Practical Volatility Modeling for Financial Market Risk Management

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Abstract

Being able to choose most suitable volatility model and distribution specification is a more demanding task. This paper introduce an analyzing procedure using the Kullback-Leibler information criteria (KLIC) as a statistical tool to evaluate and compare the predictive abilities of possibly misspecified density forecast models. The main advantage of this statistical tool is that we use the censored likelihood functions to compute the tail minimum of the KLIC, to compare the performance of a density forecast models in the tails. We include an illustrative simulation and an empirical application to compare a set of distributions, including symmetric/asymmetric distribution, and a family of GARCH volatility models. We highlight the use of our approach to a daily index, the Kuala Lumpur Composite index (KLCI).

Our results shows that the choice of the conditional distribution appear to be a more dominant factor in determining the adequacy of density forecasts than the choice of volatility model. Furthermore, the results support the Skewed for KLCI return distribution.

Keywords Density forecast, Conditional distribution, Forecast accuracy, KLIC, GARCH models

Paper type Research paper

1. Introduction

In recent years, there has been increasing concern among researchers, practitioners and regulators over how to evaluate models of financial risk. It is observed that the research on evaluating density forecast models has been very versatile since the seminal paper of Diebold *et al.*(1998a) and the key device is the probability integral transform (PIT), which has a long history. The literature usually cites (Rosenblatt, 1952), for the basic result, and the approach features in several expositions from different points of view. For instance, Diebold *et al.*(1998a;1998b), Diebold *et al.*(1999), (Clements and Smith, 2000;2002) and (Berkowitz, 2001) have applied this transformation to evaluating density forecasts. (Bai, 2003), (Corradi and Swanson, 2005) and Bao *et al.*(2006) have applied it to testing the parametric specification of conditional distributions of dynamic models.

Let $\{Y_t\}_{t=1}^{n}$denote a time series and I_{t-1} represent the information set at time $t-1$. Let $F_t(./I_{t-1})$ be the forecast distribution of Y_t given the information I_{t-1}. Deibold et al. (1998a) shows that the transformed variables (PIT), $Z_t = F_t(Y_t/I_{t-1})$, $t = 1, \dots, n$ are $iid\ U(0,1)$ if and only if the forecasts are correct. At the evaluation stage they, suggested that visual assessment of the PIT series is useful, a histogram of the series is generally used because of the ease of verification of the requirement that PIT is uniform over the unit interval. Deibold et al. (1998a) method is very convenient and useful for financial risk management, as well as for macroeconomic forecasting because it transforms the problem of evaluating the conditional density into the problem of testing the properties of PIT. As an example, Diebold applied this approach to the Survey of Professional Forecasters. (Hong, 2001) develop a joint test for uniformity and serial independence by comparing a kernel estimate of the joint density function of Z_{t-1} and Z_t with the uniform density on the unit square. The test is limited to standard normal distribution, regardless of the fact of the data true distribution.

Berkowitz (2001) argued that consistent nonparametric tests typically require the availability of large data sets to achieve accurate inference. He advocates simple parametric tests and extend Diebold *et al.*(1998a) framework by utilizing a second transformation that implies data normality, if a sequence of density forecast is correct. Berkowitz (2001) first transforms the $PITs$ to $Z_t^* = \Phi^{-1} Z_t$ in which $\Phi(.)$ is the distribution function of the standard normal distribution, and then imposes a linear AR structure on the Z_t^*'s. Thompson (2002) provides theoretical justification for the graphical procedures used in papers such as Diebold *et al.*(1998a) by describing a family of specification tests for uniformity and serial independence based on the empirical distribution function and/or the sample periodogram. Likewise, (Bai, 2003) proposes a kolmogorov type test based on the comparison of the empirical distribution function and the cumulative distribution function(CDF). As a consequence of using estimated parameters, the limiting distribution of the test reflects the contribution of parameter estimation error and is not nuisance parameter free. To overcome this problem, Bai (2003) uses a novel approach based on a martingalization argument to construct a modified Kolmogorov test which has a nuisance parameter free limiting distribution. This test found to have power against violations of uniformity but not against violations of independence. While (Corradi and Swanson, 2005), their approach is to compare the CDF of density forecast model to the empirical distribution. The distance is measured by the mean square error of the CDF and the empirical distribution function, integrated out over different quantiles of the CDF.

Despite the burgeoning interest in and evaluation of volatility forecasts, a clear consensus on which distribution and/or volatility model specification to use has not yet been reached even for finance practitioners and risk professionals. However there has been much less effort in comparing alternative density forecast models. Considering the recent empirical evidence on volatility clustering and asymmetry and heavy-tailed in financial return series, we believe that using a formal test, in the context of density forecasts, will contribute to the existing literature (Tay and Wallis, 2000).

Therefore the main aim of this paper is to able risk managers and economist to choose the most suitable volatility model and distribution specifications, by a rigorous density forecast comparison methodology. We utilize the Kullback-Leibler Information Criterion ($KLIC$) as a unified test of evaluate, compare and to assess which volatility model and/or distribution are statistically more appropriate to mimic the time series behavior of a return series. This generality follows from appreciation, that the (Berkowitz, 2001) Likelihood Ratio (LR) test can be related to the $KLIC$ (Bao *et al.,* 2006), a well-respected measure of "distance" between two densities.

The structure of the remainder of this paper is as follows. We review the statistical evaluation of individual density forecasts using the $PITs$ in section 2 and develop the distance measure based on the $KLIC$ and LR test for candidate models as well as region test in section 3 and 4 respectively. Hypotheses testing and model comparison are discussed in section 5. Section 6 shows how the $KLIC$ can be used to compare statistically the accuracy of two competing density forecasts applied to simulated data. Section 7 contains an application to empirical data. Section 8 concludes the paper.

2. Probability Integral Transform

For a sample of n one-step-ahead forecasts and the corresponding outcomes, the probability integral transform of the realized variables with respect to the forecast densities is defined as

$$\begin{aligned} z_t &= \int_{-\infty}^{x_t} f_t(u)du \\ &= F_t(x_t)\,; \qquad t = R+1, \dots, T \end{aligned} \tag{1}$$

Let $g_t(x_t)$ be the true density of x_t, and let $f_t(x_t)$ be a density forecast of x_t, and let z_t be the probability integral transform of x_t with respect to $f_t(x_t)$. It is well known that if $f_t(.)$ coincides with the true density $g_t(x_t)$, then the sequence $\{z_t\}_{t=1}$ are $iid\ U$ $[0,1]$. If the transformed time series $\{z_t\}$ is not $iid\ U[0,1]$, then $f_t(.)$ is not an optimal density forecast model (Diebold *et al.*,1999). This can be proofed by assuming that $\partial F_t^{-1}(z_t)/\partial z_t$ is continuous and non-zero over the support of x_t, z_t has unit interval with density;

$$\begin{aligned} q_t(z_t) &= \left|\frac{\partial F_t^{-1}(z_t)}{\partial z_t}\right| g_t\big(F_t^{-1}(z_t)\big) \\ &= \frac{g_t\big(F_t^{-1}(z_t)\big)}{f_t\big(F_t^{-1}(z_t)\big)} \end{aligned}$$

Where $f_t(x_t) = \partial F_t^{-1}(x_t)/\partial x_t$ and $x_t = F_t^{-1}(z_t)$. Therefore, in particular, a key fact; if $f_t(x_t) = g_t(x_t)$, then $z_t \in (0,1)$ and $q_t(z_t)$ is simply the $U(0,1)$ density. This idea dates at least to Rosenblatt (1952). Therefore, a natural test of optimality of a density forecast model is to test the $iid\ U[1,0]$ properties of the series$\{z_t\}$.

Since our objective, is to compare the out-of-sample predictive abilities among competing density forecast models. Suppose that, there are $l+1$ models $(k = 0,1,\dots,l)$ in a set of competing models, possibly misspecified. Let the density forecast model k $(k = 0,1,\dots,l)$ be denoted by $f_{x,t}(x)$. We dived the whole sample into two sub-samples $\{Z_t\}_{t=1}^{R}$ and $\{Z_t\}_{t=R+1}^{T}$, the first sample to estimate the unknown parameters and the second sub-sample to check if the corresponding outcomes, the probability integral transform of the realized variables with respect to the forecast densities ($PITs$) are $iid\ N(0,1)$. That is, we first construct

$$\begin{aligned} z_{k,t} &= \int_{-\infty}^{x_t} f_{k,t}(u)du \\ &= F_{k,t}(x_t)\,; \qquad t = R+1, \dots, T \end{aligned} \tag{2}$$

Taking the inverse normal transform of the PIT is

$$Z_{k,t}^{*} = \Phi^{-1} z_{k,t} \tag{3}$$

and $\Phi(.)$ is the CDF of the standard normal. In other words, testing the departure of $\left\{Z_{k,t}^{*}\right\}_{t=1}^{T}$ from $iid\ N(0,1)$ is equivalent to testing the distance of the forecasted density from the true –unknown- density.

3. Distance Measure

The test for adequacy of a postulated distribution may be appropriately measured by Kullback Information Criterion (Kullback and Leibler, 1951) divergence measure between two conditional densities, $D(g;f) =$

$E[lng_t(x_t) - lnf_t(x_t)]$, where the expectation is with respect to the true distribution. Following (Vuong, 1989), we define the distance between a model and the true density as the minimum of $KLIC$

$$D_{KLIC}(g; f) = \int g_t(x_t) ln \left\{ \frac{g_t(x_t)}{f_t(x_t)} \right\} dx \quad \text{or} \tag{4}$$

$$D_{KLIC}(g; f) = E[lng_t(x_t) - lnf_t(x_t)] \tag{5}$$

The smaller $D(g; f)$, the closer the density forecast is to the true density; $D(g; f) = 0$ if and only if $f_t(x_t) = g_t(x_t)$. However, $D(g; f)$ is generally unknown, since we cannot observe $g_t(.)$ and hence the expectation, it can be consistently estimated by

$$D_{KLIC}(g; f) = \frac{1}{T} \sum_{t=1}^{T} E[lng_t(x_t) - lnf_t(x_t)] \tag{6}$$

But we still do not know $g(.)$. Moreover, and importantly, the true density $g(.)$ may exhibit structural change, as indicated by its time subscript. For this, we utilize the LR test statistics and the probability integral transform (PIT) of the actual realizations of the process with respect to the model's density forecast and hence to compare possibly misspecified models in terms of their distance to the true model.

4. Relating LR Test to the $KLIC$

Re-interpreting Bekowitz (2001) LR test as test of whether the $KLIC$ "distance" between the true density and the forecast density equals zero. Note the following equivalence (Berkowitz, 2001):

$$ln\left[g_t(x_t)/f_{k,t}(x_t)\right] = ln\left[p_t\left(z_{k,t}^*\right)/\phi\left(z_{k,t}^*\right)\right] \tag{7}$$

where $p_t(.)$ is the unknown density of $z_{k,t}^*$, $\phi(.)$ is the standard normal density. In other words, testing the departure of $\left\{Z_{k,t}^*\right\}_{t=1}^{T}$from $iid\ N(0,1)$ is equivalent to testing the distance of the forecasted density from the true –unknown- density $g_t(x_t)$. Testing whether $p(.)$is $iid\ N(0,1)$ is both more convenient and more sensible than testing the distance between $g_t(x_t)$ and $f_{k,t}(x_t)$ since we do not know $g_t(x_t)$. The test statistics D_{KLIC} is proportional to the LR test of Berkowitz (2001), assuming normality of ε_t. Specifying $\left\{z_{k,t}^*\right\}_{t=1}^{T}$ as an $AR(1)$ process

$$z_{k,t}^* = \rho Z_{t-1}^* + \varepsilon_t \tag{8}$$

where $Var(\varepsilon_t) = \sigma^2$, ρ is a vector of parameters, and ε_t is iid distributed. In Berkowitz (2001), $\varepsilon_t$is assumed to be normally distributed. Actually, if we specify $p(.)$such as iid and normal, then our comparison based on the distance measure D_{KLIC}will suffer the same criticism of the LR test of Berkowitz, as pointed out by (Clements and Smith, 2000; Bao *et al*., 2006). A remedy to such criticism is to consider more general forms for $p_t\left(z_{k,t}^*\right)$. Bao *et al.(*2006) suggested the use of the seminonparametric (SNP) density of (Gallant and Nychka, 1987) for ε_t. in the AR process of the order K

$$p_t(\varepsilon_t; \theta) = \frac{[\sum_{k=0}^{K} R_K \varepsilon_t^K]^2 \phi(\varepsilon_t)}{\int_{-\infty}^{+\infty} [\sum_{k=0}^{K} R_K z_t^K]^2 \phi(z_t) dz} \tag{9}$$

A change of variables using the location-scale transformation, $y = R\epsilon + \mu$, where R is an upper triangular matrix and μ is an M-vector. The change of variable formula applied to the location-scale transformation, the density of $z_{k,t}^*$is

$$p_t\left(z_{k,t}^*\right) = \frac{p_t\left[\left(z_{k,t}^* - \rho Z_{t-1}^*\right)/\sigma\right]}{\sigma} \tag{10}$$

thus, the estimated minimum $KLIC$ divergence measure is

$$D_{KLIC} = \frac{1}{T} \sum_{t=1}^{T} \left[ln\left(\frac{p_t\left[\left(z_{k,t}^* - \rho Z_{t-1}^*\right)/\sigma\right]}{\sigma} \right) - ln\phi\left(z_{k,t}^*\right) \right] \tag{11}$$

The LR test statistics of the adequacy of the density forecast model $f_{k,t}(.)$ in (Berkowitz 2001) is simply the above formula with $p(.) = \phi(.)$.

Rather than evaluating the performance of the whole density we can also evaluate in any regions of particular interest. Risk managers and other practitioner in finance care more about the extreme values in the lower tail (larger loss) than about the values in other regions of the distribution (small loss/gain). Therefore, a density forecast model that accurately predicts tail events, is of more interest in finance. For a complete evaluation of these forecasts, we need to integrate this approach with testing procedures applicable to the tails of the distribution. To do so, D_{KLIC}distance measure can be easily modified for the tail parts. We focus on the lower tails only. Therefore, we define

$$p_{k,t}^{\tau}\left(z_{k,t}^{*\tau}\right) = \begin{cases} \Phi^{-1}(\alpha) \equiv \tau & if\ z_{k,t}^{*} \geq \tau \\ z_{k,t}^{*\tau} & if\ z_{k,t}^{*} < \tau \end{cases} \tag{12}$$

Let $1(.)$ denote and indicator function that takes 1 if its argument is true and 0 otherwise, the distribution function for $z_{k,t}^{*\tau}$ can be constructed as

$$p_{k,t}^{\tau}\left(z_{k,t}^{*\tau}\right) = \left[1 - p\left(\frac{\tau - \rho Z_{t-1}^{*}}{\sigma}\right)\right]^{1\left(z_{k,t}^{*\tau} \geq \tau\right)} \left[\frac{p_t\left[\left(z_{k,t}^{*} - \rho Z_{t-1}^{*}\right)/\sigma\right]}{\sigma}\right]^{1\left(z_{k,t}^{*\tau} < \tau\right)} \tag{13}$$

Therefore, the teal minimum D_{KLIC} divergence can be estimated analogously

$$D_{KLIC}^{\tau} = \frac{1}{T}\sum_{t=1}^{T}\left[lnp_{k,t}^{\tau}\left(z_{k,t}^{*\tau}\right) - ln\phi^{\tau}\left(z_{k,t}^{*\tau}\right)\right] \tag{14}$$

where $\phi^{\tau}\left(z_{k,t}^{*\tau}\right) = [1 - \Phi(\tau)]^{1(z_{k,t}^{*} \geq \tau)}\left[\phi\left(z_{k,t}^{*}\right)\right]^{1(z_{k,t}^{*} < \tau)}$

5. Model Comparison

Model comparison between a benchmark model: 0; and competing models: $k = 1, \ldots, l$ can be conveniently formulated by exploiting the framework of West (1996) and White (2000). To test the null hypothesis that $g_t(x_t) = f_{k,t}(x_t)$, Consider the loss differential

$$d_t = \left[lng_t(x_t) - lnf_{k,t}(x_t)\right] = \left[lnp_t\left(z_{k,t}^{*}\right) - ln\phi\left(z_{k,t}^{*}\right)\right] \tag{15}$$

the null hypothesis of the density forecast being correctly specified is then

$$H_0 = E(d_t) = 0 \Rightarrow D_{KLIC} = 0 \tag{16}$$

Pairwise comparison: model k is no better than the benchmark model (model 0), the null hypothesis is

$$H_1 = E\left(d_{k,t}\right) \leq 0 \tag{17}$$

Multiple comparisons: can any one of the competing models beat the benchmark model?

$$H_2 = \max_{1 \leq k \leq l} E\left(d_{k,t}\right) \leq 0 \tag{18}$$

The sample mean $\bar{d}$ is defined as:

$$\bar{d} = D_{KLIC} = \frac{1}{T}\sum_{t=1}^{T}\left[ln\, p_t\left(z_{k,t}^{*}\right) - ln\phi\left(z_{k,t}^{*}\right)\right] \tag{19}$$

To test the hypothesis about $\bar{d}$ by a suitable central limit theorem we have the limiting distribution $\sqrt{T}\left(\bar{d} - \mathrm{E}(\mathrm{d_t})\right) \rightarrow N(0, \Omega)$ where in general expression the covariance matrix Ω is rather complicated because it allow for parameter uncertainty (West 1996). However, ignoring parameter uncertainty (which asymptotically we can as the sample size used to estimate the model's parameter grows relative to T; West (1996, Theorem 4.1)) Ω reduces to the long run covariance matrix associated with d_t or 2π the spectral density of $\left(\bar{d} - \mathrm{E}(\mathrm{d_t})\right)$ at frequency zero as is the case showed by Diebold and Mariano (1995). This long run covariance matrix S_d is defined as $S_d = \gamma_2 + 2\sum_{j=1}^{\infty}\gamma_j$, where $\gamma_j = E\left(d_t d_{t-j}\right)$. Alternatively, to this asymptotic test, White (2000) suggested and justified using "bootstrap realty check", a small sample test based on the bootstrap is called the "$realty\ check\ p - value$" for data snooping. This would involve re-sampling the test statistic $\bar{d} = D_{KLIC}$ by creating R bootstrap samples from $\{\mathrm{d_t}\}_{t=1}^{T}$ accounting for dependence by using the so-called stationary bootstrap that resample using blocks of random length. In practice bootstrap the following statistics to get the "$reality\ check\ p - value$" for H_2:

$$\bar{V_n} = \max_{1 \leq k \leq l} \sqrt{n}\left[\bar{d} - \mathrm{E}(\mathrm{d_k})\right] \tag{20}$$

where $\mathrm{E}(\mathrm{d_k})$ is set to be zero. Hansen's (2001), argues that White's p-value are consider as an upper bound of the true p-value, therefore he modified the "$reality\ check\ p - value$" that depends on the variance of $\bar{d}_k$.

6. Applications to Empirical Data

In this section, we study density forecasts of Kula Lumpur Composite Index (KLCI) daily returns for the period of 02/01/1991 to 31/12/2004, with a total sample size of (T= 3653). For forecasting propose, an out-of-sample density forecast of size n=1591, through 26/11/1998-31/12/2004. We use this data set to compare $20(4 \times 5)$ density forecast models. A model in each cell, corresponding to a particular density specification in conjunction with a particular volatility specification, is regarded as a benchmark model and it is compared with the remaining 19 models. The number of bootstraps is set to 1000 and the mean block length 4, which corresponds to q=0.25 in (White, 2000). As a reminder to the reader that the lag length for AR is chosen to be $AR(3) - SNP(8)$ for the AR model in (7), according to Akaike and Bayesian information criteria (AIC,BIC).

6.1 Empirical Results

$D_{KLIC}$and its censored versions as defined in (4) with three different values of τ ($\alpha = 100\%, 10\%\ and\ 5\%$) are reported in Tables 1, 2 and 3.

The results for 100% tail (whole distribution) of the KLCI series presented in Table 1, the best specification obtained by the model of $Skewed - t\ +\ APARCH$ where the D_{KLIC} =0.00561 and the reality check-p- value = 0.994, followed by $Student - t\ +\ GARCH$ then $Skewed - t\ +\ GJR$ with D_{KLIC} 0.00586 and 0.00593 and reality check-p- value of 0.952 and 0.899 respectively. These results are in line with previous empirical studies on emerging markets (Hassan and Shamiri, 2007).

Table 1:The D_{KLIC} distance measure and the Reality Check-p- values for KLCI series: Whole distribution

	GARCH	EGARCH	GJR	APARCH
Normal	0.01665	0.01994	0.01715	0.01702
	0.7332	0.5621	0.6000	0.6001
	0.6990	0.4491	0.59871	0.6000
Student-t	0.00586	0.01513	0.01110	0.01251
	0.9528	0.7766	0.8867	0.8745
	0.8924	0.7036	0.87756	0.8062
GED	0.07421	0.01570	0.01186	0.07454
	0.0267	0.6968	0.89113	0.0234
	0.0059	0.6202	0.79663	0.0036
Skewed-t	0.00929	0.01003	0.00593	0.00561
	0.8873	0.8911	0.89914	0.9949
	0.8261	0.8001	0.88765	0.9765
Skewed GED	0.01982	0.02321	0.01803	0.01983
	0.6432	0.0376	0.6831	0.6623
	0.62012	0.0311	0.6067	0.6109

For each combination of distribution and volatility, the first number is D_{KLIC} , second and third are the reality check-p value of White's (2000) and Hansen's (2001) test, respectively.

Table 2, reports the 10% tail, the $Skewed - t$ generates the best density forecast model in combination with most volatility models considered ($EGARCH, APARCH\ and\ GJR$), with large reality check-p- value and small D_{KLIC}. All the remaining models are clearly dominated by these models as indicated by the small reality check-p- values of those models. All other distributions do not provide adequate density forecast models in combination with any of the $GARCH$ models, the normal and GED distributions are among the worst for the whole sample.

Table 2:The D_{KLIC}distance measure and the Reality Check-p- values for KLCI series: 10% lower tail

	GARCH	EGARCH	GJR	APARCH
	0.06776	0.06319	0.06996	0.06928
Normal	0.7101	0.5885	0.4871	0.5401
	0.6905	0.4997	0.4391	0.5004
	0.03717	0.04247	0.04144	0.04223
Student-t	0.7899	0.5987	0.6161	0.5989
	0.7001	0.5999	0.5969	0.5999
	0.06276	0.04201	0.04012	0.06279
GED	0.4576	0.6799	0.7115	0.4138
	0.4420	0.6143	0.6548	0.4131
	0.00668	0.00124	0.00591	0.00113
Skewed-t	0.5791	1.0000	0.7212	1.0000
	0.5428	1.0000	0.6892	1.0000

	0.04940	0.05111	0.04887	0.04949
Skewed GED	0.5855	0.5533	0.5926	0.5825
	0.5563	0.5094	0.5423	0.5221

- For each combination of distribution and volatility model, the first number is D_{KLIC} and second number is the reality check-p- value of White's (2000).
- The critical value of χ^2= 19.81 at 10%.
- The critical value of D_{KLIC} = 0.0058 at 10%.

The results for 5% tail in Table 3, shows that S$kewed-t$ with $APARCH$ attains again the smallest D_{KLIC}, followed by $Skewed-t$ with $EGARCH$. These two models have a D_{KLIC} smaller than 0.0066, indicating their adequacy as a forecast model. All other alternative models are not adequate, with very low reality check-p-values and large D_{KLIC}.

Table 3:The D_{KLIC} distance measure and the Reality Check-p- values for KLCI series: 5% lower tail

	GARCH	**EGARCH**	**GJR**	**APARCH**
	0.06245	0.06461	0.06498	0.06424
Normal	0.6391	0.3991	0.3010	0.3199
	0.6417	0.2018	0.2005	0.1022
	0.02958	0.03545	0.03411	0.03497
Student-t	0.7891	0.5781	0.6191	0.6977
	0.7551	0.5441	0.6021	0.6219
	0.05760	0.03446	0.03149	0.05775
GED	0.0000	0.7191	0.8090	0.0000
	0.0000	0.6821	0.7811	0.0000
	0.01331	0.00534	0.01324	0.00490
Skewed-t	0.3781	1.0000	0.3551	1.0000
	0.3087	0.9988	0.3042	1.0000
	0.04367	0.04587	0.04237	0.04387
Skewed GED	0.5883	0.5030	0.5985	0.5833
	0.5745	0.5005	0.5784	0.5725

- For each combination of distribution and volatility, the first number is D_{KLIC} and second number is the reality check-p- value of White's (2000).
- The critical value of χ^2= 22.36 at 5%.
- The critical value of D_{KLIC} = 0.0066 at 5%.

We notice, the results drawn from the Tables 1, 2 and 3 (100%, 10% and 5%) that, the worst distribution model is the Normal, which does not produce any adequate density forecast model with any combination of the four $GARCH$ models. We also note, that the distributional model exhibits much more robust performance across the different combination with the volatility models, *vice versa* is not true. That is, a good distributional model can often become a very bad choice with other distributional models. Once a good distributional model has been chosen, the choice of $GARCH$ models may not be important. Therefore, the distribution choice is much more important than the volatility model choice.

7. Conclusions

The issue described in this paper stem from the fact that the prediction produced by a density forecasting model can rarely be compared to the true generating distribution in real world problems. Instead, only a single instance of the generating distribution (actual outcome) is available to the forecaster to optimize and evaluate their model. Therefore, using the true density as a point of reference it is possible to rank densities relative to the true density to determine the best model.

In this paper we analyzed and used the Kullback-Leibler Information Criterion ($KLIC$) as a unified statistical tool to evaluate, and compare density forecasts. Computation of the $KLIC$ is facilitated by exploiting its relationship with the well-known Berkowitz LR test for the evaluation of individual density forecasts based on the $PITs$. To compare the performance of density forecast models in the tails, we also use a censored LR statistics to estimate the tail minimum D_{KLIC}.We have found that D_{KLIC} provides a useful and statistically powerful tool to compare competing density forecasts.

Empirical findings based on the daily $KLCI$ return series confirm the recent evidence on heavy-tailed and asymmetry in financial return distributions. $Skewed-t$, a distribution that captures these two properties, appears to produce the best density forecast in tails. Our findings based on the empirical data confirm that successful density forecast depends much more heavily on the choice of distributional model than the choice of volatility model. Moreover, the D_{KLIC}testing approach appears to deliver extremely good power of dedicating inadequate model.

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