Licensing probabilistic Patents: The duopoly case.

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LICENSING PROBABILISTIC PATENTS: THE DUOPOLY CASE

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Abstract. In this work we study licensing games of non drastic innovations under the shadow of probabilistic patents. We study the situation of a insider innovator that get a new reduction cost innovation and acts in a duopoly market under Cournout competition.

When the property rights are not ironclad the potential licensee additional to the option of use the backstop technology instead of the new technology ,has the option of infringe the patent. Under infringement the patent holder can sue the infringer in a court and if its successful could get a order of damages payment. Then when the infringer decides about what kind of technology to use the infringement is always better than to use the backstop technology then a difference of the ironclad licensing games probabilistic rights, change the threats points and makes attractive for the patent holder just to license big innovations under the Lost Profit rule.

1. Introduction

Once a researcher gets a innovation and obtain a patent, this patent becomes in a right to sue others for unauthorized use of the innovation, this is a consequence that the Patent office (or another entity responsible for give property rights) not necessarily verify that the procedure or knowledge qualifies for be a innovation i.e. in the sense of a innovative step. That means that maybe the innovation could be obvious or just a copy of a existence knowledge technology.

Once the innovation is patented, the patent holder has right to get a monopoly, but we are interested in the type of innovation that do not creates a new product for a direct commercialization instead we are interested in innovations that reduces the costs of production of an existing product, under Cournout competition lower costs are converted in more quantity supply of goods and hopefully in more profits. It is could be possible too that if the innovation is enough big the other firms in the industry could not compete in such case the innovation is called drastic.

Is well know that sometimes the patent holder decides to share the property right with others (licensing), a license is a settlement that permits to a third part uses the innovation in exchange of a pecuniary payment, commonly are used: Fixed fees, royalty rates and auctions as compensation mechanisms. The amounts of such payments are calculated in base to the threat point of the patent holder and the firms in the industry.

In this context some basic questions are:

(1) Which mechanism he will use to give patents? (fixed fees, royalty rates, and so on).
(2) How many licenses he will offer? (maybe no one).
(3) How much the patent holder can ask for his innovation?

(4) How much this responses changes when the patent holder is inside the market or outside the market?
(5) How the industry structure and the kind of competition affect the results?

There is a huge literature that have analyzed this question, the common approach had been the game theoretical analysis, in this approach the patent holder and one or several incumbent firms are participants in a dynamic game of three stages: At the first stage of the game the patent holder decides how much ask for the licenses and how many licenses he will offer. At the second stage the incumbents or potential licensees decide to get the license of continues using the backstop technology, and finally in the last stage the firms compete in the market [1].

Nowadays uncertainty over the property rights plays a important role in licensing, at first, because changes the threat points of the patent holder and the other firms. Because the property rights are not necessarily ironclad, infringement are in some cases profitable for the incumbent firms, in such cases the patent holder can enforce the property rights by using the legal system, where needs to prove infringement, if the patent holder is successful in to prove infringement, the court can authorize the pay of damages in such way that the patent holder be compensated by the infringement of the incumbent(s). Another that could be complementary or substitute is an injunction order, in this case the infringer need to stop the infringement action.

One question that jumps to the arena is How the incumbent firms reacts under probabilistic property rights?. In the ironclad case the firms can just choose between the use of the backstop technology (the best technology without the use of the innovation) or buy a license (it is offer a license by the patent holder), but in the case of probabilistic property rights, infringe is a option too.

Even thought there is a long standing interest and extensive discussions on patent damage rules in the law literature, formal and rigorous economics analyses on this issue are virtually non-existent with the exceptions of Shankerman and Scotchmer , Anton and Yao.

Some recent articles had study consequences of damages rules over the infringement decision of the incumbent firm:

(1) The choose of backstop technology or the option of infringement from the incumbent firm had been analyzed for the cases of a new product [2] and for a cost reducing innovation [1].
(2) case of vertical competition [5][3]
(3) probabilistic property rights just using injunction Farrell and Shapiro [3]

The objective of this work is to fill the gap between liability rules and licensing, even that in most of the cases it is proved that the incumbent prefers to infringe instead to use the backstop technology, one question that by now it is not completely understood is to know if under just liability rules how the patent holder reacts, meaning to respond the following question:

(1) It is better for the patent holder uses fixed fee or a royalty rate?
(2) in which cases the patent holder prefers not to offer a license?
(3) What are the advantages of LP and UE in to deter infringement and give incentives to the innovation?

[1] see [4] and [6] for a survey about licensing games under ironclad rights

We build a simple dynamic game based in the specification of Wang and Anton and Yao, the game has a three stage dynamic specification and the concept used to solve such model is the Sub Game Perfect Nash Equilibrium (SPNE). Our results shows that Under fixed-fee licensing, firm 1 will never license its innovation to firm 2 no matter which liability rule is used (lost profits or unjust enrichment) and under royalty licensing, firm 1 will license its innovation to firm 2 if the innovation is greater and just if the liability rule is LP.

2. Licensing probabilistic patents

We consider a situation called two-supplier world in which there are two suppliers of a product: firm 1 and firm 2. Both firms produced the same good under a fixed marginal cost c, then the cost function is $f(Q_i) = CQ_i$ where $0 < C < A < \infty$ and Q_i is the quantity offered by the firm $i = 1, 2$.

The firm 1 gets a right innovation that reduces the cost from C to $C - \epsilon$, where $0 < \epsilon < C$ and the firm 1 have the option to license this technology to the firm 2.

A licensing game consists of three stages. In the first stage the firm 1 decides to licenses (l) or not licenses (n) his innovation, whether licenses, he sets a fixed fee (F) or a royalty rate(r). In the second stage the firm 2 decides between three alternatives: 1) accept the offer of the patent holder $(\mathcal{L})$; 2) reject the offer and use the backstop technology $(\mathcal{N})$; 3) Infringe the patent $(\mathcal{I})$ (see figure below).

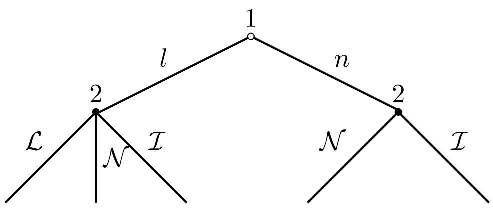

Figure 1. Extensive game form the first and second stages

If the firm 2 uses the new technology without permission, the firm 1 will try to enforces his rights. We assume that the firm 1 has the probability $\theta \in (0, 1)$ of success in the sue and if this happens the firm 2 pays damages by D.

The Damages could be calculated by liability rule as Lost Profits rule (LP) or Unjust Enrichment rule (UE).

In the last stage the two firms compete *ala Cournot* and face a inverse linear demand $P = A - Q$, where $Q = Q_1 + Q_2$ is the total quantity offered by the two firms, and Q_1 and Q_2 are the quantities produced by the patent holder and the incumbent. at the very beginning the two firms p compete facing.

Let us denote by Π_i the profit on the firm $i = 1, 2$, $\pi_i = \Pi_i/(A - C)^2$, $q_i = Q_i/(A - C)$ and $\gamma = \epsilon/(A - C)$.

The concept used for solve this game will be Sub Game Perfect Nash Equilibrium (SPNE), then at first we will solve the third stage , afterwards the second stage and so on.

3. Competition stage

We will began deducing the profits of both firms under different situations, first if the firm 2 decides to use the backstop technology ($\mathcal{N}$),they offer the following quantities $q_1^{\mathcal{N}} = \dfrac{1+2\gamma}{3}, q_2^{\mathcal{N}} = \dfrac{1-\gamma}{3}$ and get get the following profits

(1)
$$\pi_1^{\mathcal{N}} = \begin{cases} \left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\dfrac{1+\gamma}{2}\right)^2 & \text{if } 1\leq\gamma \end{cases} ; \pi_2^{\mathcal{N}} = \begin{cases} \left(\dfrac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ 0 & \text{if } 1\leq\gamma \end{cases}$$

Notice that if $\gamma \geq 1$ then the firm 2 cannot compete in this market, this case is known as an *drastic innovation*, in this case the firm 1 becomes a monopolist, we get that $q_1^{\mathcal{N}} = \dfrac{1+\gamma}{2}, q_2^{\mathcal{N}} = 0$.

A more complex situation emerges under the case that the firm 2 decides to infringe the patent, in this case the patent holder can try to enforce the property rights. We assume that if the patent holder is successful in the court, the court will ask for damages.

$$(2) \qquad \pi_1 = (1-q_1-q_2+\gamma)q_1 + \theta d(q_1,q_2)\,;\pi_2 = (1-q_1-q_2+\gamma)q_2 - \theta d(q_1,q_2)$$

In the two cases the profit is based in two components (see eq.(2)), the first is the part gained by the sales and the second part are the damages, the damages could be calculated in different ways, the most common way to do it is using the Lost Profits rule (**LP**) or the unjust enrichment rule (**UE**), in this work we just analyze this two rules.

The idea behind LP is to compensate the share of profit lost by the patent holder caused by the infringement, then we use the $\pi_1^{\mathcal{N}}$ as the profile profit because under this scenario there is no infringement. Under this assumption the damages in the LP case are

$$d^{LP} = \max\left\{\pi_1^{\mathcal{N}} - (1-q_1-q_2+\gamma)q_1, 0\right\}$$

The solution of the LP case is not trivial and deserves an special treatment (see Anton and Yao [1]), in equilibrium the firm 2 will offer the quantity

$$(3) \qquad q_2^{\mathcal{I},\mathcal{LP}} = \begin{cases} q_2^{\mathcal{N}} & \text{if } 0<\gamma\leq\theta/(3-2\theta) \\ (1-\theta)\dfrac{1+\gamma}{3-\theta} & \text{if } \theta/(3-2\theta)<\gamma \end{cases}$$

When $q_2^{\mathcal{I},\mathcal{LP}} = q_2^{\mathcal{N}}$, the patent holder gets the same profit that in the situation of no infringement if $0 < \gamma \leq \theta/(3-2\theta)$, in the words of Anton and Yao [1] the infringer acts as a *Passive Infringer*, because the firm 1 cannot see the effects of the infringement cause on its profits, and the situation when $q_2^{\mathcal{I},\mathcal{LP}} > q_2^{\mathcal{N}}$ the firm 2 acts as an *Active Infringer* if $\gamma > \theta/(3-2\theta)$.

Then by using the last result is straightforward to obtain that

$$
\begin{aligned}
\pi_1^{\mathcal{I},\mathcal{LP}} &= \begin{cases} \left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } 0<\gamma\le\theta/(3-2\theta) \\ (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2+\theta\left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } \theta/(3-2\theta)<\gamma<1 \\ (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2+\theta\left(\dfrac{1+\gamma}{2}\right)^2 & \text{if } 1\le\gamma \end{cases} \\
\pi_2^{\mathcal{I},\mathcal{LP}} &= \begin{cases} \left(\dfrac{1+2\gamma}{3}\right)\left(\dfrac{1-\gamma}{3}\right) & \text{if } 0<\gamma\le\theta/(3-2\theta) \\ \left(\dfrac{1+\gamma}{3-\theta}\right)^2-\theta\left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } \theta/(3-2\theta)<\gamma<1 \\ \left(\dfrac{1+\gamma}{3-\theta}\right)^2-\theta\left(\dfrac{1+\gamma}{2}\right)^2 & \text{if } 1\le\gamma \end{cases}
\end{aligned} \tag{4}
$$

Now in the case of UE, the damages are calculated in base to excess of profit for the firm 2 respect to the position without infringement, then the damage expression d in the eq (2) is changed by

$$d^{UE}=(1-q_1-q_2+\gamma)q_2-\pi_2^{\mathcal{N}}$$

that it is always positive we obtain in a straightforward way the following profits:

$$
\begin{aligned}
\pi_1^{\mathcal{I},\mathcal{UE}} &= \begin{cases} \left(\dfrac{1+\gamma}{3-\theta}\right)^2-\theta\left(\dfrac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\dfrac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1\le\gamma \end{cases} \\
\pi_2^{\mathcal{I},\mathcal{UE}} &= \begin{cases} (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2+\theta\left(\dfrac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1\le\gamma \end{cases}
\end{aligned} \tag{5}
$$

In the competition stage the offer from the patent holder(amounts or fixed fee or royalty rate) are known. In the case that the patent holder offers a fixed fee F, we obtain the following profits

$$\pi_1^{\mathcal{L},\mathcal{F}}=\left(\frac{1+\gamma}{3}\right)^2+f\,;\pi_2^{\mathcal{L},\mathcal{F}}=\left(\frac{1+\gamma}{3}\right)^2-f \tag{6}$$

where $f=F/(A-C)$, and in the case where the patent holder offered a royalty rate R, we get that

$$\pi_1^{\mathcal{L},\mathcal{R}}=\left(\frac{1+\gamma+r}{3}\right)^2+r\frac{1+\gamma-2r}{3}\,;\pi_2^{\mathcal{L},\mathcal{R}}=\left(\frac{1+\gamma-2r}{3}\right)^2 \tag{7}$$

where $r=R/(a-c)$.

Apparently in both cases the profit levels do not depend on γ, but in fact they do, the amount of a fixed and royalty rates are determined by comparison with the profits in the case of infringement as we will see in the next sections.

4. Second stage

In the second stage of the game the firm 2 have to decide among three situations use the backstop technology $\mathcal{N}$, become a licensee $\mathcal{L}$, or infringe the patent $\mathcal{I}$.

The firm takes its decision depend on the expected payoffs in each situation($\pi_2^{\mathcal{N}}$, $\pi_2^{\mathcal{L}}$ and $\pi_2^{\mathcal{I}}$).

Lemma 1. *If the courts calculates damages using the* LP *rule or the* UE *rule* $\pi_2^{\mathcal{I}} > \pi_2^{\mathcal{N}}$.

The Lemma 1 says that that in any situation infringement is always best than uses the backstop technology, meaning that the liability rules do not deter infringement, in the case of the unjust enrichment rule this result is expected because the damage is calculated as the excess of profit from the infringer compared with the situation of no infringement, then the incumbent could be think why do not infringe? if at worst get the same as he would not infringe the patent in the worst case. A more general proof of this fact could be find it in Anton and Yao [1] for the case of Lost Profits rule.

The Lemma 1 makes irrelevant the action $\mathcal{N}$ for the incumbent firm 2, but even due we cannot order completely the actions of the firm 2, because we need to determine the values of the profits under the license option that depends of the fixed fee or a royalty rate.

By now we have as given the royalty or fixed fee, then we will study if there are the values of the fixed fee (royalty rate) that makes the license option as good as the infringement option for the firm 2, then by using (6) and (7), we can induce that in the case of fixed fee the value that make license as good as infringe is $\underline{f} = \left(\frac{1+\gamma}{3}\right)^2 - \pi_2^I$, notice too that if $\underline{f}$ is negative there is no positive fixed fee or royalty that makes the license option as good as infringe.

Then in the case of UE we get that

$$\underline{f}^{\mathcal{UE}} = \begin{cases} \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1-\gamma}{3}\right)^2 & \text{if } 0 < \gamma < 1 \\ \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1 \leq \gamma \end{cases} \tag{8}$$

Unfortunately this expression is not greater that zero in all the cases, but is straightforward to prove that $\underline{f}^{\mathcal{UE}} > 0$ if

$$\gamma > \frac{12 - 5\theta + \theta^2 - 2\sqrt{(3-\theta)^2(3+\theta)}}{6 - 7\theta + \theta^2}$$

Now in the case of lost profits we get that

$$\underline{f}^{\mathcal{LP}} = \begin{cases} \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+2\gamma}{3}\right)\left(\frac{1-\gamma}{3}\right) & \text{if } 0 < \gamma \leq \theta/(3-2\theta) \\ \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+2\gamma}{3}\right)^2 & \text{if } \theta/(3-2\theta) < \gamma < 1 \\ \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+\gamma}{2}\right)^2 & \text{if } 1 \leq \gamma \end{cases} \tag{9}$$

Notice too that is straightforward to prove that $\underline{f} > 0$ in this case, and in consequence of the past results and because

$$\underline{r} = (1 + \gamma - 3\sqrt{\pi_1^{\mathcal{I}}})/2$$

, we establish that

Lemma 2. *In the case of LP always exits and fixed fee or royalty rate that makes the action $\mathcal{L}$ more profitable fro the incumbent firm. In the case of UE there are this fixed fee or royalty rate just if* $\gamma > \dfrac{12 - 5\theta + \theta^2 - 2\sqrt{(3-\theta)^2(3+\theta)}}{6 - 7\theta + \theta^2}$.

Then under certain circumstances $\pi_2^{\mathcal{L}} > \pi_2^{\mathcal{I}}$ and under this liability rules $\pi_2^{\mathcal{I}} > \pi_2^{\mathcal{N}}$, then we just need to find which is the optimal decision of the patent holder: to offer a license to the incumbent firm or do not.

5. Licensing stage

Now we will see how much the patent holder ask for give the right as we saw in the case of UE not always licensing with a positive fixed fee or positive royalty is possible, first we will see which is the optimal fixed fee or royalty that maximizes the patent holder's profit. In the case of the fixed fee as we now see equation 6 the profit of the patent holder is $\pi_1^{\mathcal{L},\mathcal{F}} = \left(\dfrac{1+\gamma}{3}\right)^2 + f$ then the maximum fixed fee that the patent holder can ask is the solution then $f = \max\{\underline{f}, 0\}$. then using this results and the formulas of the eq (8) and (9).

$$\pi_1^{\mathcal{L},\mathcal{F},\mathcal{UE}} = \begin{cases} 2\left(\dfrac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\dfrac{1-\gamma}{3}\right)^2 & \text{if } \gamma_1 < \gamma < 1 \\ 2\left(\dfrac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2 & \text{if } \gamma \geq 1 \end{cases} \tag{10}$$

The another case is unreachable because the firm 2 always will get more profit by infringing instead of licensing in any scheme of a positive fixed fee, in the case of lost profits we get that

$$\pi_1^{\mathcal{L},\mathcal{F},\mathcal{LP}} = \begin{cases} 2\left(\dfrac{1+\gamma}{3}\right)^2 - \left(\dfrac{1+2\gamma}{3}\right)\left(\dfrac{1-\gamma}{3}\right) & \text{if } \gamma \leq \dfrac{\theta}{3-2\theta} \\ 2\left(\dfrac{1+\gamma}{3}\right)^2 - \left(\dfrac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } \gamma > \dfrac{\theta}{3-2\theta} \\ 2\left(\dfrac{1+\gamma}{3}\right)^2 - \left(\dfrac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\dfrac{1+\gamma}{2}\right)^2 & \text{if } \gamma \geq 1 \end{cases} \tag{11}$$

Now at this moment we have a complete panorama of the situation under fixed fees, our next question to be answered is When the patent holder decides to license the innovation under fixed fees by assuming that infringement is punishes by using liability rules as Lost Profits and Unjust Enrichment, the Proposition 1 says that the patent holder finds more profitable no to license because under other circumstance obtain a greater benefit.

Proposition 1. *The patent holder will never license using a fixed fee*

In the case of royalties (see eq 7)we get that $\pi_1^{\mathcal{L},\mathcal{R}} = \left(\frac{1+\gamma+r}{3}\right)^2 + r\frac{1+\gamma-2r}{3}$ this function reach the maximum when r is equal to $(1+\gamma)/2$, but because $\underline{r} = (1+\gamma-3\sqrt{\pi_1^{\mathcal{I}}})/2$ and $\pi_1^{\mathcal{I},\mathcal{LP}}$ and $\pi_1^{\mathcal{I},\mathcal{UE}}$ are greater than 0, we get that

$$r = \underline{r}$$

.

Now using this result and by direct comparison is straightforward to prove that

Proposition 2. *The patent holder will never license using a royalty rate under UE (drastic or not), but when the innovation is non-drastic and* $\gamma > \frac{\theta(3-2\theta)+3\sqrt{(3-\theta)^2(2-\theta)}}{18-15\theta+4\theta^2}$ *the patent holder license its innovation using a royalty rate just under LP rule. If the innovation is drastic the patent holder always license by using a royalty rate under LP, but never license under UE.*

In the following graph is observable in dark the areas where the patent holder offers a license payed by a royalty rate, notice that only big innovations and licensed by using royalty rates, this could be an explanation of why incumbent firms have a high rate of infringement.

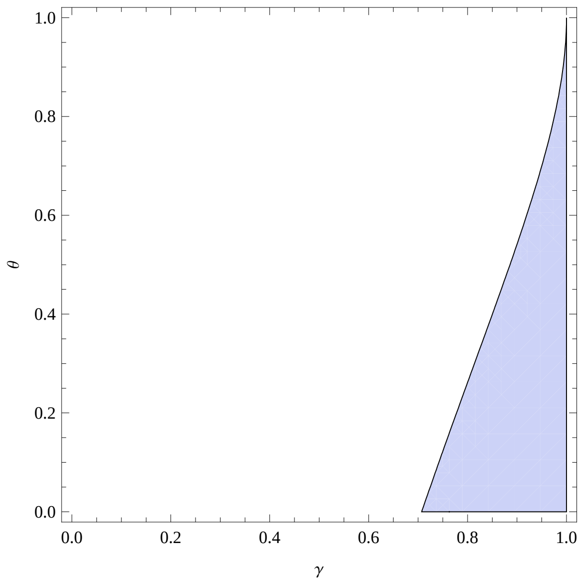

FIGURE 2. $\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} > 0$

6. CONCLUSIONS

One work that is related with this work is the work of Wang [7], in this work he analyzes the ironclad case, his results are sumarizes in three points:

(1) Under fixed-fee licensing, firm 1 will license its innovation to firm 2 if and only if $\gamma < 2/3$. In particular, firm 1 will become a monopoly when the innovation is drastic.

(2) Under royalty licensing, firm 1 will license its innovation to firm 2 if the innovation is non-drastic. In the case of a drastic innovation, firm 1 will become a monopoly.
(3) With either a non-drastic or a drastic innovation, licensing by means of a royalty is at least as good as licensing by means of a fee for the patent-holding firm (firm 1), and licensing by means of a fee is at least as good as licensing by means of a royalty for consumers

our results show that

(1) Under fixed-fee licensing, firm 1 will never license its innovation to firm 2.
(2) Under royalty licensing, firm 1 will license its innovation to firm 2 if the innovation is greater and just if the liability rule is LP.

The reason for this differences is consequence that liability rules and probabilistic patents changes the threats points in the bargaining for licenses, in fact under probabilistic rights the incumbent obtain a greater profit without licensing in comparison with the case of ironclad patents and no licensing.

For another part, the liability rules becomes a transfer system between the infringer and the patent holder making in this way a more profitable infringement for the infringer and for the patent holder.

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Proofs

Proof Lemma 1. For the case under the LP rule, by using the equations (1) and (4) and when $\gamma \leq \theta/(3-2\theta)$, $\pi_2^{\mathcal{I},\mathcal{LP}} = \left(\frac{1+2\gamma}{3}\right)\left(\frac{1-\gamma}{3}\right) > \left(\frac{1-\gamma}{3}\right)^2 = \pi_2^{\mathcal{N}}$

When $\gamma > \theta/(3-2\theta)$,

$$G(\gamma,\theta) = \pi_2^{\mathcal{I},\mathcal{LP}} - \pi_2^{\mathcal{N}} = \left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1+2\gamma}{3}\right)^2 - \left(\frac{1-\gamma}{3}\right)^2$$

, now notice that $G_{11} = \left(\frac{1}{3-\theta}\right)^2 - \frac{4\theta+1}{9}$, because at $\theta = 0$ $G_{11} = 0$ and because $dG_{11}/d\theta = 2(3-\theta)^{-3} - 4/9 < (2)^{-2} - 4/9 < 0$, $G_{11} < 0$ for $\theta \in (0,1)$, then G is concave in γ for $\theta \in (0,1)$.

$G(1,\theta) = \left(\frac{2}{3-\theta}\right)^2 - \theta$, moreover $G_2(1,\theta) = 8(3-\theta)^{-3} - 1 < 0$ for $\theta \in (0,1)$, $G(1,0) = \left(\frac{2}{3}\right)^2$ and $G(1,1) = 0$ then by continuity $G(1,\theta) > 0$ for $\theta \in (0,1)$.

$$G(\theta/(3-2\theta),\theta) = \left(\frac{1}{3-2\theta}\right)^2 - \theta\left(\frac{1}{3-2\theta}\right)^2 - \left(\frac{1-\theta}{3-2\theta}\right)^2 = \frac{\theta(1-\theta)}{(3-2\theta)^2} > 0$$

Then finally because G is concave in γ and $G(\theta/(3-2\theta),\theta), G(1,\theta) > 0$, we conclude that $G > 0$ for $\gamma > \theta/(3-2\theta)$ and $\theta \in (0,1)$

In the UE case by using (5) and (1) we get that

$$\pi_2^{\mathcal{I},\mathcal{UE}} = (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1-\gamma}{3}\right)^2 > \left(\frac{1-\gamma}{3}\right)^2 = \pi_2^{\mathcal{N}}$$

meaning that under the UE the incumbent firm always will prefer infringe the patent over use the backstop technology.

□

Proof Proposition 1. Our next question is to know if $\pi_1^{\mathcal{L},\mathcal{F}} > \pi_1^{\mathcal{I}}$, in the case of UE we get that

$$\pi_1^{\mathcal{L},\mathcal{F},\mathcal{UE}} - \pi_1^{\mathcal{I},\mathcal{UE}} = 2\left(\frac{1+\gamma}{3}\right)^2 - (2-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 = \frac{\theta(2\theta-3)(1+\gamma)^2}{3^2(3-\theta)^2} < 0 \tag{12}$$

Then in the case of UE the patent holder prefer not to license.

Now in the case of LP when $\gamma \leq \frac{\theta}{3-2\theta}$ we get that

(13)
$$\pi_1^{\mathcal{L},\mathcal{F},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} = 2\left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+2\gamma}{3}\right)\left(\frac{1-\gamma}{3}\right) - \left(\frac{1+\gamma}{3}\right)^2 = -\frac{\gamma(1+\gamma)}{3^2} < 0$$

and in the case that $\gamma > \frac{\theta}{3-2\theta}$

(14)
$$\begin{aligned}\pi_1^{\mathcal{L},\mathcal{F},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} &= 2\left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+2\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1+2\gamma}{3}\right)^2 \\ &= -\frac{\theta(3-2\theta)}{3^2(3-\theta)^2} < 0\end{aligned}$$

□