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Abstract

What happens when information reaches the human brain? In economics, a black-box approach to information processing in the brain is generally taken with an implicit assumption that information, once it reaches the brain, is accurately processed. In sharp contrast, research in brain sciences has established that when information reaches the brain, a mental template or schema (neural substrate of knowledge) is first activated, which influences information absorption. Schemas are created through a resource intensive process in which finite brain resources are allocated to different tasks, with resource allocation in the brain having an impact on the structure of schemas. In this article, we explore the implications of this richer view from brain sciences for the capital asset pricing model (CAPM). We show that two versions of CAPM arise depending on how the brain resources are allocated in schema creation. In one version, the relationship between beta and expected returns is flat along with features akin to value, size, and momentum effects. In the second version, the relationship between beta and expected return is strongly positive with an implied risk-free rate which could be negative. Novel predictions emerging from this approach are: momentum is negatively correlated with value, size, and betting-against-beta, and stocks that command a lion's share of investor attention have lower risk-adjusted returns.

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Resource Allocation in the Brain and the Capital Asset Pricing Model

What happens when information reaches the human brain? In economics, a black-box approach to information absorption is typically taken with an implicit assumption that information, when it reaches the brain, is accurately processed. In sharp contrast, research in brain sciences has established that when information reaches the brain, a mental template or schema, is first activated, which influences information absorption.[1] Brain imaging studies show that schema formation is a resource-intensive process that involves different regions of the brain talking to each other[2]; however, these schemas, once formed, make subsequent processing of schema-consistent information a lot faster.[3] In this article, we study the implications of this richer view from brain sciences for the capital asset pricing model (CAPM).

A schema can be conceived as a scaffold or a blueprint,[4] representing a pre-existing knowledge structure. Neurologically, it is a brain template that involves systems of neurons across various brain regions talking to each other, with each system constituting a particular unit in the schema. That is, schemas contain units as well as relationships between these units. For example, for a car schema, units could be car body and wheel, with the relationship that car body contains four wheels. For a firm schema, units could be expected cash flow levels and associated risks with a specific relationship between these units. Schemas, by only containing the essential details, simplify the world. They direct attention to relevant aspects, and speed-up processing of information that fits within the schema.

New schemas are created by attempting to appropriately modify a related schema. Brain organizes knowledge in a network of such interconnected schemas. For example, a child may initially only have a schema for a horse (large with four legs, hair, and a tail).

[1] There is a large body of literature in neuroscience that explores various facets of schemas and how they influence information absorption (for a review, see van Kesteren et al (2012), Gilboa and Marlatte (2017), Spalding et al (2015) and references therein).
[2] See Ohki and Takei (2018) and references therein.
[3] Sweegers et al (2015)
[4] See Hampson and Morris (1996) or Anderson (2000) for a detailed review of schema theory.

However, when she encounters a cow, a new schema for a cow could be created by modifying the horse schema. Similarly, relevant to our context, an investor analysing a firm for the first time, may create a new schema for the firm by altering the schema of a similar firm that she has analysed before. Studying the implications of such a schema-creation process for CAPM is the subject of this article.

Research in brain sciences has established that there is brain specialization with different brain systems performing different tasks and competing for scarce resources that are allocated by a 'central executive system' (CES) located in the lateral prefrontal cortex (see Alonso et al (2014) and references therein). This suggests that, while modifying an existing schema to create a new one, each unit in a schema is exclusively worked on by a distinct system of neurons. Each system makes demands for resources with task performance dependent on resource allocation. For relatively simple schemas (such as for a cow or a car), the resource constraint is not binding and all units in an existing schema are fully adjusted to create accurate units in the new schema. However, for sufficiently complex schemas such as a firm schema, the resource constraint is likely to be binding. In the context of CAPM, keeping things simple, expected cash flows and risk of the cash flows are the two key units in the schema of a given firm. So, each unit is worked on by a different system of neurons while modifying an existing schema to create a new one. With a binding resource constraint, how the scarce brain resources are split across the two units matters.

In this article, we consider two ways in which scarce brain resources can be allocated towards the two units while creating a new schema for a firm:[5]

1) More resources are allocated to the brain system working on expected cash flows.

2) More resources are allocated to the brain system working on risk of the cash flows.

It follows that there are two types of traders: 1) Traders (cashflow-schema traders), who are better at processing cashflow information than risk information as they have a schema with a more accurate cashflow unit. Such traders either underreact or overreact to

[5] The third case in which the resources are split evenly across the two units is absorbed in the first case. This is because task complexity (number of distinct mental operations needed) is higher for risk estimation, which is known to lower task performance (Alonso et al 2014). So, with resources evenly split, risk unit is relatively more inaccurate. Hence, the same outcomes as in the first case are obtained in this case.

risk news. 2) Traders (risk-schema traders) who are better at processing risk-related information as they have a schema with a more accurate risk unit. Such traders either underreact or overreact to cashflow news.

We show that which trader type is marginal matters for CAPM. When a cashflow-schema trader is marginal, a version of CAPM is obtained (cashflow-schema CAPM), which displays a flatter relationship between stock beta and expected excess returns. High-alpha-of-low-beta along with features akin to value, size, and momentum effects arise in this version. Two novel predictions are: momentum is negatively correlated with value, size, and betting-against-beta, and stocks that command a lion's share of investor and analyst attention have lower risk-adjusted returns.

When a risk-schema trader is marginal, another version of CAPM arises (risk-schema CAPM). In this version, there is a strong positive relationship between beta and expected excess return with an implied risk-free rate that could be negative. Stocks that do better in the first version (low beta, small) do worse in the second version. We hypothesize that, normally, the marginal investor is expected to be a cashflow-schema trader (Basu et al 2013). However, consistent with the findings in Hendershott et al (2019) and Savor and Wilson (2014), there are specific times when the risk-schema trader is marginal such as at-open and on macroeconomic announcement days, leading to a steeper relationship between beta and average stock returns at such times.

Schemas, even though imperfect, generate price predictions that are rank-order correct: riskier stocks are priced lower than less risky stocks. This is a pretty good showing given the resource constraint. A key advantage is that substantially less brain resources are needed to process information that fits within a schema, a process known as assimilation in cognitive science literature. Modifying or adapting a schema (accommodation) is much more resource intensive.[6] So, diverting more brain resources to a particular schema (accommodation) has a substantial opportunity cost. Hence, schemas, once established, are resistant to change.

[6] See van Kesteren and Meeter (2020) for a discussion on assimilation and accommodation processes in the context of brain processes.

2. CAPM adjusted for Resource Allocation in the Brain

We take a modern derivation of CAPM (such as in Frazzini and Pedersen (2014)) and add a twist to it, which is incorporating the implications of information processing through a schema as created by a resource-constrained brain. As in Frazzini and Pedersen (2014), we consider an overlapping generations (OLG) economy. Each agent lives for two periods. Agents that are born at t aim to maximize their utility of wealth at $t+1$. Their utility functions are identical and exhibit mean-variance preferences. They trade securities $s = 1, \cdots, S$ where security s pays dividends d_t^s and has n_S^* shares outstanding, and invest the rest of their wealth in a risk-free asset that offers a rate of r_F.

The market is described by a representative agent who maximizes:

$$\max n'\{E_t(P_{t+1} + d_{t+1}) - (1 + r_F)P_t\} - \frac{\gamma}{2} n'\Omega_t n$$

where P_t is the vector of prices, Ω_t is the variance-covariance matrix of $P_{t+1} + d_{t+1}$, and γ is the risk-aversion parameter.

It follows that the price of a security, s, is given by:

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma Cov(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \tag{2.1}$$

where security s payoff is $X_{t+1}^s = P_{t+1}^s + d_{t+1}^s$

and the aggregate market payoff is:

$$X_{t+1}^M = n_1^*(P_{t+1}^1 + d_{t+1}^1) + n_2^*(P_{t+1}^2 + d_{t+1}^2) + \cdots\cdots\cdots + n_S^*(P_{t+1}^S + d_{t+1}^S).$$

2.1 Schema Creation

As discussed in the introduction, schema is a mental template that contains units as well as a relationship between units. With mean-variance preferences, the relevant units are expected cash flows and the risk of cash flows, with risk measured by covariance of cash flows with the aggregate market cash flows. We define a firm-schema as follows: "*It is a* *knowledge structure about a firm's expected cashflows and associated risks, which facilitates processing new information to evaluate one's willingness-to-pay (WTP*)".

So, given (2.1), a firm-schema has the following general form:

$$WTP = \frac{A\ function\ of\ expected\ cashflows - (riskaversion)(A\ function\ of\ risk)}{1 + riskfree\ rate}$$

To understand the process of schema creation, we consider how a typical stock analyst behaves while analysing a firm. Stock analysis is usually done at firm-level cashflows, which are then transformed to the level of an individual security. We denote firm-level earnings or cashflows by π_{t+1}^s where the number of outstanding shares is n_s^*. Earnings-per-share (EPS) is then given by: $EPS_{t+1} = \frac{\pi_{t+1}^s}{n_s^*}$. Denoting the price-earnings (P/E) ratio, inclusive of dividends, for firm s by c_s:

$$X_{t+1}^s = P_{t+1}^s + d_{t+1}^s = c_s(EPS_{t+1}) = c_s \frac{\pi_{t+1}^s}{n_s^*}$$

We assume that when a trader analyses the cash flows of a firm s for the first time, she creates a schema by modifying the schema for a similar firm q that she has analysed earlier. The two units that constitute a schema for a firm are: expected cashflows and the risk of cashflows. So, the process of creating a new schema by modifying an existing schema requires modifications in these two units.

For expected cash flow levels, the modification is:

$$E'(\pi_{t+1}^s) = E(\pi_{t+1}^q) - m_1 D_1$$

where $D_1 = E(\pi_{t+1}^q) - E(\pi_{t+1}^s)$ is the correct adjustment needed, and $0 \leq m_1 \leq 1$, captures the fraction of correct adjustment reached. If the brain is not resource constrained, then $m_1 = 1$, which corresponds to full or correct adjustment. On the other hand, $m_1 < 1$, indicates that the resource constraint in the brain is binding.

Transforming to the level of EPS:

$$\frac{E'(\pi_{t+1}^s)}{n_s^*} = \frac{E(\pi_{t+1}^q)}{n_q^*}\frac{n_q^*}{n_s^*} - \frac{m_1 D_1}{n_s^*}$$

$$\Rightarrow E'(EPS_{t+1}^{s}) = \frac{n_q^*}{n_s^*} E\left(EPS_{t+1}^{q}\right) - m_1\left(\frac{n_q^*}{n_s^*} E\left(EPS_{t+1}^{q}\right) - E(EPS_{t+1}^{s})\right)$$

$$\Rightarrow E'(EPS_{t+1}^{s}) = (1 - m_1)\frac{n_q^*}{n_s^*} E\left(EPS_{t+1}^{q}\right) + m_1 E(EPS_{t+1}^{s})$$

Similarly, the schema-unit for the risk of cash flows is obtained as follows:

$$Cov'(\pi_{t+1}^{s}, X_{t+1}^{M}) = Cov\left(\pi_{t+1}^{q}, X_{t+1}^{M}\right) - m_2 D_2$$

$$\Rightarrow \frac{Cov'(\pi_{t+1}^{s}, X_{t+1}^{M})}{n_s^*} = \frac{Cov\left(\pi_{t+1}^{q}, X_{t+1}^{M}\right)}{n_q^*}\frac{n_q^*}{n_s^*} - m_2\left(\frac{Cov\left(\pi_{t+1}^{q}, X_{t+1}^{M}\right)}{n_q^*}\frac{n_q^*}{n_s^*} - \frac{Cov(\pi_{t+1}^{s}, X_{t+1}^{M})}{n_s^*}\right)$$

$$\begin{aligned}\Rightarrow Cov'(EPS_{t+1}^{s}, X_{t+1}^{M}) & \\ &= Cov\left(EPS_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} \\ &- m_2\left(Cov\left(EPS_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} - Cov(EPS_{t+1}^{s}, X_{t+1}^{M})\right)\end{aligned}$$

$$\Rightarrow Cov'(EPS_{t+1}^{s}, X_{t+1}^{M}) = (1 - m_2)Cov\left(EPS_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} + m_2\left(Cov(EPS_{t+1}^{s}, X_{t+1}^{M})\right)$$

Following the behavior of a typical stock analyst, we define the notion of similar firms as having the following two properties:

1) Firms that are in the same line of business, and
2) Have the same P/E ratios.

P/E ratios (inclusive of dividends) for s and q are c_s and c_q, and applying the above properties, the firms are in the same line of business with similar P/E ratios:

$$c_c \approx c_q = c$$

So, the schema-unit for risk of the cash flows is estimated as:

$$Cov'(cEPS_{t+1}^{s}, X_{t+1}^{M}) = (1 - m_2)Cov\left(cEPS_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} + m_2\left(Cov(cEPS_{t+1}^{s}, X_{t+1}^{M})\right)$$

$$\Rightarrow Cov'(X_{t+1}^{s}, X_{t+1}^{M}) = (1 - m_2)Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} + m_2\left(Cov(X_{t+1}^{s}, X_{t+1}^{M})\right)$$

$$\Rightarrow Cov'(X_{t+1}^{s}, X_{t+1}^{M})$$
$$= Cov(X_{t+1}^{s}, X_{t+1}^{M}) + (1-m_2)\left(Cov\left(X_{t+1}^{q}, X_{t+1}^{M}\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^{s}, X_{t+1}^{M})\right) \quad (2.2)$$

Similarly, the schema-unit for expected cash flow levels can be written as:

$$E'(cEPS_{t+1}^{s}) = (1-m_1)\frac{n_q^*}{n_s^*}E\left(cEPS_{t+1}^{q}\right) + m_1 E(cEPS_{t+1}^{s})$$

$$\Rightarrow E'(X_{t+1}^{s}) = (1-m_1)\frac{n_q^*}{n_s^*}E\left(X_{t+1}^{q}\right) + m_1 E(X_{t+1}^{s})$$

$$\Rightarrow E'(X_{t+1}^{s}) = E(X_{t+1}^{s}) + (1-m_1)\left(E\left(X_{t+1}^{q}\right)\frac{n_q^*}{n_s^*} - E(X_{t+1}^{s})\right) \quad (2.3)$$

(2.2) and (2.3) capture the following two properties associated with resource allocation in the brain (see Alonso et al (2014)):

1) When a new schema is created by modifying an existing schema, the process is broken down into separate tasks, with each unit worked on by a separate system of neurons. Each system communicates its resource requirements to CES, which allocates finite brain resources between systems.

2) The resource constraint is generally binding for complex schemas with task performance dependent on how much of resources are allocated to that particular task.

2.2 Cashflow-Schema CAPM

Schema creation is a resource intensive process. A separate system of neurons is allocated to each unit in the schema of a firm, with allocation of brain resources to the two units determined by CES. In this section, we assume that more resources are devoted to the unit for expected cash flows when compared with the unit for risk of the cash flows. That is, $m_1 > m_2$. We refer to such traders as having a cashflow-schema, and the CAPM so obtained is referred to as the cashflow-schema CAPM. In section 2.4, we consider the other case where $m_2 > m_1$ (with such traders referred to as risk-schema traders).

Without loss of generality, we set $m_1 = 1$, it then follows that $m_2 = m < 1$. Suppose, there is a firm q that had been analysed earlier, and its schema is modified to create a schema for firm s.

The share price of firm q is given by (from 2.1):

$$P_t^q = \frac{E\left(X_{t+1}^q\right) - \gamma Cov\left(X_{t+1}^q, X_{t+1}^M\right)}{1 + r_F} \tag{2.4}$$

And, the share price of firm s is given by (using 2.2):

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma\left\{Cov(X_{t+1}^s, X_{t+1}^M) + (1-m)\left(Cov\left(X_{t+1}^q, X_{t+1}^M\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^s, X_{t+1}^M)\right)\right\}}{1 + r_F} \tag{2.5}$$

A schema is handy as it lowers the resource requirement for processing information that fits within a schema (for example, information about $E(X_{t+1}^s)$ or $Cov(X_{t+1}^s, X_{t+1}^M)$). Using $E^{Assimilation}$ to denote the brain energy requirements for assimilation in the schema, and E to denote the requirements without a schema:

Assimilation: $E^{Assimilation} \ll E$

However, this resource saving comes at a cost, which is underreaction and overreaction to risk-news. There is underreaction to firm-specific risk news as:

$$\frac{\partial P_t^s}{\partial Cov(X_{t+1}^s, X_{t+1}^M)} = -\frac{\gamma m}{1 + r_F} \tag{2.5.1}$$

(2.5.1) shows underreaction to firm-specific risk-news as $m < 1$ when compared with rational expectations $(m = 1)$. There is also overreaction to irrelevant risk news as:

$$\frac{\partial P_t^s}{\partial Cov\left(X_{t+1}^q, X_{t+1}^M\right)} = -\frac{\gamma(1-m)\frac{n_q^*}{n_s^*}}{1 + r_F} \tag{2.5.2}$$

(2.5.2) shows overreaction to irrelevant risk news as $m < 1$ when compared with rational expectations $(m = 1)$.

In contrast with assimilation, the process of modifying a schema (accommodation) is much more resource intensive. Accommodation involves spending resources to modify m. Using $E^{Accommodation}$ to denote the energy required for processing information about m:

$$E^{Accommodation} \gg E^{Assimilation}$$

So, a schema, once established, is modified (accommodation) only if the additional benefits of doing so justify the high opportunity cost (high energy cost) involved.

From (2.4) and (2.5), the expected returns of s and q are then (with $R_F = 1 + r_F$):

$$E\left[R_{t+1}^q\right] = R_F + \frac{\gamma}{P_t^q} Cov\left(X_{t+1}^q, X_{t+1}^M\right) \tag{2.6}$$

$$E[R_{t+1}^s] = R_F + \frac{\gamma}{P_t^s}\left\{Cov(X_{t+1}^s, X_{t+1}^M) + (1-m)\left(Cov\left(X_{t+1}^q, X_{t+1}^M\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^s, X_{t+1}^M)\right)\right\} \tag{2.7}$$

To fix ideas, initially it is useful to assume that there are just two firms in the market, s and q before generalizing to N firms. Multiplying (2.6) by $w_q = \frac{n_q^* P_t^q}{P_t^M}$, which is the weight of firm q in the market portfolio ($P_t^M$is the price of aggregate market portfolio), multiplying (2.7) by $w_s = \frac{n_s^* P_t^s}{P_t^M}$, and adding:

$$E[R_{t+1}^M] = R_F + \frac{\gamma}{P_t^M}\{Var(X_M) + (1-m)\left(Cov\left(X_{t+1}^q, X_{t+1}^M\right)n_q^* - Cov(X_{t+1}^s, X_{t+1}^M)n_s^*\right)\}$$

$$\Rightarrow \gamma = \frac{(E[R_{t+1}^M] - R_F)P_t^M}{\{Var(X_M) + (1-m)\left(Cov\left(X_{t+1}^q, X_{t+1}^M\right)n_q^* - Cov(X_{t+1}^s, X_{t+1}^M)n_s^*\right)\}} \tag{2.8}$$

Substituting (2.8) in (2.6) and re-arranging/simplifying leads to the modified CAPM equation for q:

$$E\left[R_{t+1}^q\right] = R_F + (E[R_{t+1}^M] - R_F) \cdot \beta_q \cdot \left(\frac{1}{1 + (1-m)\left(w_q\beta_q - w_s\beta_s\right)}\right) \tag{2.9}$$

where $\beta_q = \frac{Cov(R_{t+1}^q, R_{t+1}^M)}{Var(R_{t+1}^M)}$ and $\beta_s = \frac{Cov(R_{t+1}^s, R_{t+1}^M)}{Var(R_{t+1}^M)}$

Substituting (2.8) in (2.7) leads to:

$$E[R_{t+1}^s] = R_F + (E[R_{t+1}^M] - R_F) \cdot \beta_s \cdot \left(\frac{1 + (1-m)\left(\frac{w_q \beta_q}{w_s \beta_s} - 1\right)}{1 + (1-m)\left(w_q \beta_q - w_s \beta_s\right)} \right) \tag{2.10}$$

(2.9) and (2.10) are modified CAPM expressions when schemas are created with a binding resource constraint (and with more brain resources allocated to the schema-unit concerned with expected cash flows). Note that (2.9) and (2.10) revert to the classical CAPM expression when $m = 1$ (resource constraint in the brain is not binding).

Generalizing to N firms with several q firms spawning new schemas of many s firms, the corresponding CAPM expressions for q and s firms are:

$$E\left[R_{t+1}^q\right] = R_F + (E[R_{t+1}^M] - R_F) \cdot \beta_q \cdot \left(\frac{1}{1 + (1-m)\left(\sum_q \sum_s \left(w_q \beta_q - w_s \beta_s\right)\right)} \right) \tag{2.11}$$

$$E[R_{t+1}^s] = R_F + (E[R_{t+1}^M] - R_F) \cdot \beta_s \cdot \left(\frac{1 + (1-m)\left(\frac{w_q \beta_q}{w_s \beta_s} - 1\right)}{1 + (1-m)\left(\sum_q \sum_s \left(w_q \beta_q - w_s \beta_s\right)\right)} \right) \tag{2.12}$$

It is intriguing to note that CAPM expressions with finite brain resources have the same form as the classical CAPM with only one difference: a factor that multiplies β appears. When the resource constraint is not binding, $m = 1$, the multiplicative factor equals 1, so we revert back to the classical CAPM expression.

2.3 High-alpha-of-low-beta, value, size, and momentum effects

(2.12) and (2.11) show that the classical CAPM is a special case of a schema-adjusted CAPM. In schema-adjusted CAPM, there is an additional multiplicative factor, which multiplies β.

This factor reduces to 1 when the resource constraint is not binding. In other words, the schema-adjusted CAPM reduces to the classical CAPM when $m = 1$.

For a firm s whose schema is created by modifying the schema of a similar firm (same line of business with similar P/E ratios) q, this additional multiplicative factor is equal to:

$$f = \left(\frac{1+(1-m)\left(\frac{w_q\beta_q}{w_s\beta_s}-1\right)}{1+(1-m)\left(\sum_q\sum_s\left(w_q\beta_q - w_s\beta_s\right)\right)}\right) \tag{2.13}$$

Firms to which investors and analysts devote most of their time are likely to be ones that spawn new schemas for other firms. Investor and analyst attention is strongly asymmetric with large, prominent firms (high market capitalizations) getting a lion's share (Fang and Peress 2009). This motivates the following assumption:

- Within a group of firms whose schemas are spawned by the same firm, q, the following holds: $w_q\beta_q > w_s\beta_s$ for all s

It follows that $f > 0$. Proposition 1 shows the emergence of high-alpha-of-low-beta in the cashflow-schema CAPM

Proposition 1 (High-alpha-of-low-beta) ***In a given cross-section of stocks, a stock with low beta outperforms a stock with large beta on a risk-adjusted basis, all else equal.***

Proof

Suppose there are two stocks s and s' such that $\beta_s < \beta_{s'}$. Risk-adjusted return on s is given by:

$$\frac{E[R^s_{t+1}] - R_F}{\beta_s} = \left\{1 + (1-m)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)\right\} \times \frac{1}{g} \times (E[R^M_{t+1}] - R_F)$$

where g is a constant in any given cross-section: $g = 1 + (1-m)\left(\sum_q\sum_s\left(w_q\beta_q - w_s\beta_s\right)\right)$

Risk-adjusted return on s' is given by:

$$\frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}} = \left\{1 + (1-m)\left(\frac{w_q \beta_q}{w_s \beta_{s'}} - 1\right)\right\} \times \frac{1}{g} \times (E[R_{t+1}^{M}] - R_F)$$

As β_s and $\beta_{s'}$ appear in the denominator on R.H.S, it follows that:

$\frac{E[R_{t+1}^{s}] - R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}}$ ■

One can also see the size effect in the cashflow-schema CAPM as proposition 2 shows.

Proposition 2 (Size effect) ***In a given cross-section of stocks, a stock with a lower weight in the market portfolio outperforms a stock with a higher weight on a risk-adjusted basis, all else equal***

Proof

Suppose there are two stocks s and s' such that $w_s < w_{s'}$. Following the same steps as in the proof of proposition 1, it is easy to see that $\frac{E[R_{t+1}^{s}] - R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}}$. ■

The cashflow-schema CAPM not only explains the high-alpha-of-low-beta and size-effect, but also the value effect. Value effect refers to the finding that a stock with low price to fundamentals tends to outperform a stock with high price to fundamentals. Suppose there are two stocks s and s' that have the same fundamentals (expected cash flows and the risk of the cash flows). That is, $E(X_{t+1}^{s}) = E\left(X_{t+1}^{s'}\right)$, and $Cov(X_{t+1}^{s}, X_{t+1}^{M}) = Cov\left(X_{t+1}^{s'}, X_{t+1}^{M}\right)$. Assume that $P_s < P_{s'}$.

If there is a value effect, then it must be so that

$$\frac{E[R_{t+1}^{s}] - R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}}$$

To see if the above is true, start from:

$P_s = \frac{E(X_{t+1}^s)-\gamma\left\{Cov(X_{t+1}^s,X_{t+1}^M)+(1-m)\left(Cov(X_{t+1}^q,X_{t+1}^M)\frac{n_q^*}{n_s^*}-Cov(X_{t+1}^s,X_{t+1}^M)\right)\right\}}{1+r_F} < P_{s'} =$
$\frac{E(X_{t+1}^{s'})-\gamma\left\{Cov(X_{t+1}^{s'},X_{t+1}^M)+(1-m)\left(Cov(X_{t+1}^q,X_{t+1}^M)\frac{n_{q'}^*}{n_{s'}^*}-Cov(X_{t+1}^{s'},X_{t+1}^M)\right)\right\}}{1+r_F}$. Assuming the same fundamentals across the two stocks, $E(X_{t+1}^s) = E\left(X_{t+1}^{s'}\right)$, and $Cov(X_{t+1}^s, X_{t+1}^M) = Cov\left(X_{t+1}^{s'}, X_{t+1}^M\right)$, it follows that: $Cov\left(X_{t+1}^q, X_{t+1}^M\right)\frac{n_q^*}{n_s^*} > Cov\left(X_{t+1}^{q'}, X_{t+1}^M\right)\frac{n_{q'}^*}{n_{s'}^*}$

$$\Rightarrow Cov\left(X_{t+1}^q, X_{t+1}^M\right)\frac{n_q^*}{n_s^*} - Cov(X_{t+1}^s, X_{t+1}^M) > Cov\left(X_{t+1}^{q'}, X_{t+1}^M\right)\frac{n_{q'}^*}{n_{s'}^*} - Cov\left(X_{t+1}^{s'}, X_{t+1}^M\right)$$

$$\Rightarrow Cov(X_{t+1}^s, X_{t+1}^M)\left\{\frac{Cov\left(X_{t+1}^q, X_{t+1}^M\right)}{Cov(X_{t+1}^s, X_{t+1}^M)}\frac{n_q^*}{n_s^*} - 1\right\}$$
$$> \ Cov\left(X_{t+1}^{s'}, X_{t+1}^M\right)\left\{\frac{Cov\left(X_{t+1}^{q'}, X_{t+1}^M\right)}{Cov\left(X_{t+1}^{s'}, X_{t+1}^M\right)}\frac{n_{q'}^*}{n_{s'}^*} - 1\right\}$$

$$\Rightarrow \frac{Cov\left(X_{t+1}^q, X_{t+1}^M\right)}{Cov(X_{t+1}^s, X_{t+1}^M)}\frac{n_q^*}{n_s^*} > \frac{Cov\left(X_{t+1}^{q'}, X_{t+1}^M\right)}{Cov\left(X_{t+1}^{s'}, X_{t+1}^M\right)}\frac{n_{q'}^*}{n_{s'}^*}$$

$$\Rightarrow \frac{w_q\beta_q}{w_s\beta_s} > \frac{w_{q'}\beta_{q'}}{w_{s'}\beta_{s'}} \qquad (2.14)$$

It follows immediately from (2.14) that:

$$\frac{E[R_{t+1}^s] - R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}}$$

Proposition 3 follows.

Proposition 3 (Value effect) ***In a given cross-section of stocks, a stock with low price to fundamentals outperforms a stock with high price to fundamentals on a risk-adjusted basis.***

It is intriguing that features akin to value, and size, as well as high-alpha-of-low-beta can all be seen in the cashflow-schema CAPM that has the same form as the classical CAPM except for the appearance of a factor, f, which multiplies beta. This multiplicative factor is larger for small size stocks, for low beta stocks, and for value stocks.

2.3.1 The Momentum Effect

It is clear from (2.5) that schema-based stock price predictions are rank-order correct for all schemas spawned by the same firm q. That is, riskier stocks are priced lower than less risky stocks. If an investor decides to improve the schema of a particular firm by devoting more resources to it, there is an opportunity cost involved as these resources need to be diverted from elsewhere. So, unless the updated schema delivers additional benefits that exceed the opportunity costs, schema modification is not optimal. The additional benefits depend on the likelihood of the updated schema determining the stock price, which obviously depends on whether other investors are also updating the same schema or not. Such coordinated schema updating may take place for stocks that have experienced positive shocks recently (momentum winners), with resources potentially being diverted away from the assimilation processes of momentum losers. Hence, m goes up for momentum winners, which further increases their prices as can be seen from (2.5). The positive shocks followed by further price increases due to schema updating is akin to the momentum effect. A novel prediction arising from this mechanism is discussed in section 3.

2.4 Risk-Schema CAPM

When schema of a firm is being created by modifying an existing schema, the two schema-units that need to be adjusted are expected cash flows and the risk of cash flows. In the previous sections, we considered the case when more brain resources are allocated to expected cash flows. In this section, we consider the other case: when more brain resources are allocated to the system of neurons working on the risk of cash flows. In (2.2) and (2.3), this means the following: $m_1 < m_2$. Without loss of generality, we set $m_2 = 1$, it follows that $m_1 = m < 1$.

The stock of firm s, whose schema is obtained by modifying the schema of a similar firm (same line of business and P/E ratios) q, is priced as:

$$P_t^s = \frac{E(X_{t+1}^s) + (1-m)\left(\frac{n_q^*}{n_s^*}E\left(X_{t+1}^q\right) - E(X_{t+1}^s)\right) - \gamma Cov(X_{t+1}^s, X_{t+1}^M)}{1+r_F} \qquad (2.15)$$

The stock of firm q, whose schema is modified to obtain the schema for s, is priced as:

$$P_t^q = \frac{E\left(X_{t+1}^q\right) - \gamma Cov\left(X_{t+1}^q, X_{t+1}^M\right)}{1+r_F} \qquad (2.16)$$

Following the same set of steps as in section 2.2, the following generalized CAPM expressions for s and q stocks are obtained:

$$E[R_{t+1}^s] = R_F + \beta_s\left[E[R_{t+1}^M] - R_F + \frac{(1-m)}{P_t^M}\sum_q\sum_s\left(n_q^*E\left(X_{t+1}^q\right) - n_s^*E(X_{t+1}^s)\right)\right]$$
$$- \frac{(1-m)}{P_t^s}\left\{\frac{n_q^*}{n_s^*}E\left(X_{t+1}^q\right) - E(X_{t+1}^s)\right\} \qquad (2.17)$$

$$E\left[R_{t+1}^q\right] = R_F + \beta_q\left[E[R_{t+1}^M] - R_F + \frac{(1-m)}{P_t^M}\sum_q\sum_s\left(n_q^*E\left(X_{t+1}^q\right) - n_s^*E(X_{t+1}^s)\right)\right] \qquad (2.18)$$

where P_t^M is the value of aggregate market portfolio. The above can be simplified further by defining expected market capitalization inclusive of dividends as: $E\left(w_{t+1}^q\right) = \frac{n_q^*E\left(X_{t+1}^q\right)}{P_t^M}$ and $E(w_{t+1}^s) = \frac{n_s^*E(X_{t+1}^s)}{P_t^M}$:

$$E[R_{t+1}^s] = R_F + \beta_s\left[E[R_{t+1}^M] - R_F + (1-m)\sum_q\sum_s\left(E\left(w_{t+1}^q\right) - E(w_{t+1}^s)\right)\right]$$
$$-(1-m)\left\{\frac{E\left(w_{t+1}^q\right) - E(w_{t+1}^s)}{w_t^s}\right\} \qquad (2.19)$$

where $w_t^s = \frac{n_s^*P_t^s}{P_t^M}$ is the weight of stock s in the market portfolio.

$$E\left[R_{t+1}^q\right] = R_F + \beta_q\left[E[R_{t+1}^M] - R_F + (1-m)\sum_q\sum_s\left(E\left(w_{t+1}^q\right) - E(w_{t+1}^s)\right)\right] \qquad (2.20)$$

Given evidence that large firms (large market capitalizations) get a lion's share of investor and analyst attention (Fang and Peress 2009), it is likely that they are the ones spawning schemas of other firms. Hence, we assume that $E\left(w_{t+1}^{q}\right) > E(w_{t+1}^{s})$.

It is immediately obvious that, in risk-schema CAPM, the relationship between beta and excess stock return is steeper than what the classical CAPM predicts as beta is multiplied by a factor larger than excess market return. Larger the beta, bigger the improvement over classical CAPM prediction.

Furthermore, the implied risk-free rate is smaller than what the classical CAPM predicts and could even be negative:

$$R_F' = R_F - (1-m)\left\{\frac{E\left(w_{t+1}^{q}\right) - E(w_{t+1}^{s})}{w_t^{s}}\right\} \tag{2.21}$$

It is straightforward to see that large size (market capitalization) stocks do better in this version as the implied risk-free rate is larger for them.

Proposition 4 formalizes the key differences between the two versions of CAPM.

Proposition 4 (Differences between the two versions) ***CAPM when more brain resources are allocated to expected cash flows (Cashflow Schema CAPM) differs from the CAPM when more brain resources are allocated to the risk of cash flows (Risk schema CAPM) in the following ways:***

1) ***The former has a flatter relationship between beta and expected returns, whereas the latter has a steeper relationship between beta and expected returns.***
2) ***The implied risk-free rate is smaller in the latter and could be negative.***
3) ***Small size, and low beta stocks do better in the former whereas large size, and high beta stocks do better in the latter.***

3. Novel Predictions

Two novel predictions follow from the approach developed here. The first prediction follows from considering the explanation for the momentum effect. Using $m^* > m$ to denote the updated value of m for a stock, s', which is a momentum winner, the updated factor multiplying beta is:

$$f = \left(\frac{1 + (1 - m^*) \left(\frac{w_q \beta_q}{w_{s'} \beta_{s'}} - 1 \right)}{1 + (1 - m) \left(\sum_q \sum_{s \neq s'} \left(w_q \beta_q - w_s \beta_s \right) \right) + (1 - m^*) \left(w_q \beta_q - w_{s'} \beta_{s'} \right)} \right) \tag{2.22}$$

So, when resources are devoted to updating the scheme of a momentum winner, the factor that multiplies its beta, f, falls. As this factor is responsible for value, size, and high-alpha-of-low-beta effects, it follows that value, size, and betting-against-beta get weaker for the momentum winner. Hence, the first novel prediction is as follows: momentum is negatively correlated with value, size, and betting-against-beta. Intriguingly, negative correlations between momentum and value (Asness et al 2013) as well as momentum and size (Rabener 2017) have already been noted in the empirical literature.

The second novel prediction follows from comparing the multiplicative factor for a q stock with the multiplicative factor of an s stock. As q stock belongs to a prominent firm that commands a lion's share of investor and analyst attention, it likely spawns schemas of s firms. From (2.11) and (2.12): the multiplicative factor is smaller for q stock; hence, the second novel prediction is that q stocks or stocks that command a lion's share of investor and analyst attention (media attention could be a proxy for that) have lower risk-adjusted returns. The findings in Fang and Peress (2009) are consistent with this prediction.

4. Discussion and Conclusions

Research in brain sciences has established that knowledge construction in the brain takes place via schema creation and modification. We show that incorporating this richer view from brain sciences into CAPM leads to two distinct versions of CAPM. One version potentially provides a unified explanation for high-alpha-of-low-beta, value, size, and momentum effects, and generates two novel predictions. The second version has a steeper relationship between beta and average stock returns with an implied risk-free rate which is likely to be negative. Intriguingly, Hendershott et al (2019) note that at-open, the relationship between beta and average stock return is steeper. Similarly, Savor and Wilson (2014) have noted a steeper relationship as well on macroeconomic announcement days. Could it be that we observe the second version of CAPM at such specific times due to risk-schema traders being marginal? This line of thinking is promising as, arguably, trades on such specific times are driven by risk-reductions, to which the cashflow schema traders underreact, making risk-schema traders marginal.

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