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Learning With Friends:

A Theoretical Note On The Role of Network Externalities In Human Capital Models For The New Industry

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Abstract

Contemporary literature on how individuals learn in the 21st-century reveal critical differences from learning patterns in the mid-20th century—a period in which celebrated, pioneering works of Mincer, Becker and Ben-Porath on human capital were developed. Education and learning theories have evolved, but the prevailing human capital theories have not. Given continued technological progress, and the rise in available knowledge through the Internet, learning in networks is a distinct feature of the 21st-century industry. The connectivist theory of learning in the digital age is explored and substantiated. Using optimal control theory and dynamic optimisation, we define optimal conditions for knowledge generation and growth of learning networks. We find that knowledge per learner grows exponentially when the obsolescence rate of knowledge is less than the departure rate of learners from the learning network. We also find that a learning network will continue to grow as long as learners are sufficiently impatient and that technology sufficiently becoming obsolete faster. Furthermore, we show a positive relationship between the size of the network and wealth on knowledge. That is, as long as the remaining wealth on knowledge is increasing, the learning network will continue to grow over time. We present insights for policy consideration that address the necessary and sufficient conditions for sustained knowledge generation and the growth of the learning network.

JEL Codes: J24, O15, M53

Key Words: human capital, learning, endogenous growth, industry 4.0, networks

An illustration

Suppose you are a contact centre agent whose main job is to entertain customer enquiries by phone. One day, you find out that your multinational company will roll-out a voice-enabled

artificial intelligence solution for all its clients, including the account you are handling. In no time, your job will become obsolete. Now suppose you are a general practice physician. Majority of your patients see you for a regular check-up, which includes standard blood chemistry analysis. Then, you find out that a startup has started to mobilise hundreds, if not thousands, of wellness experts accessible via a mobile app. These wellness experts, equipped with smart devices that can draw blood onsite, delivers the initial blood chemistry results *and* analysis available in less time than if the procedure were done in a typical hospital. This patient is only then likely to see you for grave concerns if any at all. Now consider the situation in which you are a legal counsellor specialising in contract law. You find out from Mashable, Geek.com, Popular Mechanics, and artificiallawyer.com that a artificial intelligence (AI) or robot coded to interpret contracts beat not one but 20 contract lawyers in terms of accuracy rate (AI scored 95%, lawyers on average scored 85%) and time to complete task (AI took only 26 seconds while lawyers finished the job in 92 minutes on average).

Change is disruptive

The impact of artificial intelligence and automation, two of the nine pillars of Industry 4.0, on the labour market is unprecedented. Enabled by the Internet of Things (IoT), Industrial Internet of Things (IIoT), Cloud-based manufacturing, and smart manufacturing, the new industrial revolution at hand promises a seamless, intelligent and automated production flow across processes that facilitate economic activity. This ongoing transformation of industry results in higher levels of efficiency and changes in the relationships between actors on both the supply and demand sides of the market, which now include the machine playing a critical role in achieving overall productivity (Vaidya et al., 2018; Gilchrist, 2016).

The disruption in the labour market is imminent. In the Philippines where the business process outsourcing sector contributes to about 8% of the country's gross domestic product and employs over 1.4 million full-time employees, about 900,000 workers face the risk of obsolescence due to automation (AT Kearney, 2018; ASEAN, 2017; Karthik et al., 2017). In the ASEAN-5 (Cambodia, Indonesia, the Philippines, Thailand, and Vietnam), nearly three in five jobs face a high risk of automation (Chang and Phu, 2016). Globally, in what is considered the “worst-case scenario”, a study revealed that almost 800 million could be displaced of which 400 million would require new training for entirely new job categories (Manyika et al., 2017).

Characterised by the fusion of physical, biological, and virtual worlds, Industry 4.0 is expected to change not only the production and consumption of goods and services but also the way people live and view the world around them (Schwab, 2017). With more extensive connectivity through the Internet, it not surprising how new business models, concepts, and patterns of behaviour emerge inducing shifts in the demand for new skills and the nature of jobs in the 21st-century industry (see World Bank 2019 and World Economic Forum 2018 for a detailed discussion).

A case for learning

The increasingly brisk pace of innovation in the industry at costs made more accessible as the adoption of “best practices” in productivity-enhancing measures create an incentive for even more firms to implement artificial intelligence and automation, among other labour-saving initiatives enabled by Industry 4.0. With valid reasons to fear technological displacement characteristic of the transition in the short run, individuals must learn, and for firms and institutions to provide learning opportunities that respond to the transformative effects of the new industry on virtually every sector (Acemoglu and Restrepo, 2018; Acemoglu and Restrepo, 2017).

Stiglitz and Greenwald (2014), in their groundbreaking work, *Creating a Learning Society: A New Approach to Growth, Development, and Social Progress*, underline the importance of learning especially in episodes of rapid productivity increases that have both microeconomic and macroeconomic ramifications. They maintain that shifting the production possibilities frontier further out through an increase in investments in capital and people require the necessary stabilisation that firm-level and government policies on knowledge generation provide as instability brought about by episodic yet transformative changes in technology is adverse to learning itself. Doing so results in “fuller and more efficient utilisation of resources” and lead to “systematically higher rates of productivity increase.”

Without learning, individuals would be left behind in the transition to newer innovations and are likely to take on new jobs requiring lower productivity and, consequently, at lower wage rates. Technological unemployment, once a contentious topic in the discussion of labour market dynamics in past industrial revolutions (cf. Schwab, 2016), is no longer theoretical but a practical reality today.

Updating human capital theory

Celebrated models of human capital by Becker (1962), Ben-Porath (1967), and Mincer (1958) provide the fundamental framework for understanding schooling and training decisions on the part of both the individual and the firm. Pivotal work on human capital theory in the mid-20th century have responded to essential questions about the role of learning in maximising the optimising individual’s lifetime earnings and the role of education in determining the potential of success in the workforce.

At the time of their landmark work on human capital theory, technological advances in manufacturing enabled higher levels of efficiency in the mass production of goods accompanied

by the further lowering of costs of transportation and communication and broadening access to education. Determinants of success—investments in formal education, time in training, types of on-the-job training, and the role of parental investments, to name a few factors—in the labour market motivated much of the formal enquiry into human capital theory. While results and insights from studies over half a century ago still enjoy some relevance today, changes in the way individuals learn in the 21st-century are just as essential if theory serves the role of explaining contemporary behaviour and phenomena.

If existing theories no longer fully or only partially explain human capital dynamics, then new theories must be developed. In the theories proposed by Becker, Ben-Porath and Mincer, formal education (i.e., by schooling), informal education (i.e., on-the-job training), the role of parental human capital in the formation of non-cognitive skills (otherwise known as “soft skills”) are well established both theoretically and empirically (see Heckman and Kautz 2012 for studies related to non-cognitive skills). Killingsworth (1982) introduced the “learning by doing” model in an attempt to marry it with Becker and Ben-Porath’s investment in training approach, establishing the role of experience (i.e., time on performing work itself) analytically as an additional source of human capital stock aside from formal training and episodes of formal schooling.

Learning networks and connectivism

These existing models of human capital formation have a gap that fails to recognise the learning that occurs in social networks and communities of practice as they tend to focus on the individual accumulating human capital and not on the individual as a member of a network of other learners. Learning in networks is not entirely new; that is, learning in social networks is well established in the literature. Extensive discussions on Bayesian learning and topology of social

networks provide substantial contributions to the theory of learning in the modern world (see Mossel et al. 2015 and Acemoglu et al. 2008 for a sampling of the literature on learning networks).

However, most of the available literature on learning networks (or learning in social networks), sophisticated as they come, are focused mainly on the dynamics of learning itself and not how learning in networks accrues to the human capital formation of the individual and the network as a whole. One insight from Acemoglu et al. (2008) that we find interesting as it is relevant to this paper is the finding that, as the social network becomes sufficiently large, individuals converge to taking the right action conditional on private beliefs being unbounded, proving the existence of asymptotic learning in the network. They demonstrated that as long as private beliefs are unbounded, there would be asymptotic learning in almost every rational network. This paper's results complement these findings; however, only in terms of the necessary size of the network to maximise learning accrued to the wealth on knowledge.

Learning theories are also evolving as new ways of creating, consuming, and sharing knowledge emerge as shaped and influenced by the Internet, where network externalities are inherent and native. Today, individuals produce and consume knowledge in social networks whether in their communities of practice (i.e., sometimes offline) or their digital communities (e.g., via online platforms like Facebook, e-mail, ResearchGate). Intrinsic in this learning is the interaction with other learners who are also involved in activities that produce and consume knowledge. Furthermore, the rise in available knowledge and the continued technological progress at a faster pace of innovation given Industry 4.0 provide an even stronger impetus to understanding human capital dynamics under the conditions of learning networks.

Today, connectivism is one of the most respected theories on learning networks found first in the groundbreaking work of Siemens (2005), "Connectivism: a learning theory for the digital

age” which recognises learning as a network externality that is shaped by advances in technology and an increase in the level of socialisation. He cites the limitations of existing theories of learning, particularly behaviourism, cognitivism, and constructivism, in explaining emergent phenomena brought about by technological progress. He argues that an entirely new approach is necessary when underlying conditions have changed so significantly as seen in the impact of technology and new sciences—chaos and networks—on learning.

In his theory, learning is a “process that occurs within nebulous environments of shifting core elements—not entirely under the control of the individual” (Siemens, 2005). Further, he identifies that learning and knowledge are determined in the “diversity of opinions” through a process of “connecting specialised nodes or information sources.” Consistent with the approach that integrates technology with learning, he posits that “learning may reside in non-human appliances” (e.g., on networks, databases, platforms). With the individual as a learner at the core of connectivism, personal knowledge is a product of the network and, in turn, feeds into organisations and institutions through networks and back to the individual. This process of learning in networks presents cycles of knowledge creation and sharing, ensuring that a learner is up-to-date in the requirements of their field as reinforced by the connections formed along the way. The process is subject to decentralisation and the emergence of self-organising networks consistent with the theory’s alignment with complexity science.

The relevance of Siemens’ theory of connectivism has tremendous value in the attempt to update human capital theories that are in tune with the changes in the ways individuals learn today. More than ever, connectivism deserves further exploration in economics to further substantiate the theory’s lack of analytical foundations and empirical testing for which it has received criticism (Clara and Barbera, 2013; Bell, 2011). Some scholars have also questioned the robustness of the

theory in terms of its philosophical and epistemological significance (Clara and Barbera, 2013). While connectivism may have limitations, several scholars in the field of education theory across different domains recognise its usefulness in understanding learning dynamics in a digitally-enabled society (Goldie, 2016; Clara and Barbera, 2013; Kop and Hill, 2008).

Opportunity

This theoretical note aims to address the gap in existing human capital theories taking into account that the individual is creating, consuming and sharing learning in a network that is consistent with practical observations of most learners today.

It seeks to contribute to the body of knowledge on learning social networks and substantiate the central feature of connectivism—network learning—with an analytical exposition of human capital dynamics within a network of learners. Furthermore, the paper seeks to establish optimal conditions which enable knowledge to accumulate subject to the rate of technological progress and the relationship between the size of the network and its effect on overall wealth on knowledge that may be accrued to the network.

This paper lays the groundwork for further studies in learning networks with multiple agents (n -individuals and n -firms) in a variety of market structures and different types of competitive games (e.g., Cournot, Stackelberg, Bertrand), and the welfare economics of innovative learning individuals and firms.

Analytics

Let the return to education at time t be defined as follows,

$$r(t) = N(t)[1 - s(t)]H(t)e^{-\rho t}$$

where $N(t)$ is the population of learners in the learning network, $s(t)$ be the amount of time spent learning by one learner, and $H(t)$ be the knowledge generated from the learning network. In order to capture network externalities effects, we assume that the size of the learning network affects an individual's return to education.

The individual seeks to optimise lifetime returns to education, formally:

$$\max \int_0^T N(t)[1 - s(t)]H(t)e^{-\rho t}$$

subject to the law of motion of knowledge generation $\dot{H}(t) = s(t)H(t) - \delta_H H(t)$ and population growth of the learning network $\dot{N}(t) = s(t)N(t) - \delta_N N(t)$, where δ_H is obsolescence rate of knowledge and δ_N is the departure rate from the learning network. We assume here that growth of the learning community is made endogenous by the time spent learning $s(t)$ by the optimising learner. That is, the learning network grows faster when an individual spends more time engaged in learning.

The Hamiltonian is given by

$$H = N(t)[1 - s(t)]H(t)e^{-\rho t} + \lambda(t)[s(t)H(t) - \delta_H H(t)] + \mu(t)[s(t)N(t) - \delta_N N(t)]$$

with the following first-order condition equations

$$\frac{\partial H}{\partial s(t)} = -N(t)H(t)e^{-\rho t} + \lambda(t)H(t) + \mu(t)N(t) = 0 \quad (1)$$

$$\frac{\partial H}{\partial H(t)} = N(t)[1 - s(t)]e^{-\rho t} + \lambda(t)[s(t) - \delta_H] = -\dot{\lambda}(t) \quad (2)$$

$$\frac{\partial H}{\partial N(t)} = [1 - s(t)]H(t)e^{-\rho t} + \mu(t)[s(t) - \delta_N] = -\dot{\mu}(t) \quad (3)$$

$$\frac{\partial H}{\partial \lambda(t)} = s(t)H(t) - \delta_H H(t) = \dot{H}(t) \quad (4)$$

$$\frac{\partial H}{\partial \mu(t)} = s(t)N(t) - \delta_N N(t) = \dot{N}(t) \quad (5)$$

with the following boundary conditions: $s(T) = 0$, $\lambda(T) = 0$, $\mu(T) = 0$, $H(0) = H_0$, and $N(0) = N_0$.

Combining equations (4) and (5), we obtain the following result

$$H(t) = N(t)C e^{-(\delta_H - \delta_N)t} \quad (6)$$

where $C = H_0/N_0$ or endowed knowledge per learner. Taking the derivative of equation (6) with respect to $N(t)$ implies that the marginal contribution to the stock of knowledge by an additional

learner to the network is positive albeit diminishing over time. We can express equation (6) in terms of knowledge per learner as follows

$$h(t) = h_0 e^{-(\delta_H - \delta_N)t}$$

where $h \equiv H/N$. The equation above then implies that knowledge per learner will either grow exponentially, when the obsolescent rate of knowledge is less than the departure of learners from the learning network, or decay. The former is more likely than the latter under Industry 4.0 as technological innovations are quick to replace existing ones at an increasing rate.

Equations (4) and (5) both imply

$$\lambda(t)H(t) = R(T) - R(t) = \mu(t)N(t) \tag{7}$$

where $R(t) = \int r(t)dt$ can be interpreted as wealth accumulated by the learner on knowledge at time t . Since the co-state variables $\lambda(t)$ and $\mu(t)$ can be interpreted as the marginal cost of knowledge and learner, respectively, then equation (7) implies that the total cost of education and of the learning network at time t should be equal to the wealth on knowledge to be gained at the remaining periods until the terminal period.

Denote the remaining wealth on knowledge to be gained from time t to T as $W(t)$. Substituting this in equation (1) and using equation (6), we obtain an expression of the population of the learning community

$$N(t) = \sqrt{\frac{2W(t)e^{(\rho+\delta_H-\delta_N)t}}{H_0/N_0}} \quad (8)$$

Here, we observe that the learning network will continue to grow as long as $\rho + \delta_H$ is greater than δ_N . That is, as long as the learners are sufficiently impatient, characterized by a high discount rate ρ (i.e., the greater ρ , the more they will delay learning) and that knowledge sufficiently becomes obsolete faster due to faster technological progress, represented by a high δ_H , the learning community will grow in size over time. We also note the positive relation between $N(t)$ and $W(t)$. That is, as long as remaining wealth on knowledge is increasing the learning network will grow over time. Since $W(T) = 0$, then at the terminal period we must have $N(T) = H(T) = 0$. Hence, at some point in time, the learning network will decline and no further knowledge will be generated.

Insights for policy

The analytical exposition of learning networks as a characteristic feature of the 21st-century industry reveal insights for policy consideration particularly in addressing the need to continuously grow the size of the network, lowering the departure rate from learning networks, and using the brisk pace of innovation as leverage to make knowledge more accessible to learning networks.

First, the priority is to grow the learning network faster than the departure of learners from the network. This imperative ensures the continued generation of knowledge and the consequential wealth on knowledge generated for the rest of the network. As the rate of technological obsolescence increases, there is an increased likelihood of knowledge obsolescence as well. Given the brisk pace of innovation in the context of the fourth industrial revolution, universities, the private sector and government must consider policies that encourage the participation of adults in

learning networks. One such solution is making continuing education programmes more accessible to the public by affording workers the flexibility to convert a portion of the total work hours into training time through accredited continuing education partners. Universities and other higher education institutions are encouraged to pursue a closer partnership with the private sector in bringing such lifelong learning programmes closer to workers. The government, on the other hand, can strengthen its technical and vocational education and training programmes (TVET) by expanding modules to lifelong learning skills with facilities that increase the likelihood of socialisation among learners both offline and on digital platforms.

Second, as the analytics show, there is a need to create continued learning incentives for workers, which can lower the departure rate from a learning network. Incentives like gaining recognition through gamified experiences, blended learning methodologies, and increased community interactions can lower the rate of departure from a learning network that is not caused by mandatory retirement from work. Additionally, providing an environment in which communities of practice can converge both offline and on digital platforms may induce salience within the learning network. Firms can encourage the participation of workers in industry-level professional organisations to strengthen the learning network among individuals within similar job categories and those who may not be in the same job categories but work within the same industry.

Finally, while there is virtually no measure that can decrease the rate of innovation, there is merit in considering policies that guarantee the adoption of newer, more efficient platforms in the creation, delivery and consumption of knowledge in the public domain. Solutions that make useful knowledge nonexcludable and nonrivalrous to more individuals use the brisk pace of innovation as leverage for learning networks to thrive. As technology achieves higher levels of efficiency in the knowledge process, learning networks can create, deliver and consumer

knowledge at a faster rate as well. If inefficient methods in the production and consumption of knowledge continue to characterise the state of learning, learning networks would be suboptimal, which may increase the departure rate from the network itself. Private-public partnerships may be considered in making knowledge more accessible to learning networks using the most efficient platforms available in the market which universities, firms and the general public can benefit from at lower or no cost to them.

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Appendix

From equation (4), we have

$$\int \frac{1}{H(t)} dH(t) = \int [s(t) - \delta_H] dt$$

$$\ln H(t) = \int s(t) dt - \delta_H t + C_H$$

$$H(t) = A_H e^{-\delta_H t} e^{S(t)}$$

where $A_H = e^{C_H}$ and $S(t) = \int s(t) dt$. Similarly, we obtain from equation (5) the following expression

$$N(t) = A_N e^{-\delta_N t} e^{S(t)}$$

Combining both results, we derive equation (6)

$$\begin{aligned} H(t) &= A_H e^{-\delta_H t} \left[\frac{N(t)}{A_N e^{-\delta_N t}} \right] \\ &= N(t) C e^{-(\delta_H - \delta_N)t} \end{aligned}$$

where $C = A_H/A_N$. Using the boundary conditions for H and N , we have at time $t = 0$ the following expression $H(0) = N(0)C$ which implies that $C = H_0/N_0$.

Equation (2) can be expressed as a linear differential equation in $\lambda(t)$

$$\dot{\lambda}(t) + \lambda(t)[s(t) - \delta_H] = -[1 - s(t)]N(t)e^{-\rho t}$$

The general solution is given by

$$\lambda(t) = e^{-[s(t)-\delta_H+C_H]t} \left[C_\lambda - \int N(t)[1 - s(t)]e^{-\rho t} e^{s(t)-\delta_H+C_H} dt \right]$$

$$\lambda(t) = H(t)^{-1} \left[C_\lambda - \int N(t)[1 - s(t)]H(t)e^{-\rho t} dt \right]$$

$$\lambda(t)H(t) = C_\lambda - R(t)$$

where $R(t) = \int r(t)dt$. Using the boundary condition $\lambda(T) = 0$, we have at time $t = T$ the following expression $0 = C_\lambda - R(T)$ or $C_\lambda = R(T)$. Hence, we have $\lambda(t)H(t) = R(T) - R(t)$. Similar derivations can be done to show that $\mu(t)N(t) = R(T) - R(t)$.

Since $\mu(t)N(t) = \lambda(t)H(t) = W(t)$, where $W(t) = R(T) - R(t)$, then we can express equation (1) as $-N(t)H(t)e^{-\rho t} + 2W(t) = 0$ or, equivalently,

$$N(t)H(t)e^{-\rho t} = 2W(t)$$

Using the result that $H(t) = N(t)C e^{-(\delta_H - \delta_N)t}$, we can further rewrite equation (1) as follows

$$N(t)^2 C e^{-(\rho + \delta_H - \delta_N)t} = 2W(t)$$

Solving for $N(t)$ gives equation (8)

$$N(t) = \sqrt{\frac{2W(t)e^{(\rho+\delta_H-\delta_N)t}}{H_0/N_0}}$$

Bio

Paul John M. Peña is an assistant professorial lecturer of corporate planning, the economics of innovation and economics for the liberal arts at De La Salle University where he is currently completing his doctorate in economics. His research interests are in human capital development, the economics of innovation, and market dynamics of the fourth industrial revolution. Before joining the academia, he was the chief executive officer of Ignite Delta and Partners Manila, chief experience officer of Dentsu Jayme Syfu, head of brand at Philip Morris International (Philippines), a chief digital officer of Leo Burnett, digital marketing director of Globe Telecom and marketing director of Zing.vn. He majored in the Humanities at the University of Asia and the Pacific.

Dickson A. Lim is an assistant professor of economics at the De La Salle University where he is currently the graduate studies coordinator for the School of Economics. His research interests are in labour economics, monetary economics, and complexity economics. He earned his doctorate in economics from De La Salle University. He previously worked for the International Monetary Fund.

Declaration

The authors have no conflicts of interest in pursuing this research and any of its extension as of this writing.