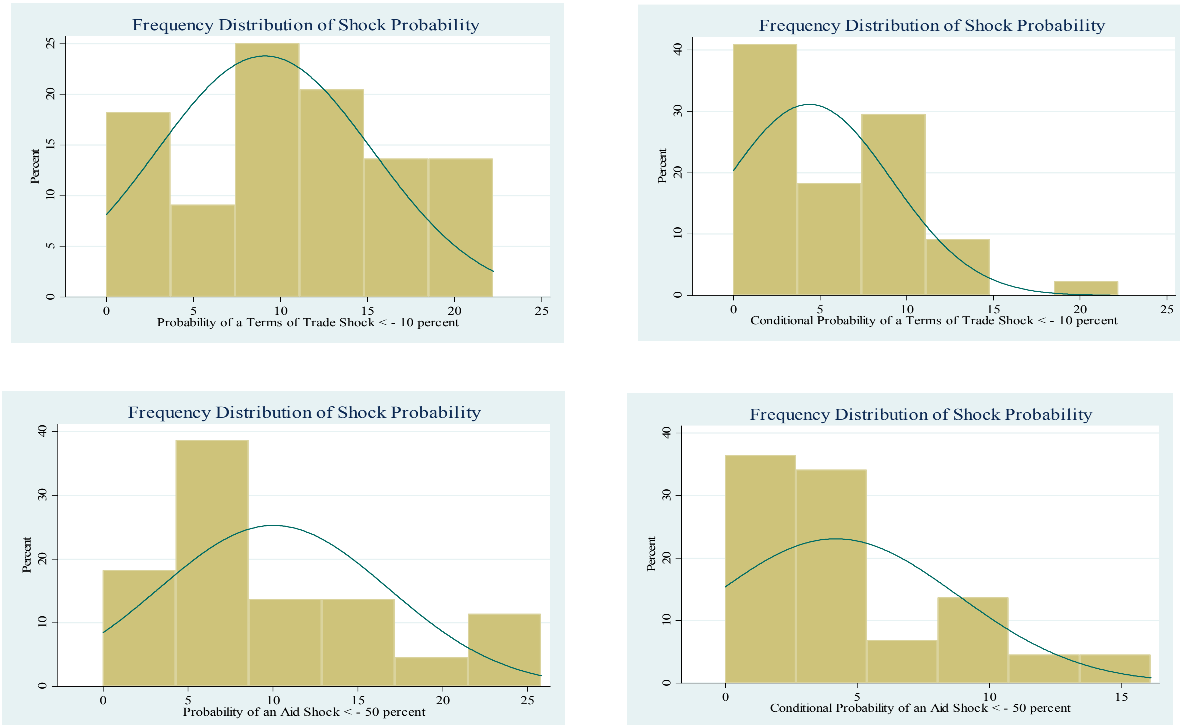
Countries in SSA face substantially different shocks compared to industrial and emerging market countries. The main shocks facing low income countries in Africa are: a sharp change in their terms of trade due to exogenous movements in the prices of key exports/imports and a change in the net aid flows (defined as Net Official Development Assistance Grants less Food and Technical assistance) received by them.

For the purposes of our analysis we define shocks in terms of annual percentage changes in terms of trade or aid flows facing a country. To further capture the idea of `large changes' we define a terms of trade shock as an year on year decline in the terms of trade index larger than 10 percent in absolute terms. Similarly, an aid shock is defined as a decline in the net aid inflow of 50 percent or more in absolute terms. These thresholds are based on the probability distribution of the two shocks so that such events happen 20-25 percent of the time for an average SSA country[2] (Figures 4 and 5)

Eighty percent of the SSA countries face a 10 percent or larger negative terms of trade shock at least 5 percent of the time or more while 80 percent of these countries face a 50 percent or larger decline in aid flow at least 5 percent of the time or more. The average size of a terms of trade shock in SSA countries is 21 percent while the average size of aid shock is 181 percent (roughly 4-5 percent of GDP). Average probability of a terms of trade shock as defined above, in SSA countries, is about 20 percent and that of an aid shock is 10 percent.

[2] Exploring the sensitivity of these results to the precise threshold values can be interesting future work.

Figure 4. Frequency Distributions of Key Parameters

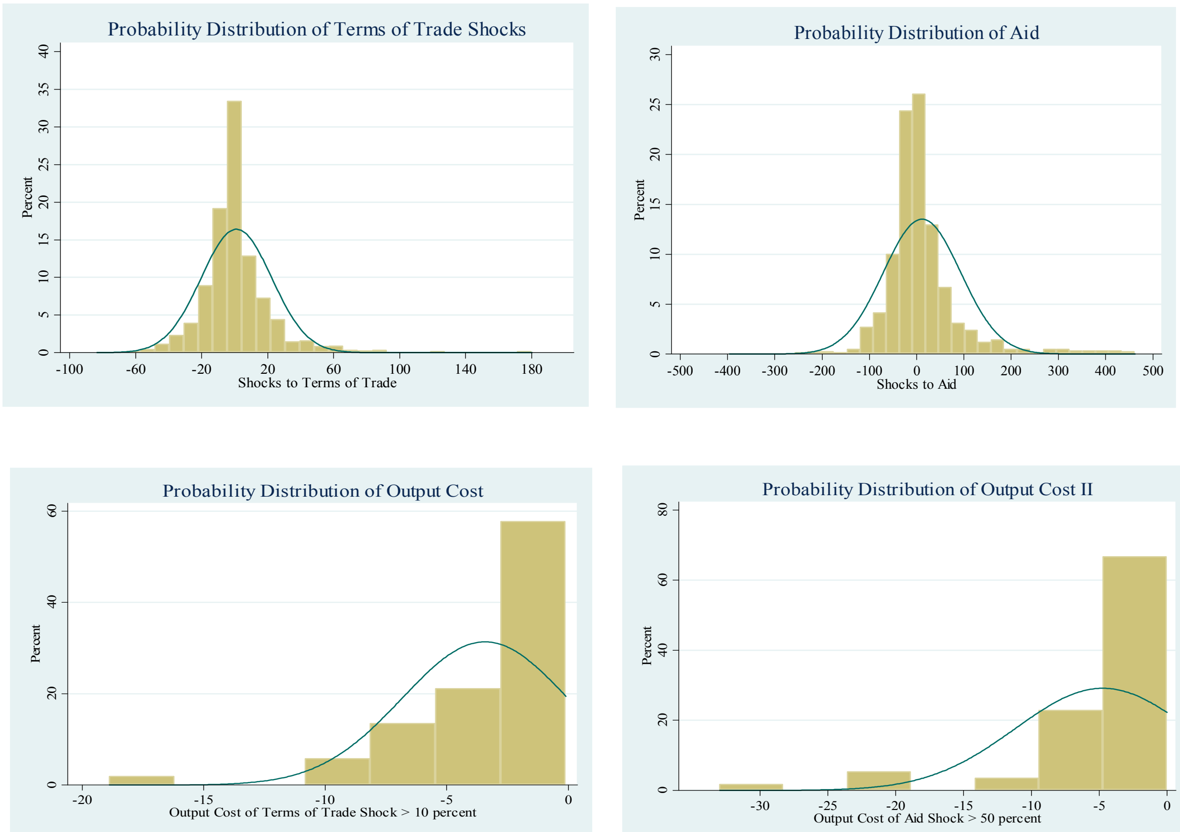


The loss in output and consumption due to such shocks can be significant, especially for countries with low levels of foreign reserves. The average loss in output, as measured by the reduction in GDP growth following a shock, associated with a 10 percent or larger terms of trade shock is 1.5 percent (based on the sample)[3]. Same is true for the output loss associated with an aid shock of 50 percent or more. About 40 percent of aid shocks are associated with an output cost lying between 0.5 to 4 percent and another 20 percent with an output loss between 5-6 percent. About 40 percent of the terms of trade shocks are associated with output cost of 2 percent or more.

However, the actual response of output and consumption to a shock varies significantly with the level of reserve holdings of a country. Countries with a high level of reserves to GDP ratio (those in the top 25th percentile) showed very little effect of a terms of trade or aid shock on their output and consumption. On the other hand, countries with low reserves to GDP ratio (bottom 25th percentile) showed a significant decline in their output growth and an even more dramatic decline in their per capita consumption.

[3] Other variables, such as exchange rate and fiscal stance might determine the cost of the shocks and reserve accumulation and there is possibility of some endogeneity between aid shocks and output falls. A thorough empirical analysis of these issues is beyond the scope of this paper.

Figure 5. Frequency Distributions of Key Parameters



Figures 6 and 7 provide some evidence on the response of absorption, output, and foreign reserves to large terms-of-trade and aid shocks over the period 1980-2006. We classify countries as 'Low-Reserve' (LR henceforth) or 'High Reserve' (HR henceforth) based on their average reserve-to-GDP ratio during 2000-06. Of the 44 SSA countries, eleven countries whose average reserve-to-GDP ratio was in the bottom 25th percentile were called LR countries. Overall average reserve to GDP ratio for this group during this period was 4.2 percent.

Countries in the HR group had an average reserve to GDP ratio higher than 16 percent during 2000-06 and the overall average for the period was about 28 percent (7 times more than that for LR countries).

We then identify the 'shocks', as defined in the beginning, for these countries over a period of 27 years (1980-2006).[4] Next we look at the behavior of GDP growth, domestic absorption (defined as domestic consumption plus gross capital investment per capita.) and foreign

[4] The frequency distribution of shocks may have changed over the period reflecting changes in the patterns of aid and terms-of-trade shocks.

reserves over a five-year `event' window centered around the shock occurring at time zero. This is done for aid and terms of trade shocks separately. Events that occur inside the five-year window of the previous shock episode are excluded. The solid lines in the panels are the path of these variable in response to a terms of trade / aid shock while the broken lines give the one standard error band.

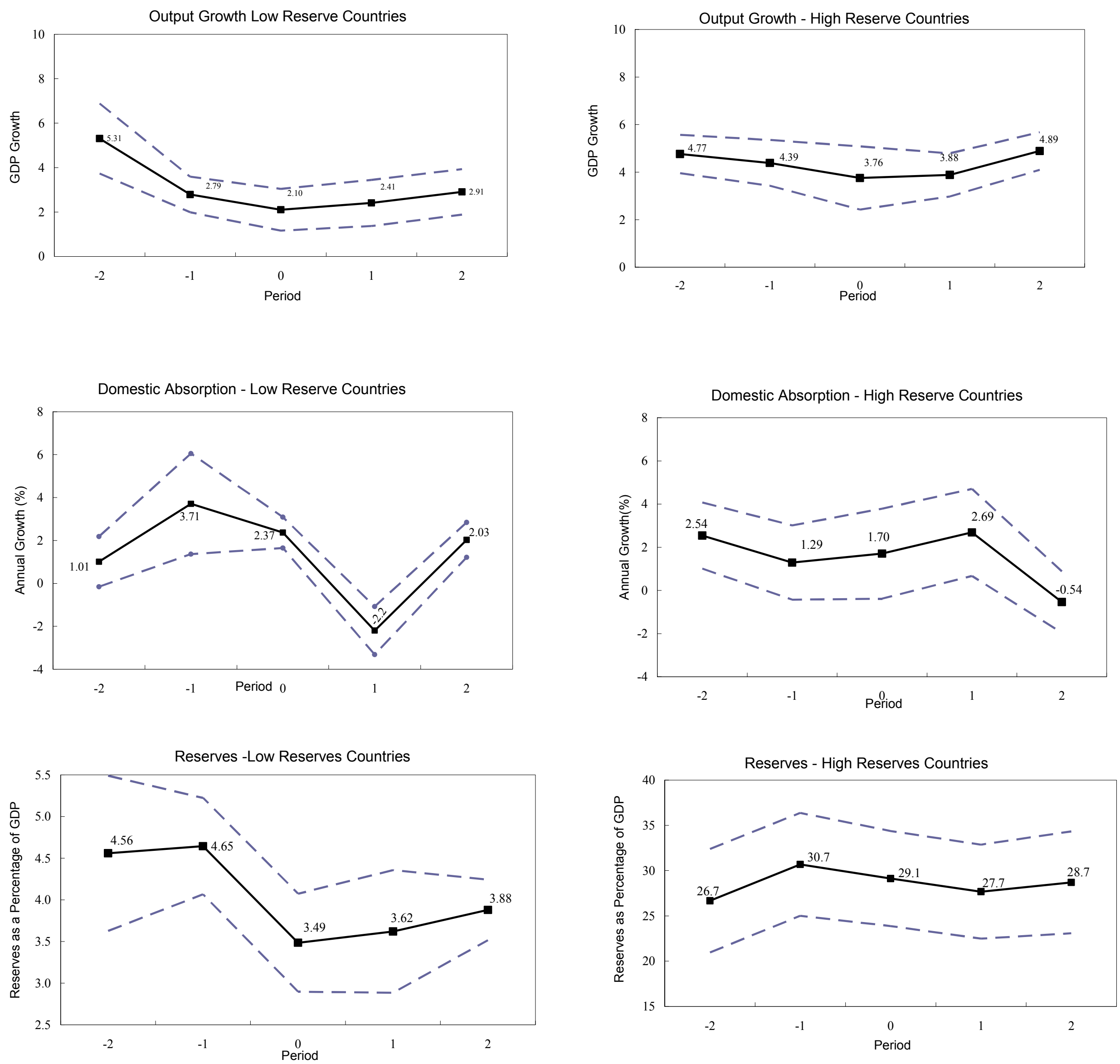
Looking at the top column in Figure 6 we can see that LR countries face a more significant decline in their GDP growth due to a terms of trade shock as compared to the HR countries. The difference is even more striking when it comes to the response of per capita absorption. For LR countries growth in per capita absorption declines significantly (it is in fact negative for one year after the shock) and does not return to the pre-shock level even after 2 years. On the other hand there is no significant change in domestic absorption for HR countries in response to a large TOT shock. It appears that countries with a higher level of reserves are better able to cushion the effect of a shock by utilizing their reserves in the event of a shock. Last column of the same panel, which plots the movement in reserves as a percentage of GDP around the shock, gives some evidence to support this view. Both LR and HR countries accumulate reserves during 'normal' times (i.e. before the shock) and draw down these reserves during a TOT shock (as shown by a fall in reserve level at time zero). However, while this fall in reserves amounts to about 1 percent of GDP for LR countries, for HR countries this reduction amounts to roughly 1.6 percent of their GDP. Looking at the cumulative reduction in reserves over the two years starting with the shock, the reduction in reserves for LR countries is only 0.98 percent while it is about 3 percent for HR countries.

Clearly it is more difficult for LR countries to cushion their output and consumption in the event of a shock. In fact, unlike HR countries which continue to draw down their reserves for another year after the shock the LR countries are seen to start building their reserves immediately after the shock. One potential reason for this may be that with already low level of reserves, LR countries can not draw down on these any further without inducing panic in the domestic and international financial markets and thereby increasing the risk of an economic crisis.

Looking at the case of a large aid `shock' we find similar results. Output and domestic absorption are affected more in case of LR countries except that in this case the decline in domestic absorption occurs one year after the shock. This may reflect the difference between the timing of disbursement and utilization of aid proceeds. Also, as was the case with a terms of trade shock, absorption is affected more severely than output indicating the role of consumption smoothing. To the best of our knowledge the empirical evidence provided above has not been recorded anywhere else and exploring the relationship between the level of reserve holdings and response of key macro-economic variables would be a fruitful area of future research.

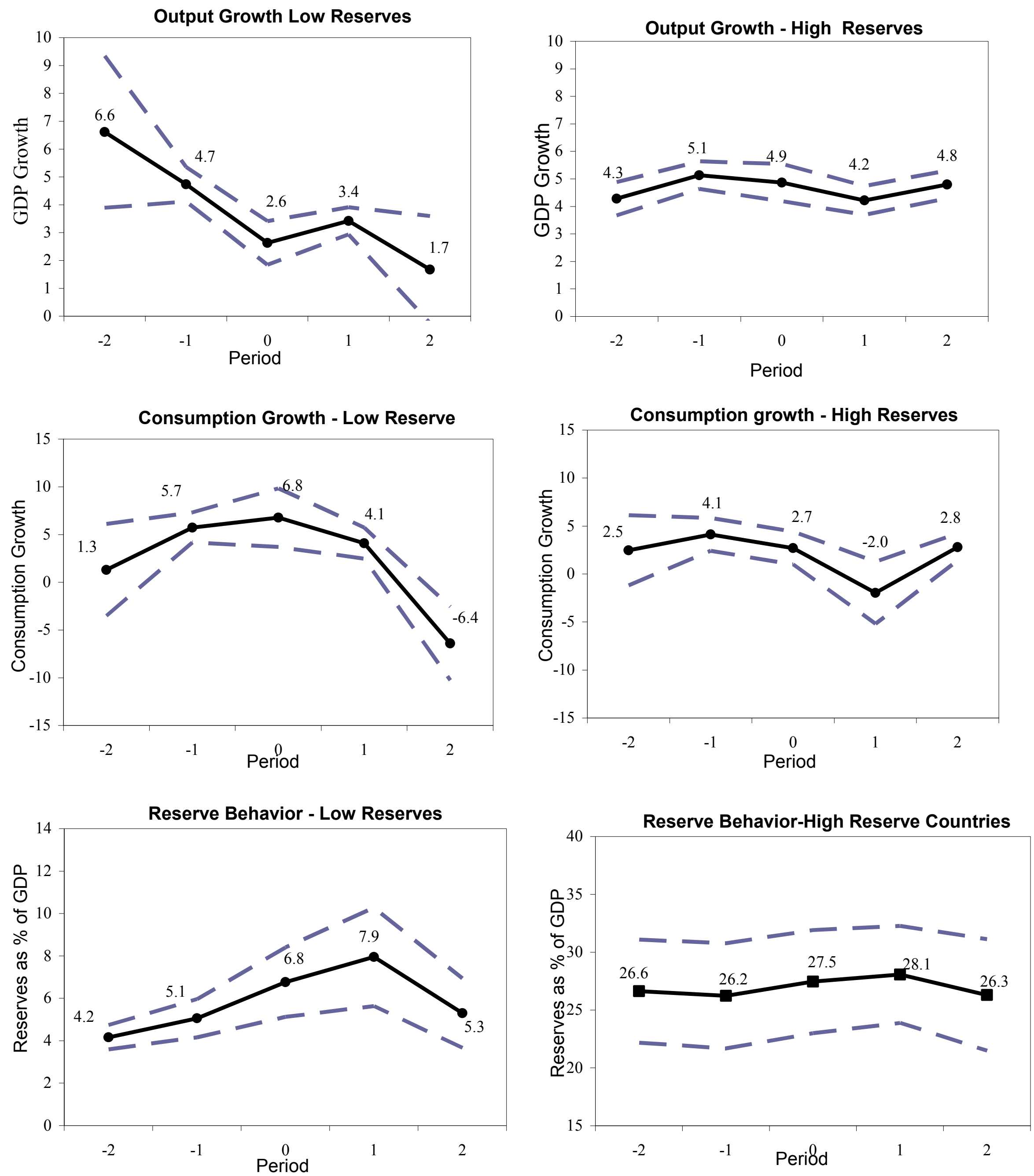
To sum up, SSA countries are routinely faced with abrupt changes in the terms of trade and aid flows, which have contributed to higher volatility in aggregate output and, in extreme cases, to economic crisis. Assessing the adequacy of reserves, thus, requires understanding the role of reserves in smoothing domestic absorption in response to external shocks, the object of the next section.

Figure 6. Response of Key Macro Economic Variables to a Large TOT Shock



Source: Authors' computations based on I.M.F, *International Financial Statistics* and World Bank, *World Development Indicators*.

Figure 7. Response of Key Macroeconomic Variables to a Large Aid Shock



Source: Authors' computations based on I.M.F, *International Financial Statistics* and World Bank, *World Development Indicators*.

IV. Small Open Economy with Two Goods

Consider a Small Open Economy (SOE) with two goods—one tradable and another non-tradable. The economy follows a deterministic path for the output of two goods, disturbed only by exogenous shocks to the terms of trade. Besides it gets a unilateral transfer of tradable good from abroad called "Aid" which grows at the same rate as output and is stochastic in nature. To elaborate further, a shock to the terms of trade is defined as a fall of ten percent or more in the terms of trade from the 'normal' level, which is set equal to one. Aid shock is also defined as an unforeseen drop in aid flows of five percent or more. In periods with no shocks, terms of trade and aid flow remain at their normal level. Probabilities of the two shocks are exogenously given as π_{TOT} and π_{Aid} and the two shocks are independent of each other. When either of these shocks occur, two things happen – one, output growth falls below its 'normal level' and second, in the case of external private borrowing, there is not roll over of short-term private external debt. Thus, aid and T.O.T shocks are always accompanied by a 'Sudden-Stop' in capital. The domestic economy is composed of the private sector and the government. We present two cases: with and without private external borrowing.

A. No External Private Borrowing

The representative private consumer is subject to the following budget constraints:

$$C_t^T = T \times Y_t^T + Z_t + A_t \tag{1.1}$$

$$C_t^N = Y_t^N \tag{1.2}$$

Here C_t^T is the consumption of tradable good in period t and C_t^N is the consumption of non-tradable good. Y_t^T and Y_t^N are the period t output of tradable and non-tradable goods respectively. T is the terms of trade. A_t is the unilateral transfer of tradable good from abroad which is given directly to the representative private consumer and consumed in the same period. Z_t is the transfer of tradable good by the government. In every period the consumption of non-tradable good is equal to its output (for simplicity we assume that the non-tradable good can not be saved). The consumption of tradable good on the other hand equals the sum of the output of tradable good, government transfer of tradable good and aid flow from abroad every period.

Combining equations 1.1 and 1.2 we get the over budget constraint for the consumer,

$$C_t^T + P_t^N \times C_t^N = T \times Y_t^T + Z_t + A_t + P_t^N \times Y_t^N \tag{1.3}$$

P_t^N is the endogenously determined price of non-tradable good. Output of tradable as well as non-tradable good grows at the same constant rate `g' until a terms of trade or aid shock occurs. The shock is associated with a fall in output by a fraction γ. After the shock the output goes back to its long-run path.

Denoting the periods before, during and after the shock with the sub-scripts *b*, *d* and *a* we can write the following equations summarizing our assumptions,

$$Y_{t,b}^T = (1+g)^t Y_0^T,\ Y_{t,d}^T = (1-\gamma)(1+g)^t Y_0^T,\ Y_{t,a}^T = (1-\gamma)(1+g)^t Y_0^T \tag{1.4}$$

$$Y_{t,b}^N = (1+g)^t Y_0^N,\ Y_{t,d}^N = (1-\gamma)(1+g)^t Y_0^N,\ Y_{t,a}^N = (1-\gamma)(1+g)^t Y_0^N \tag{1.5}$$

$$A_{t,b} = (1+g)^t A_0,\ A_{t,d} = (1-\gamma_{Aid})(1+g)^t A_0,\ A_{t,a} = (1-\gamma_{Aid})(1+g)^t A_0 \tag{1.6}$$

The government can issue a long-term security that does not have to be repaid during the shock (there is just one long-term security though there are two shocks). The long-term security issued by the government is a bond that yields one unit of good in every period until the shock occurs. The security stops yielding any income after the shock.

The pre-shock price of the security is equal to the present discounted value of the one unit of good it pays in the next period plus the expected market value of the security,

$$P = \frac{1}{1+r+\delta}\left[1+(1-\pi_s).P\right]$$

Where $\pi_s = \pi_{TOT} + \pi_{Aid} - \pi_{TOT} \times \pi_{Aid}$ is the probability of a shock (to aid, terms of trade or both) occurring in any period, r is the short term interest rate (equal to the discount rate of the representative consumer) and δ is the term premium. This implies,

$$P = \frac{1}{r+\delta+\pi_s} \tag{1.7}$$

Equation 1.7 uses the fact that the price of the long-term security is constant before the sudden stop and falls to zero when sudden stop occurs.

The government issues the long-term security to finance a stock of reserves,

$$R_t = PN_t \tag{1.8}$$

Where N_t is the number of securities issued by the government in period t. Government's budget constraint is given by:

$$Z_t + R_t + N_{t-1} = P\left(N_t - N_{t-1}\right) + \left(1+r\right)R_{t-1} \tag{1.9}$$

Equation 1.8 can be used to substitute out N_t and N_{t-1} from equation 1.9 to get the following expression,

$$Z_t^b = -\left(\frac{1}{P} - r\right)R_{t-1} = -\left(\delta + \pi_s\right)R_{t-1} \tag{1.10}$$

Negative transfer implies that the government taxes the representative consumer in order to pay for the cost of carrying the reserves, which is proportional to the term premium plus the probability of a shock.

If and when the shock occurs, the government transfers the reserves (net of last payment on the long-term security) to help the representative consumer,

$$Z_t^d = \left(1 - \delta - \pi_s\right)R_{t-1} \tag{1.11}$$

After the shock the governments become inactive and R_t, N_t and Z_t are all equal to zero. Using equations 1.10 and 1.11 we can get the expressions for tradable consumption before and after the shock,

$$C_{t,b}^T = T \times Y_{t,b}^T - \left(\delta + \pi_s\right)R_{t-1} + A_t \tag{1.12}$$

$$C_{t,TOT}^T = (1 - \gamma_{TOT}) \times T \times Y_{t,d}^T + \left(1 - \delta - \pi_s\right)R_{t-1} + A_t \tag{1.13}$$

$$C_{t,Aid}^T = T \times Y_{t,d}^T + \left(1 - \delta - \pi_s\right)R_{t-1} + \left(1 - \gamma_{Aid}\right) \times A_t \tag{1.14}$$

$$C_{t,TOT,Aid}^T = \left(1 - \gamma_{TOT}\right) \times T \times Y_{t,d}^T + \left(1 - \delta - \pi_s\right)R_{t-1} + \left(1 - \gamma_{Aid}\right) \times A_t \tag{1.15}$$

$$C_{t,a}^T = T \times Y_{t,a}^T + A_t \tag{1.16}$$

Increasing R_{t-1} raises the consumption of tradable good in period t if there is a shock and lowers it if there is no shock. Expression for the consumption of non-tradable is as follows,

$$C_{t,b}^N = Y_{t,b}^N,\ C_{t,d}^N = Y_{t,d}^N,\ C_{t,a}^N = Y_{t,a}^N \tag{1.17}$$

Government chooses R so as to maximize the welfare of the representative consumer,

$$U_t = \sum_{s=0}^{\infty} E_0\left[\left(1+r\right)^{-s} u\left(C_{t+s}\right)\right], \tag{1.18}$$

Where the flow utility function has a constant relative risk aversion σ,

$$u\left(C\right) = \frac{C^{1-\sigma} - 1}{1-\sigma},$$

And C is the aggregate consumption,

$$C_t^i = \left(C_{t,i}^T\right)^{\alpha}\left(C_{t,i}^N\right)^{1-\alpha}; \; i = b,\, TOT,\, Aid$$

C_s^i is the commonly used Cobb-Douglas consumption aggregator with constant elasticity of substitution between the tradable and the non-tradable goods and α is the share of tradable good in total consumption. The discount rate of representative consumer is equal to the rate of interest r. This ensures that the consumer's maximization problem is well defined in the absence of endogenous discount rate or upward sloping interest rate function. Since R_t only affects the level of consumption in the next period, government's problem is to choose the level of R_t in each period that maximizes the level of expected utility $u\left(C_{t+1}\right)$ next period.

$$R_t = \arg\max\left(E_0\left[u\left(C_{t+1}\right)\right]\right) = \arg\max(1-\pi_s)\times u\left(C_{t+1}^b\right) + \left(\pi_{TOT} - \pi_{TOT}\times\pi_{Aid}\right)\times u\left(C_{t+1}^{TOT}\right)$$
$$+\left(\pi_{Aid} - \pi_{TOT}\times\pi_{Aid}\right)\times u\left(C_{t+1}^{Aid}\right) + \pi_{TOT}\times\pi_{Aid}\times u\left(C_{t+1}^{TOT,Aid}\right)$$

The first order condition for the problem is,

$$\left(1-\pi_s\right)\left(\delta+\pi_s\right)u_T^{'}\left(C_{t+1}^b\right) = \left(1-\delta-\pi_s\right)\begin{bmatrix}\left(\pi_{TOT}-\pi_{TOT}\times\pi_{Aid}\right)\times u_T^{'}\left(C_{t+1}^{TOT}\right)\\ +\left(\pi_{Aid}-\pi_{TOT}\times\pi_{Aid}\right)\times u_T^{'}\left(C_{t+1}^{Aid}\right)\\ +\pi_{TOT}\times\pi_{Aid}\times u_T^{'}\left(C_{t+1}^{TOT,Aid}\right)\end{bmatrix} \tag{1.19}$$

The right-hand side of equation 1.19 is the expected marginal utility of reserves conditional on a shock. The left-hand side is the probability of no shock times the expected marginal cost of reserves conditional on there being no shock. When the level of reserves is chosen optimally, the marginal utility of holding reserves is equal to the marginal cost of holding them.

Denoting the marginal rate of substitution between consumption in the event of a shock and consumption in the absence of a shock by p_t we can write,

$$p_t = \frac{E_0\left[u_T^{'}\left(C_t^d\right)\right]}{E_0\left[u_T^{'}\left(C_t^b\right)\right]} \tag{1.20}$$

Where `$d$' denotes `during the shock'. Equation 1.20 says that when reserves are set optimally, this price should be constant and equal to,

$$p=\frac{\delta+\pi_s}{1-\delta-\pi_s}=\frac{1}{1-\delta-\pi_s}-1 \tag{1.21}$$

Some Notations,

$$\begin{aligned}
C_t^{TOT} &= \left(C_{t,TOT}^{T}\right)^{\alpha}\left(C_{t,TOT}^{N}\right)^{1-\alpha},\\
C_t^{Aid} &= \left(C_{t,Aid}^{T}\right)^{\alpha}\left(C_{t,Aid}^{N}\right)^{1-\alpha},\\
C_t^{TOT,Aid} &= \left(C_{t,TOT,Aid}^{T}\right)^{\alpha}\left(C_{t,TOT,Aid}^{N}\right)^{1-\alpha}\\
C_t^{b} &= \left(C_{t,b}^{T}\right)^{\alpha}\left(C_{t,b}^{N}\right)^{1-\alpha}
\end{aligned} \tag{1.22}$$

$$E_0\left[u_T^{'}\left(C_t^d\right)\right]=\alpha\begin{bmatrix}\left(\pi_{TOT}-\pi_{TOT}\times\pi_{Aid}\right)\times\left(\left(C_t^{TOT}\right)^{-\sigma}\left(C_{t,TOT}^{T}\right)^{\alpha-1}\left(C_{t,TOT}^{N}\right)^{1-\alpha}\right)\\ +\left(\pi_{Aid}-\pi_{TOT}\times\pi_{Aid}\right)\times\left(\left(C_t^{Aid}\right)^{-\sigma}\left(C_{t,Aid}^{T}\right)^{\alpha-1}\left(C_{t,Aid}^{N}\right)^{1-\alpha}\right)\\ +\pi_{TOT}\times\pi_{Aid}\times\left(\left(C_t^{TOT,Aid}\right)^{-\sigma}\left(C_{t,TOT,Aid}^{T}\right)^{\alpha-1}\left(C_{t,TOT,Aid}^{N}\right)^{1-\alpha}\right)\end{bmatrix} \tag{1.23}$$

And,

$$E_0\left[u_T^{'}\left(C_t^b\right)\right]=\alpha\left(1-\pi_s\right)\times\left(\left(C_t^b\right)^{-\sigma}\left(C_{t,b}^{T}\right)^{\alpha-1}\left(C_{t,b}^{N}\right)^{1-\alpha}\right) \tag{1.24}$$

equations 1.21, 1.22, 1.23 and 1.24 we can simulate the optimal level of reserves for different values of parameters and shocks.

B. External Private Borrowing

We modify the above framework to allow private external borrowing. We assume that the representative agent can engage in short-term borrowing from abroad and that the short-term external debt (which is a fraction λ of tradable good output) grows at the same rate as output until a shock occurs. This debt has to be repaid in terms of tradable good even when there is a shock to the terms of trade or aid, with an interest rate r. There is no default. In the event of a shock the agent repays its outstanding external debt but can not engage in fresh borrowing during or after the shock (no debt roll over). This assumption is necessary for keeping the reserve management problem meaningful since without the no-debt-roll-over condition private agent will be able to smooth over its consumption by engaging in external borrowing. These assumptions can be expressed as follows,

$$L_t^b=\lambda(1+g)^t T\times Y_0^T,\ L_t^d=L_t^a=0 \tag{1.25}$$

Consumption of tradable good in each period is equal to,

$$C_t^T = T \times Y_t^T + L_t - (1+r) L_{t-1} + Z_t + A_t$$

Thus, the expression for tradable good consumption before, during and after a shock is,

$$C_{t,b}^T = T \times Y_{t,b}^T + L_t - (1+r) L_{t-1} - (\delta + \pi_s) R_{t-1} + A_t \quad (1.26)$$

$$C_{t,TOT}^T = (1-\gamma_{TOT}) \times T \times Y_{t,d}^T - (1+r) L_{t-1} + (1-\delta-\pi_s) R_{t-1} + A_t \quad (1.27)$$

$$C_{t,Aid}^T = T \times Y_{t,d}^T - (1+r) L_{t-1} + (1-\delta-\pi_s) R_{t-1} + (1-\gamma_{Aid}) \times A_t \quad (1.28)$$

$$C_{t,TOT,Aid}^T = (1-\gamma_{TOT}) \times T \times Y_{t,d}^T - (1+r) L_{t-1} + (1-\delta-\pi_s) R_{t-1} + (1-\gamma_{Aid}) \times A_t \quad (1.29)$$

$$C_{t,a}^T = T \times Y_{t,a}^T + A_t \quad (1.30)$$

Replacing these modified expressions in equations 1.22, 1.23 and 1.24 will give us optimal level of reserves with short term external borrowing.

V. Simulation Results

The two-good model presented in the above paragraphs does not allow for an analytical solution. We therefore use numerical techniques to solve for the level of optimal reserves as a function of output. Simulation results presented below assume external private borrowing[5].

A. Choice of Parameters

Table A.1 gives the value of key parameters used for benchmark simulations. These parameter values have been calibrated using data for 44 SSA countries. Average size of a terms of trade 'shock' across SSA countries was about 21 percent while that of an aid shock was 1.8 percent (4-5 percent of GDP). These were used to calibrate γ_{TOT} and γ_{Aid}. Similarly average probabilities of a terms of trade shock and an aid shock were used to calibrate π_{TOT} and π_{Aid} respectively. Aid as a share of G.D.P is equal to 4 percent (equal to average aid to GDP ratio for the 44 SSA countries between (1975-2005). Output cost of terms of trade and aid shocks, γ_1 and γ_2, were calibrated based on the impulse responses discussed in section III. Potential output growth, g, is set to be 5 percent based on the average growth rate for Africa over last 5 years. Share of short term debt in GDP, λ, was again calibrated using country-specific data for the 44 SSA countries for 2007. Risk free short-term rate of return, r, is set at 5 percent – same as the federal funds rate during 1987-2005. Term premium, δ,

[5] An interesting future extension would be to look at a case where there is no private external borrowing.

which reflects the cost of holding reserves, is set at 1.5 percent. This is equal to the value used by Jeanne and Ranciere (2005). One can argue that the cost of holding foreign reserves is higher in low-income African countries than in emerging market countries and therefore the term premium should be slightly higher for these countries. This value is also likely to differ across countries, and if available, country specific term-premia should be used. The remaining parameters are obtained from other low-income country studies.

B. Jeanne-Ranciere and the Two-Good Model

In this section we compare the results from Jeanne-Ranciere model with those from the Two-Good model. To make the comparison meaningful we do not incorporate aid shocks as of yet in the Two-Good model. In fact, we assume aid flows to be equal to zero for the purposes of this exercise. In effect there is just one shock which takes a different interpretation in the case of a Two-Good model and has a direct effect on the level of tradable income. Figure 8 plots the level of optimal reserves (expressed as a percentage of G.DP.) against key parameters for both the models (remaining parameters are kept constant and same across the two models). For both the models the level of optimal reserves increases with the probability and the output cost of the shock and declines with the term premium δ. Optimal level of reserves also go up with the size of terms of trade shock in case of the Two-Good model. However, for relevant ranges of the key parameters , the Two-Good model suggests a higher level of optimal reserves when compared to the Ranciere model.

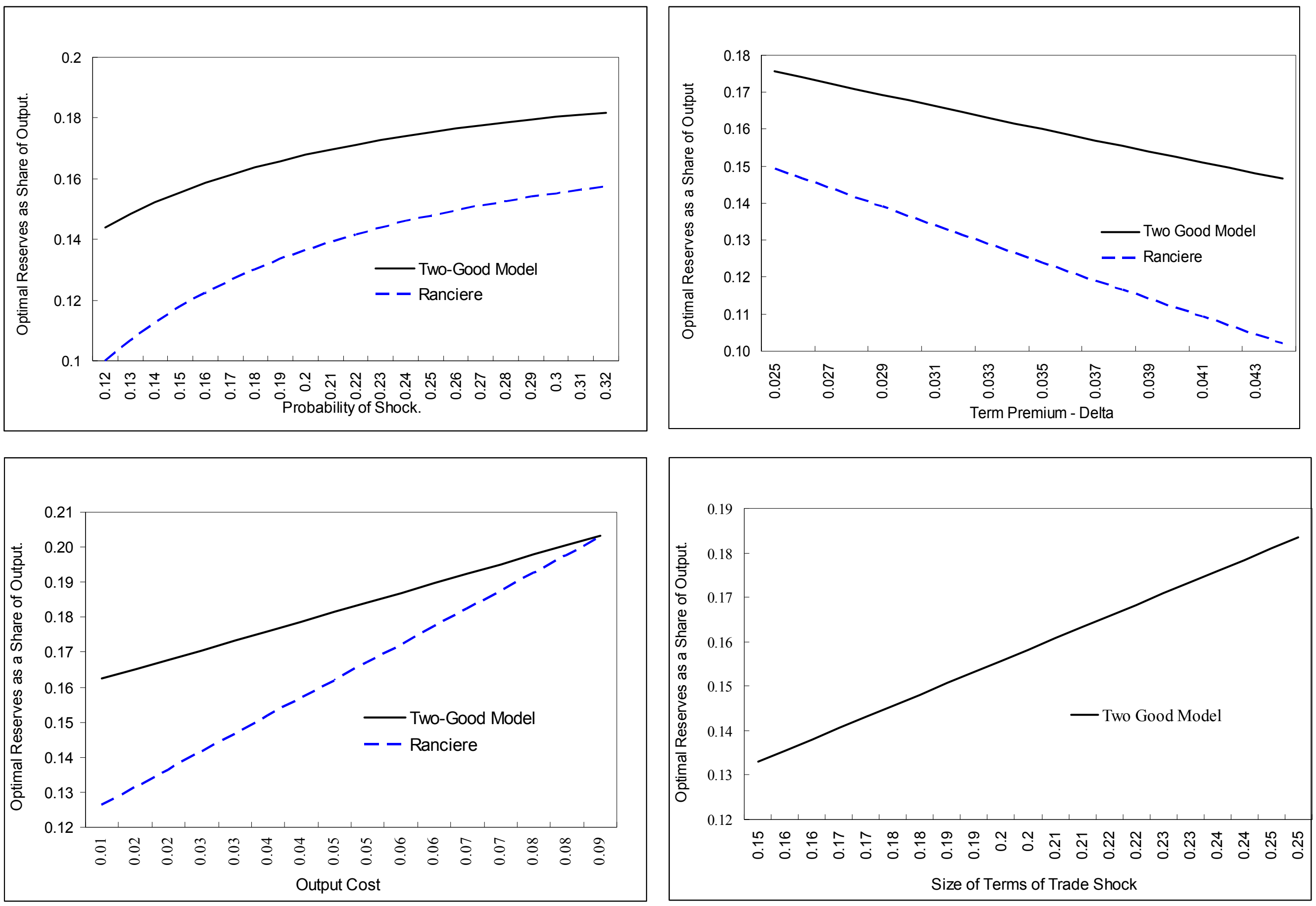
To understand these results we look at the *key differences between the Ranciere model and the two-goods model*. Firstly, the Two-Good model allows us to take in to account the direct effect of a fall in the terms of trade on the ability of the developing country to import tradable goods. This can be called as the – 'direct income effect' of a fall in the relative price of a country's exports.

Second, return to an additional dollar of reserves in terms of non-shock dollar is lower in the model with two-goods. This is because a terms of trade shock affects both non-tradable and tradable goods output (and consumption), but reserves can only be used to cushion the decline in tradable goods consumption and a lower non-tradable consumption implies a lower marginal utility of tradable consumption.

Finally, the substitution of tradable consumption between shock and non-shock states is easier in the case of the two-goods model, since tradable goods only account for half the total consumption (its impact on the overall consumption and total utility is half). In other words, the return, in terms of utility, to having an additional dollar in reserves and hence having smaller variation in the tradable goods consumption before and during the shock is smaller when compared to the case with a single good which is also used for accumulating reserves.

While the first point implies a higher optimal level of reserves in the two-goods model, the last two points imply a lower level of optimal reserves. For the relevant range of key parameters in question the first effect dominates the last two effects and hence the optimal level of reserves as a share of output is higher in the model with two goods. This in turn implies that under the benchmark parameters, the fall in consumption due to a terms of trade shock would be smaller in the two-goods model. For the set of benchmark parameters given in Table A .1, the Two-Good Model suggests an optimal level of reserve that is greater by about 2 percent of GDP than the one suggested by the Ranciere model.

Figure 8. Optimal Reserve Behavior – Jeanne-Ranciere vs. Two-Good Model [1,2]

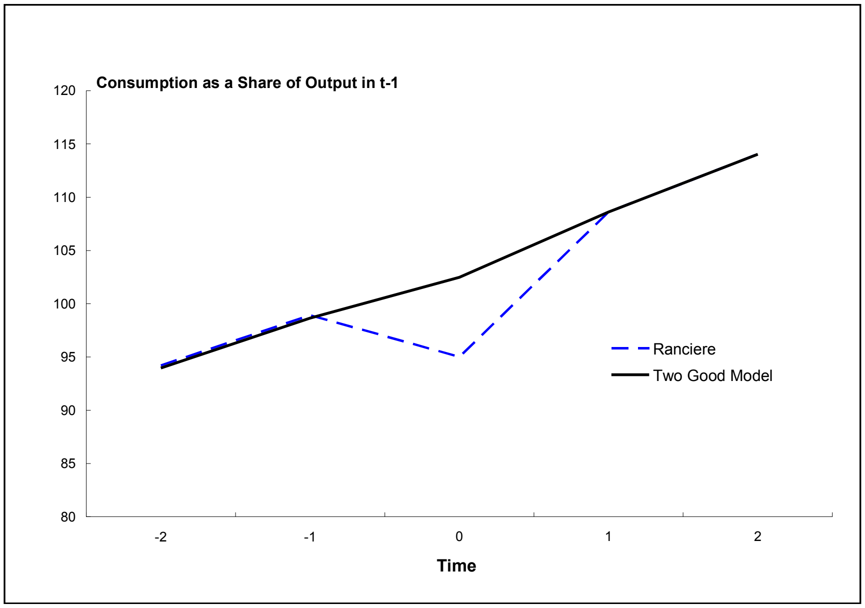


[1] Broken line shows the results for Jeanne-Ranciere while the solid line shows the Two-Good model.
[2] Optimal Reserves are expressed as a percentage of output.

Figure 9 below shows the path of consumption (expressed as a percentage of GDP in the year before the shock) in the event of a terms of trade shock of average size at time zero when optimal reserves are chosen under the two-goods model. We compare it with the path under the Ranciere model for illustrative purposes. In the one-good model consumption falls by about 4 percent whereas it falls by only 1.5 percent in the case of the two-goods model.

Clearly, for countries facing exogenous shocks such as a terms-of-trade fall, it is important to take in to account those shocks while assessing the adequacy of their reserve holdings.

Figure 9. Path of Consumption - Ranciere vs. Two-Good Model

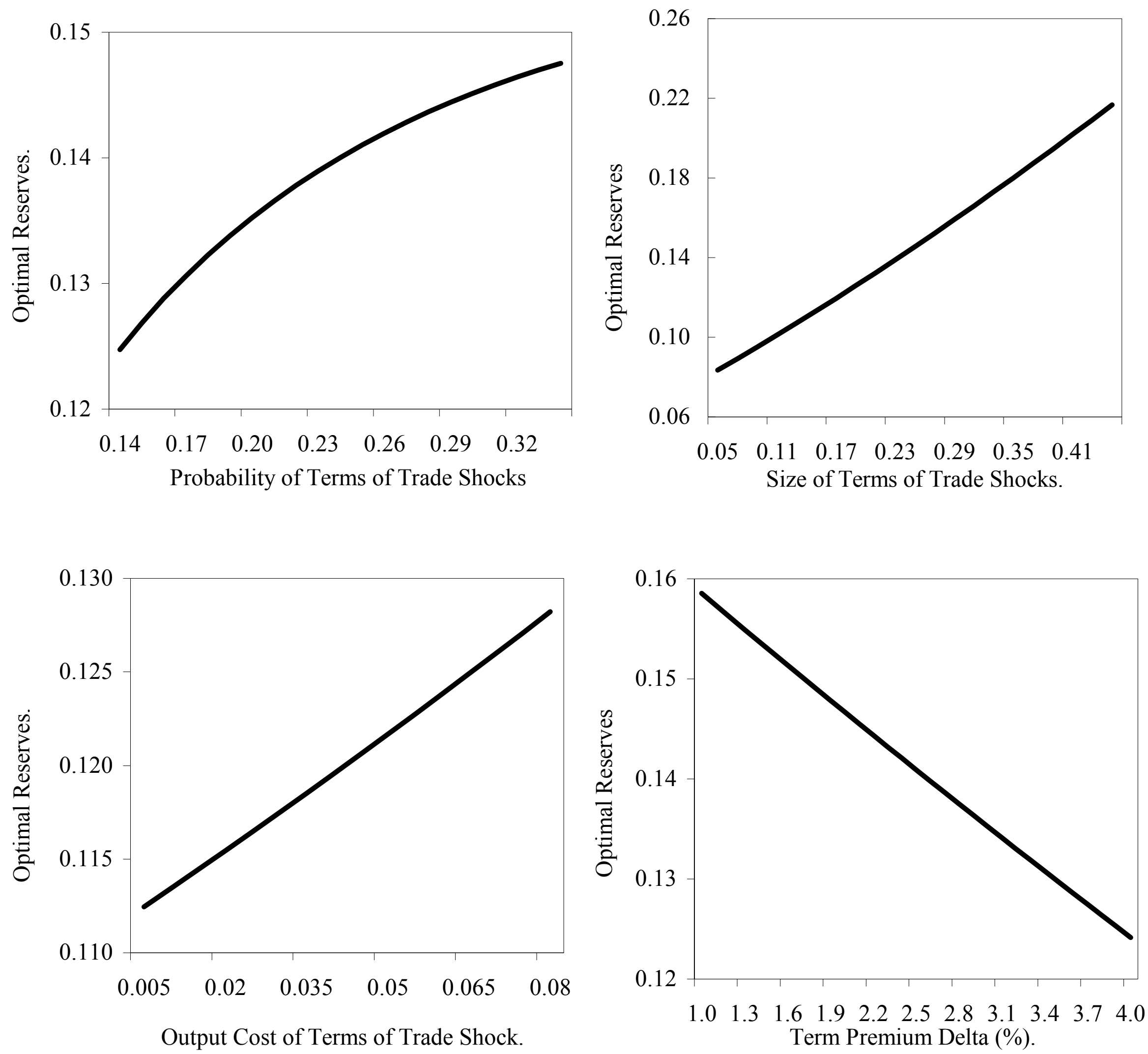


C. Adding Aid Shocks to the Framework

In this section we add the aid shocks to our Two-Good model and present the results for the full model. The two shocks in the model occur independently of each other. We use this assumption to make calculations simple since there is empirical evidence to show that the two shocks are uncorrelated (Dhasmana, 2007). Figures 10 and 11 show the results from the complete model. As before, the optimal level of reserves increases with the probability and size of terms of trade and aid shock. A higher probability of shock implies a higher expected loss in the utility due to the shock and therefore a higher optimal level of reserves. Similar argument holds for the case of size of shocks. Since the loss of consumption that a country will have to face in the event of a shock is directly related to the actual size of the shock, a country which is subject to large movements in its terms of trade or aid flows, should optimally hold a larger stock of reserves.

The optimal level of reserves also increases with the output cost associated with each shock. Along with the actual size of the shock, the output loss associated with each shock determines the fall in consumption and therefore, the loss in marginal utility, associated with it. Finally, increase in the term premium lowers the optimal level of reserves that the countries should hold. Just like any other form of insurance, the cheaper it is for a country to hold reserves as a cover against exogenous shocks, the greater is the optimal amount of reserves it should hold, other things remaining constant. In terms of our model, the term premium parameter determines the cost of carrying reserves in terms of reduced consumption before the shock. The higher the term premium, the bigger the cut in pre-shock consumption that the country has to take in order to finance higher consumption during the shock and therefore, lower the optimal level of reserves.

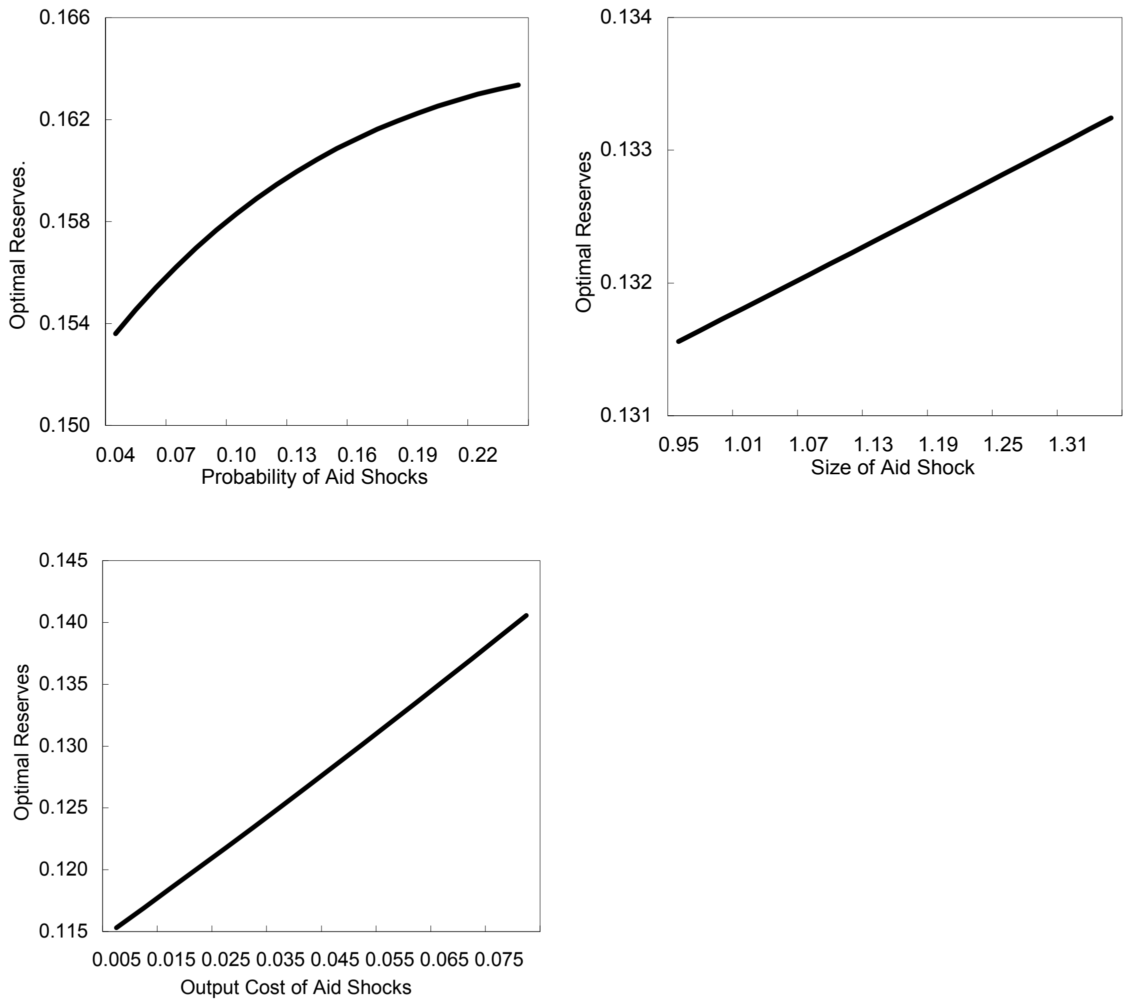
Figure 10. Optimal Reserve Behavior—Two Good Model with both TOT and Aid Shock [I][1]



[1]Optimal Reserves are expressed as a percentage of output.

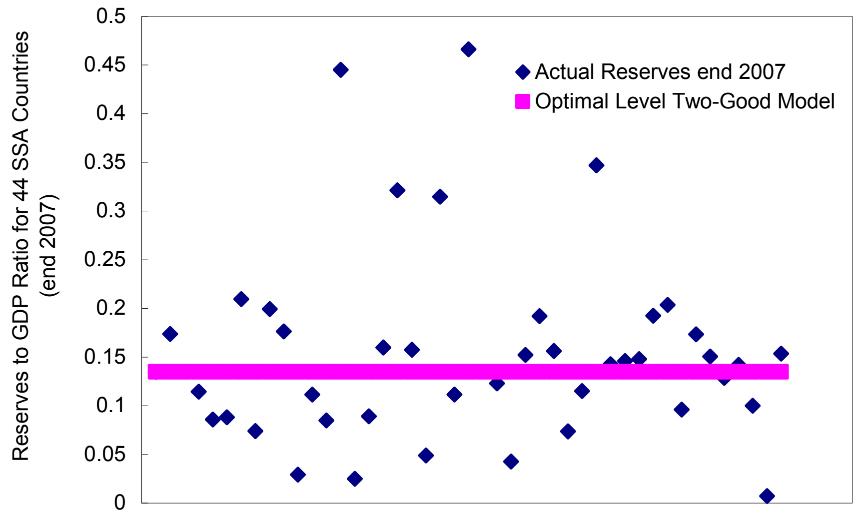
For the benchmark parameters, the optimal level of reserves given by the Two-Good model is equal to 11 to 12 percent of total output. Thus, according to the Two-Good model, a typical SSA country facing terms of trade and aid shock should hold foreign reserves equal to about 11-12 percent of it's G.D.P. Figure 12 plots the actual level of reserves to GDP ratio for SSA countries at the end of year 2007 along with the optimal. We can see that while many SSA countries had adequate or more than adequate reserves at the end of 2007 according to our model, there were a few whose reserve levels were significantly below the benchmark optimal given by the same model. This may be a cause of concern since SSA countries are often subject to multiple shocks at the same time and holding reserves significantly below the optimum can make them even more vulnerable to the possibility of economic crisis due to exogenous shocks.

Figure 11. Optimal Reserve Behavior—Two Good Model with both TOT and Aid Shock [II][1]



[1]Optimal Reserves are expressed as a percentage of output.

Figure 12. Actual Level of Reserves to GDP ratio for SSA countries

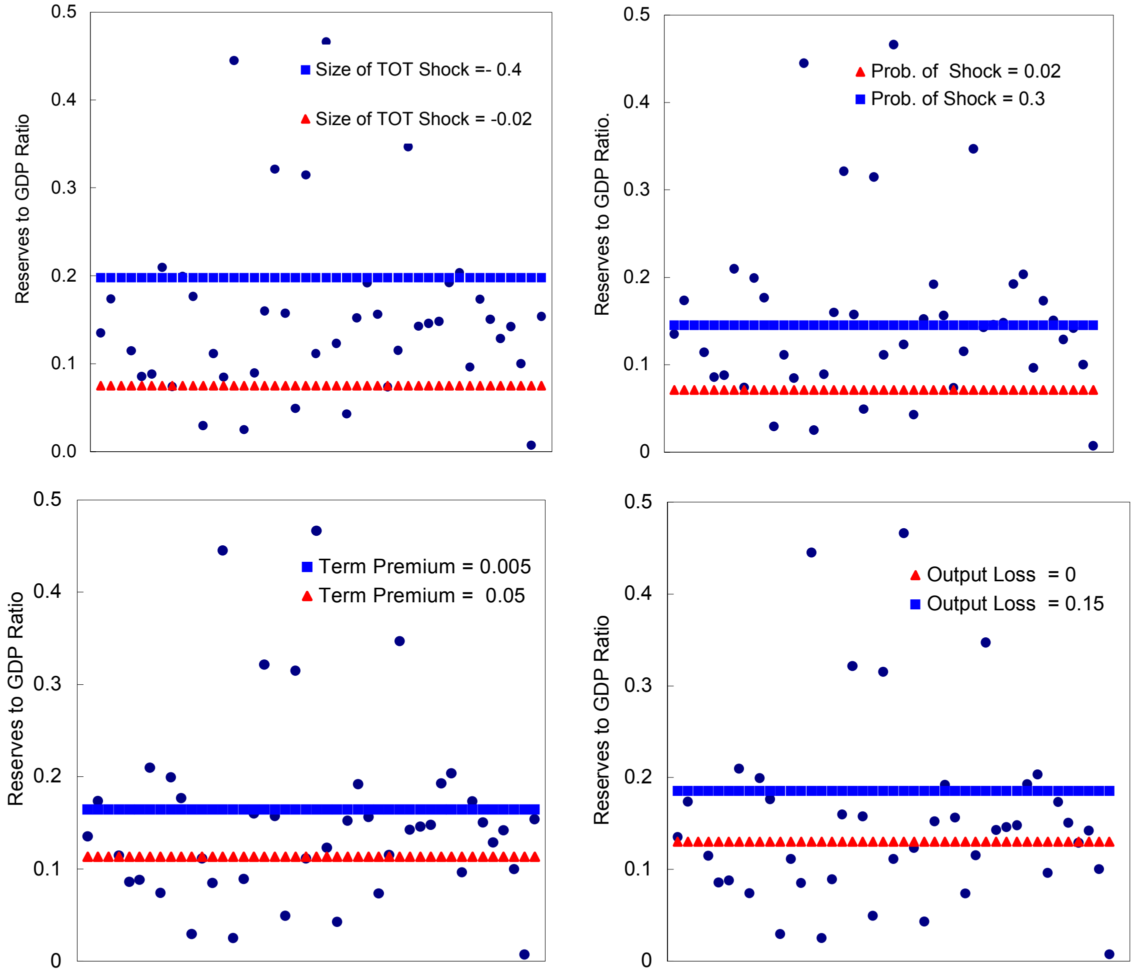


However, results based on average parameter values can be misleading if there is large variation across countries. We therefore present sensitivity analysis and some country specific results in the next two sections.

D. Sensitivity Analysis.

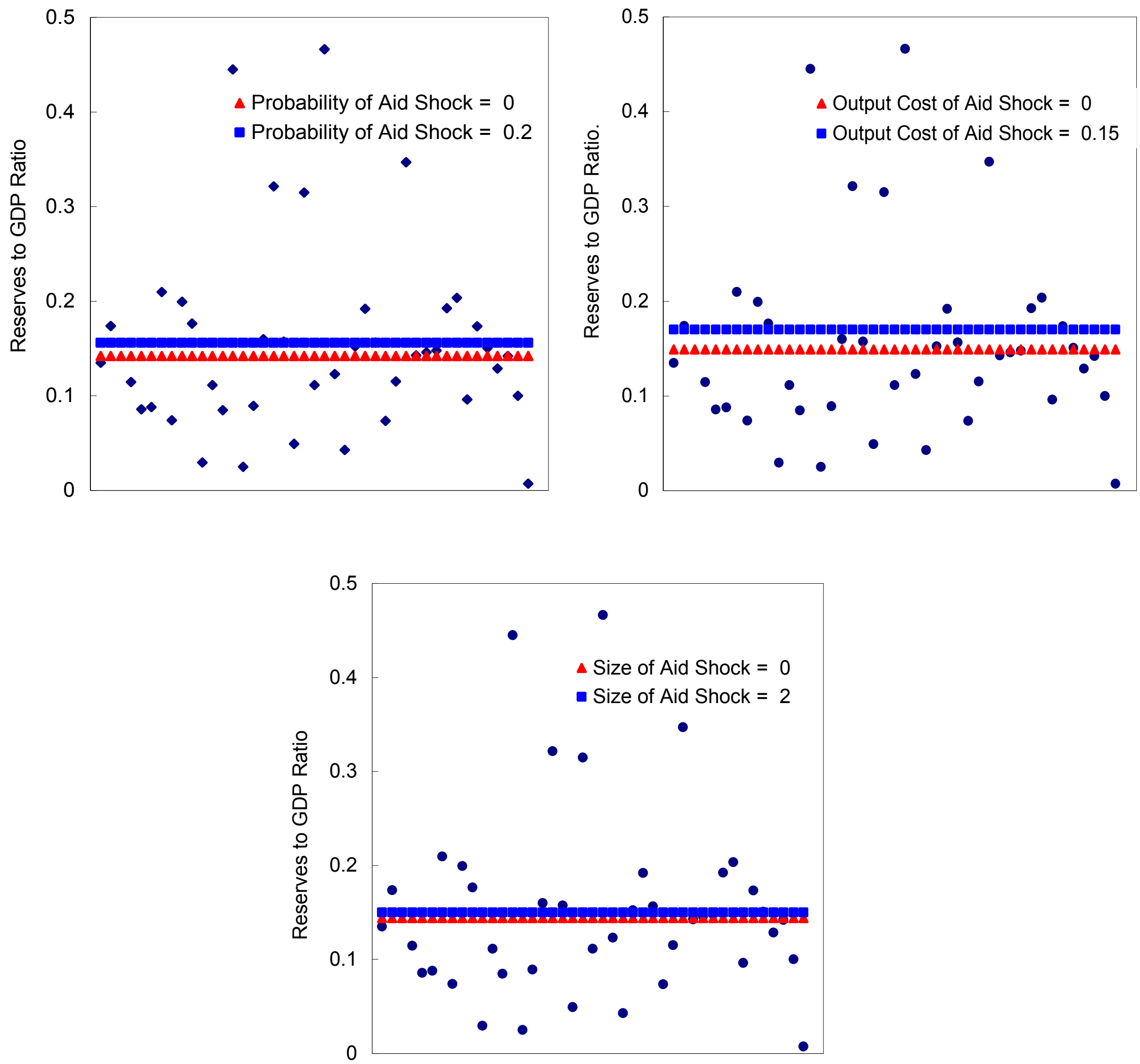
Figures 13 and 14 show the sensitivity of the level of optimal reserves to the choice of some key parameters. As in the figure above, we plot the actual reserve to GDP ratio for SSA countries at the end of year 2007 along with the optimal level determined by our model for alternative values of key parameters. The top column of Figure 11 shows the sensitivity of optimal reserves to the size and probability of TOT shocks. The red line in the top right hand figure of Figure 13 shows the optimal level of reserves if the average size of a terms of trade shock is 2 percent while the blue line above it shows the optimal level of reserves when the size of terms of trade shocks is 40 percent. Clearly, the choice of probability parameter makes a significant difference in terms of determining whether a country has adequate reserves or not. Same holds true for the size and output cost associated with a TOT shock and the size of term premium. Figure 14 presents similar exercise for the choice of aid shock parameters. Given that aid is only a small percentage of output, the results for optimal reserves are less sensitive to the choice of aid shock parameters. However, for countries with a much greater dependence on aid (either for consumption or for investment), the choice of optimal reserve level is likely to be more sensitive to the size, probability and output cost of aid shock.

Figure 13. Sensitivity of Optimal Reserves to Key Parameters[/1]



Sources: WEO and AFR Database.
[1]The Graphs above show the sensitivity of optimal reserves to the size and probability of TOT shocks, output cost associated with them and the term premium.

Figure 14. Sensitivity of Optimal Reserves to Key Parameters[1]



Sources: WEO and AFR Database
[1]The graphs above show the sensitivity of optimal reserves to the size and probability of Aid shocks and the output cost associated with them.

E. Country Specific Applications with Sensitivity Analysis

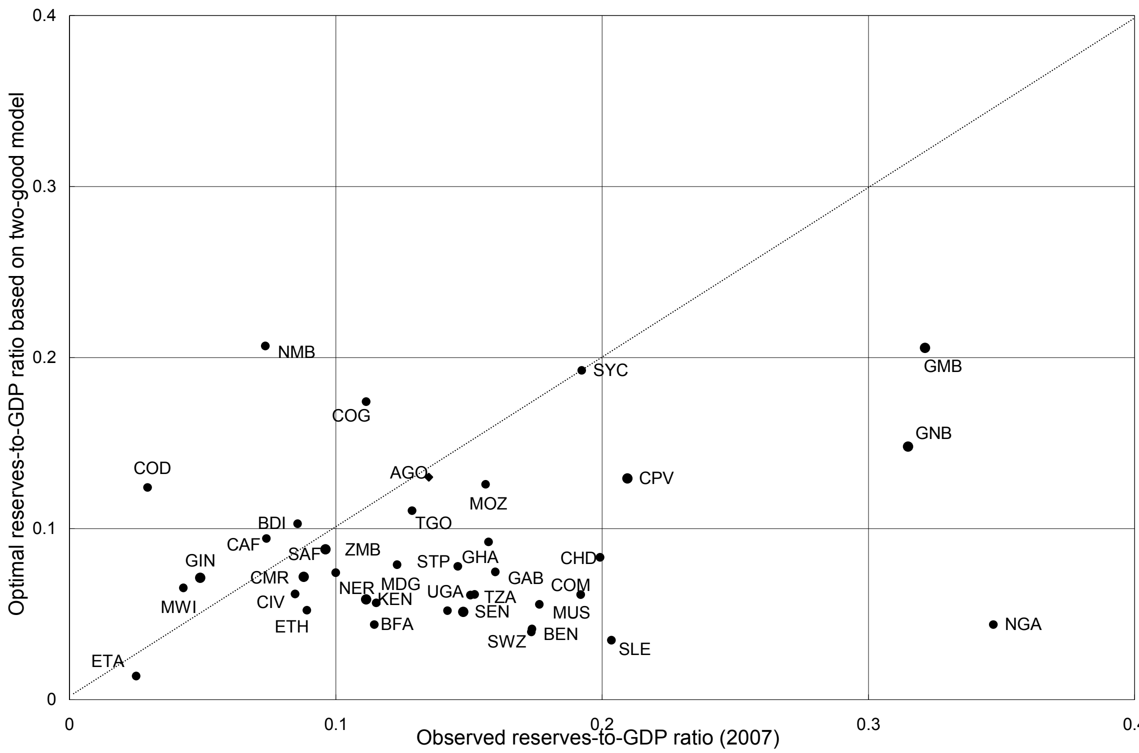
The results above show that the choice of key parameters can affect the level of reserves that would be optimal for a particular country. In this section we use some country specific parameters alongside a few common parameters to obtain country specific estimates of optimal reserves for the SSA countries. We use data from 1980-2006 to estimate the probability of terms of trade and aid `shocks' and the share of tradable goods in consumption ($\alpha$) for each of the 44 SSA countries. The probability of a `shock' is simply the number of shock events during 1980-2007 divided by the total number of years for each country whereas the share of tradable good is calculated by multiplying the share of consumption in domestic demand with the share of imports in total consumption. Term premium δ is set equal to 1.5 for all countries (same as that used for the emerging markets by Jeanne and Ranciere). Figure 15 below plots the level of actual reserves at the end of year 2007 against the optimal reserve level, as determined by our model, for SSA countries. Both, the actual and the optimal level of reserves are expressed as a ratio of GDP.

The broken blue line is the 45 degree line which identifies countries holding an actual reserve level equal to the optimal for them. Countries to the right of this blue line hold more reserves than the optimal level indicated by our model. This can be due to several reasons – an un-anticipated surge in price of oil or other major exports increasing the government revenues or domestic money supply (thereby forcing government to undertake sterilization operations) or excessive dependence on a non-renewable resource for export revenues (e.g. diamonds in Botswana) that is likely to be exhausted in foreseeable future.

Countries to the left of the 45 degree line are those carrying fewer reserves than suggested by our model. A few of these, such as Burundi and the Democratic Republic of Congo seem to have reserve levels that are way below the optimal. Deciding upon the actual reasons and remedies for inadequate reserve accumulation requires us to look at each country separately.

These results should be interpreted with caution. The simulations suggest that a few SSA countries do not currently carry reserves consistent with the expected output costs associated with expected terms-of-trade or aid shocks. But while the use of a small open economy two-goods model allows us to simulate the optimal level of reserves across a broad spectrum of shocks and output costs, the “optimal level” of reserves is sensitive to the choice of key parameters such as the risk aversion, the term premium and the probability of shocks, and results in figure 15, which are illustrative, yield the results of model simulations for a given set of parameters.

Figure 15. Reserve Adequacy for African Countries Using Two-Good Model



Sources: WEO and AFR database

Figure 16 below illustrates the application of our model for assessing reserve behavior for individual countries, illustrated by a set of two countries. It plots the actual and `optimal' level of reserves along with the 'reserve gap'—all expressed as a percentage of GDP, for Angola and Congo, D.R.for the years 2000-07.

The reserve gap is defined as the difference between optimal level of reserves, as suggested by our model and the actual level of reserves. We use a combination of country specific and common parameters to simulate optimal level of reserves for each country over time. In particular, the probability of aid and terms of trade shocks, ratio of short term debt to GDP and share of imports in consumption are country specific while the other parameters are common across countries. Of the country specific parameters, share of short term debt in GDP and that of imports in consumption are allowed to vary across time whereas the shock probabilities remain constant.

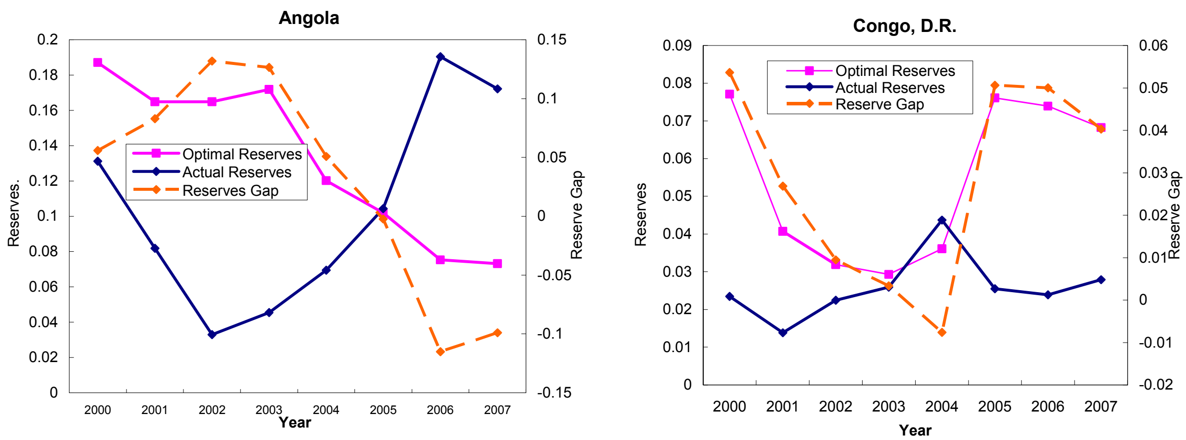
Having thus calculated the level of optimal reserve as a share of output for each country we subtract the actual level of reserves, also expressed as a share of output., to obtain the reserve gap. The following main results emerge from this exercise:

- The decline in the reserve gap is most significant for **Angola,** where the optimal level of reserves has declined in line with falling short term debt and the actual level of reserves has increased owing to a windfall of oil revenues in recent years. As a result Angola has had more reserves than suggested by the two good model for the last two years, partly due to uncertainty about future oil revenues.

- **Congo, D.R.** maintained a roughly constant level of reserve to GDP ratio (around 2.5 percent) but saw an increase in the optimal level of reserves, with the reserve gap increasing by some 4 percent of GDP over the last 8 years.

To summarize, assessing the adequacy of reserves held by a country requires us to look in to country specific factors affecting the reserve accumulation behavior. The two-good model, with exogenous terms of trade and aid shocks is a good starting point in this direction which can be built upon by using more country specific information on factors affecting reserve accumulation.

Figure 16. Country Specific Application—Illustrative Examples



Source: WEO

VI. Conclusion

Sustaining adequate level of reserves is a key policy consideration for SSA countries. These countries continue to face risks arising from abrupt changes in the terms of trade and aid flows, which contribute to a higher volatility in aggregate output and, in extreme cases, to economic crisis. In this setting, maintaining adequate level of foreign reserves can be an important element in helping to reduce macroeconomic volatility, particularly so if there are no alternative sources of financing.

The consumption smoothing role of reserves is particularly important in SSA countries as suggested by the preliminary empirical evidence presented in the paper. Countries with very low level of reserves are more strongly affected by exogenous shocks than the others. Assessing adequacy should therefore be informed by a country's vulnerability to shocks, the magnitude of shocks, and the opportunity cost of holding reserves. Our model is a first step in the direction of developing a framework for reserve adequacy in SSA which takes in to account this consumption smoothing role of reserves and compares the insurance value of foreign reserves in the event of exogenous shocks against their `carrying costs'.

The use of a small open economy two-goods model allows us to simulate the optimal level of reserves across a broad spectrum of shocks and output costs, against which one can contrast actual holdings of SSA countries. It is clear that the "optimal level" of reserves is sensitive to the choice of key parameters such as the term premium and probability of shocks and therefore results based on average values of parameters can be misleading. We therefore use available country-specific information to obtain optimal reserve levels for SSA countries. The simulations suggest that a few SSA countries do not currently carry reserves consistent with the expected output costs associated with expected terms-of-trade or aid shocks.

The Two-Good model provides us with a benchmark against which we can compare the actual reserve holdings of a country. To fully understand the reserve accumulation behavior of a country, however, one would also need to take into account other factors such as, for instance, the nature of its exchange rate regime, the degree of monetization. There is inevitably no one-size-fits-all level of reserves for all SSA countries. The actual choice of reserve levels to hold depends on a number of factors and should therefore be studied within the context of overall macroeconomic policy framework in a country.

A few interesting extensions of the paper would include relaxing the restriction of no-saving-in-terms-of non-tradable good and introducing capital accumulation and production in the economy. This simple model can also be extended to incorporate other shocks besides terms of trade and aid shocks.

Appendix

Equation 1.20, 1.23 and 1.24 can be written as,

$$\frac{\delta+\pi_s}{1-\delta-\pi_s}=\frac{\begin{bmatrix}(\pi_{TOT}-\pi_{TOT}\times\pi_{Aid})\times\left(\left(C_t^{TOT}\right)^{-\sigma}\left(C_{t,TOT}^T\right)^{\alpha-1}\left(C_{t,TOT}^N\right)^{1-\alpha}\right)\\+(\pi_{Aid}-\pi_{TOT}\times\pi_{Aid})\times\left(\left(C_t^{Aid}\right)^{-\sigma}\left(C_{t,Aid}^T\right)^{\alpha-1}\left(C_{t,Aid}^N\right)^{1-\alpha}\right)\\+\pi_{TOT}\times\pi_{Aid}\times\left(\left(C_t^{TOT,Aid}\right)^{-\sigma}\left(C_{t,TOT,Aid}^T\right)^{\alpha-1}\left(C_{t,TOT,Aid}^N\right)^{1-\alpha}\right)\end{bmatrix}}{(1-\pi_s)\times\left(\left(C_t^b\right)^{-\sigma}\left(C_{t,b}^T\right)^{\alpha-1}\left(C_{t,b}^N\right)^{1-\alpha}\right)}$$

Using the expression for consumption aggregator in 1.22 we can write,

$$\frac{\delta+\pi_s}{1-\delta-\pi_s}=\frac{\begin{bmatrix}(\pi_{TOT}-\pi_{TOT}\times\pi_{Aid})\times\left(\left(C_{t,TOT}^T\right)^{\{(1-\sigma)\alpha-1\}}\left(C_{t,d}^N\right)^{(1-\alpha)(1-\sigma)}\right)\\+(\pi_{Aid}-\pi_{TOT}\times\pi_{Aid})\times\left(\left(C_{t,Aid}^T\right)^{\{(1-\sigma)\alpha-1\}}\left(C_{t,d}^N\right)^{(1-\alpha)(1-\sigma)}\right)\\+\pi_{TOT}\times\pi_{Aid}\times\left(\left(C_{t,TOT,Aid}^T\right)^{\{(1-\sigma)\alpha-1\}}\left(C_{t,d}^N\right)^{(1-\alpha)(1-\sigma)}\right)\end{bmatrix}}{(1-\pi_s)\times\left(\left(C_{t,b}^T\right)^{\{(1-\sigma)\alpha-1\}}\left(C_{t,b}^N\right)^{(1-\alpha)(1-\sigma)}\right)}$$

Further $\frac{C_{t,d}^N}{C_{t,b}^N}=(1-\gamma)$ so that,

$$\frac{\delta+\pi_s}{1-\delta-\pi_s}\times\frac{1}{(1-\gamma)^{(1-\alpha)(1-\sigma)}}=\frac{\begin{bmatrix}(\pi_{TOT}-\pi_{TOT}\times\pi_{Aid})\times\left(C_{t,TOT}^T\right)^{\{(1-\sigma)\alpha-1\}}\\+(\pi_{Aid}-\pi_{TOT}\times\pi_{Aid})\times\left(C_{t,Aid}^T\right)^{\{(1-\sigma)\alpha-1\}}\\+\pi_{TOT}\times\pi_{Aid}\times\left(C_{t,TOT,Aid}^T\right)^{\{(1-\sigma)\alpha-1\}}\end{bmatrix}}{(1-\pi_s)\times\left(C_{t,b}^T\right)^{\{(1-\sigma)\alpha-1\}}}$$

In the special case when there are no aid shocks we can rewrite the above equations using the expressions for tradable consumption given in equations 1.12-1.16 ,

$$\frac{\delta+\pi_{TOT}}{1-\delta-\pi_{TOT}}\times\frac{1}{(1-\gamma)^{(1-\alpha)(1-\sigma)}}=\frac{\left[\pi_{TOT}\times\left((1-\gamma_{TOT})\times T\times Y_{t,d}^T+(1-\delta-\pi_{TOT})R_{t-1}+A_t\right)^{\{(1-\sigma)\alpha-1\}}\right]}{(1-\pi_{TOT})\times\left(T\times Y_{t,b}^T-(\delta+\pi_{TOT})R_{t-1}+A_t\right)^{\{(1-\sigma)\alpha-1\}}}$$

Solving this expression for R_{t-1} we can get the optimal level of reserves as a function of pre-shock tradable output and aid flow,

$$R_{t-1} = k_1 \times Y_{t,b}^{T} + k_2 \times A_t$$

Where ,

$$k_1 = \left[\frac{\left(Cons - (1-\gamma_{TOT}) \times (1-\gamma)\right)}{Cons \times (\delta + \pi_{TOT}) + (1 - \delta - \pi_{TOT})} \right],$$

$$k_2 = \frac{Cons - 1}{Cons \times (\delta + \pi_{TOT}) + (1 - \delta - \pi_{TOT})},$$

And,

$$Cons = \left(\frac{\delta + \pi_{TOT}}{1 - \delta - \pi_{TOT}} \times \frac{1}{(1-\gamma)^{(1-\alpha)(1-\sigma)}} \times \frac{1 - \pi_{TOT}}{\pi_{TOT}} \right)^{\frac{1}{(1-\sigma)\alpha - 1}}$$

Table A1. Benchmark Parameters

Parameters for Terms of Trade Shock	**Benchmark Value**
Size of the Shock, γ_{TOT}	0.219
Output loss due to the TOT shock, γ_1	0.015
Coefficient of Risk Aversion, σ	2
Share of Tradable, α	0.5
Probability of TOT Shock, π_{TOT}	0.209
Term Premium, δ	0.015
Potential Output Growth, g	0.05
Risk free Rate of Return, r	0.05
Short Term Debt as a Share of Output, λ	0.204
Aid as a share of GDP	0.04
Size of Aid Shock, γ_{Aid}	1.81
Output loss due to Aid shock, γ_2	0.015
Probability of Aid Shock, π_{Aid}	0.10

Table A2. Simulation Parameters for Countries

	Probability TOT Shock	Probability Aid Shock	Share of Import in Consumption	Share of Short-Term Debt in Tradables
Angola	39.3	7.1	56.86	0.037
Benin	28.6	7.1	20.54	0.027
Botswana	25.0	21.4	37.73	0.350
Burkina Faso	25.0	3.6	22.16	0.021
Burundi	35.7	3.6	32.22	0.120
Cameroon	25.0	3.6	27.56	0.078
Cape Verde	28.6	3.6	41.55	0.118
Central African Rep.	17.9	3.6	20.74	0.279
Chad	17.9	3.6	50.00	0.000
Comoros	35.7	14.3	23.91	0.063
Congo, Dem. Rep. of	10.7	7.1	48.05	0.126
Congo, Republic of	17.9	21.4	101.79	0.032
Côte d'Ivoire	14.3	7.1	43.65	0.000
Equatorial Guinea	32.1	14.3	88.28	0.001
Eritrea	3.6	3.6	27.81	0.029
Ethiopia	28.6	7.1	27.01	0.019
Gabon	10.7	32.1	49.83	0.048
Gambia, The	17.9	10.7	44.83	0.293
Ghana	21.4	7.1	48.79	0.022
Guinea	17.9	10.7	30.82	0.083
Guinea-Bissau	21.4	14.3	41.59	0.194
Kenya	17.9	3.6	30.10	0.032
Lesotho	3.6	7.1	70.11	0.024
Madagascar	35.7	7.1	41.22	0.007
Malawi	25.0	7.1	33.45	0.025
Mali	17.9	3.6	31.59	0.032
Mauritius	3.6	28.6	64.69	0.022
Mozambique	14.3	7.1	41.76	0.151
Namibia	25.0	10.7	47.73	0.253
Niger	28.6	3.6	27.90	0.019
Nigeria	25.0	32.1	32.65	0.008
Rwanda	39.3	3.6	24.62	0.011
São Tomé & Príncipe	32.1	10.7	45.00	0.000
Senegal	10.7	7.1	36.37	0.019
Seychelles	39.3	25.0	117.34	0.074
Sierra Leone	3.6	14.3	26.38	0.074
South Africa	3.6	3.6	33.70	0.221
Swaziland	0.0	39.3	70.58	0.000
Tanzania	32.1	3.6	31.95	0.003
Togo	21.4	0.0	51.92	0.023
Uganda	35.7	7.1	26.80	0.012
Zambia	32.1	7.1	39.48	0.007
Average	21.5	10.4	42.51	0.094

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