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Article

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Abstract:

Mainstream economic growth models downplay the role of energy, while attributing most of growth to an exogenous residual – total factor productivity (TFP). This makes them unsuitable to tackle the challenge of marrying sustainability and economic development targets. Meanwhile, research suggests that measuring energy at the stage where it's actually productive (*useful*), and in *exergy* terms (in thermodynamics, the potential to do work), unlocks new insights concerning energy's strong link with economic output. In this work we test for relationship linking TFP and final-to-useful (F-to-U) exergy efficiency, resorting to both observational and statistical methods (cointegration). Several models are considered, assessing the impact of: a) disaggregating capital inputs (i.e. buildings, stationary, non-stationary); b) quality-adjusting labour; c) disaggregating F-to-U exergy efficiency (stationary and non-stationary end-uses). Results for Portugal (1960-2014) show that TFP can be proxied by changes in F-to-U exergy efficiency, namely for stationary end-uses. This link is strengthened when disaggregate capital, and schooling-corrected labour measures are considered. When TFP is estimated as a function of F-to-U exergy efficiency, virtually all of long-term economic growth is explained by directly measurable capital, labour, and exergy efficiency in production. Resulting models provide satisfactory explanations of economic growth, founded on energy use and efficiency.

Keywords: economic growth; total factor productivity; energy efficiency; useful exergy; aggregate production function; cointegration analysis

JEL: C51; D24; O11; O13; O47; Q43

1. Introduction

European and international commitments towards decarbonization put forward by the European Commission, individual member-states, and other countries demand a drastic reduction in greenhouse gas (GHG) emissions by 2050. These commitments put the pressure on policymakers and businesses to identify the pathways to achieve decarbonization goals without compromising socioeconomic development.

One of the key energy options for decarbonization concerns the substitution of fossil fuel energy resources for renewable energy resources. Lowering the costs of renewables in the short-term will help lower the costs of decarbonization efforts, while also having long-term benefits such as reduced air pollution and increased access to energy. On the other hand, another major key option for decarbonization – shown to have a greater impact on GHG emissions than changing the supply-side of energy consumption – is concerned with reducing the amount of energy required to provide products and services, through improvements in energy efficiency, the "hidden" energy resource. According to the International Energy Agency, improving the energy efficiency of buildings, industry, and transportation could lead to a 1/3 reduction of the world's energy needs by 2050, and help control GHG emissions.[1] Together, energy efficiency and renewable energy are the twin pillars of sustainable energy policy [Prindle et al., 2007].

To discuss the effects of climate change mitigation actions, from the demand side, one must understand the role played by energy use and energy efficiency in the economy. However, this link is severed in basic mainstream neoclassical growth theory, which provides an explanation of economic growth built on capital accumulation and human labour, neglecting energy resources and, furthermore, attributing most of growth to an unexplained term – total factor productivity (TFP). These models are unsuited to tackle the challenges posed by a growing commitment to reduce GHG emissions and achieve sustainability goals, without compromising economic development.

Alternatively, it has been proposed (and empirically tested) that the role of energy use and efficiency in economic production and growth is better understood by adopting a more appropriate (and thermodynamically defined) measure to account for productive energy uses – *useful exergy* [Sousa et al., 2017]. Research suggests that useful exergy use and efficiency is closely related to TFP and economic growth, in the long-run [Brockway et al., 2018]. Thus, a deeper understanding of the possible relationships linking this energy metric with economic output, TFP, and the traditional factors of production (capital and labour) may open the door to better explanations for what drives economic growth in the long-run. This represents an improvement over the "standard" neoclassical growth models, in two ways: 1) accounting for observed economic growth without the need for an unexplained, residual term; 2) acknowledging the essentiality of energy use and efficiency in economic production and development. Improvements on economic growth modelling built on energy resources are more suited to benefit policymakers and businesses in tackling the challenges of marrying sustainability and economic development goals.

In this work we contribute to the development of improved models of economic growth, built on energy resources, by investigating the relationships linking the efficiency with which an economy converts final exergy into a useful form, and the exogenous total factor productivity term, representing the portion of observed growth not explained by the traditional factors of production. If a strong relationship is found between these two variables, and TFP can be proxied by final-to-useful exergy efficiency, then a better explanation for observed economic growth can be achieved, without downplaying the role of energy in the economy.

This article is structured as follows: subsections 1.1 and 1.2 review the role of energy inputs (and efficiency) acknowledged by both the mainstream neoclassical theory of economic growth, and alternatives proposed, for example, within the field of ecological economics, while also presenting *useful exergy* as the appropriate metric to account for productive energy uses in the economy; section 2 details the methods; section 3 presents the obtained results, which are discussed in section 4; section 5 concludes.

1.1. Mainstream economic growth theory and the role of energy

The most basic of mainstream neoclassical growth theory – the Solow-Swan model [Solow, 1956; Swan, 1956] – attempts to explain long-run economic growth as a result of capital accumulation and increases in human labour inputs. However, it has been shown that the combination from these two factors of production alone is insufficient to fully account for historical economic growth, and an exogenous residual term, often called total factor productivity (or TFP), is included in these models. Moreover, it has been shown that TFP is often the major driver of historical economic growth within industrialized economies [Easterly & Levine, 2001] (e.g. for the U.S. it accounts for approximately 85% of growth between 1909-1949 [Solow, 1957]).

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[1] It would also have a national security benefit by reducing the rate at which domestic energy resources are depleted, and reduce the level of energy imports from foreign countries.

At the core of the Solow-Swan model is a neoclassical aggregate production function (APF), representing the technological relationship between the factors of production (capital K and human labour L) and the economic output (Y) that can be produced by the combination of these factors. The Cobb-Douglas formulation for the APF is widely used due to its simplicity and attractive properties – Equation (1).

$$Y = A \cdot K^{\alpha_K} \cdot L^{\alpha_L} \tag{1}$$

In Equation (1), TFP is represented by a multiplier term A. The constant exponents α_K and α_L represent the output elasticities for each factor of production[2], a measure of their productive power.

Unlike the factors of production K and L, which are directly measurable from national accounts and statistics, the TFP term A is an exogenous residual, traditionally estimated as the ratio of empirically observed economic output (e.g. gross domestic product, or GDP), and the weighed combination of factors of production – Equation (2).

$$A = \frac{Y}{K^{\alpha_K} \cdot L^{\alpha_L}} \tag{2}$$

Because of the way it is calculated, the TFP term absorbs many components, some wanted (technical and/or organizational innovation), others unwanted (measurement errors, omitted variables, aggregation bias, etc.) [Hulten, 2001]. This has led to TFP being dubbed "a measure of our ignorance" [Abramovitz, 1956]. Further efforts in growth accounting have found that improvements on the measurement of the traditional factors of production – acknowledging qualitative differences, as well as quantity – leads to a deflation of the TFP residual term [Griliches & Jorgenson, 1967]. The seminal work of Griliches & Jorgenson (1967) has been implemented in a more general input-output framework [Jorgenson, Ho & Stiroh, 2008; Jorgenson, Gollop & Fraumeni, 2016], constituting the core methodology for the mainstream EU KLEMS growth accounts [O'Mahony et al., 2008]. One of the key characteristics of the EU KLEMS database is the adoption of accurate measures for both capital and labour inputs – taking into account the heterogeneity of these inputs, and the differences in the services provided by distinct types of capital (e.g. ICT and non-ICT) and labour (skilled, unskilled) – in order to achieve less biased estimates of TFP. The more "traditional" growth accounting methods assume that the marginal productivity of all types of labor and capital are the same and equal to the aggregate wage and capital shares (a "cost-share theorem"), which in turn leads to estimates of TFP contribution to growth which will overstate "true" TFP if the composition of either of factor inputs shifts over time towards types of higher quality (i.e. composition effects). On the other hand, the EU KLEMS allows for shifts in quality of factor inputs, e.g. to a workforce with a higher share of high skilled workers or to the purchase of specific capital asset classes – such as ICT – which potentially have higher marginal productivities. The EU KLEMS approach leads to a significant reduction in the contribution of TFP to overall economic growth (e.g. the TFP growth rates estimated in EU KLEMS for the Euro Area in 2004 are less than half of those calculated using the "traditional" approach) [Koszerek et al., 2007]. The offset in estimated TFP with the EU KLEMS approach is due to higher contributions from capital (estimated on the basis of a disaggregated capital stock) and from labour (estimated on the basis of a disaggregated labour force). Thus, by adopting measures for both capital and labour services rather than simply a measure of the quantity of either of these inputs essentially means that the efficiency gains from using these inputs (i.e. que quality improvements) are included as part of the contribution of these inputs to growth, rather than being part of the residual TFP component. The adoption of growth accounting methodologies such as the one used in EU KLEMS have been facilitated by the increased availability of quality-adjusted measures for both capital and labour inputs in current mainstream databases (e.g. the UK Office for National Statistics (ONS),

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[2] Following the constant returns to scale property that $\alpha_K + \alpha_L = 1$.

which made available estimates for capital services [Oulton & Wallis, 2015] and the Barro-Lee Educational Attainment Database, which provides detailed schooling datasets that can be used in the construction of quality-adjusted labour measures). However, a significant portion of TFP still cannot be "explained away", even when accounting for quality-adjusted measures for factors of production [van Ark, 2014].

Besides relying on an exogenous component to justify most of economic growth in developed countries, another major criticism raised against the basic neoclassical growth model concerns the absence of energy inputs, implying that economic growth is independent from energy use. In the Solow-Swan approach, this is justified by the aforementioned "cost-share theorem", which corresponds each factors' output elasticity with how much that factor receives as payments from total income. In national accounts, the combination of payments to capital and labour inputs virtually exhausts all of income (with capital receiving approximately 30% in the form of interest and rents, and labour receiving the remaining 70% in the form of wages and salaries). Furthermore, the share of payments attributed to each of the two "traditional" factors of production – capital and labour – is approximately stable for long periods of time, which constitutes a stylized fact for developed economies [Kaldor, 1957]. This stylized fact, in turn, supports the assumption that output elasticities in the Cobb-Douglas aggregate production function formulation – Equation (1) – are constant. Payments to energy inputs, when represented at all in national accounts, are roughly equated with revenues from energy industries, a very small share (< 10%) of income. It then follows, by neoclassical assumption, that the productive power of energy – when compared with capital and labour – must be correspondingly small. Energy is also considered, in mainstream economic growth theory, to be an intermediate input to production, generated by the primary factors of production (capital and labour), and used up entirely in production.

1.2. Productive energy as a driver of growth: theory and evidence

The neglect of energy as a relevant factor of production creates difficulties for the basic neoclassical growth models when it comes to justifying economic recessions resulting from energy crises in the past decades (e.g. the oil shocks of the 1970s) [Kümmel et al., 2008]. It also means that economic models such as this are not suitable to tackle the challenges posed by the commitment to reduce greenhouse gas (GHG) emissions and the transition towards a carbon neutral economy, without compromising economic development.

Outside mainstream economic growth theory, especially in the field of ecological economics, there is an acknowledgement that a suitable theory of economic growth should take into account the essentiality of energy resources to economic production, and improvements in energy efficiency as a driver of total factor productivity and economic growth.

Authors Robert Ayres and Benjamin Warr were among the first to emphasize that energy resources are productive in the economy, by conceptualizing a positive feedback cycle through which cheaper energy resources, due to discoveries, economies of scale and experience, ultimately lead to lower costs (and higher demand) for products and services, higher wages, and substitution of mechanical power produced from energy resource inputs, thus driving further increases in scale, experience, learning, and still lower costs [Ayres, 2001; Ayres & Warr, 2005].

This perspective implies that energy resources are, simultaneously, a major driver of economic growth, and a constraint, since there is no substitute for energy in production. Based on this outlook, some growth models have treated energy as an independent factor of production, alongside capital and labor, and were able to reproduce past economic growth without the need for a multiplier TFP component [Tintner et al., 1977; Kümmel et al., 2000; Lindenberger & Kümmel, 2002; Ayres & Warr, 2005]. The estimated output elasticities from these models suggest that the productive power of energy is higher than its cost share, and hence are not consistent with the neoclassical cost share theorem [Kümmel et al., 2008]. Other growth models posit minimum energy requirements to a given level of economic production [Stern, 1997] and include realistic constraints on substitution possibilities between energy, capital, and labor [d'Arge & Kogiku, 1973; Gross & Veendorp 1990; ven den Bergh & Nijkamp, 1994; Lindenberger & Kümmel, 2011]. All these studies conclude that future economic growth will be hampered by scarce energy resources.

The debate of whether or not energy should play a more relevant role in economic growth theories is accompanied by the debate on how energy inputs to production should be measured and aggregated, in order to accurately represent productive energy uses in the economy.

The stage at which energy flows become productive to the economy is the *useful* stage, which corresponds to energy actually delivered to perform an ultimate function in the economy, that has economic value (e.g. the light emitted, as opposed to the heat radiated, by an electricity-consuming lightbulb). The *useful* stage is distinguished from the *final* stage (energy transformed in the energy sector, and sold to consumers) and the *primary* stage of energy flows (energy contained in natural resources, prior to any transformation process).

Even at the useful stage, energy can be used in different forms (e.g. heat, work, light, etc.), which are qualitatively different. Work is the most valuable form of energy, because it has the highest efficiency of conversion to any other form[3]. The maximum amount of work that can be performed by a given quantity of energy as the target subsystem approaches thermodynamic equilibrium with a reference state, reversibly, constitutes a comparable measure across different energy forms, acknowledging differences in both quantity and quality. In thermodynamics, this metric is called the exergy content of energy, and it is definable not just to more familiar forms of energy but also to all materials. Useful exergy then, constitutes an appropriate metric to account for actually productive energy uses in the economy, comparable and aggregated across different energy forms.

Recent work on societal level exergy analysis has produced novel data for useful exergy for several countries [Warr et al., 2010; Serrenho et al., 2014]. Empirically, a common relationship between useful exergy consumption and economic output (represented by their ratio, *useful exergy intensity*) is found for industrialized economies. The methods for societal useful exergy accounting have been refined and employed to the Portuguese economy [Serrenho et al., 2016], uncovering a remarkably stable long-run useful exergy intensity for at least the past 50 years[4] – Figure 1.

These empirical results suggest that economic output is more closely linked to useful exergy consumption than other energy/exergy variables (primary, final), and have motivated research on the relationships linking useful exergy consumption, economic growth, and traditional factors of production such as capital and labour [Warr & Ayres, 2010]. The existence and robustness of statistically significant (and economically plausible) long-run relationships between economic output, capital inputs, human labour, and useful exergy inputs has been investigated resorting to appropriate econometric techniques, such as cointegration analysis. For the Portuguese economy, the cointegration approach has shown that a plausible Cobb-Douglas based growth model cannot be obtained unless useful exergy inputs are included in the cointegration space [Santos et al., 2018]. The same model adjusts very well to historical output trends without the need for an exogenous TFP component, but imposes a constraint on substitutability between the three factors of production: capital, human labour, and useful exergy. Furthermore, one of the most interesting aspects of this approach, focused on the Portuguese economy, is that the essentiality of useful exergy inputs to production is reconciled with the neoclassical "cost-share theorem" assumption that corresponds the output elasticities for capital and human labour inputs to the respective share of payments received by these factors from total income. Thus, the results found in [Santos et al., 2018] suggest that the recognition of energy resources as fundamental to economic production and growth does not

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[3] For example, 1 kWh of work can be converted into up to 1 kWh of heat at 30 ºC, while the same quantity of heat can only be converted into at most 0.066 kWh of work.

[4] There is also evidence for a remarkably stable useful exergy intensity for the last 150 years in Portugal [Serrenho et al., 2016], despite dramatic changes in the composition of useful exergy consumed throughout this period.

necessarily imply a rejection of neoclassical assumptions, namely that output elasticities can be corresponded with the traditional factors' cost shares.

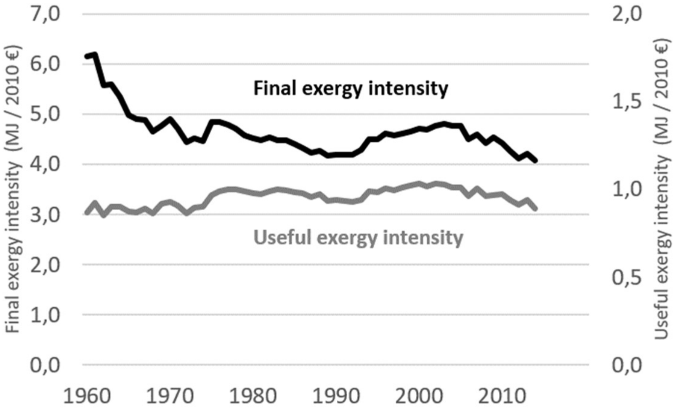

Figure 1 - Final and useful exergy intensity: Portugal 1960-2014.

Besides cointegration analysis, Granger non-causality tests have been conducted to determine the direction of causality between useful exergy consumption and economic output [Warr & Ayres, 2010, Santos et al., 2018]. These tests have uncovered evidence for unidirectional causality running from useful exergy consumption to economic output, thus suggesting that growth is driven by an increased availability of useful exergy ("growth hypothesis"), which can be attained both through either increased consumption of final exergy quantity, or increases in final-to-useful exergy efficiency.

The aforementioned results are consistent with the view that the quantitative amount of energy consumed, and the efficiency with which energy is converted, transformed, and delivered to end-uses, are important components of economic growth. Other recent works have focused specifically on the relationship between exergy efficiency and economic growth. In the work by Matthew Heun and Paul Brockway [Heun & Brockway, 2019], for example, a societal exergy analysis is conducted for both Ghana and the UK, and the obtained results suggest that thermodynamic primary-to-useful exergy efficiency and economic output are positively correlated. Although the authors do not assess the direction of causality in the efficiency-output link, they suggest two possible mechanisms to explain it: 1) higher economic output means higher investment in more exergy efficient final-to-useful equipment; 2) higher exergy efficiency leads to lower expenses with energy consumption, which in turn lead to more money available to invest in remaining economic activities, thus increasing output. It should be noted that both these mechanisms could also co-exist.

Also for the UK, a recent effort to develop an econometric economy-wide model that explicitly includes thermodynamics efficiency and useful exergy consumption as explicit integral components [Sakai et al., 2019] provides additional empirical evidence for the view that energy efficiency is a driver of economic growth. The MARCO (MAcroeconometric Resource COnsumption) model uses a counterfactual simulation approach to isolate and quantify the effect of thermodynamics efficiency gains on economic growth. Thermodynamic efficiency is understood by the authors as referring to the key final-to-useful exergy conversion stage, in which rarely studied at the economy-wide level, but where most of the thermodynamic exergy losses occur. Results obtained with the MARCO model suggest that between 1971 and 2013, a quarter of UK economic growth is due to gains in the final-to-useful exergy efficiency. The MARCO model also finds that useful exergy consumption is the energy variable with the highest impact on UK economic growth, surpassing both final exergy and energy prices. These results provide empirical support to the aforementioned argument that energy measured using the useful exergy metric reveals a closer link between energy consumption and economic growth.

The MARCO model results also support the earlier modelling work by Benjamin Warr and Robert Ayres and their Resource EXergy Services (REXS) model [Warr & Ayres, 2006], which also finds that thermodynamic efficiency gains have been (and will be) a significant driver of US economic growth. The REXS model eliminates the need for an exogenous TFP component to explain historical economic growth, instead redefining TFP (or technological progress) as a measure of the efficiency of conversion of energy into a useful form – useful exergy. Being a traditional model, REXS assumes that economic output is a measure of independent inputs, linked by an aggregate production function. The simplest model tested by the authors that includes energy inputs – an energy-augmented Cobb-Douglas production function – is able to explain historical US economic growth well for the 20th century, only when the energy inputs are measured as useful exergy (as opposed to primary energy). Statistical fits are significantly improved when adopting a linear-exponential (LINEX) production function originally proposed in [Kümmel, 1989]. However, the non-standard LINEX formulation has found little acceptance in the economics community, having been criticized for not satisfying standard concavity considerations [Saunders, 2008]. Nevertheless, despite the choice of aggregate production function, the results from the REXS model support the view that most of the contribution from "technical progress" to growth can be attributed to advances in energy/exergy efficiency.

The present work aims to provide further empirical validation for the strong relationship linking energy, measured in exergy terms at the useful stage, and economic growth. This is done by testing long-term correlation between total factor productivity and final-to-useful exergy efficiency, through appropriate econometric methods. If a robust relationship is found, exogenous TFP can be proxied by directly measurable final-to-useful exergy efficiency, which leads to a better model for economic growth, that takes into account the important role of energy used productively in the economy.

In assessing the possible relationship linking TFP and final-to-useful (F-to-U) exergy efficiency, our starting point will be looking at observational evidence – i.e. similarities in the long-term behavior of both variables, that may constitute stylized facts – and then employ the statistically robust techniques from econometric to test for the validity of these stylized facts. TFP and F-to-U exergy efficiency are tested for cointegration, and Granger causality between them. These techniques have been previously applied to study the link between energy/exergy and economic output [Stern, 2000; Cleveland et al., 2000; Ghali & El-Sakka, 2004; Stresing et al., 2008; Warr & Ayres, 2010; Santos et al., 2018].

As in the aforementioned MARCO model [Sakai et al., 2019], we focus on exergy efficiency at the final-to-useful stage, since this is where most thermodynamic conversion losses occur, and for the amounting theoretical and empirical evidence supporting a closer relationship between the useful stage of energy flows and economic output and growth. We adopt the exergy metric to account for energy flows at both the final and useful stages since it is the only metric which allows a correct aggregation of energy flows in physical terms, by weighing the potential work that can be performed by a given energy quantity.

We take into account that more accurate measures for historical exogenous TFP will affect the estimation of a significant relationship between this variable and F-to-U exergy efficiency. Thus, we look at the impact of adopting more appropriate measures for the traditional factors of production – capital and labour – on estimates for historical TFP (and consequently, on the relationship linking TFP and F-to-U exergy efficiency). We consider measures for capital and labour that take into account qualitative differences in these inputs. In line with the methods adopted in the EU KLEMS database growth accounting procedures, we adopt, for capital, a disaggregated measure based on differences in the depreciation of heterogeneous capital asset categories. As for human labour, we acknowledge qualitative differences by introducing a human capital corrected measure for these inputs.

While the major goal of this work is essentially to establish that there is a significant relationship linking TFP and F-to-U exergy efficiency, at the aggregate level, we also consider the disaggregation of F-to-U exergy efficiency, distinguishing between stationary and non-stationary end-uses. This disaggregation allows us to form a more complete picture of the changes in useful exergy consumption and efficiency in the economy, and which – if any – specific useful exergy end-use is more closely related to the evolution of TFP, and economic growth, in the long-run. Understanding this can lead to important insights for policymakers to act upon, in promoting sustainable economic development.

Finally, given the aforementioned innovative aspect of our analysis, and in order to facilitate the econometric procedures and comparison between different modelling choices, we opt to restrict ourselves to a simple formulation for our economic models. Thus, all the models presented in our work are based on the very tractable and intuitive (albeit simplistic) Cobb-Douglas aggregate production function satisfying, among other properties, the neoclassical "cost-share theorem". As shown in previous work […], the recognition and assessment of the importance of energy resources to economic growth does not necessarily require the rejection of neoclassical assumptions such as the "cost-share theorem".

The aforementioned lines of investigation are detailed in the next section, and applied to the Portuguese economy, between 1960 and 2014.

2. Materials and Methods

The conceptual macroeconomic model adopted in this work is represented in Figure 2.

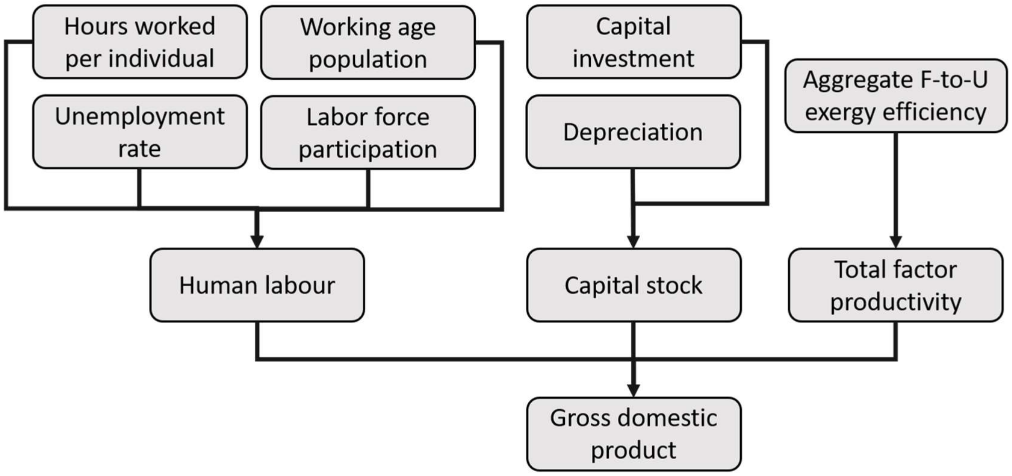

Figure 2 - Conceptual macroeconomic model.

In Figure 2, boxes represent variables in the model, and arrows the direction of estimation. This work focuses mostly on the relationships linking aggregate final-to-useful exergy efficiency with total factor productivity, or TFP (shown on the right of Figure 2), and the relationship linking capital stock, human labor inputs, and TFP, with the observed level for gross domestic product (GDP) – i.e. an aggregate production function. Below we discuss each of the variables pertaining to these relationships.

2.1. Final-to-useful exergy efficiency

Exergy accounting studies at the final and useful stages of exergy flows conducted for single or groups of countries are generally organized by energy carrier (*coal & coal products*; *oil & oil products*; *natural gas*; *combustible renewables*; *electricity & CHP heat*; *food & feed*; *other non-conventional carriers*), institutional sector (*industry*; *transport*; *other, including residential*; *non-energy uses*; and *energy industries own-uses*), and end-use (*heat, high, medium, and low temperature*; *mechanical drive*; *light*; *other electrical uses*; and *muscle work*).

Aggregate final-to-useful exergy efficiency (ε) is computed as follows:

$$\varepsilon = \frac{X_U}{X_F} \tag{3}$$

In Equation (3) X_U and X_F are, respectively, total useful and final exergy consumed.

The aggregate final-to-useful exergy efficiency represented in Equation (3) is also disaggregated at the end-use level, in terms of *stationary* end-uses, and *non-stationary* end-uses – Equation (4). This disaggregation and computation of final-to-useful exergy efficiencies for both stationary and non-stationary end-uses requires disaggregate data on final and useful exergy consumption by end-use categories.

We consider *stationary* exergy devices as any device having as a final function high, medium, or low temperature heating, heat cogeneration, cooling, lighting, electronics, electrolysis, and stationary mechanical drive. On the other hand, we consider *non-stationary* exergy devices as any device having as a final function non-stationary mechanical drive, e.g. vehicles.

$$\varepsilon_n = \frac{X_{U,n}}{X_{F,n}} \tag{4}$$

With $n =$ stationary (st), non-stationary (nst).

2.2. Capital inputs to production

The available stock of capital to production is estimated, each year, through the application of the perpetual inventory method (PIM), represented by Equation (5):

$$K_t = K_{t-1} + i_{t-1} \cdot Y_{t-1} - \delta_{t-1} \cdot K_{t-1} \tag{5}$$

Where K_t is the level of capital stock, at a given year t, i_t is capital investment, as a percentage of GDP Y, and δ_t is the rate of capital depreciation. The PIM can be used to compute aggregate capital stock, or disaggregate capital stock, provided an initial stock, investment shares and depreciation rates are available for each type of capital asset.

On a first approach, both investment and capital stock are taken as aggregates only, without accounting for asset heterogeneity. Aggregate investment expenditure (accounted as total gross fixed capital formation – GFCF) and aggregate capital stock are taken directly from the European Commission's annual macroeconomic database (AMECO) [AMECO, 2019], which applies the PIM assuming a given initial value for capital stock (1960), and a constant aggregate (i.e. equal across different capital assets) annual rate of depreciation.

We also consider disaggregate capital stock inputs to production, which are computed in three distinct categories: *stationary capital*, *non-stationary capital*, and *buildings*. This is done by crossing data available in three databases: AMECO; the European Commission's EU KLEMS database [Jäger et al., 2016]; the Groningen Growth and Development Centre Penn World Tables – Version 9.1 (PWT9.1) [Feenstra et al., 2015].

The AMECO database provides investment time series (as GFCF), between 1960 and 2014, for the following asset categories: *construction* (including dwellings and non-residential construction and civil engineering); *metal products and machinery*; *transport*; and *other*.

The EU KLEMS database provides a larger disaggregation of capital investment by asset types, but for a shorter period of time (1995-2014). The asset categories contemplated in the EU KLEMS database are: *dwellings*; *other buildings and structures*; *transport*; *other machinery equipment and weapons*; *ICT equipment* (subdivided in *computer hardware* and *telecommunication equipment*); *intellectual property products* (subdivided in *computer software and databases*, *research and development*, and *mineral exploration/artistic originals*); *cultivated assets*.

Both the AMECO and EU KLEMS databases assume a constant aggregate depreciation rate δ. The PWT9.1 database, on the other hand, provides constant depreciation rates for different asset categories, namely: *structures* (residential and non-residential); *transport equipment*; *software*; *other* *machinery and assets*; *computers*; *communication equipment*. Table 1 shows the constant depreciation rate associated with each asset category in PWT9.1.

Table 1 - Constant specific depreciation rates (δ), by PWT9.1 capital asset category.

Asset category	Constant depreciation rate (δ)
Structures (residential and non-residential)	2.0%/year
Transport equipment	18.9%/ year
Software	31.5%/ year
Other machinery & assets	12.6%/ year
Computers	31.5%/ year
Communication equipment	11.5%/ year

Through a correspondence between asset categories across the three aforementioned databases, time series for capital stock, between 1960 and 2014, with specific (constant) depreciation rates for each PWT9.1 asset category, are obtained. This is done by first corresponding capital investment in AMECO asset categories with capital investment in EU KLEMS asset categories, for overlapping years (1995-2014). The EU KLEMS categories are then corresponded with PWT9.1 asset categories, in order to assign a specific (constant) depreciation rate to each asset type. Some categories can be directly corresponded, while others require additional assumptions. For the asset categories of *computers* and *software*, since no specific data is available prior to 1995, it was assumed that capital investment in these categories was zero in 1960[5], and grew linearly up until 1995. For the asset category *communication equipment*, a similar assumption was made: no capital investment prior[6] to 1877, and linear growth from there up until 1995.

Figure 3 summarizes the capital disaggregation process undertaken in this work. All GFCF data from the AMECO and EU KLEMS databases is measured in constant price 2010 €.

For each asset type, the initial capital stock, for the year 1960, is computed according to the disequilibrium approach found in [Berlemann & Wesselhöft, 2014] – Equation (6):

$$K_{n,1960} \approx \frac{I_{n,1960}}{g_{I_n} + \delta_n} \tag{6}$$

Where for each category of capital n, the initial capital stock $K_{n,1960}$ is approximately equal to the level of capital investment in that asset in that initial year $I_{n,1960}$ divided by the sum of the average growth rate of capital investment in that asset for the first 5 years since 1960 (g_{I_n}) and the constant depreciation rate corresponding to that type of asset.

Hence, for each asset category n, Equation (5) becomes

$$K_{n,t} = K_{n,t-1} + i_{n,t-1} \cdot Y_{t-1} - \delta_{n,t-1} \cdot K_{n,t-1} \tag{7}$$

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[5] The first computer in Portugal was installed in 1961.

[6] The first experiments with the telephone began in this year in Portugal.

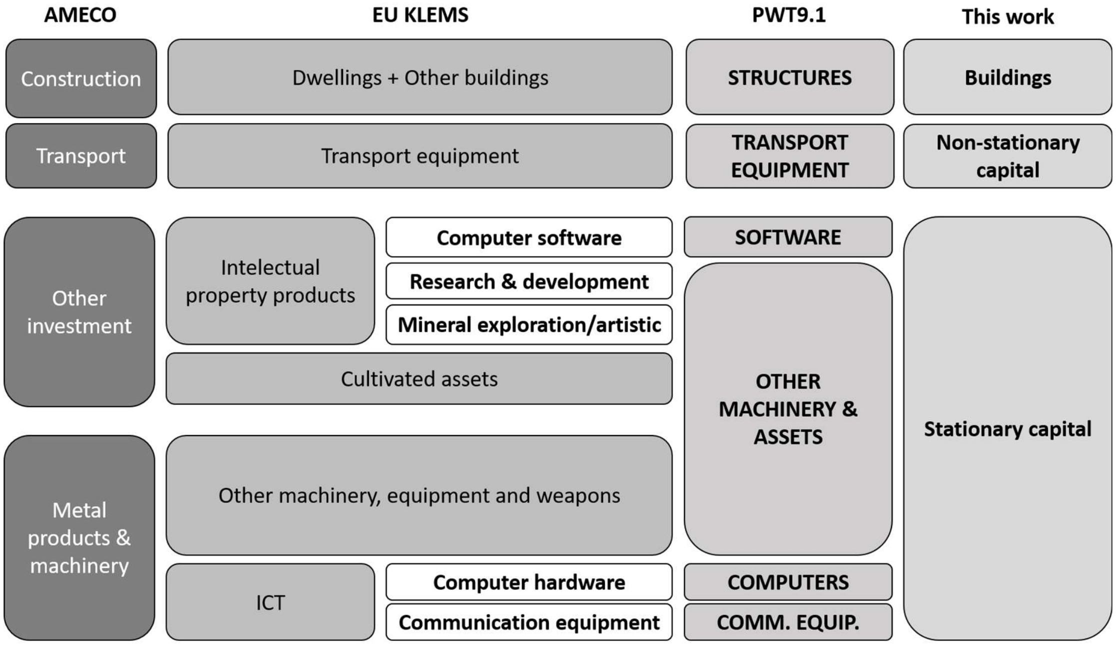

Figure 3 - Correspondence between capital asset categories across databases (AMECO, EU KLEMS, PWT9.1) and the present work.

After computing capital stock for each asset category in the PWT9.1, these categories are grouped according to the three distinct capital categories defined in this work: *stationary capital* (K_{st}), *non-stationary capital* (K_{nst}), and *buildings* (K_{build}) – see Figure 3. Total capital stock, for any given year t, is given by

$$K = K_{st} + K_{nst} + K_{build}$$

(8)

The process described above, by which we disaggregate capital inputs, implies that aggregate capital estimated as the sum of defined disaggregate capital categories – Equation (8) – will differ from aggregate capital obtained directly from the AMECO database – see Appendix A. In Sections 2.4 and 2.5 we detail when and where each of these two aggregate measures for capital inputs are adopted in our analysis.

2.3. Human labour

The quantity of human labour inputs to production is accounted in this work as the total number of hours worked by total engaged individuals in the economy. This quantity is given by Equation (9):

$$L = h \cdot (1 - ur) \cdot Pop_{15} \quad \cdot PR$$

(9)

Where h is the average annual number of hours worked per engaged individual, ur is the unemployment rate, Pop_{15-64} is the working age population, defined between the ages of 15 and 64, and PR is the labour force participation rate.

On a first approach, no qualitative differences in human labour inputs are considered. We later take into account qualitative differences in labour inputs, proxying the quality of labour by a human capital index hc, a function of the expected returns to additional years of schooling [Caselli, 2005; Psacharopoulos & Patrinos, 2004] – Equations (10) and (11).

$$\varphi(s) = \begin{cases} 0.134 \cdot s; \quad \text{if } s \leq 4 \\ 0.134 \cdot 4 \ + 0.101 \cdot (s-4); \quad \text{if } \ 4 < s \leq 8 \\ 0.134 \cdot 4 \ + 0.101 \cdot 4 \ + \ 0.068 \cdot (s-8); \quad \text{if } \ s \leq 8 \end{cases}$$

(10)

$$hc = e^{\varphi(s)}$$

(11)

Where s corresponds to years of schooling, and $\varphi(s)$ are the average returns depending on the years of schooling. Quality-adjusted human labour inputs L' are then given by Equation (12):

$$L' = hc \cdot h \cdot (1 - ur) \cdot Pop_{15-64} \cdot PR$$

(12)

2.4. Aggregate production function

The aggregate production function adopted throughout is the work is the Cobb-Douglas formulation which, in its most traditional form, is written as Equation (1) – Section 1. By directly observing the levels, over time, of economic output Y, and both capital and labor inputs (K, L), total factor productivity A can be indirectly estimated, from Equation (1), as shown in Equation (2) – Section 1.

On a first approach, the simplest form for the aggregate Cobb-Douglas production function – Equation (1) – is adopted, and TFP is hence computed from Equation (2). As previously mentioned, in this approach aggregate capital stocks are taken directly from the AMECO database, and human labour inputs are unadjusted for qualitative differences – Equation (9). Also, in the first approach, the output elasticities corresponding to capital (α_K) and labour (α_L) inputs are assumed constant and corresponded with the average share of payments, in total income, allocated to each of these two factors of production. For capital, the output elasticity is measured as the ratio of gross operating surplus (adjusted for the self-employed) by total income, and for labour, as the ratio of total compensation of employees by total income.

With the disaggregation of capital inputs considered in Section 2.2 – Equation (7)– the Cobb-Douglas aggregate production function formulation is rewritten as

$$Y = [A_{st} \cdot K_{st}]^{\alpha_{Kst}} \cdot [A_{nst} \cdot K_{nst}]^{\alpha_{Knst}} \cdot {K_{build}}^{\alpha_{Kbuild}} \cdot L'^{\alpha_L}$$

(13)

Where capital is disaggregated according to the aforementioned categories of *stationary* capital (K_{st}), *non-stationary* capital (K_{nst}), and *buildings* (K_{build}). Human labour inputs are here assumed to be quality-adjusted (L'), computed from Equation (12).

In Equation (13), total factor productivity is capital-augmenting, and disaggregated in terms of technological improvements to *stationary* capital (A_{st}), and *non-stationary* capital (A_{nst}). It is assumed here that technological improvements represented by the total factor productivity multipliers are related only to these two categories of capital inputs, since these are the categories of capital assets that actively convert final to (economically productive) useful exergy, with a given efficiency.

Analogously to Equation (2), an estimate for aggregate total factor productivity is obtained from Equation (13) by rewriting as

$$A' = A_{st}^{\alpha_{Kst}} \cdot A_{nst}^{\alpha_{Knst}} = \frac{Y}{K_{st}^{\alpha_{Kst}} \cdot K_{nst}^{\alpha_{Knst}} \cdot K_{build}^{\alpha_{Kbuild}} \cdot L'^{\alpha_L}}$$

(14)

Note that aggregate total factor productivity estimated in this way differs from the values computed from Equation (2), hence it is distinguished by an apostrophe.

Concerning the output elasticities in Equations (13) and (14), there is no readily available data regarding the disaggregation of shares in total income allocated to capital in terms of the categories considered in this work (*stationary, non-stationary, buildings*). For that reason, the output elasticities corresponding to the three capital inputs categories are computed by approximating the payments to each type of capital inputs by the consumption of fixed capital (i.e. the depreciated value each year) for each category of capital:

$$\alpha_{K_{n,t}} = \alpha_K \frac{K_{n,t} \cdot \delta_n}{\sum_n K_{n,t} \cdot \delta_n} \tag{15}$$

Where $\alpha_{K_{n,t}}$ is the output elasticity for capital category n, at time t, and $K_{n,t} \cdot \delta_n$ is the consumption of fixed capital for capital category n, at time t, given by multiplying the (constant) depreciation rate δ_n by the corresponding capital stock $K_{n,t}$. Total capital depreciation at time t is given by the denominator $\sum_n K_{n,t} \cdot \delta_n$. The multiplicative term α_K corresponds to the output elasticity for total aggregate capital inputs, estimated as the share of total income paid to total capital inputs, in national accounts (see Appendix B). The value for the output elasticities for each category of capital inputs in Equation (13) is assumed constant over time and equal to the average of time-varying output elasticities computed using Equation (15).

2.5. Total factor productivity and final-to-useful exergy efficiency

One of the major aims of the present work is to explore the extent to which total factor productivity and final-to-useful exergy efficiency are linked, and that the former can be written as a function of the latter.

On a first approach, this link is established in a very simplistic way, merely by empirically comparing the behavior of both the TFP and aggregate final-to-useful exergy efficiency time series, over time, in log-space and normalized to their corresponding initial values, and computing the coefficient c that is the ratio of the two – Equation (16).

$$c = \frac{\ln(A)}{\ln(\varepsilon)} \tag{16}$$

Where A correspond to the time series for total factor productivity computed from Equation (2), and ε to the aggregate final-to-useful exergy efficiency time series computed from Equation (3).

The coefficient c in Equation (16) is found to be roughly constant over time (see Section 3.5) and on a first approach total factor productivity A can be written as a function of aggregate final-to-useful exergy efficiency:

$$A = \varepsilon^c \tag{17}$$

Consequently, economic output is written as a function also of final-to-useful exergy efficiency, substituting Equation (17) in Equation (1):

$$Y = \varepsilon^c \cdot K^{\alpha_K} \cdot L^{\alpha_L} \tag{18}$$

In this initial approach, the TFP term A and economic output Y in Equations (1), (2), (17), and (18), are computed considering aggregate capital directly taken from the AMECO database, human labour inputs unadjusted for qualitative differences – Equation (12) – and aggregate final-to-useful exergy efficiency – Equation (3). This is the most basic approach to assess the possible relationship between TFP and final-to-useful exergy efficiency, requiring only macroeconomic data readily available from international databases.

However, as a consequence of the disaggregate PIM conducted in Equation (6), the aggregate capital stock estimate – Equation (8) – will differ from the one obtained directly from the AMECO. This, along with the estimation of quality-adjusted human labour inputs – Equation (12) – means that the estimated TFP term A given by Equation (2) will also differ. We compute and compare the TFP A – Equation (2) – adopting both the AMECO and our own aggregate capital estimates, and unadjusted versus quality-adjusted human labour inputs, and we link both estimates with aggregate final-to-useful exergy efficiency – Equations (16) and (17). The results are compared in the Results section.

Analogously to the previous approach, in this work we also compute the TFP term A', obtained from the disaggregation of capital inputs in the aggregate production function – Equations (13) and (14) – and test its link with aggregate final-to-useful exergy efficiency, assuming a constant exponent c'.

$$A' = \varepsilon^{c'} \tag{19}$$

Moreover, in the present work we also test for models linking the TFP term A with disaggregate final-to-useful exergy efficiencies for stationary and non-stationary end-uses – Equation (4) – by assuming

$$A = \varepsilon_{st}^{c_1} \cdot \varepsilon_{nst}^{c_2} \tag{20}$$

Where each final-to-useful exergy efficiency is weighed by a corresponding constant exponent.

On the other hand, when considering both the disaggregation of capital inputs and the disaggregation of final-to-useful exergy efficiency, we express total factor productivity as the product of the TFP terms associated with each category – *stationary* (A_{st}) and *non-stationary* (A_{nst}) – as in Equation (14). Each of these components is assumed to be a function of its corresponding final-to-useful exergy efficiency, i.e.

$$\begin{aligned} A_{st} &= \varepsilon_{st}^{c'_1} \\ A_{nst} &= \varepsilon_{nst}^{c'_2} \end{aligned} \tag{21}$$

Where c'_1 and c'_2 are assumed to be constants. From Equations (14) and (21), the aggregate TFP term A' can then be written as a function of both *stationary* and *non-stationary* final-to-useful exergy efficiencies:

$$A' = \varepsilon_{st.}^{c'_1 \cdot \alpha_{Kst.}} \cdot \varepsilon_{nst.}^{c'_2 \cdot \alpha_{Knst.}} \tag{22}$$

Substituting Equations (21) and (22) in the disaggregate Cobb-Douglas production function of Equation (13) gives

$$Y = \varepsilon_{st.}^{c'_1 \cdot \alpha_{Kst.}} \cdot \varepsilon_{nst.}^{c'_2 \cdot \alpha_{Knst.}} \cdot K_{st.}^{\alpha_{Kst.}} \cdot K_{nst.}^{\alpha_{Knst.}} \cdot K_{build.}^{\alpha_{Kbuild.}} \cdot L'^{\alpha_L} \tag{23}$$

The formulation for the production function represented in Equation (23) distinguishes among three categories of capital inputs (*stationary capital* K_{st}, *non-stationary capital* K_{nst}, and *buildings* K_{build}), each with a corresponding output elasticity, and between two types of final-to-useful exergy efficiency (*stationary* ε_{st}, and *non-stationary* ε_{nst}), each with its own contribution to TFP.

Besides the simplistic approach to assess the relationship between TFP and aggregate final-to-useful exergy efficiency in Equation (16), in the present work we aim to obtain more statistically robust estimates for the coefficient c, by adopting econometric techniques of cointegration analysis. According to this approach, two or more non-stationary time series are cointegrated if there is a linear relationship between them that is itself stationary. If cointegration exists, this linear relationship forms a statistically significant long-run relationship between the variables.

To compare the explanatory power to observed TFP estimates from the disaggregation of both capital inputs to production and final-to-useful exergy efficiencies, the models tested for cointegration in the present work correspond to Equations (17)[7], (19), (20), and (22). That is:

$$\ln(A) = u_1 \cdot \ln(\varepsilon) \tag{24}$$

$$\ln(A) = u_2 \cdot \ln(\varepsilon_{st.}) + u_3 \cdot \ln(\varepsilon_{nst.}) \tag{25}$$

$$\ln(A') = u_4 \cdot \ln(\varepsilon) \tag{26}$$

$$\ln(A') = u_5 \cdot \ln(\varepsilon_{st.}) + u_6 \cdot \ln(\varepsilon_{nst.}) \tag{27}$$

Where u_1, u_4 correspond to constants c and c' in Equations (17) and (19), u_2 and u_3 correspond to constants c_1 and c_2 in Equation (20), and u_5 and u_6 correspond to $c'_1 \cdot \alpha_{Kst}$ And $c'_2 \cdot \alpha_{Knst}$ in Equation (22).

Lifting the natural logarithm in Equations (24-(27), we obtain:

$$A_1 = \varepsilon^{u_1} \tag{28}$$

$$A_2 = \varepsilon_{st.}^{u_2} \cdot \varepsilon_{nst.}^{u_3} \tag{29}$$

$$A'_1 = \varepsilon^{u_4} \tag{30}$$

$$A'_2 = \varepsilon_{st.}^{u_5} \cdot \varepsilon_{nst.}^{u_6} \tag{31}$$

Cointegration testing in this work is conducted using the software JMulti [Lütkepohl, Krätzig & Phillips, 2004], following the usual steps: 1) testing the time series for non-stationarity, through the Augmented Dickey-Fuller (ADF), Schmidt-Philips (SP), and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) unit root tests, both in levels (including a time trend) and first differences (without a time trend); 2) specifying the vector autoregression (VAR), through residual analysis (Portmanteau test) and lag length criteria; 3) testing for cointegration through the Johansen trace and max-eigenvalue tests, considering the hypothesis of both an intercept and linear trend in the cointegration space, or only an intercept; 4) specifying the vector error-correction model (VECM) based on the cointegration

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[7] Note that for the model based on Equation (17), we test only the variant in which aggregate capital is obtained from the disaggregation process conducted in Section 2.2 – Equation (7) – , and human labour inputs are quality-adjusted.

tests outcome, and conducting Granger non-causality tests to assess the causal effects between variables.

The aggregate production functions and economic output estimates corresponding to Equations (28)-(31) are presented below:

$$Y_1 = \varepsilon^{u_1} \cdot K^{\alpha_K} \cdot L'^{\alpha_L} \tag{32}$$

$$Y_2 = \varepsilon_{st.}^{u_2} \cdot \varepsilon_{nst.}^{u_3} \cdot K^{\alpha_K} \cdot L'^{\alpha_L} \tag{33}$$

$$Y'_1 = \varepsilon^{u_4} \cdot K_{st}^{\alpha_{Kst}} \cdot K_{nst}^{\alpha_{Knst}} \cdot K_{build}^{\alpha_{Kbuild}} \cdot L'^{\alpha_L} \tag{34}$$

$$Y'_2 = \varepsilon_{st}^{u_5} \cdot \varepsilon_{nst}^{u_6} \cdot K_{st}^{\alpha_{Kst}} \cdot K_{nst}^{\alpha_{Knst}} \cdot K_{build}^{\alpha_{Kbuild}} \cdot L'^{\alpha_L} \tag{35}$$

Hence, in the present work we test a total of 6 models assessing the possible relationships linking TFP and final-to-useful exergy efficiency. For clarity, these models and their differences in estimation are summarized in Table 2. These models tend from a more simplistic approach (Model A), towards an increasingly more complex approach, featuring adjustment in the quality of human labour inputs, and the disaggregation of both capital inputs and final-to-useful exergy efficiencies (Model F).

As an example, consider model D. For this model, capital inputs are measured as aggregate stock, as the sum of the capital categories defined in Section 2.2 – Equation (8). Human labour inputs are quality-adjusted, computed from Equation (12). Final-to-useful exergy efficiency is measured as disaggregated in the categories defined in Section 2.1. Historically observed TFP is computed from Equation (2), and the possible link between this TFP and disaggregate final-to-useful exergy efficiency is assessed by testing Equation (29) for cointegration[8]. Estimated economic output is given by Equation (33).

Table 2 - Summary of models tested. Column (1) identifies the model; columns (2-3) indicate the measure adopted for capital and labor inputs; column (4) indicates how final-to-useful exergy efficiency is estimated; columns (5-6) indicate how historical TFP is computed, and how it is estimated as a function of final-to-useful exergy efficiency; column (7) indicates whether the link between TFP and F-to-U efficiency is assessed through cointegration analysis or not; column (8) indicates how economic output is estimated.

Model	**Capital inputs**	**Labour inputs**	**F-to-U exergy efficiency**	**TFP**		**Coint.**	**Output**
				Obs.	**Est.**		
A	Agg. – AMECO	Unadj. – Eq. (9)	Agg. – Eq. (3)	Eq. (2)	Eq. (16)-(17)	No	Eq. (18)
B	Agg. – Eq. (8)	Quality-adj. – Eq. (12)	Agg. – Eq. (3)	Eq. (2)	Eq. (16)-(17)	No	Eq. (18)
C	Agg. – Eq. (8)	Quality-adj. – Eq. (12)	Agg. – Eq. (3)	Eq. (2)	Eq. (28)	Yes	Eq. (32)
D	Agg. – Eq. (8)	Quality-adj. – Eq. (12)	Disagg. – Eq. (4)	Eq. (2)	Eq. (29)	Yes	Eq. (33)
E	Disagg. – Eq. (7)	Quality-adj. – Eq. (12)	Agg. – Eq. (3)	Eq. (14)	Eq. (30)	Yes	Eq. (34)
F	Disagg. – Eq. (7)	Quality-adj. – Eq. (12)	Disagg. – Eq. (4)	Eq. (14)	Eq. (31)	Yes	Eq. (35)

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[8] Actually, testing the equivalent equation in natural logarithm – Equation (25) – for cointegration.

3. Results

This section presents the main results from the analysis proposed in this paper. A detailed discussion of results is included in Section 4.

Figure 4 represents the evolution of final-to-useful exergy efficiency for the Portuguese economy, both aggregate, and disaggregated by stationary and non-stationary end-uses – as determined through Equation (4).

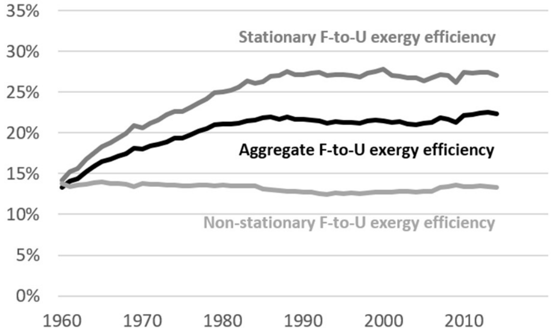

Figure 4 - Stationary, non-stationary, and aggregate final-to-useful exergy efficiencies: Portugal 1960-2014.

Figure 5 (a) compares historical GDP with the estimated contribution from the traditional factors of production – capital and labour – in a Cobb-Douglas aggregate production function, for all models considered in our analysis. Figure 5 (b) compares the estimated historical TFP for each model considered in our analysis.

Figure 6 presents the normalized ratio of natural logarithms of time series for TFP and aggregate final-to-useful exergy efficiency from Equation (16), for models A and B. The average values for the coefficient c for each of these models are given in Table 3.

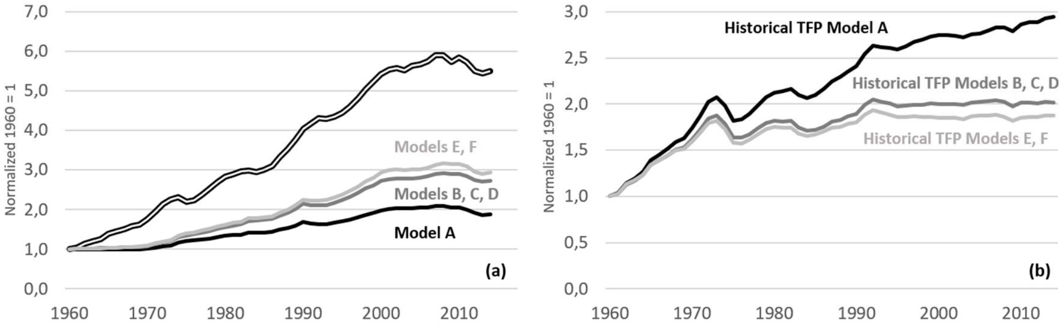

Figure 5 – (a) Historical GDP (compound line) and the estimated contribution from the capital-labour contribution according to Models A, B, C, D, E, and F; (b) Historical TFP computed for Models A, B, C, D, E, and F.

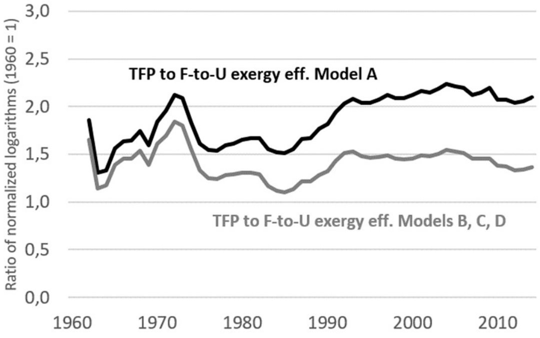

Figure 6 - Ratio of TFP to aggregate final-to-useful exergy efficiency for Models A, B, C, and D.

Unit root tests conducted on TFP time series – estimated from Equations (2) and (14) – and final-to-useful exergy efficiency time series – aggregate and disaggregate – suggest that these time series are all non-stationary in levels and stationary in first differences. That is, the aforementioned time series are integrated of the same order – order one I(1) – and can be tested for cointegration following the Johansen approach.

Cointegration tests conducted following the Johansen approach on Equations (24-27) conclude that there is at most one cointegration relationship linking the variables, and the corresponding cointegration coefficients are represented in Table 3. VAR and VECM models, for each case in Equations (24-27), are specified in order to verify no serial correlation between variables.

Table 3 - Estimated coefficients for Equations (16-17) and cointegration coefficients for all models considered. Errors in parenthesis.

Model	Eq.	Coefficient	Cointegration	Lags	Value
A	(16-(17)	c	No	-	1.869
B	(16-(17)	c	No	-	1.405
C	(24)	u_1	Yes	1	1.373 (0.055)
D	(25)	u_2	Yes	2	1.304 (0.081)
		u_3			2.383 (0.786)
E	(26)	u_4	Yes	1	1.240 (0.045)
F	(27)	u_5	Yes	9	1.190 (0.047)
		u_6			2.551 (0.445)

Granger non-causality tests conducted for each of the models corresponding to Equations (24-27) conclude that there is a unidirectional causal effect running from final-to-useful exergy efficiencies towards total factor productivity in models C, D, E. As for model F, there is evidence for bidirectional causality between final-to-useful exergy efficiencies and TFP.

Detailed results concerning the econometric analysis conducted in this work – unit root tests, cointegration tests, and Granger non-causality tests – are included in Appendix C.

Based on the results obtained from cointegration analysis, Equations (28)-(31) are used to obtain estimated TFP series, for each model. Figure 7 presents the adjustments to historical TFP obtained with each of the six models tested in our work. As shown in Table 2, adjustments to TFP obtained with models A and B are obtained by estimating coefficient c, as in Equations (16)-(17). On the other hand, adjustments to TFP obtained with models C, D, E, F are obtained from the relationships identified through cointegration analysis.

Figure 8 presents the adjustments to historical GDP obtained with each of the six models tested in our work. As shown in Table 2, adjustments to GDP obtained with models A and B are obtained by Equation (18), using estimated TFP as computed by estimating the coefficient(s) c reported in Table 3. Adjustments to GDP obtained with models C, D, E, F are obtained from Equations (32)-(35), using estimated TFP as computed with identified cointegration relationships, with cointegration coefficients reported in Table 3.

Table 4 presents growth accounting exercises conducted for each of the six models considered in our analysis. Represented growth rates correspond to average values for the entire period 1960-2014. Capital and labour contributions are determined by multiplying (constant) output elasticities by average growth rates, for each factor. Historical average TFP growth rates are determined by the ratio of observed GDP and capital-labour Cobb-Douglas production function. Estimated average TFP growth rates are computed from the TFP-final-to-useful exergy efficiency relationship for each model. Unexplained GDP growth is determined first as average historical TFP growth divided by average historical GDP growth, and then as historical average TFP growth minus estimated average TFP growth, divided by average historical GDP growth.

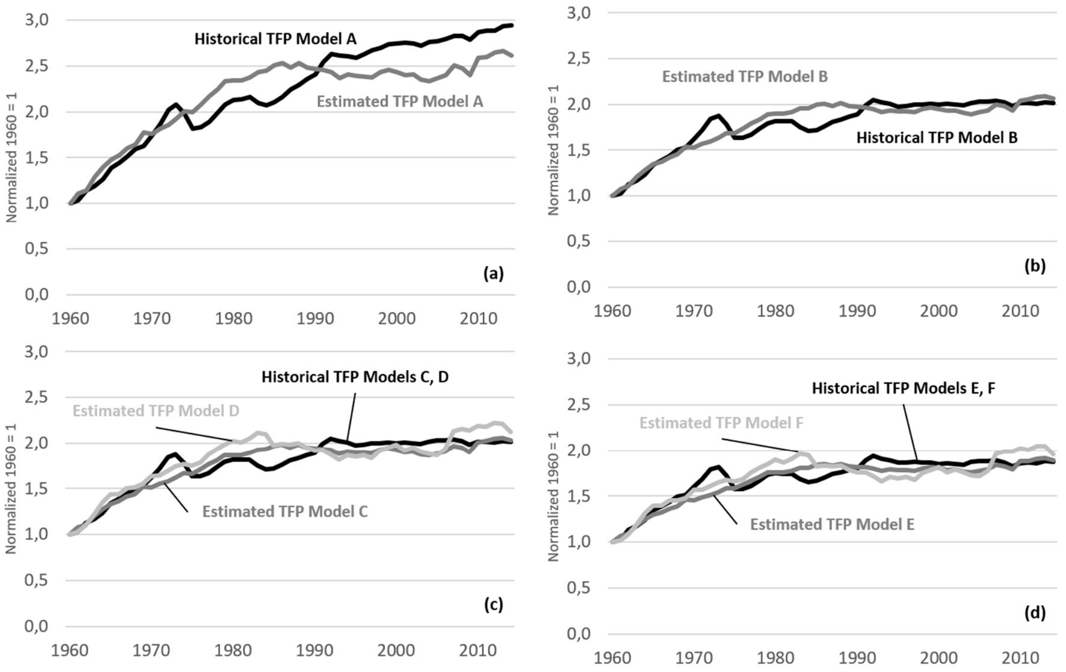

Figure 7 – Historical and estimated TFP for (a) Model A; (b) Model B; (c) Models C and D; (d) Models E and F.

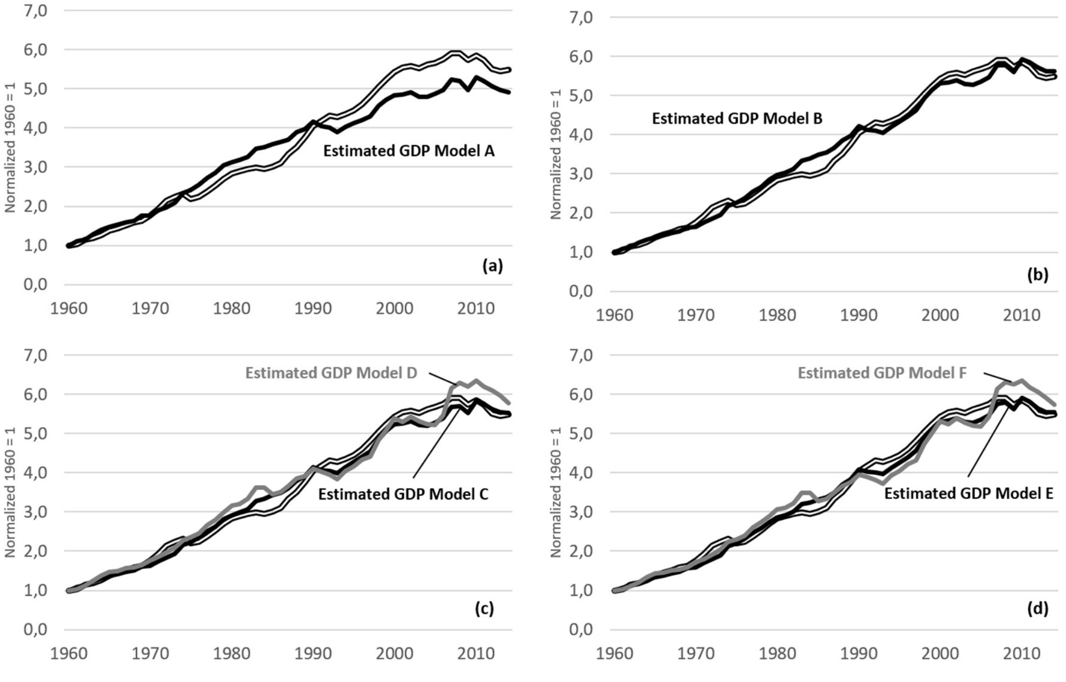

Figure 8 – Historical GDP (compound line) and estimated GDP according to (a) Model A; (b) Model B; (c) Models C and D; (d) Models E and F.

Table 4 – Growth accounting for all models. Column (1) indicates the model; Column (2) shows GDP growth rate; Columns (3-4) show the contributions to GDP growth from capital and labour, respectively; Columns (5-6) show, respectively, historical TFP growth and how much of GDP growth it represents; Columns (7-8) show, respectively, TFP growth estimated from final-to-useful exergy efficiency, and how much of GDP growth is left unexplained. All growth rates are averages for the period 1960-2014.

Model	GDP (%)	Capital (%)	Labour (%)	Hist.TFP (%)	Unexplained GDP growth (%)	Est. TFP (%)	Unexplained GDP growth (%)
A	3.20	0.98	0.19	2.03	63.35	1.79	7.48
B	3.20	0.87	1.01	1.33	41.44	1.35	−0.60
C	3.20	0.87	1.01	1.33	41.44	1.32	0.36
D	3.20	0.87	1.01	1.33	41.44	1.40	−2.28
E	3.20	1.01	1.01	1.19	37.18	1.19	0.10
F	3.20	1.01	1.01	1.19	37.18	1.25	−1.88

The next section provides a detailed and structured discussion of the results presented here.

4. Discussion

4.1. Historical final-to-useful exergy efficiency and TFP

Reproducing some of the results previously obtained in [Serrenho et al., 2016], Figure 4 shows that aggregate final-to-useful (F-to-U) exergy efficiency in Portugal is characterized by a pronounced increase at the beginning of the period (1960-80). This rise in efficiency is brought by the electrification process that took place in the country, as well as increasing high temperature heat uses, consequences of the second industrialization era in Portugal. After 1980, aggregate F-to-U exergy efficiency stabilizes at 22-23%, due to a significant increase in automobile uses that have offset the continuing – albeit slowing down – electrification process.

Regarding disaggregate F-to-U exergy efficiency for the Portuguese economy, Figure 4 reveals that, throughout the studied period between 1960 and 2014, only stationary F-to-U exergy efficiency increased. This increase is more significant between 1960 and 1980, since electrification occurred firstly in industries, providing electric stationary mechanical drive with 70-80% F-to-U exergy efficiency to the economy. The broader use of high temperature heat, with 34-44% F-to-U exergy efficiency, also contributed to the rise of stationary F-to-U exergy efficiency during the 1960-80 period. From 1980 to 1988, the rise in stationary F-to-U exergy efficiency decelerates, and from 1988 until 2014 stagnates around 27%. As for non-stationary F-to-U exergy efficiency, it is remarkably stable – almost constant – throughout the whole considered time period of 1960-2014. This stability is probably due to the less than significant increases in internal combustion engines (ICE) F-to-U exergy efficiencies, as well as the low penetration of electrical vehicles up to 2014.

The contributions from the so-called "traditional" factors of production – capital and labour – to historical GDP, for each model, compared in Figure 5 (a) reveal that most of the evolution in historical GDP for the Portuguese economy is unaccounted for by these inputs. The average GDP growth rate and corresponding average capital and labour contributions, for the period 1960-2014, can be consulted in Table 4.

For Model A and Models B, C, and D, TFP is computed from Equation (2), using capital from the AMECO database and unadjusted labour (Model A), and using estimated capital and quality-adjusted labour, estimated from the methodologies detailed in Sections 2.2 and 2.3, respectively (Models B, C, D). Improving the measurement of the "traditional" inputs to production significantly reduces the magnitude of the TFP residual term: from over 60% of observed average GDP growth to just over 40% - Table 4. Most of this reduction comes from acknowledging qualitative differences in human labour inputs, as opposed to a merely quantitative measure, since the average growth rate for capital is in fact lower for Models B, C, and D than for Model A (a consequence from the asset-specific constant depreciation rates assigned in Section 2.2).

For Models E and F, TFP is computed from Equation (14), using disaggregate capital estimates for buildings, stationary, and non-stationary assets, and quality-adjusted labour. The sum of average growth rates for disaggregate capital (weighted by each corresponding constant output elasticity – see Appendix B - Table B. 1) is higher than the average rate of aggregate capital, hence the TFP component is reduced, for Models E and F, below 40% of average observed GDP growth - Table 4. Thus, the disaggregation of capital by asset type (and corresponding output elasticities) provides an improved measurement of the contribution of each type of capital asset to economic growth, thus reducing the magnitude of the TFP residual term. However, even with improved measures adopted for capital and human labour inputs, TFP is still the largest component of observed economic growth for the period 1960-2014 in Portugal.

Figure 5 (b) explicitly shows the historical TFP estimates for each model considered in our analysis, and is another way of looking at the results from Figure 5 (a) discussed above. Figure 5 (b) also hints at a possible relationship between historical TFP and final-to-useful exergy efficiency time series. Like aggregate final-to-useful exergy efficiency – Figure 4 – historical TFP shows a significant increase in the earlier decades of the studied time period, and a subsequent deceleration and stagnation towards the end of the period.

4.2. Final-to-useful exergy efficiency as a proxy for TFP

The six models developed in this work aim at establishing a relationship in which final-to-useful (F-to-U) exergy efficiency (or efficiencies) act as a proxy for changes in historical TFP, and consequently GDP growth.

The models can be organized as follows: models for which the TFP-efficiency relationship is estimated purely from observational methods (Models A and B); models for which the TFP-efficiency relationship is estimated through more robust statistical methods – i.e. cointegration analysis (Models C, D, E, and F). Within each aforementioned group, the models can be further distinguished according to the adopted measures for the traditional factors of production – capital and labour – and F-to-U exergy efficiency. Regarding the models estimated using cointegration analysis (C, D, E, F), we are particularly interested in assessing the effects of aggregate versus disaggregate measures for both capital and F-to-U exergy efficiency in explaining historical TFP and GDP growth.

4.2.1. Observational models (A and B)

For Models A and B, the relationship between historical TFP and aggregate F-to-U exergy efficiency is computed from purely observational methods, without resorting to cointegration analysis – Table 2. For these models, the relationship between the two variables is characterized by the coefficient c, given by the ratio of the (normalized) natural logarithms of both time series – Equations (16(17). From Figure 6, we can see that the underlying assumption that this coefficient c is approximately constant over time is much more befitting to Model B than to Model A. The value estimated for the c coefficient is higher for Model A (1.863) than for Model B (1.405) – Table 3.

Comparing between historical TFP and estimated TFP for Models A and B – Figure 7 (a-b) – we conclude that for both models we can obtain a satisfactory reproduction of historical TFP from the behavior of aggregate F-to-U exergy efficiency. This suggests that aggregate F-to-U exergy efficiency is a good proxy for historical TFP. The relationship seems to be much closer for Model B, strengthened by quality-adjusting (i.e. adopting better measures) for the traditional inputs to production – capital and labour.

For Model A, historical TFP grows, on average, 2.03%, while TFP estimated as a function of aggregate F-to-U exergy efficiency grows, on average, 1.79% - Table 4. This suggests that, for Model A, about 88% of historical TFP is proxied by the behavior in aggregate F-to-U exergy efficiency. Regarding GDP growth, by acknowledging aggregate F-to-U exergy efficiency as a proxy for unexplained TFP, Model A is able to account for over 90% of average GDP growth in Portugal, compared with less than 40% of GDP growth explained with just the traditional factors of production.

For Model B, historical TFP grows less, on average, than for Model A, since more of historical GDP growth is accounted for by the (quality-adjusted measures of) traditional factors of production (namely education-adjusted labour). Historical TFP grows on average 1.33% for Model B, and TFP estimated as a function of aggregate F-to-U exergy efficiency grows, on average, 1.35%. This leads to an "over-explanation" of average historical TFP growth by Model B, which is propagated to the explanation of average historical GDP growth. This "over-explaining" of average TFP and GDP growth can probably be attributed to the purely observational method used to estimate TFP from F-to-U exergy efficiency in Model B. Nevertheless, it is clear from both Table 4 and Figure 8 (a-b) that Model B provides a better explanation for historical TFP and GDP in Portugal, when compared to Model A.

Overall, the results obtained from the two observational models considered in our analysis suggest a close relationship linking historical TFP and aggregate final-to-useful exergy efficiency, and that using exergy efficiency as a proxy for TFP leads to a more satisfactory explanation for observed economic growth. Furthermore, the adoption of improved measures for the traditional factors of production – particularly labour – leads to more accurate computation of the unexplained TFP component of economic growth and this, in turn, leads to a strengthened link between TFP and aggregate final-to-useful exergy efficiency.

4.2.2. Cointegration models (C, D, E, F)

For Models C, D, E, and F, the relationship between historical TFP and aggregate final-to-useful (F-to-U) exergy efficiency (Models C, E) or disaggregate – stationary and non-stationary – F-to-U efficiencies (Models D, F), is obtained through cointegration analysis. For all the aforementioned models, the relationship between TFP and F-to-U exergy efficiency (or efficiencies) is characterized by the cointegration coefficients u_i $(i = 1,2,3,4,5,6)$ – Equations (24-(27).

The evidence for at most one cointegration relationship between TFP and exergy efficiency is robust, according to the Johansen tests, and this supports the argument for a long-run, statistically significant relationship linking these variables. On the other hand, the results from the Granger-causality tests conducted on these models – suggesting that it is F-to-U exergy efficiency that Granger-causes TFP – support the argument that TFP can be written as a function of F-to-U exergy efficiency.

Aggregate versus disaggregate final-to-useful exergy efficiency

Models C and E test for the relationship between TFP and aggregate F-to-U exergy efficiency, while Models D and F test for the relationship between TFP and disaggregate (stationary and non-stationary) F-to-U exergy efficiencies. We can directly compare Model C (E) with Model D (F), regarding the effects of adopting, respectively, aggregate or disaggregate F-to-U exergy efficiencies in the estimation of TFP and GDP.

Model C can also be directly compared with Model B, the only difference between the two being the estimation method for their respective coefficients (observational, for Model B; cointegration, for Model C). The cointegration coefficient obtained for Model C is similar (and within the margin of error) to the c coefficient computed for Model B – see Table 3. The result obtained with Model C is statistically more robust, while still providing validation for the observational evidence that motivated this work – explored in the specification of Models A and B – that historical TFP and aggregate F-to-U exergy efficiency behave similarly, over time, for Portugal, and that the former can be proxied by the latter.

Disaggregation of F-to-U exergy efficiency between stationary and non-stationary end-uses does not invalidate the identification of at most one cointegration relationship linking these efficiencies with historical TFP. The estimated cointegration coefficients for Models E and F – Table 3 – suggest that changes in non-stationary F-to-U exergy efficiency (2.383 and 2.551 in Models D and F, respectively) have a much higher impact on TFP than changes in stationary F-to-U exergy efficiency (1.304 and 1.190 in Models D and F, respectively). This seems strange, since – from Figure 4 – non-stationary F-to-U exergy efficiency barely changes for the time period 1960-2014 (in fact, the most significant change is that is slightly decreases throughout this period), which contrasts with the increase (and later stagnation) verified in historical TFP. When considering these results, one should also note that the cointegration coefficients estimated for non-stationary F-to-U exergy efficiency in Models D and F have associated errors one order of magnitude higher than cointegration coefficients for stationary or aggregate F-to-U exergy efficiency.

We should take into consideration that – from Figure 4 – most of the long-term behavior of aggregate F-to-U exergy efficiency can be attributed to the long-term behavior of F-to-U exergy efficiency for stationary end-uses – both reflecting the evolution of historical TFP for this period. Hence, the high value for the coefficients associated with non-stationary F-to-U exergy efficiency in the cointegration relationships of Models D and F (u_3 and u_6, respectively) should be taken with a grain of salt. Since non-stationary F-to-U exergy efficiency remains virtually unchanged throughout the 1960-2014 period for the Portuguese economy, a high coefficient associated with this variable does not imply that it has a high impact on TFP growth throughout the same period. In fact, as an explanatory variable of TFP, non-stationary F-to-U exergy efficiency appears to be redundant. Note that cointegration coefficients estimated for stationary F-to-U exergy efficiency, in Models D and F (u_2 and u_5, respectively), are very similar in value to those estimated for aggregate F-to-U exergy efficiency, in Models C and E (u_1 and u_4, respectively). This implies that the information relevant to proxy historical TFP from F-to-U exergy efficiency is virtually all contained within the behavior of F-to-U exergy efficiency for stationary end-uses. In other words, for the Portuguese economy, TFP – and hence economic – growth has been driven, in the past decades, by changes in the final-to-useful exergy efficiency of stationary end-use devices.

Regarding the "over-explaining" of average TFP and GDP historical growth from Models D and F – Table 4 – it is likely due to the inclusion of the – possibly redundant – non-stationary F-to-U exergy efficiency variable in these models. In both of these models, average estimated TFP growth is higher than average historical TFP growth, which in turn leads to negative values for the percentage of historical GDP growth left unexplained by these models (-2.28% for Model D, and -1.88% for Model F). On the other hand, Models C and E constitute the two models in our analysis that best estimate historical TFP and GDP growth, based on measures for F-to-U exergy efficiency. These two models leave a very small percentage of historical GDP growth unexplained (0.36% for Model C, and 0.10% for Model E), when TFP is proxied by aggregate F-to-U exergy efficiency.

Overall, models for which the relationship between historical TFP and measures for final-to-useful exergy efficiency are estimated through cointegration analysis result in a statistically robust confirmation of the long-term relationship between these variables observed for Portugal, and expressed in observational terms in Models A and B. The major result from the cointegration analysis and Granger non-causality tests conducted in Models C, D, E, and F is that final-to-useful exergy efficiency is a good proxy for TFP change in the Portuguese economy, for the period 1960-2014.

Regarding the insights obtained from adopting aggregate versus disaggregate F-to-U exergy efficiency measures in our cointegration models, the results seem to confirm that most of the changes in TFP – and consequently GDP – proxied by F-to-U exergy efficiency are due to changes in F-to-U exergy efficiency for stationary end-uses. This is in line with the observations made with Figure 4, in which non-stationary F-to-U exergy efficiency remains approximately constant throughout the 1960-2014 period, while stationary end-uses are responsible for virtually all of the behavior in aggregate F-to-U exergy efficiency. Thus, the inclusion of a non-stationary F-to-U exergy efficiency variable in Models D and F leads to these models over-explaining average historical GDP growth, while Models C and E – with aggregate F-to-U exergy efficiency measures – provide the best explanations for historical average GDP growth.

Aggregate versus disaggregate capital stock

Models C and D adopt aggregate measures for capital stock, while Models E and F adopt disaggregate measures for capital stock, obtained through the methodology detailed in Section 2.2. We can directly compare Model C (D) with Model E (F), regarding the adoption, respectively, of aggregate or disaggregate measures of capital stock in the estimation of TFP and GDP.

Disaggregate capital stock, overall, has a higher average contribution to average GDP growth for the period 1960-2014 – Table 4. This leads to a slightly better explanation of average historical GDP growth obtained with the traditional factors of production (by about 5%). On the other hand, the adoption of disaggregate measures for capital stock also affects the estimation of the cointegration relationship between TFP and exergy efficiency – Table 3. Adopting disaggregate measures for capital stock results in a smaller cointegration coefficient for aggregate final-to-useful exergy efficiency, in Model E (1.240, compared with 1.373 in Model C), and in stationary final-to-useful exergy efficiency, in Model F (1.190, compared with 1.304 in Model D). The cointegration coefficient for non-stationary final-to-useful exergy efficiency slightly increases when disaggregate capital stock is considered, on Model F (2.551, compared with 2.383 in Model D). The errors associated with each cointegration coefficient are slightly smaller for all models that adopt disaggregate capital stock measures. In that sense, it could possibly be argued that the adoption of disaggregate capital stock measures in our models strengthens the identified cointegration relationship between TFP and final-to-useful exergy efficiencies. However, this affirmation requires further investigation.

Regarding the explanation of historical average GDP growth, Models C and E can be compared in terms of the choice of measure – aggregate or disaggregate – for capital inputs. As mentioned in the previous heading, these two models provide the best explanations for historical average GDP growth in our analysis – less than 1% of GDP growth unexplained. There is a slight difference between the results obtained with the two models, due to the adoption of aggregate versus disaggregate capital measures. In Model C, unexplained average historical GDP growth amounts to approximately 0.36%, while for Model E, unexplained GDP growth amounts to only 0.10%.

Thus, of all the models considered in our analysis, the results suggest that the one providing the best explanation for historical average GDP growth in Portugal, for the period 1960-2014, is a model for which capital measures are disaggregated according to buildings, stationary, and non-stationary assets; human labour is adjusted for qualitative differences via a schooling-based human capital index; and TFP is proxied by an aggregate final-to-useful exergy efficiency measure. With this model – Model E – we are able to explain virtually all of historical average GDP growth.

5. Conclusions

The main conclusion from the present work is that total factor productivity – the residual term obtained after discounting the contributions from the traditional factors of production from observed economic growth – can be adequately proxied by energy efficiency, from the final to the useful stage of energy flows, and measured in exergy terms. By estimating TFP as a function of directly observable final-to-useful exergy efficiency, all models considered in our analysis provide a better explanation for historical economic development in the Portuguese economy: one that is grounded in thermodynamics, as well as economics.

Our work follows in the footsteps of recent research that emphasizes adopting an appropriate metric to account for energy used productively (i.e. useful exergy) in order to capture the real importance of energy resources to economic production and development. The present work contributes to the arguments posed by Robert Ayres, Benjamin Warr, and others, that energy (exergy) efficiency is a driver of economic growth, by providing further empirical and statistical evidence to support them.

For the Portuguese economy, the range of models tested in our work provides observational evidence for a significant relationship linking historical TFP and F-to-U exergy efficiency. This insight is then strengthened by the econometric methods (i.e. cointegration, Granger causality) employed to identify a statistically significant long-run relationship linking the two time series. For all tested models, satisfactory estimates for historical TFP are obtained as a function of F-to-U exergy efficiency, and in turn this leads to better explanations for historical GDP growth, that do not rely on an exogenous residual term to justify most of economic development.

Improving on the measures for the traditional factors of production –considering disaggregate capital inputs, and schooling-corrected labour inputs – leads to improved computation of the historical TFP term, and as a result, this leads to a stronger relationship linking TFP and F-to-U exergy efficiency. So, the more accurately we measure TFP, the more of it can be explained as a function of final-to-useful exergy efficiency.

For the Portuguese economy, the long-term behavior of aggregate final-to-useful exergy efficiency is found to be mostly due to the changes in F-to-U exergy efficiency concerning stationary end-uses, rather than non-stationary end-uses, for which F-to-U exergy efficiency is mostly constant throughout the 1960-2014 period. This may imply, for the relationship linking F-to-U exergy efficiencies and TFP, that a satisfactory proxy for historical TFP can be obtained just from the changes in stationary F-to-U exergy efficiency, at least for Portugal. This insight needs to be further investigated in future work.

The methodology described – and applied to the Portuguese economy – in our work should be applied to other countries and groups of countries, in order to cement the explanatory power of final-to-useful exergy efficiency for TFP and GDP growth. Furthermore, efforts should be made to focus the study on longer periods of time, subject to data availability.

Also, we identify certain limitations with the methodology presented here, which should be addressed in future iterations of this type of analysis. First, the measures adopted for capital inputs can be improved, by considering not stocks but the services provided by different categories of assets. Second, further disaggregation of both capital, final-to-useful exergy efficiency, and even human labour measures should be considered in the future, in order to assess the contribution from each subcategory of these factors of production to economic development. Third, additional statistical analysis should be conducted on the relationship linking TFP and final-to-useful exergy efficiency, for example by testing for the possible exclusion of variables from the cointegration space.

Finally, the insights obtained with the present work could be implemented in the development of scenarios for future economic development, based on projections for changes in final-to-useful exergy efficiencies. Such scenario-building exercises would provide policymakers with relevant information to tackle the challenges posed by assuring economic growth within sustainability and environmental concerns.

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Appendix A – Capital and human labour inputs

The disaggregation of capital investment, by asset category, according to the three main databases consulted in this work – AMECO, EU KLEMS, and PWT9.1 – is represented in Figure A. 1, Figure A. 2, and Figure A. 3 respectively.

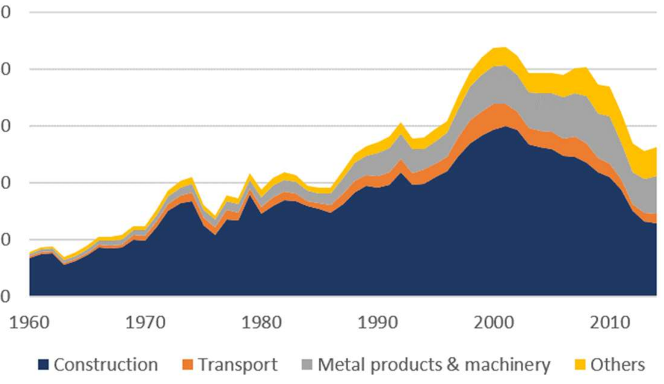

Figure A. 1 - Disaggregate capital investment by asset type according to the AMECO database for Portugal 1960-2014.

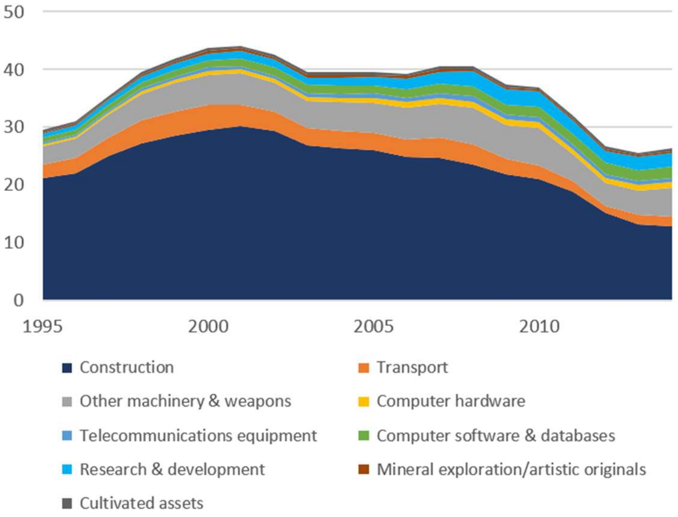

Figure A. 2 - Disaggregate capital investment by asset type according to the EU KLEMS database for Portugal 1995-2014.

The disaggregation of historical trends for investment in each capital category defined in this work (*stationary*, *non-stationary*, *buildings*), it is depicted in Figure A. 4. Several stages of the Portuguese economy can be identified from these series.

Up until 1974 there is a growing capital investment, which started before 1960, during the dictatorship period in Portugal, known as the Estado Novo. In 1974 the democratic revolution occurs and, along with the international crisis brought by the oil shock of 1973, causes a drop in capital investment. Between the years 1978-79 and 1983-84, Portugal has been under economic stabilization agreements with the International Monetary Fund (IMF), and hence the drops in capital investment preceding these periods. In 1985, Portugal joins the European Economic Community (EEC), and up until 1992 verifies a significant increase in capital investment. After that year, however, a new recession causes a new drop in capital investment.

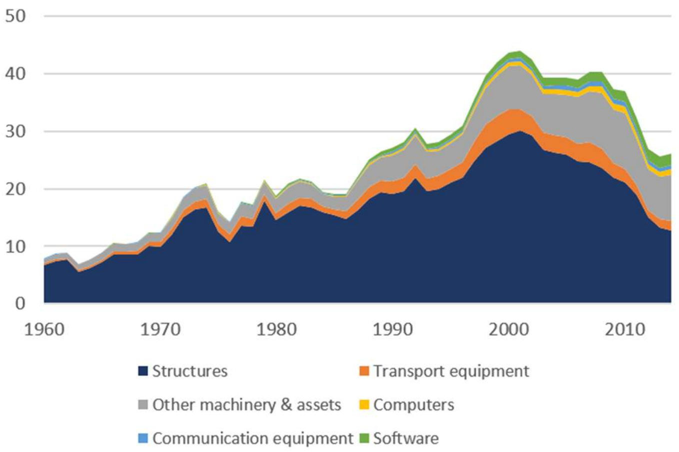

Figure A. 3 - Disaggregate capital investment by asset type according to the PWT9.1 database for Portugal 1960-2014.

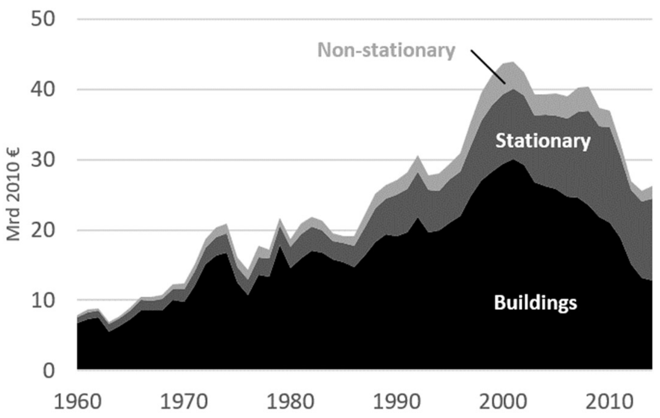

Figure A. 4 - Total investment expenditure disaggregated by non-stationary, stationary, and buildings categories: Portugal 1960-2014.

The road towards joining the euro currency, in 1999, is marked by rising capital investment, the basis of which is internal demand, and not exports. With interest rates dropping due to the prospects of joining the euro currency, there is a rapid growth of internal demand, and a decrease in private saving. While non-tradable goods sectors grew, investment in tradable goods sectors decreased, which contributed to a slow productivity growth. The Portuguese economy entered an external indebtedness period which became worse with the joining of the euro currency. Besides a difficult adjustment to its new monetary context, the extension of the European Union (EU) to Central and Eastern European countries, in 2004 and 2007, affected the Portuguese economy, impacting future investment and commercial trade. The appearance of China, India, and other emerging low-cost economies in the global market created competition in areas of intensive skilled labour, in which the Portuguese economy specialized. Finally, in 2008 the country suffers an economic crisis, due to internal and international factors, and capital investment falls abruptly. In 2011, the country undergoes its last, to date, IMF intervention.

The stock of capital inputs for each capital category defined in this work (*stationary*, *non-stationary*, *buildings*) is presented in Figure A. 5.

It can be observed how capital stock in structures has a very significant weight in total capital, which can also be noted for capital investment in this category – Figure A. 4. Despite the decrease in capital investment in structures, from 1960 to 2014, it still constitutes the largest portion of investment. Also, the fact that capital depreciation for structures is significantly lower than for other capital categories contributes to the disproportional accumulation of this type of capital.

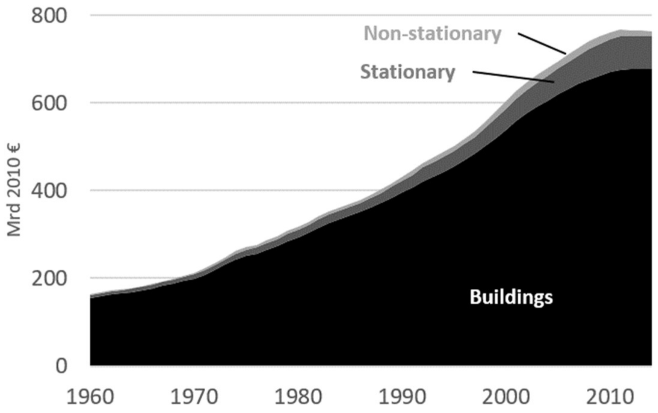

Figure A. 5 - Total capital stock disaggregated by non-stationary, stationary, and buildings categories: Portugal 1960-2014.

It should, however, be noted that the stock for stationary capital grows significantly throughout the 1960-2014 period.

Figure A. 6 below shows the difference between aggregate capital inputs as obtained directly from the AMECO database, and aggregate capital inputs estimated after the process of disaggregation of these inputs described in Section 2.2 – Equation (8). Figure A. 6(a) represents the differences in levels, while Figure A. 6(b) represents the difference in annual growth rate.

From Figure A. 6 we can observe how the initial capital stock determined for estimated aggregate capital stock (from the estimation of initial values for capital stock for each disaggregate category defined in Section 2.2) is higher than the initial value for capital stock according to the AMECO database. This causes the estimated aggregate capital stock series to assume higher values throughout the whole period, despite the annual growth rate for aggregate capital stock obtained directly from the AMECO database being higher for most of the considered period.

The latter observation can be justified by the fact that there is an increase in the capital asset categories of machinery, transport, and information and communication technologies (ICT), throughout time, that is reflected in the lower growth rates for estimated aggregate capital stock, which considers distinct depreciation rates for these categories – see Table 1. These depreciation rates are quite high, and lead to slower annual growth of these capital categories, and hence aggregate capital. As for the AMECO aggregate capital stock series, the depreciation rate adopted is constant, and identical for all asset categories, hence these differences in the speed of asset depreciation are not considered.

Overall, the estimated series for aggregate capital stock, following the disaggregation process of Section 2.2, is more rigorous than the one presented in the AMECO database.

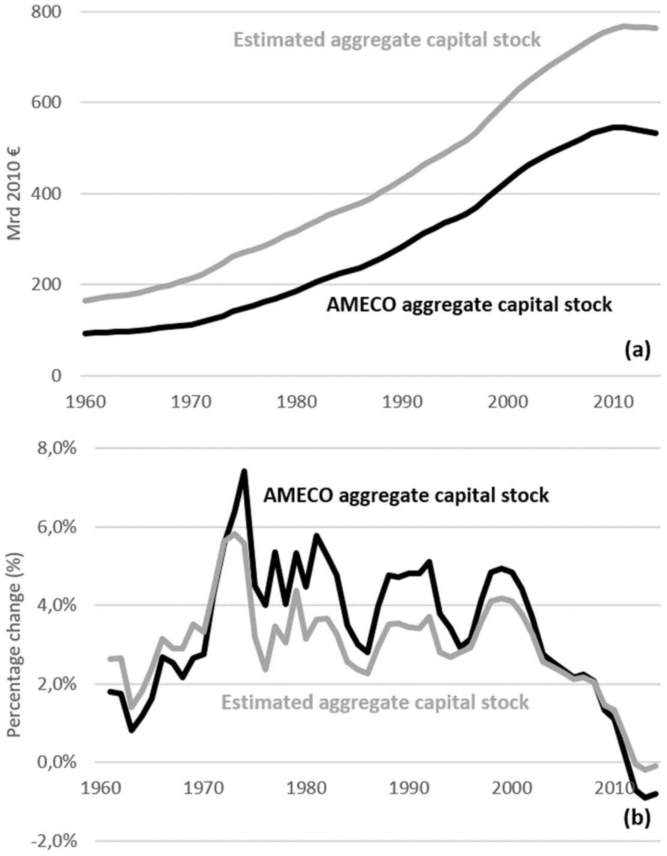

Figure A. 6 - Aggregate capital stock levels (a) and annual growth rates (b) for the Portuguese economy 1960-2014, according to the AMECO database, and the estimates presented in this work – Equation (8).

In our work, the AMECO aggregate capital stock are used only in the most basic approach to establish a relationship between TFP and final-to-useful exergy efficiency, and compute GDP. This is because this is the simplest approach to the issue, which requires only macroeconomic data readily available from international databases. For the rest of our work, including cointegration procedures, only aggregate capital stock computed after the disaggregation process of Section 2.2 is adopted.

Human labour historical trends and projections for the future are presented in Figure A. 7. Both unadjusted human labour input measures (total hours worked) and quality-adjusted measures (through a human capital index) are shown.

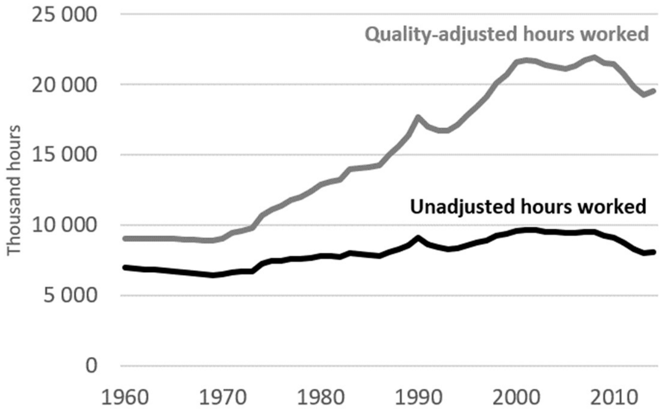

Figure A. 7 - Unadjusted and quality-adjusted human labor measures: Portugal 1960-2014.

Besides an increase in the total number of hours worked in the Portuguese economy, between 1960-2014, an increase in the productivity of human labour can also be observed, as the quality-adjusted labour (proxied by increases in years of schooling) rises significantly.

Appendix B – Output elasticities

Figure B. 1 shows empirically observed cost shares, for Portugal, corresponding to payments to capital and human labour inputs, for the period 1960-2014.

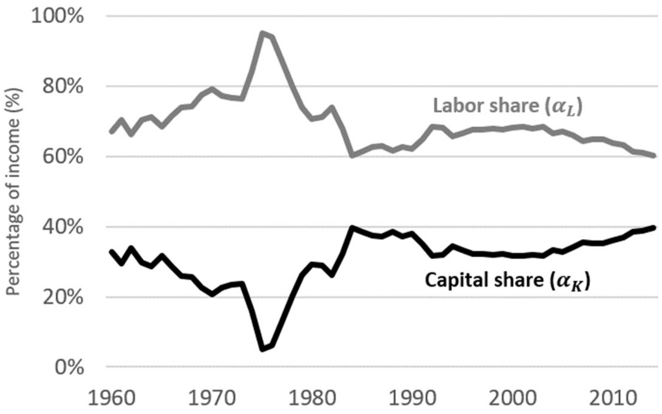

Figure B. 1 - Shares of payments to capital and labor in total income: Portugal 1960-2014.

It can be observed, from Figure B. 1, how despite the very large fluctuations that characterize the cost shares in this country[9], these shares return to average values which are consistent with the empirically observed stylized fact that human labour should receive approximately 2/3 of income in payments, and capital the remaining 1/3. The (constant) output elasticities of capital and human labour in the aggregate production function of Equation (18) correspond to the average values for empirically observed cost shares for these factors of production, i.e. approximately 30% (or $\bar{\alpha}_K \cong 0.3$) for capital, and 70% (or $\bar{\alpha}_L \cong 0.7$) for human labour.

Time-varying output elasticities for each defined category of disaggregated capital stock (stationary, non-stationary, buildings) are estimated from Equation (15), and represented in Figure B. 2.

Figure B. 2(a) shows time-varying output elasticities for each capital stock category, estimated through Equation (15), assuming a time-varying aggregate capital stock output elasticity. Figure B. 2(b) shows time-varying output elasticities for each capital stock category, estimated through Equation (15), but assuming a constant (average) aggregate capital stock output elasticity. The average values for the output elasticities corresponding to each capital stock category represented in Figure B. 2 are shown in Table B. 1. The average values do not change significantly whether a time-varying or constant aggregate capital stock output elasticity is assumed in the calculations with Equation (15). Output elasticities are assumed constant and equal to their corresponding average values when used in the Cobb-Douglas aggregate production function formulations defined by Equations (31)-(34). The sum of estimated average output elasticities for the three defined categories of capital stock give the aggregate output elasticity for capital inputs (0.3), while labor inputs have an output elasticity of 0.7, thus satisfying constant returns to scale.

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[9] Brought by the Democratic Revolution that took place in 1974, along with external factors such as the oil crisis.

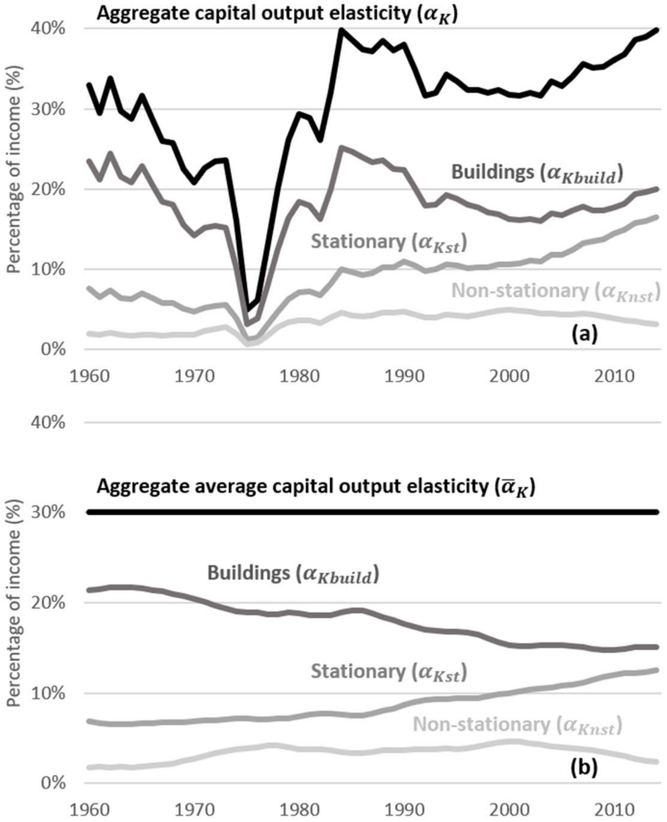

Figure B. 2 - Estimated time-varying output elasticities for capital stock categories (stationary, non-stationary, buildings) for Portugal 1960-2014. (a) Relative to a time-varying aggregate capital stock output elasticity. (b) Relative to an average (constant) aggregate capital stock output elasticity.

Table B. 1 - Estimated output elasticities for each category of capital assets - Equation (8).

Asset category		Estimated average output elasticity ($\bar{\alpha}$)
Buildings	(α_{Kbuild})	0.180
Stationary capital	(α_{Kest})	0.086
Non-stationary capital	(α_{Knest})	0.034
Aggregate capital	(α_K)	0.300

Appendix C – Additional results

Table C. 1 and Table C. 2 show the test statistics obtained from the unit root tests conducted on the variables representing historical TFP for Models B, C, D, and E, F, as well as aggregate final-to-useful exergy efficiency (ε), and stationary and non-stationary final-to-useful exergy efficiencies ($\varepsilon_{st.}$ And $\varepsilon_{nst.}$, respectively).

The unit root tests are the Augmented Dickey-Fuller test (ADF), the Schmidt-Phillips test, and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. The null hypothesis for the ADF and Schmidt-Phillips tests is that a unit root is present in the time series sample (i.e. the time series is non-stationary), while the null hypothesis for the KPSS test is that the time series sample is stationary.

Table C. 1 shows test statistics for time series in levels. Table C. 2 shows test statistics for time series in first differences. For all time series in levels, a trend term and intercept are included in the unit root test. For all time series in first differences, only the intercept term is included.

*Table C. 1 – Unit root test statistics for variables in Levels. All variables in natural logarithms. * Rejection of the null hypothesis at the 1% level.*

Variable	Lags	ADF	Schmidt-Phillips	KPSS
TFP (Models B, C, D)	1	-3.971*	-1.186	0.428*
TFP (Models E, F)	1	-4.235*	-1.131	0.427*
ε	1	-4.064*	-0.122	0.608*
$\varepsilon_{st.}$	1	-4.064*	-0.020	0.652*
$\varepsilon_{nst.}$	1	-0.980	-1.278	0.447*

*Table C. 2 – Unit root test statistics for variables in 1st differences. All variables in natural logarithms. * Rejection of the null hypothesis at the 1% level.*

Variable	Lags	ADF	Schmidt-Phillips	KPSS
TFP (Models B, C, D)	2	-3.379	-4.870*	0.609
TFP (Models E, F)	2	-3.441*	-5.010*	0.594
ε	4	-3.537*	-5.711*	0.799*
$\varepsilon_{st.}$	5	-3.071	-7.341*	0.840*
$\varepsilon_{nst.}$	1	-4.424*	-6.530*	0.221

While the test statistics results are not consistent across different unit root tests, we can infer from the Schmidt-Phillips and KPSS test results that all considered time series are likely non-stationary in levels, and stationary in first differences, i.e. the time series are integrated of order one I(1). The test results from the ADF unit root tests have low statistical power and can be safely dismissed.

Table C. 3 presents the test statistics concerning the Johansen cointegration tests conducted on Models C, D, E, and F. The null hypothesis for each Johansen test are: 1) there are no cointegration vectors (CV) linking the variables (None); 2) there is at most 1 cointegration vector linking the variables (at most 1); there are at most 2 cointegration vectors linking the variables (At most 2). For models with only two variables (Model C, Model E), we test the null hypothesis that there are no cointegration vectors, or at most 1 cointegration vector. For models with three variables (Model D, Model F), we test the null hypothesis that there are no cointegration vectors, at most 1 cointegration vector, or at most 2 cointegration vectors. For two-variable models, if the null hypothesis of At most 1 cointegration vector is rejected, the included time series are stationary. For three-variable models, if the null hypothesis of At most 2 cointegration vectors is rejected, the included time series are stationary.

For all models considered in our analysis, the null hypothesis of no cointegration vectors linking the variables is rejected. However, the null hypothesis of at most 1 cointegration vector linking the variables is accepted at the 1% significance level, for all models. This suggests that all models considered in our analysis have at most 1 cointegration relationship linking TFP and final-to-useful exergy efficiency variables. The fact that the At most 1 (At most 2) null hypothesis is never rejected for two-variable (three-variable) models validates the unit root tests conducted on each time series, indicating that these time series are all non-stationary in levels and stationary in first differences (i.e. integrated of order one).

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*Table C. 3 – Johansen cointegration test statistics. All variables in natural logarithms. * Rejection of null hypothesis at the 1% significance level.*

Model	Nº of CV	Lags	LR	p-value	90%	95%	99%
$(TFP_{B,C,D}; \varepsilon)$	None*	1	75.85	0.0000	17.98	20.16	24.69
	At most 1		3.50	0.5021	7.60	9.14	12.53
$(TFP_{E,F}; \varepsilon)$	None*	2	45.54	0.0000	17.98	20.16	24.69
	At most 1		8.18	0.0775	7.60	9.14	12.53
$(TFP_{B,C,D}; \varepsilon_{st.}; \varepsilon_{nst.})$	None*	1	77.36	0.0000	32.25	35.07	40.78
	At most 1		13.09	0.3650	17.98	20.16	24.69
	At most 2		5.45	0.2462	7.60	9.14	12.53
$(TFP_{E,F}; \varepsilon_{st.}; \varepsilon_{nst.})$	None*	9	61.65	0.0000	32.25	35.07	40.78
	At most 1		23.45	0.0158	17.98	20.16	24.69
	At most 2		8.74	0.0602	7.60	9.14	12.53

Table C. 4 – Granger non-causality test statistics for Models C, D, E, F.

Model C			**Model E**		
$\varepsilon \rightarrow TFP_{B,C,D}$	Test-statistic	3.1714	$\varepsilon \rightarrow TFP_{E,F}$	Test-statistic	3.4677
	p-value	0.0467		p-value	0.0197
$TFP_{B,C,D} \rightarrow \varepsilon$	Test-statistic	0.3933	$TFP_{E,F} \rightarrow \varepsilon$	Test-statistic	1.2460
	p-value	0.6760		p-value	0.2983
Model D			**Model F**		
$\varepsilon_{st.} \rightarrow TFP_{B,C,D}, \varepsilon_{nst.}$	Test-statistic	2.9968	$\varepsilon_{st.} \rightarrow TFP_{E,F}, \varepsilon_{nst.}$	Test-statistic	11.3943
	p-value	0.0211		p-value	0.0000
$\varepsilon_{nst.} \rightarrow TFP_{B,C,D}, \varepsilon_{st.}$	Test-statistic	0.1238	$\varepsilon_{nst.} \rightarrow TFP_{E,F}, \varepsilon_{st.}$	Test-statistic	4.3438
	p-value	0.9737		p-value	0.0002
$TFP_{B,C,D} \rightarrow \varepsilon_{st.}, \varepsilon_{nst.}$	Test-statistic	1.4595	$TFP_{E,F} \rightarrow \varepsilon_{st.}, \varepsilon_{nst.}$	Test-statistic	11.1731
	p-value	0.2185		p-value	0.0000
$\varepsilon_{st.}, \varepsilon_{nst.} \rightarrow TFP_{B,C,D}$	Test-statistic	1.3274	$\varepsilon_{st.}, \varepsilon_{nst.} \rightarrow TFP_{E,F}$	Test-statistic	4.2864
	p-value	0.2634		p-value	0.0002

Table C. 4 presents the test statistics and corresponding p-values for Granger non-causality tests conducted on Model C, D, E, and F. The null hypothesis is that there is no Granger-causality in the direction indicated by the arrow. If the p-value is below 0.10, 0.05, or 0.01 the null hypothesis is rejected with a 10%, 5%, or 1% significance level, respectively.

For all models represented in Table C. 4, the evidence for final-to-useful exergy efficiency time series Granger-causing TFP time series is stronger than in the opposite direction. For Models C, E, there is unidirectional Granger-causality from aggregate F-to-U exergy efficiency to the TFP time series. For Model D, there is unidirectional Granger-causality from stationary F-to-U exergy efficiency to TFP, but no Granger-causality from non-stationary F-to-U exergy efficiency to TFP. However, for Model F, there is evidence for bidirectional Granger-causality between both stationary/non-stationary F-to-U exergy efficiency and TFP.

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