Wild-Bootstrapped Variance Ratio Test for Autocorrelation in the Presence of Heteroskedasticity

Jeong, Jinook and Kang, Byunguk

Yonsei University

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by

Jinook Jeong[1]
School of Economics
Yonsei University

and

Byunguk Kang
School of Economics
Yonsei University

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Abstract

The Breusch-Godfrey's LM test is one of the most popular tests for autocorrelation. However, it has been shown that the LM test may be erroneous when there exist heteroskedastic errors in regression model. Some remedies recently have been proposed by Godfrey and Tremayne (2005) and Shim et al. (2006). This paper suggests wild-bootstrapped variance ratio test for autocorrelation in the presence of heteroskedasticity. We show through a Monte Carlo simulation that our wild-bootstrapped VR test has better small sample properties and is robust to the structure of heteroskedasticity.

Keywords: variance-ratio test, Breusch-Godfrey's LM test, autocorrelation, heteroskedasticity, wild bootstrap

JEL classifications: C12, C15

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[1] Corresponding author: Professor Jinook Jeong, School of Economics, Yonsei University, Seoul, Korea; Tel.: +82-2-2123-2493; e-mail address: jinook@yonsei.ac.kr

I. Introduction

Breusch-Godfrey's LM test (B-G LM test hereafter) is one of the most popular tests for autocorrelation in regression models, as it is simple and is unrestricted by the dynamics of error term.[2] Recently, it has been shown that B-G LM test may be misleading in the presence of variance break. Hyun et al. (2006) present a Monte Carlo result where the empirical size of B-G LM test is distorted with a variance break. Shim et al. (2006) propose a modified LM test and a modified variance-ratio (VR) test to remedy B-G LM test in the presence of a variance break. However, the remedies by Shim et al. (2006) are not free of limitations. First, their modified LM test would not be valid with a more general form of heteroskedasticity but a variance break. For example, a multiplicative heteroskedasticity, which is a more practical structure of heteroskedasticity, may not be properly dealt with by their modified LM test. For another example, their modified LM test may not be valid with 'multiple' variance breaks. Second, the finite sample properties of the tests suggested by Shim et al. (2006) may not be as good as the asymptotic properties, as the test statistics are basically two-step estimators. Since the first-stage estimates are error-ridden, the second-stage estimates could suffer from the cumulative errors.

We suggest a refined VR test using wild bootstrap as the remedy for various types of heteroskedasticity. Wild bootstrap has been originally suggested by Härdle and Mammen (1990) and been widely used to deal with heteroskedasticity.[3] For example, Godfrey and Tremayne (2005) apply wild bootstrap for B-G LM test under heteroskedasticity. They show the empirical size and power of B-G LM test are improved with wild bootstrap. One interesting feature of their method is the use of

[2] Breusch (1978) and Godfrey (1978)

[3] For example, see Mammen (1993), Davidson and Flachaire (2001), and Flachaire (2003) among others.

heteroskedasticity-consistent covariance estimates in computing the LM statistic. They first correct the covariance for heteroskedasticity and then apply wild bootstrap to the heteroskedasticity-corrected LM statistic. We will show that wild bootstrap of our VR test statistic does not need such pre-correction. Without using heteroskedasticity-consistent covariance estimates, our wild-bootstrapped VR test has better power than Godfrey-Tremayne's LM test in finite samples. We also show through Monte Carlo simulations that our wild-bootstrapped VR test is accurate in the presence of general form of heteroskedasticity, and its finite sample property is significantly better than previously proposed tests even in the presence of a variance break.

II. Model

We consider the following linear regression model:

$$y_t = X_t'\beta + u_t, \qquad t = 1, 2, ..., T \tag{1}$$

where X_t is $k \times 1$ vector, $E(u_t) = 0$ and $E(u_t u_s) = 0$ for $t \neq s$. Our concern is whether u_t is serially independent. The standard LM test by Breusch and Pagan works well for the autocorrelation test if u_t is homoskedastic. However, the standard B-G LM test may not be valid if u_t is not homoskedastic. Hyun et al. (2006) consider a variance break case as follows.

$$u_t = \sigma_t \eta_t \tag{2}$$

$$\sigma_t^2 = \sigma_1^2 1[t \leq \tau T] + \sigma_2^2 1[t > \tau T]$$

where η_t is $IID(0,1)$ and break timing, τ is between 0 and 1. In this model, variance of u_t, σ_t^2 is given by σ_1^2 for $t \leq \tau T$ and changes into σ_2^2 after τT. Hyun et al. (2006) report that the empirical size of B-G LM test is distorted and that the magnitude of distortion depends on the degree of break, break timing, and the lag length (p) of the error term.

For this problem with variance break, Shim et al. (2006) suggest two tests, the modified LM test based on feasible generalized least squares (FGLS) and the modified variance-ratio (VR) test. The FGLS-based LM test procedure is as follows.

i) Estimate the break point by quasi-maximum likelihood estimation by Bai et al. (1998) or the least squares method by Bai (1993).

ii) According to the estimated break, estimate σ_1 and σ_2 from the subsamples.

iii) Using the estimated variances, apply FGLS to estimate the FGLS residuals.

iv) Compute the LM statistic using the FGLS residuals, and apply the asymptotic $\chi^2(p)$ distribution for the test.

It is noted that the FGLS-based LM test is not robust to the structure of heteroskedasticity. If the number of break points is not known a priori, or if other forms of heteroskedasticity than variance break exist in the data, the FGLS-based LM test would be invalid.

On the other hand, the modified VR test employs the VR test by Lo and MacKinley (1988, 1989). The VR test is originally designed for serial correlation in a time series variable. It examines if the variance of 'q-period return' is exactly q times higher than the variance of 'one-period return.' If it is, that implies there is no evidence of serial correlation. Lo and MacKinley (1988) also propose a heteroskedasticity-robust form of VR test, using White's correction. Shim et al. (2006) modify the heteroskedasticity-robust VR test for a regression model. The modified VR test is robust to the structure of heteroskedasticity, but its finite sample properties need to be scrutinized as it depends on an asymptotic correction. As we will show later through a Monte Carlo simulation, the modified VR test over-rejects the true null hypothesis of no autocorrelation in small samples and shows low power in small samples.

The test designed by Godfrey and Tremayne (2005) does not have the small sample problems of Shim et al. (2006). They employ heteroskedasticity-consistent covariance matrix estimates (HCCME) to get heteroskedasticity-robust LM statistic and use wild bootstrap to estimate the empirical distribution of the statistic, because finite distribution of this LM statistic is quite different from its asymptotic distribution. The test shows empirical sizes that are very close to pre-specified type I error even in small sample. The empirical powers of this test, however, are not very good. Especially, when the lag length is misspecified, the test shows poor powers in small samples.

Besides the above variance break setup, we consider a more general form of heteroskedasticity case. Let us define the error term as:

$$u_t = \varepsilon_t \eta_t \tag{3}$$

where η_t's are white noise, and ε_t is a heteroskedastic factor creating various types of heteroskedasticity. For example, if ε_t is a binary variable, it results in a variance break. If ε_t is a function of a pre-determined variable, it results in multiplicative heteroskedasticity. We will show later that the original LM test would fail in the presence of heteroskedasticity such as (3), and that the wild-bootstrap VR test can provide a remedy for LM test.

III. Wild-bootstrapped VR Test

A number of bootstrap procedures have been proposed for heteroskedastic models.[4] Wu (1986) proposes so-called 'weighted bootstrap' for heteroskedastic data and shows that the variance estimate by weighted bootstrap is consistent under mild

[4] For a review, see Jeong and Maddala (1993).

conditions. The basic idea of weighted bootstrap is to transform the heteroskedastic residuals into homoskedastic ones in the resampling process. Liu (1988) extends Wu's weighted bootstrap to the non-regression contexts. The wild bootstrap proposed by Härdle and Mammen (1990) is a generalized version of Wu's weighted bootstrap and there exist a number of alternative procedures depending on how to transform the residuals into homoskedastic ones. Davidson and Flachaire (2001) propose a wild bootstrap based on Rademacher distribution, and Flachaire (2003) shows that the Rademacher version of wild bootstrap has better finite sample properties than any previous versions of wild bootstrap. The Rademacher version of wild bootstrap applied to VR statistic is:

i) Compute the residuals, $\hat{u}_t = y_t - X_t\hat{\beta}$ by OLS.

ii) Compute $VR(q) = \frac{\hat{\sigma}(q)^2}{\hat{\sigma}(1)^2} - 1$, where $\hat{\sigma}(1)^2 = \frac{1}{T-1}\sum_{t=1}^{T}\hat{u}_t^2$, $\hat{\sigma}(q)^2 = \frac{T}{q(T-q+1)(T-q)}\sum_{t=q}^{T}\hat{u}_t(q)^2$, and $\hat{u}_t(q) = \sum_{i=0}^{q-1}\hat{u}_{t-i}$.

iii) Construct a fake y^* by the formula $y_t^* = X_t\hat{\beta} + \{\hat{u}_t/(1-h_t)^{1/2}\}v_t \quad t = 1,2,...,T$ where $h_t = X_t(X'X)^{-1}X_t'$ and v_t is a random variable with mean 0 and variance 1. The wild bootstrap takes v_t from Rademacher distribution as

$v_t = 1$ with a probability $\frac{1}{2}$

$v_t = -1$ with a probability $\frac{1}{2}$

iv) Reestimate β^* using X and y^*, and compute a bootstrapped VR statistic, $VR(q)^B$.

v) Repeat (iii)-(iv) m times to approximate the distribution of VR statistic by a bootstrap distribution $\hat{\mathrm{F}}_{\mathrm{B}}$, putting mass (1/m) at each point $VR(q)_1^B$,

$VR(q)_2^B$, ..., $VR(q)_m^B$.

Using the above procedure, we can construct a heteroskedasticity-robust empirical (bootstrapped) distribution of VR statistic. We expect the bootstrapped distribution will give us a more accurate critical point than the asymptotic normal distribution for two reasons. First, the asymptotic normal distribution is invalid in the presence of heteroskedasticity, while the bootstrapped distribution is robust to heteroskedasticity. Second, as bootstrapped distribution is more accurate in small samples than asymptotic approximations under mild conditions, the wild-bootstrapped VR test is expected to have better finite sample properties.

When we design the bootstrap test procedure, it is crucial to construct the bootstrapped distribution under the null hypothesis. In the case of autocorrelation test using wild bootstrap, because the residual is multiplied by a random variable (v_t) having zero mean, unit variance, and no serial correlations, the autocorrelation vanishes in bootstrapped data. Accordingly, the null hypothesis of no autocorrelation is imposed on the bootstrapped distribution, even though there is the autocorrelation in the original error terms.

IV. Monte Carlo Simulation

In this section, we compare through simulations the sizes and powers of our wild-bootstrapped VR test to the original LM test by Breusch (1978) and Godfrey (1978), the FGLS-based LM test, the modified VR test by Shim et al. (2006) and wild-bootstrapped HR-LM test by Godfrey and Tremayne (2005). We consider two types of heteroskedasticity: variance break defined in (2) and general heteroskedasticity defined in (3).

1. Variance Break Case

First, let us examine the empirical sizes of the tests. We generate the error terms u_t's to have variance break but no autocorrelation by (2). The fundamental error terms, η_t's are generated from independent N(0,1) and we fix $\sigma_1 = 1$ and handle the variance shift using σ_2. Two cases can be considered about σ_2: decreasing variance ($\sigma_2 < \sigma_1$) and increasing variance ($\sigma_2 > \sigma_1$). For brevity of presentation, we present the results from decreasing variance case.[5] We take (0.25, 0.4, 0.6, 0.8, 1) as σ_2. Various break timings (τ) are considered from 0.05 to 0.95. y_t's are generated by

$$y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + u_t \qquad t = 1, 2, \ldots, T \tag{4}$$

where $\beta_0 = \beta_1 = \beta_2 = 1$ and X_{1t}, X_{2t} follow $IIN(0,1)$ with $\mathrm{cov}(X_{1t}, X_{2t}) = 0$. We examine three sample sizes: T=20, T=50, and T=100. Three different lag lengths (p) of the autoregressive serial correlation are employed: p=1, p=4, and p=8. The nominal size is set to 5%. The number of simulations is 1000, and the number of bootstrap resamples is 1,000.

Table 1(A)-(C) confirm the findings of Hyun et al. (2006). Table 1(A) shows the empirical sizes of B-G LM test in the above setup for p=1, Table 1(B) for p=4, and Table 1(C) for p=8, respectively. For Tables 1(A)-(C), the sample size is set at T=100. As shown in the Tables, the empirical sizes of the original LM test are severely distorted in the presence of variance break. The size distortion becomes more severe as the variance reduction ratio after the break $(= \sigma_2 / \sigma_1)$ intensifies. The size distortion also depends on the location (τ) of the break as Hyun et al. (2006) show.

Table 2(A)-(C) show the results of the modified VR test (p=1 and T=20, 50, 100 respectively). When the sample size is as large as 100, Table 2(C) reaffirms the

[5] There exist no qualitative differences in increasing variance case. The simulation results are available from authors upon request.

findings of Shim et al. (2006). The empirical sizes are significantly more accurate than the original LM test in Table 1(C). For example, when τ=0.2 and σ_2=0.25, the size of the modified VR test is 5.4% in Table 2(C) while the size of the original LM test is 23.6% in Table 1(A). When the sample size becomes smaller, however, the empirical sizes of the modified VR test become inaccurate. When T=50, the empirical sizes in Table 2(B) show tendency for over-rejection, sometimes as high as 8.4%. When T=20, the modified VR test seriously over-rejects the null hypothesis as shown in Table 2(A). In many cases, the empirical sizes are higher than 10%. This is not surprising considering the VR test uses an asymptotic standard normal distribution, and uses an asymptotic correction to deal with heteroskedasticity. We will show later that the wild-bootstrapped tests have significantly better small sample properties than the modified VR test.

The FGLS-based LM test works better than the modified VR test in this variance break case. It is natural for FGLS-based LM test to outperform the modified VR test. The FGLS procedure specifically reflects the variance break pattern in the first stage, while the modified VR test corrects heteroskedasticity in a general way. However, the FGLS-based LM test show weak performance when the sample size is small and the lag length is large. Table 3(A)-(C) show the empirical sizes of FGLS-based LM test for p=8, and T=20, 50, and 100, respectively. We can see the sizes are inaccurate especially when T=20 and p=8. We will show later that the two wild-bootstrapped tests have accurate sizes even when the lag length is as high as p=8.

On the other hand, the two wild-bootstrapped tests, our proposed wild-bootstrapped VR test and the wild-bootstrapped HR-LM test by Godfrey and Tremayne (2005) have very accurate empirical sizes in the presence of variance break. Tables 4-

9 show the empirical sizes of the two tests. Table 4(A)-(C) are for the cases of p=1, Tables 5(A)-(C) are for p=4, and Tables 6(A)-(C) are for p=8. As shown in the Tables, the empirical sizes of the wild-bootstrap tests are all close to the nominal size. Even when the sample size is as small as 20 and the lag length is as high as 8, Table 6(A) shows that the empirical sizes of the tests are quite accurate. From the Tables 4-6, it is clear that the wild-bootstrapped tests are robust to the degree of heteroskedasticity (variance reduction rate), sample size, and the length of lag.

Let us now examine the powers of alternative tests. To consider the power, we impose autocorrelation as well as heteroskedasticity on error term in the following way:

$$u_t = \rho u_{t-1} + \sigma_t \eta_t \qquad (5)$$

$$\sigma_t^2 = \sigma_1^2 1[t \le \tau T] + \sigma_2^2 1[t > \tau T]$$

It is noted that the error term now follows AR(1) process and its variance has a break at τT. In the simulation, we use 0.7 for the value of ρ. All the other parameters are the same as before.

Tables 7-9 present the empirical powers of the two wild-bootstrapped tests. We only present the results of the wild-bootstrapped tests because the other two tests fail to control the size in our simulation. Tables 7(A)-(B) are for the cases of lag length, p=1, Tables 8(A)-(B) are for p=4, and Tables 9(A)-(B) are for the cases of p=8.[6] First, we notice from the Tables 7-9 that the empirical powers of the wild-bootstrapped tests are robust to heteroskedasticity. No matter how high the variance reduction rate $(= \sigma_2 / \sigma_1)$ is, the empirical powers are almost the same as the homoskedasticity case of $\sigma_2 / \sigma_1 = 1$. This implies that the wild-bootstrapped tests have resolved the problem of

[6] We don't present the cases of T=100 because the powers of T=100 are very close to 1 in almost every case. The results are available from the authors upon request.

the original LM test: size distortion and power reduction due to heteroskedasticity.

Second, we observe from Table 7-9 that our wild-bootstrapped VR test has superior small sample properties to the wild-bootstrapped HR-LM test by Godfrey and Tremayne (2005) in all cases. The wild-bootstrapped VR test show significantly higher powers and is more robust to the structure of lag length than wild-bootstrapped HR-LM test. Both the tests show worse power as lag length grows, which is natural considering we impose AR(1) structure in the simulations. However, even when the lag length is misspecified, the powers of our wild-bootstrapped VR test are considerably higher than the wild-bootstrapped HR-LM test. For example, as seen in Tables 9(A), the powers of the wild-bootstrapped VR (around 0.450 on average) are almost four times higher than the powers of the wild-bootstrapped HR-LM test (around 0.120 on average), in the case of p=8 and T=20.

2. General Heteroskedasticity Case

Now, let us consider a more general form of heteroskedasticity than variance break. All the other parameters of the simulation are the same as variance break case in the earlier section except for u_t. The error term now is defined by (3), where η_t's follow $IIN(0,1)$. To create heteroskedasticity, a random variable ε_t is drawn from $IIN(0,1)$ and sorted by three different rules. First, we sort ε_t's by the absolute values of ε_t, to create continuously increasing heteroskedasticity. Second, we sort ε_t by the negative of the absolute value, to create continuously decreasing heteroskedasticity. Last, we sort ε_t by its raw value, to create heteroskedasticity with an increase after a decrease. Three forms of heteroskedasticity generated by the rules are shown in Figure 1(A)-(C).

Table 10(A)-(C) show the empirical sizes of the alternative tests in each case of continuous heteroskedasticity: increasing variances, decreasing variances, and increasing after decreasing, respectively. As seen in Tables 10(A)-(C), the wild-bootstrapped VR test and the wild-bootstrapped HR-LM test control the size quite successfully. The two tests show stable empirical sizes respectively from 0.043 to 0.070 and from 0.032 to 0.065. On the other hand, the original LM test, the FGLS-based LM test, and the modified VR test all tend to over-reject the true null hypothesis in most cases and under-reject the null in the cases of small sample and large lag length.[7]

Table 11(A)-(C) show the empirical powers of the two wild-bootstrapped tests. Considering the sizes of the other tests are considerably distorted, we only present the empirical powers of the wild-bootstrapped VR test and the wild-bootstrapped HR-LM test. To compute powers, we impose autocorrelation as well as heteroskedasticity on the error term in the following way:

$$u_t = \rho u_{t-1} + \varepsilon_t \eta_t \tag{6}$$

Note that the error term now follows AR(1) process and its variance has a continuous heteroskedasticity factor. In the simulation, we set $\rho = 0.7$. All the other parameters are the same as before.

We observe from Table 11(A)-(C) that the empirical powers of the two tests in general heteroskedasticity case show similar patterns to the ones in variance break case. The wild-bootstrapped VR test always shows significantly higher power and is much more robust to misspecification of lag length than the wild-bootstrapped HR-LM test by Godfrey and Tremayne (2005). For example, as seen in Tables 11(B), the power of the wild-bootstrapped VR is 0.490, almost four times higher than the one of the wild-

[7] But the empirical sizes of the modified VR test become much more accurate in larger samples than T=500. The results are available from the authors upon request.

bootstrapped HR-LM test, 0.132, in the case of p=8 and T=20.

VI. Conclusion

We propose a wild-bootstrapped VR test for autocorrelation in the presence of heteroskedasticity. We also compare the finite sample properties of the test to the existing tests, such as the original Breusch-Godfrey's LM test, the FGLS-based LM test by Shim et al. (2006), the modified VR test by Shim et al. (2006) and the wild-bootstrapped HR-LM test by Godfrey and Tremayne (2005). The Monte Carlo simulation shows that the wild-bootstrapped VR test outperforms the existing tests both in size and power.

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Table 1(A) Empirical sizes of the original LM test (T= 100, p=1, $\alpha = 5\%$)

$\frac{\sigma_2}{\sigma_1}$ \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.205	0.258	0.236	0.225	0.178	0.134	0.113	0.089	0.078	0.058	0.060
0.4	0.102	0.151	0.143	0.126	0.113	0.115	0.092	0.083	0.075	0.054	0.054
0.6	0.051	0.075	0.088	0.080	0.066	0.059	0.067	0.066	0.053	0.058	0.066
0.8	0.054	0.071	0.055	0.061	0.057	0.046	0.069	0.046	0.068	0.051	0.056
1.0	0.058	0.052	0.052	0.057	0.040	0.040	0.045	0.053	0.064	0.055	0.049

Table 1(B) Empirical sizes of the original LM test (T= 100, p=4, $\alpha = 5\%$)

$\frac{\sigma_2}{\sigma_1}$ \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.273	0.400	0.433	0.375	0.265	0.202	0.155	0.109	0.088	0.080	0.048
0.4	0.076	0.174	0.201	0.211	0.188	0.138	0.128	0.093	0.090	0.065	0.056
0.6	0.041	0.056	0.090	0.101	0.081	0.086	0.082	0.063	0.053	0.055	0.048
0.8	0.041	0.046	0.044	0.060	0.057	0.052	0.052	0.040	0.047	0.057	0.043
1.0	0.056	0.045	0.052	0.048	0.042	0.038	0.040	0.050	0.045	0.048	0.050

Table 1(C) Empirical sizes of the original LM test (T= 100, p=8, $\alpha = 5\%$)

$\frac{\sigma_2}{\sigma_1}$ \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.146	0.378	0.490	0.418	0.333	0.235	0.137	0.109	0.069	0.048	0.042
0.4	0.042	0.101	0.185	0.224	0.192	0.166	0.135	0.084	0.068	0.049	0.035
0.6	0.034	0.047	0.058	0.090	0.076	0.093	0.060	0.053	0.053	0.044	0.042
0.8	0.037	0.037	0.037	0.034	0.046	0.042	0.040	0.051	0.044	0.049	0.039
1.0	0.047	0.041	0.034	0.040	0.045	0.044	0.049	0.049	0.037	0.035	0.038

Table 2(A) Empirical sizes of the modified VR test (T= 20, p=1, $\alpha = 5\%$)

τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.242	0.138	0.102	0.098	0.071	0.092	0.083	0.073	0.071	0.078	0.084
0.4	0.133	0.098	0.109	0.091	0.080	0.082	0.083	0.072	0.067	0.089	0.078
0.6	0.095	0.099	0.075	0.079	0.079	0.076	0.082	0.069	0.075	0.073	0.083
0.8	0.095	0.071	0.078	0.081	0.084	0.083	0.076	0.086	0.068	0.081	0.083
1.0	0.090	0.072	0.084	0.074	0.078	0.091	0.098	0.090	0.069	0.081	0.089

Table 2(B) Empirical sizes of the modified VR test (T= 50, p=1, $\alpha = 5\%$)

τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.084	0.075	0.061	0.060	0.050	0.057	0.051	0.068	0.063	0.045	0.053
0.4	0.077	0.054	0.066	0.055	0.049	0.053	0.058	0.065	0.069	0.064	0.063
0.6	0.060	0.067	0.048	0.053	0.058	0.065	0.064	0.072	0.065	0.059	0.046
0.8	0.048	0.055	0.070	0.054	0.061	0.073	0.063	0.062	0.063	0.054	0.052
1.0	0.061	0.063	0.063	0.071	0.055	0.059	0.052	0.067	0.056	0.054	0.068

Table 2(C) Empirical sizes of the modified VR test (T= 100, p=1, $\alpha = 5\%$)

τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.072	0.062	0.054	0.055	0.049	0.047	0.059	0.052	0.053	0.044	0.057
0.4	0.069	0.059	0.056	0.051	0.043	0.064	0.051	0.060	0.062	0.042	0.060
0.6	0.048	0.055	0.067	0.046	0.043	0.041	0.059	0.052	0.050	0.056	0.065
0.8	0.059	0.065	0.055	0.065	0.058	0.050	0.064	0.049	0.068	0.040	0.057
1.0	0.064	0.055	0.058	0.055	0.049	0.049	0.051	0.055	0.062	0.052	0.055

Table 3(A) Empirical sizes of the FGLS-based LM test (T= 20, p=8, $\alpha = 5\%$)

σ_2/σ_1 \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.005	0.004	0.006	0.000	0.004	0.005	0.002	0.008	0.006	0.010	0.006
0.4	0.004	0.003	0.007	0.005	0.008	0.010	0.007	0.010	0.006	0.010	0.010
0.6	0.004	0.006	0.008	0.004	0.005	0.007	0.005	0.004	0.003	0.009	0.007
0.8	0.007	0.009	0.007	0.009	0.005	0.009	0.012	0.006	0.006	0.008	0.006
1.0	0.011	0.006	0.007	0.009	0.006	0.003	0.007	0.007	0.011	0.007	0.002

Table 3(B) Empirical sizes of the FGLS-based LM test (T= 50, p=8, $\alpha = 5\%$)

σ_2/σ_1 \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.032	0.035	0.034	0.021	0.030	0.028	0.019	0.034	0.025	0.032	0.032
0.4	0.029	0.029	0.021	0.042	0.023	0.030	0.036	0.018	0.024	0.029	0.024
0.6	0.021	0.024	0.021	0.021	0.025	0.023	0.024	0.026	0.024	0.041	0.026
0.8	0.025	0.023	0.018	0.023	0.033	0.025	0.027	0.026	0.028	0.023	0.040
1.0	0.021	0.036	0.028	0.027	0.027	0.027	0.023	0.016	0.024	0.031	0.016

Table 3(C) Empirical sizes of the FGLS-based LM test (T= 100, p=8, $\alpha = 5\%$)

σ_2/σ_1 \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.25	0.026	0.041	0.036	0.034	0.027	0.035	0.033	0.037	0.044	0.042	0.038
0.4	0.033	0.034	0.025	0.048	0.031	0.047	0.041	0.035	0.041	0.044	0.031
0.6	0.036	0.040	0.040	0.031	0.039	0.040	0.032	0.039	0.046	0.041	0.035
0.8	0.032	0.039	0.034	0.030	0.034	0.033	0.037	0.039	0.038	0.040	0.038
1.0	0.038	0.034	0.031	0.036	0.035	0.039	0.036	0.040	0.031	0.030	0.034

Table 4(A) Empirical sizes of the wild-bootstrapped tests (T=20, p=1, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.052	0.039	0.051	0.057	0.041	0.047	0.064	0.049	0.054	0.070	0.058
	0.4	0.045	0.047	0.053	0.047	0.050	0.054	0.052	0.052	0.046	0.050	0.055
	0.6	0.052	0.056	0.055	0.058	0.060	0.055	0.048	0.055	0.053	0.054	0.055
	0.8	0.054	0.047	0.051	0.044	0.060	0.054	0.056	0.068	0.037	0.052	0.049
	1.0	0.052	0.045	0.063	0.056	0.063	0.064	0.062	0.048	0.047	0.055	0.063
VR	0.25	0.041	0.051	0.066	0.068	0.054	0.068	0.064	0.041	0.047	0.059	0.048
	0.4	0.043	0.050	0.064	0.063	0.050	0.057	0.052	0.041	0.038	0.055	0.049
	0.6	0.047	0.052	0.053	0.051	0.054	0.049	0.049	0.048	0.058	0.042	0.054
	0.8	0.051	0.039	0.047	0.049	0.044	0.053	0.046	0.061	0.034	0.052	0.051
	1.0	0.046	0.040	0.053	0.044	0.047	0.053	0.043	0.049	0.043	0.053	0.060

Table 4(B) Empirical sizes of the wild-bootstrapped tests (T=50, p=1, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.042	0.050	0.053	0.052	0.045	0.044	0.043	0.062	0.057	0.051	0.053
	0.4	0.052	0.046	0.048	0.055	0.045	0.044	0.059	0.057	0.052	0.050	0.053
	0.6	0.043	0.063	0.041	0.041	0.049	0.050	0.052	0.062	0.052	0.041	0.041
	0.8	0.044	0.043	0.068	0.051	0.055	0.058	0.055	0.049	0.055	0.053	0.050
	1.0	0.058	0.066	0.057	0.053	0.038	0.047	0.043	0.059	0.048	0.049	0.054
VR	0.25	0.058	0.074	0.067	0.064	0.053	0.054	0.044	0.070	0.058	0.043	0.050
	0.4	0.057	0.050	0.056	0.070	0.055	0.051	0.058	0.057	0.058	0.056	0.052
	0.6	0.040	0.062	0.039	0.049	0.057	0.053	0.055	0.062	0.062	0.050	0.045
	0.8	0.039	0.048	0.060	0.049	0.054	0.059	0.053	0.061	0.056	0.054	0.044
	1.0	0.050	0.064	0.059	0.055	0.041	0.047	0.046	0.055	0.053	0.044	0.055

Table 4(C) Empirical sizes of the wild-bootstrapped tests (T=100, p=1, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.037	0.052	0.055	0.052	0.050	0.041	0.056	0.052	0.046	0.048	0.053
	0.4	0.060	0.057	0.056	0.048	0.047	0.058	0.049	0.062	0.058	0.044	0.049
	0.6	0.045	0.054	0.056	0.041	0.039	0.038	0.058	0.057	0.044	0.053	0.060
	0.8	0.055	0.066	0.052	0.056	0.051	0.050	0.059	0.041	0.063	0.043	0.055
	1.0	0.053	0.050	0.051	0.052	0.041	0.052	0.042	0.056	0.061	0.053	0.050
VR	0.25	0.046	0.054	0.060	0.062	0.053	0.046	0.056	0.049	0.046	0.048	0.053
	0.4	0.059	0.062	0.062	0.054	0.044	0.065	0.055	0.063	0.060	0.041	0.056
	0.6	0.046	0.054	0.059	0.047	0.041	0.044	0.058	0.054	0.053	0.056	0.058
	0.8	0.052	0.064	0.053	0.056	0.053	0.049	0.063	0.046	0.059	0.041	0.050
	1.0	0.056	0.051	0.047	0.053	0.042	0.052	0.049	0.051	0.062	0.050	0.048

Table 5(A) Empirical sizes of the wild-bootstrapped tests (T=20, p=4, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.046	0.048	0.035	0.049	0.040	0.046	0.047	0.045	0.060	0.044	0.047
	0.4	0.048	0.049	0.033	0.045	0.053	0.040	0.051	0.067	0.070	0.048	0.059
	0.6	0.051	0.050	0.037	0.049	0.052	0.051	0.043	0.046	0.047	0.046	0.055
	0.8	0.046	0.051	0.054	0.058	0.049	0.054	0.049	0.054	0.067	0.048	0.045
	1.0	0.050	0.059	0.055	0.051	0.050	0.055	0.044	0.041	0.043	0.043	0.051
VR	0.25	0.044	0.057	0.062	0.067	0.058	0.058	0.045	0.064	0.054	0.053	0.052
	0.4	0.053	0.044	0.043	0.056	0.063	0.072	0.058	0.052	0.051	0.053	0.044
	0.6	0.045	0.041	0.049	0.064	0.056	0.048	0.047	0.042	0.062	0.048	0.057
	0.8	0.048	0.050	0.059	0.058	0.044	0.053	0.055	0.044	0.064	0.064	0.057
	1.0	0.054	0.054	0.047	0.056	0.044	0.041	0.048	0.045	0.053	0.052	0.057

Table 5(B) Empirical sizes of the wild-bootstrapped tests (T=50, p=4, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.049	0.062	0.040	0.057	0.064	0.047	0.051	0.055	0.043	0.054	0.048
	0.4	0.051	0.054	0.043	0.054	0.047	0.044	0.046	0.054	0.055	0.053	0.038
	0.6	0.046	0.040	0.051	0.048	0.046	0.053	0.055	0.063	0.041	0.039	0.051
	0.8	0.061	0.067	0.050	0.060	0.051	0.055	0.054	0.049	0.054	0.062	0.043
	1.0	0.044	0.055	0.061	0.053	0.049	0.045	0.046	0.056	0.053	0.058	0.062
VR	0.25	0.038	0.074	0.057	0.072	0.057	0.054	0.059	0.048	0.061	0.058	0.050
	0.4	0.053	0.064	0.044	0.068	0.051	0.050	0.055	0.052	0.063	0.049	0.059
	0.6	0.039	0.045	0.046	0.062	0.052	0.050	0.046	0.049	0.043	0.047	0.048
	0.8	0.058	0.054	0.060	0.059	0.053	0.037	0.050	0.060	0.048	0.063	0.046
	1.0	0.054	0.049	0.047	0.048	0.059	0.047	0.063	0.054	0.046	0.059	0.059

Table 5(C) Empirical sizes of the wild-bootstrapped tests (T=100, p=4, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.044	0.043	0.046	0.051	0.043	0.059	0.050	0.052	0.047	0.047	0.062
	0.4	0.045	0.049	0.047	0.040	0.056	0.048	0.041	0.054	0.031	0.059	0.049
	0.6	0.043	0.050	0.053	0.041	0.044	0.053	0.046	0.048	0.051	0.051	0.040
	0.8	0.044	0.045	0.060	0.045	0.051	0.052	0.058	0.048	0.040	0.056	0.056
	1.0	0.044	0.054	0.055	0.064	0.052	0.062	0.055	0.062	0.039	0.045	0.048
VR	0.25	0.061	0.057	0.066	0.054	0.058	0.043	0.051	0.061	0.053	0.053	0.047
	0.4	0.053	0.058	0.058	0.051	0.058	0.045	0.045	0.074	0.048	0.043	0.052
	0.6	0.038	0.037	0.058	0.048	0.047	0.074	0.051	0.059	0.050	0.055	0.041
	0.8	0.053	0.037	0.067	0.057	0.052	0.054	0.057	0.056	0.051	0.062	0.049
	1.0	0.052	0.045	0.053	0.054	0.050	0.060	0.059	0.043	0.038	0.060	0.044

Table 6(A) Empirical sizes of the wild-bootstrapped tests (T=20, p=8, $\alpha = 5\%$)

Test	τ / σ_2/σ_1	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.053	0.050	0.030	0.041	0.041	0.040	0.053	0.050	0.054	0.047	0.049
	0.4	0.034	0.051	0.046	0.060	0.044	0.040	0.047	0.054	0.054	0.049	0.038
	0.6	0.048	0.053	0.051	0.044	0.052	0.041	0.052	0.055	0.044	0.057	0.055
	0.8	0.069	0.053	0.044	0.046	0.054	0.050	0.043	0.056	0.052	0.063	0.036
	1.0	0.045	0.047	0.061	0.057	0.048	0.039	0.062	0.041	0.059	0.052	0.038
VR	0.25	0.043	0.041	0.040	0.034	0.043	0.043	0.055	0.072	0.054	0.051	0.042
	0.4	0.048	0.042	0.057	0.062	0.043	0.061	0.050	0.044	0.059	0.042	0.053
	0.6	0.065	0.052	0.053	0.053	0.060	0.056	0.046	0.047	0.043	0.060	0.062
	0.8	0.063	0.050	0.054	0.065	0.048	0.051	0.058	0.040	0.046	0.054	0.058
	1.0	0.041	0.051	0.054	0.052	0.049	0.060	0.043	0.049	0.061	0.054	0.051

Table 6(B) Empirical sizes of the wild-bootstrapped tests (T=50, p=8, $\alpha = 5\%$)

Test	τ / σ_2/σ_1	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.049	0.060	0.048	0.053	0.050	0.045	0.042	0.045	0.050	0.049	0.052
	0.4	0.044	0.056	0.048	0.046	0.044	0.057	0.036	0.061	0.062	0.048	0.048
	0.6	0.052	0.048	0.048	0.037	0.043	0.047	0.045	0.053	0.051	0.048	0.051
	0.8	0.058	0.063	0.039	0.052	0.053	0.051	0.044	0.046	0.049	0.053	0.043
	1.0	0.055	0.050	0.048	0.044	0.046	0.056	0.058	0.054	0.059	0.033	0.052
VR	0.25	0.054	0.047	0.068	0.058	0.058	0.065	0.049	0.053	0.054	0.054	0.049
	0.4	0.043	0.052	0.050	0.065	0.053	0.059	0.045	0.050	0.052	0.063	0.047
	0.6	0.067	0.048	0.054	0.056	0.058	0.053	0.068	0.046	0.047	0.055	0.043
	0.8	0.068	0.052	0.051	0.045	0.045	0.051	0.056	0.054	0.057	0.038	0.053
	1.0	0.044	0.039	0.032	0.056	0.064	0.062	0.048	0.056	0.054	0.048	0.037

Table 6(C) Empirical sizes of the wild-bootstrapped tests (T=100, p=8, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.053	0.056	0.042	0.045	0.047	0.042	0.045	0.065	0.057	0.046	0.060
	0.4	0.057	0.056	0.051	0.047	0.047	0.049	0.046	0.055	0.040	0.048	0.054
	0.6	0.048	0.041	0.053	0.042	0.051	0.051	0.050	0.048	0.049	0.044	0.053
	0.8	0.032	0.047	0.047	0.044	0.051	0.060	0.041	0.035	0.048	0.066	0.059
	1.0	0.056	0.037	0.051	0.051	0.046	0.051	0.059	0.043	0.067	0.064	0.044
VR	0.25	0.052	0.070	0.067	0.051	0.049	0.056	0.056	0.057	0.047	0.050	0.060
	0.4	0.061	0.058	0.060	0.059	0.055	0.061	0.062	0.050	0.048	0.053	0.044
	0.6	0.056	0.048	0.047	0.049	0.058	0.059	0.056	0.049	0.052	0.044	0.042
	0.8	0.052	0.041	0.048	0.042	0.045	0.057	0.049	0.061	0.051	0.051	0.068
	1.0	0.048	0.048	0.056	0.060	0.062	0.046	0.058	0.043	0.059	0.048	0.056

Table 7(A) Empirical power of the wild-bootstrapped tests (T=20, p=1, $\rho = 0.7$, $\alpha = 5\%$)

Test	σ_2/σ_1 \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.524	0.517	0.464	0.474	0.451	0.500	0.478	0.479	0.493	0.560	0.562
	0.4	0.550	0.503	0.539	0.505	0.477	0.471	0.505	0.528	0.518	0.513	0.556
	0.6	0.502	0.526	0.509	0.482	0.497	0.504	0.538	0.554	0.530	0.547	0.533
	0.8	0.505	0.506	0.513	0.535	0.542	0.535	0.516	0.548	0.530	0.509	0.506
	1.0	0.544	0.533	0.548	0.511	0.535	0.539	0.546	0.524	0.500	0.521	0.510
VR	0.25	0.642	0.614	0.589	0.610	0.604	0.632	0.608	0.616	0.618	0.678	0.678
	0.4	0.671	0.649	0.647	0.638	0.604	0.604	0.644	0.646	0.627	0.644	0.682
	0.6	0.627	0.639	0.632	0.598	0.627	0.624	0.665	0.661	0.666	0.646	0.653
	0.8	0.636	0.647	0.631	0.641	0.671	0.660	0.637	0.643	0.641	0.635	0.620
	1.0	0.658	0.646	0.660	0.634	0.650	0.640	0.677	0.643	0.602	0.638	0.633

Table 7(B) Empirical power of the wild-bootstrapped tests (T=50, p=1, $\rho = 0.7$, $\alpha = 5\%$)

Test	σ_2/σ_1 \ τ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.940	0.906	0.869	0.894	0.902	0.938	0.960	0.970	0.981	0.992	0.988
	0.4	0.981	0.972	0.951	0.953	0.947	0.967	0.972	0.987	0.980	0.991	0.981
	0.6	0.982	0.977	0.985	0.975	0.974	0.984	0.972	0.984	0.983	0.984	0.986
	0.8	0.988	0.988	0.983	0.983	0.990	0.991	0.983	0.978	0.990	0.987	0.983
	1.0	0.989	0.991	0.983	0.982	0.987	0.992	0.990	0.989	0.986	0.982	0.992
VR	0.25	0.965	0.931	0.908	0.928	0.932	0.958	0.974	0.977	0.987	0.995	0.991
	0.4	0.985	0.982	0.967	0.971	0.963	0.981	0.979	0.994	0.987	0.995	0.986
	0.6	0.988	0.982	0.993	0.985	0.985	0.993	0.980	0.992	0.992	0.989	0.994
	0.8	0.992	0.991	0.988	0.986	0.994	0.994	0.990	0.987	0.993	0.992	0.989
	1.0	0.990	0.994	0.992	0.986	0.991	0.994	0.996	0.991	0.991	0.988	0.996

Table 8(A) Empirical power of the wild-bootstrapped tests (T=20, p=4, $\rho = 0.7$, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.241	0.195	0.213	0.201	0.205	0.210	0.214	0.238	0.222	0.237	0.243
	0.4	0.218	0.222	0.207	0.208	0.209	0.217	0.218	0.240	0.234	0.263	0.235
	0.6	0.207	0.228	0.206	0.206	0.202	0.229	0.226	0.251	0.254	0.221	0.238
	0.8	0.216	0.214	0.222	0.212	0.219	0.234	0.216	0.229	0.224	0.222	0.211
	1.0	0.239	0.217	0.238	0.224	0.214	0.200	0.206	0.232	0.237	0.213	0.227
VR	0.25	0.498	0.595	0.561	0.480	0.571	0.542	0.534	0.562	0.560	0.538	0.519
	0.4	0.484	0.561	0.539	0.502	0.541	0.523	0.540	0.559	0.559	0.530	0.529
	0.6	0.540	0.541	0.526	0.489	0.522	0.550	0.533	0.548	0.556	0.513	0.545
	0.8	0.540	0.520	0.521	0.515	0.502	0.524	0.514	0.539	0.526	0.534	0.514
	1.0	0.515	0.499	0.543	0.493	0.529	0.529	0.512	0.520	0.542	0.519	0.538

Table 8(B) Empirical power of the wild-bootstrapped tests (T=50, p=4, $\rho = 0.7$, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.843	0.798	0.706	0.707	0.725	0.764	0.823	0.857	0.899	0.921	0.914
	0.4	0.889	0.858	0.818	0.801	0.817	0.830	0.862	0.885	0.901	0.908	0.931
	0.6	0.902	0.926	0.872	0.898	0.891	0.902	0.896	0.904	0.909	0.920	0.931
	0.8	0.911	0.894	0.913	0.908	0.911	0.902	0.903	0.911	0.903	0.914	0.921
	1.0	0.913	0.904	0.920	0.917	0.900	0.906	0.918	0.897	0.921	0.912	0.932
VR	0.25	0.956	0.884	0.828	0.844	0.864	0.902	0.912	0.933	0.957	0.957	0.952
	0.4	0.962	0.942	0.901	0.900	0.916	0.914	0.930	0.945	0.952	0.955	0.953
	0.6	0.957	0.961	0.940	0.954	0.950	0.936	0.949	0.949	0.955	0.950	0.957
	0.8	0.965	0.948	0.953	0.948	0.959	0.949	0.947	0.947	0.961	0.959	0.952
	1.0	0.942	0.951	0.947	0.960	0.945	0.948	0.954	0.947	0.963	0.949	0.959

Table 9(A) Empirical power of the wild-bootstrapped tests (T=20, p=8, $\rho = 0.7$, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.107	0.123	0.095	0.121	0.116	0.088	0.116	0.119	0.126	0.127	0.128
	0.4	0.134	0.102	0.086	0.107	0.113	0.105	0.095	0.120	0.136	0.127	0.126
	0.6	0.110	0.112	0.134	0.104	0.108	0.108	0.109	0.117	0.115	0.136	0.146
	0.8	0.124	0.118	0.119	0.118	0.119	0.142	0.130	0.123	0.129	0.128	0.134
	1.0	0.117	0.112	0.150	0.118	0.107	0.111	0.122	0.121	0.142	0.116	0.135
VR	0.25	0.439	0.477	0.461	0.470	0.460	0.484	0.522	0.483	0.471	0.434	0.431
	0.4	0.417	0.467	0.450	0.436	0.431	0.471	0.455	0.461	0.453	0.456	0.430
	0.6	0.397	0.441	0.451	0.466	0.433	0.455	0.458	0.455	0.478	0.459	0.440
	0.8	0.442	0.440	0.440	0.416	0.458	0.447	0.444	0.453	0.447	0.480	0.432
	1.0	0.434	0.440	0.424	0.450	0.436	0.421	0.427	0.431	0.433	0.450	0.424

Table 9(B) Empirical power of the wild-bootstrapped tests (T=50, p=8, $\rho = 0.7$, $\alpha = 5\%$)

Test	τ / $\frac{\sigma_2}{\sigma_1}$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
HR-LM	0.25	0.663	0.582	0.546	0.520	0.554	0.580	0.640	0.709	0.739	0.764	0.768
	0.4	0.743	0.668	0.652	0.656	0.655	0.658	0.695	0.729	0.720	0.759	0.776
	0.6	0.750	0.771	0.717	0.722	0.716	0.735	0.756	0.742	0.760	0.758	0.793
	0.8	0.765	0.745	0.774	0.746	0.752	0.760	0.763	0.798	0.777	0.787	0.808
	1.0	0.764	0.776	0.780	0.780	0.772	0.760	0.764	0.759	0.776	0.749	0.788
VR	0.25	0.889	0.840	0.702	0.686	0.740	0.775	0.778	0.799	0.820	0.814	0.823
	0.4	0.848	0.838	0.771	0.760	0.766	0.778	0.807	0.847	0.804	0.849	0.820
	0.6	0.854	0.857	0.800	0.792	0.813	0.819	0.814	0.802	0.824	0.828	0.836
	0.8	0.842	0.817	0.857	0.833	0.828	0.830	0.815	0.856	0.844	0.859	0.849
	1.0	0.825	0.843	0.831	0.829	0.836	0.813	0.823	0.818	0.837	0.837	0.842

Figure 1. The heteroskedasticity forms considered in the simulations

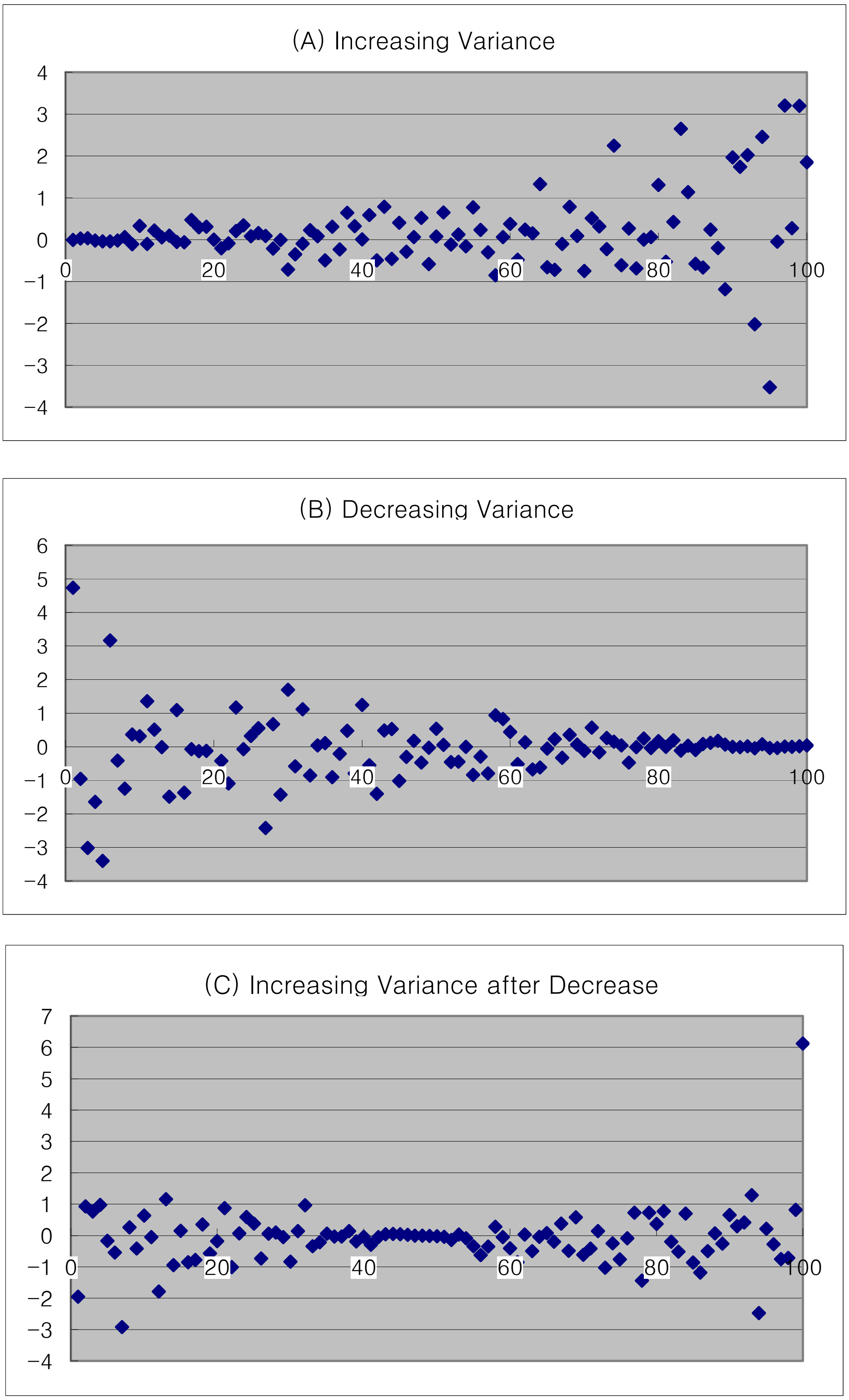

Table 10(A) Empirical size of five tests in continuously increasing variance ($\alpha = 5\%$)

test	B-G LM			FGLS-based LM			Modified VR			wild-bootstrapped HR-LM			wild-bootstrapped VR		
P \ T	20	50	100	20	50	100	20	50	100	20	50	100	20	50	100
1	0.146	0.186	0.212	0.075	0.096	0.105	0.112	0.091	0.059	0.043	0.047	0.053	0.056	0.055	0.056
4	0.239	0.332	0.391	0.075	0.115	0.150	0.071	0.044	0.038	0.048	0.049	0.050	0.058	0.049	0.057
8	0.224	0.469	0.566	0.028	0.112	0.156	0.060	0.039	0.029	0.054	0.055	0.047	0.051	0.047	0.054

Table 10(B) Empirical size of four tests in continuously decreasing variance ($\alpha = 5\%$)

test	B-G LM			FGLS-based LM			Modified VR			wild-bootstrapped HR-LM			wild-bootstrapped VR		
P \ T	20	50	100	20	50	100	20	50	100	20	50	100	20	50	100
1	0.084	0.151	0.190	0.049	0.081	0.108	0.113	0.081	0.067	0.047	0.052	0.046	0.068	0.060	0.052
4	0.051	0.188	0.287	0.035	0.058	0.106	0.062	0.043	0.044	0.039	0.037	0.043	0.056	0.050	0.059
8	0.000	0.136	0.347	0.002	0.027	0.100	0.072	0.044	0.027	0.037	0.044	0.061	0.065	0.060	0.055

Table 10(C) Empirical size of four tests in continuously increasing variance after decreasing ($\alpha = 5\%$)

test	B-G LM			FGLS-based LM			Modified VR			wild-bootstrapped HR-LM			wild-bootstrapped VR		
P \ T	20	50	100	20	50	100	20	50	100	20	50	100	20	50	100
1	0.123	0.155	0.219	0.077	0.129	0.160	0.175	0.101	0.088	0.065	0.051	0.051	0.063	0.061	0.058
4	0.058	0.208	0.298	0.044	0.161	0.263	0.054	0.047	0.041	0.034	0.064	0.040	0.043	0.070	0.047
8	0.014	0.145	0.345	0.003	0.119	0.285	0.060	0.019	0.022	0.032	0.048	0.049	0.045	0.048	0.066

Table 11(A) Empirical power of wild-bootstrapped tests in continuously increasing variance ($\rho = 0.7$, $\alpha = 5\%$)

T / P	wild-bootstrapped HR-LM			wild-bootstrapped VR		
	20	50	100	20	50	100
1	0.422	0.884	0.990	0.521	0.909	0.993
4	0.173	0.697	0.963	0.438	0.837	0.975
8	0.124	0.545	0.917	0.364	0.703	0.918

Table 11(B) Empirical power of wild bootstrapped tests in continuously decreasing variance ($\rho = 0.7$, $\alpha = 5\%$)

T / P	wild-bootstrapped HR-LM			wild-bootstrapped VR		
	20	50	100	20	50	100
1	0.541	0.958	1.000	0.643	0.967	1.000
4	0.280	0.782	0.987	0.575	0.891	0.993
8	0.132	0.586	0.923	0.490	0.774	0.944

Table 11(C) Empirical power of wild bootstrapped tests in continuously increasing variance after decrease ($\rho = 0.7$, $\alpha = 5\%$)

T / P	wild-bootstrapped HR-LM			wild-bootstrapped VR		
	20	50	100	20	50	100
1	0.322	0.844	0.995	0.429	0.876	0.995
4	0.153	0.659	0.931	0.374	0.780	0.966
8	0.113	0.503	0.896	0.305	0.660	0.932