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## Abstract

What happens when information reaches the human brain? In economics, a black-box approach to information processing in the brain is generally taken with an implicit assumption that information, once it reaches the brain, is accurately processed. In sharp contrast, research in brain sciences has established that when information reaches the brain, a mental template or schema (neural substrate of knowledge) is first activated, which influences information absorption. Schemas are created through a resource intensive process in which finite brain resources are allocated to different tasks, with resource allocation in the brain having an impact on the structure of schemas. In this article, we explore the implications of this richer view from brain sciences for the capital asset pricing model (CAPM). We show that two versions of CAPM arise depending on how the brain resources are allocated in schema creation. In one version, the relationship between beta and expected returns is flat along with value and size effects. In the second version, the relationship between beta and expected return is strongly positive with an implied risk-free rate which could be negative. The two version CAPM provides a unified explanation for a series of empirical findings including high-alpha-of-low-beta, size and value effect as well as strongly positive relationship between beta and average stock returns at specific times such as on macroeconomic announcement days, and at market open. As certain morbidities, such as autism, are thought to be associated with lack of schemas that attenuate information, a laboratory experiment with high functioning autism sufferers might be our best bet at observing the classical CAPM in its full glory.

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# Resource Allocation in the Brain and the Capital Asset Pricing Model

What happens when information reaches the human brain? In economics, a black-box approach to information absorption is typically taken with an implicit assumption that information, when it reaches the brain, is accurately processed. In sharp contrast, research in brain sciences has established that when information reaches the brain, a mental template or schema, is first activated, which influences information absorption.[1] Brain imaging studies show that schema formation is a resource-intensive process that involves different regions of the brain talking to each other[2]; however, these schemas, once formed, make subsequent processing of schema-consistent information a lot faster by attenuating schema-inconsistent information.[3] In this article, we study the implications of this richer view from brain sciences for the capital asset pricing model (CAPM).

A schema can be conceived as a scaffold or a blueprint,[4] essentially representing a set of preconceived ideas. Neurologically, it is a brain template that involves systems of neurons across various brain regions talking to each other, with each system constituting a particular unit in the schema. That is, schemas contain units as well as relationships between these units. For example, for a car schema, units could be car body and wheel, with the relationship that car body contains four wheels. For a firm schema, units could be expected cash flow levels and associated risks with a specific relationship between these units. Schemas, by only containing the essential details, simplify the world. They direct attention to relevant aspects, and speed-up processing of information that fits within the schema.

When received information does not fit within an existing schema and it can't be ignored, then the brain may create a new schema by attempting to appropriately modify a

[1] There is a large body of literature in neuroscience that explores various facets of schemas and how they influence information absorption (for a review, see van Kesteren et al (2012), Gilboa and Marlatte (2017), Spalding et al (2015) and references therein).
[2] See Ohki and Takei (2018) and references therein.
[3] Sweegers et al (2015)
[4] See Hampson and Morris (1996) or Anderson (2000) for a detailed review of schema theory.

related schema. Brain organizes knowledge in a network of such interconnected schemas. For example, a child may initially only have a schema for a horse (large with four legs, hair, and a tail). However, when she encounters a cow, a new schema for a cow could be created by modifying the horse schema. Similarly, relevant to our context, an investor analysing a firm that she has not analysed before, may create a new schema for the firm by altering the schema of a similar firm that she has analysed before. Studying the implications of such a schema-creation process for CAPM is the subject of this article.

Research in brain sciences has established that there is brain specialization with different brain systems performing different tasks and competing for scarce resources that are allocated by a 'central executive system' (CES) located in the lateral prefrontal cortex (see Alonso et al (2014) and references therein). This suggests that, while modifying an existing schema to create a new one, each unit in a schema is exclusively worked on by a distinct system of neurons. Each system makes demands for resources with task performance dependent on resource allocation. For relatively simple schemas (such as for a cow or a car), the resource constraint is not binding and all units in an existing schema are fully adjusted to create accurate units in the new schema. However, for sufficiently complex schemas such as a firm schema, the resource constraint is likely to be binding. In the context of CAPM, keeping things simple, expected cash flows and risk of the cash flows are the two key units in the schema of a given firm. So, each unit is worked on by a different system of neurons while modifying an existing schema to create a new one. With a binding resource constraint, how the scarce brain resources are split across the two units matters.

In this article, we consider two ways in which scarce brain resources can be allocated towards the two units while creating a new schema for a firm:

1) More resources are allocated to the brain system working on expected cash flows.

2) More resources are allocated to the brain system working on risk of the cash flows.

It follows that there are two types of traders. Traders, who are more adept at processing cash flow information as they have schemas with more resources devoted to cash flows, and traders who are better at processing risk-related information as they have schemas with more resources devoted to risk of the cash flows. Note that neither type of trader completely ignores information, it is just that their relative skill in processing a certain type of information differs depending on the structure of schemas that they have.

We show that which trader type is marginal matters for CAPM. When a cashflow-schema trader is marginal, a version of CAPM is obtained (cashflow-schema CAPM), which displays a flatter relationship between stock beta and expected excess returns. Betting-against-beta anomaly is observed along with the value and size effects. Hence, a unified explanation for betting-against-beta, value, and size anomalies emerges in this version.

When a risk-schema trader is marginal, another version of CAPM arises (risk-schema CAPM). In this version, there is a strong positive relationship between beta and expected excess return with an implied risk-free rate that could be negative. Stocks that do better in the first version (low beta, small) do worse in the second version.

Generally, the marginal investor is expected to be a cashflow-schema trader due to the importance given to earnings or cashflow news (Basu et al 2013). However, there are specific times when the marginal investor is expected to be a risk-schema trader: (i) when risks fall, or (ii) when cashflows fall. When risks fall, both cashflow-schema traders and risk-schema traders increase their demand; however, the increase in demand from risk-schema traders is larger making them net buyers. When cashflows levels fall, both cashflow-schema traders and risk-schema traders reduce their demand; however, the demand reduction from risk-schema traders is smaller, making them net buyers.

As risk reductions as well as cashflow reductions make risk-schema traders marginal, we predict a steeper relationship between beta and average stock returns at such times. Risk reductions can happen, at least for some traders, when macro announcements about interest rates, unemployment, and inflation are made. Cashflow level reductions are associated with weak aggregate spending in the economy generally indicated by low inflation. Hence, we predict a steeper relationship between beta and average stock returns on macro announcement days, and during periods when inflation is low. Indeed, this is what Savor and Wilson (2014) and Cohen et al (2005) find.

In general, whenever trades are triggered by risk-reduction, we expect a strongly positive relationship between beta and average stock returns as the marginal trader is a risk-schema trader at such times. For example, at market open, there is quite a bit of uncertainty about the opening price, which can be substantially different from the previous close. This uncertainty keeps most traders on the sidelines. Only traders who manage to reduce this uncertainty for themselves (perhaps by considering what happened in other markets across the globe while this particular market was closed) increase their demand. As risk reduction increases the demand by risk-schema traders more, they become net-buyers at open. Hence, we predict that, at market open, the relationship between beta and average excess stock return is strongly positive. Again, this is consistent with the empirical findings in Hendershott et al (2019).

## 2. CAPM adjusted for Resource Allocation in the Brain

We take a modern derivation of CAPM (such as in Frazzini and Pedersen (2014)) and add a twist to it, which is incorporating the implications of information processing through a schema as created by a resource-constrained brain.

As in Frazzini and Pedersen (2014), we consider an overlapping generations (OLG) economy with agents having identical beliefs. Each agent lives for two periods. Agents that are born at $t$ aim to maximize their utility of wealth at $t+1$. Their utility functions are identical and exhibit mean-variance preferences. They trade securities $s = 1, \cdots, S$ where security $s$ pays dividends $d_t^s$ and has $n_s^*$ shares outstanding, and invest the rest of their wealth in a risk-free asset that offers a rate of $r_F$.

The market is described by a representative agent who maximizes:

$$\max n'\{E_t(P_{t+1} + d_{t+1}) - (1 + r_F)P_t\} - \frac{\gamma}{2} n'\Omega_t n$$

where $P_t$ is the vector of prices, $\Omega_t$ is the variance-covariance matrix of $P_{t+1} + d_{t+1}$, and $\gamma$ is the risk-aversion parameter.

It follows that the price of a security, $s$, is given by:

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma Cov(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \tag{2.1}$$

where security $s$ payoff is $X_{t+1}^{s} = P_{t+1}^{s} + d_{t+1}^{s}$

and aggregate market payoff is:

$$X_{t+1}^{M} = n_1^*(P_{t+1}^{1} + d_{t+1}^{1}) + n_2^*(P_{t+1}^{2} + d_{t+1}^{2}) + \cdots\cdots\cdots + n_S^*(P_{t+1}^{S} + d_{t+1}^{S}).$$

### 2.1 Schema Creation

As discussed in the introduction, schema is a mental template that contains units as well as a relationship between units. With mean-variance preferences, the relevant units are expected cash flows and the risk of cash flows, with the risk measured by covariance of cash flows with the aggregate market cash flows. We define a firm-schema as follows: "It is a set of preconceived ideas about expected cash flow levels and their risks that help in processing new information to evaluate one's willingness-to-pay (WTP)". So, a firm-schema has the following general form:

$$WTP = \frac{A\ function\ of\ expected\ cashflows - (riskaversion)(A\ function\ of\ risk)}{1 + riskfree\ rate}$$

To understand the process of schema creation, we consider how a typical stock analyst behaves while analysing a firm. Stock analysis is usually done at firm-level cash flows, which are then transformed to the level of an individual security. We denote firm-level earnings or cash flows by $\pi_{t+1}^{s}$ where the number of outstanding shares is $n_s^*$. Earnings-per-share (EPS) is then given by: $EPS_{t+1} = \frac{\pi_{t+1}^{s}}{n_s^*}$. Denoting the price-earnings (P/E) ratio, inclusive of dividends, for firm $s$ by $c_s$:

$$X_{t+1}^{s} = P_{t+1}^{s} + d_{t+1}^{s} = c_s(EPS_{t+1}) = c_s \frac{\pi_{t+1}^{s}}{n_s^*}$$

We assume that when a trader analyses the cash flows of a firm $s$ for the first time, she creates a schema by modifying the schema for a similar firm $q$ that she has analysed earlier. The two units that constitute a schema for a firm are: expected cash flows and the risk of cash flows. So, the process of creating a new schema by modifying an existing schema requires modifications in these two units.

For expected cash flow levels, the modification is:

$$E'(\pi^s_{t+1}) = E(\pi^q_{t+1}) - m_1 D_1$$

where $D_1 = E(\pi^q_{t+1}) - E(\pi^s_{t+1})$ is the correct adjustment needed, and $0 \leq m_1 \leq 1$, captures the fraction of correct adjustment reached. If the brain is not resource constrained, then $m_1 = 1$, which corresponds to full or correct adjustment. On the other hand, $m_1 < 1$, indicates that the resource constraint is binding.

Transforming to the level of EPS:

$$\frac{E'(\pi^s_{t+1})}{n^*_s} = \frac{E(\pi^q_{t+1})}{n^*_q}\frac{n^*_q}{n^*_s} - \frac{m_1 D_1}{n^*_s}$$

$$\Rightarrow E'(EPS^s_{t+1}) = \frac{n^*_q}{n^*_s}E(EPS^q_{t+1}) - m_1\left(\frac{n^*_q}{n^*_s}E(EPS^q_{t+1}) - E(EPS^s_{t+1})\right)$$

$$\Rightarrow E'(EPS^s_{t+1}) = (1 - m_1)\frac{n^*_q}{n^*_s}E(EPS^q_{t+1}) + m_1 E(EPS^s_{t+1})$$

Similarly, the schema-unit for the risk of cash flows is obtained as follows:

$$Cov'(\pi^s_{t+1}, X^M_{t+1}) = Cov(\pi^q_{t+1}, X^M_{t+1}) - m_2 D_2$$

$$\Rightarrow \frac{Cov'(\pi^s_{t+1}, X^M_{t+1})}{n^*_s} = \frac{Cov(\pi^q_{t+1}, X^M_{t+1})}{n^*_q}\frac{n^*_q}{n^*_s} - m_2\left(\frac{Cov(\pi^q_{t+1}, X^M_{t+1})}{n^*_q}\frac{n^*_q}{n^*_s} - \frac{Cov(\pi^s_{t+1}, X^M_{t+1})}{n^*_s}\right)$$

$$\begin{aligned}\Rightarrow Cov'(EPS^s_{t+1}, X^M_{t+1}) \\ &= Cov(EPS^q_{t+1}, X^M_{t+1})\frac{n^*_q}{n^*_s} \\ &\quad - m_2\left(Cov(EPS^q_{t+1}, X^M_{t+1})\frac{n^*_q}{n^*_s} - Cov(EPS^s_{t+1}, X^M_{t+1})\right)\end{aligned}$$

$$\Rightarrow Cov'(EPS^s_{t+1}, X^M_{t+1}) = (1 - m_2)Cov(EPS^q_{t+1}, X^M_{t+1})\frac{n^*_q}{n^*_s} + m_2\left(Cov(EPS^s_{t+1}, X^M_{t+1})\right)$$

Following the behavior of a typical stock analyst, we define the notion of similar firms as having the following two properties:

1) Firms that are in the same line of business, and

2) Have the same P/E ratios.

P/E ratios (inclusive of dividends) for $s$ and $q$ are $c_s$ and $c_q$, and applying the above properties, the firms are in the same line of business with similar P/E ratios:

$c_c \approx c_q = c$

So, the schema-unit for risk of the cash flows is estimated as:

$$Cov'(cEPS_{t+1}^{s}, X_{t+1}^{M}) = (1 - m_2)Cov\left(cEPS_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} + m_2\left(Cov(cEPS_{t+1}^{s}, X_{t+1}^{M})\right)$$

$$\Rightarrow Cov'(X_{t+1}^{s}, X_{t+1}^{M}) = (1 - m_2)Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} + m_2\left(Cov(X_{t+1}^{s}, X_{t+1}^{M})\right)$$

$$\Rightarrow Cov'(X_{t+1}^{s}, X_{t+1}^{M}) = Cov(X_{t+1}^{s}, X_{t+1}^{M}) + (1 - m_2)\left(Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^{s}, X_{t+1}^{M})\right) \quad (2.2)$$

Similarly, the schema-unit for expected cash flow levels can be written as:

$$E'(cEPS_{t+1}^{s}) = (1 - m_1)\frac{n_q^*}{n_s^*}E\left(cEPS_{t+1}^{q}\right) + m_1E(cEPS_{t+1}^{s})$$

$$\Rightarrow E'(X_{t+1}^{s}) = (1 - m_1)\frac{n_q^*}{n_s^*}E\left(X_{t+1}^{q}\right) + m_1E(X_{t+1}^{s})$$

$$\Rightarrow E'(X_{t+1}^{s}) = E(X_{t+1}^{s}) + (1 - m_1)\left(E\left(X_{t+1}^{q}\right)\frac{n_q^*}{n_s^*} - E(X_{t+1}^{s})\right) \quad (2.3)$$

(2.2) and (2.3) capture the following two properties associated with resource allocation in the brain (see Alonso et al (2014)):

1) When a new schema is created by modifying an existing schema, the process is broken down into separate tasks, with each unit worked on by a separate system of neurons. Each system communicates its resource requirements to CES, which allocates finite brain resources between systems.

2) The resource constraint is generally binding for complex schemas with task performance dependent on how much of resources are allocated to that particular task.

## 2.2 Cashflow-Schema CAPM

Schema creation is a resource intensive process. A separate system of neurons is allocated to each unit in the schema of a firm, with allocation of brain resources to the two units determined by CES. In this section, we assume that more resources are devoted to the unit for expected cash flows when compared with the unit for risk of the cash flows. That is, $m_1 > m_2$. We refer to such traders as having a cashflow-schema, and the CAPM so obtained is referred to as the cashflow-schema CAPM. In section 2.4, we consider the other case where $m_2 > m_1$ (with such traders referred to as risk-schema traders).

Without loss of generality, we set $m_1 = 1$, it then follows that $m_2 = m < 1$. Suppose, there is a firm $q$ that had been analysed earlier, and its schema is modified to create a schema for firm $s$.

The share price of firm $q$ is given by (from 2.1):

$$P_t^q = \frac{E\left(X_{t+1}^q\right) - \gamma Cov\left(X_{t+1}^q, X_{t+1}^M\right)}{1 + r_F} \tag{2.4}$$

And, the share price of firm $s$ is given by (using 2.2):

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma \left\{Cov(X_{t+1}^s, X_{t+1}^M) + (1-m)\left(Cov\left(X_{t+1}^q, X_{t+1}^M\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^s, X_{t+1}^M)\right)\right\}}{1 + r_F} \tag{2.5}$$

The expected returns of $s$ and $q$ are then (with $R_F = 1 + r_F$):

$$E\left[R_{t+1}^q\right] = R_F + \frac{\gamma}{P_t^q} Cov\left(X_{t+1}^q, X_{t+1}^M\right) \tag{2.6}$$

$$E[R_{t+1}^s] = R_F + \frac{\gamma}{P_t^s}\left\{Cov(X_{t+1}^s, X_{t+1}^M) + (1-m)\left(Cov\left(X_{t+1}^q, X_{t+1}^M\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^s, X_{t+1}^M)\right)\right\} \tag{2.7}$$

To fix ideas, initially it is useful to assume that there are just two firms in the market, $s$ and $q$ before generalizing to $N$ firms. Multiplying (2.6) by $w_q = \frac{n_q^* P_t^q}{P_t^M}$, which is the weight of firm $q$ in the market portfolio ($P_t^M$ is the price of aggregate market portfolio), multiplying (2.7) by $w_s = \frac{n_s^* P_t^s}{P_t^M}$, and adding:

$$E[R_{t+1}^M] = R_F + \frac{\gamma}{P_t^M}\{Var(X_M) + (1-m)\big(Cov\big(X_{t+1}^q, X_{t+1}^M\big)n_q^* - Cov(X_{t+1}^s, X_{t+1}^M)n_s^*\big)\}$$

$$\Rightarrow \gamma = \frac{(E[R_{t+1}^M] - R_F)P_t^M}{\{Var(X_M) + (1-m)\big(Cov\big(X_{t+1}^q, X_{t+1}^M\big)n_q^* - Cov(X_{t+1}^s, X_{t+1}^M)n_s^*\big)\}} \qquad (2.8)$$

Substituting (2.8) in (2.6) and re-arranging/simplifying leads to the modified CAPM equation for $q$:

$$E\big[R_{t+1}^q\big] = R_F + (E[R_{t+1}^M] - R_F) \cdot \beta_q \cdot \left(\frac{1}{1 + (1-m)\big(w_q\beta_q - w_s\beta_s\big)}\right) \qquad (2.9)$$

where $\beta_q = \frac{Cov\left(R_{t+1}^q, R_{t+1}^M\right)}{Var\left(R_{t+1}^M\right)}$ and $\beta_s = \frac{Cov\left(R_{t+1}^s, R_{t+1}^M\right)}{Var\left(R_{t+1}^M\right)}$

Substituting (2.8) in (2.7) leads to:

$$E[R_{t+1}^s] = R_F + (E[R_{t+1}^M] - R_F) \cdot \beta_s \cdot \left(\frac{1 + (1-m)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)}{1 + (1-m)\big(w_q\beta_q - w_s\beta_s\big)}\right) \qquad (2.10)$$

(2.9) and (2.10) are modified CAPM expressions when schemas are created with a binding resource constraint (and with more brain resources allocated to the schema-unit concerned with expected cash flows). Note that (2.9) and (2.10) revert to the classical CAPM expression when $m = 1$ (resource constraint in the brain is not binding).

Generalizing to $N$ firms with several $q$ firms spawning new schemas of several $s$ firms, the corresponding CAPM expressions for $q$ and $s$ firms are:

$$E\left[R_{t+1}^{q}\right] = R_F + (E[R_{t+1}^{M}] - R_F) \cdot \beta_q \cdot \left(\frac{1}{1 + (1-m)\left(\sum_q \sum_s \left(w_q \beta_q - w_s \beta_s\right)\right)}\right) \quad (2.11)$$

$$E[R_{t+1}^{s}] = R_F + (E[R_{t+1}^{M}] - R_F) \cdot \beta_s \cdot \left(\frac{1 + (1-m)\left(\frac{w_q \beta_q}{w_s \beta_s} - 1\right)}{1 + (1-m)\left(\sum_q \sum_s \left(w_q \beta_q - w_s \beta_s\right)\right)}\right) \quad (2.12)$$

It is intriguing to note that CAPM expressions with finite brain resources have the same form as the classical CAPM with only one difference: a factor that multiplies $\beta$ appears. When the resource constraint is not binding, $m = 1$, the multiplicative factor equals 1, so we revert back to the classical CAPM expression.

### 2.3 High-alpha-of-low-beta, value, and size effects

(2.12) and (2.11) show that the classical CAPM is a special case of a schema-adjusted CAPM. In schema-adjusted CAPM, there is an additional multiplicative factor, which multiplies $\beta$. This factor reduces to 1 when the resource constraint is not binding. In other words, the schema-adjusted CAPM reduces to the classical CAPM when $m = 1$.

For a firm $s$ whose schema is created by modifying the schema of a similar firm (same line of business with similar P/E ratios) $q$, this additional multiplicative factor is equal to: $f = \left(\frac{1+(1-m)\left(\frac{w_q \beta_q}{w_s \beta_s}-1\right)}{1+(1-m)\left(\sum_q \sum_s \left(w_q \beta_q - w_s \beta_s\right)\right)}\right)$ (2.13)

Firms to which investors and analysts devote most of their time are likely to be ones that spawn new schemas for other firms. Investor and analyst attention is strongly asymmetric with large, prominent firms (high market capitalizations) getting a lion's share (Fang and Peress 2009). This motivates the following assumption:

- Within a group of firms whose schemas are spawned by the same firm, $q$, the following holds: $w_q \beta_q > w_s \beta_s$

It follows that $f > 0$. Proposition 1 shows the emergence of high-alpha-of-low-beta in the cashflow-schema CAPM

**Proposition 1 (High-alpha-of-low-beta)** ***In a given cross-section of stocks, a stock with low beta outperforms a stock with large beta on a risk-adjusted basis, all else equal.***

**Proof**

Suppose there are two stocks $s$ and $s'$ such that $\beta_s < \beta_{s'}$. Risk-adjusted return on $s$ is given by:

$$\frac{E[R_{t+1}^{s}] - R_F}{\beta_s} = \left\{1 + (1-m)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)\right\} \times \frac{1}{g} \times (E[R_{t+1}^{M}] - R_F)$$

where $g$ is a constant in a given cross-section of stocks.

$$g = 1 + (1-m)\left(\sum_q \sum_s \left(w_q\beta_q - w_s\beta_s\right)\right)$$

Risk-adjusted return on $s'$ is given by:

$$\frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}} = \left\{1 + (1-m)\left(\frac{w_q\beta_q}{w_s\beta_{s'}} - 1\right)\right\} \times \frac{1}{g} \times (E[R_{t+1}^{M}] - R_F)$$

As $\beta_s$ and $\beta_{s'}$ appear in the denominator on R.H.S, it follows that:

$$\frac{E[R_{t+1}^{s}]-R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right]-R_F}{\beta_{s'}} \blacksquare$$

One can also see the size effect in the cashflow-schema CAPM as proposition 2 shows.

**Proposition 2 (Size effect)** ***In a given cross-section of stocks, a stock with a lower weight in the market portfolio outperforms a stock with a higher weight on a risk-adjusted basis, all else equal***

**Proof**

Suppose there are two stocks $s$ and $s'$ such that $w_s < w_{s'}$. Following the same steps as in the proof of proposition 1, it is easy to see that $\frac{E[R_{t+1}^{s}]-R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right]-R_F}{\beta_{s'}}$.

■

The cashflow-schema CAPM not only explains the high-alpha-of-low-beta and size-effect, but also the value effect. Value effect refers to the finding that a stock with low price to fundamentals tends to outperform a stock with high price to fundamentals. Suppose there are two stocks $s$ and $s'$ that have the same fundamentals (expected cash flows and the risk of the cash flows). That is, $E(X_{t+1}^{s}) = E\left(X_{t+1}^{s'}\right)$, and $Cov(X_{t+1}^{s}, X_{t+1}^{M}) = Cov\left(X_{t+1}^{s'}, X_{t+1}^{M}\right)$. Assume that $P_s < P_{s'}$.

If there is a value effect, then it must be so that

$$\frac{E[R_{t+1}^{s}] - R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}}$$

To see if the above is true, start from:

$P_s = \frac{E(X_{t+1}^{s})-\gamma\left\{Cov(X_{t+1}^{s},X_{t+1}^{M})+(1-m)\left(Cov(X_{t+1}^{q},X_{t+1}^{M})\frac{n_q^*}{n_s^*}-Cov(X_{t+1}^{s},X_{t+1}^{M})\right)\right\}}{1+r_F} < P_{s'} = \frac{E(X_{t+1}^{s'})-\gamma\left\{Cov(X_{t+1}^{s'},X_{t+1}^{M})+(1-m)\left(Cov(X_{t+1}^{q},X_{t+1}^{M})\frac{n_{q'}^*}{n_{s'}^*}-Cov(X_{t+1}^{s'},X_{t+1}^{M})\right)\right\}}{1+r_F}$. Assuming the same fundamentals across the two stocks, $E(X_{t+1}^{s}) = E\left(X_{t+1}^{s'}\right)$, and $Cov(X_{t+1}^{s}, X_{t+1}^{M}) = Cov\left(X_{t+1}^{s'}, X_{t+1}^{M}\right)$, it follows that: $Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} > Cov\left(X_{t+1}^{q'}, X_{t+1}^{M}\right)\frac{n_{q'}^*}{n_{s'}^*}$

$$\Rightarrow Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^{s}, X_{t+1}^{M}) > Cov\left(X_{t+1}^{q'}, X_{t+1}^{M}\right)\frac{n_{q'}^*}{n_{s'}^*} - Cov\left(X_{t+1}^{s'}, X_{t+1}^{M}\right)$$

$$\Rightarrow Cov(X_{t+1}^{s}, X_{t+1}^{M}) \left\{ \frac{Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)}{Cov(X_{t+1}^{s}, X_{t+1}^{M})} \frac{n_q^*}{n_s^*} - 1 \right\}$$

$$> Cov\left(X_{t+1}^{s'}, X_{t+1}^{M}\right) \left\{ \frac{Cov\left(X_{t+1}^{q'}, X_{t+1}^{M}\right)}{Cov\left(X_{t+1}^{s'}, X_{t+1}^{M}\right)} \frac{n_{q'}^*}{n_{s'}^*} - 1 \right\}$$

$$\Rightarrow \frac{Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)}{Cov(X_{t+1}^{s}, X_{t+1}^{M})} \frac{n_q^*}{n_s^*} > \frac{Cov\left(X_{t+1}^{q'}, X_{t+1}^{M}\right)}{Cov\left(X_{t+1}^{s'}, X_{t+1}^{M}\right)} \frac{n_{q'}^*}{n_{s'}^*}$$

$$\Rightarrow \frac{w_q \beta_q}{w_s \beta_s} > \frac{w_{q'} \beta_{q'}}{w_{s'} \beta_{s'}} \tag{2.14}$$

It follows immediately from (2.14) that:

$$\frac{E[R_{t+1}^{s}] - R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}}$$

Proposition 3 follows.

**Proposition 3 (Value effect)** ***In a given cross-section of stocks, a stock with low price to fundamentals outperforms a stock with high price to fundamentals on a risk-adjusted basis.***

It is intriguing that value and size effects as well as high-alpha-of-low-beta can all be seen in the cashflow-schema CAPM that has the same form as the classical CAPM except for the appearance of a factor, $f$, which multiplies beta. This multiplicative factor is larger for small size stocks, for low beta stocks, and for value stocks.

Stocks with original schemas that spawn schemas for other stocks have a generalized CAPM expression given in (2.11). That is, for such stocks, beta is multiplied by a factor less than 1, and a comparison of (2.11) and (2.12) clearly shows that such stocks have lower risk-adjusted returns when compared with stocks with derived schemas. Original schemas are likely to be associated with stocks that command greater investor and analyst attention.

Proposition 4 follows.

**Proposition 4 (Attention effect)** ***Large, prominent stocks that receive a lion's share of investor attention have lower risk-adjusted return when compared with stocks that receive less investor attention.***

Proposition 4 is consistent with the empirical findings in Fang and Peress (2009). Next, we consider the case when more brain resources are allocated to risk of the cash flows when compared with expected cash flows.

**2.4 Risk-Schema CAPM**

When schema of a firm is being created by modifying an existing schema, the two schema-units that need to be adjusted are expected cash flows and the risk of cash flows. In the previous sections, we considered the case when more brain resources are allocated to expected cash flows. In this section, we consider the other case: when more brain resources are allocated to the system of neurons working on the risk of cash flows. In (2.2) and (2.3), this means the following: $m_1 < m_2$. Without loss of generality, we set $m_2 = 1$, it follows that $m_1 = m < 1$.

The stock of firm $s$, whose schema is obtained by modifying the schema of a similar firm (same line of business and P/E ratios) $q$, is priced as:

$$P_t^s = \frac{E(X_{t+1}^s) + (1-m)\left(\frac{n_q^*}{n_s^*}E\left(X_{t+1}^q\right) - E(X_{t+1}^s)\right) - \gamma Cov(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \qquad (2.15)$$

The stock of firm $q$, whose schema is modified to obtain the schema for $s$, is priced as:

$$P_t^q = \frac{E\left(X_{t+1}^q\right) - \gamma Cov\left(X_{t+1}^q, X_{t+1}^M\right)}{1 + r_F} \qquad (2.16)$$

Following the same set of steps as in section 2.2, the following generalized CAPM expressions for $s$ and $q$ stocks are obtained:

$$E[R_{t+1}^{s}] = R_F + \beta_s \left[ E[R_{t+1}^{M}] - R_F + \frac{(1-m)}{P_t^M} \sum_q \sum_s \left( n_q^* E\left(X_{t+1}^{q}\right) - n_s^* E(X_{t+1}^{s}) \right) \right] - \frac{(1-m)}{P_t^s} \left\{ \frac{n_q^*}{n_s^*} E\left(X_{t+1}^{q}\right) - E(X_{t+1}^{s}) \right\} \tag{2.17}$$

$$E\left[R_{t+1}^{q}\right] = R_F + \beta_q \left[ E[R_{t+1}^{M}] - R_F + \frac{(1-m)}{P_t^M} \sum_q \sum_s \left( n_q^* E\left(X_{t+1}^{q}\right) - n_s^* E(X_{t+1}^{s}) \right) \right] \tag{2.18}$$

where $P_t^M$ is the value of aggregate market portfolio. The above can be simplified further by defining expected market capitalization inclusive of dividends as: $E\left(w_{t+1}^{q}\right) = \frac{n_q^* E\left(X_{t+1}^{q}\right)}{P_t^M}$ and $E(w_{t+1}^{s}) = \frac{n_s^* E(X_{t+1}^{s})}{P_t^M}$:

$$E[R_{t+1}^{s}] = R_F + \beta_s \left[ E[R_{t+1}^{M}] - R_F + (1-m) \sum_q \sum_s \left( E\left(w_{t+1}^{q}\right) - E(w_{t+1}^{s}) \right) \right] - (1-m) \left\{ \frac{E\left(w_{t+1}^{q}\right) - E(w_{t+1}^{s})}{w_t^s} \right\} \tag{2.19}$$

where $w_t^s = \frac{n_s^* P_t^s}{P_t^M}$ is the weight of stock s in the market portfolio.

$$E\left[R_{t+1}^{q}\right] = R_F + \beta_q \left[ E[R_{t+1}^{M}] - R_F + (1-m) \sum_q \sum_s \left( E\left(w_{t+1}^{q}\right) - E(w_{t+1}^{s}) \right) \right] \tag{2.20}$$

Given evidence that large firms (large market capitalizations) get a lion's share of investor and analyst attention (Fang and Peress 2009), it is likely that they are the ones spawning schemas of other firms. Hence, we assume that $E\left(w_{t+1}^{q}\right) > E(w_{t+1}^{s})$.

It is immediately obvious that, in risk-schema CAPM, the relationship between beta and excess stock return is steeper than what classical CAPM predicts as beta is multiplied by a factor larger than excess market return. Larger the beta, bigger the improvement over classical CAPM prediction.

Furthermore, the implied risk-free rate is smaller than what the classical CAPM predicts and could even be negative:

$$R'_F = R_F - (1-m)\left\{\frac{E\left(w_{t+1}^{q}\right) - E(w_{t+1}^{s})}{w_t^s}\right\} \tag{2.21}$$

It is straightforward to see that large size (market capitalization) stocks do better in this version as the implied risk-free rate is larger for them.

Proposition 5 formalizes the key differences between the two versions of CAPM.

**Proposition 5 (Differences between the two versions)** ***CAPM when more brain resources are allocated to expected cash flows (Cashflow Schema CAPM) differs from the CAPM when more brain resources are allocated to the risk of cash flows (Risk schema CAPM) in the following ways:***

1) ***The former has a flatter relationship between beta and expected returns, whereas the latter has a steeper relationship between beta and expected returns.***
2) ***The implied risk-free rate is smaller in the latter and could be negative.***
3) ***Small size, and low beta stocks do better in the former whereas large size, and high beta stocks do better in the latter.***

## 3. Discussion and Conclusions

Depending on which trader type is marginal, either cashflow-schema-CAPM or risk-schema-CAPM is observed. Using the behavior of a typical stock analyst as a guide, normally one expects to observe cashflow-schema-CAPM, as most of the resources of a typical stock analyst are devoted to estimating future cash flows. However, there are specific times when the marginal trader is expected to be a risk-schema trader. We expect this to happen when traders are prompted by risk-reductions or cashflow reductions. Using the superscript, $i$, for risk-schema traders, and the superscript, $j$, for cashflow-schema traders, we can map the change in their respective willingness-to-pay for $s$ as follows:

$$\frac{\partial WTP^{is}}{\partial Cov^{i}(X_{t+1}^{s}, X_{t+1}^{M})} = \frac{-\gamma}{1+r_F} < 0 \tag{2.22}$$

$$\frac{\partial WTP^{js}}{\partial Cov^j(X_{t+1}^s, X_{t+1}^M)} = \frac{-\gamma m}{1 + r_F} < 0 \tag{2.23}$$

$$\frac{\partial WTP^{is}}{\partial E^i(X_{t+1}^s)} = \frac{m}{1 + r_F} > 0 \tag{2.24}$$

$$\frac{\partial WTP^{js}}{\partial E^j(X_{t+1}^s)} = \frac{1}{1 + r_F} > 0 \tag{2.25}$$

As risk-reductions increase the willingness-to-pay of risk-schema traders by more, whereas, reductions in expected cashflows reduces their willingness-to-pay by less, it follows that the marginal trader is expected to be a risk-schema trader under these scenarios. Hence, as discussed in the introduction, when trades are prompted by risk-reductions (actual or perceived), as on macro announcement days or at market open, we expect a stronger relationship between beta and average stock returns. This is consistent with empirical findings (Savor and Wilson 2014, Hendershott et al 2019).

When trades are prompted by reductions in expected cashflow levels, such as when there is a negative shock to aggregate spending (generally associated with low inflation), we expect a stronger relationship between beta and average returns due to the marginal investor being a risk-schema trader. The findings in Cohen et al (2005) are consistent with this prediction.

Research in brain sciences has made it clear that reliance on schemas is essentials for a normal functioning human being (see Spalding et al (2015) and references therein). Without schemas to attenuate information, and focus on the relevant bits, even simple tasks such as putting fuel in a car or setting a dinner table becomes overly exhausting as commonly experienced by individuals with autism (APA 2013). If reliance on schemas is responsible for deviations from classical CAPM, then a laboratory experiment with subjects who do not rely on schemas (such as people with high functioning autism) on a delayed time-scale (to avoid information overload) is our best bet at observing the classical CAPM in its full glory.

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