Daily commuting

Berliant, Marcus

Washington University in St. Louis

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Marcus Berliant[†‡]

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Abstract

Workers generally commute on a daily basis, so we model commuting as a repeated game. The folk theorem implies that for sufficiently large discount factors the repeated commuting game has as a Nash equilibrium any strategy profile that is at least as good as the maximin strategy for a commuter in the one shot game, including the efficient ones. This result applies whether the game is static, in the sense that only routes are chosen as a strategy by commuters, or dynamic, where both routes and times of departure are chosen. Our conclusions pose a challenge to congestion pricing. We examine evidence from St. Louis to determine what equilibrium strategies are actually played in the repeated commuting game.

JEL number: R41 Keywords: Repeated game; Nash Equilibrium; Commuting; Folk theorem

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†Department of Economics, Washington University, Campus Box 1208, 1 Brookings Drive, St. Louis, MO 63130-4899 USA. Phone: (314) 935-8486, Fax: (314) 935-4156, e-mail: berliant@wustl.edu

‡Institute of Economic Research, Kyoto University.

1 Introduction

1.1 Motivation and Related Literature

What happens to commuting behavior when a commute is repeated daily? Does behavior, namely route and departure time choice, differ dramatically from that observed in the simple context where the commuters know that they only have to commute once? One shot commuting is the exclusive focus of the extant literature.

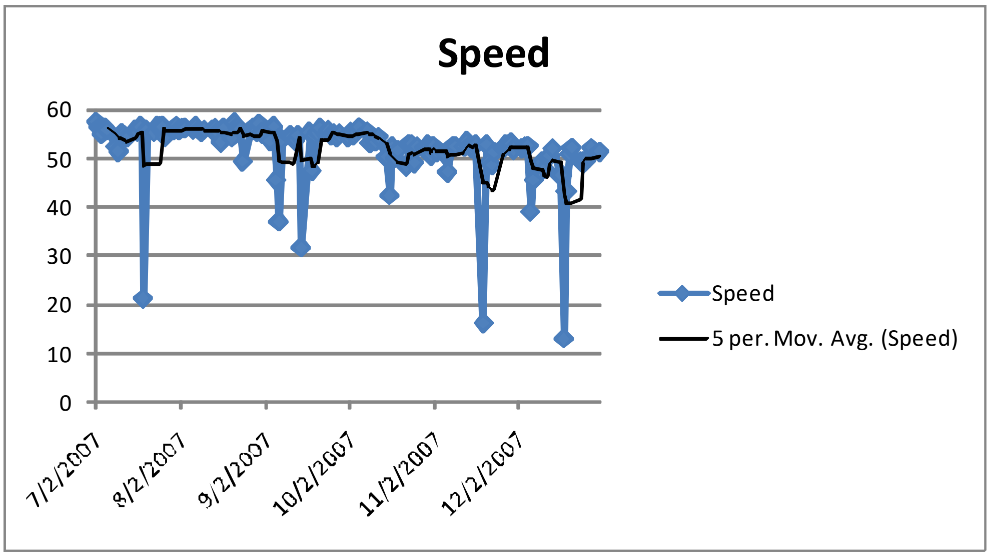

Figure 1: Evening rush hour (5-6 PM) I-64 westbound weekdays .3 miles west of Hampton Avenue

Empirical motivation for our work comes from Figure 1. The vertical axis represents speed in miles per hour, whereas the horizontal axis represents evening rush hour dates in the latter half of 2007. A major commuting highway in St. Louis was shut down on January 2, 2008. Why did rush hour traffic speed on the highway *decrease* during the last three months before closure relative to previous dates? We shall return to this in section 3.3 below. But first, we discuss the basic literature on commuting.

Beckmann et al. (1956) introduced a model of rush hour without time, but with commuter delay a function of the number of cars using a link. The classical work of Vickrey (1963, 1969) analyzed congestion as an externality, Pigouvian taxes, and infrastructure. Arnott et al (1993) examined exogenous bottlenecks and Pigouvian taxes, whereas Sandholm (2001) models road congestion as a potential game. Sandholm (2007) considers an evolutionary approach to setting optimal tolls in the case where commuters are identical (so they have the same home and work locations) using an evolutionary process to refine Nash equilibrium. Daniel et al (2009) implements the commuting model experimentally. All of this literature considers only one shot commuting.

The main difference between our work and most of the literature is that we address different questions. *That is, our primary purpose is to study the equilibria of the commuting game repeated daily rather than as a one shot game.*

1.2 Outline

Our results and the outline of the balance of the paper are as follows. In Section 2, we give our notation and outline the static and dynamic models of one shot commuting. The static model has no time, as only route is a choice. The dynamic model adds a time dimension where departure time is a commuter choice in addition to route. In Section 3, we study Nash equilibria of each of the two models when they are repeated daily. By applying the folk theorem, we find that the set of equilibria is much larger than in the one shot game, be it static or dynamic. *It is important that researchers consider this expansion of the equilibrium set when analyzing their models.* The repeated game structure yields many more equilibria, even when the folk theorem does not apply, than the one shot game structure studied in the literature. Evidence relevant to repeated game strategies used by commuters in St. Louis is examined. Finally, Section 4 gives our conclusions.

2 The Commuting Game

The details and extensive analysis of the ***one shot*** *commuting game, which is the stage game for the commuting game repeated daily, can be found in Berliant (2020).* Both the static game, where only a route is chosen by commuters, and the dynamic game, where both route and departure time are chosen by commuters, are analyzed there. Here will shall be brief, so that the focus can be on the new results derived from the repeated game.

The measure space of commuters is given by $([0,1], \mathcal{C}, \mu)$ where $[0,1]$ is the set of commuter types, $\mathcal{C}$ is the collection of Lebesgue measurable subsets of $[0,1]$, and μ is a positive measure absolutely continuous with respect to Lebesgue measure on $[0,1]$. All references to measurability are to this measure space.

There is a finite set of *nodes*, denoted by $m, n = 1, 2, ...N$, and a finite set of links between nodes. The set of all nodes is denoted by $\mathcal{N} = \{1, 2, ..., N\}$. The capacity of a direct link between nodes m and n is given by $x_{mn} \in [0, \infty]$, where $x_{nn} = \infty$; if a direct link between nodes m and n does not exist, then $x_{mn} = 0$.

We assume that commuters have an inelastic demand for one trip per day to work.

To complete the game-theoretic structure, the strategies and payoffs of the commuters must be specified. In the static game, there is no choice of departure time; there is only route choice. Each commuter has a fixed origin node and a fixed destination node. There is a measurable *origin map* $O : C \to \mathcal{N}$ assigning a departure node to each commuter, and a measurable *destination map* $D : C \to \mathcal{N}$ assigning a destination node to each commuter.

Let π_k be the map that projects a vector onto its coordinate k. A *route*, denoted by r, is a vector of integer length $\ell \geq 2$. The set of all routes is denoted by $\mathfrak{R}$:

$$\begin{aligned} \mathfrak{R}^\ell &= \left\{ r \in \mathcal{N}^\ell \mid \text{for } i = 1, 2, ..., \ell - 1, x_{\pi_i(r)\pi_{i+1}(r)} > 0 \right\} \\ \mathfrak{R} &\equiv \bigcup_{\ell=2}^{\infty} \mathfrak{R}^\ell \end{aligned}$$

We assume that there is some route between a pair of nodes if there is a positive measure of commuters with that origin and that destination. A *commuting route structure* is a pair (l, R) where l is a *commuting length map*, namely a measurable map $l : C \to \{2, 3, ...\}$, and R is a measurable map $R : C \to \mathfrak{R}$, such that for $i = 1, 2, ..., l(c) - 1$, $x_{\pi(i)\pi(i+1)} > 0$, and almost surely for $c \in C$, $\pi_1(R(c)) = O(c)$ and $\pi_{l(c)}(R(c)) = D(c)$.

Given a commuting route structure (l, R), its *flow* $f \in \mathbb{R}_+^{N^2}$ is given by:

$$f(m, n) = \mu(\{c \in C \mid \exists k \in \{1, 2, ..., l(c) - 1\} \text{ with } \pi_k(R(c)) = m \text{ and } \pi_{k+1}(R(c)) = n\}) \\ \text{for } m, n = 1, 2, ...N.$$

If the link is congested, then the travel time increases. For example, it could increase in proportion to the flow of commuters, $f(m, n)$. More specifically, if the flow of commuters doubles, then travel time on the link is doubled. We will use other examples below.

More generally, we can allow traffic to slow down according to any well-behaved function of the number of commuters on a link and link capacity. Therefore, we specify the function $\upsilon : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_{++}$ where $\upsilon(f, x)$ is the speed of traffic with flow f on a link with capacity x. We assume that for fixed x, v is continuous and non-increasing in f. Define the length of the link (m, n) to be $\lambda(m, n)$.

Although it is difficult to discuss travel time in a static model that is inherently atemporal, the travel time is calculated in a sort of steady state. Under this interpretation, f is the measure of commuters (repeatedly) passing through the link on their route.

The *time cost of a commuting structure* (l, R) *for commuter* c is

$$\theta(l, R, c) = \qquad (1)$$

$$\sum_{\{(m,n)\in\mathcal{N}\times\mathcal{N} | \pi_i(R(c))=m, \pi_{i+1}(R(c))=n \text{ for some } 0\leq i\leq l(c)-1\}} \frac{\lambda(m, n)}{\upsilon\left(f(m, n)\right), x_{mn})}$$

Thus, $-\theta$ is the objective or payoff function for each commuter.

A *Nash equilibrium of the static model* is a commuting structure (l, R) such that almost surely for $c \in C$, there is no route r of length ℓ for commuter c such that

$$\theta(l, R, c) > \sum_{\{(m,n)\in\mathcal{N}\times\mathcal{N} | \pi_i(r)=m, \pi_{i+1}(r)=n \text{ for some } 0\leq i\leq \ell-1\}} \frac{\lambda(m, n)}{\upsilon\left(f(m, n), x_{mn}\right)}$$

Rosenthal (1973), Sandholm (2001), and Konishi (2004) are excellent references for existence and uniqueness of Nash equilibrium in this model.

Example 1: Please refer to Figure 1. Consider measure 4 commuters who must transit from home at node A to work at node M.

$$\begin{array}{ccccccccccccccccc}
 & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot & & \\
 & \nearrow & B & \searrow & & \nearrow & E & \searrow & & \nearrow & H & \searrow & & \nearrow & K & \searrow & \\
\cdot & & & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot \\
A & \searrow & C & \nearrow & D & \searrow & F & \nearrow & G & \searrow & I & \nearrow & J & \searrow & L & \nearrow & M \\
 & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot & &
\end{array}$$

Figure 1: Nash Equilibrium that is not Pareto Optimal

Everyone must pass through nodes D, G, and J on the way from node A to node M. There are parallel routes in a series of four. The nodes B, C, E, F, H, I, K, and L appear simply to distinguish among the links and routes. For upper links ABD, DEG, GHJ, and JKM, the time a commuter spends on a link is given by f, where f is the measure of commuters using the link.

For lower links ACD, DFG, GIJ, and JLM, the time a commuter spends on a link is given by 2, independent of the measure of commuters using the link. Although it is unnecessary, if the length of each link is 1, speed can be computed by taking the reciprocals of the time on a link. Nash equilibrium has measure 2 using every link, for a total travel time of 8 independent of route. The identity of the users of any route is irrelevant. Turn next to a strict Pareto improvement. It will transfer measure 1 from each upper link to each lower link. In other words, measure 1 uses an upper link whereas measure 3 uses the corresponding lower link. However, the identity of the users matters. The commuters who use link ABD will only use lower links after travelling link ABD. Similarly, the commuters who use link DEG will only use lower links, including link ACD, for the remainder of their trip. In this way, use of upper links rotates among the commuters. The total travel time of each commuter is 7. This is not a Nash equilibrium, because each consumer would prefer to use upper links rather than the lower ones, for a total travel time of 3.

The classical Braess (1968) paradox provides another class of examples. That work shows that in a static model, adding new links to a network can cause Nash equilibrium travel time to increase. For our purposes, the opposite experiment works. If one begins with a network Nash equilibrium and then allows a planner to prohibit travel on some links, a Pareto improvement can be created. It is not a Nash equilibrium unless the prohibition is in place.

The dynamic model adds departure times to the static model. Departure times and routes are strategic choices of the commuters, whereas arrival times and arrival penalties are the consequences. Berliant (2020) provides examples in dynamic models where Nash equilibria are strictly Pareto dominated by strategy profiles that are not Nash equilibria. They have a flavor different from the examples in the static model, as they rely on mis-coordination of departure times in Nash equilibrium.

3 The Repeated Commuting Game

3.1 The Commuting Folk Theorem and the Commuting Anti-Folk Theorem

In repeated games with a continuum of players, the commuting game is a very nice special case. There are two important theories of equilibrium behavior, both quite famous, namely the *Folk Theorem* and the *Anti-Folk Theorem.* The conclusions of the two theorems are in a sense opposites. The first says that any individually rational, feasible payoffs can be supported by a Nash equilibrium of the repeated game. Included in this set are the Pareto efficient payoffs. The second theorem says that only Nash equilibria of the one shot game are equilibria of the repeated game. The critical issue in the determining which theorem applies is what players can observe about the strategies used by other players in past plays of the stage game. The formalities can get technical; see Kaneko (1982), Massó and Rosenthal (1989), and Massó (1993). So we describe them in a relatively informal manner.

The crux of the matter is this: Fixing one particular individual, after finitely many plays of the stage game, can a positive measure of players observe that individual player's past behavior? If there is such a set of positive measure for each fixed individual, then the folk theorem applies. If *no* individual's behavior can be detected by a set of players of positive measure, then the anti-folk theorem applies. Note that these two cases are not exhaustive. In the end, which theorem might apply is an empirical matter. There is some evidence that, in other contexts, the folk theorem is relevant; see, for instance, Lee (1999).

With a finite number of strategies (routes and possibly departure times), *it is not far-fetched to think that any particular individual's strategy is observable by those who use the same departure node.*[1] In the next subsection, we give a second reason, called the "snowball effect," why defection from equilibrium strategies might be observable in the dynamic commuting game. But for now, let us focus on the *implications* of the two theorems for daily commuting.

We begin by assuming observability and apply the folk theorem. Here we examine two repeated games. The first has the static model repeated every day, namely a countable infinity of repetitions. The second has the dynamic model repeated every day. The main results, using Kaneko (1982, Propositions 2.1 and 2.1'),[2] are that if commuters have discount factors sufficiently close to one, in other words they do not discount the future much, then there is a huge variety of equilibria. The usual folk theorem holds, so any individually

[1] At this point, it is useful to take versions of strategies such that if a set of measure zero plays a particular strategy, then no commuter plays it.

[2] Kaneko (1982) actually says that any feasible, individually rational stage game strategy profile can be achieved as a repeated game equilibrium. Payoffs can be derived from this strategy.profile. So in summarizing the results, we use strategy profile and payoff interchangeably.

rational, feasible strategy profile (not necessarily a Nash equilibrium in the one shot game) can be obtained as a Nash equilibrium of the repeated game. Kaneko (1982, Proposition 2.1") proves The Perfect Folk Theorem, where we can restrict even to subgame perfect Nash equilibrium and obtain similar results. The equilibrium strategy profiles are supported by various punishment strategies, that apply if the prescribed equilibrium strategy is not followed by a player. Thus, the one day equilibrium is just one of many. Moreover, on the equilibrium path, one only observes the prescribed equilibrium strategies, not the punishments. *Thus, one expects to see the one shot equilibrium played, perhaps, but also (for example) the efficient strategies.*

In the static model, the implication is that any feasible routing strategy that gives utility at least as high as the maximin payoff for the one shot game for each commuter can be achieved as a constant (over time) Nash equilibrium strategy for the infinitely repeated game with no discounting. If we modify this so that the utility of the strategy in the one shot game is at least ϵ greater than the maximin utility, then the prescribed strategy can be achieved as a Nash equilibrium strategy in the infinitely repeated game with a discount factor sufficiently close to 1. Example 1 is applicable here. In that example, there is a Pareto improvement over Nash equilibrium that will not be a Nash equilibrium for the one shot game. However, it can be supported as a (subgame perfect) Nash equilibrium in the repeated game with discount factor sufficiently close to 1. Standard strategies that support this are the threat of Nash reversion. As we have described, the Braess paradox gives further examples of Pareto improvements over one shot Nash equilibrium that can be supported in repeated games.

Consider the repeated dynamic commuting game. That is, the dynamic commuting game is played daily. What payoffs are attainable? We shall apply a folk theorem, so the set of payoffs attainable as Nash equilibria in the repeated game is related to the payoffs attainable in the one shot game. Specifically, for large enough discount factors in the repeated game, all feasible payoffs at least as high as the maximin payoff for the one shot game (that are not necessarily Nash equilibria of the one shot game) are attainable as Nash equilibria of the repeated game. In fact, we can show that any payoff that is feasible in the one shot game can be attained as a Nash equilibrium of the repeated game. This result is achieved by simply computing the maximin payoff of the one shot game. It will be $-\infty$. Why? Consider one individual. The worst case scenario for that individual in the one shot commuting game is that everyone else who lives at the same node "blockades" them at time zero. That is, the strategy used by everyone else is to depart at time 0 along the same route as the deviating commuter, whoever and whatever that may be. Then local congestion is infinite, so nobody ever reaches the destination or even moves at all, independent of what the commuter in question does (namely, what departure time strategy they follow). Time to destination is infinite.[3] So any feasible route and departure time strategy for the one shot game can be supported as a Nash equilibrium of the infinitely repeated game without discounting. If we modify this so that the utility of the prescribed strategy profile in the one shot game is above $-\infty$, then the prescribed strategy can be achieved in the infinitely repeated game with a discount factor sufficiently close to 1.

Of course, if no individual's behavior is observable, then the anti-folk theorem applies to both the static and dynamic models, so the only Nash equilibria of the repeated game are the Nash equilibria of the one shot game (Kaneko, 1982, Propositions 2.3 and 2.3').

One might argue that commuting is not a good context for repeated game punishment strategies. However, evolutionary foundations of the folk theorem studied in Vasin (1999, 2006) show how these many equilibria can obtained as globally stable outcomes of various game-theoretic dynamical systems, including replicator dynamics and selection dynamics, justifying our interest. Beyond that, grim trigger strategies are not terribly complicated. The basic idea is that commuters play, say, a Pareto optimal strategy every period until they observe something they didn't expect on their drive to work, at which time everyone institutes a punishment strategy. In equilibrium, use of the punishment strategy is never actually observed. This punishment strategy will be subgame perfect, and thus credible. Stochastic elements, such as random weather or accidents, can easily be added to the model. Such elements are common in the folk theorem literature; see, for example, Fudenberg and Yamamoto (2011).

[3] In fact, in the dynamic commuting game, there is sometimes an exogenous departure grid, in that only a finite number of departure times can be chosen and actual departure time is randomized over a small time interval containing the chosen time. If the departure grid is sufficiently fine relative to the measure of commuters departing from each origin node, then other commuters can always make any particular commuter arrive as late as desired. Thus, by choosing the grid to be sufficiently fine, any given payoff is above the maximin payoff. The upshot is that any feasible departure time and route strategy gives a payoff that is at least as high as the maximin payoff for a grid chosen sufficiently fine.

3.2 Finite Commuters vs. Continuum of Commuters: The Snowball Effect

Here we consider the relevance of models with a continuum of commuters, such as the one we have used. Of course, they are only relevant in the case that they are mathematically convenient approximations to the equilibria of models with a large but finite number of commuters.

With a finite number of commuters, the anti-folk theorem becomes irrelevant, as the folk theorem applies because there is generally no issue of observability of strategies. With a continuum of commuters without observability of strategies, the anti-folk theorem applies. Due to this apparent discontinuity in the set of equilibria as the number of commuters tends to infinity, it is imperative to examine the continuity properties of the Nash equilibrium correspondence.

Let us put aside the static commuting game. Given the discussion of the previous subsection, we consider two cases in the context of the repeated dynamic commuting game: when individual strategies are observable and when individual strategies are unobservable.

When individual strategies are observable, for example by commuters departing from the same node, the commuting folk theorem applies to both the model with a finite number of commuters and a continuum of commuters. Thus, there is no issue of a discontinuity as the number of commuters tends to infinity.

When individual strategies are not observable, there is the potential for such a discontinuity. The set of equilibria can contract from the set of individually rational, feasible strategies to the set of one shot Nash equilibria. In the model with a continuum of commuters, when an individual commuter changes their strategy, there will be no change in what is observed by other agents, say their commuting time, so there is no basis on which to punish deviators. Thus, the anti-folk theorem applies. But now consider the model with a finite number of commuters. Even if the number of commuters is large, deviations from a prescribed along-the-equilibrium-path-strategy can be detected (for instance by commuters on the same route using the same departure time on the equilibrium path since their commuting time changes) and therefore can be punished. This explains the contraction of the equilibrium set. However, one can easily argue that *as the number of agents gets large, these individual commuter deviations become undetectable, as their effects are small* and indistinguishable from noise. For example, an analog would be to assume perfect competition in the context of a finite number of agents, where the error from this assumption is small for large economies.

If this were true, then there would be no substantial error in simply using the limit commuting game with a continuum of commuters without observability. *The big problem here is that the effect of one commuter deviations in large but finite commuting games are not small.* To see this, consider a simple example with 2 nodes, home and work, and 1 link of length 2. Variants of this example are described more formally in Berliant (2020). Everyone commutes once a day between home and work. If density (cars per mile) at a time and place on a link is f, then speed is $1 + 1/f$ whereas volume is speed multiplied by density, or $1 + f$. There is measure 2 commuters. Set arrival time to 2; if consumers arrive late, the penalty is large. Consider the Nash equilibrium where departures are uniform on times $[0, 1]$, volume is 2, and density is $f = 1$. So speed is 2 and the last commuter arrives at work at time 2. If a single commuter deviates from this strategy, it is undetectable.

Next, instead of using a continuum of commuters, consider a large but finite number. In the case of a large but finite number of commuters, it's natural to think of a (fine) grid of a finite number of evenly spaced departure times in $[0, 1]$. Pure commuter strategies are uniformly distributed over departure times so that departure density is again 1. Speed and volume are 2. Suppose that a commuter changes their strategy from the second departure time in the grid to the first, reducing density and volume at the second departure time and increasing density and volume at the first departure time. This will slow down the first cohort. The second cohort will quickly catch up, slowing down both cohorts. The third cohort will catch up to the first two, and so forth. This "snowball effect" will not only be detectable (even if individual strategies aren't), but it also substantially changes the behavior of the entire system due to one commuter's deviation. Such a "snowball effect" is simply not possible in the commuting game with a continuum of commuters.

It is logical to inquire next whether this effect disappears as the number of commuters tends to infinity. The issue here, as in classical urban economics, is how one takes limits as the number of commuters tends to infinity. If the number of commuters is simply increased whereas the road capacities remain constant, some densities tend to infinity and some speeds tend to zero, so the system halts. Allowing road capacity to tend to infinity seems unrealistic. The last possibility, that seems implicit in urban transportation models generally, is that one commuter in the finite model is represented by a continuum of identical commuters of positive measure (say 1) in the continuum model. In that case, deviation by a set of measure zero of commuters does not make economic sense (though it does make mathematical sense), as it does not correspond to any type of behavior in the model with a finite number of commuters. Under this interpretation, deviations can only occur for coalitions of commuters of measure 1, and we are back to the snowball effect.

With this interpretation of the dynamic model, neither the snowball effect nor the folk theorem should be a surprise.

In summary, the behavior of the dynamic commuting game with a continuum of commuters where individual commuter strategies are undetectable differs from the behavior of the commuting game with a large but finite number of commuters where individual strategies are unobserved. For this reason, we view the repeated dynamic commuting game with a continuum of commuters where the individual strategies are undetectable, and the associated anti-folk theorem, as irrelevant.

In the general game theory literature with a continuum of players, only the extreme cases of observability (the folk theorem) and unobservability (the anti-folk theorem) have been investigated thoroughly. There are two more important points to be made about the application of the theory of repeated games to the commuting game. First, the commuters who observe a defection are not necessarily those who punish. In the simple one route example, of course any commuter who observes a defection can punish. However, with many routes, this might not be possible. If individual strategies are observable to all, for example neighbors departing at the same time and along the same route, then of course we are back in the context of the folk theorem. An alternative assumption is that any defection causes a snowball effect, in that a positive measure of commuters is affected. Then it is assumed that if a positive measure of commuters is affected, this is observable to all and the deviators can be punished.

The problem with this idea is that there is literally no snowball effect with a continuum of commuters, but only with a large but finite (or countable) number. In fact, this is the reason there is a discontinuity of the Nash equilibrium correspondence in the limit as the number of commuters goes to infinity. A sufficient condition for a snowball effect in large but finite games close to the game with a continuum of commuters of interest is: For the given one shot strategy profile that is to be supported as a repeated game Nash equilibrium, at any time on any link with a positive local density of commuters, speed is strictly decreasing in density. Under this condition on the strategy profile, whenever a commuter deviates, there is a snowball effect; this is detected and punished by everyone, for example by using a blockade in the next period.

In summary, our conclusion is that although the snowball effect is not present in commuting games with a continuum of commuters, it is present in the large but finite games nearby. We also assume that effects on any positive measure of commuters are observable to all. Thus, under a sufficient condition on strategies to be supported in the repeated game, it makes sense to say that the *consequence* of any individual deviation from a prescribed strategy is observable, and thus the folk theorem is applicable to such strategies in the repeated dynamic game with a continuum of commuters.

Therefore, be it from observations of neighboring commuters or the snowball effect, the folk theorem in the model with a continuum of commuters seems relevant.

A messy alternative to our framework would employ a finite or countable number of commuters and either Nash equilibrium in mixed strategies or ε-equilibrium in pure strategies. The drawbacks of this approach are tractability and consistency with the balance of the literature on commuting. However, the advantages of this approach are that the snowball effect could be made explicit.

3.3 Evidence

In this subsection, we examine evidence, in the context of the repeated commuting game, that can tell us whether commuters are playing a one shot Nash equilibrium in all periods, or whether other strategies, possibly more efficient, are used. The idea for the analysis is similar to that used in Lee (1999), but now in the context of commuting.

Consider a repeated game with a termination date that is finite and known to the players. In general, it is expected that only one shot Nash equilibrium will be played every period, since backward induction leads to the unravelling of other possible equilibrium strategies.

However, as described in Lee (1999, p. 123), there are various theories involving small changes in the classical repeated game model that lead to a kind of folk theorem in finitely repeated games. This is exploited by the empirical work in the field.[4] Next we proceed to try to determine which

[4] If the folk theorem only applied to infinitely repeated games, those wishing to determine which strategies are played in equilibrium would be waiting a long time for data.

equilibrium strategy is reflected in commuting data.

If for example the players are myopic and playing one shot Nash equilibrium, then it is expected that behavior will not change as the repeated game termination date approaches. If the players are using strategies other than one shot Nash, for example they are participating in some tacit collusion as the folk theorem might predict, then one expects to see such strategies played when the termination date of the game is not near, but reversion to one shot Nash equilibrium close to the termination date.

How does this work in the context of the repeated commuting game? On January 2, 2008, reconstruction was begun on I-64 (state route 40), a major east-west commuting corridor in St. Louis. A portion was completely shut down. Parts were reopened a year later, though other (adjacent) parts were shut down at that time.[5] We take this to be the termination of a repeated, daily commuting game. This closure was announced years in advance, so it was not a shock to commuters. We examine rush hour traffic speed and volume for locations that were closed on this date.

If commuters were playing one shot Nash equilibrium strategies, one would expect to see the same rush hour traffic speed daily until close to the closure. Near the time of the closure, traffic volume would drop off and speed would increase as commuters explored alternate routes to be used after closure.

If commuters were playing a strategy other than one shot Nash, for example Pareto dominant over one shot Nash, then one would expect to see high traffic speed when the closure is not imminent, followed by lower traffic speed as the closure date approaches and one shot Nash is played,[6] followed by an increase in speed near the closure date due to commuters exploring alternate routes.

Thus, it is detection of this counterintuitive decrease in traffic speed as the closure date approaches that can distinguish among the equilibria of the system.

Before presenting the data, it is useful to recall the fundamental identity of traffic, namely: Traffic volume is equal to speed times density. We have obtained data on volume and speed, so density can be calculated. But there are two important points to be made. First, volume is not terribly informative on its own in general, as there can be two equilibria with the same volume, one with low speed and high density, the other with low density and high speed. Second, the externality actually perceived by commuters is in speed, so we focus on that.

[5] The entire highway was reopened on December 7, 2009.

[6] The exact timing depends on both the model used for the folk theorem in the finitely repeated game and the discount factor.

We have obtained data from two sensor locations, one toward the east end (closer to the downtown area) of the closure, the other at the west end.[7] Let's examine the east location first, studying evening then morning rush hour. The figures graph average traffic speed and total volume in the hour by date. We have deleted weekends, but we have not deleted holidays that fall on weekdays.

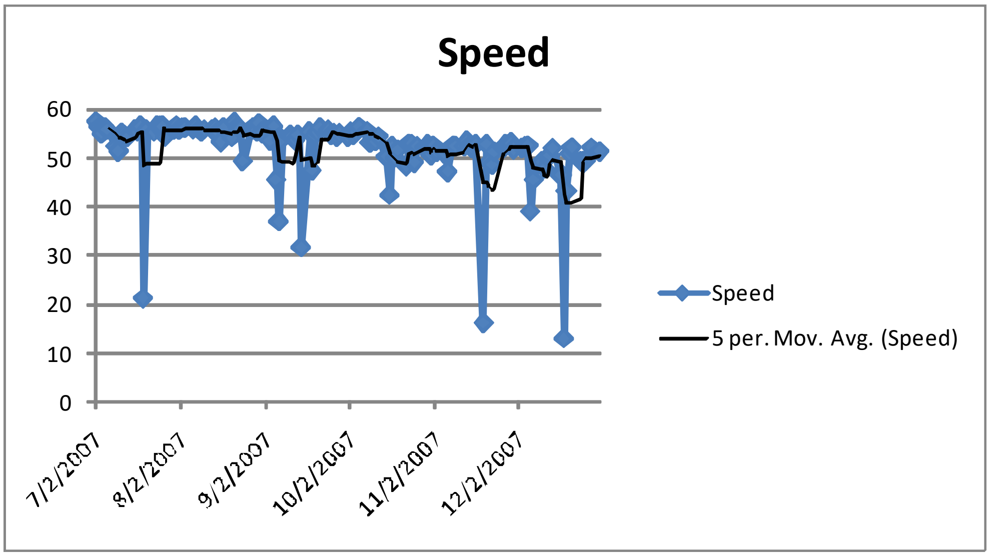

Figure 1 (again): Evening rush hour (5-6 PM) I-64 westbound weekdays .3 miles west of Hampton Avenue

[7] The author was offered more data than the one calendar year at two sensors actually provided, but at the cost of relinquishing rights to all future work (whether related to this project or not), as well as other considerations.

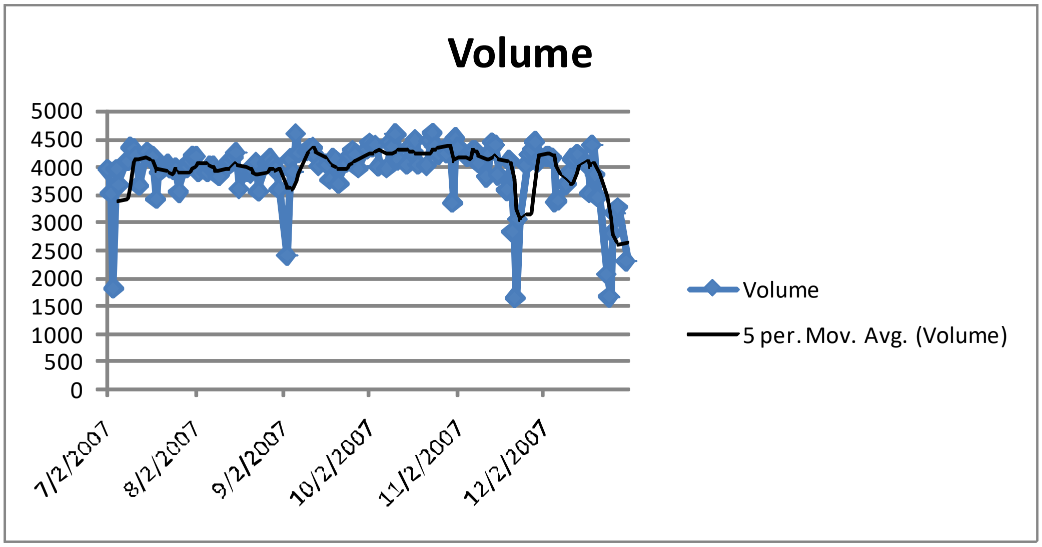

Figure 2: Evening rush hour (5-6 PM) I-64 westbound weekdays .3 miles west of Hampton Avenue

Notice that in early October, there is a decrease in speed and an attendant increase in volume, as seen in Figures 1 and 2. The outliers in the data are obviously accidents. For morning rush hour, as seen in Figure 3, there is a similar effect, though not as large in magnitude and with speed increasing over the Thanksgiving holiday.

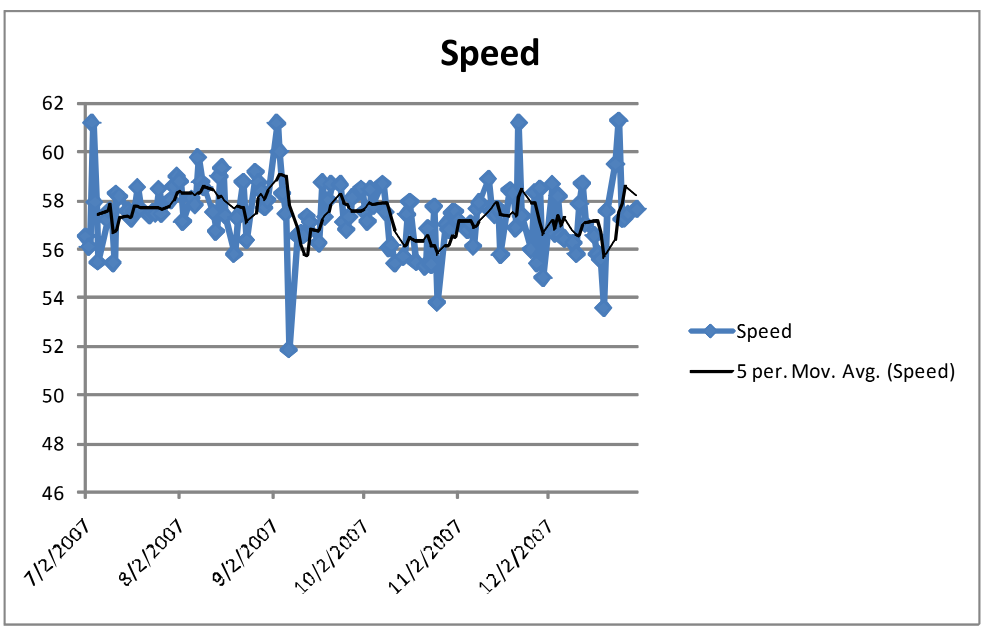

Figure 3: Morning rush hour (8-9 AM) I-64 eastbound weekdays .3 miles west of Hampton Avenue

Evening rush hour for the west sensor is displayed in Figure 4. In general, for the west sensor (more distant from the central business district), traffic moves at the speed limit. We conjecture that this is due to the fact that traffic in this area is not congested enough to cause speeds to drop below the speed limit during rush hours.

Many questions about the data arise at this point. Does weather cause traffic to slow down in the fall? There are two responses. In the author's experience, most of the inclement driving weather occurs in St. Louis during the time period from mid-December to mid-February. Moreover, the sensors at the west end of the shut down, that show no decrease in speed, serve as a nice control for weather, as St. Louis is very flat and thus weather seems to be common to most of the area. Can holiday shopping can account for the increased traffic? Most stores used for shopping are now located in malls well outside the city, along with most of the area's population. Does an increasing accident rate in the fall cause the decrease in speed rather than the theory we have put forth? If this were the case, we would observe a *decrease* in volume accompanying the decrease in speed. Instead, we observe an increase in volume in the data. Could the effect we observe be due to intertemporal substitution between commuting with a car and commuting with mass transit, where commuters take advantage of the expressway when it is open? Conceptually, this would depend on the elasticity of substitution between commuting mode choices. This elasticity of substitution has been found to be quite low; see, for example, Chung (1979). Is the change in commuting speed seasonal? In theory, one could look at commuting in 2006. However, a low probability event occurred that year that disrupted commuting and corrupted data throughout the fall - the Cardinals (unexpectedly) won the World Series.

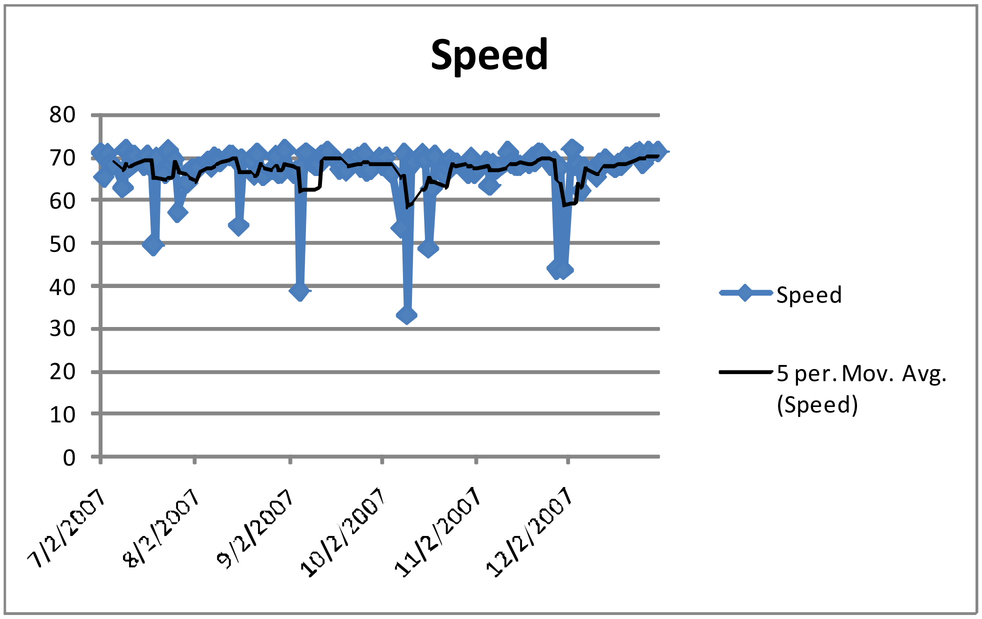

Figure 4: Evening rush hour (5-6 PM) I-64 westbound weekdays .7 miles west of Brentwood Boulevard

In summary, there is some evidence that commuters are not playing one shot Nash equilibrium. They revert to one shot Nash equilibrium strategies at around 2 1/2 to 3 months from the end of the game.

4 Conclusions

We have examined the set of Nash equilibria in the infinitely repeated versions of both the static and dynamic commuting games, and use the folk theorem to obtain these large sets. Why have the additional equilibria from the repeated game, *including efficient equilibria*, been ignored by the literature? It would be interesting to see if classical assertions of transportation economists that "commuting equilibria are inefficient due to a congestion externality" hold in the real world. We have presented some preliminary evidence from the shutdown of an expressway in St. Louis that commuters do not always play one shot Nash equilibrium. We have also discussed the application of the anti-folk theorem to our specific game, namely conditions under which the Nash equilibria of the infinitely repeated game are the Nash equilibria of the one shot game.

The commuting folk theorem poses a direct challenge to congestion pricing. If commuters are already playing equilibrium strategies that are efficient *without tolls, congestion pricing can mess this up.* If the commuters are playing efficient strategies, then the introduction of congestion pricing can jolt the system to a new repeated game equilibrium, for example one that is Pareto dominated. In order to make this idea formal, congestion pricing would have to be added to the model, likely as a penalty additive in the utility function as in Sandholm (2007). Then a folk theorem would be applied to this extended model. The equilibria of the models with and without congestion pricing could be compared. It is expected that all individually rational, feasible payoffs would be equilibria of the repeated games under a sufficiently high discount factor. The additional notation and complexity does not seem worth the trouble. However, an example is in order.

Example 1 (continued): Suppose that the commuters are commuting happily each day using Pareto optimal strategies supported by (say) the threat of Nash reversion if they deviate. Suddenly, one day, they experience a Pigouvian congestion tax, namely marginal damages at the optimum. Suppose that the utility function is additive in money. The Pigouvian tax is specified as follows. It is 1 for all of the upper links, such as ABD, and 0 for all the lower links, such as ACD. The Pareto optimum itself is a Nash equilibrium of the repeated game with Pigouvian taxes, but so is any other feasible strategy profile that is at least as good as the one shot Nash equilibrium, that has travel time of 8.

The folk theorem and anti-folk theorem can also be applied to repeated versions of other one shot models in the literature, such as Arnott et al. (1993). For the bottleneck type of model, again the punishment strategy of interest involves everyone arriving at the bottleneck simultaneously, at the earliest possible time. It would be very interesting to explore experimental complements to our theory and data; for example, see Daniel et al (2009). Which equilibrium of the repeated game is selected in the laboratory?

The model could also be extended to allow elastic demand for travel to work.

Future work includes examining the repeated dynamic model with myopic commuters. The set of Nash equilibria will include the equilibria from the one shot game, but not as many as in the repeated commuting game with a discount factor close to 1.

The repeated commuting model should be applied to real world commuting. Since it can accommodate an arbitrary (exogenous) route structure, it has both positive and normative content, especially regarding Pareto improvements. For example, it can be used to perform cost benefit analysis with respect to changing public infrastructure.

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