Agricultural Revolution and Industrialization

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Abstract

This study explores how agricultural technology affects the endogenous takeoff of an economy in the Schumpeterian growth model. Due to the subsistence requirement for agricultural consumption, an improvement in agricultural technology leads to a reallocation of labor from the agricultural sector to the industrial sector. Therefore, the agricultural improvement expands the firm size in the industrial sector, which determines the incentives for innovation and triggers an endogenous transition from stagnation to growth. Calibrating the model to US data for a quantitative analysis, we find that without the reallocation of labor from agriculture to the industrial sector in the early 19th century, the takeoff of the US economy would have been delayed by about four decades.

JEL classification: O30, O40
Keywords: agricultural technology, endogenous takeoff, innovation, economic growth

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The spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it. [...] The introduction of the turnip [...] made possible a change in crop rotation which [...] brought about a tremendous rise in agricultural productivity. As a result, more food could be grown with much less manpower. Manpower was released for capital construction. The growth of industry would not have been possible without the turnip and other improvements in agriculture. Nurkse (1953, p. 52-53)

1 Introduction

According to Nurkse (1953), improvements in agricultural technology that released labor from agriculture to the industrial sector were crucial for the industrial revolution. The industrial revolution sparked the great divergence discussed in Pomeranz (2001) and centuries of sustained economic growth. Modern growth economics has developed ambitious models that capture these facts and allow investigation of the forces driving the process. Typically such models build on the theory of endogenous technological change pioneered by Romer (1990). Since at its core the theory has dynamic increasing returns, it identifies the size of the market in which firms operate as a (if not the) crucial factor determining the incentives for innovation. A spectacular application of these ideas is the Unified Growth Theory of Galor and Weil (2000); see also Galor (2005, 2011). Models in this tradition produce an endogenous takeoff and allow study of the transition of an economy from stagnation to growth. Following these two influential branches of studies on economic growth, Peretto (2015) develops a Schumpeterian growth model of endogenous takeoff in which firm size determines the incentives for innovation; see Cohen and Klepper (1996a, b) and Laincz and Peretto (2006) for evidence. In this study, we use this model to formalize Nurkse's idea and then quantitatively evaluate the effects of agricultural technology.

In Peretto (2015), firm size is increasing in the population size and decreasing in the number of firms. Therefore, holding other things constant, a larger population tends to give rise to an earlier transition from stagnation to growth. However, countries with large population, such as China and India, did not experience an early industrial takeoff partly because the vast majority of their population was in agriculture, which is not relevant for firm size in the industrial sector. Therefore, we introduce an agricultural sector into the Schumpeterian growth model of endogenous takeoff. By preserving the analytical tractability of the Peretto model, we derive a closed-form solution of the equilibrium growth rate throughout the entire transition from stagnation to the balanced growth path. We show that it is the evolution of firm size in the industrial sector that determines the incentives for innovation. We use this growth-theoretic framework to explore how agricultural technology affects the endogenous takeoff of an economy. In summary, we find that an increase in the level of agricultural technology leads to an earlier takeoff of the economy. The intuition of this result can be explained as follows.

Due to the subsistence requirement for agricultural consumption, the industrial sector is small relative to the agricultural sector when the level of agricultural technology is low. An improvement in agricultural technology leads to a reallocation of labor from the agricultural sector to the industrial sector. As a result, the agricultural improvement expands firm size in the industrial sector, which determines the incentives for innovation. As firm size in the industrial sector becomes sufficiently large, firms start to invest in innovation, which in turn triggers an endogenous transition from stagnation to growth.

We also calibrate the model to US data in order to perform a quantitative analysis. At the beginning of the 19th century, the agricultural share of the US workforce was about 80%; see Baten (2016). Then, the agricultural share of the US workforce decreased to about 70% in 1830 and 60% in 1840; see Lebergott (1966) and Weiss (1986). We find that this reallocation of labor from agriculture to the industrial sector was a useful driving force for the takeoff of the US economy. Without this reallocation of labor from agriculture to the industrial sector, the takeoff of the US economy would have been delayed by about four decades. Finally, we derive a formula from our model to show that a one-fifth increase in industrial employment causes an earlier takeoff by about a decade.

This study relates to the literature on endogenous technological change. Romer (1990) is the seminal study and develops the first R&D-based growth model in which innovation is driven by the invention of new products (i.e., horizontal innovation). Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) develop the Schumpeterian growth model in which innovation is driven by the quality improvement of products (i.e., vertical innovation). Peretto (1994, 1998, 1999), Smulders and van de Klundert (1995) and Howitt (1999) combine the two dimensions of innovation to develop the second-generation Schumpeterian growth model.[1] The present study contributes to this literature by incorporating an agricultural sector into the Schumpeterian growth model with vertical and horizontal innovation to show that it is the firm size in the industrial sector that determines the incentives for innovation. We find that the scale-invariant property arising from the two dimensions of innovation in the industrial sector is important in allowing the allocation of resources to affect the endogenous takeoff but not economic growth in the long run.

This study also relates to the literature on endogenous takeoff. Seminal studies in this literature include Galor and Weil (2000) and Galor and Moav (2002), who develop unified growth theory.[2] Unified growth theory shows that the quality-quantity tradeoff in childrearing and the accumulation of human capital enable an economy to escape from the Malthusian trap and experience an endogenous transition from stagnation to growth.[3] The present study explores technological progress as an alternative channel for the endogenous takeoff of an economy. The Schumpeterian growth model of endogenous takeoff is based on Peretto (2015).[4] We incorporate an agricultural sector into the Peretto model and show that it is the firm size in the industrial sector that affects the endogenous takeoff. In other words, we formalize the idea of Nurkse (1953) and Murphy *et al.* (1989) in a dynamic general equilibrium model, which allows us to quantify the effect of agricultural technology on the industrialization of an economy.[5]

[1] Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008, 2010) and Ang and Madsen (2011) provide supportive empirical evidence for the second-generation Schumpeterian model.

[2] See also Jones (2001) and Hansen and Prescott (2002) for other early studies on endogenous takeoff.

[3] See Galor and Mountford (2008), Galor *et al.* (2009) and Ashraf and Galor (2011) for evidence and Galor (2011) for a comprehensive review of unified growth theory.

[4] Chu *et al.* (2020) explore the effects of patent protection on endogenous takeoff in the Peretto model.

[5] See Lagakos and Waugh (2013) for an interesting study that explores the large productivity differences in agriculture across countries.

The rest of this paper is organized as follows. Section 2 describes the Schumpeterian growth model. Section 3 explores the effects of agricultural technology. Section 4 performs a quantitative analysis. Section 5 concludes.

2 A Schumpeterian model of endogenous takeoff

The Schumpeterian growth model of endogenous takeoff is based on Peretto (2015). The model features both vertical and horizontal innovation. In the Peretto model, whether the economy experiences innovation depends on the firm size in the economy. By incorporating an agricultural sector with subsistence consumption into the model, we show that it is the firm size in the industrial sector that determines innovation and the endogenous takeoff of the economy.

2.1 Household

The economy features a representative household,[6] which has a population size of L_t that grows at an exogenous rate $\lambda > 0$. The household has a Stone-Geary utility function:

$$U = \int_0^\infty e^{-(\rho-\lambda)t} \left[\ln c_t + \beta \ln(q_t - \eta)\right] dt, \tag{1}$$

where c_t denotes per capita consumption of an industrial good (numeraire) and q_t denotes per capita consumption of an agricultural good for which there is a subsistence consumption level determined by the parameter $\eta > 0$.[7] The parameter $\rho > \lambda$ is the subjective discount rate, whereas the parameter $\beta > 0$ determines the importance of agricultural consumption relative to industrial consumption.

The household maximizes utility subject to an asset-accumulation equation:

$$\dot{a}_t = (r_t - \lambda)a_t + w_t - c_t - p_t q_t, \tag{2}$$

where a_t is the value of assets owned by each member of the household, and r_t is the real interest rate. Each member of the household supplies one unit of labor to earn the wage income w_t. p_t is the price of the agricultural good. Standard dynamic optimization yields the growth path of industrial consumption c_t given by

$$\frac{\dot{c}_t}{c_t} = r_t - \rho \tag{3}$$

and also the demand for agricultural consumption q_t given by

$$q_t = \eta + \frac{\beta c_t}{p_t}. \tag{4}$$

[6] See Chu and Peretto (2019) for an analysis of heterogeneous households in the Peretto model.

[7] This is a common feature of structural change models; see Matsuyama (1992), Laitner (2000) and Kongsamut *et al.* (2001). These interesting studies focus on the implications of structural change on economic growth but not endogenous takeoff.

2.2 Agricultural production

The production function of the agricultural good follows from Lagakos and Waugh (2013):

$$Q_t = AL_{q,t}, \tag{5}$$

where the parameter $A > \eta$ denotes the level of agricultural technology. $L_{q,t}$ is the amount of labor allocated to the agricultural sector.[8] This sector is perfectly competitive, and the zero-profit condition is given by

$$w_t = p_t A, \tag{6}$$

which equates the wage rate to the value of the marginal product of agricultural labor.

2.3 Industrial production

This sector is also perfectly competitive. The production function of the industrial good is

$$Y_t = \int_0^{N_t} X_t^{\theta}(i) \left[Z_t^{\alpha}(i) Z_t^{1-\alpha} L_{y,t}/N_t^{1-\sigma}\right]^{1-\theta} di, \tag{7}$$

where $\{\theta, \alpha, \sigma\} \in (0,1)$. $X_t(i)$ is the quantity of non-durable intermediate goods $i \in [0, N_t]$. The productivity of $X_t(i)$ depends on its own quality $Z_t(i)$ as well as the average quality of all intermediate goods $Z_t \equiv \int_0^{N_t} Z_t(j)\, dj/N_t$, which captures technology spillovers. The parameter α determines the private return to quality, and hence, $1-\alpha$ determines the degree of technology spillovers. The parameter σ determines a congestion effect $1-\sigma$ of variety. As we will show, the social return to variety is measured by σ.

Profit maximization yields the conditional demand functions for $L_{y,t}$ and $X_t(i)$ as follows:

$$w_t = (1-\theta)\frac{Y_t}{L_{y,t}}, \tag{8}$$

$$X_t(i) = \left(\frac{\theta}{P_t(i)}\right)^{1/(1-\theta)} \frac{Z_t^{\alpha}(i) Z_t^{1-\alpha} L_{y,t}}{N_t^{1-\sigma}}, \tag{9}$$

where $P_t(i)$ is the price of $X_t(i)$. Perfect competition implies that firms pay $(1-\theta) Y_t = w_t L_{y,t}$ for industrial labor and $\theta Y_t = \int_0^{N_t} P_t(i) X_t(i)\, di$ for intermediate goods.

2.4 Intermediate goods and in-house R&D

There is a continuum of differentiated intermediate goods $i \in [0, N_t]$. A monopolistic firm produces differentiated intermediate good i with a linear technology that requires $X_t(i)$ units of the industrial good to produce $X_t(i)$ units of intermediate good i. In other words, the marginal cost for the monopolistic firm in industry i to produce $X_t(i)$ with quality $Z_t(i)$ is one. The monopolistic firm also needs to incur $\phi Z_t^{\alpha}(i) Z_t^{1-\alpha}$ units of the industrial good as

[8] For tractability, we do not consider land in our model. See Vollrath (2011) for an interesting study that explores the effects of land intensity and labor intensity in agriculture on industrialization.

a fixed operating cost. To improve the quality of its products, the firm devotes $I_t(i)$ units of the industrial good to in-house R&D. We specify the innovation function as

$$\dot{Z}_t(i) = I_t(i). \tag{10}$$

The firm's (before-R&D) profit flow at time t is

$$\Pi_t(i) = [P_t(i) - 1] X_t(i) - \phi Z_t^{\alpha}(i) Z_t^{1-\alpha}. \tag{11}$$

The value of the monopolistic firm in industry i is

$$V_t(i) = \int_t^{\infty} \exp\left(-\int_t^s r_u du\right) [\Pi_s(i) - I_s(i)] ds. \tag{12}$$

The monopolistic firm maximizes (12) subject to (10) and (11). We solve this dynamic optimization problem in the proof of Lemma 1 and find that the unconstrained profit-maximizing markup ratio is $1/\theta$. However, competitive firms can also produce $X_t(i)$ with the same quality $Z_t(i)$ as the monopolistic firm,[9] but they face a higher unit cost of production given by $\mu > 1$. To price the competitive firms out of the market, the monopolistic firm sets its price as follows:

$$P_t(i) = \min\{\mu, 1/\theta\}. \tag{13}$$

For the rest of the analysis, we assume $\mu < 1/\theta$ implying that $P_t(i) = \mu$.

Following the standard approach in the literature, we consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0, N_t]$ and the size of each intermediate-good firm is identical across all industries $X_t(i) = X_t$.[10] From (9) and $P_t(i) = \mu$, the quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_{y,t}}{N_t^{1-\sigma}}. \tag{14}$$

We define the following transformed variable:

$$x_t \equiv \theta^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}} = \mu^{1/(1-\theta)} \frac{L_t}{L_{y,t}} \frac{X_t}{Z_t}, \tag{15}$$

which is a state variable determined by the ratio $L_t/N_t^{1-\sigma}$. In Lemma 1, the rate of return on quality-improving R&D is increasing in the firm size measured by $x_t l_{y,t}$, where $l_{y,t} \equiv L_{y,t}/L_t$ is the industrial share of labor, which captures the idea that agricultural labor does not contribute to the firm size in the industrial sector.

Lemma 1 *The rate of return on quality-improving in-house R&D is*

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t l_{y,t} - \phi\right]. \tag{16}$$

Proof. See Appendix A. ■

[9] Here we assume diffusion of technologies from the monopolistic firm to competitive firms in each industry.

[10] Symmetry also implies $\Pi_t(i) = \Pi_t$, $I_t(i) = I_t$ and $V_t(i) = V_t$.

2.5 Entrants

We follow previous studies to assume that entrants have access to aggregate technology Z_t in order to ensure symmetric equilibrium at any time t. A new firm pays δX_t units of the industrial good to enter the market with a new variety of intermediate goods and set up its operation. $\delta > 0$ is an entry-cost parameter. The asset-pricing equation is given by

$$r_t = \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{17}$$

When entry is positive, the entry condition is given by

$$V_t = \delta X_t. \tag{18}$$

Substituting (10), (11), (15), (18) and $P_t = \mu$ into (17) yields the return on entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\delta}\left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t l_{y,t}}\right) + z_t + \frac{\dot{x}_t}{x_t} + \frac{\dot{l}_{y,t}}{l_{y,t}}, \tag{19}$$

where $z_t \equiv \dot{Z}_t / Z_t$ is the growth rate of aggregate quality.

2.6 Equilibrium

The equilibrium is a time path of allocations $\{a_t, q_t, c_t, Y_t, X_t, I_t, L_{y,t}, L_{q,t}\}$ and prices $\{r_t, w_t, p_t, P_t, V_t\}$ such that

- the household consumes $\{q_t, c_t\}$ to maximize utility taking $\{r_t, w_t, p_t\}$ as given;
- competitive firms produce Q_t to maximize profits taking $\{w_t, p_t\}$ as given;
- competitive firms produce Y_t to maximize profits taking $\{w_t, P_t\}$ as given;
- intermediate-good firms choose $\{P_t, I_t\}$ to maximize V_t taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the value of all existing monopolistic firms adds up to the value of the household's assets such that $a_t L_t = N_t V_t$;
- the labor market clears such that $L_{q,t} + L_{y,t} = L_t$;
- the market of the agricultural good clears such that $q_t L_t = A L_{q,t}$; and
- the market of the industrial good also clears such that

$$Y_t = c_t L_t + N_t \left(X_t + \phi Z_t + I_t\right) + \dot{N}_t \delta X_t. \tag{20}$$

2.7 Aggregation

Substituting (9) and $P_t = \mu$ into (7) and imposing symmetry yield the aggregate production function of the industrial good as follows:

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{\sigma} Z_t L_{y,t}. \tag{21}$$

The growth rate of industrial output per capita (i.e., $y_t = Y_t/L_t$) is

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \sigma n_t + z_t + \frac{\dot{l}_{y,t}}{l_{y,t}}, \tag{22}$$

which is determined by the variety growth rate $n_t \equiv \dot{N}_t/N_t$, the quality growth rate z_t and the growth rate of the industrial labor share $l_{y,t}$.

2.8 Labor allocation

Equating the labor demand functions in (6) and (8) yields

$$p_t = \frac{(1-\theta)\,Y_t}{AL_{y,t}}. \tag{23}$$

Then, substituting the supply of Q_t in (5) and the relative price p_t in (23) into the demand function for q_t in (4) yields the industrial labor share $l_{y,t}$ as

$$l_{y,t} = \frac{L_{y,t}}{L_t} = \left(1 + \frac{\beta}{1-\theta}\frac{c_t}{y_t}\right)^{-1}\left(1 - \frac{\eta}{A}\right), \tag{24}$$

which shows that for a given consumption-output ratio c_t/y_t, the industrial labor share $l_{y,t}$ is *increasing* in agricultural technology A. This pattern of sectoral reallocation is due to the subsistence consumption parameter $\eta > 0$ under which an improvement in agricultural technology releases labor from agriculture to the industrial sector.

3 Agricultural revolution and endogenous takeoff

The dynamics of the economy is determined by the dynamics of the state variable x_t, which is stable and gradually increases from an initial value x_0 towards a steady-state value x^* if the following parameter condition holds:

$$\delta\phi > \frac{1}{\alpha}\left[\mu - 1 - \delta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right] > \mu - 1. \tag{25}$$

In the proof of Proposition 1, we show that the economy begins in a pre-industrial era in which the growth rate of industrial output per capita is zero. Then, the economy enters the first industrial era in which new products are being developed and the growth rate of industrial output per capita becomes positive. After that, the growth rate rises further as the economy enters the second industrial era in which the quality of products is being improved (i.e., vertical innovation) while new products continue to be developed (i.e., horizontal innovation).[11] Finally, the economy converges to the balanced growth path that features both vertical and horizontal innovation. Figure 1 illustrates the dynamic path of the equilibrium growth rate g_t from stagnation to long-run growth g^*. In what follows, we show that an agricultural revolution gives rise to an earlier takeoff of the economy.

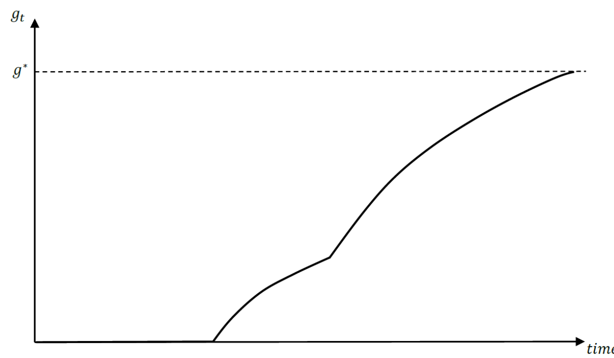

Figure 1: Dynamic path of economic growth

3.1 The pre-industrial era

The economy begins in a pre-industrial era, in which neither horizontal innovation nor vertical innovation has been activated. In this pre-industrial era, the firm size $x_t l_{y,t}$ is so small that monopolistic firms cannot earn a positive profit; i.e.,

$$x_t l_{y,t} < \phi \mu^{1/(1-\theta)}/(\mu - 1) \Leftrightarrow \Pi_t < 0.$$

Therefore, all intermediate goods N_0 are produced by competitive firms, which do not incur the operating cost and simply price the intermediate goods at their marginal cost such that $P_t(i) = \mu$. In this case, the intermediate-good sector generates zero profit, and the value of monopolistic firms is zero. As a result, industrial consumption in the pre-industrial era is

$$c_t = w_t l_{y,t} = (1 - \theta) y_t, \tag{26}$$

which implies a consumption-output ratio of $c_t / y_t = 1 - \theta$.

Substituting (26) into (24) yields the equilibrium level of industrial labor share in the pre-industrial era:

$$l_y = \frac{1}{1 + \beta} \left(1 - \frac{\eta}{A} \right), \tag{27}$$

[11] Here we consider the realistic case in which the creation of products happens before the quality improvement of products.

which is stationary and increasing in the level of agricultural technology A. The growth rate of industrial output per capita is then

$$g_t = \sigma n_t + z_t + \frac{\dot{l}_{y,t}}{l_{y,t}} = 0 \tag{28}$$

because $n_t = z_t = \dot{l}_{y,t}/l_{y,t} = 0$ in the pre-industrial era. Finally, the inequality for $\Pi_t < 0$ can now be written as the following condition:

$$A < \frac{\eta}{1 - \frac{1}{\mu - 1}(1+\beta)\mu^{1/(1-\theta)}\phi/x_t}. \tag{29}$$

In other words, when the level of agricultural technology is low, the economy remains in an equilibrium with zero economic growth because the firm size in the industrial sector is not large enough to provide sufficient incentives for innovation.

3.2 The first industrial era

Suppose agricultural technology improves and the level of A increases to a point in which the firm size $x_t l_{y,t}$ becomes sufficiently large for horizontal innovation (but not vertical innovation) to occur. We refer to this as the first industrial era. When horizontal innovation is activated, the consumption-output ratio c_t/y_t jumps to the steady-state value in Lemma 2.

Lemma 2 *When variety innovation is activated, the consumption-output ratio jumps to*

$$\frac{c_t}{y_t} = \frac{(\rho - \lambda)\delta\theta}{\mu} + 1 - \theta. \tag{30}$$

Proof. See Appendix A. ■

In this case, the equilibrium level of industrial labor share is given by

$$l_y^* = \frac{1}{1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)}\left(1 - \frac{\eta}{A}\right). \tag{31}$$

which continues to be stationary and increasing in agricultural technology A. In the first industrial era, the firm size $x_t l_y^*$ is sufficiently large for horizontal innovation to be viable, which requires[12]

$$A > \frac{\eta}{1 - \frac{1}{\mu - 1 - \delta(\rho - \lambda)}\left[1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\mu^{1/(1-\theta)}\phi/x_t}. \tag{32}$$

[12] There exists an intermediate range of A between (29) and (32), in which monopolistic firms in N_0 could earn positive profits but do not invest in innovation yet. We follow Chu and Peretto (2019) to assume that intermediate goods are produced by competitive firms until innovation occurs because only innovation allows the industrial sector to be monopolized. In Appendix B, we consider an extension of the model that does not rely on this assumption and show that the dynamics of the economy actually becomes less realistic.

Intuitively, when the level of agricultural technology increases, the economy reallocates labor from the agricultural sector to the industrial sector. If the firm size in the industrial sector is large enough, it would provide sufficient incentives for innovation to occur.

When the inequality in (32) holds, the variety growth rate can be derived from (19) as[13]

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta}\left(\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t l_y^*}\right) + \lambda - \rho > 0, \tag{33}$$

which is increasing in agricultural technology A via industrial labor share l_y^* and also depends on the state variable x_t. The growth rate of industrial output per capita is $g_t = \sigma n_t > 0$. Figure 2 presents the time path of the growth rate g_t when A increases at time t and causes the economy to escape from the pre-industrial era and enter the first industrial era.

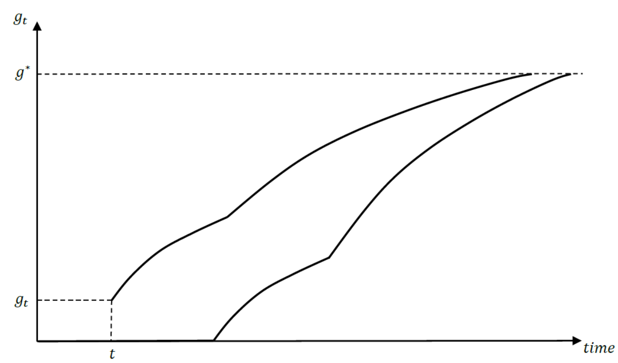

Figure 2: Agricultural improvement (case 1)

3.3 The second industrial era

Suppose agricultural technology A increases to an even higher level in which the firm size $x_t l_y^*$ becomes sufficiently large for both horizontal and vertical innovation to occur simultaneously. We refer to this as the second industrial era. Given the presence of horizontal innovation, the consumption-output ratio c_t/y_t continues to be given by the steady-state value in Lemma 2, whereas the industrial labor share l_y^* is given by (31). In this era, the firm size $x_t l_y^*$ is sufficiently large for vertical innovation to be also viable, which requires

$$A > \frac{\eta}{1 - \left[1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\Omega/x_t}. \tag{34}$$

where the composite parameter Ω is defined as

$$\Omega \equiv \underset{\omega}{\arg\text{solve}}\left\{\left[\frac{\mu-1}{\mu^{1/(1-\theta)}}\omega - \phi\right]\left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta\omega}\right] = \rho - \sigma\left(\rho - \lambda\right)\right\}.$$

[13]Here we use $r_t^e = r_t = \rho + g_t = \rho + \sigma n_t$, $z_t = 0$ and $\dot{x}_t/x_t = \lambda - (1-\sigma)n_t$.

When the inequality in (34) holds, the growth rate of industrial output per capita is determined by the rate of return on quality-improving in-house R&D such that[14]

$$g_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t l_y^* - \phi \right] - \rho > 0, \tag{35}$$

which is also increasing in agricultural technology A via industrial labor share l_y^* and depends on the state variable x_t. Equation (35) implies that the quality growth rate z_t is given by

$$z_t = g_t - \sigma n_t, \tag{36}$$

where g_t is derived in (35) and the variety growth rate n_t can be derived from (19) as[15]

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t l_y^*} \right) + \lambda - \rho > 0. \tag{37}$$

Equations (35)-(37) determine the quality growth rate z_t as a function of the firm size $x_t l_y^*$. The inequality in (34) ensures that $z_t > 0$.[16] Figure 3 presents the time path of the growth rate of industrial output per capita when A increases at time t and causes the economy to escape from the pre-industrial era and directly enter the second industrial era.

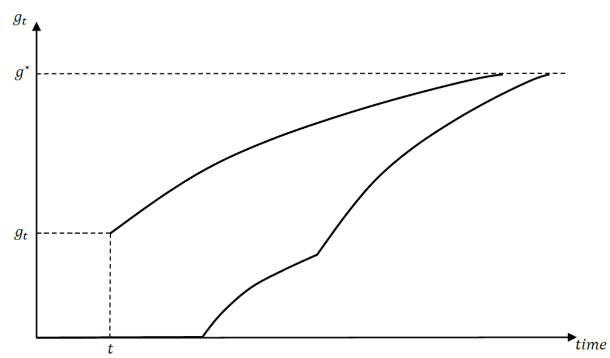

Figure 3: Agricultural improvement (case 2)

3.4 Balanced growth path

In the long run, the state variable x_t converges to its steady-state value x^*, implying that the variety growth rate n_t converges to $\lambda/(1-\sigma)$.[17] We can then compute the steady-state equilibrium value of the firm size $x^* l_y^*$ given by[18]

$$x^* l_y^* = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta [\rho + \sigma\lambda/(1-\sigma)]}. \tag{38}$$

[14] Here we use $r_t^q = r_t = \rho + g_t$.

[15] Here we use $r_t^e = r_t = \rho + g_t = \rho + \sigma n_t + z_t$ and $\dot{x}_t / x_t = \lambda - (1-\sigma) n_t$.

[16] Derivations are available upon request.

[17] Recall that $x_t \equiv \theta^{1/(1-\theta)} L_t / N_t^{1-\sigma}$, and the growth rate of L_t is λ.

[18] Here we use (35)-(37) to derive n_t and then set $n^* = \lambda/(1-\sigma)$ to derive the steady-state $x^* l_y^*$.

The steady-state firm size $x^* l_y^*$ in turn determines the steady-state equilibrium growth rate of industrial output per capita as

$$g^* = \alpha \left[(\mu - 1) \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \sigma\lambda/(1-\sigma)\right]} - \phi \right] - \rho > 0, \tag{39}$$

which is independent of agricultural technology A due to the scale-invariant property of the Schumpeterian growth model with endogenous market structure.

Proposition 1 summarizes all the results in Section 3.

Proposition 1 *The economy begins in a pre-industrial era without innovation. Then, the economy enters the first industrial era with horizontal innovation. Finally, the economy enters the second industrial era with both vertical and horizontal innovation before converging to the balanced growth path. An improvement in agricultural technology gives rise to an earlier takeoff but does not affect long-run economic growth.*

Proof. See Appendix A. ■

4 Quantitative analysis

In the early 19th century, the agricultural share of the US workforce decreased from about 80% to 60%.[19] We perform a counterfactual analysis on how large an effect this reallocation of labor from agriculture to the industrial sector had on the takeoff of the US economy.

Recall that the firm size, which determines the timing of the takeoff, is captured by

$$x_t l_{y,t} = x_t(1 - l_{q,t}), \tag{40}$$

where $l_{q,t} \equiv L_{q,t}/L_t$ is the agricultural labor share. In the proof of Proposition 1, we show that the takeoff occurs when $x_t l_{y,t}$ reaches the threshold $\phi\mu^{1/(1-\theta)}/[\mu - 1 - \delta(\rho - \lambda)]$.[20] A decrease in the agricultural labor share $l_{q,t}$ from 80% to 60% translates to an increase in the industrial labor share $l_{y,t}$ from 20% to 40%,[21] which expands the firm size $x_t l_{y,t}$ by a factor of 2 for a given x_t. In the pre-industrial era, the state variable x_t increases at the population growth rate λ. In the US, the long-run population growth rate is 1.8%.[22] Therefore, without the increase in the industrial labor share $l_{y,t}$, the state variable x_t would take

$$t = \frac{\ln 2}{\lambda} = \frac{0.7}{1.8\%} = 39 \text{ years}$$

[19] See Baten (2016), Lebergott (1966) and Weiss (1986).

[20] See (A10) in Appendix A.

[21] Here we are putting manufacturing and services together as the industrial sector that requires innovation; see e.g., United Nations (2011) for a review on the importance of innovation in the services sector. Kongsamut *et al.* (2001) show that manufacturing and services require the same technology growth rate in order for a balanced growth path to exist in their model.

[22] Data source: Maddison Project Database.

to increase by a factor of 2. In other words, without the reallocation of labor from agriculture to the industrial sector in the early 19th century, the takeoff of the US economy would have been delayed by about four decades. Furthermore, we can derive the following formula:[23]

$$t = \frac{\ln(1+\chi)}{\lambda} \approx \frac{\chi}{\lambda} \text{ years,} \tag{41}$$

where χ is the percent change in $l_{y,t}$. For example, given a population growth rate λ of 1.8%, a one-fifth increase in industrial employment causes an earlier takeoff by about a decade.

We now calibrate the rest of the model to data in the US economy in order to perform a quantitative analysis. In addition to the population growth rate λ, the model also features the following parameters: $\{\rho, \alpha, \sigma, \beta, \theta, \delta, \phi, \mu\}$.[24] We set the discount rate ρ to a conventional value of 0.05. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers $1-\alpha$ to 0.833 and the social return of variety σ to 0.25. Then, we calibrate β using the current agricultural share of GDP in the US, which is about 1%.[25] Furthermore, we calibrate $\{\theta, \delta, \phi\}$ by matching the following moments of the US economy: 60% for the labor income share of GDP, 62% for the consumption share of GDP, and 1% for the long-run growth rate of technology. Finally, we calibrate the markup ratio μ by matching the average growth rates of the simulated path from our model and the historical path in the US.

Table 1: Calibrated parameter values

λ	ρ	α	σ	β	θ	δ	ϕ	μ
0.018	0.050	0.167	0.250	0.016	0.404	2.547	1.212	1.630

To explore how well our model matches the historical path of the growth rate in the US, we first use historical data to calibrate a time path for the subsistence ratio η/A. Specifically, we calibrate the initial value of η/A using an agricultural labor share of 80% at the beginning of the 19th century; see Baten (2016). Then, we use an agricultural labor share of 60% in 1840 and 53% in 1860 in Lebergott (1966) and Weiss (1986) and also an agricultural share of GDP of 30% in 1900, 20% in 1920-1930, 10% in 1950 and 2% in 1980 in Kongsamut *et al.* (2001) to compute a piecewise linear path of η/A. Based on this imputed path of η/A, Figure 4 simulates the path of the agricultural share of GDP, which decreases from about 70% in the early 19th century to 1% at the end of the 20th century as in the US data.

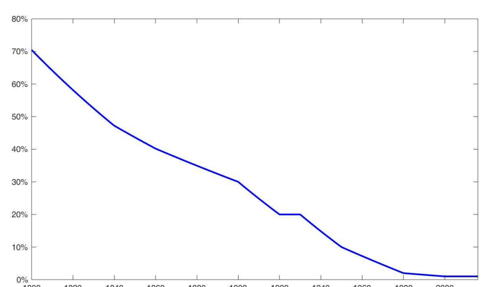

Figure 4: Agricultural share of GDP

[23] It is useful to note that the approximation $\ln(1+\chi) \approx \chi$ only holds when χ is small.

[24] There is also the subsistence ratio η/A, which we will calibrate using historical data.

[25] Here we assume that the subsistence requirement is no longer binding in modern days; i.e., $\eta/A \to 0$.

Figure 5 presents the simulated path of the growth rate of industrial output per worker and the HP-filter trend of the US growth rate[26] along with a simulated path of the growth rate without agricultural improvement (i.e., η/A remains at its initial value). Here we pick an initial value x_0 such that the takeoff of the economy occurs before the mid-19th century. Following the occurrence of horizontal innovation, vertical innovation also starts to happen half a decade later. After that the economy keeps growing and reaches a growth rate as high as 3% due to the expansion of the industrial sector, which helps to accelerate the rate of innovation. Around the time of the Great Depression in the 20th century, there is a pause in the reallocation of labor from agriculture to the industrial sector, which translates into a temporary slow down in technological progress before a recovery. Before the end of the 20th century, the growth rate of the economy gradually falls towards the long-run growth rate due to the deceleration of sectoral reallocation. This simulated pattern replicates the data reasonably well with the average growth rate increasing from 1.08% in the 19th century to 2.24% in the 20th century before decreasing to 1.04% in the 21st century, whereas the corresponding data are 1.20%, 2.12% and 1.13% in the 19th, 20th and 21st centuries respectively. In contrast, the simulated path of the growth rate without agricultural improvement cannot capture this inverted-U pattern in the data.

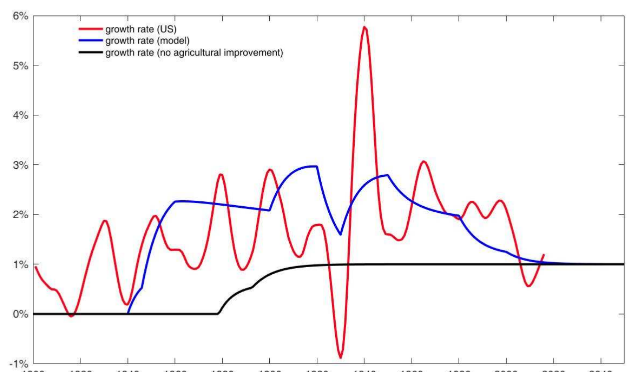

Figure 5: Economic growth

5 Conclusion

In this study, we have developed a Schumpeterian growth model with both agricultural and industrial sectors in which the firm size in the industrial sector determines the endogenous takeoff of an economy. This growth-theoretic framework is motivated by the observation that countries with large population, such as China and India, did not experience an early industrial takeoff. Our explanation is that the vast majority of their population being in

[26] Unfortunately, we don't have historical data on labor productivity growth in the US, so we use data on the growth rate of output per capita as a proxy.

agriculture did not contribute to the firm size in the industrial sector. We have used our model to explore how an agricultural revolution affects the timing of endogenous takeoff and found that a sectoral reallocation that expands the firm size in the industrial sector leads to an earlier transition from stagnation to growth. Our quantitative analysis indicates that the decline in the agricultural share of the US workforce in the early 19th century contributed to the takeoff of the US economy. Without the reallocation of labor from agriculture to the industrial sector, the takeoff of the US economy would have been delayed by four decades.

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Appendix A

Proof of Lemma 1. The current-value Hamiltonian for monopolistic firm i is

$$H_t(i) = \Pi_t(i) - I_t(i) + \zeta_t(i)\dot{Z}_t(i) + \xi_t(i)[\mu - P_t(i)], \tag{A1}$$

where $\xi_t(i)$ is the multiplier on $P_t(i) \leq \mu$. We substitute (9)-(11) into (A1) and derive

$$\frac{\partial H_t(i)}{\partial P_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial P_t(i)} = \xi_t(i), \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial I_t(i)} = 0 \Rightarrow \zeta_t(i) = 1, \tag{A3}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha\left\{[P_t(i) - 1]\left[\frac{\theta}{P_t(i)}\right]^{1/(1-\theta)}\frac{L_{y,t}}{N_t^{1-\sigma}} - \phi\right\} Z_t^{\alpha-1}(i)\, Z_t^{1-\alpha} = r_t\zeta_t(i) - \dot{\zeta}_t(i). \tag{A4}$$

If $P_t(i) < \mu$, then $\xi_t(i) = 0$. In this case, $\partial\Pi_t(i)/\partial P_t(i) = 0$ yields $P_t(i) = 1/\theta$. If the constraint on $P_t(i)$ is binding, then $\xi_t(i) > 0$. In this case, we have $P_t(i) = \mu$. Therefore, we have proven (13). Then, the assumption $\mu < 1/\theta$ implies $P_t(i) = \mu$. Substituting (A3), (15) and $P_t(i) = \mu$ into (A4) and imposing symmetry yield (16), where $l_{y,t} \equiv L_{y,t}/L_t$. ■

Proof of Lemma 2. The value of assets owned by each member of the household is

$$a_t = V_t N_t / L_t. \tag{A5}$$

If $n_t > 0$, then $V_t = \delta X_t$ in (18) holds. Substituting (18) and $\mu X_t N_t = \theta Y_t$ into (A5) yields

$$a_t = \delta X_t N_t / L_t = (\theta/\mu)\,\delta Y_t / L_t = (\theta/\mu)\,\delta y_t, \tag{A6}$$

which implies that a_t/y_t is constant. Substituting (A6), (3) and (8) into (2) yields

$$\begin{aligned}\frac{\dot{y}_t}{y_t} = \frac{\dot{a}_t}{a_t} &= r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - c_t - p_t q_t}{a_t} \\ &= \frac{\dot{c}_t}{c_t} + \rho - \lambda + \frac{(1-\theta)\,\mu}{\delta\theta} - \frac{\mu}{\delta\theta}\frac{c_t}{y_t},\end{aligned} \tag{A7}$$

where we have also used $w_t L_{q,t} = p_t Q_t$. Equation (A7) can be rearranged as

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = \frac{\mu}{\delta\theta}\frac{c_t}{y_t} - \frac{(1-\theta)\,\mu}{\delta\theta} - (\rho - \lambda), \tag{A8}$$

which shows that the dynamics of c_t/y_t is characterized by saddle-point stability such that c_t/y_t jumps to its steady-state value in (30) whenever $n_t > 0$. ■

Proof of Proposition 1. In the pre-industrial era, the firm size $x_t l_y$ is not sufficiently large for horizontal and vertical innovation to be viable such that the variety growth rate and the quality growth rate are both zero (i.e., $n_t = z_t = 0$). In this case, the industrial labor share l_y is given by (27) and the state variable $x_t = \theta^{1/(1-\theta)} L_t / N_0^{1-\sigma}$ increases at the population growth rate λ. Therefore, in the pre-industrial era, the dynamics of x_t is simply

$$\dot{x}_t = \lambda x_t > 0. \tag{A9}$$

In the first industrial era, the firm size $x_t l_y^*$ becomes sufficiently large for horizontal innovation (but not vertical innovation) to be viable such that $n_t > 0$ and $z_t = 0$. In this case, the variety growth rate n_t is given by (33), which is positive if and only if[27]

$$x_t > \frac{\mu^{1/(1-\theta)} \phi / l_y^*}{\mu - 1 - \delta(\rho - \lambda)} \equiv x_N > x_0, \tag{A10}$$

where l_y^* is given by (31) and increasing in A. The dynamics of $x_t = \theta^{1/(1-\theta)} L_t / N_t^{1-\sigma}$ is

$$\dot{x}_t = [\lambda - (1-\sigma) n_t] \, x_t = \frac{1-\sigma}{\delta} \left\{ \frac{\phi \mu^{1/(1-\theta)}}{l_y^*} - \left[\mu - 1 - \delta \left(\rho + \frac{\sigma \lambda}{1-\sigma} \right) \right] x_t \right\} > 0, \tag{A11}$$

which uses (33) for n_t.

In the second industrial era, the firm size $x_t l_y^*$ becomes sufficiently large for both horizontal and vertical innovation to be viable such that $n_t > 0$ and $z_t > 0$. In this case, the quality growth rate z_t is given by (36), which is positive if and only if[28]

$$x_t > \frac{\Omega}{l_y^*} \equiv x_Z > x_N, \tag{A12}$$

where l_y^* is given by (31) and the composite parameter Ω is defined as before:

$$\Omega \equiv \underset{\omega}{\arg \text{solve}} \left\{ \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} \omega - \phi \right] \left[\alpha - \frac{\mu^{1/(1-\theta)} \sigma}{\delta \omega} \right] = \rho - \sigma \left(\rho - \lambda \right) \right\}.$$

We use (35)-(37) to derive n_t and the linearized dynamics of $x_t = \theta^{1/(1-\theta)} L_t / N_t^{1-\sigma}$ as

$$\dot{x}_t = \frac{1-\sigma}{\delta} \left\{ \left[(1-\alpha)\,\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma} \right) \right] \frac{\mu^{1/(1-\theta)}}{l_y^*} - \left[(1-\alpha)\,(\mu - 1) - \delta \left(\rho + \frac{\sigma \lambda}{1-\sigma} \right) \right] x_t \right\} \geq 0, \tag{A13}$$

where we have used $\sigma \mu^{1/(1-\theta)} / \left(x_t l_y^* \right) \cong 0$.

Given (25), the autonomous dynamics of x_t is stable and captured by (A9), (A11) and (A13). Given an initial value x_0, the state variable x_t increases according to (A9) until x_t reaches the first threshold x_N, which is decreasing in A via l_y^*. Then, x_t increases according to (A11) until x_t reaches the second threshold x_Z, which is also decreasing in A via l_y^*. Finally, x_t increases according to (A13) until x_t converges to its steady-state value x^* in (38). ■

[27] It is useful to note that (A10) is equivalent to (32).

[28] It is useful to note that (A12) is equivalent to (34).

Appendix B

In this appendix, we extend the baseline model to allow for the possibility that in the pre-industrial era (i.e., $n_t = z_t = 0$), monopolistic profits become positive (i.e., $\Pi_t > 0$) before the takeoff occurs. When $n_t = 0$, the entry condition in (18) does not hold. However, the asset-pricing equation in (17) still holds and becomes

$$r_t = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t}, \tag{B1}$$

where $I_t = z_t = 0$. We use (A5) and $n_t = 0$ to derive $\dot{a}_t/a_t = \dot{V}_t/V_t - \lambda$ and then substitute this equation into (2) to obtain

$$\frac{\dot{V}_t}{V_t} - \lambda = \frac{\dot{a}_t}{a_t} = r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - p_t q_t - c_t}{a_t}. \tag{B2}$$

Substituting (B1) into (B2) yields

$$c_t = \frac{\Pi_t}{V_t} a_t + w_t l_{y,t} = \frac{N_t}{L_t} \Pi_t + (1-\theta)\, y_t, \tag{B3}$$

where we have used (A5), $w_t l_{q,t} = p_t q_t$ and $w_t l_{y,t} = (1-\theta)\, y_t$. Then, substituting (11) and $P_t = \mu$ into (B3) yields

$$c_t = \frac{N_t X_t \left(\mu - 1 - \phi Z_t/X_t\right)}{L_t} + (1-\theta)\, y_t = \theta \mu^{\theta/(1-\theta)} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t l_{y,t}}\right) y_t + (1-\theta)\, y_t, \tag{B4}$$

where the second equality uses $\theta Y_t = \mu N_t X_t$ and (15). The consumption-output ratio is

$$\frac{c_t}{y_t} = \theta \mu^{\theta/(1-\theta)} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t l_{y,t}}\right) + 1 - \theta, \tag{B5}$$

which would increase from (26) to (30) if the firm size $x_t l_{y,t}$ increases from $\phi\mu^{1/(1-\theta)}/(\mu-1)$ to $\phi\mu^{1/(1-\theta)}/[\mu - 1 - \delta(\rho - \lambda)]$. Finally, we substitute (B5) into (24) and manipulate the equation to obtain the equilibrium firm size:

$$x_t l_{y,t} = \frac{\frac{\beta\theta\phi}{1-\theta}\mu^{\theta/(1-\theta)} + \left(1 - \frac{\eta}{A}\right) x_t}{1 + \beta\left(1 + \frac{\theta}{1-\theta}\frac{\mu-1}{\mu}\right)}, \tag{B6}$$

which continues to be increasing in the level of agricultural technology A.

Given that the dynamics of x_t is still given by (A9) in the pre-industrial era, the firm size $x_t l_{y,t}$ gradually increases towards the threshold in (A10) to trigger the takeoff as before. The only difference is that as x_t increases overtime, $l_{y,t}$ in (B6) is gradually decreasing from l_y in (27) to l_y^* in (31) (instead of jumping from l_y to l_y^* at the time of the takeoff). This additional dynamics in $l_{y,t}$ gives rise to negative growth in the industrial output per capita before the takeoff, which is less realistic than the dynamics in the baseline model.