Who gains and who loses from congestion pricing in a monocentric city with a bottleneck?

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Who gains and who loses from congestion pricing in a monocentric city with a bottleneck?*

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Abstract

This study develops a model in which heterogeneous commuters choose their residential locations and departure times from home in a closed monocentric city with a bottleneck located at the entrance to the central business district (CBD). We show that commuters sort themselves both temporally and spatially according to their income, value of time, and flexibility at the equilibria with and without an optimal congestion pricing. These two equilibria exhibit fundamentally different properties, indicating that congestion pricing alters the urban spatial structure. We then consider two cases wherein rich commuters are either flexible or inflexible and demonstrate that (a) rich commuters reside farther from the CBD in the former case and closer to the CBD in the latter case; (b) congestion pricing makes cities denser and more compact in the former, whereas it causes cities to become less dense and to expand spatially in the latter; and (c) in both cases, pricing helps rich commuters but hurts poor commuters. We further reveal that although expanding the capacity of the bottleneck generates a Pareto improvement when commuters do not relocate, it can lead to an unbalanced distribution of benefits among commuters: commuters residing closer to the CBD gain, while those residing farther from the CBD lose. This suggests that expanding capacity financed by the revenue from congestion pricing could be regressive in a city where rich commuters are inflexible.

Keywords: peak-load pricing; residential location; distributional effects;

1 Introduction

Peak-period traffic congestion has long been a serious problem and an important policy issue for many cities. Congestion pricing is the widely known tool to alleviate traffic congestion, but it has hardly been implemented in practice mainly because of a concern

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about its distributional effects. Pricing might make some commuters worse off since it reduces travel time but increases monetary costs.

The literature on traffic congestion has been devoted to examining the distributional impacts of peak-load pricing and to proposing a measure that generates a Pareto improvement (e.g., Vickrey, 1973; Cohen, 1987; Arnott et al., 1994; van den Berg and Verhoef, 2011a,b, 2014; Liu et al., 2015; Hall, 2018). The standard approach is to analyze the bottleneck model with heterogeneous commuters, implying that commuters are assumed *not to relocate*. Since it is well recognized that alleviating traffic congestion changes the spatial distribution of residents in the long-run, we can say that the literature focuses on the short-run effects of congestion pricing.

Traditional models of urban spatial structure, which are based on the monocentric city model (Alonso, 1964; Mills, 1967; Muth, 1969), have succeeded in predicting the empirically observed patterns of residential location based on the trade-off between land rent and commuting costs. These traditional models, however, mostly describe traffic congestion by using *static* congestion models, in which congestion at a location depends only on the total traffic demand (i.e., the total number of commuters passing a location), regardless of the time-of-use pattern (e.g., Kanemoto, 1980; Wheaton, 1998; Anas et al., 1998). This indicates that these models do not capture *peak-period* traffic congestion that takes the form of queuing at a bottleneck.

Several studies have incorporated a spatial dimension into the bottleneck model by embedding the dynamic bottleneck congestion into a simple monocentric city model (Arnott, 1998; Gubins and Verhoef, 2014; Takayama and Kuwahara, 2017; Fosgerau et al., 2018; Fosgerau and Kim, 2019). Their models, however, consider homogeneous commuters, thereby being inapplicable to examining the long-run distributional impacts of peak-load pricing. The only exception is Takayama and Kuwahara (2017) who incorporate heterogeneity in commuters' income (value of time) and flexibility. They demonstrate that congestion pricing helps rich commuters but hurts poor commuters. This result, however, essentially depends on the assumption of quasi-linear preferences since this assumption makes the income elasticity of the demand for land equal to zero, which is inconsistent with empirical evidence (Wheaton, 1977; Glaeser et al., 2008). In other words, they suppose that the expenditure on land is the same across all commuters regardless of their income levels and does not change even if a congestion toll is imposed. This means that their model ignores the effects of peak-period congestion and congestion pricing on commuters' land use pattern. Therefore, the long-run distributional effects of peak-load pricing have yet to be clarified.

This study develops a model of trip timing and residential location choices of heterogeneous commuters that resolves the limitations of the literature discussed above. We consider a closed monocentric city with a bottleneck located at the entrance to the CBD as in Gubins and Verhoef (2014) and Fosgerau et al. (2018) and employ a utility function that allows the income elasticity of the demand for land to be positive. We show that commuters sort themselves both temporally and spatially depending not only on their income and value of time, but also on their flexibility. We also find that cities with more flexible commuters are less dense, i.e., cities expand outward as commuters become more flexible.

This study then investigates the long-run effects of an optimal peak-load pricing.[1] We show that congestion pricing changes commuting costs, thereby altering commuters' lot sizes and spatial distribution. In addition, even if congestion pricing generates a Pareto improvement in the short-run (i.e., if commuters do not relocate), it does not necessarily lead to a Pareto improvement in the long-run. This occurs for the following reasons: improvements in commuting cost increase the lot size of commuters residing near the CBD; this causes the city to expand outward; the spatial expansion of the city increases commuting distance of commuters residing farther from the CBD. To demonstrate concretely the distributional effects of pricing, we analyze the model for two cases wherein rich commuters are either flexible or inflexible. This analysis clarifies that (a) rich commuters reside farther from the CBD in the former and closer to the CBD in the latter; (b) congestion pricing makes cities denser and more compact in the former, whereas it causes cities to become less dense and to expand spatially in the latter; and (c) in both cases, pricing helps rich commuters but hurts poor commuters.

We further reveal that although the bottleneck capacity expansion generates a Pareto improvement in the short-run, it can lead to an unbalanced distribution of benefits among commuters in the long-run: commuters residing closer to the CBD gain and commuters residing farther from the CBD lose. This occurs because decreasing commuting costs causes the city to spatially expand, thereby increasing commuting distance of commuters residing farther from the CBD. Thus, the capacity expansion financed by the revenue from congestion pricing could be progressive when rich commuters are flexible, while regressive when rich commuters are inflexible.[2]

This study proceeds as follows. Section 2 presents a model in which heterogeneous commuters choose their departure times from home and residential locations in a monocentric city. Sections 3 and 4 characterize equilibria with and without an optimal congestion pricing, respectively. Section 5 clarifies the effects of peak-load pricing. Section 6 concludes the study.

[1] The tradable network permit scheme (Wada and Akamatsu, 2013; Akamatsu and Wada, 2017), which resolves important issues for implementing congestion pricing, has the same effect as an optimal peak-load pricing. Therefore, its long-run effects are identical to those obtained in this paper. Similar schemes have been proposed by, e.g., Verhoef et al. (1997), Yang and Wang (2011), Nie (2012), He et al. (2013), and Nie and Yin (2013).

[2] In the model of Takayama and Kuwahara (2017), commuters sort spatially according to their value of time but not to flexibility and commuters with a high value of time must reside closer to the CBD. Furthermore, the capacity expansion helps all commuters. Therefore, the results of this study are essentially different from those obtained in Takayama and Kuwahara (2017).

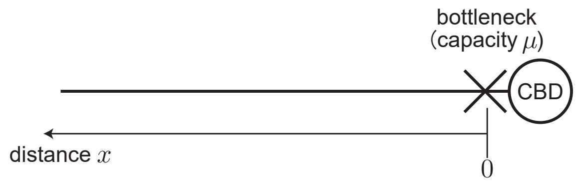


Figure 1: Urban spatial structure

2 The model

2.1 Assumptions

We consider a long narrow city with a spaceless CBD, in which all job opportunities are located. The CBD is located at the edge of the city and a residential location is indexed by distance x from the CBD (see Figure 1). In the city, land is uniformly distributed with unit density along a road. As is common in the literature, the land is owned by absentee landlords.[3] The road has a single bottleneck with capacity s at the entrance to the CBD (i.e., $x = 0$). If arrival rates at the bottleneck exceed its capacity, a queue develops. To model queuing congestion, we employ first-in-first-out (FIFO) and a point queue, in which vehicles have no physical length as in standard bottleneck models (Vickrey, 1969; Arnott et al., 1993). Free-flow travel time per unit distance is assumed to be constant at $\tau > 0$ (i.e., free-flow speed is $1/\tau$).

There are G groups of commuters, who differ in their income, value of (travel) time, and schedule delay cost for arriving at work earlier or later than desired. The number of commuters of group $i \in \mathcal{G} \equiv \{1, 2, \cdots, G\}$, whom we call "commuters i," is fixed and denoted by N_i. They have a common desired arrival time t^* at work. The commuting cost of commuter i who resides at x and arrives at work at time t is the sum of travel time cost $\alpha_i\{q(t) + \tau x\}$ and schedule delay cost $d_i(t - t^*)$:

$$c_i(x, t) = \alpha_i\{q(t) + \tau x\} + d_i(t - t^*), \tag{1a}$$

$$d_i(t - t^*) = \begin{cases} \beta_i(t^* - t) & \text{if} \quad t \leq t^*, \\ \gamma_i(t - t^*) & \text{if} \quad t \geq t^*, \end{cases} \tag{1b}$$

where $\alpha_i > 0$ is the value of time of commuters i, $q(t)$ denotes the queuing time of commuters arriving at work at time t, and τx represents the free-flow travel time of commuters residing at x. $\beta_i > 0$ and $\gamma_i > 0$ are the marginal early and late delay costs, respectively.

This study imposes the following assumptions about the value of time and the marginal schedule delay costs, which is common to the literature employing a bottleneck model with commuter heterogeneity (e.g., Vickrey, 1973; Arnott et al., 1992, 1994; van den Berg and Verhoef, 2011b; Hall, 2018).

[3] We can make the alternative assumption that the land is publicly owned and that the aggregate land rent is equally redistributed to all commuters. As we demonstrate in Appendix E, the results under this assumption (public land ownership) are essentially identical to those obtained with absentee landlords.

Assumption 1

(i) $\alpha_i > \beta_i$ *for all* $i \in \mathcal{G}$.

(ii) $\gamma_i/\beta_i = \eta > 1$ *for all* $i \in \mathcal{G}$.

Assumption 1 (i) requires that the value of time α_i is higher than the marginal early delay cost β_i for all commuters $i \in \mathcal{G}$. This assumption implies that commuters prefer to wait at the office rather than wait in traffic. If this condition is violated, there is no equilibrium that satisfies the FIFO property (i.e., vehicles must leave the bottleneck in the same order as their arrival at the bottleneck). Assumption 1 (ii) means that commuters with a high early delay cost also have a high late delay cost.

It is well known that the primary source of heterogeneity in the value of time $(\alpha_i)_{i\in\mathcal{I}}$ is variation in their income $(y_i)_{i\in\mathcal{I}}$.[4] Thus, we suppose that commuters with a high (low) value of time are assumed to be *rich (poor)*.

Assumption 2 *If* $\alpha_i \geq \alpha_j$, *then* $y_i \geq y_j$.

Each commuter consumes a numéraire good and land. The preferences of commuter i who resides at x and arrives at work at time t are represented by the Cobb-Douglas utility function

$$u(z_i(x,t), a_i(x,t)) = \{z_i(x,t)\}^{1-\mu}\{a_i(x,t)\}^{\mu}, \tag{2}$$

where $\mu \in (0,1)$, $z_i(x,t)$ denotes consumption of the numéraire good, and $a_i(x,t)$ is the lot size. As in the standard bottleneck models (e.g., Vickrey, 1969; Hendrickson and Kocur, 1981; Arnott et al., 1990b; Fosgerau and de Palma, 2012) and Fosgerau et al. (2018), we treat the travel time cost $\alpha_i\{q(t) + \tau x\}$ and the schedule delay cost $d_i(t - t^*)$ as money metric and let them enter the budget constraint for analytical simplicity.[5] The budget constraint is then given by

$$y_i = z_i(x,t) + \{r(x) + r_{\mathrm{A}}\}\, a_i(x,t) + c_i(x,t), \tag{3}$$

where $r_A > 0$ is the exogenous agricultural rent and $r(x) + r_{\mathrm{A}}$ denotes land rent at x.

The first-order conditions of the utility maximization problem give

$$z_i(x,t) = (1-\mu)I_i(x,t), \qquad a_i(x,t) = \frac{\mu I_i(x,t)}{r(x) + r_{\mathrm{A}}}, \qquad I_i(x,t) \equiv y_i - c_i(x,t), \tag{4}$$

where $I_i(x,t)$ denotes the income net of commuting cost earned by commuters i who reside at x and arrive at work at t. Substituting this into the utility function, we obtain

[4]Other sources of heterogeneity in the value of time include trip purpose (work or recreation), time of day, physical or psychological amenities available during travel, and the total duration of the trip (Small and Verhoef, 2007).

[5]This assumption implies that commuters' working hours are treated as given.

the indirect utility function

$$v(I_i(x,t), r(x) + r_{\mathrm{A}}) = (1-\mu)^{1-\mu}\mu^{\mu} I_i(x,t)\{r(x) + r_{\mathrm{A}}\}^{-\mu}. \tag{5}$$

2.2 Equilibrium conditions

Similar to models in Gubins and Verhoef (2014) and Takayama and Kuwahara (2017), we assume commuters make short-run decisions about day-specific trip timing and long-run decisions about residential location. In the short-run, commuters i minimize commuting cost $c_i(x,t)$ by selecting their arrival time t at work taking their residential location x as given. In the long-run, each commuter i chooses a residential location x so as to maximize his/her utility. We therefore present the short- and long-run equilibrium conditions.

2.2.1 Short-run equilibrium conditions

In the short-run, commuters determine only their day-specific arrival time t at work, which implies that the number $N_i(x)$ of commuters i residing at x (spatial distribution of commuters) is assumed to be a given. It follows from (1) that the commuting cost $c_i(x,t)$ of commuters i consists of a cost $\alpha_i \tau x$ of free-flow travel time depending only on residential location x and a bottleneck cost $c_i^b(t)$ owing to queuing congestion and a schedule delay depending only on arrival time t at work:

$$c_i(x,t) = c_i^b(t) + \alpha_i \tau x, \tag{6a}$$

$$c_i^b(t) \equiv \alpha_i q(t) + d_i(t - t^*). \tag{6b}$$

This implies that each commuter i chooses arrival time t so as to minimize his/her bottleneck cost $c_i^b(t)$. Therefore, short-run equilibrium conditions coincide with those in the standard bottleneck model, which are given by the following three conditions:

$$\begin{cases} c_i^b(t) = c_i^{b*} & \text{if} \quad n_i(t) > 0 \\ c_i^b(t) \geq c_i^{b*} & \text{if} \quad n_i(t) = 0 \end{cases} \quad \forall i \in \mathcal{G}, \tag{7a}$$

$$\begin{cases} \sum_{k \in \mathcal{G}} n_k(t) = s & \text{if} \quad q(t) > 0 \\ \sum_{k \in \mathcal{G}} n_k(t) \leq s & \text{if} \quad q(t) = 0 \end{cases} \quad \forall t \in \mathbb{R}_+, \tag{7b}$$

$$\int n_i(t)\mathrm{d}t = N_i \qquad \forall i \in \mathcal{G}, \tag{7c}$$

where $n_i(t)$ denotes the number of commuters i who arrive at work at time t (i.e., arrival rate of commuters i at the CBD) and c_i^{b*} is the short-run equilibrium bottleneck cost of commuters i.

Condition (7a) represents the no-arbitrage condition for the choice of arrival time t. This condition means that, at the short-run equilibrium, no commuter can reduce the bottleneck cost by altering arrival time unilaterally. Condition (7b) is the capacity constraint of the bottleneck, which requires that the total departure rate $\sum_{k\in\mathcal{G}} n_k(t)$ at the bottleneck equals capacity s if there is a queue; otherwise, the total departure rate is (weakly) lower than s. Condition (7c) is flow conservation for commuting demand.

These conditions give $n_i(t)$, $q(t)$, and c_i^{b*} at the short-run equilibrium as functions of $(N_i)_{i\in\mathcal{G}}$.[6] The short-run equilibrium commuting cost $c_i^*(x)$ and the income net of commuting cost $I_i(x)$ of commuters i residing at x are given by

$$c_i^*(x) = c_i^{b*} + \alpha_i \tau x, \tag{8a}$$

$$I_i(x) \equiv y_i - c_i^*(x). \tag{8b}$$

2.2.2 Long-run equilibrium conditions

In the long-run, each commuter i chooses a residential location x so as to maximize indirect utility (5). Thus, long-run equilibrium conditions are expressed as the following complementarity problems:

$$\begin{cases} v(I_i(x), r(x)+r_\mathrm{A}) = v_i^* & \text{if} \quad N_i(x) > 0 \\ v(I_i(x), r(x)+r_\mathrm{A}) \leq v_i^* & \text{if} \quad N_i(x) = 0 \end{cases} \qquad \forall x \in \mathbb{R}_+,\ \forall i \in \mathcal{G}, \tag{9a}$$

$$\begin{cases} \sum_{k\in\mathcal{G}} a(I_i(x), r(x)+r_\mathrm{A}) N_k(x) = 1 & \text{if} \quad r(x) > 0 \\ \sum_{k\in\mathcal{G}} a(I_i(x), r(x)+r_\mathrm{A}) N_k(x) \leq 1 & \text{if} \quad r(x) = 0 \end{cases} \qquad \forall x \in \mathbb{R}_+ \tag{9b}$$

$$\int_0^\infty N_i(x)\,\mathrm{d}x = N_i \qquad \forall i \in \mathcal{G}, \tag{9c}$$

where v_i^* is the long-run equilibrium utility level of commuters i and $a(I_i(x), r(x)+r_\mathrm{A})$ denotes the lot size of commuters i at location x, which is given by

$$a(I_i(x), r(x)+r_\mathrm{A}) = \frac{\mu I_i(x)}{r(x)+r_\mathrm{A}}. \tag{10}$$

Condition (9a) is the equilibrium condition for commuters' choice of residential location. This condition implies that, at the long-run equilibrium, no commuter has incentive to change residential location unilaterally. Condition (9b) is the land market clearing condition. This condition requires that, if total land demand $\sum_{k\in\mathcal{G}} a(I_k(x), r(x)+r_\mathrm{A})N_k(x)$ for housing at x equals supply 1, land rent $r(x)+r_\mathrm{A}$ is (weakly) larger than agricultural rent r_A. Condition (9c) expresses the population constraint.

As is discussed in Takayama and Kuwahara (2017), *traditional bid-rent approach (Alonso, 1964; Kanemoto, 1980; Fujita, 1989; Duranton and Puga, 2015) is equivalent to our approach using complementarity problems* (for the proof, see Appendix A.1). Specifically, long-run equilibrium conditions (9) coincide with those of the bid-rent approach. Therefore, even if we use the traditional bid-rent approach, we obtain the same results as those presented in this study.

[6] Note that the short-run equilibrium conditions depend on $(N_i)_{i\in\mathcal{G}}$ but not on $N_i(x)$.

3 Equilibrium

3.1 Short-run equilibrium

The short-run equilibrium conditions (7) coincide with those in the standard bottleneck model, as discussed above. Therefore, we can invoke the results of studies utilizing the bottleneck model to characterize the short-run equilibrium (Arnott et al., 1994; Lindsey, 2004; Iryo and Yoshii, 2007; Liu et al., 2015). In particular, the following properties of the short-run equilibrium are useful for investigating the properties of our model.

Lemma 1 (Lindsey, 2004; Iryo and Yoshii, 2007) *Suppose Assumption 1 (i). Then, the short-run equilibrium has the following properties:*

(a) The short-run equilibrium bottleneck cost c_i^{b*} is uniquely determined.

(b) The short-run equilibrium number $(n_i^*(t))_{i\in\mathcal{G}}$ of commuters arriving at time t coincides with the solution of the following linear programming problem:

$$\min_{(n_i(t))_{i\in\mathcal{G}}} \sum_{i\in\mathcal{G}} \int \frac{d_i(t-t^*)}{\alpha_i} n_i(t)\mathrm{d}t \tag{11a}$$

$$s.t. \quad \sum_{i\in\mathcal{G}} n_i(t) \leq s \quad \forall t \in \mathbb{R}, \tag{11b}$$

$$\int n_i(t)\ \mathrm{d}t = N_i \quad \forall i \in \mathcal{G}, \tag{11c}$$

$$n_i(t) \geq 0 \quad \forall i \in \mathcal{G},\ \forall t \in \mathbb{R}. \tag{11d}$$

Let us define time-based cost as the cost converted into equivalent travel time. Since that cost for commuters i is given by dividing the cost by α_i, we say that $\frac{d_i(t-t^*)}{\alpha_i}$ represents the time-based schedule delay cost of commuters i. Therefore, Lemma 1 (b) shows that, at the short-run equilibrium, the total *time-based* schedule delay cost is minimized, but the total schedule delay cost is not necessarily minimized.[7]

We let $\mathrm{supp}\,(n_i^*) = \{t \in \mathbb{R}_+ \mid n_i^*(t) > 0\}$ be the support of the short-run equilibrium number $n_i^*(t)$ of commuters i who arrive at work at t. From Lemma 1 (b), we have

$$\mathrm{supp}\,(\textstyle\sum_{i\in\mathcal{G}} n_i^*) = [t^{\mathrm{E}}, t^{\mathrm{L}}], \tag{12}$$

where t^{E} and t^{L} denote the earliest and latest arrival times of commuters, which satisfy

$$t^{\mathrm{L}} = t^{\mathrm{E}} + \frac{\sum_{i\in\mathcal{G}} N_i}{s}. \tag{13}$$

This indicates that, at the short-run equilibrium, a rush hour in which queuing congestion occurs must be a single time interval.

[7]As will be shown in Section 4.1, under an optimal peak-load toll, the total schedule delay cost (the social cost of commuting) is minimized at the short-run equilibrium.

By using short-run equilibrium condition (7a), we obtain

$$\frac{c_i(t_i)}{\alpha_i} + \frac{c_j(t_j)}{\alpha_j} \leq \frac{c_i(t_j)}{\alpha_i} + \frac{c_j(t_i)}{\alpha_j} \qquad \forall t_i \in \mathrm{supp}\,(n_i^*),\ t_j \in \mathrm{supp}\,(n_j^*). \tag{14}$$

Substituting (6b) into this, we have

$$\begin{cases} \left(\frac{\beta_i}{\alpha_i} - \frac{\beta_j}{\alpha_j}\right)(t_i - t_j) \geq 0 & \text{if} \quad \max\{t_i, t_j\} \leq t^* \\ \left(\frac{\gamma_i}{\alpha_i} - \frac{\gamma_j}{\alpha_j}\right)(t_i - t_j) \leq 0 & \text{if} \quad \min\{t_i, t_j\} \geq t^* \end{cases} \qquad \forall i, j \in \mathcal{G}. \tag{15}$$

This leads to the following proposition as given in Arnott et al. (1994) and Liu et al. (2015):

Proposition 1 *Suppose Assumption 1. Then, at the short-run equilibrium, commuters with a high marginal time-based schedule delay cost (β_i/α_i) arrive closer to their preferred arrival time t^*.*

This proposition indicates that the short-run equilibrium has the following properties: if marginal schedule delay cost of commuters i is lower than that of commuters j (i.e., $\beta_i/\alpha_i < \beta_j/\alpha_j$), early-arriving commuters i arrive at the CBD earlier than early-arriving commuters j and late-arriving commuters i arrive at the CBD later than late-arriving commuters j. This occurs because commuters with a lower time-based schedule delay cost avoid queuing time rather than a schedule delay.

By using Proposition 1, we can explicitly obtain the short-run equilibrium bottleneck cost. For the moment, we assume, without loss of generality, that commuters with small i have a (weakly) higher marginal time-based schedule delay cost:

Assumption 3 $\frac{\beta_{i-1}}{\alpha_{i-1}} \geq \frac{\beta_i}{\alpha_i}$ *for all $i \in \mathcal{G}\backslash\{1\}$.*

Under this assumption, commuters with smaller i arrive (weakly) closer to their preferred arrival time t^*. Therefore, the short-run bottleneck cost c_i^{b*} of commuters i is derived by following the procedure employed in literature employing a bottleneck model with commuter heterogeneity (see, e.g., van den Berg and Verhoef, 2011b):

$$c_i^{b*} = \frac{\eta}{1+\eta}\left\{\beta_i \frac{\sum_{k=1}^{i} N_k}{s} + \alpha_i \sum_{k=i+1}^{G} \frac{\beta_k}{\alpha_k}\frac{N_k}{s}\right\} \qquad \forall i \in \mathcal{G}. \tag{16}$$

This indicates that commuters with high value of travel time or high schedule delay cost incur higher bottleneck costs at the short-run equilibrium.

We see from the results of this subsection that the indirect utility (5) is uniquely determined. Therefore, in the following subsection, we characterize the long-run equilibrium by using the properties of the complementarity problems (9).

3.2 Long-run equilibrium

We examine the properties of urban spatial structure at the long-run equilibrium. From (9b) and (10), we have

$$r(x) + r_A = R(I(x)) = \begin{cases} \mu I(x) & \text{if} \quad \mu I(x) \geq r_{\mathrm{A}}, \\ r_{\mathrm{A}} & \text{if} \quad \mu I(x) \leq r_{\mathrm{A}}, \end{cases} \tag{17a}$$

$$I(x) \equiv \sum_{i \in \mathcal{G}} I_i(x) N_i(x), \tag{17b}$$

where $I(x)$ denotes the total income net of commuting cost in location x. Substituting this into (5), the indirect utility is expressed as

$$v_i(x) = (1-\mu)^{1-\mu} \mu^{\mu} I_i(x) \{R(I(x))\}^{-\mu} \tag{18}$$

Therefore, the long-run equilibrium conditions in (9) are rewritten as

$$\begin{cases} v_i(x) = v_i^* & \text{if} \quad N_i(x) > 0 \\ v_i(x) \leq v_i^* & \text{if} \quad N_i(x) = 0 \end{cases} \qquad \forall x \in \mathbb{R}_+, \ \forall i \in \mathcal{G}, \tag{19a}$$

$$\int_0^{\infty} N_i(x)\, \mathrm{d}x = N_i \qquad \forall i \in \mathcal{G}. \tag{19b}$$

The equilibrium conditions (9) or (19) are equivalent to the Karush-Kuhn-Tucker (KKT) conditions of the following optimization problems, which can be used to examine the uniqueness of the long-run equilibrium:

Lemma 2

(a) The spatial distribution $(N_i(x))_{i \in \mathcal{G}}$ of commuters is a long-run equilibrium if and only if it satisfies the KKT conditions of the following optimization problem:

$$\max_{(N_i(x))_{i \in \mathcal{G}}} \ P((N_i(x))_{i \in \mathcal{G}}) = P_1((N_i(x))_{i \in \mathcal{G}}) + P_2((N_i(x))_{i \in \mathcal{G}}) \tag{20a}$$

$$\text{s.t.} \quad \int_0^{\infty} N_i(x) \mathrm{d}x = N_i \quad \forall i \in \mathcal{G}, \tag{20b}$$

$$N_i(x) \geq 0 \quad \forall i \in \mathcal{G}, \forall x \in \mathbb{R}_+, \tag{20c}$$

where $P_1((N_i(x))_{i \in \mathcal{G}})$ and $P_2((N_i(x))_{i \in \mathcal{G}})$ are expressed as

$$P_1((N_i(x))_{i \in \mathcal{G}}) = \int_0^{\infty} \sum_{i \in \mathcal{G}} v(I_i(x), R(I(x)))\, N_i(x)\, \mathrm{d}x, \tag{20d}$$

$$P_2((N_i(x))_{i \in \mathcal{G}}) = (1-\mu)^{-\mu} \mu^{\mu} \int_0^{\infty} \left\{ R(I(x))^{1-\mu} - r_{\mathrm{A}}^{1-\mu} \right\} \mathrm{d}x. \tag{20e}$$

(b) The set of utility level $(v_i^*)_{i \in \mathcal{G}}$ and land rent $r(x) + r_{\mathrm{A}}$ is a long-run equilibrium if

and only if it satisfies the KKT conditions of the following optimization problem:

$$\min_{r(x),(v_i^*)_{i\in\mathcal{G}}} \quad D((v_i^*)_{i\in\mathcal{G}}, r(x)) = D_1((v_i^*)_{i\in\mathcal{G}}) + D_2(r(x)) \tag{21a}$$

$$\text{s.t.} \quad v_i^* \geq v(I_i(x), r(x) + r_{\mathrm{A}}) \quad \forall i \in \mathcal{G}, \forall x \in \mathbb{R}_+, \tag{21b}$$

$$r(x) \geq 0 \quad \forall x \in \mathbb{R}_+, \tag{21c}$$

where $D_1((v_i^*)_{i\in\mathcal{G}})$ *and* $D_2(r(x))$ *are expressed as*

$$D_1((v_i^*)_{i\in\mathcal{G}}) = \sum_{i\in\mathcal{G}} N_i v_i^* \tag{21d}$$

$$D_2(r(x)) = (1-\mu)^{-\mu}\mu^{\mu} \int_0^{\infty} \left\{ [r(x) + r_{\mathrm{A}}]^{1-\mu} - r_{\mathrm{A}}^{1-\mu} \right\} \mathrm{d}x \tag{21e}$$

Proof The KKT conditions of problem (20) correspond to the long-run equilibrium conditions (19). The KKT conditions of problem (21) correspond to the conditions (9a). Thus, we have Lemma 2.

Since the long-run equilibrium conditions are represented by (19), the model of commuters' location choice can be viewed as *a multiple population game* in which the set of population is $\mathcal{G}$, the set of players of population i is $[0, N_i]$, the strategy set is $\mathbb{R}_+$, and the payoff is $(v_i(x))_{i\in\mathcal{G}}$. Furthermore, $P((N_i(x))_{i\in\mathcal{G}})$ is *a potential function* of the game since $\frac{\partial P((N_i(x))_{i\in\mathcal{G}})}{\partial N_i(x)} = v_i(x)$ for all $i \in \mathcal{G}$ and $x \in \mathbb{R}_+$. Therefore, Lemma 2 (a) suggests that *a long-run equilibrium of our model can be considered a Nash equilibrium of the potential game with a continuous strategy set,* which is studied in Cheung and Lahkar (2018).

The objective function $P((N_i(x))_{i\in\mathcal{G}})$ of the optimization problem (20) is concave, but it is not strictly concave. This implies that the equilibrium spatial distribution of commuters $(N_i^*(x))_{i\in\mathcal{G}}$ is not necessarily unique. However, by using Lemma 2 (b), we can show the uniqueness of $r(x)$ and $(v_i^*)_{i\in\mathcal{G}}$.

Lemma 3 *The long-run equilibrium land rent* $r(x) + r_{\mathrm{A}}$ *and utility level* $(v_i^*)_{i\in\mathcal{G}}$ *are uniquely determined.*

Proof See Appendix B.

By using the equilibrium condition (19a), we can see that there is no vacant location between any two populated locations, as shown in Lemma 4.

Lemma 4 *The long-run equilibrium number* $\sum_{i\in\mathcal{G}} N_i^*(x)$ *of commuters residing at* x *has the following properties:*

(a) the support of $\sum_{i\in\mathcal{G}} N_i^*(x)$ *is given by*

$$\mathrm{supp}\left(\textstyle\sum_{i\in\mathcal{G}} N_i^*\right) = [0, X^{\mathrm{B}}], \tag{22}$$

where X^{B} denotes the residential location for commuters farthest from the CBD (i.e., city boundary).

(b) the land rent $r(x) + r_{\mathrm{A}}$ satisfies

$$r(x) + r_{\mathrm{A}} = \mu I(x) > r_{\mathrm{A}} \quad \forall x \in \mathrm{supp}\,(\textstyle\sum_{i \in \mathcal{G}} N_i^*) \backslash \{X^{\mathrm{B}}\}, \tag{23a}$$

$$r(X^{\mathrm{B}}) + r_{\mathrm{A}} = \mu I(X^{\mathrm{B}}) = r_{\mathrm{A}}. \tag{23b}$$

Proof See Appendix C.

It follows immediately from Lemma 4 that the indirect utility $v_i(x)$ of commuters i is given by

$$v_i(x) = (1-\mu)^{1-\mu} I_i(x) \{I(x_i)\}^{-\mu} \qquad \forall i \in \mathcal{G},\ \forall x \in [0, X^{\mathrm{B}}]. \tag{24}$$

This implies that the optimization problem (20) is rewritten as

$$\max_{(N_i(x))_{i \in \mathcal{G}}} \frac{1}{1-\mu} \int_0^{X^{\mathrm{B}}} \sum_{i \in \mathcal{G}} v_i(x) N_i(x)\, \mathrm{d}x \tag{25a}$$

$$\text{s.t.} \quad \int_0^{X^{\mathrm{B}}} N_i(x) \mathrm{d}x = N_i \quad \forall i \in \mathcal{G}, \tag{25b}$$

$$N_i(x) \geq 0 \quad \forall i \in \mathcal{G}, \forall x \in [0, X^{\mathrm{B}}], \tag{25c}$$

This shows that the total utility is maximized in the long-run and thus the long-run equilibrium is Pareto optimal. Note that since the short-run equilibrium bottleneck cost c_i^{b*} is taken as given, this does not indicate that the equilibrium is efficient but instead indicates that market failures in the model are caused only by traffic (bottleneck) congestion.

The long-run equilibrium condition (9a) yields

$$v_i(x_i) \cdot v_j(x_j) \geq v_i(x_j) \cdot v_j(x_i) \qquad \forall x_i \in \mathrm{supp}\,(N_i^*), \forall x_j \in \mathrm{supp}\,(N_j^*), \forall i, j \in \mathcal{G}, \tag{26}$$

where $N_i^*(x)$ denotes the long-run equilibrium number of commuters i residing at x. Substituting (24) into this, we have

$$\left\{ \frac{y_i - c_i^{b*}}{\alpha_i} - \frac{y_j - c_j^{b*}}{\alpha_j} \right\} (x_i - x_j) \geq 0 \qquad \forall x_i \in \mathrm{supp}\,(N_i^*), \forall x_j \in \mathrm{supp}\,(N_j^*), \forall i, j \in \mathcal{G}. \tag{27}$$

This condition implies that if $\frac{I_i(x)}{\alpha_i} > \frac{I_j(x)}{\alpha_j}$, then $x_i \geq x_j$ at the long-run equilibrium,[8]

[8] Let $\Psi_i(x, v_i^*)$ denote bid-rent function of commuters i. Then, as shown in Appendix A.2, $\Psi_i(x, v_i^*)$ is steeper than $\Psi_j(x, v_j^*)$ if and only if the condition $I_i(x)/\alpha_i > I_j(x)/\alpha_j$ holds. Therefore, we can say that Proposition 2 is consistent with the standard results obtained in the literature studying the traditional location model (e.g., Kanemoto, 1980; Fujita, 1989; Duranton and Puga, 2015).

which yields the following proposition.

Proposition 2 *Commuters with a high time-based income net of commuting cost ($I_i(x)/\alpha_i$) reside farther from the CBD at the long-run equilibrium.*

This proposition states that *commuters sort themselves spatially depending not only on their income and value of time, but also on their flexibility.* This is because commuters with a high income net of commuting cost consume a larger amount of land and commuters with a high value of time want to reduce their free-flow travel time cost.

Proposition 2 also indicates that if $\frac{y_i - c_i^{b*}}{\alpha_i} \neq \frac{y_j - c_j^{b*}}{\alpha_j}$ for all $i, j \in \mathcal{G}$, $(N_i^*(x))_{i \in \mathcal{G}}$ is uniquely determined. If there exist $i, j \in \mathcal{G}$ such that $\frac{y_i - c_i^{b*}}{\alpha_i} = \frac{y_j - c_j^{b*}}{\alpha_j}$, $(N_i^*(x))_{i \in \mathcal{G}}$ is not unique because the locations of commuters i and j are interchangeable without affecting their utilities.

By using Proposition 2, we examine properties of the long-run equilibrium. For this, we assume, without loss of generality, that commuters with small i have lower time-based income net of commuting cost:

Assumption 4 $\frac{I_{i-1}(x)}{\alpha_{i-1}} \leq \frac{I_i(x)}{\alpha_i}$ *for all* $i \in \mathcal{G} \backslash \{1\}$.

For the moment, we also assume that all commuters $i-1$ reside closer than every commuter i for examining the properties of $r(x)$ and $(v_i^*)_{i \in \mathcal{G}}$ at the long-run equilibrium, each of which is uniquely determined. Let X_i denote the location for commuters i residing nearest the CBD. Then, this assumption means that commuters i reside in $[X_i, X_{i+1}]$ (i.e., $\mathrm{supp}\,(N_i^*) = [X_i, X_{i+1}]$). Therefore, we have $v_i(x) = v_i(X_i)$ for all $x \in \mathrm{supp}\,(N_i^*)$. This, together with the population constraint (19b), yields the following lemma

Lemma 5 *Suppose Assumption 4 and* $\mathrm{supp}\,(N_i^*) = [X_i, X_{i+1}]$ *for any* $i \in \mathcal{G}$*. Then, the long-run equilibrium land rent at location* X_i *is given by*

$$r(X_i) + r_{\mathrm{A}} = r_i \equiv \sum_{k=i}^{G} \alpha_k \tau N_k + r_{\mathrm{A}}. \tag{28}$$

Proof See Appendix D.

Substituting this into (61), we obtain X_i as follows:

$$X_1 = 0, \qquad X_{i+1} = \sum_{j=1}^{i} \left[\{r_{j+1}\}^{-\mu} - \{r_j\}^{-\mu} \right] \{r_{i+1}\}^{\mu} \frac{y_j - c_j^{b*}}{\alpha_j \tau} \quad \forall i \in \mathcal{G}, \tag{29}$$

From these results, we have the following lemma:

Lemma 6 *Suppose Assumption 4. Then, at the long-run equilibrium,* *(a) the city boundary X^{B} is given by*

$$X^{\mathrm{B}} = \sum_{i \in \mathcal{G}} \left[\{r_{i+1}\}^{-\mu} - \{r_i\}^{-\mu} \right] \{r_{\mathrm{A}}\}^{\mu} \frac{y_i - c_i^{b*}}{\alpha_i \tau} \tag{30}$$

where r_i is represented as (28).

(b) the long-run equilibrium utility level $(v_i^*)_{i \in \mathcal{G}}$, land rent $r(x) + r_{\mathrm{A}}$, and lot size $a_i(x)$ are given by

$$v_i^* = (1-\mu)^{1-\mu} \mu^{\mu} \alpha_i \left[\{r_{i+1}\}^{-\mu} \frac{y_i - c_i^{b*}}{\alpha_i} - \sum_{j=1}^{i} \left[\{r_{j+1}\}^{-\mu} - \{r_j\}^{-\mu} \right] \frac{y_j - c_j^{b*}}{\alpha_j} \right] \qquad \forall i \in \mathcal{G}, \tag{31a}$$

$$r(x) + r_{\mathrm{A}} = (1-\mu)^{\frac{1-\mu}{\mu}} \mu \left\{ \frac{I_i(x)}{v_i^*} \right\}^{\frac{1}{\mu}} \qquad \forall x \in \mathrm{supp}\,(N_i^*), \tag{31b}$$

$$a_i(x) = (1-\mu)^{-\frac{1-\mu}{\mu}} \{I_i(x)\}^{-\frac{1-\mu}{\mu}} \{v_i^*\}^{\frac{1}{\mu}} \qquad \forall x \in \mathrm{supp}\,(N_i^*). \tag{31c}$$

We see from Lemma 6 (a) that the city boundary X^{B} increases with an increase in the time-based income net of bottleneck cost $(\frac{y_i - c_i^{b*}}{\alpha_i})$. This shows that *the spatial size of the city is affected not only by commuters' income and value of time, but also by their flexibility.* Furthermore, cities with richer or more flexible commuters are less dense. That is, *cities expand outward as commuters become richer or more flexible.*

From Lemma 6 (b), we have

$$\frac{\mathrm{d}\{r(x) + r_{\mathrm{A}}\}}{\mathrm{d}x} = -\frac{\alpha_i \tau}{a_i(x)} < 0 \qquad \forall x \in \mathrm{supp}\,(N_i^*), \tag{32}$$

which is known as the Alonso-Muth condition. This states that, at the long-run equilibrium, the marginal commuting cost $\alpha_i \tau$ equals the marginal land cost saving $-\frac{\mathrm{d}\{r(x)+r_{\mathrm{A}}\}}{\mathrm{d}x} a_i(x)$. Thus, the land rent $r(x) + r_{\mathrm{A}}$ decreases with distance x from the CBD.

Lemma 6 (b) also allows us to examine the long-run effect of the bottleneck capacity expansion. It follows from (16) that the short-run equilibrium bottleneck cost c_i^{b*} decreases with the bottleneck capacity s. That is, *in the short-run, the capacity expansion generates a Pareto improvement.* However, we can see by differentiating the equilibrium utility level $(v_i^*)_{i \in \mathcal{G}}$ with respect to the capacity that there can exist $i \in \mathcal{G}$ such that $\frac{\mathrm{d}v_i^*}{\mathrm{d}s} < 0$. More specifically, since we have

$$\frac{\mathrm{d}v_i^*}{\mathrm{d}s} = (1-\mu)^{1-\mu} \mu^{\mu} \alpha_i \left[-\{r_{i+1}\}^{-\mu} \frac{1}{\alpha_i} \frac{\mathrm{d}c_i^{b*}}{\mathrm{d}s} + \sum_{j=1}^{i} \left[\{r_{j+1}\}^{-\mu} - \{r_j\}^{-\mu} \right] \frac{1}{\alpha_j} \frac{\mathrm{d}c_j^{b*}}{\mathrm{d}s} \right], \tag{33a}$$

$$\frac{\mathrm{d}v_1^*}{\mathrm{d}s} = -(1-\mu)^{1-\mu} \mu^{\mu} \{r_1\}^{-\mu} \frac{\mathrm{d}c_1^{b*}}{\mathrm{d}s} > 0, \tag{33b}$$

$$\frac{1}{\alpha_{i-1}}\frac{\mathrm{d}v_{i-1}^{*}}{\mathrm{d}s} > \frac{1}{\alpha_i}\frac{\mathrm{d}v_i^{*}}{\mathrm{d}s} \qquad \forall i \in \mathcal{G}\backslash\{1\}, \tag{33c}$$

the capacity expansion cannot lead to a Pareto improvement in the long-run if there exists $i \in \mathcal{G}$ such that

$$\frac{\{r_{i+1}\}^{-\mu}}{\alpha_i}\frac{\mathrm{d}c_i^{b*}}{\mathrm{d}s} > \sum_{j=1}^{i}\frac{\{r_{j+1}\}^{-\mu}-\{r_j\}^{-\mu}}{\alpha_j}\frac{\mathrm{d}c_j^{b*}}{\mathrm{d}s}. \tag{34}$$

That is, if (34) holds for some i, *commuters residing closer to the CBD gain, but those residing farther from the CBD lose from the capacity expansion.* This is due to the fact that the expansion increases the city boundary X^{B}, thereby increasing commuting distance of commuters residing farther from the CBD.

The results obtained thus far are summarized as follows.

Proposition 3

(a) The spatial size of the city depends on commuters' income, value of time, and flexibility. Furthermore, cities with richer or more flexible commuters are less dense.

(b) The bottleneck capacity expansion generates a Pareto improvement in the short-run, but it can lead to an unbalanced distribution of benefits in the long-run: commuters residing closer to the CBD gain and those residing farther from the CBD lose.

4 Optimal peak-load pricing

Studies utilizing the standard bottleneck model show that queuing time is a pure deadweight loss. Hence, in our model, there is no queue at the social optimum, and the social optimum is achieved by imposing an optimal peak-load toll (e.g., Arnott, 1998; Gubins and Verhoef, 2014; Takayama and Kuwahara, 2017). This section examines the effect of an optimal pricing by analyzing equilibrium under this pricing policy.

4.1 Short-run equilibrium

An optimal congestion toll $p(t)$ eliminates queuing congestion. Thus, the commuting cost $c_i^o(x,t)$ of commuters i is given by

$$c_i^o(x,t) = c_i^{bo}(t) + \alpha_i \tau x, \tag{35a}$$

$$c_i^{bo}(t) \equiv p(t) + d_i(t - t^*). \tag{35b}$$

Superscript o describes variable under the optimal congestion toll.

Since we consider heterogeneous commuters, the congestion toll $p(t)$ does not equal the queuing time cost $\alpha_i q(t)$ at the no-toll equilibrium, and it is set so that travel demand $n^o(t)$

at the bottleneck equals supply (i.e., capacity) s. Therefore, the short-run equilibrium conditions are expressed as

$$\begin{cases} c_i^{bo}(t) = c_i^{bo*} & \text{if} \quad n_i^o(t) > 0 \\ c_i^{bo}(t) \geq c_i^{bo*} & \text{if} \quad n_i^o(t) = 0 \end{cases} \qquad \forall i \in \mathcal{G},\ \forall t \in \mathbb{R}, \tag{36a}$$

$$\begin{cases} \sum_{i\in\mathcal{G}} n_i^o(t) = s & \text{if} \quad p(t) > 0 \\ \sum_{i\in\mathcal{G}} n_i^o(t) \leq s & \text{if} \quad p(t) = 0 \end{cases} \qquad \forall t \in \mathbb{R}, \tag{36b}$$

$$\int n_i^o(t)\ \mathrm{d}t = N_i \qquad \forall i \in \mathcal{G}. \tag{36c}$$

Condition (36a) is the no-arbitrage condition for commuters' arrival time choices. Condition (36b) denotes the bottleneck capacity constraints, which assure that queuing congestion is eliminated at the equilibrium. Condition (36c) provides the flow conservation for commuting demand. These conditions give $n_i^o(t), p(t), c_i^{bo*}$ at the short-run equilibrium.

As in the case without the congestion toll, by invoking the results of studies employing the bottleneck model, we have the following lemma.

Lemma 7 (Lindsey, 2004; Iryo and Yoshii, 2007) *Suppose Assumption 1 (i). Then, the short-run equilibrium under the congestion toll has the following properties:*

(a) The bottleneck cost c_i^{bo*} is uniquely determined.

(b) The short-run equilibrium number $(n_i^{o*}(t))_{i\in\mathcal{G}}$ of commuters arriving at time t coincides with the solution of the following linear programming problem:

$$\min_{(n_i^o(t))_{i\in\mathcal{G}}} \sum_{i\in\mathcal{G}} \int d_i(t-t^*)\, n_i^o(t)\ \mathrm{d}t \tag{37a}$$

$$s.t. \quad \sum_{i\in\mathcal{G}} n_i^o(t) \leq s \quad \forall t \in \mathbb{R}, \tag{37b}$$

$$\int n_i^o(t)\ \mathrm{d}t = N_i \quad \forall i \in \mathcal{G}, \tag{37c}$$

$$n_i^o(t) \geq 0 \quad \forall i \in \mathcal{G},\ \forall t \in \mathbb{R}. \tag{37d}$$

Lemma 7 (b) suggests that total schedule delay cost is minimized at the short-run equilibrium under the congestion toll. Note that total schedule delay cost equals total commuting cost minus total toll revenue. Hence, Lemma 7 (b) indicates that, *in the short-run, the optimal congestion toll minimizes the social cost of commuting.*

From the short-run equilibrium condition (36a), we have

$$c_i^{bo}(t_i) + c_j^{bo}(t_j) \leq c_i^{bo}(t_j) + c_j^{bo}(t_i) \qquad \forall t_i \in \mathrm{supp}\,(n_i^{o*}),\ \forall t_j \in \mathrm{supp}\,(n_j^{o*}),\ \forall i,j \in \mathcal{G}. \tag{38}$$

Substituting (35b) into this, we have

$$\begin{cases} (\beta_i - \beta_j)(t_i - t_j) \geq 0 & \text{if} \quad \max\{t_i, t_j\} \leq t^*, \\ (\gamma_i - \gamma_j)(t_i - t_j) \leq 0 & \text{if} \quad \min\{t_i, t_j\} \geq t^*. \end{cases} \tag{39}$$

Therefore, we obtain the following proposition.

Proposition 4 *Suppose Assumption 1. Then, at the short-run equilibrium, commuters with a high marginal schedule delay cost (β_i) arrive closer to their preferred arrival time t^*.*

Propositions 1 and 4 show that *the equilibrium bottleneck cost under the congestion toll c_i^{bo*} generally differs from the no-toll equilibrium bottleneck cost c_i^{b*} when we consider commuter heterogeneity in the value of time.* To see this concretely, we assume, without loss of generality, that commuters with small i have a (weakly) higher marginal schedule delay cost:

Assumption 5 $\beta_{i-1} \geq \beta_i$ *for all* $i \in \mathcal{G}\backslash\{1\}$.

Then, we can obtain the short-run equilibrium bottleneck cost c_i^{bo*} and commuting cost $c_i^{o*}(x)$ under the toll in the same manner as in (16).

$$c_i^{bo*} = \frac{\eta}{1+\eta}\left\{\beta_i \frac{\sum_{k=1}^{i} N_k}{s} + \sum_{k=i+1}^{G} \beta_k \frac{N_k}{s}\right\} \qquad \forall i \in \mathcal{G}, \tag{40a}$$

$$c_i^{o*}(x) = c_i^{bo*} + \alpha_i \tau x. \tag{40b}$$

This shows that inflexible commuters have higher bottleneck costs at the equilibrium under the toll, which is fundamentally different from the properties of the no-toll equilibrium bottleneck cost c_i^{b*}.

4.2 Long-run equilibrium

We characterize the urban spatial structure at the long-run equilibrium under the toll by using the short-run equilibrium bottleneck cost c_i^{bo*}. In the long-run, the difference between cases with and without pricing appears only in the income net of commuting cost. Specifically, under the congestion toll, the income net of commuting cost is expressed as

$$I_i^o(x) \equiv y_i - c_i^{o*}(x), \qquad I^o(x) \equiv \sum_{i \in \mathcal{G}} I_i^o(x) N_i(x). \tag{41}$$

The long-run equilibrium conditions are thus represented as (9) with the use of (41).

Without loss of generality, let us introduce the following assumption, as in the case without the toll.

Assumption 6 $\frac{I_{i-1}^o(x)}{\alpha_{i-1}} \leq \frac{I_i^o(x)}{\alpha_i}$ *for all* $i \in \mathcal{G}\backslash\{1\}$.

Then, following the same procedure as in Section 3.2 reveals the following properties of the long-run equilibrium with an optimal congestion pricing.

Lemma 8 *Under the congestion toll, the long-run equilibrium has the following properties.*

(a) *Let* $\mathrm{supp}\,(N_i^{o*})$ *be the support of the long-run equilibrium number* $N_i^{o*}(x)$ *of commuters residing at* x. *Then, for any* $x_i \in \mathrm{supp}\,(N_i^{o*})$ *and* $x_j \in \mathrm{supp}\,(N_j^{o*})$,

$$\left\{ \frac{y_i - c_i^{bo*}}{\alpha_i} - \frac{y_j - c_j^{bo*}}{\alpha_j} \right\} (x_i - x_j) \geq 0. \tag{42}$$

(b) *Suppose Assumption 6. Then, the city boundary* $X^{o\mathrm{B}}$ *and equilibrium utility level* $(v_i^{o*})_{i\in\mathcal{G}}$ *are uniquely determined and are given by*

$$X^{o\mathrm{B}} = \sum_{i\in\mathcal{G}} \left[\{r_{i+1}\}^{-\mu} - \{r_i\}^{-\mu} \right] \{r_{\mathrm{A}}\}^{\mu} \frac{y_i - c_i^{bo*}}{\alpha_i \tau}, \tag{43a}$$

$$v_i^{o*} = (1-\mu)^{1-\mu}\mu^{\mu}\alpha_i \left[\{r_{i+1}\}^{-\mu} \frac{y_i - c_i^{bo*}}{\alpha_i} - \sum_{j=1}^{i} \left[\{r_{j+1}\}^{-\mu} - \{r_j\}^{-\mu} \right] \frac{y_j - c_j^{bo*}}{\alpha_j} \right] \quad \forall i \in \mathcal{G}, \tag{43b}$$

where r_i *is represented as* (28).

(c) *The spatial distribution* $(N_i^o(x))_{i\in\mathcal{G}}$ *of commuters is a long-run equilibrium if and only if it satisfies the KKT conditions of the following optimization problem:*

$$\max_{(N_i(x))_{i\in\mathcal{G}}} \frac{1}{1-\mu} \int_0^{X^{o\mathrm{B}}} \sum_{i\in\mathcal{G}} v_i^o(x) N_i(x)\, \mathrm{d}x \tag{44a}$$

$$\text{s.t.} \quad \int_0^{X^{o\mathrm{B}}} N_i(x)\mathrm{d}x = N_i \quad \forall i \in \mathcal{G}, \tag{44b}$$

$$N_i(x) \geq 0 \quad \forall i \in \mathcal{G}, \forall x \in [0, X^{o\mathrm{B}}], \tag{44c}$$

where $v_i^o(x)$ *is expressed as*

$$v_i^o(x) = (1-\mu)^{1-\mu} I_i^o(x) \{I^o(x_i)\}^{-\mu} \qquad \forall i \in \mathcal{G},\ \forall x \in [0, X^{o\mathrm{B}}]. \tag{44d}$$

Lemmas 8 (a) and (b) show that the urban spatial structure at the long-run equilibrium under the congestion toll has the same properties as the case without pricing: commuters with a high time-based income net of commuting cost reside farther from the CBD; cities expand outward as commuters become richer or more flexible. Furthermore, imposing an optimal congestion toll can lead to changes in the city boundary and the spatial sorting pattern of commuters since it alters the short-run bottleneck costs of commuters when commuters are heterogeneous in their value of time.

From Lemma 8 (b), we can also see that the capacity expansion causes the city to physically expand outward. Furthermore, although the expansion generates a Pareto improvement in the short-run, it does not necessarily lead to a Pareto improvement in the long-run like the case without pricing.

Lemma 8 (c), together with Lemma 7 (b), demonstrates that the equilibrium with pricing corresponds to the social optimum given that the social cost of commuting is minimized in the short-run and a Pareto optimal distribution of commuters is achieved in the long-run.

This lemma yields the following proposition.

Proposition 5

(a) Commuters with a high time-based income net of commuting cost ($I_i^o(x)/\alpha_i$) reside farther from the CBD at the long-run equilibrium under an optimal peak-load toll.

(b) Imposing an optimal peak-load toll alters the urban spatial structure if commuters are heterogeneous in their value of time.

(c) The bottleneck capacity expansion generates a Pareto improvement in the short-run, but it can lead to an unbalanced distribution of benefits in the long-run: commuters residing closer to the CBD gain and those residing farther from the CBD lose.

5 Comparison between equilibria with and without pricing

5.1 Short- and long-run equilibria

In the previous sections, we have investigated the properties of equilibria with and without pricing and have shown that the urban spatial structure changes with the imposition of an optimal congestion toll. This section compares these equilibria to demonstrate the effects of the congestion toll concretely. Note that its effects essentially depend on the distributions of income, values of time, and schedule delays. We set $(y_i)_{i \in \mathcal{G}}$, $(\alpha_i)_{i \in \mathcal{G}}$, and $(\beta_i)_{i \in \mathcal{G}}$ such that the relationship between residential location and commute timing choices is consistent with the empirical evidence provided by Fosgerau and Kim (2019).[9] Specifically, by supposing Assumptions 1–6, we consider a situation in which commuters who reside farther from the CBD arrive at work farther from t^* at the equilibria with and without pricing.

Under this setting, commuters with small i are inflexible and have a high marginal time-based schedule delay cost. Therefore, they are willing to pay in travel time or money to reduce schedule delay, thereby arriving closer to their preferred arrival time t^* at the short-run equilibrium. The difference between short-run equilibrium bottleneck costs with

[9]Fosgerau and Kim (2019) show empirically that commuters traveling a longer distance tend to arrive at work at the edge of the morning peak time while ones traveling a shorter distance at the peak time.

and without pricing is thus given by

$$c_i^{bo*} - c_i^{b*} = \frac{\eta}{1+\eta} \sum_{k=i+1}^{G} (\alpha_k - \alpha_i) \frac{\beta_k}{\alpha_k} \frac{N_k}{s} \qquad \forall i \in \mathcal{G}. \tag{45}$$

This clearly shows that the sign of $c_i^{bo*} - c_i^{b*}$ depends on the difference in commuters' value of time.

Commuters with small i have a low time-based income net of commuting costs both before and after imposing the toll. This implies that they reside closer to the CBD at the long-run equilibrium. Therefore, we have

$$X^{o\mathrm{B}} - X^{\mathrm{B}} = \sum_{i \in \mathcal{G}} \left[\{r_{i+1}\}^{-\mu} - \{r_i\}^{-\mu} \right] \{r_{\mathrm{A}}\}^{\mu} \frac{c_i^{b*} - c_i^{bo*}}{\alpha_i \tau}, \tag{46a}$$

$$v_i^{o*} - v_i^{*} = (1-\mu)^{1-\mu} \mu^{\mu} \left[\{r_{i+1}\}^{-\mu} \{c_i^{b*} - c_i^{bo*}\} - \sum_{j=1}^{i} \left[\{r_{j+1}\}^{-\mu} - \{r_j\}^{-\mu} \right] \frac{\alpha_i}{\alpha_j} \{c_j^{b*} - c_j^{bo*}\} \right] \quad \forall i \in \mathcal{G}. \tag{46b}$$

(46a) indicates that the spatial size of the city can expand or shrink by imposing the toll due to changes in the short-run bottleneck cost. (46b) shows that the difference between the bottleneck costs with and without pricing affects the commuters' benefits from the imposition of the toll.

The difference of the equilibrium utility level (46b) also shows that even if congestion pricing generates a Pareto improvement in the short-run (i.e., $c_i^{bo*} \leq c_i^{b*}$ for all $i \in \mathcal{G}$), it does not necessarily lead to a Pareto improvement in the long-run (i.e., $v_i^{o*} \geq v_i^{*}$ for all $i \in \mathcal{G}$). This can occur in the following mechanism: improvements in the bottleneck cost increase the income net of commuting cost and the lot size of commuters residing near the CBD; this causes the city to expand outward; the spatial expansion of the city increases the commuting distance of commuters residing farther from the CBD, which decreases their income net of commuting cost.

To see the effects of an optimal peak-load pricing more concretely, we introduce an additional assumption on the value of time in the following subsection. Specifically, we analyze the following two cases:

Case A: rich commuters are flexible

Case B: rich commuters are inflexible

5.2 Simple examples

5.2.1 Case A: rich commuters are flexible

We first introduce the following assumption in addition to Assumptions 1–6.

Assumption 7 *$\alpha_{i-1} < \alpha_i$ for all $i \in \mathcal{G} \backslash \{1\}$.*

Note that Assumptions 2–7 are not too restrictive. Indeed, if the income y_i of commuters i is proportional to their value of time α_i (i.e., $y_i = \phi\alpha_i$), Assumptions 5 and 7 (i.e., $\beta_{i-1} \geq \beta_i$ and $\alpha_{i-1} < \alpha_i$ for all $i \in \mathcal{G}\backslash\{1\}$) are sufficient conditions for these assumptions to hold.

In Case A, rich commuters are flexible and have a lower marginal time-based schedule delay cost. This implies that rich commuters tend to avoid queuing time and paying the toll rather than schedule delay. Thus, they arrive farther from their preferred arrival time t^* at the short-run equilibria with and without pricing. The short-run equilibrium bottleneck costs with and without pricing satisfy

$$c_{i-1}^{bo*} - c_{i-1}^{b*} > c_i^{bo*} - c_i^{b*} \qquad \forall i \in \mathcal{G}\backslash\{1\}, \tag{47a}$$

$$c_G^{bo*} - c_G^{b*} = 0. \tag{47b}$$

We see from (47) that *congestion pricing increases short-run equilibrium bottleneck costs of all commuters other than richest ones.* This reflects the fact that poor commuters pay a higher toll and that the richest commuters are those who face no queuing cost at the equilibrium without pricing and face no toll at the equilibrium with pricing.

The toll decreases the income net of commuting cost, which leads to a decrease in lot size and the spatial size of city. This can be seen by substituting (47) into (46a). This means that *the city becomes denser with pricing,* which is same as the standard results of traditional location models considering static congestion (Kanemoto, 1980; Wheaton, 1998; Anas et al., 1998).

We see from (46b) and (47) that the difference between the equilibrium utility levels with and without pricing satisfy

$$v_{i-1}^{o*} - v_{i-1}^{*} < v_i^{o*} - v_i^{*} \qquad \forall i \in \mathcal{G}\backslash\{1\}, \tag{48a}$$

$$v_1^{o*} - v_1^{*} < 0, \tag{48b}$$

$$v_G^{o*} - v_G^{*} > 0. \tag{48c}$$

This shows that *rich commuters gain and poor commuters lose from pricing in Case A.* This occurs because the spatial shrinkage of the city reduces the commuting distance, which helps commuters residing farther from the CBD.

These results establish the following proposition.

Proposition 6 *Suppose Assumptions 1–7. Then,*

(a) rich commuters arrive farther from their preferred arrival time and reside farther from the CBD at the equilibria with and without pricing;

(b) an optimal congestion pricing weakly increases the bottleneck costs of all commuters, which causes the city to become denser and more compact;

(c) rich commuters gain and poor commuters lose from imposing the toll.

5.2.2 Case B: rich commuters are inflexible

In Case B, we assume that rich commuters are inflexible and have a higher marginal time-based schedule delay cost. That is, we suppose Assumptions 1–6 and 8.

Assumption 8 $\alpha_{i-1} > \alpha_i$ *for all* $i \in \mathcal{G}\backslash\{1\}$.

Note that Assumptions 2–6 and 8 are also not too restrictive. Indeed, if the income y_i of commuters i is given by $\phi\alpha_i + \psi$ with $\phi > 0$ and $\psi > c_1^{bo*}$, the sufficient conditions for these assumptions to hold are given by Assumptions 3 and 8 (i.e., $\frac{\beta_{i-1}}{\alpha_{i-1}} \geq \frac{\beta_i}{\alpha_i}$ and $\alpha_{i-1} > \alpha_i$ for all $i \in \mathcal{G}\backslash\{1\}$).

In Case B, rich commuters are willing to pay in travel time or money to reduce schedule delay, thereby arriving closer to their preferred arrival time t^* at the short-run equilibria with and without pricing. Thus, the short-run equilibrium bottleneck costs with and without pricing satisfy

$$c_{i-1}^{bo*} - c_{i-1}^{b*} < c_i^{bo*} - c_i^{b*} \qquad \forall i \in \mathcal{G}\backslash\{1\}, \tag{49a}$$

$$c_G^{bo*} - c_G^{b*} = 0. \tag{49b}$$

The conditions in (49) shows that, *in the short-run, a Pareto improvement is achieved by imposing an optimal congestion toll.* This happens because rich commuters experience larger queuing time at the no-toll equilibrium and imposing the toll eliminates all queuing.

The conditions in (49) also indicate that the toll increases their income net of commuting cost. This leads to increases in their lot size, thereby increasing the city boundary (i.e., $X^{o\text{B}} > X^{\text{B}}$). This can be confirmed by substituting (49) into (46a). This means that *the city becomes less dense with pricing,* which contrasts with the standard results of traditional location models that consider static congestion (Kanemoto, 1980; Wheaton, 1998; Anas et al., 1998).

By using (49), we obtain the following conditions on the equilibrium utility level, which reveal that *rich commuters gain and poor commuters lose from pricing in Case B.*

$$v_{i-1}^{o*} - v_{i-1}^{*} > v_i^{o*} - v_i^{*} \qquad \forall i \in \mathcal{G}\backslash\{1\}, \tag{50a}$$

$$v_1^{o*} - v_1^{*} > 0, \tag{50b}$$

$$v_G^{o*} - v_G^{*} < 0. \tag{50c}$$

This is due to the fact that the spatial expansion of the city increases the commuting distance, thereby increasing commuting cost of poor commuters who reside farther from the CBD.

We summarize the results as the following proposition.

Proposition 7 *Suppose Assumptions 1–6 and 8. Then,*

(a) rich commuters arrive closer to their preferred arrival time and reside closer to the CBD at the equilibria with and without pricing.

(b) *an optimal congestion pricing generates a Pareto improvement in the short-run, but it causes the city to become less dense and to spatially expand outward in the long-run.*

(c) *rich commuters gain and poor commuters lose from imposing the toll.*

5.3 Numerical examples

We numerically analyze the model to show the effects of an optimal congestion toll and the bottleneck capacity expansion. In this analysis, we use the following parameter values:

$$G = 4, \quad \mu = 0.25, \quad \tau = 2 \text{ (min/km)}, \quad r_{\mathrm{A}} = 1000, \quad (N_i)_{i\in\mathcal{G}} = (100, 1000, 1000, 100). \tag{51a}$$

The values of y_i, α_i, β_i, η are set to be consistent with Assumptions 2–8 and the empirical result (Small, 1982):

$$\text{Case A:} \quad \begin{cases} (y_i)_{i\in\mathcal{G}} = (90, 120, 150, 240), & (\alpha_i)_{i\in\mathcal{G}} = (0.3, 0.4, 0.5, 0.8), \\ (\beta_i)_{i\in\mathcal{G}} = (0.28, 0.25, 0.25, 0.05), & \eta = 4.0. \end{cases} \tag{51b}$$

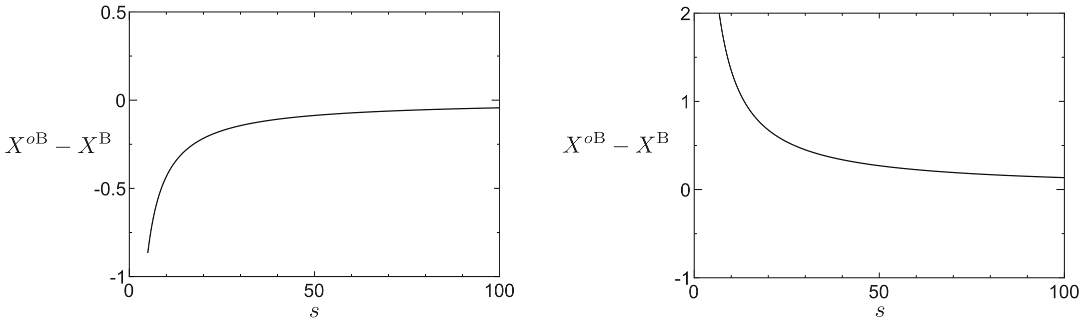
$$\text{Case B:} \quad \begin{cases} (y_i)_{i\in\mathcal{G}} = (230, 220, 195, 185), & (\alpha_i)_{i\in\mathcal{G}} = (0.8, 0.7, 0.45, 0.35), \\ (\beta_i)_{i\in\mathcal{G}} = (0.75, 0.65, 0.35, 0.02), & \eta = 4.0. \end{cases} \tag{51c}$$

We conduct comparative statics with respect to bottleneck capacity s. As we can see from Figure 2, imposing the toll results in a denser urban spatial structure in Case A, whereas it leads to spatial expansion of the city in Case B. Figures 3 and 4 indicate that, in both cases, congestion pricing leads to an unbalanced distribution of benefits among commuters: rich commuters gain and poor commuters lose. These results are consistent with those presented in Section 5.2.

Figures 3 (b) and 4 (b) also show that $\frac{\mathrm{d}v_4^*}{\mathrm{d}s} < 0$ and that $\frac{\mathrm{d}v_4^{o*}}{\mathrm{d}s} < 0$. This implies that a Pareto improvement is not achieved by expanding the bottleneck capacity in both cases. More specifically, commuters residing farthest from the CBD lose from a capacity improvement. This is also consistent with Propositions 3 and 5.

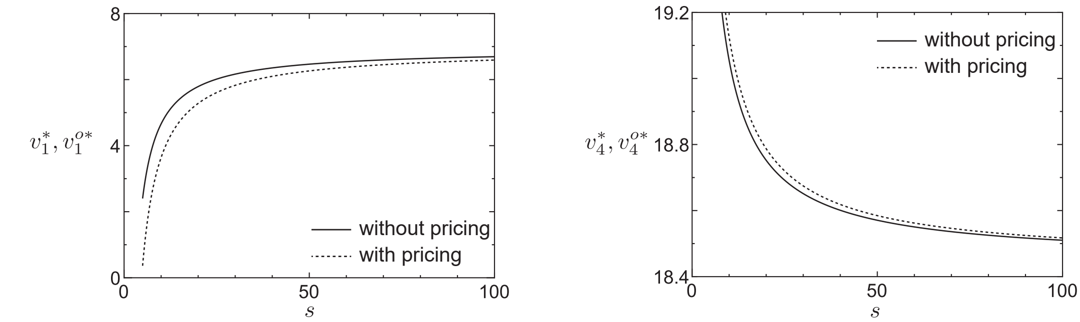
6 Conclusion

This study develops a model in which heterogeneous commuters choose their departure times from home and residential locations in a closed monocentric city. We show that commuters sort themselves both temporally and spatially according to their income, value of time, and flexibility. We also reveal that the imposition of an optimal congestion toll causes the city to spatially shrink or expand—this can help rich commuters but hurt poor commuters. Furthermore, although the bottleneck capacity expansion generates a Pareto



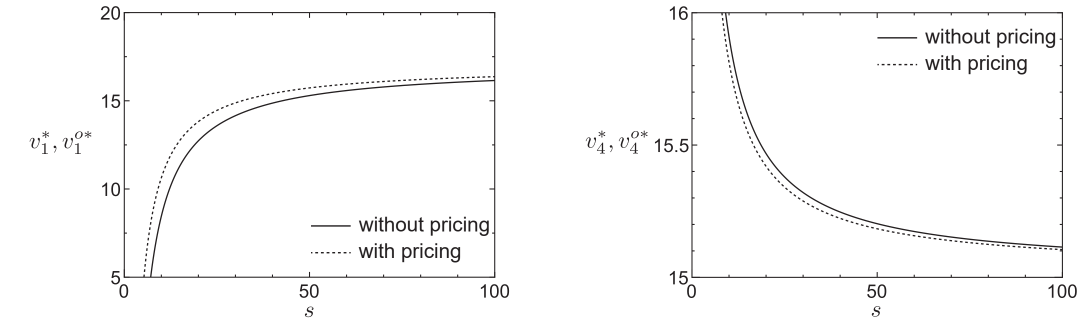
(a) Case A (b) Case B

Figure 2: Difference between city boundaries with and without pricing



(a) commuters 1 (the poorest commuters) (b) commuters 4 (the richest commuters)

Figure 3: Utility of commuters in Case A



(a) commuters 1 (the richest commuters) (b) commuters 4 (the poorest commuters)

Figure 4: Utility of commuters in Case B

improvement in the short-run, it can lead to an unbalanced distribution of benefits among commuters in the long-run: commuters residing closer to the CBD gain and commuters residing farther from the CBD lose. This implies that if poor commuters reside farther from the CBD, the capacity expansion financed by the revenue from congestion pricing could be regressive.

In this paper, we consider a city with a single bottleneck. Therefore, we need to examine the robustness of our result by analyzing a model with multiple bottlenecks (Kuwahara, 1990; Akamatsu et al., 2015; Fosgerau and Kim, 2019). In addition, it would be valuable for future research to study toll-revenue redistribution schemes that lead to a Pareto improvement. It is also important to investigate effects of policies other than optimal congestion pricing, such as step tolls (Arnott et al., 1990a; Laih, 1994, 2004; Xiao et al., 2011; Lindsey et al., 2012), TDM measures (Mun and Yonekawa, 2006; Takayama, 2015) for alleviating traffic congestion, and urban policies (e.g., urban growth boundary, floor-to-area ratio regulations) to substitute for congestion pricing (Brueckner, 2007; Anas and Rhee, 2007; Pines and Kono, 2012)

A Equivalence between the bid-rent and complementarity approaches

A.1 Equilibrium conditions

We show that long-run equilibrium conditions (9) coincide with those of the bid-rent approach. The condition (9a) can be rewritten as

$$\begin{cases} r(x) + r_{\mathrm{A}} = \Psi_i(x, v_i^*) & \text{if} \quad N_i(x) > 0 \\ r(x) + r_{\mathrm{A}} \geq \Psi_i(x, v_i^*) & \text{if} \quad N_i(x) = 0 \end{cases} \quad \forall x \in \mathbb{R}_+, \quad \forall i \in \mathcal{G}. \tag{52}$$

$\Psi_i(x, v_i^*)$ is given by

$$\Psi_i(x, v_i^*) = \left\{ \frac{(1-\mu)^{1-\mu}\mu^{\mu} I_i(x)}{v_i^*} \right\}^{\frac{1}{\mu}}. \tag{53}$$

Furthermore, since $\max_{a_i}[I_i(x) - \{v_i^*\}^{1/(1-\mu)} a_i^{-\mu/(1-\mu)}]/a_i = \Psi_i(x, v_i^*)$,[10] $\Psi_i(x, v_i^*)$ can be interpreted as the bid-rent function of commuters i.[11] This shows that conditions in (9b), (9c), and (52) are the equilibrium conditions of the bid-rent approach (see, e.g., Fujita, 1989, Definition 4.2).

A.2 Relative steepness of bid-rent curves

As is shown in Fujita (1985), we can say that $\Psi_i(x, v_j^*)$ is steeper than $\Psi_j(x, v_i^*)$ if and only if the following condition holds:

$$\frac{\partial \Psi_i(x, v_i^*)}{\partial x} < \frac{\partial \Psi_j(x, v_j^*)}{\partial x} \quad \text{whenever} \quad \Psi_i(x, v_i^*) = \Psi_j(x, v_j^*). \tag{54}$$

Differentiating the bid-rent function $\Psi_i(x, v_i^*)$ with respect to location x, we have

$$\frac{\partial \Psi_i(x, v_i^*)}{\partial x} = -\frac{\Psi_i(x, v_i^*)^{1/\mu}}{\mu} \frac{\alpha_i \tau}{I_i(x)}. \tag{55}$$

[10] $\{v^*\}^{1/(1-\mu)} a_i^{-\mu/(1-\mu)}$ represents the amount of numéraire good that is necessary to achieve utility level v^* when the lot size of the house is a_i.

[11] As shown in, e.g., Fujita (1989), this maximization problem defines the bid-rent function.

Therefore, the condition (54) can be rewritten as

$$\frac{I_i(x)}{\alpha_i} > \frac{I_j(x)}{\alpha_j}. \tag{56}$$

B Proof of Lemma 3

The optimization problem (21) is equivalent to

$$\min_{r(x)} \sum_{i \in \mathcal{G}} N_i \max_x v(I_i(x), r(x) + r_{\mathrm{A}}) + D_2(r(x)) \tag{57a}$$

$$\text{s.t.} \quad r(x) \geq 0 \quad \forall x \in \mathbb{R}_+. \tag{57b}$$

Since the objective function of this problem is strictly convex, $r(x)$ is uniquely determined. Furthermore, the uniqueness of $r(x)$ implies that the indirect utility $v(I_i(x), r(x) + r_{\mathrm{A}})$ is uniquely determined. Therefore, $(v_i^*)_{i \in \mathcal{G}}$ is also uniquely determined.

C Proof of Lemma 4

For any $x^a, x^b (> x^a) \in \mathrm{supp}\,(\sum_{i \in \mathcal{G}} N_i^*)$, there is no $x^c \in (x^a, x^b)$ such that $\sum_{i \in \mathcal{G}} N_i^*(x^c) = 0$ since the indirect utility is given by (18). Thus, we obtain Lemma 4 (a).

Differentiating the indirect utility with respect to location x, we have

$$\frac{\mathrm{d}v_i(x)}{\mathrm{d}x} = \begin{cases} v_i(x) \left\{ -\frac{\alpha_i \tau}{I_i(x)} - \frac{\mu}{I(x)} \frac{\mathrm{d}I(x)}{\mathrm{d}x} \right\} & \text{if} \quad \mu I(x) \geq r_{\mathrm{A}}, \\ v_i(x) \left\{ -\frac{\alpha_i \tau}{I_i(x)} \right\} < 0 & \text{if} \quad \mu I(x) \leq r_{\mathrm{A}}. \end{cases} \tag{58a}$$

Therefore, at the long-run equilibrium, the total income net of commuting costs satisfies

$$\begin{cases} \frac{\mathrm{d}I(x)}{\mathrm{d}x} < 0 \\ \mu I(x) \geq r_{\mathrm{A}} \end{cases} \qquad \forall x \in \mathrm{supp}\,(\textstyle\sum_{i \in \mathcal{G}} N_i^*). \tag{59}$$

Furthermore, it follows from the long-run equilibrium condition (9a) that $I(X^{\mathrm{B}})$ also satisfies

$$\mu I(X^{\mathrm{B}}) = r_{\mathrm{A}}. \tag{60}$$

Thus, we have Lemma 4 (b).

D Proof of Lemma 5

At the long-run equilibrium, the indirect utility (5) satisfies $v_i(x) = v_i(X_{i+1})$ for all $x \in \mathrm{supp}\,(N_i^*)$, and this condition gives

$$\frac{r(x) + r_{\mathrm{A}}}{r(X_{i+1}) + r_{\mathrm{A}}} = \left\{ \frac{I_i(x)}{I_i(X_{i+1})} \right\}^{\frac{1}{\mu}}. \tag{61}$$

Furthermore, from (9b) and Proposition 2, we have $a_i(x) = \frac{1}{N_i^*(x)}$. Substituting these into (4), we obtain $N_i^*(x)$ as follows:

$$N_i^*(x) = \frac{1}{\mu}\{I_i(x)\}^{\frac{1-\mu}{\mu}}\{I_i(X_{i+1})\}^{-\frac{1}{\mu}}\{r(X_{i+1}) + r_{\mathrm{A}}\}. \tag{62}$$

Therefore, the population constraint (19b) can be rewritten as

$$\begin{aligned} N_i &= -\frac{1}{\alpha_i \tau}\{r(X_{i+1}) + r_{\mathrm{A}}\}\left[1 - \left\{\frac{I_i(X_i)}{I_i(X_{i+1})}\right\}^{\frac{1}{\mu}}\right] \\ &= -\frac{r(X_{i+1}) - r(X_i)}{\alpha_i \tau}. \end{aligned} \tag{63}$$

Since $r(X_{I+1}) = 0$, we have Lemma 5.

E The case of public land ownership

We consider the case of public land ownership, in which the aggregate land rent is equally redistributed to all commuters. In this case, the budget constraint of commuter i who resides at x and arrives at work at time t is given by

$$y_i + \frac{R}{\sum_{j \in \mathcal{G}} N_j} = z_i(x,t) + \{r(x) + r_{\mathrm{A}}\}a_i(x,t) + c_i(X,t), \tag{64}$$

where $R = \int_0^{X^{\mathrm{B}}} r(x)\mathrm{d}x$ is the aggregate land rent.

The land rent $r(x) + r_{\mathrm{A}}$ is obtained by following the same procedure as in the case wherein land is owned by absentee landlords:

$$r(x) + r_{\mathrm{A}} = \left\{\zeta \frac{I_i^\star(x)}{v_i^\star}\right\}^{\frac{1}{\mu}} \tag{65}$$

where $\zeta \equiv (1-\mu)^{1-\mu}\mu^\mu$ and

$$I_i^\star(x) = I_i^{b\star} - \alpha_i \tau x, \tag{66a}$$

$$I_i^{b\star} = y_i + \frac{R}{\sum_{j \in \mathcal{G}} N_j} - c_i^{b\star}, \tag{66b}$$

$$v_i^\star = \zeta I_i^\star(X_{i+1}^\star) r_{i+1}^{-\mu} = \zeta I_i^\star(X_i^\star) r_i^{-\mu} \tag{66c}$$

$$X_i^\star = \sum_{j=1}^{i-1} \left[\{r_{j+1}\}^{-\mu} - \{r_j\}^{-\mu}\right] \{r_i\}^{\mu} \frac{I_j^{b\star}}{\alpha_j \tau}, \tag{66d}$$

$$c_i^{b\star} = \begin{cases} c_i^{b*} & \text{without pricing,} \\ c_i^{bo*} & \text{with pricing.} \end{cases} \tag{66e}$$

It follows from this that the aggregate land rent R satisfies

$$R = \frac{\mu}{1+\mu} \sum_{i \in \mathcal{G}} \frac{1}{\alpha_i \tau} \left\{ r_i I_i^\star(X_i^\star) - r_{i+1} I_i^\star(X_{i+1}^\star) \right\} - r_{\mathrm{A}} X^{\star \mathrm{B}}, \tag{67}$$

where $X^{\star B} = X_{G+1}^\star$. After some tedious calculations, this equation is rewritten as

$$R = \frac{\mu}{1+\mu} \left\{ R + \sum_{i \in \mathcal{G}} N_i (y_i - c_i^{b\star}) \right\} - \frac{1}{1+\mu} r_{\mathrm{A}} X^{\star B}. \tag{68}$$

Therefore, we have

$$R = \frac{\sum_{i \in \mathcal{G}} \left(\mu N_i - [\{r_{i+1}\}^{-\mu} - \{r_i\}^{-\mu}] \{r_{\mathrm{A}}\}^{1+\mu} \frac{1}{\alpha_i \tau} \right) (y_i - c_i^{b*})}{1 + \sum_{i \in \mathcal{G}} [\{r_{i+1}\}^{-\mu} - \{r_i\}^{-\mu}] \{r_{\mathrm{A}}\}^{1+\mu} \frac{1}{\alpha_i \tau \sum_{j \in \mathcal{G}} N_j}}. \tag{69}$$

We can investigate the properties of equilibria with and without pricing in the case of public land ownership by replacing the income y_i of the main text with $y_i + R/\sum_{j \in \mathcal{G}} N_j$. We conduct comparative statics with respect to bottleneck capacity s under the same parameter values as in Section 5.3. We see from Figures 5–7 that the results in the case of public land ownership are qualitatively the same as those obtained with absentee landlords.

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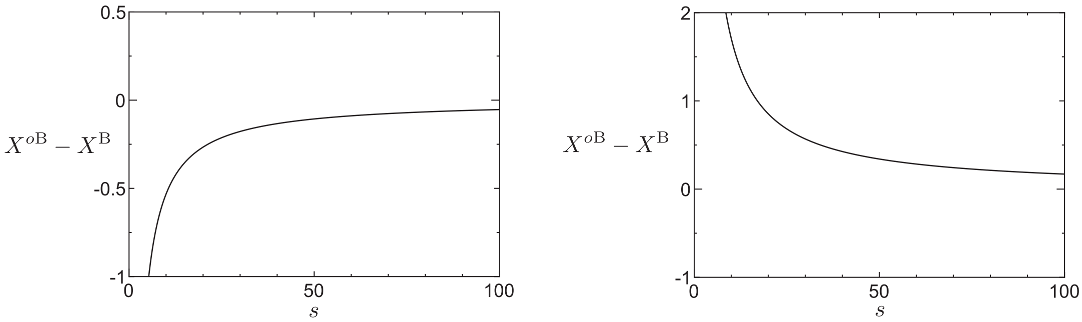
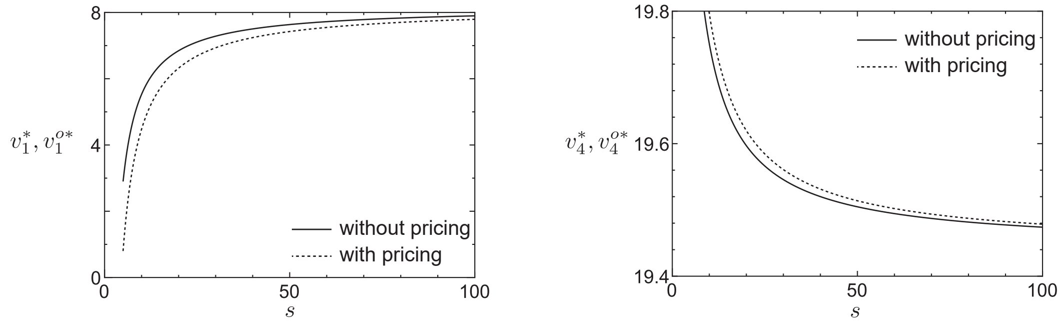


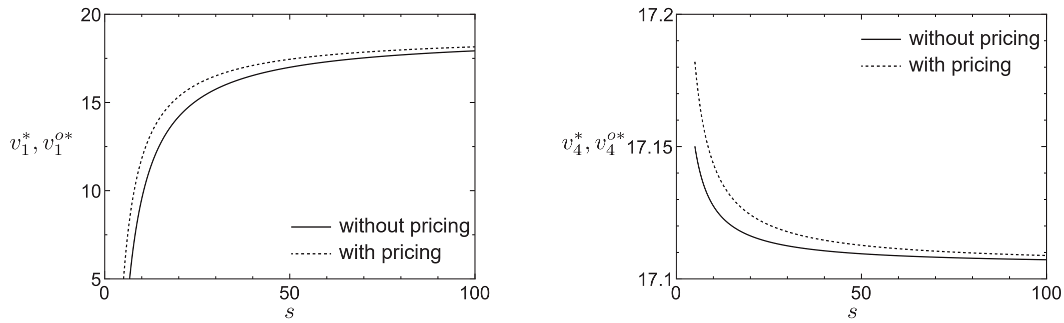
(a) Case A (b) Case B

Figure 5: Difference between city boundaries with and without pricing



(a) commuters 1 (the poorest commuters) (b) commuters 4 (the richest commuters)

Figure 6: Utility of commuters in Case A



(a) commuters 1 (the richest commuters) (b) commuters 4 (the poorest commuters)

Figure 7: Utility of commuters in Case B

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