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Abstract

This paper investigates the potential problem of 'pseudo-exogenous' instruments in regression models. We show that the performance of Hausman test is deteriorated when the instruments are asymptotically exogenous but endogenous in finite samples, through Monte Carlo simulations.

Keywords: Hausman test, endogeneity, instrumental variable

JEL classifications: C12, C15

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I. Introduction

When there exist endogenous explanatory variables in a regression model, the least squares estimator fails to achieve consistency. To identify the endogeneity of the explanatory variables, Hausman test is widely employed. Hausman test works pretty well, but it is not free of problems. Meepagala (1992) shows that the power of Hausman test decreases as the sample size becomes smaller. Staiger and Stock (1994) show that 'weak' instruments weaken the power of Hausman test. Wong (1996) proposes a bootstrap procedure to improve the finite sample properties of Hausman test when the instruments are weak.

This paper identifies another potential problem of Hausman test. When the instruments of IV estimation are correlated with the error term of the regression, although the correlation converges to zero eventually, the finite sample performance of Hausman test becomes seriously deteriorated. Let us call such instruments, which are asymptotically exogenous but endogenous in the finite sample, 'pseudo-exogenous' instruments. Pseudo-exogenous instruments, of course, do not affect the asymptotic distribution of Hausman test. However, as we will show through a series of Monte Carlo experiments, the empirical sizes and powers of Hausman test could be considerably inaccurate in finite samples. Especially, we will show the empirical power function of Hausman test actually 'collapses' in some cases.

One of the most popularly used instruments is the fitted value of the endogenous variable from the reduced form regression. This so-called 2SLS (two-stage least squares) is widely used as it gives a proper instrument. Such a fitted value is by construction a pseudo-exogenous instrument. The correlation between the fitted value and the error term is asymptotically zero, but may not be zero in finite samples.

II. Hausman Test with the Pseudo-exogenous Instrument

Let us consider the following model.

$$y = x\beta + u$$

where x is an $(N \times 1)$ vector of explanatory variable, u is an $(N \times 1)$ vector of error terms, and y is an $(N \times 1)$ vector of dependent variable. Suppose there exists an $(N \times 1)$ vector of the instrumental variable, z. We are interested in testing H_0: "x is exogenous" against H_1: "x is endogenous." By a similar derivation as in Bound *et al.* (1995), it is straightforward that

$$\text{plim}\hat{\beta}_{ols} = \text{plim}(x'x)^{-1}x'y = \beta + \frac{\sigma_{xu}}{\sigma_x^{\ 2}}$$

$$\text{plim}\hat{\beta}_{iv} = \text{plim}(z'x)^{-1}z'y = \beta + \frac{\sigma_{zu}}{\sigma_{xz}}$$

where $\sigma_{xu} \equiv \text{plim}(\frac{x'u}{N})$, $\sigma_{zu} \equiv \text{plim}(\frac{z'u}{N})$, $\sigma_{xz} \equiv \text{plim}(\frac{x'z}{N})$ and $\sigma_x^{\ 2} \equiv \text{plim}(\frac{x'x}{N})$.

First, suppose that z is exogenous so that $\frac{\sigma_{zu}}{\sigma_{xz}} = 0$. Let us define $q \equiv \text{plim}(\hat{\beta}_{iv} - \hat{\beta}_{ols})$. If x is exogenous (i.e. H_0 is true) as well as z, then $\frac{\sigma_{xu}}{\sigma_x^{\ 2}} = \frac{\sigma_{zu}}{\sigma_{xz}} = 0$ and $q = 0$. If z is still exogenous but x is endogenous (i.e. H_1 is true), then $\frac{\sigma_{zu}}{\sigma_{xz}} = 0$ and $q = -\frac{\sigma_{xu}}{\sigma_x^{\ 2}}$. Now Hausman statistic H is

$$H \equiv (\hat{\beta}_{iv} - \hat{\beta}_{ols})'\left[\text{Var}(\hat{\beta}_{iv}) - \text{Var}(\hat{\beta}_{ols})\right]^{-1}(\hat{\beta}_{iv} - \hat{\beta}_{ols})$$

which converges to zero under H_0, and diverges from zero under H_1. It has been shown by Hausman (1978) that Hausman statistic has an asymptotic χ^2 distribution under H_0.

Second, suppose z is not exogenous at all. In this case, Hausman test is not defined well. If z is not exogenous, then $\frac{\sigma_{zu}}{\sigma_{xz}} \neq 0$. Thus, even when x is exogenous (i.e. H_0 is true), q is not zero any longer but $\frac{\sigma_{zu}}{\sigma_{xz}}$. When x is endogenous (i.e. H_1 is true), q is now $\frac{\sigma_{zu}}{\sigma_{xz}} - \frac{\sigma_{xu}}{\sigma_x^2}$. Thus, Hausman statistic H no longer converges to zero under H_0, and is no longer guaranteed to diverge from zero under H_1. Accordingly, the null distribution of H is no longer an asymptotic χ^2.

Third, let us consider the case of 'pseudo-exogenous' instrumental variables, which are asymptotically exogenous but endogenous in the finite sample. In this case, Hausman test is well-defined in large samples, but could be problematic in small samples. Although $q \equiv \text{plim}(\hat{\beta}_{iv} - \hat{\beta}_{ols}) = 0$ under H_0 and $q \equiv \text{plim}(\hat{\beta}_{iv} - \hat{\beta}_{ols}) \neq 0$ under H_1, as $E(z'u) \neq 0$ in finite samples, the Hausman statistic, H, may not be close enough to zero under H_0 and/or distinguished enough from zero under H_1. The finite sample distribution of Hausman statistic may not be χ^2 either. The following section examines the effect of the pseudo-exogenous instrument on Hausman test through simulations.

III. Monte Carlo Simulation

To substantiate the effect of the pseudo-exogenous instrument, a Monte Carlo study is performed. Consider the following data generating process (DGP):

$$y = x\beta + u$$

where

$$\begin{pmatrix} x_i \\ z_i \\ u_i \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} s_x^2 & s_{xz} & s_{xu} \\ s_{xz} & s_z^2 & s_{zu} \\ s_{xu} & s_{zu} & s_u^2 \end{bmatrix} \right) = N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xz} & \rho_{xu} \\ \rho_{xz} & 1 & 20\rho_{zu}/N \\ \rho_{xu} & 20\rho_{zu}/N & 1 \end{bmatrix} \right)$$

where i = 1, 2, …, N. For simplicity, s_x^2, s_z^2, s_u^2 and β are all set to one. To make the instrument 'pseudo-exogenous,' s_{zu} is defined as $s_{zu} \equiv \frac{20\rho_{zu}}{N}$. Note that ρ_{zu}, the correlation coefficient between z and u in a sample size of 20 (N=20), is not set to always zero so that z may not be 'fully exogenous' in finite samples. As the sample size increases, however, s_{zu} converses to zero (i.e. the instrument becomes exogenous). Thus, z is a 'pseudo-exogenous' instrument. The correlation coefficient between x and z, ρ_{xz} is set to 0.7 so that we can avoid the so-called 'weak instrument' problems. Four alternative sample sizes are considered: 20, 50, 100, and 500 for comparisons. The simulation has been performed 1,000 times.

Table 1 and Figure 1 present the empirical sizes of Hausman test. First, we notice that the empirical size of Hausman test is not accurate in small samples even when the instrument is perfectly exogenous ($\rho_{zu}=0$). For instance, when N=20 and $\rho_{zu}=0$, the rejection rate is only 0.008 while the nominal size is 0.05. Second, when the instrument is pseudo-exogenous (i.e. $\rho_{zu}\neq 0$), the empirical sizes of Hausman test are seriously distorted. For example, when $N=20$ and $\rho_{zu}=0.7$, Hausman test rejects the true null hypothesis 925 times out of 1,000 simulations: the empirical size is 92.5% while the nominal size is 5%. Such size distortion fades away as the sample size grows, but the empirical size is far from accurate even when N=500: the empirical size is 13.6% while the nominal size is 5%.[2]

Table 1 Empirical sizes of Hausman test ($\alpha = 5\%$)

N	ρ_{zu}				
	0	0.1	0.3	0.5	0.7
20	0.008	0.014	0.145	0.638	0.925
50	0.035	0.052	0.162	0.436	0.781

[2] We experimented how big the sample size (N) should be to achieve an accurate empirical size in the same setup of simulation. We found the rejection rate converged to the nominal size only after N reached 20,000.

100	0.035	0.047	0.108	0.269	0.467
500	0.058	0.047	0.066	0.095	0.136

Figure 1 Empirical sizes of Hausman test ($\alpha = 5\%$)

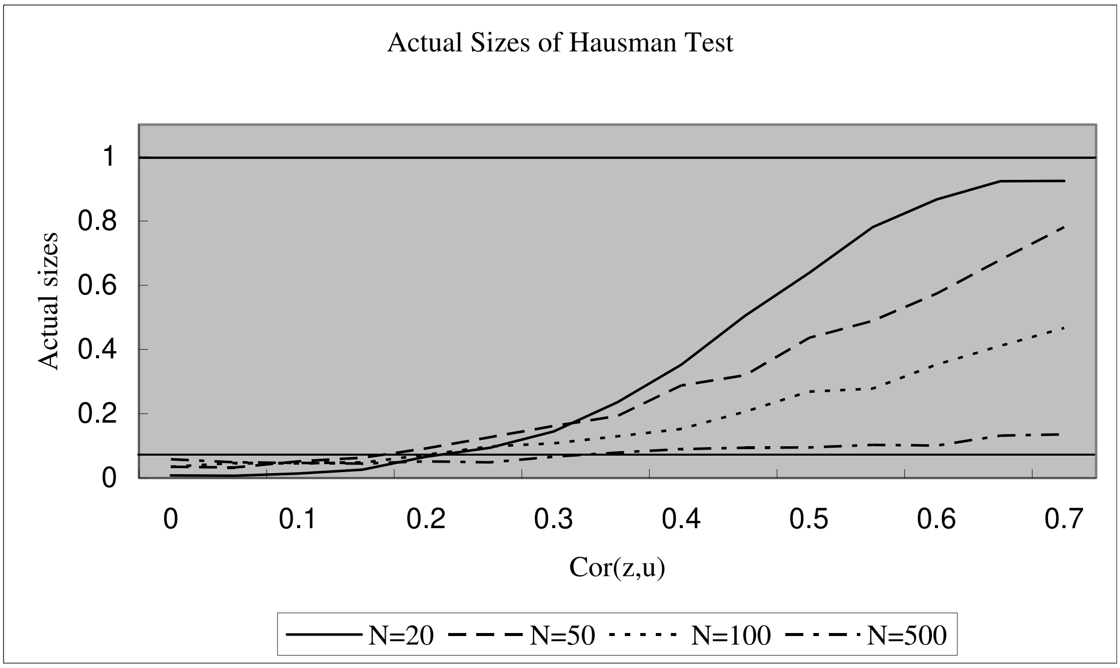


Table 2 shows the empirical powers of Hausman test. First of all, it is obvious that Hausman test does not work well in small samples when the instrument is pseudo-exogenous. For example, even when ρ_{xu}=0.7, Hausman test does not easily reject the false null hypothesis H_0: $\rho_{xu} = 0$ in N = 20 cases, if $\rho_{zu} \neq 0$: the empirical powers are 7.6% (for $\rho_{zu} = 0.3$), 1.4% (for $\rho_{zu} = 0.5$), and 12.3% (for $\rho_{zu} = 0.7$). Generally, the empirical powers are extremely low when N = 20, only a few exceptions showing higher powers than 10% in Table 2. When the sample size is 50, the empirical powers become a bit higher, but still show powers lower than 10% in quite a few cases. Even when the sample size is 100, Hausman test rejects only 13.7% of the false null hypothesis in some case ($\rho_{xu} = 0.1$ and $\rho_{zu} = 0.7$). It should be noted that even when the pseudo-exogeneity of the instrument is quite weak (such as $\rho_{zu} = 0.1$), the empirical power of Hausman test could be pretty low in small samples. For example, in the case of ρ_{xu} = 0.5 and N = 20, the empirical power of Hausman test is 0.342 for ρ_{zu}=0.0 but the power decreases to 0.120 for ρ_{zu} =0.1. It implies that, in small samples, Hausman test could be distorted by a weak correlation between the instrument and the error term even though they are asymptotically independent.

Table 2 Empirical power of Hausman test ($\alpha = 5\%$)

ρ_{xu}	N	ρ_{zu}				
		0	0.1	0.3	0.5	0.7
0.1	20	0.010	0.008	0.079	0.468	0.890
	50	0.070	0.038	0.058	0.157	0.465
	100	0.149	0.088	0.039	0.042	0.137
	500	0.607	0.527	0.473	0.331	0.272
0.3	20	0.054	0.008	0.008	0.140	0.712
	50	0.493	0.316	0.100	0.035	0.068
	100	0.859	0.787	0.547	0.329	0.151
	500	1.000	1.000	1.000	1.000	0.999
0.5	20	0.342	0.120	0.011	0.031	0.383
	50	0.986	0.953	0.697	0.296	0.070
	100	1.000	1.000	0.999	0.990	0.934
	500	1.000	1.000	1.000	1.000	1.000
0.7	20	0.900	0.746	0.076	0.014	0.123
	50	1.000	1.000	1.000	0.981	0.774
	100	1.000	1.000	1.000	1.000	1.000
	500	1.000	1.000	1.000	1.000	1.000

Second, the empirical powers of Hausman test show quite irregular variations as ρ_{zu} increases. Intuitively, the power is expected to become lower as the magnitude of the instrument's endogeneity (= ρ_{zu}) becomes higher. As shown in Table 2, however, the empirical powers do not support such intuition. Sometimes the power becomes *higher* as ρ_{zu} becomes higher (for example, see the case of ρ_{xu} =0.3 and N=20, among others), while sometimes the power becomes lower as expected. It is apparent from Table 2 that the power variations show no consistency at all. The reason why the empirical powers are inconsistent is explained in section II. Although $(\hat{\beta}_{iv} - \hat{\beta}_{ols})$ converges to zero under H_0 and to a positive number under H_1 in large samples, it could well be non-zero under H_0 and zero under H_1 in small samples. As a result, Hausman statistic is not defined well.

The empirical power functions depicted in Figures 2-5 confirm such irregularities. Figure 2 presents the empirical power function of Hausman test when N = 20 for various values of ρ_{zu}. Unlike a typical power function, they do not either approach to the nominal size under H_0, nor approach to 1 under extreme H_1. When the magnitude of pseudo-exogeneity is high (for example, $\rho_{zu} = 0.7$), the power function goes even the opposite way. As sample size grows, such an odd behavior weakens a little. However, even when N = 50 and N = 100, the power function 'collapses' at around $\rho_{zu} = 0.35$ and $\rho_{zu} = 0.15$, respectively.

Figure 2. Power function of Hausman test ($N=20, \ \alpha=5\%$)

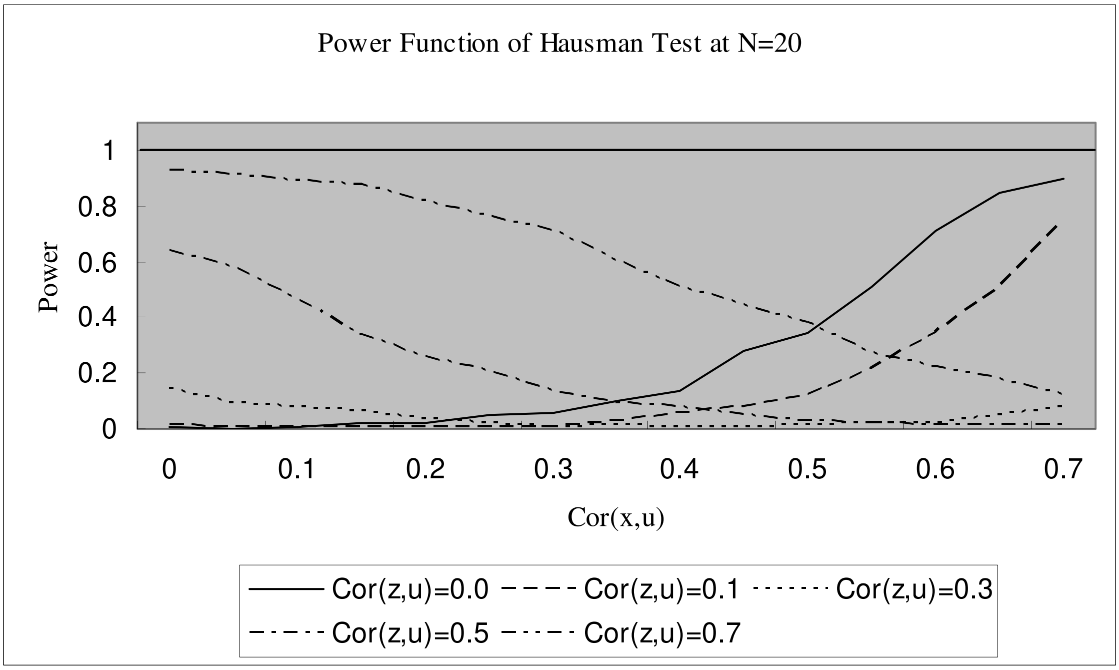


Figure 3. Power function of Hausman test ($N=50, \ \rho_{xz}=0.7, \ \alpha=5\%$)

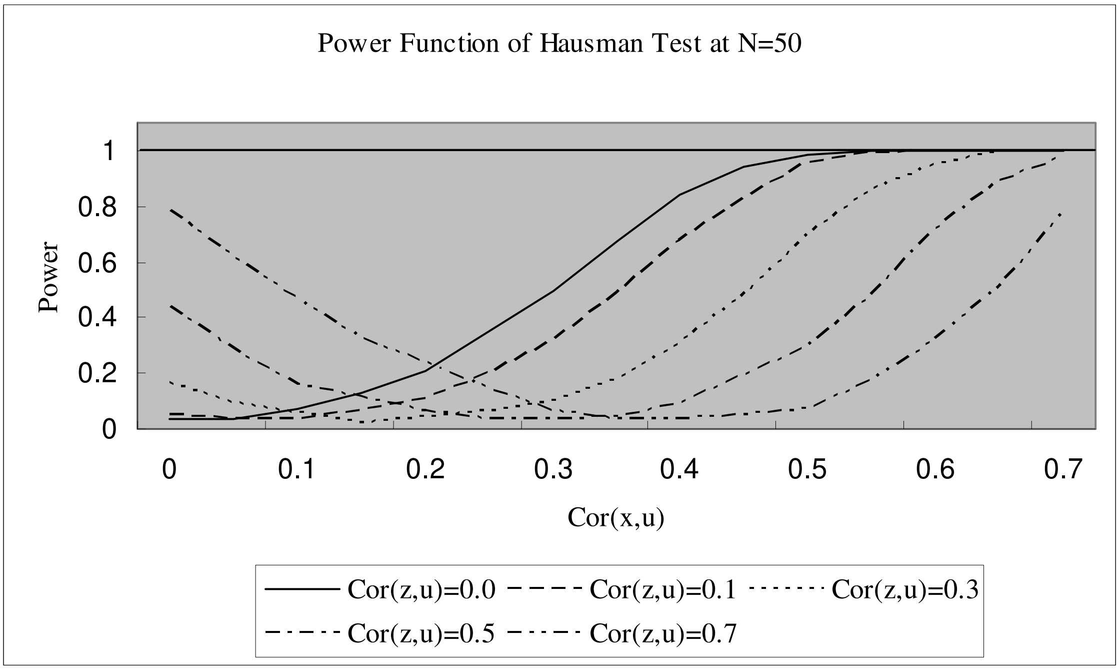


Figure 4. Power function of Hausman test ($N = 100, \ \alpha = 5\%$)

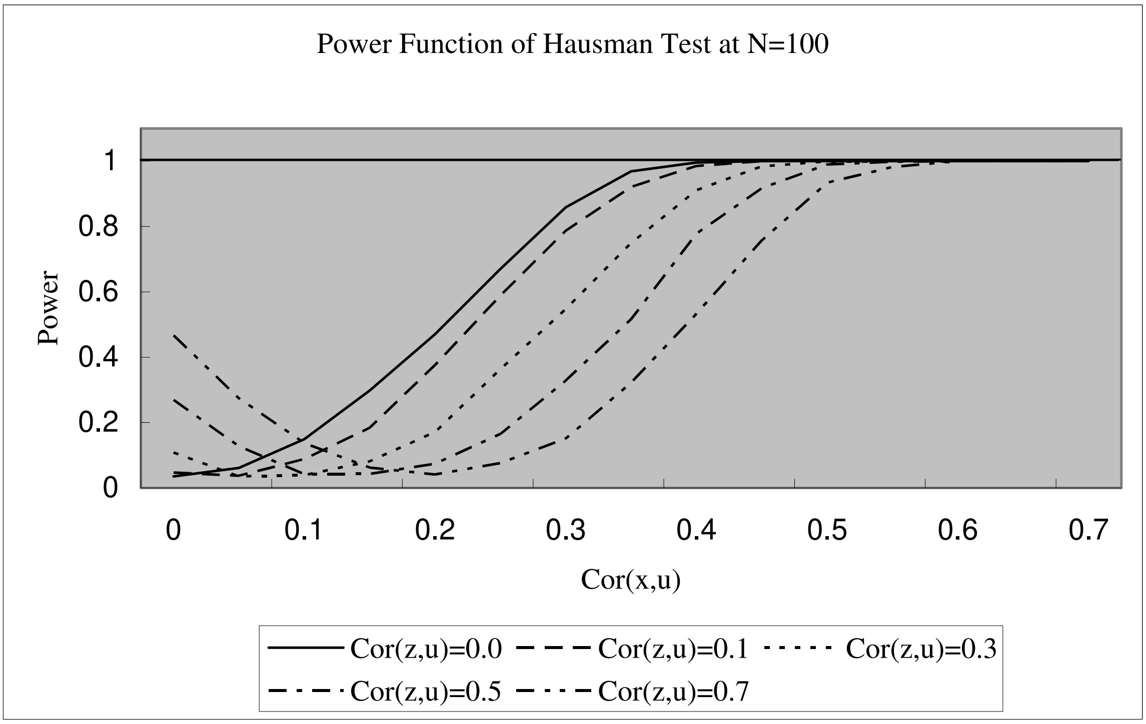
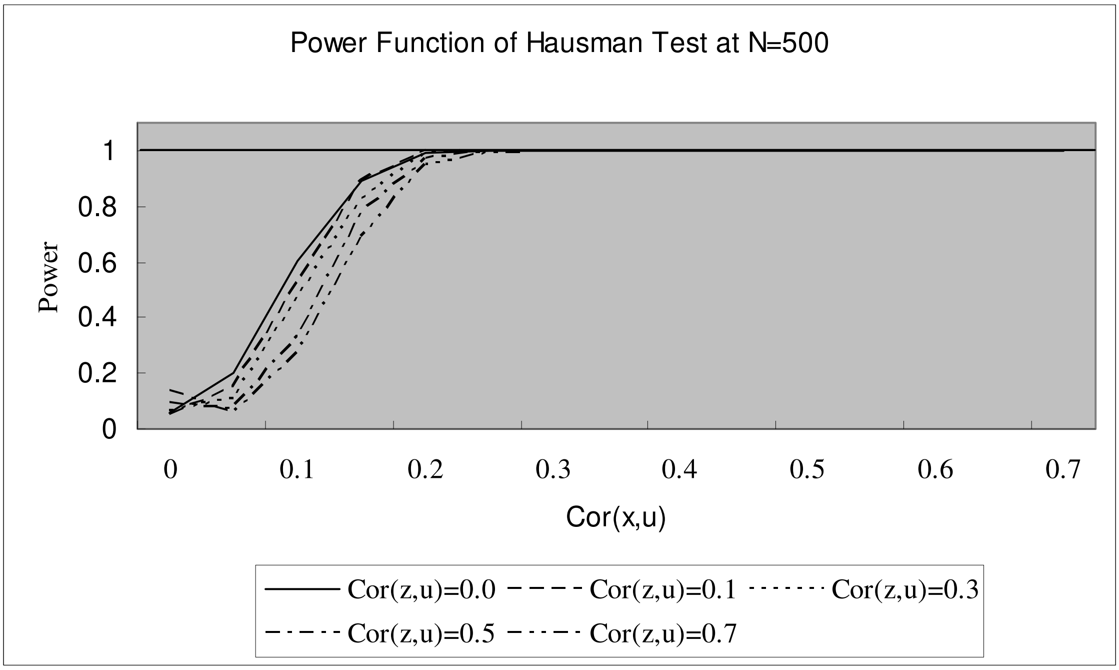


Figure 5. Power function of Hausman test ($N = 500, \ \alpha = 5\%$)



IV. Conclusion

While the problems of 'weak' instruments in IV estimation have been thoroughly studied [3], the problems that 'endogenous' instruments may create have not been studied to a great extent. This paper examines the effects of 'pseudo-exogenous' instruments on Hausman test in finite samples. We show that the size and power of Hausman test could be very inaccurate in finite samples when the instruments are pseudo-exogenous. Researchers need to be cautious about the exogeneity of the instruments when they use IV estimation in practice.

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[3] For examples, Maddala and Jeong (1992) or Bound et al. (1995).