Introduction to sub-interval analysis. Estimations for the centers of gravity

Harin, Alexander

Modern University for the Humanities

18 May 2020

Introduction to sub-interval analysis.
Estimations for the centers of gravity

Alexander Harin
aaharin@gmail.com
Modern University for the Humanities

An introduction to a sub-interval analysis (SI analysis or SIA) namely to a SI arithmetic is presented. Prerequisites and possible applications of the SIA are reviewed. A system of definitions of the SIA is formulated. New basic formulae are obtained. Some examples are considered including estimations of the minimal values of forbidden zones for measurements in behavioral economics. The article is concentrated mainly on estimations for the centers of gravity.

Contents

1. Introduction

A sub-interval analysis (SI analysis or S-I analysis or SIA or S-IA) was founded in 2011-2012 in reports and working papers of the author of the present article (see, e.g., Harin 2011.a-2011.d and Harin 2012.a-2012.e) as a new branch of the interval analysis (see, e.g., Moore 1966 and Shary 2020). At present the sub-interval analysis is only partially intersected with the traditional interval analysis, is essentially beyond its scope and is developed independently from it.

This article begins a systematic introduction to the basics of the SI analysis. The basics of a SI arithmetic are considered here in particular.

The prerequisites of the sub-interval analysis are the interval analysis and the needs of the tools for consideration of a lot of situations that are far beyond the scope of the traditional interval analysis.

Possible fields of applications of the SIA can include, e.g., accounting and audit, decision theory, databases, econometrics, image analysis and pre-recognition, long-term processes, micro- and macroeconomics, etc. The realized considerations have confirmed the usefulness of the applications of the SIA.

2. Main definitions, notations, and expressions

2.1. Main definitions and notations

Consider an interval $\boldsymbol{X} = [a, b] : 0 < (b\text{-}a) < \infty$. Consider a set of points $\{x_s\}$: s =0, 1, …, S : 1, $0 < S < \infty$, on this interval such that

$$\underline{a \equiv x_0 < x_1} < x_2 < \ldots < x_s < x_{s+1} < \ldots < x_S \equiv b\,.$$

This set of points divides the interval $\boldsymbol{X}$ into a set of S adjacent sub-intervals $\{\boldsymbol{X_s}\}$. Due to this division, the interval $\boldsymbol{X}$ may be denoted as $\boldsymbol{X_{1..S}}$.

The boundaries of $\{\boldsymbol{X_s}\}$ can be defined by various manners, for example by $\boldsymbol{X_s} \equiv [x_{s\text{-}1}, x_s)$ except of the far right sub-interval $\boldsymbol{X_S} \equiv [x_{S\text{-}1}, x_S] \equiv [x_{S\text{-}1}, b]$. The main condition of such definitions of the division is that any point of the interval should unambiguously belong to only one sub-interval.

So the interval $\boldsymbol{X_{1..S}}$ is divided into a set of adjacent sub-intervals $\{\boldsymbol{X_s}\}$.

The lengths of the sub-intervals may be denoted as L_s. They can be normalized by the length of the whole interval $L_{1..S} = b\text{-}a$ and we have $l_s = L_s/L_{1..S}$ and $l_{1..S} \equiv 1$.

Suppose a set of quantities $\{W_s\}$: $W_s \geq 0$, s =1, 2, … S : $1<S<\infty$ and

$$\sum_{s=1}^{S} W_s = W \equiv W_{1..S} < \infty\,.$$

For the purposes of the SI analysis, the quantities $\{W_s\}$ may be named as weights of the sub-intervals and may be normalized by the whole weight $W_{1..S}$ as $w_s = W_s/W_{1..S}$ and $w_{1..S} \equiv 1$. The normalized (or relative) weights and also $W_{1..S} \equiv 1$ will be used here as a rule due to their convenience.

Generally W_s may be assumed as, e.g., pointwise (see also the Section 4).

2.2. Main natural expressions

2.2.1. Center of gravity

One of the main natural expressions of the interval analysis is the rule for summation $k_1\boldsymbol{X_1} + k_2\boldsymbol{X_2} = [\, k_1\underline{X_1} + k_2\underline{X_2},\, k_1\overline{X_1} + k_2\overline{X_2}\,]$.

The main natural expressions of the sub-interval arithmetic describe the coordinates of the interval $\boldsymbol{G_{1..S}}$ for the center of gravity and are determined analogously to the interval analysis. The bottom (left) boundary $\underline{G_{1..S}}$ of the interval $\boldsymbol{G_{1..S}}$ for the center of gravity is

$$\underline{G_{1..S}} = \sum_{s=1}^{S} w_s \underline{X_s} \,.$$

The top (right) boundary $\overline{G_{1..S}}$ of the interval $\boldsymbol{G_{1..S}}$ for the center of gravity is

$$\overline{G_{1..S}} = \sum_{s=1}^{S} w_s \overline{X_s} \,.$$

The length of the interval for the center of gravity $\Delta G_{1..S} \equiv \text{leng}\, \boldsymbol{G_{1..S}}$ is equal to the difference between its top $\overline{G_{1..S}}$ and bottom $\underline{G_{1..S}}$ boundaries

$$\Delta G_{1..S} = \overline{G_{1..S}} - \underline{G_{1..S}} \,.$$

The expressions for center of gravity can be normalized by the length of the whole interval $L_{1..S}$ and we have $\boldsymbol{g_{1..S}} = \boldsymbol{G_{1..S}}/L_{1..S}$ and $\Delta g_{1..S} = \Delta G_{1..S}/L_{1..S}$.

2.2.2. Analogs of the moments

One may define analogs of the moments of n-th order for a set of quantities $\{w(x_k)\}$ relative to a point $x_{reference}$, where $1 \le n < \infty$ and $1 \le k < \infty$, as

$$M(x_{reference})^n = \sum_{k=1}^{K} w(x_k)(x_k - x_{reference})^n \,.$$

The bottom (left) boundary $\underline{M^n}$ of the interval $\boldsymbol{M^n}$ for the moments of n-th order of $\{w(x_s)\}$ relative to a point $x_{reference}$ is

$$\underline{M(x_{reference})^n} = \sum_{k=1}^{K} w(x_k)\underline{(x_k - x_{reference})^n} \,.$$

The top (right) boundary $\overline{M^n}$ of the interval $\boldsymbol{M^n}$ for the moments of n-th order of the set of quantities $\{w_s\}$ relative to a point $x_{reference}$ is

$$\overline{M(x_{reference})^n} = \sum_{k=1}^{K} w(x_k)\overline{(x_k - x_{reference})^n} \,.$$

The center of gravity corresponds to an analog of the expectation. If $\{w_s\}$ is the distribution of a random quantity then the above formulae determine the boundaries for the intervals for its moments.

3. Main equalities

3.1. Novosyolov formula

Let us estimate the length of the interval for the center of gravity of an interval that is divided by some sub-intervals. This length may be treated as a measure of precision of determination of the center of gravity. On that ground I will use a notation $\Delta G_{1..S}$ for the length of the interval for the center of gravity.

The length of the interval for the center of gravity $\Delta G_{1..S}$ can be easily transformed to a new formula

$$\Delta G_{1..S} \equiv \overline{G_{1..S}} - \underline{G_{1..S}} = \sum_{s=1}^{S} w_s \overline{X_s} - \sum_{s=1}^{S} w_s \underline{X_s} = \sum_{s=1}^{S} w_s \left(\overline{X_s} - \underline{X_s} \right) = \sum_{s=1}^{S} w_s L_s \ .$$

I have named the formula

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s L_s \qquad (1)$$

in honor of my mentor A.A. Novosyolov. It is referred to as the **Novosyolov formula** or shortly **N-formula**.

Let us try to derive one more formula.

The simplest case for a preliminary consideration is an interval $\boldsymbol{X_{1..2}}$ divided by two sub-intervals $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ under the condition that the weight of only one of the sub-intervals is known. However this example is not correct because we should know also the whole weight for the Novosyolov formula.

So let us start from the simplest correct case of three sub-intervals $\boldsymbol{X_1}$, $\boldsymbol{X_2}$, and $\boldsymbol{X_3}$ of an interval $\boldsymbol{X_{1..3}}$.

3.2. Weight-formula

The maximal possible length $\Delta G_{1..3}$ of the interval for the center of gravity of an interval $\mathbf{X_{1..3}}$ is the length $L_{1..3}$ of the whole interval. Let us consider this length $L_{1..3}$ as the limit point of overestimation.

The Novosyolov formula for the three sub-intervals is the sum of the three summands

$$\Delta G_{1..3} = w_1 l_1 + w_2 l_2 + w_3 l_3 .$$

Let us start from the statement that only one of the weights, e.g. the first one is known (and the total weight and all the lengths are known), and transform it equivalently to the limit point of overestimation

$$w_1 l_1 \equiv w_1 l_1 + w_1 l_2 + w_1 l_3 - w_1 l_2 - w_1 l_3 = w_1 L_{1..3} - w_1 l_2 - w_1 l_3 .$$

Such equivalent transformation for the total sum is

$$\begin{aligned} \Delta G_{1..3} &= w_1 l_1 + w_2 l_2 + w_3 l_3 = \\ &= w_1 L_{1..3} + w_2 L_{1..3} + w_3 L_{1..3} - \\ &- w_1 (l_2 + l_3) - w_2 (l_1 + l_3) - w_3 (l_1 + l_2) = \\ &= L_{1..3} - w_1 (l_2 + l_3) - w_2 (l_1 + l_3) - w_3 (l_1 + l_2) \end{aligned} .$$

So we can obtain the formula for the general case

$$\Delta G_{1..S} = L_{1..S} - \sum_{s=1}^{S} w_s \sum_{m \in [1,S], | m \neq s} l_m . \qquad (2)$$

In proper cases it can be written also in a simplified form as

$$\Delta G_{1..S} = L_{1..S} - \sum_{s=1}^{S} w_s (L_{1..S} - l_s) .$$

This equality, formula may be named as a Weight-formula of mass formula or shortly M-formula.

3.3. Length-formula

The Novosyolov formula includes not only the weights but the lengths as well. We may try to obtain one more formula.

Consider once more the same example of three sub-intervals but from the point of view of the lengths

$$w_1 l_1 \equiv w_1 l_1 + w_2 l_1 + w_3 l_1 - w_2 l_1 - w_3 l_1 = W_{1..3} l_1 - (w_2 + w_3) l_1 = l_1 - (w_2 + w_3) l .$$

Such equivalent transformation for the total sum is

$$\begin{aligned} &\Delta G_{1..3} = w_1 l_1 + w_2 l_2 + w_3 l_3 = \\ &= W_{1..3} l_1 + W_{1..3} l_2 + W_{1..3} l_3 - \\ &- (w_2 + w_3) l_1 - (w_1 + w_3) l_2 - (w_1 + w_2) l_3 = \\ &= L_{1..3} - (w_2 + w_3) l_1 - (w_1 + w_3) l_2 - (w_1 + w_2) l_3 \end{aligned} .$$

The formula differs from the case of the weights.

The Novosyolov formula for the three sub-intervals is the sum of the three summands

$$\Delta G_{1..3} = w_1 l_1 + w_2 l_2 + w_3 l_3 .$$

Let us consider the case when only one of the lengths, e.g. the first one is known (and the total length and all the weights are known), and transform it equivalently to the limit point of overestimation. The equivalent transformation for the total sum is

$$\Delta G_{1..3} = L_{1..3} - (w_2 + w_3) l_1 - (w_1 + w_3) l_2 - (w_1 + w_2) l_3 .$$

So we can obtain the formula for the general case

$$\Delta G_{1..S} = L_{1..S} - \sum_{s=1}^{S} l_s \sum_{p \in [1,S], | p \neq s} w_p . \tag{3}$$

In proper cases it can be written also in a simplified form as

$$\Delta G_{1..S} = L_{1..S} - \sum_{s=1}^{S} l_s (W_{1..S} - w_s) .$$

This equality, formula may be named as a Length-formula or exPanse-formula or sPace-formula or shortly P-formula.

3.4. Ring of formulae

So we can write an ensemble of the formulae

$$\begin{aligned}\Delta G_{1..S} &= \sum_{n=1}^{S} w_n l_n = \\ &= L_{1..S} - \sum_{m=1}^{S} w_m \sum_{s\in[1,S],|s\neq m} l_s = . \\ &= L_{1..S} - \sum_{p=1}^{S} l_p \sum_{s\in[1,S],|s\neq p} w_s\end{aligned} \tag{4}$$

I have named it as a “Ring of formulae”.
In proper cases it can be written also in a simplified form as

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s l_s = L_{1..S} - \sum_{s=1}^{S} w_s (L_{1..S} - l_s) = L_{1..S} - \sum_{s=1}^{S} l_s (W_{1..S} - w_s)\,.$$

4. Limited density situations

The above considerations I have assumed that the weights of the considered quantities can be pointwise. But actually the pointwise values are an abstraction, idealization or approximation. As a rule, real values cannot be pointwise even in the microcosm of the elementary particles.

In particular, the densities of the pointwise values of the considered quantities are assumed to be infinite. Or, these densities are assumed to be sufficiently high to neglect the lengths that are occupied by these quantities in comparison with the lengths of the sub-intervals.

Maximal densities $\rho_{max} < \infty$ of the weights can be introduced for real considerations of real situations. Such limited densities decrease evidently the interval uncertainty that is the inherent feature of the sub-intervals. The less the maximal density, the less the interval uncertainty. In the limit when

$$\rho_{\max} = \frac{W_{1..S}}{L_{1..S}},$$

the interval uncertainty is equal to zero.

The densities of the weights of the considered quantities can be also limited from below by a certain minimal density $\rho_{min} > 0$. This minimal density can be considered as a non-zero background. Such a limited density decreases evidently the interval uncertainty of the sub-intervals as well. The more the minimal density, the less the interval uncertainty. In the limit when

$$\rho_{\min} = \frac{W_{1..S}}{L_{1..S}},$$

the interval uncertainty is equal to zero too.

5. Situations with incomplete information

A valuable feature of the SI arithmetic is the possibility to estimate situations with incomplete information. Let us consider basic manifestations of this feature.

5.1. Theorem of interval character of incomplete knowledge

Let us consider and prove one of possible variants of a general theorem about interval character of incomplete knowledge. It can be useful, e.g., to analyze and estimate long-term processes and unfinished series of measurements.

Theorem of interval character of incomplete knowledge. Discrete finite case. Suppose that a set of quantities $\{w(x_k)\} : w(x_k) > 0$, is defined on a discrete set of points $\{x_k\} : k = 1, 2, \dots K : K < \infty$, of an interval $\boldsymbol{X}=[a, b]$. If the quantities $\{w(x_k)\}$ are exactly known in a subset $\{x_{k.exact}\}$ of the set $\{x_k\}$, that will be referred to as the subset of "exact" point (we denote the subset of these "exact" points as $\{x_{k.exact}\}$), except of at least two points that will be referred to as the "inexact" points $x_{inexact1}$ and $x_{inexact2}$, that is

$$\{w(x_k)\} = \{w(x_{k.exact})\} \bigcup w(x_{inexact1}) \bigcup w(x_{inexact2}),$$

the distance between these two "inexact" points is $|x_{inexact2} - x_{Inexact1}| \geq 2l_{min} > 0$ and the quantities $w(x_{inexact1})$ and $w(x_{inexact2})$ may vary in a certain non-zero interval Δ such that $\max(w(x_{inexact1})) - \min(w(x_{inexact1})) \geq \Delta w > 0$ and $\max(w(x_{inexact2})) - \min(w(x_{inexact2})) \geq \Delta w > 0$, then any analog of finite moment for the set $\{w(x_k)\}$ is known within the accuracy not better than a non-zero interval.

Proof. As long as the distance between the two "inexact" points is non-zero $|x_{inexact2} - x_{Inexact1}| \geq 2l_{min} > 0$, then for any point reference point $x_{reference}$ at least one of two "inexact" points, say $x_{inexact1}$, is evidently remoted from this point not less than $|x_{inexact1} - x_{reference}| \geq l_{min}$.

Let us denote the exactly known parts of moment analogs $M(x_{reference})^n$ as

$$M_{exact}(x_{reference})^n = \sum_{k=1}^{K-2} w(x_{k.exact})(x_{k.exact} - x_{reference})^n .$$

The general expression for the analogs of the moments can be rewritten as

$$\begin{aligned} M(x_{reference})^n &= M_{exact}(x_{reference})^n \\ &+ w(x_{inexact1})(x_{inexact1} - x_{reference})^n + w(x_{inexact2})(x_{inexact2} - x_{reference})^n . \end{aligned}$$

As long as the distance $|x_{reference} - x_{inexact1}|$ is not less than l, then we obtain

$$\begin{aligned} \Delta M(x_{reference})^n &\equiv |\overline{M(x_{reference})^n} - \underline{M(x_{reference})^n}| \\ &\geq [\max(w(x_{inexact1})) - \min(w(x_{inexact1}))] \, |x_{inexact1} - x_{reference}|^n . \\ &\geq \Delta w \times l_{min}{}^n > 0 \end{aligned}$$

Taking into account any additional "inexact" point can only increase these uncertainties. □

5.2. Underestimation and overestimation formulae. Main chain of inequalities

Let us consider the obtained formulae once more. Consider the simplest case of certain three sub-intervals $\boldsymbol{X_1}$, $\boldsymbol{X_2}$, and $\boldsymbol{X_3}$ of an interval $\boldsymbol{X_{1..3}}$.

Suppose that only l_1, w_1, $L_{1..3}$ and $W_{1..3}$ are known.

The Novosyolov formula gives

$$\Delta G_{1..3} \geq w_1 l_1 .$$

So it can be named as an underestimation formula.

The Weight-formula gives

$$\Delta G_{1..3} \leq L_{1..3} - w_1 (L_{1..3} - l_1) .$$

The Length-formula gives

$$\Delta G_{1..3} \leq L_{1..3} - (W_{1..3} - w_1) l_1 .$$

So the two last formulae can be named as overestimation formulae.

So when we know $P \leq S$ lengths, $M \leq S$ weights, and $N \leq \min(M, P)$ known both lengths and weights, we can write

$$\sum_{n=1}^{N} w_n l_n \leq \Delta G_{1..S} \leq L_{1..S} - \max \begin{cases} \sum_{m=1}^{M} w_m \sum_{s \in [1,S], | s \neq m} l_s \\ \sum_{p=1}^{P} l_p \sum_{s \in [1,S], | s \neq p} w_s \end{cases} , \qquad (5)$$

This ensemble of inequalities is the main chain of inequalities of the SI-arithmetic for situations of incomplete information.

In proper cases it can be written also in a simplified form as

$$\sum_{n=1}^{N} w_n l_n \leq \Delta G_{1..S} \leq L_{1..S} - \max \begin{cases} \sum_{m=1}^{M} w_m (L_{1..S} - l_m) \\ \sum_{p=1}^{P} l_p (W_{1..S} - w_p) \end{cases} .$$

6. Examples

6.1. Statements for minimal and maximal densities

A theorem of existence of restrictions (forbidden zones) for measurements in the behavioral economics was proved, e.g., in Harin ().The theorem may help to explain basic utility paradoxes such as the underweighting of high and the overweighting of low probabilities, risk aversion, the Allais paradox, risk premium, etc.. In particular, taking into account these restrictions diminishes the absolute values of the paradoxes and may help to partially explain them.

Sub-interval versions of this theorem are given here in addition to other considerations. Here are sub-interval versions of this theorem.

6.1.1. Limited minimal density

Consider situations when the weight density $\rho \equiv \rho_w = \rho_{weight}$ is not less than some non-zero minimal value ρ_{min}.

Statement of existence of forbidden zones for the minimal density. Suppose there is an interval $[a, b] : 0 < (b\text{-}a) < \infty$, and a quantity (weight) $w(x) \geq 0$ (of the total weight W_{total}) is defined on $[a, b]$. If the density ρ of $w(x)$ is not less than $\rho_{min} > 0$, then certain forbidden zones of the non-zero width exist near the boundaries a and b of $[a, b]$ for the total center of gravity G_{total} of $w(x)$.

Proof. Consider separately the centers of gravity for the minimal filling of the quantity and for the rest part. The minimal filling gives the center of gravity of the value $\rho_{\min} \times (b-a)$ in the center of the interval. The weight of the rest part is $W - \rho_{\min} \times (b-a)$. If the center of gravity G_{rest} of the rest part is located at $G_{rest} = x$ then (assuming, e.g., $a = 0$) the total center of gravity G_{total} is located at

$$G_{total} = x \times [W - \rho_{\min} \times (b-a)] + \frac{b-a}{2} \rho_{\min} \times (b-a).$$

When the rest part is located at one of its limit points that is at one of the boundaries of $[a, b]$, e.g., at $a = 0$ then G_{total} is located at its limit point

$$G_{total.\min} = \frac{b-a}{2} \rho_{\min} \times (b-a).$$

The difference between the coordinates of this limit point and the nearest boundary a can be named a forbidden zone for the center of gravity. The consideration for the boundary b is evidently the same. So, the width of forbidden zones (or restriction) $r_{restrict}$ is

$$r_{restrict} = \frac{(b-a)^2}{2} \rho_{\min} > 0. \qquad \Box$$

This statement is true for both the total interval and all its sub-intervals. Naturally the widths of the forbidden zones for the sub-intervals are determined by their lengths. One can say that such a minimal density restricts an “effective” length of both the interval and sub-intervals by these forbidden zones. The more the minimal density, the less the uncertainty for the center(s) of gravity. There is no uncertainty at the maximal density.

6.1.2. Limited maximal density

Consider situations when the weight density $\rho \equiv \rho_w = \rho_{weight}$ is not more than some non-zero maximal value ρ_{max}.

Statement of existence of forbidden zones for the maximal density. Suppose there is an interval $\boldsymbol{X}$ such that at least one of its boundaries, e.g., the left boundary a is finite and a quantity (weight) $w(x) \geq 0$ (of the total weight W_{total}) is defined on $\boldsymbol{X}$. If the density ρ of $w(x)$ is not more than $\rho_{max} < \infty$, then the certain forbidden zone of the non-zero width exist near the boundaries a and b of $\boldsymbol{X}$ for the total center of gravity G_{total} of $w(x)$.

Proof. The existence of the maximal finite density means that the total weight cannot be concentrated in a single point and it can be concentrated only within the length not less than W_{total} / ρ_{max}. Due to this restriction the total center of gravity cannot be located with respect to a nearer than a half of the above length. So the width $r_{restrict}$ of the forbidden zone is

$$r_{restrict} = \frac{W_{total}}{2\rho_{\max}} > 0 . \qquad \square$$

6.2. Weights versus lengths

Let us compare the results that we can obtain by means of the Weights-formula and Length-formula. Let us consider the case when the weight and length of only one sub-interval (and the total weight and length of the whole interval) are known.

Suppose we know the relative weight and length $w_1 = 0.2$ and $l_1 = 0.1$.

The weight formula gives

$$\Delta G_{1..S} = L_{1..S} - \sum_{s=1}^{S} w_s \sum_{n\in[1,S],|n\neq s} l_n = L_{1..S} - w_1(L_{1..S} - l_s) = 1 - 0.2\times 0.9 = 0.82 .$$

The length formula gives

$$\Delta G_{1..S} = L_{1..S} - l_1(W_{1..S} - w_1) = 1 - 0.1\times 0.8 = 0.92 .$$

Suppose the known the values of the relative weight and length are inversed and equal $w_1 = 0.1$ and $l_1 = 0.2$.

The weight formula gives

$$\Delta G_{1..S} = L_{1..S} - w_1(L_{1..S} - l_s) = 1 - 0.1\times 0.8 = 0.92 .$$

The length formula gives

$$\Delta G_{1..S} = L_{1..S} - l_1(W_{1..S} - w_1) = 1 - 0.2\times 0.9 = 0.82 .$$

So we see that the both of the formulae can be applied here and the exactness of the results of their application depends on the particular values of the weight and length and on the proportion between the weight and length.

6.3. Weights

Equal weights. Suppose we know that the weights of all the sub-intervals are equal to each other, that is they equals $W_s = W_{1..s}/S$ or $w_s = 1/S$. Then we have

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s L_s = \frac{1}{S}\sum_{s=1}^{S} L_s = \frac{L_{1..S}}{S}.$$

Minimal weight. Suppose we know the minimal weight W_{min} or w_{min} for the sub-intervals. Then we have

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s L_s \geq w_{\min}\sum_{s=1}^{S} L_s = w_{\min} L_{1..S} = L_{1..S}\frac{W_{\min}}{W_{1..S}}.$$

Maximal weight. Suppose we know the maximal weight W_{max} or w_{max} for the sub-intervals. Then we have

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s L_s \leq w_{\max}\sum_{s=1}^{S} L_s = w_{\max} L_{1..S} = L_{1..S}\frac{W_{\max}}{W_{1..S}}.$$

The only weight. Suppose we know the only weight W_{only} or $w_{only} \equiv w_o$ of a certain sub-interval. Then we can draw easily from the weight-formula (2)

$$\Delta G_{1..S} \leq L_{1..S} - w_o \sum_{m\in[1,S],|m\neq o} L_m .$$

Note that to draw such deductions we do not need any information on the length of the sub-intervals.

6.4. Lengths and general formulae

Equal lengths. Suppose we know that the lengths of all the sub-intervals are equal to each other, that is they equals $L_s = L_{1..s}/S$. Then we have

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s L_s = \frac{L_{1..S}}{S} \sum_{s=1}^{S} w_s = \frac{L_{1..S}}{S}.$$

Minimal length. Suppose we know the minimal length L_{min} for the sub-intervals. Then we have

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s L_s \geq L_{\min} \sum_{s=1}^{S} w_s = L_{\min}.$$

Maximal length. Suppose we know the maximal length L_{max} for the sub-intervals. Then we have

$$\Delta G_{1..S} = \sum_{s=1}^{S} w_s L_s \leq L_{\max} \sum_{s=1}^{S} w_s = L_{\max}.$$

The only length. Suppose we know the only length $L_{only} \equiv L_o$ of a certain sub-interval. Then we can draw easily from the weight-formula (2)

$$\Delta G_{1..S} \leq L_{1..S} - L_o \sum_{m \in [1,S], | m \neq o} w_m.$$

Note that to draw such deductions we do not need any information on the weights of the sub-intervals.

So we can write the following general formulae.

Equality. If we know that the lengths or weights of all the sub-intervals are equal to each other, then we have

$$\Delta G_{1..S}(equality) = \frac{L_{1..S}}{S} \qquad (6).$$

Minimum. If we know the minimal length or weight for the sub-intervals then we have

$$\Delta G_{1..S} \geq \min\left(\begin{cases} L_{\min} \\ L_{1..S} \dfrac{W_{\min}}{W_{1..S}} \end{cases} \right) \qquad (7).$$

Maximum. If we know the maximal length or weight for the sub-intervals then we have

$$\Delta G_{1..S} \geq \max\left(\begin{cases} L_{\max} \\ L_{1..S} \dfrac{W_{\max}}{W_{1..S}} \end{cases} \right) \qquad (8).$$

The only value. Suppose we know the only value either weight W_{only} (or $w_{only} \equiv w_o$) or length $L_{only} \equiv L_o$ of a certain sub-interval. Then we have

$$\Delta G_{1..S} \leq L_{1..S} - \max \begin{cases} w_o \sum\limits_{m \in [1,S], | m \neq o} L_m \\ L_o \sum\limits_{m \in [1,S], | m \neq o} w_m \end{cases} \qquad (9).$$

7. Conclusions

This article starts the systematic introduction to the sub-interval analysis. The first part of the introduction is devoted to the SI arithmetic and estimations for the centers of gravity.

Such estimations can be used to calculate or exactly evaluate intervals for the centers of gravity for time (year, month, day, …) or spatial (island, continent, state, province, city, …) or other characteristics of various objects and systems.

The valuable field of applications of SI arithmetic is the analysis and estimations for situations with incomplete information.

References

Harin, A. (2011a) About possible additions to interval arithmetic, X International conference on Financial and Actuarial Mathematics and Eventoconvergence of Technologies, Krasnoyarsk, (2011).

Harin, A. (2011b) Ruptures in the probability scale. Interval analysis. *International conference "Modern problems of applied mathematics and mechanics" devoted to 90th Anniversary from the birthday of academician N.N.Yanenko*, Novosibirsk, (2011).

Harin, A. (2011c) Interval analysis of distributions. Interval images of text, music, representations and video information, *54-th Scientific conference of MIPT "Modern problems of fundamental and applied sciences"*, Moscow, (2011).

Harin, A. (2011d) " Interval analysis of distributions and ruptures", MPRA Paper, No 35663 (2011).

Harin, A. (2012a) Subinterval analysis. First results. 15th International Symposium on Scientific Computing, Computer Arithmetic and Verified Numerical Computations, Novosibirsk, (2012).

Harin, A. (2012b) "Sub-interval analysis and possibilities of its use", 55-th Scientific conference of MIPT "Modern problems of fundamental and applied sciences", Moscow, (2012). http://mipt.ru/nauka/55conf/f_5v77jd/f_5v7b0a-arphacx5reu

Harin, A. (2012c) Interval pictures and images. Their use for preliminary analysis and recognition, *XIX International conference "Mathematics. Computer. Education"*, Dubna, (2012).

Harin, A. (2012d) About global optimization in subinterval analysis at analog of Lipschitz's condition, *XX International conference "Mathematics. Economics. Education"*, Rostov-na-Donu, (2012).

Harin, A. (2012e) About interval analogues of scalar products and Fourier series, *VII International symposium "Fourier series and their application"*, Rostov-na-Donu, (2012).

Moore, R. E. (1966) *Interval Analysis*. Prentice-Hall, Englewood Cliffs N. J., 1966.

Shary, S. P. (2020) *Finite-dimensional interval analysis*. “XYZ”, Novosibirsk, 2020.