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Abstract

In this paper, I present a rationale for an inheritance tax from the point of view of economic rents derived from ranking value and preference in the state of simultaneous optimality of all heterogeneous households. Because there are family lines that have different probabilities of persistently obtaining rents, these rents should be sufficiently taxed to achieve optimality of all heterogeneous households. Income taxes alone cannot completely remove the negative effects of these rents on optimality because it is difficult to distinguish between types of income. Therefore, an inheritance tax is necessary to complement income taxes. An important point is that the inheritance tax is not a tax on capital income—it is a tax on rents. Because sustainable heterogeneity indicates optimality in a heterogeneous population, this type of inheritance tax does not distort the economy. Rather, unless the negative effects of these rents are sufficiently removed by an inheritance tax, the economy is distorted and most households cannot achieve optimality.

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Keywords: Economic rent; Income tax; Inheritance tax; Inequality; Ranking value and preference; Sustainable heterogeneity

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1 INTRODUCTION

The justification for inheritance taxes has been studied mostly in the framework of the theory of optimal taxation. However, the conclusions of these studies are mixed and depend greatly on the assumptions used, for example, assumptions on the finiteness of life, borrowing constraints, parental behavior, government behavior, the linearity of taxation, and heterogeneity in a variety of factors. Therefore, the justifiability of an inheritance tax remains an open question.

It has been argued that, in general, an inheritance tax should not be levied because taxes on capital incomes distort the economy and inheritance taxes can be interpreted to be a kind of tax on capital incomes (Atkinson and Stiglitz, 1976; Chamley, 1986; Judd, 1985; Atkeson et al., 1999). However, an inheritance tax (or more broadly a tax on capital incomes) can be justified if we introduce certain assumptions, for example, that there are borrowing constraints or uninsured idiosyncratic heterogeneity (Aiyagari, 1995; Conesa et al., 2009), that parents behave according to particular "bequest technology" (Cremer et al., 2003), that an inheritance tax is only levied on bequests above a certain threshold (Saez, 2013), or that a government maximizes its social welfare function, which consists of general social marginal welfare weights for a population that has heterogeneous bequest tastes and labor productivities (Piketty and Saez, 2013). These assumptions may be economically important, but they may be introduced *ad hoc*. In any case, there is no current consensus on the rationale for an inheritance tax.

All of these studies, however, are confined within the traditional framework of the theory of optimal taxation. In this paper, I examine the rationale for an inheritance tax from a different perspective, specifically, from the point of view of sustainable heterogeneity (SH) shown in Harashima (2012, 2014, 2017b)[1] and rent incomes derived from ranking value and preference shown in Harashima (2017a, 2018a, 2018b, 2018c).[2]

SH is defined as the state at which all the optimality conditions of all heterogeneous households are satisfied indefinitely. SH can be achieved with appropriate government interventions even if households are heterogeneous in preferences and behave unilaterally. Conversely, unless a government appropriately intervenes, it is difficult to achieve SH. However, if households are heterogeneous only in their preferences, government interventions for SH do not necessarily necessitate an inheritance tax. Here I show that an inheritance tax is needed if households are heterogeneous in their abilities to persistently obtain rent incomes. Harashima (2019a) showed that, if these abilities are heterogeneous across households, the government has

[1] Harashima (2017b) is also available in English as Harashima (2010).
[2] Harashima (2018b) is also available in English as Harashima (2016).

to intervene with regard to rent incomes to achieve SH. Furthermore, Harashima (2017a, 2018c) showed that rent incomes originating from ranking value and preference exist widely in the economy, but they are unevenly distributed among households. Because of these unevenly distributed rents, the government needs to introduce an inheritance tax to achieve SH. An important point is that an inheritance tax achieves SH, and it is not a tax on capital incomes but on rents. Therefore, it does not distort the economy.

The paper is structured as follows. In Section 2, I discuss ranking value and preference and their relation to persistent rent incomes. I then explore SH and the necessity for government intervention (i.e., an income tax) in Section 3. In Section 4, I develop the argument for the necessity of an inheritance tax and discuss types of inheritance taxes. Finally, concluding remarks are offered in Section 5.

2 PERSISTENT RENT INCOMES

2.1 Ranking value and preference and rent incomes

2.1.1 Ranking value and preference

In this section, the concept of ranking value and preference is explained in brief on the basis of the work of Harashima (2017a, 2018a, 2018b, 2018c).[3]

2.1.1.1 Ranking value

Value is regarded as reflecting something useful. People feel, obtain, or consume value when using, enjoying, or consuming goods and services. Values derived from practical use have usually been considered in economics, but people also consume values derived from ranking. For example, if a curio is evaluated to be the best among a set of similar items, its price will become very high relative to those of the others, regardless of whether it is practically useful. Its price is so high only because it is the top-ranked item in the group. In this sense, people obtain utility not only from practical uses but also from a sense of ranking.

Therefore, value has two components: practical value and ranking value. Practical value is the value that people feel when consuming a good or service for practical purposes. Ranking value is the value that people feel from the rank of a good or service within a set of similar types of goods or services that people use, possess, or observe (e.g., the ranking of a book in a best-seller list or that of a professional baseball team in a league).

[3] Harashima (2018b) is also available in English as Harashima (2016).

2.1.1.2 Ranking preference

Suppose that goods and services have the following properties: quantity, quality, and rank. Quality is related to practical value, rank is related to ranking value, and quantity is related to both values. Suppose also that the quality and rank of each good or service are given exogenously and fixed. Here, for simplicity, I assume that there is only one type of good or service in the economy, and that all goods or services belong to this type (these goods or services are hereafter called "goods" for simplicity) and are substitutable for households' practical uses. Although the goods are substitutable from the point of view of practical uses, they are differentiated from the point of view of rank.

As first developed in Harashima (2018b), let R (= 1, 2, 3, …) be the rank of the goods. The good with rank $R = 1$ is most preferred by households, rank $R = 2$ is the next most preferred, and so on. For simplicity, it is assumed that there is no tied rank. A household's utility derived from consuming goods with rank R is

$$u\left(q_{n,R}, q_{l,R}, R\right) \ ,$$

where $q_{n,R}$ and $q_{l,R}$ are the quantity and quality of the good with rank R, respectively. For simplicity, the utility of the household is modified to

$$u(\tilde{q}_R, R) \ ,$$

where $\tilde{q}_R$ is the "quality-adjusted quantity" of the good with rank R, and $\tilde{q}_R = q_{n,R}\, q_{l,R}$.

The utility function has the following conventional characteristics:

$$\frac{\partial u(\tilde{q}_R, R)}{\partial \tilde{q}_R} > 0$$

and

$$\frac{\partial^2 u(\tilde{q}_R, R)}{\partial \tilde{q}_R^2} < 0 \ .$$

In addition, for any $r \in R$,

$$u(\tilde{q}_r, r+1) < u(\tilde{q}_r, r)$$

and

$$u(\tilde{q}_r, r+2) - u(\tilde{q}_r, r+1) > u(\tilde{q}_r, r+1) - u(\tilde{q}_r, r) \ .$$

2.1.2 Ranking value and superstars

Ranking value and preference confer monopoly powers and thereby rents (which I call "ranking monopoly rents") to the producers of high-ranking products because selling ranking values to consumers requires no additional cost; that is, the marginal cost of producing a ranking value is zero. Therefore, producers of highly ranked products can set prices above their marginal costs (see Harashima, 2017, 2018b). If households' ranking preference is strong enough, therefore, the highest-rank producer (e.g., the best player in a professional sport) can be a superstar. Harashima (2018a) showed that individual players in team sports can be also superstars and that the origin of the extremely high compensation of superstars in team sports is also ranking value and preference.

2.1.3 Product differentiation and monopoly rents

The importance of product differentiation has been emphasized in the study of business administration. The differentiation strategy is one of the three fundamental strategies in Porter's generic strategies (Porter, 1980, 1985). Harashima (2017a) showed that product differentiation is important because it provides firms monopoly powers, profits, and rents that originate from household ranking preferences. Because the strategy of product differentiation is so important (Porter, 1980, 1985) and is actually pursued by many companies, the monopoly rents generated from differentiation will be large and widespread across the economy today and in the future.

Who should receive the ranking monopoly rents that are derived from product differentiation—shareholders, ordinary employees, consumers, or executives? It seems reasonable that they should be distributed according to the level of contribution to the generation of the ranking monopoly rents, but determining the various levels of contribution by the different parties is difficult.

The monopoly rents of firms that successfully differentiate their products commonly originate in ranking value and preference in much the same way as occurs in professional sports. In addition, teams and firms share the common feature that, to obtain a higher rank, it is essential to hire more talented employees—in particular, players or executives. In these respects, a team and a firm can be seen as the same kind of economic agents or organizations in that they are the beneficiaries of households' ranking preferences. Therefore, it is highly likely that the mechanism of executive compensation is the same as that for players' salaries in team sports and that some executives will be compensated like superstars in team sports.

This does not mean that none of the ranking monopoly rent is distributed to shareholders, ordinary employees, or consumers. Indeed, it is quite likely that some of these rents are also distributed, for example, to shareholders in the form of share dividends and capital gains.

2.2 Persistent rent incomes

2.2.1 Family lines

It is likely that there are family lines (FLs) and that some of these will consist of households that are descended from common ancestors and that share similar traits. In addition, in accordance with custom and for other reasons, many groups of people mostly marry within the same or similar groups. Various abilities are exogenously given to people, but the distribution is biased. Therefore, it is reasonable to assume for simplicity that there are different FLs consisting of households that commonly share a given probability of obtaining ranking monopoly rents; moreover, this probability differs from the probabilities of households in other FLs.

Here, I examine ranking monopoly rents in an economy with FLs that are heterogeneous in the probability of obtaining those rents.

2.2.2 Homogeneous FLs

Before examining the case of heterogeneous FLs, I first examine the case of homogeneous FLs for comparison. Suppose that there are two FLs (FL1 and FL2). Each FL equally consists of N (= 1,2,3, …, N) households. A generation of each household lives only one period, but any household continues to exist over many generations indefinitely (i.e., there are equivalently indefinitely living households). Any household in both FLs equally obtains ranking monopoly rents ($z_t = z + \epsilon_t$) in period t (i.e., during the period of generation t) with probability $\frac{1}{2N}$; that is, one and only one of $2N$ households in the two FLs on average obtains z_t in period t where z is a constant and ϵ_t is i.i.d. with mean zero. In other words, any household obtains ranking monopoly rents in every $2N$ generations on average. Note that obtaining ranking monopoly rents means that, for example, in a given generation, a member of a household obtains a post as high-ranking executive in a large company and earns a very high level of compensation. If a household cannot obtain ranking monopoly rents in period t (i.e., during the period of generation t), its incomes decreases by $\frac{z_t}{2N-1}$ in this period (generation) because economic rents indicate transfers of incomes between households, and those transfers are unrelated to production costs.

For any household in either FL, the expected net ranking monopoly rents (i.e.,

the expected z_t minus expected $\frac{z_t}{2N-1}$) are

$$E(z_t) = \frac{1}{2N}z - \left(1 - \frac{1}{2N}\right)\frac{z}{2N-1} = 0\ ,$$

where *E* is the expectation operator. That is, the expected net ranking monopoly rents are zero for any household in either FL, which indicates that z_t has no persistent effect. Hence, if all households are homogeneous, any ranking monopoly rents are interpreted to be "temporarily" obtained.

2.2.3 Heterogeneous FLs

Next, I examine the case of heterogeneous FLs. In this case, the two FLs are identical except for the probability of obtaining ranking monopoly rents. Each household in FL1 obtains ranking monopoly rents in a period with probability $\frac{2}{3N}$ (in other words, it obtains them in every $\frac{3N}{2}$ generations on average). On the other hand, each household in FL2 obtains them in a period with probability $\frac{1}{3N}$ (in other words, it obtains them once in every 3*N* generations on average). The probability of obtaining rents in households in FL1 is therefore two times higher than that of households in FL2. On average, one and only one of 2*N* households in the two FLs obtains z_t in each period.

For a household in FL1, the expected ranking monopoly rents are

$$E(z_t) = \frac{2}{3N}z - \left(1 - \frac{2}{3N}\right)\frac{z}{2N-1} = \frac{z}{6N-2} > 0\ ,$$

and those for a household in FL2 are

$$E(z_t) = \frac{1}{3N}z - \left(1 - \frac{1}{3N}\right)\frac{z}{2N-1} = -\frac{z}{6N-2} < 0\ .$$

Therefore, the expected ranking monopoly rents of a household in FL1 are positive but those in FL2 are negative, and unlike the case of a homogeneous population, ranking monopoly rents have persistent effects.

At first glance, ranking monopoly rents may look like temporarily (or "luckily") obtained incomes because they can only be intermittently obtained by any household. However, they can have persistent effects as shown above and are actually "persistently"

obtained rent incomes when households are heterogeneous in the probability of obtaining ranking monopoly rents.

2.2.4 Definition of persistently obtained ranking monopoly rents

The natures of ranking monopoly rents are different depending on whether FLs are homogeneous or heterogeneous. If FLs are homogeneous, ranking monopoly rents are temporary incomes, whereas they are persistent if FLs are heterogeneous because the $E(z_t)$ of households in at least one FL must be positive if FLs are heterogeneous. The heterogeneity in abilities across FLs ultimately generates the persistent nature of ranking monopoly rents.

Consequently, we can define the "persistence" of ranking monopoly rents by whether or not the FLs are heterogeneous in their abilities. In this paper, ranking monopoly rents are considered to be persistent if FLs are heterogeneous in their abilities. The rents are temporary if the FLs are homogeneous in their abilities.

2.3 Temporary rents and other forms of persistent rents

Persistently obtained rent incomes are not limited to persistently obtained ranking monopoly rents. For example, there are rents obtained from natural monopolies or from natural resources like land and oil. I call these various kinds of persistently obtained rent incomes as a whole "persistent rent incomes."

Furthermore, the definition of persistent rent incomes given in Section 2.2.4 indicates that there are also temporary rent incomes. If the probability of obtaining any rent income is identical in all heterogeneous households, it is considered to be temporary rent income. There are many examples of temporary tent incomes, including lottery prizes and other one-time "lucky" occurrences that can affect profits and incomes.

Persistent rent incomes can be purchased by and exchanged with other households. Furthermore, a household can exchange its temporary rent incomes for various forms of persistent rent incomes; for example, they can buy shares of companies or land (some parts of share dividends and capital gains from shares are types of persistently obtained ranking monopoly rents). An owner of temporary rent incomes therefore can become an owner of persistent rent incomes.

3 SUSTAINABLE HETEROGENEITY AND INCOME TAX

3.1 Sustainable heterogeneity

In this section, I explain SH briefly on the basis of the work of Harashima (2012, 2014, 2017b, 2019a).[4]

3.1.1 The SH model

Suppose for simplicity that there are only two economies—Economy 1 and Economy 2—that are identical except for the rate of time preference (RTP). Each economy consists of its own identical households, respectively. Let θ_1 and θ_2 be the RTPs of households in Economies 1 and 2, respectively, and $\theta_1 < \theta_2$. The population growth rate is zero in both economies. The two economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Because the economies are fully open, they are integrated through trade and form a combined economy. The combined economy can be interpreted as the world economy (the international interpretation) or the national economy (the national interpretation). Usually, the concept of the balance of payments is used only for international transactions, but because both national and international interpretations are possible, this concept and terminology are also used for the national economy model in this paper.

Because a balanced growth path requires Harrod-neutral technological progress, the production function of Economy i is assumed to be

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha}$$

for $i = 1$ or 2, where $y_{i,t}$ and $k_{i,t}$ are the per capita production and capital, respectively, of Economy i in period t; A_t is the technology in period t; and α ($0 < \alpha < 1$) is a constant. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies, and the economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ are returns on investments,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \quad \text{and} \quad \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

[4] Harashima (2017b) is also available in English as Harashima (2010).

represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies, such that

$$\tau_t = \kappa\left(k_{1,t}, k_{2,t}\right) .$$

The government (or an international supranational organization under the international interpretation) can intervene in the activities of Economies 1 and 2 by transferring money between the two economies. The amount of transfer from Economy 1 to Economy 2 in period t is g_t, and it is assumed that g_t depends on capital, such that

$$g_t = \bar{g}_t k_{1,t} .$$

$\bar{g}_t$ is an exogenous variable for households and firms and is appropriately adjusted by the government (or an international supranational organization) in every period so as to achieve SH. Because $k_{1,t} = k_{2,t}$ and $\dot{k}_{1,t} = \dot{k}_{2,t}$,

$$g_t = \bar{g}_t k_{1,t} = \bar{g}_t k_{2,t} .$$

Each household in Economy 1 maximizes its expected utility

$$E \int_0^\infty u_1\left(c_{1,t}\right) \exp\left(-\theta_1 t\right) dt$$

subject to

$$\frac{dk_{1,t}}{dt} = A^{\alpha}k_{1,t}^{1-\alpha} - c_{1,t} + (1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\left(\int_0^t \tau_s\, ds + z_0\right) - \tau_t - \bar{g}_t k_{1,t}\ ,$$

and each household in Economy 2 maximizes its expected utility

$$E\int_0^{\infty} u_2(c_{2,t})\exp(-\theta_2 t)dt$$

subject to

$$\frac{dk_{2,t}}{dt} = A^{\alpha}k_{2,t}^{1-\alpha} - c_{2,t} - (1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\left(\int_0^t \tau_s\, ds + z_0\right) + \tau_t + \bar{g}_t k_{2,t}\ ,$$

where $c_{i,t}$ is the per capita consumption of Economy i in period t, u_i is the utility function of Economy i, and E is the expectation operator.

3.1.2 SH

Harashima (2017b) showed in an endogenous growth framework that, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions of both economies are satisfied (i.e., SH is achieved). Even if the government does not intervene (i.e., $\bar{g}_t = 0$), SH is achieved if the economies behave multilaterally in the sense that each economy behaves fully considering the optimality conditions of the other economy. On the other hand, if the economies behave unilaterally in the sense that an economy behaves without considering the other economy's optimality conditions, SH is not achieved unless a government appropriately intervenes. The reason why SH can be achieved in the cases of multilateral behaviors and appropriate government intervention is that the capital accumulation of the more advantaged Economy 1 is restrained by the multilateral behavior and appropriate government intervention. If SH is achieved, the growth rates of consumption in both economies are equal:

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1+\theta_2}{2}\right]\ , \qquad (1)$$

where m, v, and ϖ are positive constants, and $\varepsilon = -\frac{c_{1,t}u_1''}{u_1'} = -\frac{c_{2,t}u_2''}{u_2'}$ is the degree of relative risk aversion and is constant.

Harashima (2017b) indicated that SH also exists, even in the framework of exogenous growth (e.g., Ramsey-type growth models) with a heterogeneous population. The capital accumulation of the more advantaged Economy 1 is also restrained at SH in exogenous growth models. Hence, the capital (wealth) that a household in Economy 1 owns at SH is not k_1 but $k_1 + \Psi$, where Ψ is a negative constant, and the capital that a household in Economy 2 owns at SH is not k_2 but $k_2 - \Psi$, where $k_{i,t}$ is identical for any i through market arbitration (i.e., $k_{i,t} = k_t$ for any i).

Note that Harashima (2017b) showed that the two-economy model can be easily extended to multi-economy models, and the results in multi-economy models are basically the same as those in the two-economy model.

3.2 Government intervention

3.2.1 Government intervention when no rent income exists

Harashima (2012) showed that, if a government intervenes such that

$$\lim_{t\to\infty} \bar{g}_t = \frac{\theta_2 - \theta_1}{2} ,$$

then SH is achieved even if Economy 1 behaves unilaterally, and equation (1) is satisfied.[5] When SH is achieved, Economies 1 and 2 consist of a combined economy (Economy 1+2) with twice the population and an RTP of $\frac{\theta_1+\theta_2}{2}$. Suppose that there is a third economy with RTP of θ_3, and it is identical to Economies 1 and 2 except for its RTP. Because Economy 1+2 has twice the population of Economy 3, if a government intervenes such that the amount of (positive or negative) transfers from a household in Economy 1+2 to households in Economy 3 in period t is $g_t = \bar{g}_t k_{3,t} = \bar{g}_t k_t$,[6] SH is achieved where $k_{3,t}$ is capital in Economy 3 in period t, and

$$\lim_{t\to\infty} \bar{g}_t = \frac{\theta_3 - \frac{\theta_1+\theta_2}{2}}{3} .$$

Remember that $k_{i,t}$ is identical for any i through arbitration, as shown in Section 3.1.2. By iterating similar procedures, if a government's (positive or negative) transfers from a household in Economy 1+2+ · · · + (H – 1) to households in Economy H are made by

[5] Households in Economy 2 share the transfers equally with each other.
[6] Households in Economy 3 share the transfers equally with each other.

$$\lim_{t\to\infty} \bar{g}_t = \frac{\theta_H - \frac{\sum_{q=1}^{H-1} \theta_q}{H-1}}{H} ,$$

then SH is achieved.[7]

3.2.2 Government intervention when rent incomes exist

Harashima (2019a) showed that SH is achieved even if the government does nothing with regard to rent incomes if they are temporary. If they are persistent, however, SH cannot be achieved unless the government appropriately intervenes. In this section, I briefly explain how a government should intervene for persistent rent incomes on the basis of the work of Harashima (2019a).

Suppose for simplicity that there are only two FLs and there is only one household in each of the two FLs. The households in both FLs are identical except for RTP and the probability of obtaining rent incomes. Only the household in FL1 obtains rent incomes z_t in period t. As a result, the incomes of the household in FL2 are reduced by z_t in period t. Suppose also that the household in FL1 obtains z_t in every period; that is, z_t is persistent and that $z_t = \bar{z}k_{1,t}$ where $\bar{z}$ is a constant. In this case, if the government appropriately controls the value of $\bar{g}_t$ and

$$\lim_{t\to\infty} \bar{g}_t = \bar{z} + \frac{\theta_2 - \theta_1}{2} , \tag{2}$$

SH is achieved. That is, if rent incomes are persistent, fiscal transfers from the household in FL1 to the one in FL2 by $z_t = \bar{z}k_{1,t}$ are needed to achieve SH in addition to the transfers for heterogeneous preferences.

3.3 Income tax on persistent rent incomes

Next, I examine how a government should impose income tax on z_t based on the two-FL model used in Section 2 in which there are N households and a household in FL1 obtains z_t in a period with probability $\frac{2}{3N}$ and that in FL2 obtains z_t in a period with probability $\frac{1}{3N}$, with z_t defined as persistent rent incomes, as defined in Section 2.2.4. Suppose for simplicity that there is no technological progress and $k_{i,t}$ is constant for any t, and thereby $z_t = z + \epsilon$, as was also the case in Section 2.2.

[7] Households in Economy H share the transfers equally with each other.

The result in Section 2.2 suggests that one of the easiest ways to impose income taxes on z_t is that any household in FL1 pays income taxes $\frac{z}{6N-2}$ in any period (i.e., even in the periods when it does not obtain z_t), and money equivalent to $\frac{z}{6N-2}$ is transferred to each household in FL2 in any period (i.e., even in the periods when it obtains z_t). However, this kind of income tax will not be practically feasible because it is difficult for a government to correctly know which FL a household belongs to. In addition, this income tax will not be supported by households in FL1 because income taxes are imposed on them on z_t in any period, regardless of whether they actually obtained z_t. At the same time, households in FL2 constantly receive transfers in any period regardless of whether they obtained z_t.

Another possible way is to impose income taxes on z_t only when a household obtains z_t. These income taxes on z_t are imposed in any period but only on the household that actually obtains z_t in each period, regardless of FL. In this case, the tax rate on z_t should be 100% for the following reason. Suppose that the tax rate is χ ($0 < \chi \leq 1$) and the amount of income tax is χz_t. If $0 < \chi < 1$,

$$\lim_{t\to\infty} \bar{g}_t = \chi\bar{z} + \frac{\theta_2 - \theta_1}{2} < \bar{z} + \frac{\theta_2 - \theta_1}{2} ,$$

and therefore equation (2) cannot be satisfied. It is only when $\chi = 1$ that equation (2) can be satisfied and SH can be achieved.

4 NECESSITY OF AN INHERITANCE TAX

4.1 Difficulty in estimating z_t

Even if a government decides to impose an income tax on z_t, restricting it to only households that obtain z_t in each period, another problem arises. Before imposing an income tax on z_t, a government has to distinguish between z_t and other household income, particularly between persistent and temporary rent incomes. In other words, the government would ideally know the exact and correct value of z_t in each period. However, in reality, it is extremely difficult to distinguish between persistent and temporary rent incomes, or in other words, to estimate how much luck contributed to a household's current income. Correctly distinguishing between them would require observing and intensely analyzing the incomes of all households in all FLs over many generations in detail.

Faced with this difficulty, it seems highly likely that a government will make a

conservative estimate of the value of z_t (i.e., a relatively smaller one) because households will be much less satisfied (angrier) if z_t and the corresponding income tax are overestimated than if they are underestimated. Therefore, a government will more greatly fear the case of overestimating z_t than that of underestimating it.

4.2 Unrealized capital gains

The basic mechanism of achieving SH through government interventions is that the capital accumulation of a household in FL1 is compulsorily restrained and that in a household in FL2 is enhanced by the government. This mechanism implies that taxation implemented to achieve SH should target not only income but also capital owned by households. This aspect is important if there are incomes that cannot be easily identified by the government. Suppose that two households obtain the same amount of incomes that are recognized by the government, but in addition to these incomes, one of the two households also obtains incomes that cannot be identified by government and therefore not taxed. In this case, although the amount of income taxes imposed are the same for both households, the capital held by the two households will grow differently, which means that the mechanism of achieving SH cannot work if it only depends on an income tax.

Do uneasily identifiable incomes exist? Unfortunately, they do, for example, as unrealized capital gains. In many countries, capital gains are not taxed until they are realized because, from a practical standpoint, it is difficult to tax unrealized capital gains. Hence, if a household keeps a large amount of capital gains unrealized over generations, it can evade a large amount of income tax in the absence of government intervention. As a result, that household's capital accumulation cannot be sufficiently controlled by the government to achieve SH.

4.3 Inheritance tax as a necessity to achieve SH

If income taxes are not necessarily sufficient as a tool to achieve SH for the reasons discussed in Sections 4.1 and 4.2, another type of tax that can help achieve SH is needed. One such tax is the inheritance tax.

4.3.1 Potential of an inheritance tax

If a variable and its components fluctuate largely (in this case, income and its component parts), it can be difficult to distinguish between the component elements (in this case, persistent and temporary incomes). Flow variables can fluctuate widely, and income is a flow variable. On the other hand, stock variables generally do not fluctuate as widely, and capital is a stock variable. Hence, the difficulty in distinguishing between the components

in capital will be much lower than it is for income. For example, if the capital owned by a household in a FL persistently deviates upwards from the average over many generations, it is highly likely that that capital is being accumulated largely as a result of persistent rent incomes because a household generally will not have a run of extraordinarily good luck over many generations.

Because the tax base of an inheritance tax is made up of assets that are roughly equivalent to capital, the problem of distinguishing between the temporary and persistent components will be much smaller in the case of inheritance tax than it will be in the case of an income tax. A government will be able to at least roughly correctly estimate accumulated capital resulting from persistent rent incomes by observing a household's capital holdings. Because of this nature, an inheritance tax has the potential to be a useful supplement to the income tax to achieve SH. The importance of the ability of a government to distinguish between components is particularly important when it sets inheritance tax exemptions, as is discussed in Section 4.6.2.

4.3.2 Necessity of an inheritance tax

Suppose again that there are two FLs and that there is only one household in each FL. In addition, time is discrete and there is no technological progress. For simplicity, assume that any generation of a household in each FL equally lives for only one period, and only the household in FL1 obtains persistent rent incomes z_t and that it obtains them in every period (i.e., in every generation). The government underestimates $\bar{z}$ and cannot identify part of $\bar{z}$ for the reasons discussed in Sections 4.1 and 4.2. Therefore, income taxes are imposed according to

$$\bar{g}_t = (\bar{z} - \check{z}) + \frac{\theta_2 - \theta_1}{2}\ , \tag{3}$$

where $\check{z}$ is a constant, $\bar{z} - \check{z}$ indicates the underestimated amount of rent income, and $0 < \check{z} < \bar{z}$. Equation (3) indicates that the amount of income taxes will be less than the necessary amount for SH because equation (2) must be satisfied to achieve SH. Note that per capita capital used (not owned) by FL1 and FL2 are still $k_{1,t}$ and $k_{2,t}$, respectively, and that $k_{1,t} = k_{2,t}$.[8] Because the income tax is imposed following equation (3), however, the household in FL1 owns more capital than $k_{1,t}$, and the household in FL2 owns less capital than $k_{2,t}$.

To achieve SH, therefore, the government introduces an inheritance tax in

[8] Capitals (equivalently, machines, equipment, and so on) can be used by households (workers) even if these capitals are not owned by them.

addition to the income tax. The inheritance tax is imposed on the households in both FLs in every period (equivalently for every generation) such that the amount of inheritance tax for the household in FL1 is

$$\delta\left(k_{1,t} + \check{z}k_{1,t} - \bar{k}\right),$$

and that for FL2 is

$$\delta\left(k_{2,t} - \check{z}k_{2,t} - \bar{k}\right),$$

where δ $(0 < \delta \leq 1)$ is the constant inheritance tax rate and $\bar{k}$ is the constant tax exemption that is common to both FLs. $k_{1,t} + \check{z}k_{1,t}$ and $k_{2,t} - \check{z}k_{2,t}$ indicate the respective capital (assets) of households in FL1 and FL2 after paying the income tax $(\bar{z} - \check{z})k_{1,t}$ and receiving the transfers with regard to heterogeneous RTPs. The tax exemption is set to satisfy

$$k_{2,t} - \check{z}k_{2,t} < \bar{k} < k_{1,t} + \check{z}k_{1,t} \ .$$

Because the value of $k_{2,t} - \check{z}k_{2,t} - \bar{k}$ is negative, the household in FL2 is exempted from the inheritance tax, and it instead receives a transfer equivalent to the amount of the inheritance tax on the household in FL1 from the government. Therefore, because of the inheritance tax, the capital owned by the household in FL1 is reduced from $k_{1,t} + \check{z}k_{1,t}$ to $(1 - \delta)\left(k_{1,t} + \check{z}k_{1,t}\right)$, but that owned by the household in FL2 is unchanged (i.e., $k_{2,t} - \check{z}k_{2,t}$) before receiving the positive government transfers with regard to the inheritance tax.

For simplicity, suppose that the amount of $\check{z}k_{1,t}$ is constant such that

$$\check{z}k_{1,t} = \check{z}\tilde{k}_1 = z_R = \text{constant} \tag{4}$$

for any *t* where $\tilde{k}_{1,t}$ is $k_{1,t}$ at SH in the case that there is no persistent rent income, and that for any *t*,

$$\bar{k} < k_{1,t} + z_R = k_{1,t} + \check{z}\tilde{k}_1 \ . \tag{5}$$

Because time is discrete and there is no technological progress,

$$k_{1,t+1} = k_{1,t} + z_R - \delta\left(k_{1,t} + z_R - \bar{k}\right) = (1-\delta)\left(k_{1,t} + z_R\right) + \delta\bar{k} \ .$$

By iteration,

$$k_{1,t+2} = (1-\delta)\left(k_{1,t+1} + z_R\right) + \delta\bar{k}$$
$$\vdots$$
$$k_{1,t+m} = (1-\delta)\left(k_{1,t+m-1} + z_R\right) + \delta\bar{k} \ ,$$

and thereby,

$$k_{1,t+m} = (1-\delta)^m k_{1,t} + (1-\delta) z_R \sum_{q=1}^{m} (1-\delta)^{q-1} + \delta\bar{k} \sum_{q=1}^{m} (1-\delta)^{q-1} \ .$$

Hence,

$$\lim_{m\to\infty} k_{1,t+m} = z_R \left(\frac{1-\delta}{\delta}\right) + \bar{k} = \text{constant.} \tag{6}$$

Equation (6) indicates that the capital owned by the household in FL1 does not explode but eventually converges at a certain constant value, that is, $z_R \left(\frac{1-\delta}{\delta}\right) + \bar{k}$.

By equations (4) and (6), if the tax rate is set to be

$$\delta = \frac{\check{z}\tilde{k}_1}{\check{z}\tilde{k}_1 + \tilde{k}_1 - \bar{k}} \ , \tag{7}$$

then

$$\lim_{m\to\infty} k_{1,t+m} = \tilde{k}_1 \ . \tag{8}$$

If equations (7) and (8) are satisfied and the revenues from the inheritance tax are appropriately transferred to the household in FL2, SH is achieved. Let the tax rate that satisfies equation (7) (the "SH tax rate") be δ_{SH}. By equation (7), as the value of $\bar{k}$ approaches the value of $\tilde{k}_1$, δ_{SH} approaches its upper bound (i.e., unity), and if $\bar{k} = \tilde{k}_1$, $\delta_{SH} = 1$. If $\bar{k} = 0$, $\delta_{SH} = \frac{\check{z}}{1+\check{z}}$, and if $\bar{k} > \tilde{k}_1$, the SH tax rate cannot be set because $\delta \leq 1$.

Equation (8) indicates that the capital owned by the household in FL1 will not

deviate upwards largely from that owned by the household in FL2 if the inheritance tax rate is the SH tax rate. With the appropriate tax rate, therefore, an inheritance tax can supplement the income tax to achieve SH. In this sense, it may be seen as the final "safety net" for achieving SH.

An important point is that the state indicated by SH is efficient because all of the optimality conditions of all heterogeneous households are satisfied. In addition, the inheritance tax for achieving SH is not a tax on capital incomes, but rather it is a tax on rents. That is, the inheritance tax does not hinder efficiency. On the contrary, it is a necessity for efficiency because SH cannot be achieved without it.

4.4 Way to set the SH tax rate approximately

Equations (7) and (8) indicate that before setting δ, a government needs to know the value of $\check{z}$. It will, however, be difficult to set δ for similar reasons as those discussed in Section 4.1 for the value of z_t in the case of an income tax. If δ is incorrectly set such that $\delta < \delta_{SH}$, increases in capital of the household in FL1 will accelerate, and SH cannot be achieved. If it is incorrectly set in the opposite direction such that $\delta > \delta_{SH}$, $k_{1,t}$ will decrease to $\bar{k}$, which also means that SH cannot be achieved because $\bar{k} < k_{1,t} + z_R$ for any t as indicated in inequality (5) and thereby $\bar{k} < \tilde{k}_1$. Moreover, as shown in Section 4.3.2, if $\bar{k} > \tilde{k}_1$, the SH tax rate cannot be set. How can a government solve this problem?

Harashima (2019b)[9] showed that, in the case of heterogeneous preferences, if a government in a democratic society adjusts the transfers for SH to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to the number cast in response to decreases, then SH can be approximately achieved. In addition, in the case of heterogeneous abilities of obtaining persistent rent incomes, the value of δ will be able to be set in the same manner. If the value of δ is set in this manner, it will be approximately equal to δ_{SH} and SH will be approximately achieved.

4.5 Progressiveness

4.5.1 Does progressiveness matter?

As shown in Section 4.3, if $\bar{k} = \tilde{k}_1$, $\delta_{SH} = 1$. This implies that the progressiveness in the inheritance tax rate does not matter if $\bar{k}$ is set appropriately; that is, it is enough to set the single tax rate, $\delta_{SH} = 1$. However, in actuality, $\bar{k}$ will not be set equal to $\tilde{k}_1$ because the problem with distinguishing the persistent and temporary elements of capital, although much smaller than in the case of the income tax, will still exist to some extent.

[9] Harashima (2019b) is also available in English as Harashima (2018d).

In particular, if z_t is small, it will be difficult to distinguish the effect of persistent rent incomes from that of repeated incidences of good luck. To solve this problem, multiple inheritance tax rates may be necessary.

More importantly, households in some FLs can obtain larger amounts of rent income more often than those in other FLs, as discussed in Section 2.2. If an inheritance tax consists of only a single tax rate and this single rate is applied commonly to all heterogeneous FLs, SH may not be achieved. In practice, inheritance taxes have the property of progressiveness in many countries.

4.5.2 Rationale for progressiveness

A multi-FL model is used to examine the necessity of progressiveness because progressiveness only matters when there are multiple heterogeneous FLs with regard to persistent rent incomes. Suppose that there are M FLs (FL1, FL2, …, FLM) that are identical except for the persistent rent incomes z_t that households obtain. M is an even number for a technical reason. For simplicity, assume there is only one household in each FL, and that any generation of household in a FL equally lives only one period. If $i \leq \frac{M}{2}$, the household in FLi obtains persistent rent incomes in every period (i.e., in every generation). The amount of persistent rent income after paying the income tax is

$$\check{z}_i \tilde{k}_i = z_{R,i}$$

in every period, and $\tilde{k}_i$ is $k_{i,t}$ at SH in the case where there is no persistent rent income for any FL. It is assumed for simplicity that $z_{R,i}$ is constant for any $i \leq \frac{M}{2}$, and $z_{R,i} > z_{R,j}$ if $i < j$.[10] Conversely, if $j > \frac{M}{2}$, the incomes of household in FLj decreases by $\check{z}_i \tilde{k}_{M-j+1}$ in every period before the government transfers the money from the inheritance tax to the household in FLj. Here, suppose for simplicity that $\tilde{k}_i$ is identical for any i (= 1, 2, …, M) and therefore $\check{z}_i > \check{z}_j$ if $i < j$.

Because any generation of a household lives only one period, the inheritance tax is imposed in every period. The amount of inheritance tax for household in FLq (IT_q) for q = 1, 2, …, M is

$$IT_q = \delta\left(k_{q,t} + \check{z}_q k_{q,t} - \bar{k}\right).$$

[10] If a more general environment is assumed such that there are many households in a FL, $z_{R,i} > z_{R,j}$ means that a household in FLi can obtain persistent rent incomes more frequently than once in FLj because households can obtain rent incomes only intermittently.

The tax exemption ($\bar{k}$) is set to satisfy

$$k_{j,t} - \check{z}_i k_{M-j+1,t} < \bar{k} < k_{i,t} + \check{z}_i k_{i,t}$$

for any $i\left(\leq \frac{M}{2}\right)$ and $j\left(> \frac{M}{2}\right)$. If IT_q is negative, the inheritance tax of a household in FLq is exempted. Therefore, because of this inheritance tax, the capital owned by the household in FL$i\left(\leq \frac{M}{2}\right)$ is reduced from $\tilde{k}_{1,t} = k_{1,t} + \check{z}_i k_{1,t}$ to $(1-\delta)\left(k_{1,t} + \check{z}_i k_{1,t}\right)$. On the other hand, capital owned by the household in FL$j\left(> \frac{M}{2}\right)$ is unchanged (i.e., $k_{j,t} - \check{z}_i k_{j,t}$) before the government transfers the money from the revenue of the inheritance tax to the household in FL$j\left(> \frac{M}{2}\right)$ appropriately to achieve SH. With the transfers, the capital owned by the household in FL$j\left(> \frac{M}{2}\right)$ increases to greater than $k_{j,t} - \check{z}_i k_{j,t}$.

By a similar procedure as shown in Section 4.3, if the tax rate were set to

$$\delta = \frac{\check{z}_i \tilde{k}_i}{\check{z}_i \tilde{k}_i + \tilde{k}_i - \bar{k}}$$

for any $i\left(\leq \frac{M}{2}\right)$, then

$$\lim_{m\to\infty} k_{i,t+m} = \tilde{k}_i \tag{9}$$

could be satisfied. However, because $\check{z}_i$ is heterogeneous, there is no value of δ that can satisfy equation (9) for any $i\left(\leq \frac{M}{2}\right)$. Conversely, to make equation (9) hold for any $i\left(\leq \frac{M}{2}\right)$, the inheritance tax rate applied to each FL should be heterogeneous across FLs.

Hence, I next examine the case of heterogeneous inheritance tax rates. Let δ_i be the inheritance tax rate for FL$i\left(\leq \frac{M}{2}\right)$. Clearly, if

$$\delta_i = \frac{\check{z}_i \tilde{k}_i}{\check{z}_i \tilde{k}_i + \tilde{k}_i - \bar{k}} \tag{10}$$

is satisfied for any $i\left(\leq \frac{M}{2}\right)$, equation (9) holds simultaneously for any $i\left(\leq \frac{M}{2}\right)$. If the

government at the same time transfers the money from the inheritance tax to the household in FLj $\left(>\frac{M}{2}\right)$ appropriately, SH can be achieved. Note that if $\bar{k} > \tilde{k}_i$ for households in some FLi $\left(\leq \frac{M}{2}\right)$, a δ_i that satisfies equation (10) cannot be set for these FLs because $\delta_i \leq 1$.

Because

$$\frac{\partial \delta_i}{\partial \check{z}_i} = \frac{\tilde{k}_i\left(\tilde{k}_i - \bar{k}\right)}{\left(\check{z}_i\tilde{k}_i + \tilde{k}_i - \bar{k}\right)^2},$$

if $\tilde{k}_i - \bar{k} > 0$ for any $i\left(\leq \frac{M}{2}\right)$,

$$\frac{\partial \delta_i}{\partial \check{z}_i} > 0 .$$

Therefore, as the amount of inheritance becomes larger, the inheritance tax rate applied to it should increase to achieve SH. That is, inheritance tax rates should be progressive.

4.5.3 Another rationale

There is another rationale for the need for a progressive inheritance tax. As discussed in Section 4.5.1, because the problem of distinguishing between the persistent and temporary elements of capital still exists to some extent, if the amount of capital owned by a household is not sufficiently larger than the average, it will be difficult to distinguish the effect of persistent rent incomes on capital from the effect of repeated good luck. In this case, a government may be still cautious about setting δ_i as indicated by equation (10).

However, if the amount of capital of a household is sufficiently greater than average, it will be easier to distinguish between them because good fortune does not generally repeat for a long period (i.e., for many generations). A much larger capital amount than average indicates that it is highly likely that the capital was accumulated as a result of persistent rent incomes. As the amount of capital increases, the probability that it was accumulated owing to persistent rent incomes will also increase. Hence, it seems quite reasonable to set a higher inheritance tax rate for a larger amount of inherited capital and a lower rate for a smaller amount.

An important implication of this rationale is that inheritance tax rates should vary depending not on $\check{z}_i$ but on the amount of capital inherited.

4.6 Actual progressive inheritance tax

In many countries, inheritance tax rates do indeed vary depending on the amount of capital inherited, as discussed in Section 4.5.3 and most likely for the same reasons. This kind of inheritance tax is important as a "next best" inheritance tax while the inheritance tax whose rates vary depending on $\check{z}_i$ would be the "best" inheritance tax if it is feasible, and I examine this "next best" inheritance tax in this section.

4.6.1 Inheritance tax rates that vary depending on the amount of capital inherited

Suppose there is an inheritance tax, and the rates vary depending on $k_{i,t} + \check{z}_i k_{i,t}$. The tax rates are T_1, T_2, and T_3, and $0 < T_1 < T_2 < T_3 < 1$. The tax exemption is set such that $\tilde{k}_i > \bar{k}$ for any $i \left(\leq \frac{M}{2}\right)$. The environment assumed in this section is basically the same as that in Section 4.5.2, but there are N households in each FL, rather than one. In addition, the amount of persistent rent incomes is commonly z for any household and for any period, where z is a constant, but the probability that a household in FLi obtains z (p_i) is different across FLs such that

$$p_i = \frac{2(M+1-i)}{NM(M+1)} \tag{11}$$

while

$$N\sum_{i=1}^{M} p_i = 1\ .$$

Equation (11) indicates that as i becomes smaller, the probability becomes higher, and that a household in FLi obtains z in every $\frac{NM(M+1)}{2(M+1-i)}$ generations on average. Hence, FL1 has the highest probability among the FLs, and the probability that a household obtains z in FL1 is

$$p_1 = \frac{2}{N(M+1)}\ ,$$

which means that it obtains z once in $\frac{N(M+1)}{2}$ generations. For simplicity, also suppose

that it always obtains z exactly once in every $\frac{N(M+1)}{2}$ generations.

4.6.2 The nature of this inheritance tax

First, I examine a special case where the inheritance tax rate is only T_1 for comparison. Because the tax rate is only T_1,

$$\begin{aligned} k_{1,1} &= \left(k_{1,0} + z_R - \bar{k}\right)(1 - T_1) + \bar{k} \\ k_{1,2} &= \left(k_{1,1} - \bar{k}\right)(1 - T_1) + \bar{k} \\ k_{1,3} &= \left(k_{1,2} - \bar{k}\right)(1 - T_1) + \bar{k} \\ &\vdots \\ k_{1,t} &= \left(k_{1,t-1} - \bar{k}\right)(1 - T_1) + \bar{k}\ , \end{aligned}$$

and thereby,

$$k_{1,t} = \left(k_{1,0} + z_R - \bar{k}\right)(1 - T_1)^t + \bar{k}\ . \tag{12}$$

If T_1 is set sufficiently high, by equation (12),

$$k_{1,v} \cong \bar{k}\ , \tag{13}$$

where $v = \frac{N(M+1)}{2} - 1$ because $0 < 1 - T_1 < 1$. Because $\tilde{k}_i > \bar{k}$ for any $i\left(\leq \frac{M}{2}\right)$, the state indicated by equation (13) is inconsistent with SH and means that households in FL1 are eventually made poorer than they are at SH because of the inheritance tax. However, if T_1 is set sufficiently low, it will take a long time (many generations) to reach the state indicated by equation (13). Hence, a household in FL1 can obtain z again after $\frac{N(M+1)}{2}$ generations, that is, before reaching the state indicated by equation (13). If T_1 is set sufficiently low, therefore, a household in FL1 never reaches the state indicated by equation (13). As a result, a household in FL1 can accumulate a larger amount of $k_{1,t}$ than it would at SH and eventually monopolize all capital in the economy.

Next, I examine the case of multiple tax rates T_1, T_2, and T_3. Suppose that T_1, T_2, and T_3 are set such that

$$T_1 \text{ on } k_{i,t} \text{ when } k_{i,t} > \bar{k}_{High}$$

T_2 on $k_{i,t}$ when $\bar{k}_{High} \geq k_{i,t} > \bar{k}_{Middle}$

T_3 on $k_{i,t}$ when $\bar{k}_{Middle} \geq k_{i,t} > \bar{k}_{Low}$

so as to reach the state $k_{1,t} = \tilde{k}_1$ just after $\frac{N(M+1)}{2}$ generations where $\bar{k}_{High} > \bar{k}_{Middle} > \bar{k}_{Low}$. If T_1 is set sufficiently high, $k_{1,t}$ decreases rapidly to $\bar{k}_{High}$ but after it reaches $\bar{k}_{High}$, the tax rate applied is changed downwards to T_2, and after it reaches $\bar{k}_{Middle}$, the lowest rate T_3 is applied to the remaining $k_{1,t}$.

An important advantage of this type of inheritance tax is that a government can more easily make $k_{1,t}$ land softly on $\tilde{k}_1$ just after $\frac{N(M+1)}{2}$ generations. Because T_1 can be set sufficiently high, it will not take a long time (i.e., many generations) for $k_{1,t}$ to reach $\bar{k}_{High}$, but because T_3 can be set sufficiently low, it will take a long time (many generations) before reaching $k_{1,t} \cong \bar{k}_{Low}$. Hence, before reaching $k_{1,t} \cong \bar{k}_{Low}$, the government has enough time to adjust or fine-tune T_1 , T_2, and T_3 as needed even if the initial values of T_1 , T_2, and T_3 do not necessary exactly cause $k_{1,t} = \tilde{k}_1$ to be realized immediately after $\frac{N(M+1)}{2}$ generations. Because soft landings are possible, this "next best" inheritance tax appears to be sufficiently useful.

However, being useful does not mean that this inheritance tax system has no drawbacks. Households in FLi (≥ 2) experience periods (generations) of $k_{i,t} < \tilde{k}_i$ and as i increases, more generations will experience $k_{i,t} < \tilde{k}_i$ because commonly applied tax rates T_1 , T_2, and T_3 have been set so as to make $k_{1,t} = \tilde{k}_1$ immediately after $\frac{N(M+1)}{2}$ generations. Because the interval of obtaining z between generations of a household in FLi (≥ 2) is longer than that of a household in FL1, $k_{i,t}$ of a household in FLi (≥ 2) will be reduced to $k_{i,t} < \tilde{k}_i$ before a future generation again obtains z while any generation of a household in FL1 can enjoy $k_{1,t} \geq \tilde{k}_1$.

Nevertheless, many households in FLs other than FL1 can receive the transfers with regard to the inheritance tax from the government during some generations. In addition, and more importantly, if $\bar{k}_{Low}$ is set such that it is not too different from $\tilde{k}_i$ (but still $\tilde{k}_i > \bar{k}_{Low}$), households of FLs other than FL1 can at least keep $k_{i,t} = \bar{k}_{Low} \cong \tilde{k}_i$. It should be possible for a government to set $\bar{k}_{Low}$ such that it is not too different from $\tilde{k}_i$ because the indistinguishability problem is less severe in the case of the inheritance tax than it is in the case of the income tax. Therefore, a government will be able to roughly estimate the correct amount of accumulated capital resulting from

persistent rent incomes. As a result, differences in $k_{i,t}$ among households do not accelerate; $k_{i,t}$ will be less than $\tilde{k}_i$ during some generations, but it will be maintained around $\tilde{k}_i$ for any i (≥ 2) indefinitely.

There are many other variations of this "next best" inheritance tax. For example, some may argue that a scheme where $\bar{k}_{Low}$ is set to be the average $k_{i,t}$ of all households and T_1 , T_2, and T_3 are set to reach the state $k_{1,t} = \bar{k}_{Low}$ immediately after $\frac{N(M+1)}{2}$ generations is more favorable. Which variation should be adopted should be decided through the various political processes in democratic countries.

5 CONCLUDING REMARKS

Whether inheritance taxes can be justified economically has remained an open question. Most studies of the rationale for inheritance tax have been confined within the traditional framework of the theory of optimal taxation. In this paper, I examined it from a different perspective—that of rent incomes derived from ranking value and preference at SH (Harashima, 2010, 2012, 2014, 2016, 2017b, 2018a, 2018b).

Because there are many different FLs that have different probabilities of obtaining persistent rent incomes, SH cannot be achieved unless some of the capital accumulation of the more advantaged FLs are compulsorily restrained through government interventions. One such tool is the income tax, but it is difficult for a government to be completely successful relying only on an income tax because a government cannot perfectly distinguish between persistent rent incomes and other types of income. Therefore, another type of tax is needed. In this paper, I showed that, if an inheritance tax with an appropriate tax exemption and tax rates is introduced, the drawbacks of an income tax can be overcome and SH can be achieved. In this sense, an inheritance tax is necessary to achieve SH and may even be considered as the final safety net to achieve it.

Inheritance tax rates should be set according to the probabilities of obtaining persistent rent incomes, but this will be difficult to do in practice because it is still difficult to distinguish between the different elements of capital (e.g., persistent and temporary/lucky). A next best solution is to impose an inheritance tax with rates that vary depending on the amounts of capital (assets) households inherit. Although not perfect, SH can be approximately achieved under this type of inheritance tax system.

An important point is that the inheritance tax is not a tax on capital income—it is a tax on rents. Because SH indicates optimality in a heterogeneous population, this type of inheritance tax does not distort the economy. Rather, unless the negative effects of

these rents are sufficiently removed by an inheritance tax, the economy is distorted and most households cannot achieve optimality.

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