Risk Analysis in Investment Appraisal

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March 1994

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in
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by

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Published in "**Project Appraisal**",

Volume 9 Number 1, pages 3-18, March 1994

ABSTRACT*

This paper was prepared for the purpose of presenting the methodology and uses of the Monte Carlo simulation technique as applied in the evaluation of investment projects to analyse and assess risk. The first part of the paper highlights the importance of risk analysis in investment appraisal. The second part presents the various stages in the application of the risk analysis process. The third part examines the interpretation of the results generated by a risk analysis application including investment decision criteria and various measures of risk based on the expected value concept. The final part draws some conclusions regarding the usefulness and limitations of risk analysis in investment appraisal.

The author is grateful to Graham Glenday of Harvard University for his encouragement and assistance in pursuing this study and in the development of the RiskMaster and Riskease computer software which put into practice the concepts presented in this paper. Thanks are also due to Professor John Evans of York University, Canada, Baher El Hifnawi, Professor Glenn Jenkins of Harvard University and numerous colleagues at the Cyprus Development Bank for their assistance.

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CONTENTS

I. INTRODUCTION

The purpose of investment appraisal is to assess the economic prospects of a proposed investment project. It is a methodology for calculating the expected return based on cash-flow forecasts of many, often inter-related, project variables. Risk emanates from the uncertainty encompassing these projected variables. The evaluation of project risk therefore depends, on the one hand, on our ability to identify and understand the nature of uncertainty surrounding the key project variables and on the other, on having the tools and methodology to process its risk implications on the return of the project.

Project uncertainty

The first task of project evaluation is to estimate the future values of the projected variables. Generally, we utilise information regarding a specific event of the past to predict a possible future outcome of the same or similar event. The approach usually employed in investment appraisal is to calculate a "best estimate" based on the available data and use it as an input in the evaluation model. These single-value estimates are usually the mode[1] (the most likely outcome), the average, or a conservative estimate[2].

In selecting a single value however, a range of other probable outcomes for each project variable (data which are often of vital importance to the investment decision as they pertain to the risk aspects of the project) are not included in the analysis. By relying completely on single values as inputs it is implicitly assumed that the values used in the appraisal are certain. The outcome of the project is, therefore, also presented as a certainty with no possible variance or margin of error associated with it.

Recognising the fact that the values projected are not certain, an appraisal report is usually supplemented to include sensitivity and scenario analysis tests. Sensitivity analysis, in its simplest form, involves changing the value of a variable in order to test its impact on the final result. It is therefore used to identify the project's most important, highly sensitive, variables.

Scenario analysis remedies one of the shortcomings of sensitivity analysis[3] by allowing the simultaneous change of values for a number of key project variables thereby constructing an alternative scenario for the project. Pessimistic and optimistic scenarios are usually presented.

Sensitivity and scenario analyses compensate to a large extent for the analytical limitation of having to strait-jacket a host of possibilities into single numbers. However useful though, both tests are static and rather arbitrary in their nature.

The use of risk analysis in investment appraisal carries sensitivity and scenario analyses through to their logical conclusion. Monte Carlo simulation adds the dimension of dynamic analysis to project evaluation by making it possible build up random scenarios which are consistent with the analyst's key assumptions about risk. A risk analysis application utilises a wealth of information, be it in the form of objective data or expert opinion, to quantitatively describe the uncertainty surrounding the key project variables as probability distributions, and to calculate in a consistent manner its possible impact on the expected return of the project.

The output of a risk analysis is not a single-value but a probability distribution of all possible expected returns. The prospective investor is therefore provided with a complete risk/return profile of the project showing all the possible outcomes that could result from the decision to stake his money on a particular investment project.

Risk analysis computer programs are mere tools for overcoming the processing limitations which have been containing investment decisions to be made solely on single-value (or "certainty equivalent") projections. One of the reasons why risk analysis was not, until recently, frequently applied is that micro-computers were not powerful enough to handle the demanding needs of Monte Carlo simulation and because a tailor-made project appraisal computer model had to be developed for each case as part and parcel of the risk analysis application.

This was rather expensive and time consuming, especially considering that it had to be developed on main-frame or mini computers, often using low level computer languages. However, with the rapid leaps achieved in micro-computer technology, both in hardware and software, it is now possible to develop risk analysis programs that can be applied generically, and with ease, to any investment appraisal model.

Risk analysis is not a substitute for normal investment appraisal methodology but rather a tool that enhances its results. A good appraisal model is a necessary base on which to set up a meaningful simulation. Risk analysis supports the investment decision by giving the investor a measure of the variance associated with a project appraisal return estimate.

By being essentially a decision making tool, risk analysis has many applications and functions that extend its usefulness beyond pure investment appraisal decisions. It can also develop into a powerful decision making device in marketing, strategic management, economics, financial budgeting, production management and in many other fields in which relationships that are based on uncertain variables are modelled to facilitate and enhance the decision making process.

II. THE RISK ANALYSIS PROCESS

What is risk analysis?

Risk analysis, or "probabilistic simulation" based on the Monte Carlo simulation technique is methodology by which the uncertainty encompassing the main variables projected in a forecasting model is processed in order to estimate the impact of risk on the projected results. It is a technique by which a mathematical model is subjected to a number of simulation runs, usually with the aid of a computer. During the simulation process, successive scenarios are built up using input values for the project's key uncertain variables which are selected from multi-value probability distributions.

The simulation is controlled so that the random selection of values from the specified probability distributions does not violate the existence of known or suspected correlation relationships among the project variables. The results are collected and analysed statistically so as to arrive at a probability distribution of the potential outcomes of the project and to estimate various measures of project risk.

The risk analysis process can be broken down into the following stages as shown in Figure 1.

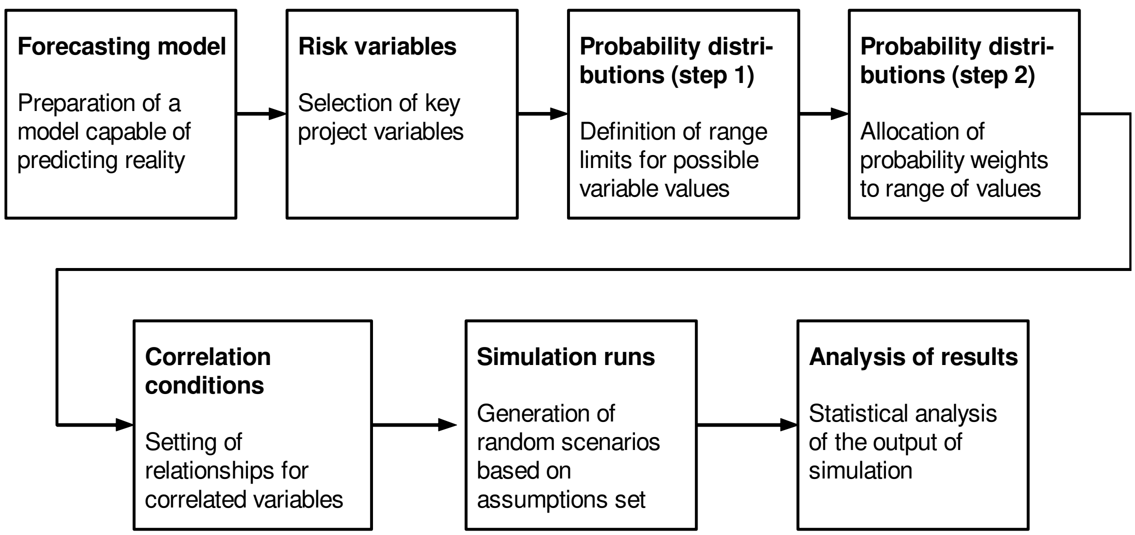
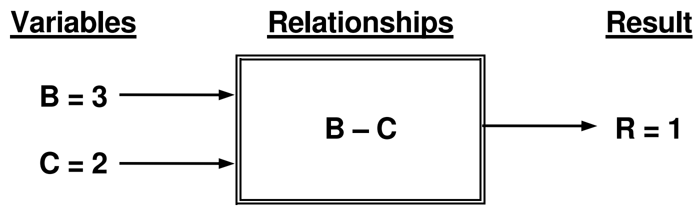


Figure 1. Risk analysis process

Forecasting model

The first stage of a risk analysis application is simply the requirement for a robust model capable of predicting correctly if fed with the correct data. This involves the creation of a forecasting model (often using a computer), which defines the mathematical relationships between numerical variables that relate to forecasts of the future. It is a set of formulae that process a number of input variables to arrive at a result. One of the simplest models possible is a single relationship between two variables. For example, if B=Benefits and C=Costs, then perhaps the simplest investment appraisal model is:



A good model is one that includes all the relevant variables (and excludes all non-relevant ones) and postulates the correct relationships between them.

Consider the forecasting model in Figure 2 which is a very simple cash flow statement containing projections of only one year[4]. It shows how the result of the model (the net cash flow) formula depends on the values of other variables, the values generated by formulae and the relationship between them. The model is made up of five variables and five formulae. Notice that there are formulae that process the result of other formulae as well as simple input variables (for instance formula F4). We will be using this simple appraisal model to illustrate the risk analysis process.

Forecasting Model

	$	Variables	Formulae
Sales price	12	V1	
Volume of sales	100	V2	
Cash inflow	1,200		F1 = V1 × V2
Materials	300		F2 = V2 × V4
Wages	400		F3 = V2 × V5
Expenses	200	V3	
Cash outflow	900		F4 = F2 + F3 + V3
Net Cash Flow	300		F5 = F1 – F4
Relevant assumptions			
Material cost per unit	3.00	V4	
Wages per unit	4.00	V5	

Figure 2. Forecasting model

Risk variables

The second stage entails the selection of the model's "risk variables". A risk variable is defined as one which is critical to the viability of the project in the sense that a small deviation from its projected value is both probable and potentially damaging to the project worth. In order to select risk variables we apply sensitivity and uncertainty analysis.

Sensitivity analysis is used in risk analysis to identify the most important variables in a project appraisal model. It measures the responsiveness of the project result vis-à-vis a change (usually a fixed percentage deviation) in the value of a given project variable.

The problem with sensitivity analysis as it is applied in practice is that there are no rules as to the extent to which a change in the value of a variable is tested for its impact on the projected result. For example, a 10% increase in labour costs may be very likely to occur while a 10% increase in sales revenue may be very unlikely. The sensitivity test applied uniformly on a number of project variables does not take into account how realistic or unrealistic the projected change in the value of a tested variable is.

In order for sensitivity analysis to yield meaningful results, the impact of uncertainty should be incorporated into the test. Uncertainty analysis is the attainment of some understanding of the type and magnitude of uncertainty encompassing the variables to be tested, and using it to select risk variables. For instance, it may be found that a small deviation in the purchase price of a given piece of machinery at year 0 is very significant to the project return. The likelihood, however, of even such a small deviation taking place may be extremely slim if the supplier is contractually obliged and bound by guarantees to supply at the agreed price. The risk associated with this variable is therefore insignificant even though the project result is very sensitive to it. Conversely, a project variable with high uncertainty should not be included in the probabilistic analysis unless its impact on the project result, within the expected margins of uncertainty, is significant.

The reason for including only the most crucial variables in a risk analysis application is twofold. First, the greater the number of probability distributions employed in a random simulation, the higher the likelihood of generating inconsistent scenarios because of the difficulty in setting and monitoring relationships for correlated variables (see Correlated variables below).

Second, the cost (in terms of expert time and money) needed to define accurate probability distributions and correlation conditions for many variables with a small possible impact on the result is likely to outweigh any benefit to be derived. Hence, rather than extending the breadth of analysis to cover a larger number of project variables, it is more productive to focus attention and available resources on adding more depth to the assumptions regarding the few most sensitive and uncertain variables in a project.

In our simple appraisal model (Figure 3) we have identified three risk variables. The price and volume of sales, because these are expected to be determined by the demand and supply conditions at the time the project will operate, and the cost of materials per unit, because the price of apples, the main material to be used, could vary substantially, again, depending on market conditions at the time of purchase. All three variables when tested within their respected margins of uncertainty, were found to affect the outcome of the project significantly.

Sensitivity and uncertainty analysis

	$	Risk variables
Sales price	**12**	← **V1**
Volume of sales	**100**	← **V2**
Cash inflow	**1,200**	
Materials	**300**	
Wages	**400**	
Expenses	**200**	
Cash outflow	**900**	
Net Cash Flow	**300**	
Relevant assumptions		
Material cost per unit	**3.00**	← **V4**
Wages per unit	**4.00**	

Figure 3. Sensitivity and uncertainty analysis

Probability distributions

Defining uncertainty

Although the future is by definition "uncertain", we can still anticipate the outcome of future events. We can very accurately predict, for example, the exact time at which daylight breaks at some part of the world for a particular day of the year. We can do this because we have gathered millions of observations of the event which confirm the accuracy of the prediction. On the other hand, it is very difficult for us to forecast with great accuracy the rate of general inflation next year or the occupancy rate to be attained by a new hotel project in the first year of its operation.

There are many factors that govern our ability to forecast accurately a future event. These relate to the complexity of the system determining the outcome of a variable and the sources of uncertainty it depends on. Our ability to narrow the margins of uncertainty of a forecast therefore depends on our understanding of the nature and level of uncertainty regarding the variable in question and the quality and quantity of information available at the time of the assessment. Often such information is embedded in the experience of the person making the prediction. It is only very rarely possible, or indeed cost effective, to conduct statistical analysis on a set of objective data for the purpose of estimating the future value of a variable used in the appraisal of a project[5].

In defining the uncertainty encompassing a given project variable one should widen the uncertainty margins to account for the lack of sufficient data or the inherent errors contained in the base data used in making the prediction. While it is almost impossible to forecast accurately the actual value that a variable may assume sometime in the future, it should be quite possible to include the true value within the limits of a sufficiently wide probability distribution. The analyst should make use of the available data and expert opinion to define a range of values and probabilities that are capable of capturing the outcome of the future event in question.

The preparation of a probability distribution for the selected project variable involves setting up a range of values and allocating probability weights to it. Although we refer to these two stages in turn, it must be emphasised that in practice the definition of a probability distribution is an iterative process. Range values are specified having in mind a particular probability profile, while the definition of a range of values for a risk variable often influences the decision regarding the allocation of probability.

Setting range limits

The level of variation possible for each identified risk variable is specified through the setting of limits (minimum and maximum values). Thus, a range of possible values for each risk variable is defined which sets boundaries around the value that a projected variable may assume.

The definition of value range limits for project variables may seem to be a difficult task to those applying risk analysis for the first time. It should, however, be no more difficult than the assignment of a single-value best estimate. In deterministic appraisal, the probable values that a project variable may take still have to be considered, before selecting one to use as an input in the appraisal.

Therefore, if a thoughtful assessment of the single-value estimate has taken place, most of the preparatory work for setting range limits for a probability distribution for that variable must have already been done. In practice, the problem faced in attempting to define probability distributions for risk analysis subsequently to the completion of a base case scenario is the realisation that not sufficient thought and research has gone into the single-value estimate in the first place.

When data are available, the definition of range limits for project variables is a simple process of processing the data to arrive at a probability distribution. For example, looking at historical observations of an event it is possible to organise the information in the form of a frequency distribution. This may be derived by grouping the number of occurrences of each outcome at consecutive value intervals. The probability distribution in such a case is the frequency distribution itself with frequencies expressed in relative rather than absolute terms (values ranging from 0 to 1 where the total sum must be equal to 1). This process is illustrated in Figure 4.

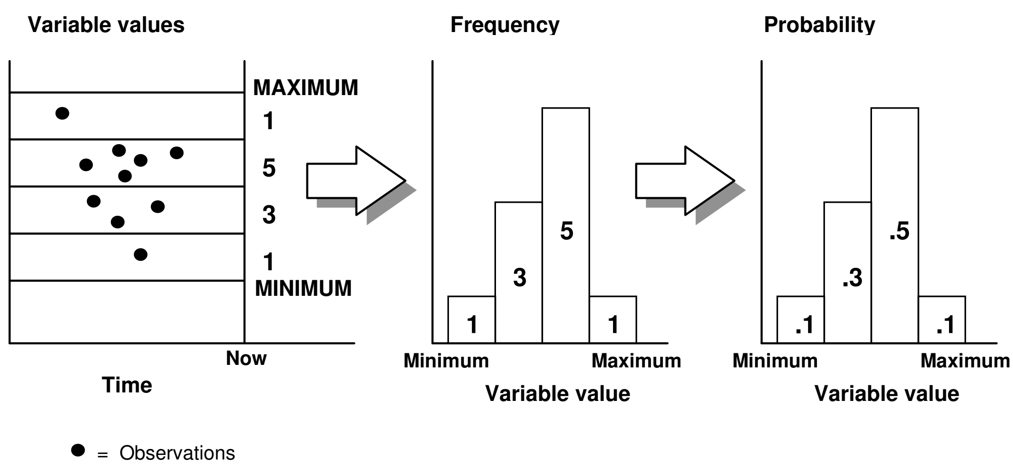


Figure 4. From a frequency to a probability distribution

It is seldom possible to have, or to afford the cost of purchasing, quantitative information which will enable the definition of range values and the allocation of probability weights for a risk variable on totally objective criteria. It is usually necessary to rely on judgement and subjective factors for determining the most likely values of a project appraisal variable. In such a situation the method suggested is to survey the opinion of experts (or in the absence of experts of people who can have some intelligible feel of the subject).

The analyst should attempt to gather responses to the question "what values are considered to be the highest and lowest possible for a given risk variable?". If the probability distribution to be attached to the set range of values (see allocating probability below) is one which concentrates probability towards the middle values of the range (for example the normal probability distribution), it may be better to opt for the widest range limits mentioned. If, on the other hand, the probability distribution to be used is one that allocates probability evenly across the range limits considered (for instance the uniform probability distribution) then the most likely or even one of the more narrow range limits considered may be more appropriate.

In the final analysis the definition of range limits rests on the good judgement of the analyst. He should be able to understand and justify the choices made. It should be apparent, however, that the decision on the definition of a range of values is not independent of the decision regarding the allocation of probability.

Allocating probability

Each value within the defined range limits has an equal chance of occurrence. Probability distributions are used to regulate the likelihood of selection of values within the defined ranges.

The need to employ probability distributions stems from the fact that an attempt is being made to forecast a future event, not because risk analysis is being applied. Conventional investment appraisal uses one particular type of probability distribution for all the project variables included in the appraisal model. It is called the deterministic probability distribution and is one that assigns all probability to a single value.

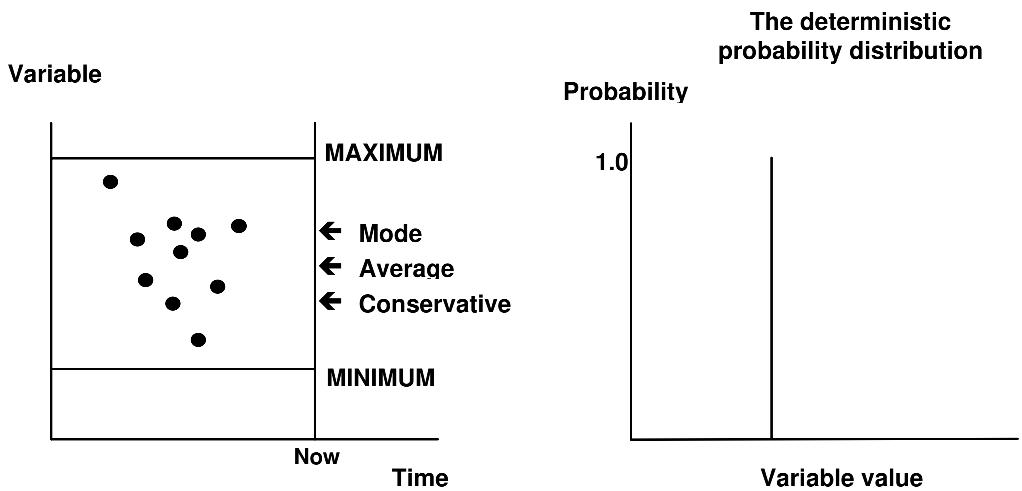


Figure 5. Forecasting the outcome of a future event: single-value estimate

In assessing the data available for a project variable, as illustrated in the example in Figure 5, the analyst is constrained to selecting only one out of the many outcomes possible, or to calculate a summary measure (be it the mode, the average, or just a conservative estimate). The assumption then has to be made that the selected value is certain to occur (assigning a probability of 1 to the chosen single-value best estimate). Since this probability distribution has only one outcome, the result of the appraisal model can be determined in one calculation (or one simulation run). Hence, conventional project evaluation is sometimes referred to as deterministic analysis.

In the application of risk analysis information contained within multi-value probability distributions is utilised. The fact that risk analysis uses multi-value instead of deterministic probability distributions for the risk variables to feed the appraisal model with the data is what distinguishes the simulation from the deterministic (or conventional) approach to project evaluation. Some of the probability distributions used in the application of risk analysis are illustrated in Figure 6.

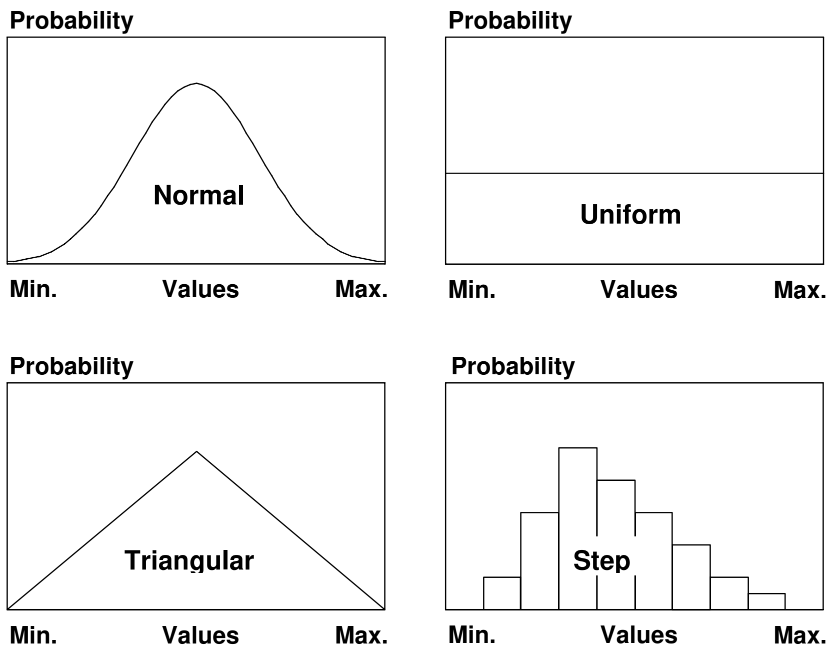


Figure 6. Multi-value probability distributions

The allocation of probability weights to values within the minimum and maximum range limits involves the selection of a suitable probability distribution profile or the specific attachment of probability weights to values (or intervals within the range).

Probability distributions are used to express quantitatively the beliefs and expectations of experts regarding the outcome of a particular future event. People who have this expertise are usually in a position to judge which one of these devices best expresses their knowledge about the subject. We can distinguish between two basic categories of probability distributions.

First, there are various types of symmetrical distributions. For example, the normal, uniform and triangular probability distributions allocate probability symmetrically across the defined range but with varying degrees of concentration towards the middle values. The variability profile of many project variables can usually be adequately described through the use of one such symmetrical distribution. Symmetrical distributions are more appropriate in situations for which the final outcome of the projected variable is likely to be determined by the interplay of equally important counteracting forces on both sides of the range limits defined; like for example the price of a product as determined in a competitive market environment (such as the sales price of apple pies in our simple example).

The second category of probability distributions are the step and skewed distributions. With a step distribution one can define range intervals giving each its own probability weight in a step-like manner (as illustrated in Figure 6). The step distribution is particularly useful if expert opinion is abundant. It is more suitable in situations where one sided rigidities exist in the system that determines the outcome of the projected variable. Such a situation may arise where an extreme value within the defined range is the most likely outcome[6].

<u>Correlated variables</u>

Identifying and attaching appropriate probability distributions to risk variables is fundamental in a risk analysis application. Having completed these two steps and with the aid of a reliable computer programme[7] it is technically possible to advance to the simulation stage in which the computer builds up a number of project scenarios based on random input values generated from the specified probability distributions (see Simulation runs below). However, proceeding straight to a simulation would be correct only if no significant correlations exist among any of the selected risk variables.

The correlation problem

Two or more variables are said to be correlated if they tend to vary together in a systematic manner. It is not uncommon to have such relationships in a set of risk variables. For example, the level of operating costs would, to a large extent, drive sales price or the price of a product would usually be expected to have an inverse effect on the volume of sales. The precise nature of such relationships is often unknown and can not be specified with a great deal of accuracy as it is simply a conjecture of what may happen in the future.

The existence of correlated variables among the designated risk variables can, however, distort the results of risk analysis. The reason for this is that the selection of input values from the assigned probability distributions for each variable is purely random. It is therefore possible that the resultant inputs generated for some scenarios violate a systematic relationship that may exist between two or more variables. To give an example, suppose that market price and quantity are both included as risk variables in a risk analysis application. It is reasonable to expect some negative covariance between the two variables (that is, when the price is high quantity is more likely to assume a low value and vice versa). Without restricting the random generation of values from the corresponding probability distributions defined for the two variables, it is almost sure that some of the scenarios generated would not conform to this expectation of the analyst which would result in unrealistic scenarios where price and quantity are both high or both low.

The existence of a number of inconsistent scenarios in a sample of simulation runs means that the results of risk analysis will be to some extent biased or off target. Before proceeding to the simulation runs stage, it is therefore imperative to consider whether such relationships exist among the defined risk variables and, where necessary, to provide such constraints to the model that the possibility of generating scenarios that violate these correlations is diminished. In effect, setting correlation conditions restricts the random selection of values for correlated variables so that it is confined within the direction and limits of their expected dependency characteristics.

Practical solution

One way of dealing with the correlation problem in a risk analysis application is to use the correlation coefficient as an indication, or proxy, of the relationship between two risk variables. The analyst therefore indicates the direction of the projected relationship and an estimate (often a reasonable guess) of the strength of association between the two projected correlated variables. The purpose of the exercise is to contain the model from generating grossly inconsistent scenarios rather than attaining high statistical accuracy. It is therefore sufficient to assume that the relationship is linear and that it is expressed in the formula:

$$Y = a + bX + e$$

where:

Y	= dependent variable,
X	= independent variable
a (intercept)	= the minimum Y value (if relationship is positive) or,
	= the maximum Y value (if relationship is negative),
b (slope)	$= \dfrac{(\text{maximum } Y \text{ value } - \text{ minimum } Y \text{ value})}{(\text{maximum } X \text{ value } - \text{ minimum } X \text{ value})}$,
e (error factor)	= independently distributed normal errors.

It is important to realise that the use of the correlation coefficient suggested here is simply that of a device by which the analyst can express a suspected relationship between two risk variables. The task of the computer programme is to try to adhere, as much as possible, to that condition[8]. The object of the correlation analysis is to control the values of the dependent variable so that a consistency is maintained with their counter values of the independent variable.

The regression equation forms part of the assumptions that regulate this relationship during a simulation process. As shown in the formula explanation above, the intercept and the slope, the two parameters of a linear regression, are implicitly defined at the time the minimum and maximum possible values for the two correlated variables are specified. Given these assumptions the analyst only has to define the polarity of the relationship (whether it is positive or negative) and the correlation coefficient (r) which is a value from 0 to 1[9].

In our simple example one negative relationship is imposed on the model. This aims at containing the possibility of quantity sold responding positively (in the same direction) to a change in price. Price (V1) is the independent variable and Volume of sales (V2) is the dependent variable. The two variables are assumed to be negatively correlated by a coefficient (r) of -0.8. The completed simulation model including the setting for correlations is illustrated in Figure 7.

Simulation model

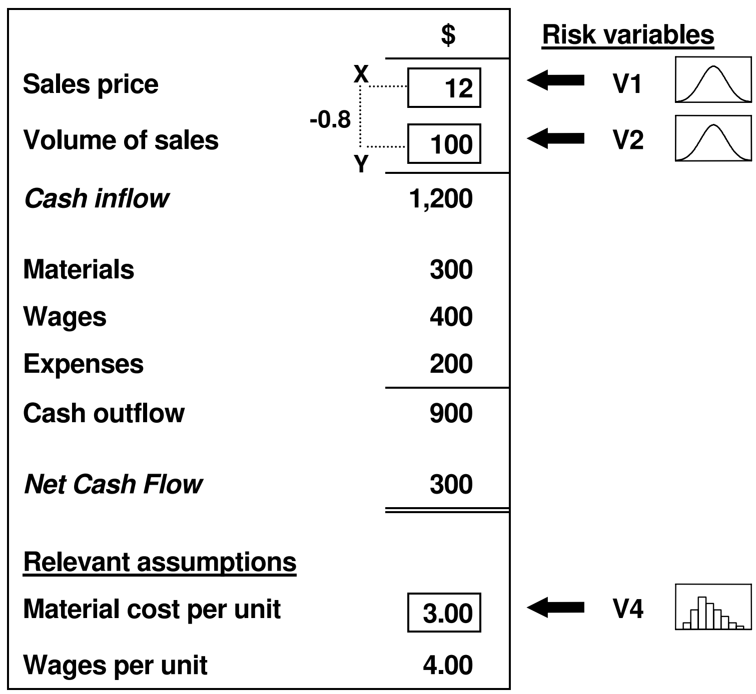


Figure 7. Simulation model

The scatter diagram in Figure 8 plots the sets of values generated during a simulation (200 runs) of our simple for two correlated variables (Sales price and Volume of sales). The simulation model included a condition for negative correlation and a correlation coefficient of -0.8. The range limits of values possible for the independent variable (sales price) were set at 8 to 16 and for the dependent variable (volume of sales) at 70 to 130[10]. Thus, the intercept and the slope of the regression line are:

$$a \text{ (intercept)} = 130$$

$$b \text{ (slope)} = \frac{(130 - 70)}{(16 - 8)} = -7.5$$

where:

a is the maximum *Y* value because the relationship is negative

b is expressed as a negative number because the relationship between the two variables is negative.

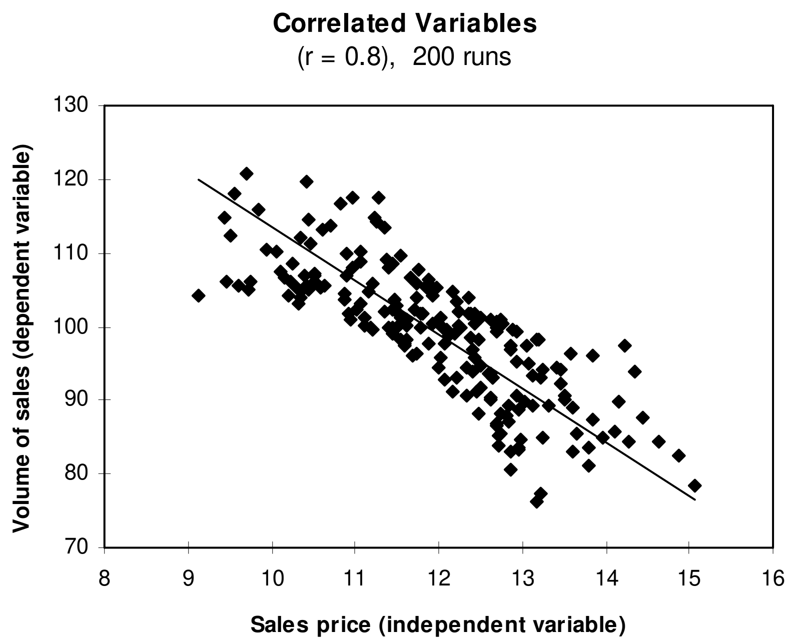


Figure 8. Scatter diagram

Simulation runs

The simulation runs stage is the part of the risk analysis process in which the computer takes over. Once all the assumptions, including correlation conditions, have been set it only remains to process the model repeatedly (each re-calculation is one run) until enough results are gathered to make up a representative sample of the near infinite number of combinations possible. A sample size of between 200 and 500 simulation runs should be sufficient in achieving this.

During a simulation the values of the "risk variables" are selected randomly within the specified ranges and in accordance with the set probability distributions and correlation conditions. The results of the model (that is the net present value of the project, the internal rate of return or in our simple example the "Net Cash Flow") are thus computed and stored following each run. This is illustrated in Figure 9 in which simulation runs are represented as successive frames of the model. Except by coincidence, each run generates a different result because the input values for the risk variables are selected randomly from their assigned probability distributions. The result of each run is calculated and stored away for statistical analysis (the final stage of risk analysis).

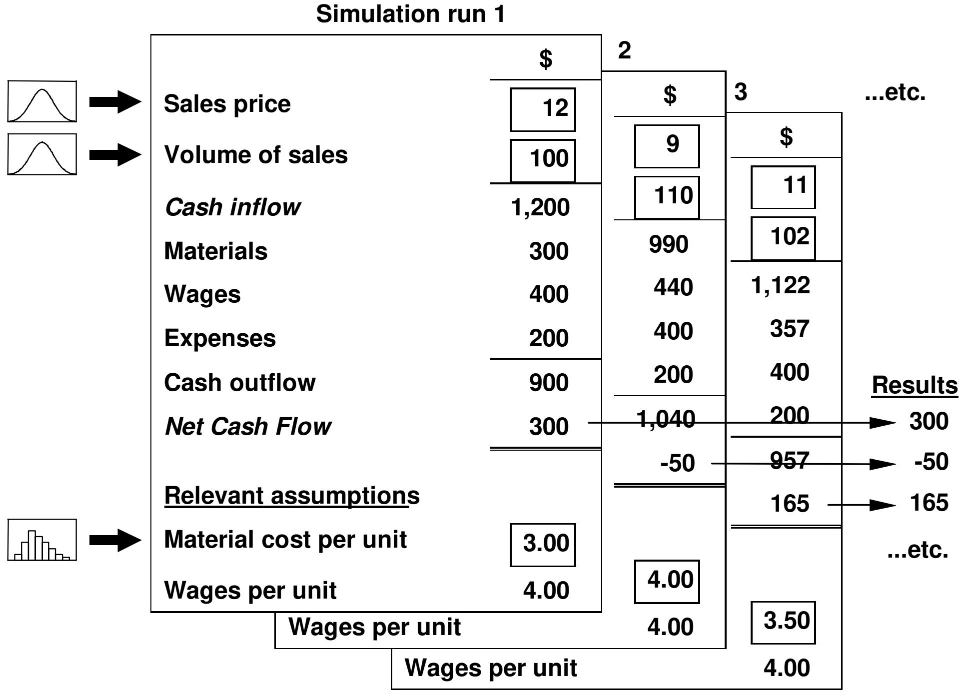


Figure 9. Simulation run

Analysis of results

The final stage in the risk analysis process is the analysis and interpretation of the results collected during the simulation runs stage. Every run represents a probability of occurrence equal to:

$$p = \frac{1}{n}$$

where:

p	=	probability weight for a single run
n	=	sample size

Hence, the probability of the project result being below a certain value is simply the number of results having a lower value times the probability weight of one run[11]. By sorting the data in ascending order it becomes possible to plot the cumulative probability distribution of all possible results. Through this, one can observe the degree of probability that may be expected for the result of the project being above or below any given value. Project risk is thus portrayed in the position and shape of the cumulative probability distribution of project returns.

Figure 10 plots the results of our simple example following a simulation process involving 200 runs. The probability of making a loss from this venture is only about 10%.

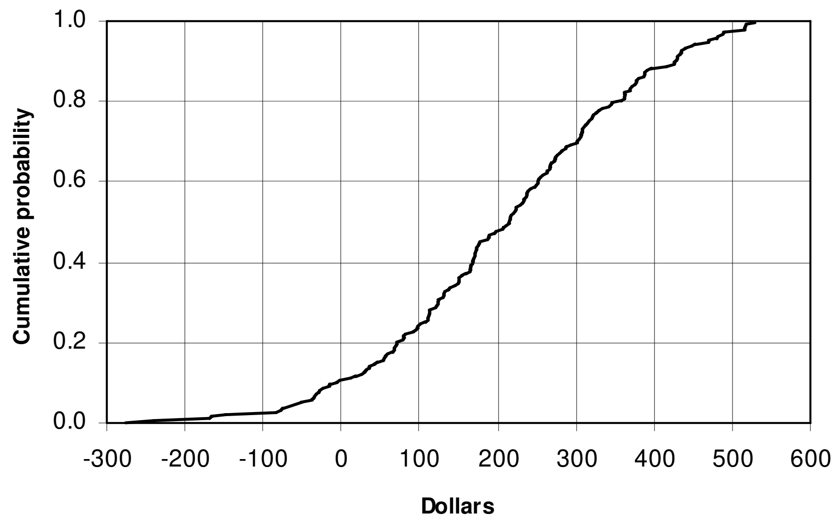


Figure 10. Distribution of results (net cash flow)

It is sometimes useful to compare the risk profiles of an investment from various perspectives. In Figure 11 the results of risk analysis, showing the cumulative probability distribution of net present values for the banker, owner and economy view of a certain project, are compared. The probability of having a net present value below zero for the economy's view case is nearly 0.4, while for that of the owner is less than 0.2. From the banker's view (or total investment perspective) the project seems quite safe as there seems to be about 95% probability that it will generate a positive NPV[12].

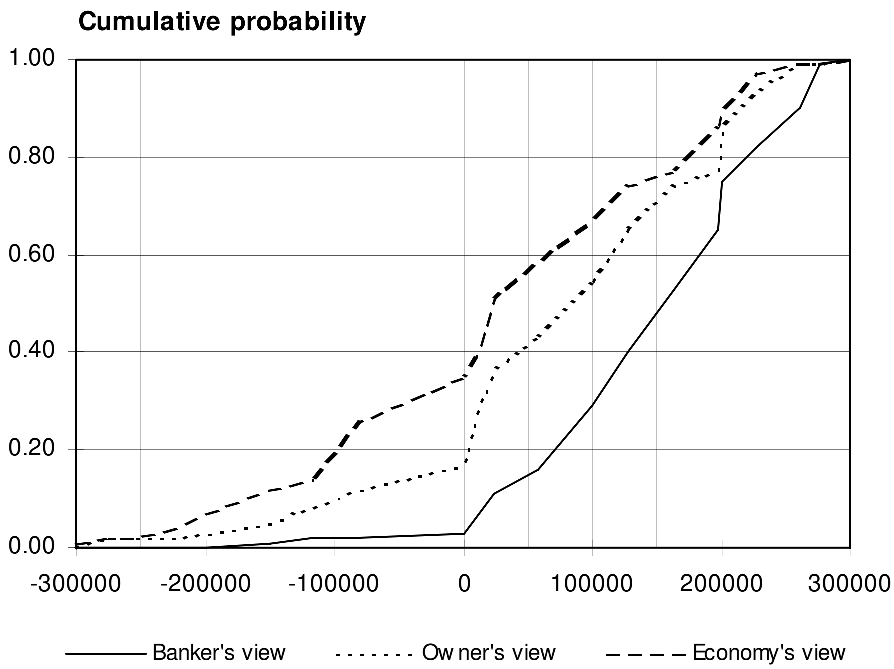


Figure 11. Net present value distribution (from different project perspectives)

III. INTERPRETING THE RESULTS OF RISK ANALYSIS

The raw product of a risk analysis is a series of results which are organised and presented in the form of a probability distribution of the possible outcomes of the project. This by itself is a very useful picture of the risk/return profile of the project which can enhance the investment decision. However, the results of risk analysis raise some interpretation issues as regards the use of the net present value criterion. They also make possible various other measures of risk which further extend the usefulness of risk analysis in investment appraisal.

Investment decision criteria

The basic decision rule for a project appraisal using certainty equivalent values as inputs and discounted at a rate adjusted for risk is simply to accept or reject the project depending on whether its NPV is positive or negative, respectively. Similarly, when choosing among alternative (mutually exclusive) projects, the decision rule is to select the one with the highest NPV, provided that it is positive. Investment criteria for a distribution of NPVs generated through the application of risk analysis are not always as clear-cut as this. We will look at two basic issues which have to do with risk analysis when used in conjunction with the NPV criterion; the choice of discount rate and the use of decision criteria.

The discount rate and the risk premium

In deterministic appraisal project risk is usually accounted for by including a risk premium in the discount rate which is used to appraise the project. The magnitude of this risk premium is basically the difference between the return usually required by investors undertaking similar projects and the risk free interest rate. The derivation of the risk premium, particularly in countries with under-developed capital markets, is subjective and, often, rather arbitrary. Brealy and Myers (R. Brealy and S. Myers 1991, page 228) have argued that the most appropriate discount rate to use in a project appraisal subjected to risk analysis is the risk-free interest rate because any other discount rate would “pre-judge [the level of] risk” in a project. Another school of thought maintains that the discount rate should include a premium for systematic (or market) risk but not for unsystematic (or project) risk.

It is not the purpose of this paper to analyse and discuss the various schools of thought on the subject. Nevertheless, the author believes that the most appropriate discount rate is the one used in the deterministic appraisal. With the application of risk analysis and the careful consideration of the risk component of the main variables of a project and their relationship, it may be possible to establish a sounder basis on which to evaluate project risk. However, being able to appreciate the level and pattern of risk involved in a project does not, by itself, mean that we can also eliminate or even reduce project risk[13]. Nor does it mean that the project looks any less (or more) risky to the outside world. The risk-free rate would therefore be most inappropriate because it would set a standard for the project which is below normal. The level of return, or hurdle, that the project is required to overcome in order to be considered worthwhile does not change simply because, as a result of risk analysis or any other tool, the investor gains a better sense of what constitutes project risk. After all, one does not change the discount rate when sensitivity or scenario analysis is applied. Risk analysis using the Monte Carlo method is fundamentally no different from scenario analysis. The only difference is that (based on the user's assumptions) the computer, rather than the analyst, builds the scenarios generated in the analysis.

Decision criteria

By using a discount rate that allows for risk, investment decision criteria normally used in deterministic analysis maintain their validity and comparability. The expected value of the probability distribution of NPVs (see Measures of risk below) generated using the same discount rate as the one used in conventional appraisal is a summary indicator of the project worth which is directly comparable (and should indeed be similar to) the NPV figure arrived at in the deterministic appraisal of the same project. Through the expected value of the NPV distribution therefore the decision criteria of investment appraisal still maintain their applicability.

However, because risk analysis presents the decision maker with an additional aspect of the project - the risk/return profile - the investment decision may be revised accordingly. The final decision is therefore subjective and rests to a large extent on the investor's attitudes towards risk.

The general rule is to choose the project with the probability distribution of return that best suits one's own personal predisposition towards risk. The “risk-lover” will most likely choose to invest in projects with relatively high return, showing less concern in the risk involved. The “risk-averter” will most likely choose to invest in projects with relatively modest but rather safe returns.

However, assuming “rational” behaviour on behalf of the decision maker the following cases may be examined. Cases 1, 2 and 3 involve the decision criterion to invest in a single project. Cases 4 and 5 relate to investment decision criteria for choosing between alternative (mutually exclusive) projects.

In every case examined both the cumulative and non-cumulative probability distributions are illustrated for comparison purposes. The cumulative probability distribution of the project returns is more useful for decisions involving alternative projects while the non-cumulative distribution is better for indicating the mode of the distribution and for understanding concepts related to expected value.

Case 1: The minimum point of the probability distribution of project return is higher than zero NPV (Figure 12).

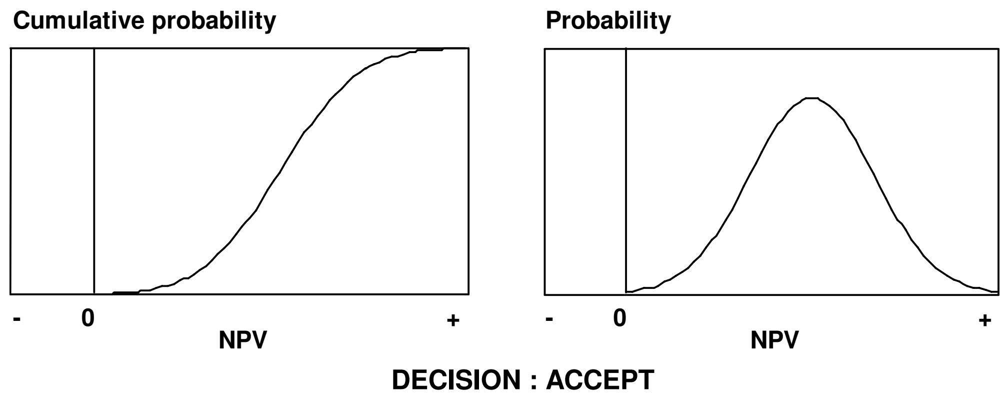


Figure 12. Case 1: Probability of negative NPV=0

Since the project shows a positive NPV even under the "worst" of cases (i.e. no probability for negative return) then clearly the project should be accepted.

Case 2: The maximum point of the probability distribution of project return is lower than zero NPV (Figure 13).

Since the project shows a negative NPV even under the "best" of cases (no probability for positive return) then clearly the project should be rejected.

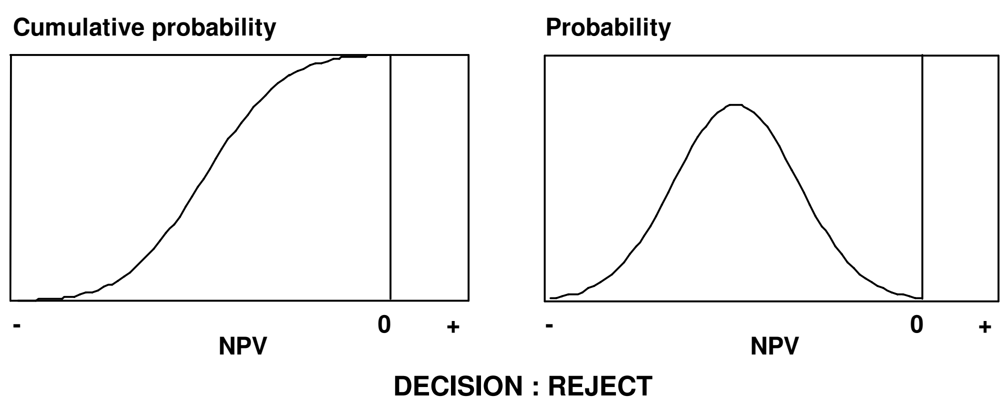
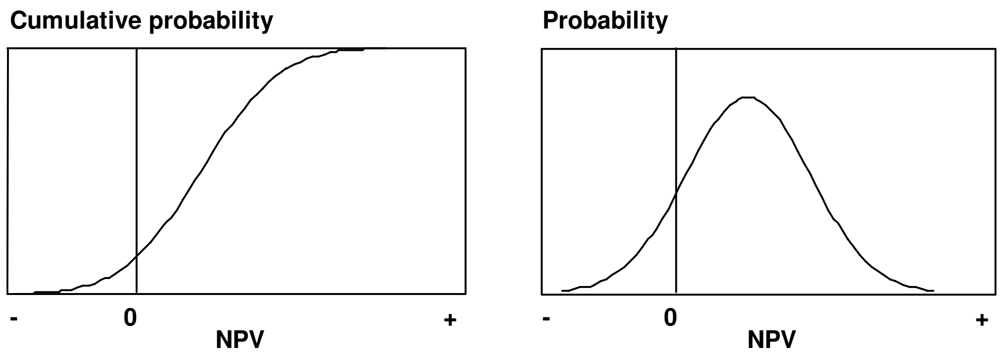


Figure 13. Case 2: Probability of positive NPV=0

Case 3: The maximum point of the probability distribution of project return is higher and the minimum point is lower than zero Net Present Value (the curve intersects the point of zero NPV - Figure 14).

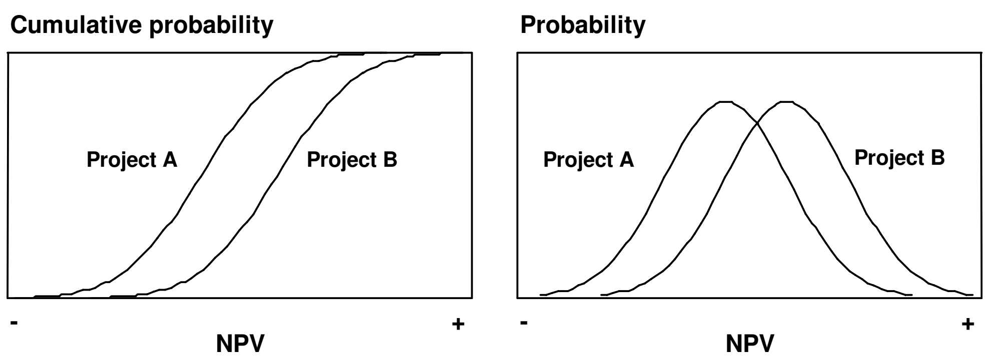
The project shows some probability of being positive as well as some probability of being negative; therefore the decision rests on the risk predisposition of the investor.



DECISION : INDETERMINATE

Figure 14. Case 3: Probability of zero NPV greater than 0 and less than 1

Case 4: Non-intersecting cumulative probability distributions of project return for mutually exclusive projects (Figure 15).



DECISION : CHOOSE PROJECT B

Figure 15. Case 4: Mutually exclusive projects
(given the same probability, one project always shows a higher return)

Given the same probability, the return of project B is always higher than the return of project A. Alternatively, given one particular return, the probability that it will be achieved or exceeded is always higher by project B than it is by project A. Therefore, we can deduce the first rule for choosing between alternative projects with risk analysis as:

Rule 1: If the cumulative probability distributions of the return of two mutually exclusive projects do not intersect at any point then always choose the project whose probability distribution curve is farther to the right.

Case 5: Intersecting cumulative probability distributions of project return for mutually exclusive projects (Figure 16).

Risk "lovers" will be attracted by the possibility of higher return and therefore will be inclined to choose project A. Risk "averters" will be attracted by the possibility of low loss and will therefore be inclined to choose project B.

Rule 2: If the cumulative probability distributions of the return of two mutually exclusive projects intersect at any point then the decision rests on the risk predisposition of the investor.

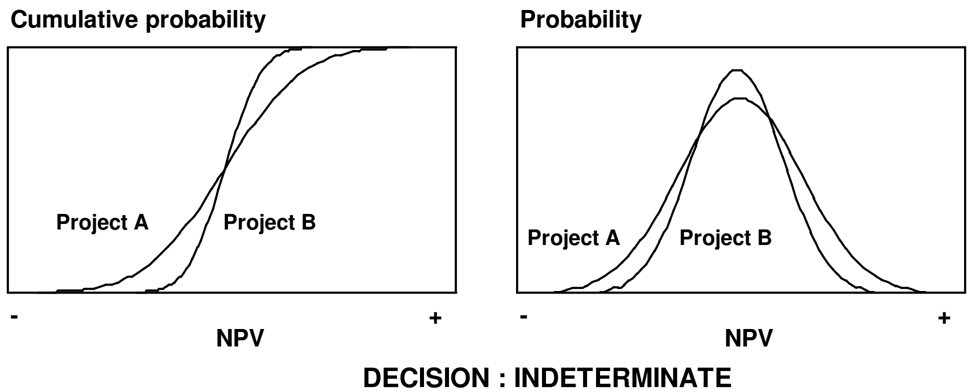


Figure 16. Case 5: Mutually exclusive projects (high return vs. low loss)

(Note: With non-cumulative probability distributions a true intersection is harder to detect because probability is represented spatially by the total area under each curve.)

Measures of risk

The results of a risk analysis application lend themselves to further analysis and interpretation through the use of a series of measures which are based on the concept of expected value.

Expected value

The expected value statistic summarises the information contained within a probability distribution. It is a weighted average of the values of all the probable outcomes. The weights are the probabilities attached to each possible outcome. In risk analysis as applied in project appraisal the expected value is the sum of the products of the generated project returns and their respective probabilities[14]. This is illustrated in the simple example of a project with four possible returns and probabilities:

Return		Probability		Expected Value
-10	x	0.2	=	-2.0
-5	x	0.3	=	-1.5
10	x	0.4	=	4.0
15	x	0.1	=	1.5
Total				2.0

The expected value of the above project is 2.0. This is derived by multiplying each return by its respective probability and summing the results. The total of all the negative returns times their respective probability is the expected loss from the project. In the above example this amounts to -3.5 (which is the sum of the "probability weighted" negative returns). The total of all the positive returns times their respective probability is the expected gain from the project. In the above example this amounts to 5.5 (which is the sum of the "probability weighted" positive returns). The expected value is, of course, the total of expected gain and expected loss.

The expected value statistic aggregates into a single number all the information that is depicted in a multi-valued probability distribution. Being a summary measure is therefore only a gross indicator of a project's worth.

Measures of risk that employ expected value concepts are the "cost of uncertainty", the "expected loss ratio" and the "coefficient of variation"; it is also used to analyse risk under conditions of limited liability.

Cost of uncertainty

The cost of uncertainty, or the value of information as it is sometimes called, is a useful concept that helps determine the maximum amount of money one should be prepared to pay to obtain information in order to reduce project uncertainty. This may be defined as the expected value of the possible gains foregone following a decision to reject a project, or the expected value of the losses that may be incurred following a decision to accept a project.

The expected gain forgone from rejecting a project is illustrated in the right-hand diagram of Figure 17 by the sum of the possible positive NPVs weighted by their respective probabilities. Similarly, the expected loss from accepting a project, indicated in the left-hand diagram, is the sum of all the possible negative NPVs weighted by their respective probabilities.

By being able to estimate the expected benefit that is likely to result from the purchase of more information, one can decide on whether it is worthwhile to postpone a decision to accept or reject a project and seek further information or whether to make the decision immediately. As a general rule one should postpone the investment decision if the possible reduction in the cost of uncertainty is greater than the cost of securing more information (including foregone profits if the project is delayed).

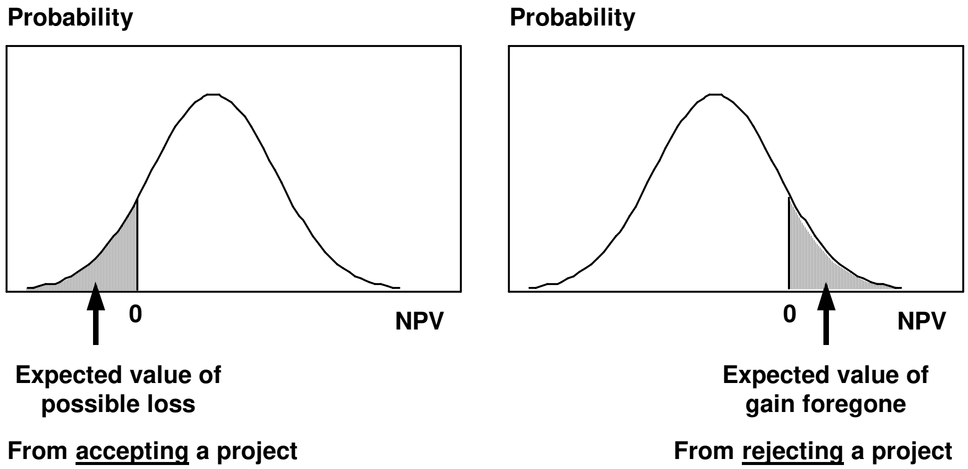


Figure 17. Cost of uncertainty

Expected loss ratio

The expected loss ratio (*el*) is a measure indicating the magnitude of expected loss relative to the project's overall expected NPV. This is expressed in the formula absolute value of expected loss divided by the sum of expected gain and absolute value of expected loss:

$$el = \frac{|\text{Expected Loss}|}{\text{Expected Gain} + |\text{Expected Loss}|}$$

It can vary from 0, meaning no expected loss, to 1, which means no expected gain. Diagrammatically, this is the probability weighted return derived from the shaded area to the left of zero NPV divided by the probability weighted return derived from the total distribution whereby the negative returns are taken as positive (see Figure 18).

A project with a probability distribution of returns totally above the zero NPV mark would compute an *el* value of 0, meaning that the project is completely unexposed to risk. On the other hand, a project with a probability distribution of returns completely below the zero NPV mark would result in an *el* of 1, meaning that the project is totally exposed to risk.

The ratio does not therefore distinguish between levels of risk for totally positive or totally negative distributions. However, within these two extreme boundaries the *el* ratio could be a useful measure for summarising the level of risk to which a project may be subjected. In the above example, the expected loss ratio is 3.5 / (5.5 + 3.5) or about 0.39.

Other methods for determining the risk exposure of a project's probability distribution of returns are possible. Such measures would vary depending on how one defines risk and on the emphasis one places on its major components. The *el* ratio is offered as an example of how one can use the results of risk analysis to assess and summarise the risk inherent in a project.

The *el* ratio defines risk to be a factor of both the shape and the position of the probability distribution of returns in relation to the “cut-off” mark of zero NPV.

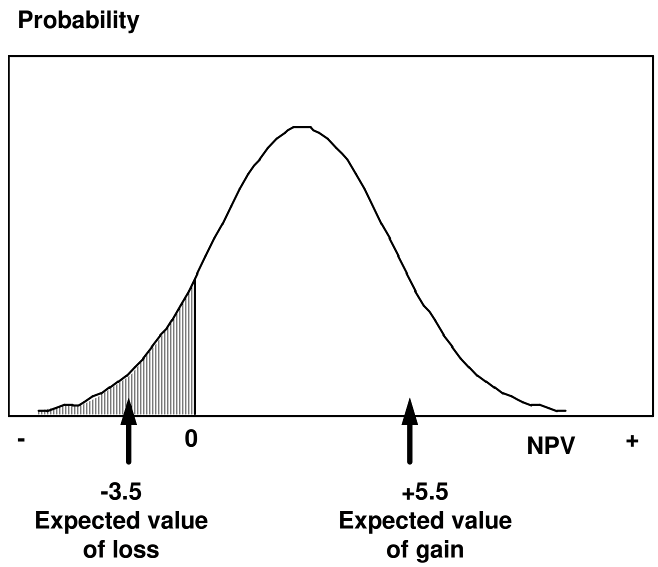


Figure 18. Expected loss ratio

Coefficient of variation

The coefficient of variation is also a useful summary measure of project risk. It is the standard deviation of the projected returns divided by the expected value. Assuming a positive expected value, the lower the coefficient of variation the less the project risk.

Conditions of limited liability

The extent of maximum loss possible under conditions of limited liability is usually defined by the legal agreements entered into by the various parties involved in a project. Looking at the investment in terms of present value the equity holders cannot lose more than the present value of their equity capital, the debt holders can only lose the present value of their loan capital, the creditors the present value of the extended credit and so on.

Consider the probability distribution of the return of a project as depicted in Figure 19.

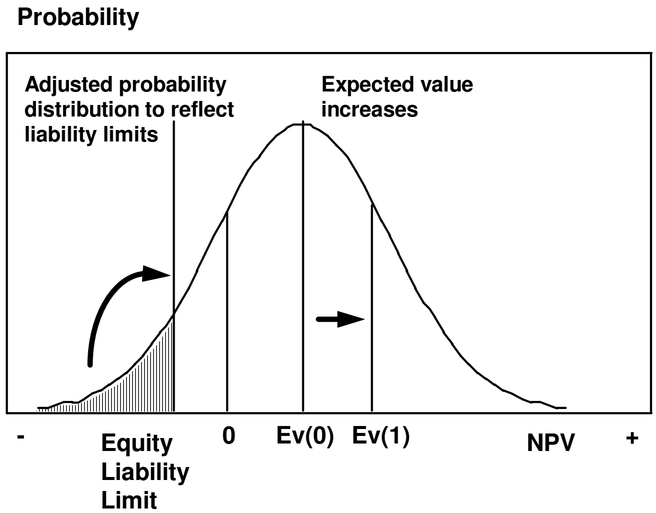


Figure 19. Risk under conditions of limited liability

From the equity holders' point of view the tail of the distribution, which is beyond their maximum liability limit as defined by the present value of equity capital invested in the project, is not relevant. The probability of the project for generating a return lower than their maximum liability limit is therefore reassigned to the point of equity liability limit as shown in the diagram. This adjustment also has the effect of raising the expected value of the project from the point of view of the equity holders, from Ev(0) to Ev(1)[15].

IV. CONCLUSION

Risk analysis is a useful tool extending the depth of project appraisal and enhancing the investment decision. Having practised the technique for a number of years the author can report the following specific advantages for risk analysis:

1. It enhances decision making on marginal projects. A project whose single-value NPV is small may still be accepted following risk analysis on the grounds that its overall chances for yielding a satisfactory return are greater than is the probability of making an unacceptable loss. Likewise, a marginally positive project could be rejected on the basis of being excessively risky, or one with a lower NPV may be preferred to another with a higher NPV because of a better risk/return profile.

2. It screens new project ideas and aids the identification of investment opportunities. Very often a new project concept is formulated that needs to be developed into a business opportunity. Before any real expenses are incurred to gather information for a full feasibility study it is possible to apply risk analysis widening the margins of uncertainty for the key project variables to reflect the lack of data. A substantial investment of human and financial resources is not incurred until the potential investors are satisfied that the preliminary risk/return profile of the project seems to be acceptable.

3. It highlights project areas that need further investigation and guides the collection of information. Risk analysis can contain the costs of investigation and fieldwork aiming at improving the accuracy of a forecast relating to particular project variables. If the cost for obtaining such information is greater than the expected benefit likely to result from the purchase of the information (see the Cost of uncertainty above), then the expense is not justified.

4. It aids the reformulation of projects to suit the attitudes and requirements of the investor. A project may be redesigned to take account for the particular risk predispositions of the investor.

5. It induces the careful re-examination of the single-value estimates in the deterministic appraisal. The difficulty in specifying range limits and probability distributions for risk analysis often resides in the fact that the projected values are not adequately researched. The need to define and support explicit assumptions in the application of risk analysis therefore forces the analyst to also critically review and revise the base-case scenario.

6. It helps reduce project evaluation bias through eliminating the need to resort to conservative estimates as a means of reflecting the analyst's risk expectations and predispositions.

7. It facilitates the thorough use of experts who usually prefer to express their expertise in terms of a probability distribution rather than having to compress and confine their opinion in a single value.

8. It bridges the communication gap between the analyst and the decision maker. The execution of risk analysis in a project appraisal involves the collection of information which to a large part reflects the acquired knowledge and expertise of top executives in an organisation. By getting the people who have the responsibility of accepting or rejecting a project to agree on the ranges and probability distributions used in risk analysis the analyst finds an invaluable communication channel through which the major issues are identified and resolved. The decision maker in turn welcomes his involvement in the risk analysis process as he recognises it to be an important management decision role which also improves his/her overall understanding of the appraisal method.

9. It supplies a framework for evaluating project result estimates. Unlike the prediction of deterministic appraisal which is almost always refuted by the actual project result, the probabilistic approach is a methodology which facilitates empirical testing.

10. It provides the necessary information base to facilitate a more efficient allocation and management of risk among various parties involved in a project. Once the various sources of risk have been assessed, project risk may be contractually allocated to those parties who are best able to bear it and/or manage it. Moreover, it enables the testing of possible contractual arrangements for the sale of the products or the purchase of project inputs between various parties until a satisfactory formulation of the project is achieved.

11. It makes possible the identification and measurement of explicit liquidity and repayment problems in terms of time and probability that these may occur during the life of the project. This becomes possible if the net-cash flow figures or other indicators of solvency included in a project appraisal model (for instance the debt service coverage ratio for each year) are monitored during the simulation process.

Finally two words of caution:

- Overlooking significant inter-relationships among the projected variables can distort the results of risk analysis and lead to misleading conclusions. The analyst should take due care to identify the major correlated variables and to adequately provide for the impact of such correlations in the simulation.

- Risk analysis amplifies the predictive ability of sound models of reality. The accuracy of its predictions therefore can only be as good as the predictive capacity of the model employed.

Notes

[1] Even if one uses the most likely value of every project variable it does not mean that the derived result will also be the most likely result (See Reutlinger, 1970, pages 25-26).

[2] A value below the most likely estimate for a variable whose impact on the cash flow of the project is positive (such as quantity sold) or a value above the most likely estimate for a variable whose impact on the net cash flow of the project is negative (such as payroll cost).

[3] Changing the value of only one project variable may create an unrealistic scenario because the variable may be correlated with other input variables.

[4] A one year cash-flow, rather than a fully projected cash-flow statement, is used so as to demonstrate as simply as possible the stages of a risk analysis application. It is assumed that the project is a once-off venture where there is no upfront capital investment or residual values (for instance producing and selling apple pies to sell in a major one time event such as the Olympic Games).

[5] Where this is possible the accuracy of the prediction will be higher under the following conditions:

- the greater the similarity of the data used to the variable to be forecast
- the bigger the sample of data
- the lower the variation of values in the data used
- the shorter the period of extrapolation from the base data.

[6] For example, the projected inflation rate of a country for a particular year may be only 2% with very low probability of dropping further; yet it is considered quite probable for the inflation rate to increase up to 7%, if popular economic measures which can cause inflationary pressures on the economy materialise.

[7] 'RiskMaster', later renamed 'RiskEase", by Master Solutions is one such software package. It is an add-in software that works with Microsoft Excel to provide risk analysis capability. The programme was originally developed by the author for the Harvard University Program in Investment Appraisal and Management (PIAM) and applies the concepts presented in this paper.

[8] Correlation analysis is usually employed to analyse a set of data to facilitate the prediction of the dependent variable from actual (or hypothetical) values of the independent variable where the regression equation and the correlation coefficient are the outputs of such analysis. In the risk analysis application described here these are merely the inputs, while the output is the generated data for the dependent variable during the simulation process.

[9] The described application of correlations to a Monte Carlo simulation refers to the method that is employed by the author in 'RiskMaster' and 'RiskEase' in order to deal with the correlation problem.

[10] It is assumed that the likelihood of occurrence of values within the defined range limits for the two variables is described by a normal probability distribution.

[11] For example, if 400 runs were generated then the probability weight is 1/400=0.0025. If 100 runs have a NPV of less than 0 then the probability for negative NPV is 100 x 0.0025=25%.

[12] An investment project can be evaluated from different view-points. In a financial appraisal the main difference between the Banker and Owner view is that the latter includes the financial flows from loan financing (loans are taken as cash inflow and payments of interest and principal as cash outflow). From the economy's perspective one uses economic rather than financial prices adjusting for taxes and subsidies and excludes loans because they do not represent real resources. For a clear exposition of investment appraisal from different perspectives see Jenkins and Harberger (1991, pages.3:10-3:20).

[13] It is of course possible to reduce risk through project re-formulation and/or to reallocate it through the design of special contracts between various parties who may be better able to absorb or deal with certain types of risk. Indeed, this is one of the most promising areas in which a risk analysis tool can be of tremendous value. See, for example, Lessard (1988) or Glenday (1989).

[14] If the simulation process generated only unique results then the probability weights would be the same for all possible outcomes (1 divided by sample size - see Analysis of results above).

[15] This type of analysis may be useful in underlining the relative risk position of particular parties involved in a project.

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