Corporate Social Responsibility and Optimal Pigouvian Taxation

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3 December 2019

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This version: April, 2020

Abstract

We formally study Pigouvian taxation in a duopoly market in which a CSR firm interacts with a profit maximizing firm. Unlike previous literature, we consider three different scenarios: (i) the CSR firm acts as a consumer-friendly firm, cares for not only its profits but also consumer surplus, as a proxy of its concern for its "stakeholders" or consumers; (ii) the CSR firm main objective is a combination of its own profit and the environment, caring for the environmental damage produced by the market in which it interacts; and (iii) the CSR firm is both consumer and environmental friendly. Finally, we compare the different Pigouvian rules derived with the first best competitive market solution and the monopoly/duopoly second best solutions.

Keywords: Corporate social responsibility, consumer-friendly firm, environment-friendly firm, Mixed Duopoly, Emission Taxation
JEL Classification: L13, L31, H23, Q50,

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1 Introduction

Corporate Social Responsibility (CSR) is currently a common practice for large and mid-cap companies around the world. For instance, the *KPMG Survey of Corporate Responsibility Reporting 2017* points out that the vast majority (93 percent) of the world's 250 largest companies by revenue based on the Fortune 500, now integrate financial and non-financial data in their annual financial reports, indicating that they believe CSR data is relevant for their investors[1]. The Sustainable Investments Institute's research for the year 2018 also found that 78 percent of the S&P 500 issued a sustainability report for the most recent reporting period, most with environmental and social performance metrics[2]. This point towards a current trend in business strategy by which firms are gradually, and increasingly, adopting corporate practices that go beyond profit-maximizing objectives, taking also into account ethical regards, community welfare and environmental sustainability as important business habits.

Very recently in economics, the industrial organization literature has also started modelling oligopoly markets in which some private firms, that we call here CSR firms, differentiate from others by maximizing its profit as well as a fraction of the market consumer surplus, in order to reflect its consumer-friendly spirit. Among the topics addressed by this literature we can mention: vertical supply chains (Goering, 2014 and Brand and Grothe, 2015); horizontal products differentiation (Matsumura and Ogawa, 2014 and Kopel and Brand, 2012) and strategic tariff policy (Wang et al. 2012, and Liu et al. 2018). Nevertheless, there are few works analyzing the environmental problem in this context.

The main aim of this work is to formally study Pigouvian taxation in a duopoly market in which a CSR firm interacts with a profit maximizing firm. Unlike previous literature, we consider three potential different scenarios; (i) the CSR firm acts as a consumer-friendly firm, cares for not only its profits but also consumer surplus, as a proxy of its concern for its "stakeholders" or consumers; (ii) the CSR firm main objective is a combination of its own profit and the environment, caring for the environmental damage produced by the market in which it interacts, and (iii) the CSR firm is both consumer and environmental friendly, caring about its profit, a share of consumer surplus and environmental damage. Previous literature typically uses the definition of a CSR firm given by case (i), assuming that it maximizes profits plus a fraction of consumer surplus (see Kopel and Brand, 2012 and Goering, 2014). Adding these additional cases allow us to evaluate more recent trends in the CSR literature in which environmental concerns have also become a priority for stakeholders and consumers (see, inter alia, Barman, 2018). As benchmark we also consider the case in which both firms in the duopoly, the CSR firm and the other private firm, only concern about material profits. In

[1] See: https://home.kpmg/content/dam/kpmg/campaigns/csr/pdf/CSR_Reporting_2017.pdf.

[2] See: https://www.weinberg.udel.edu/IIRCiResearchDocuments/2018/11/2018-SP-500-Integrated-Reporting-FINAL-November-2018-1.pd

particular, we compare the different Pigouvian rules that we derive with the first best competitive market solution in which *optimal tax rates equal marginal emissions damage* (Pigou, 1920; Baumol, 1972) and the monopoly solution in which *optimal tax rates may be less than marginal emissions damage* (Barnett, 1980).

Related literature to our work includes the following. Liu, et al. (2015) investigate the impacts of competition structures on firms' incentives for adopting strategic environmental corporate social responsibility (ECSR) certified by a Non-Governmental Organization. Leal et al (2018) put forward a Cournot duopoly model with a consumer-friendly firm and study the interplay between the strategic choice of abatement technology and the timing of government's commitment to the environmental tax policy. García et al (2018) analyze the timing of environmental policies with a consumer-friendly firm having abatement technology and compares two market-based regulatory instruments, tradable permits and emission tax regulations. Xu and Lee (2018) study CSR in Cournot markets with endogenous entry and investigates the effects of CSR on environmental taxation and welfare consequences. Finally, Villena (2020) explores what CSR motivations are better for the environment, comparing an environmental friendly CSR firm, a consumer caring CSR firm and a profit maximizing firm, in terms of the environmental damage generated in a duopoly market setting in which a CSR firm interacts with a profit maximizing firm.

2 The Model

Consider a single industry made up of two polluters: one CSR firm labeled 0 and a private firm labeled 1, which competes in quantities with homogeneous products (or perfect substitutes). Both firms have production levels of a single product output q_i, for $i = 0, 1$, with total output given by $Q = q_0 + q_1$ and an inverse demand function $f(Q)$. Both firms discharge pollution into the environment, which we denote by d_i, generating $D(\sum d_i)$ in total external environmental damages. Let total resource costs for the pollution-generating firm be represented by $c_i = c(q_i, w_i)$: where w_i represents resources devoted to pollution treatment. Assume that the firm has two ways of reducing its emissions levels d_i. It may either reduce output q_i, or it may devote more resources w_i to the treatment of pollution once it is produced, which implies that $d_i = d(q_i, w_i)$, for $i = 0, 1$. We also consider a tax on emissions, t, which works as a tax rate per unit of pollution discharged. Both firm's profit functions are then given by:

$$\pi_i(q_i, w_i) = f(Q)q_i - c(q_i, w_i) - d(q_i, w_i)t \text{ for } i = 0, 1 \tag{1}$$

As customary in the literature, we assume that the CSR firm, contrary to profit-maximizing private firms, cares for not only its profits but also for a fraction of the consumer surplus, CS, as a proxy of the firm's concern on consumers. We also consider the case in which a the CSR firm also cares for the environmental damage produced by the duopoly, D, as a proxy of the firm's concern for the environment. Hence the objective of the CSR-firm is a combination of consumers surplus, environmental damage and its own profit:

$$v_0 = \pi_0 + \theta CS - \gamma D \tag{2}$$

Let the parameter $\theta \in [0,1]$ represents the fraction or percentage of total market consumer surplus that is of concern or accrues to the socially concerned firm's stakeholders. When $\theta = 1$, all consumer's welfare is of interest to this firm while, conversely, when $\theta = 0$ the firm is not consumer friendly in our model. Similarly, the parameter $\gamma \in [0,1]$ measures the degree of concern on environmental damage by the CSR firm. When $\gamma = 1$, all environmental damage is of interest to the CSR firm while, conversely, when $\gamma = 0$ the firm is not environment friendly in our setting. We assume that θ and γ are exogenously given. This definition of CSR implies the CSR firm is willing to accept less profits to act in a more socially and environmentally concerned way. In other words, in our setting CSR is purely a costly activity (see, for instance, Goering, 2014).

We define social welfare as the difference between the sum of producer's and consumer's surplus and any technological external costs which are not accounted for in producer's surplus.[3] Particularly, in this setting we assume that social welfare will be given by the sum of consumer surplus, CS, the profits of both firms, $\pi_0 + \pi_1$, and tax revenue $T = (d_0 t + d_1 t)$, minus environmental damage, $D(d_0 + d_1)$ (Leal et al. 2018)[4]:

$$SW = CS + f(Q)(q_0 + q_1) - c_0 - c_1 - D(d_0 + d_1) \tag{3}$$

Hence, the payoff that the CSR firm maximizes is as follows:

$$v_0(q_0, w_0) = f(Q)q_0 - c(q_0, w_0) - d(q_0, w_0)t + \theta \left(\int_0^Q f(z)dz - f(Q)(Q) \right) - \gamma D(d(q_0, w_0) + d(q_1, w_1)) \tag{4}$$

Throughout the paper, we restrict attention to pure strategies. Our modelling strategy is based on a *sequential two stage game*. In the first stage the regulator chooses the emissions tax (t) that maximizes social welfare, which will be levied on the two firms. In the second stage the two firms choose their levels of production (q) and pollution abatement (w). In this sequential game of perfect information, any stage is a subgame and a strategy vector is a subgame perfect Nash equilibrium (SPNE) only if it induces a Nash equilibrium in the strategic form of every subgame. In this context, SPNE reduces to *backward induction*.

[3] Here, a real income constant measure of consumer's surplus, such as equivalent or compensating variation should be used to be strictly correct. Nevertheless, the area under a money-income constant demand curve is a good estimate of a welfare measure.

[4] Since we define social welfare as. $SW \triangleq CS + (f(Q)q_0 - c_0 - d_0 t) + (f(Q)q_1 - c_1 - d_1 t) + (d_0 t + d_1 t) - D(d_0 + d_1)$ we can notice that taxes are merely income transfers from the firms to the government, and therefore, they are canceled out.

Definition 1 *A strategy for the regulator is a tax amount $t \geq 0$ and a strategy for the firms is $\rho_i(q_i, w_i)$, where $\rho_i(\cdot)$ is a mapping from the domain of t to the domain of (q_i, w_i). Assuming that the regulator is the first mover, an equilibrium of this duopoly game is then a pair $(t, \rho_i^*(q_i^*, w_i^*))$ for $i = 0, 1$, such that:*

i. $SW(t^*, \rho_i^*(q_i^*, w_i^*)) \geq SW(t, \rho_i^*(q_i^*, w_i^*)), \forall t \geq 0$; $i = 0, 1$;

ii. $\pi_1(\rho_1^*(q_1^*, w_1^*)) \geq \pi_1(\rho_1(q_1, w_1)), \forall q_1 \geq 0, w_1 \geq 0$; *and*

iii. $v_0(\rho_0^*(q_0^*, w_0^*)) \geq v_0(\rho_i(q_0, w_0)), \forall q_0 \geq 0, w_0 \geq 0$

In other words, an equilibrium in this game imposes that: (i) the strategy of the firms be a single-valued selection from their best-response correspondences for q_i and w_i given a tax t; and (ii) the regulator chooses a tax that maximizes the social welfare function given the optimal strategy of the firms $(q_i^*, w_i^*)$for $i = 0, 1$.

Hence, we start our analysis with stage two, in which the private and public firms must choose their production (q_0, q_1) and abatement (w_0, w_1) levels, given a tax, t, defined by the regulator in stage 1. Thus, the associated optimization problem faced by the private firm in this stage is given by:

$$\max_{q_1, w_1} \ \pi_1(q_1, w_1) = f(Q)q_1 - c(q_1, w_1) - d(q_1, w_1)t \tag{5}$$

Similarly, for the CSR firm the problem becomes:

$$\max_{q_0, w_0} \ v_0(q_0, w_0) = \ f(Q)q_0 - c(q_0, w_0) - d(q_0, w_0)t + \theta\left(\int_0^Q f(z)dz - f(Q)(Q)\right) - \gamma D(d(q_0, w_0) + d(q_1, w_1)) \tag{6}$$

We denote the set of equilibria in this stage by S_2 and its typical element by the strategy profile: $S_2 = \{(q_0^*(t), w_0^*(t)); (q_1^*(t), w_1^*(t))\}$. Now with S_2 the regulator in the first stage chooses the tax rate per unit of emissions discharged, t, that maximizes the social welfare function, see (3):

$$\max_t SW = \int_0^Q f(z)dz - c(q_0^*(t), w_0^*(t)) - c(q_1^*(t), w_1^*(t)) - D(d(q_0^*(t), w_0^*(t)) + d(q_1^*(t), w_1^*(t))) \tag{7}$$

Likewise, S_1 identifies equilibria in this stage given by (t^*). We assume the following very general conditions:

Assumption 1 *The inverse demand function $f(Q)$ is twice continuously differentiable, with $f'(Q) < 0$ (whenever $f(Q) > 0$) and $lim_{Q \to \infty} f(Q) = 0$.*

Assumption 2 *Cost functions $c_i = c(q_i, w_i)$*(for $i = 0, 1$) *are increasing and twice continuously differentiable.*

Assumption 3 *The emission level functions* $d_i = d(q_i, w_i)$ (for $i = 0, 1$) *and the environmental damage function* $D(d_0 + d_1)$, *are increasing in production and decreasing in abatement effort, and twice continuously differentiable.*

Under Assumptions 1 and 2, both firms' action sets are compact since the firms would never produce quantities larger than some upper-bound. Assumption 3 is consistent with the fact that most of the literature defines environmental damage as monotonically increasing in production and decreasing in abatement effort.

3 Results

From solving the Nash Equilibrium of the second stage we obtain the following result:

Lemma 1 *Assuming that in the first stage of the game, the CSR firm and the other private firm view* t *as a parameter, we get the following first-order conditions for the profit maximization of (6) and (5), which implicitly define the strategy profile* $S_2 = \{(q_0^*(t), w_0^*(t)); (q_1^*(t), w_1^*(t))\}$*: (i)* $f(Q^*) + q_0^* \frac{\partial f(Q^*)}{\partial q_1^*} - \frac{\partial c_0^*}{\partial q_0^*} - t\frac{\partial d_0^*}{\partial q_1^*} + \theta f(Q^*)\frac{\partial Q^*}{\partial q_0^*} - \gamma \frac{\partial D}{\partial d_0^*}\frac{\partial d_0^*}{\partial q_0} = 0$*; (ii)* $\theta f(Q^*)\frac{\partial Q^*}{\partial w_0} - \frac{\partial c_0^*}{\partial w_0} - t\frac{\partial d_0^*}{\partial w_0} - \gamma \frac{\partial D}{\partial d_0}\frac{\partial d_0^*}{\partial w_0} = 0$ *; (iii)* $f(Q^*) + q_1^* \frac{\partial f(Q^*)}{\partial q_1} - \frac{\partial c_1^*}{\partial q_1} - t\frac{\partial d_1^*}{\partial q_1} = 0$*; (iv)* $-\frac{\partial c_1^*}{\partial w_1} - t\frac{\partial d_1^*}{\partial w_1} = 0$.

Let us now focus on the first stage of the game, in which the regulator faces the problem pointed out in (7), which after totally differentiating SW leads to the following FOC:
$f(Q^*)\frac{\partial Q^*}{\partial t} - \left[\frac{\partial c_0^*}{\partial q_0}\frac{dq_0^*}{dt} + \frac{\partial c_0^*}{\partial w_0}\frac{dw_0^*}{dt}\right] - \left[\frac{\partial c_1^*}{\partial q_1}\frac{dq_1^*}{dt} + \frac{\partial c_1^*}{\partial w_1}\frac{dw_1^*}{dt}\right] = \frac{\partial D}{\partial d_0}\left[\frac{\partial d_0^*}{\partial q_0}\frac{dq_0^*}{dt} + \frac{\partial d_0^*}{\partial w_0}\frac{dw_0^*}{dt}\right] + \frac{\partial D}{\partial d_1}\left[\frac{\partial d_1^*}{\partial q_1}\frac{dq_1^*}{dt} + \frac{\partial d_1^*}{\partial w_1}\frac{dw_1^*}{dt}\right]$

Combining the FOC equation of the regulator with the ones highlighted in Lemma 1, and after rearranging terms, we obtain:

$$\left(t - \frac{\partial D}{\partial d_0}(1-\gamma)\right)\left(\frac{\partial d_0^*}{\partial t}\right) + \left(t - \frac{\partial D}{\partial d_1}\right)\left(\frac{\partial d_1^*}{\partial t}\right) = \frac{\partial f(Q)}{\partial q_0}\frac{dq_0^*}{dt}(q_0 - \theta Q) + q_1 \frac{dq_1^*}{dt}\frac{\partial f(Q)}{\partial q_1} \tag{8}$$

Where $\frac{\partial d_0^*}{\partial t} = \frac{\partial d_0}{\partial q_0}\frac{dq_0^*}{dt} + \frac{\partial d_0}{\partial w_0}\frac{dw_0^*}{dt}$ and $\frac{\partial d_1^*}{\partial t} = \frac{\partial d_1}{\partial q_1}\frac{dq_1^*}{dt} + \frac{\partial d_1}{\partial w_1}\frac{dw_1^*}{dt}$,which are the equilibrium emissions levels: $d_i(q^*(t), w^*(t))$, (for $i = 0, 1$), after totally differentiating them with respect to t.

While the equation in (8) is not an explicit solution for t, because t is on both sides of the equation, Lemma 1 allows us to write q and w as functions of t. Substituting these terms into (8) then gives one equation with one unknown, t.

Given (8), assuming that Assumptions 1–3 are satisfied and that in equilibrium the outcomes of the two firms are symmetric, i.e., $q_0 = q_1 = q^*$ and $w_0 = w_1 = w^*$ we can now characterize the equilibrium in order to show some of the main results of the model exploiting some corner solutions.

First, we present the case in which both firms in the duopoly have only a profit maximizing objective, not taking into account the consumers nor the environment in their decisions, namely: $v_0 = \pi_0$ and π_1. We will use this result as our main benchmark in the discussion of main results.

Proposition 1 *The equilibrium welfare-maximizing tax when both firms in the duopoly only concern about material profits (i.e. $\theta = 0$ and $\gamma = 0$), becomes:*

$$t^* = \frac{\partial D^*}{\partial d} + \frac{q^* \frac{\partial f(Q^*)}{\partial q^*} \frac{dq^*}{dt}}{\frac{\partial d^*}{\partial q^*} \frac{dq^*}{dt} + \frac{\partial d^*}{\partial w^*} \frac{dw^*}{dt}} \tag{9}$$

From the right hand side of (9) and using the fact that $q^* \frac{\partial f(Q^*)}{\partial q^*} \frac{dq^*}{dt} = -\frac{f(Q^*)}{\eta} \frac{dq^*}{dt}$ we can obtain the following expression for the optimal tax: $t^* = \frac{\partial D^*}{\partial d} - \frac{\frac{f(Q^*)}{\eta} \frac{dq^*}{dt}}{\frac{\partial d^*}{\partial q^*} \frac{dq^*}{dt} + \frac{\partial d^*}{\partial w^*} \frac{dw^*}{dt}}$, which is dependent upon the price elasticity of demand η (with negative sign). Clearly whenever $\eta \to \infty$ then $t^* = \frac{\partial D^*}{\partial d}$, that is, as demand becomes perfectly elastic the optimal tax rate approaches marginal external damages. Nevertheless, whenever η is finite, it can be noted that the amount by which optimal tax rates fall short of marginal damages may increase as price elasticity of demand for the polluter's produce decreases, which in this case necessarily implies that a tax rate less than marginal external damages is obtained. The reason behind this result lies in the trade-off between the environmental negative externality and the welfare loss associated with the duopoly restricted output, which necessarily requires that the optimal second best tax rate must be less than marginal emissions. Thus, qualitatively, the optimal tax for this private duopoly developed here has the same structure as in the *Pigouvian tax rule under monopoly* put forward by Barnett (1980). Obviously, other things equal, the duopoly output is greater than the monopoly one, so the Pigouvian tax rule under this setting should be closer to environmental damage than the one obtained for a monopoly. In any case, the typical assumptions made by the literature namely: $\frac{dq^*}{dt} < 0$ and $\frac{dw^*}{dt} > 0$, imply that the denominator of the second term, i.e. the effect of the tax on the private firms' emissions, is negative $\frac{\partial d^*}{\partial t} < 0$. This, in turn, ensures that in this case the *Pigouvian tax rule is less than the marginal emissions damage.*

Second, we examine the case in which the objective of the CSR-firm is a combination of consumers surplus, and its own profit, that is: $v_0 = \pi_0 + \theta CS$.

Proposition 2 *The equilibrium welfare-maximizing tax for the duopoly when the CSR firm only cares about consumers, and not about the environment (i.e. $\theta > 0$ and $\gamma = 0$), becomes:*

$$\ddot{t}^* = \frac{\partial D^*}{\partial d} + \frac{(1-\theta)\, q^* \frac{\partial f(Q^*)}{\partial q^*} \frac{dq^*}{dt}}{\frac{\partial d}{\partial q^*} \frac{dq^*}{dt} + \frac{\partial d}{\partial w^*} \frac{dw^*}{dt}} \tag{10}$$

From the second term of the right hand side of (10), we can also obtain a Pigouvian tax rule depending on the demand price elasticity η, that is: $t^* = \frac{\partial D}{\partial d} - \frac{(1-\theta)\frac{f(Q^*)}{\eta}\frac{dq^*}{dt}}{\frac{\partial d}{\partial q^*}\frac{dq^*}{dt}+\frac{\partial d}{\partial w^*}\frac{dw^*}{dt}}$. From this expression, if $\theta = 1$, we obtain $\ddot{t}^* = \frac{\partial D^*}{\partial d}$ that is the optimal tax rates equal marginal emissions damage. Similarly, if $\eta \to \infty$, we also get: $\ddot{t}^* = \frac{\partial D^*}{\partial d}$. Finally if $\theta < 1$ and assuming $\frac{dq^*}{dt} < 0, \frac{dw^*}{dt} > 0$ and $\frac{\partial d^*}{\partial t} < 0$, we have that $\frac{(1-\theta)q^*\frac{\partial f(Q^*)}{\partial q^*}\frac{dq^*}{dt}}{\frac{\partial d}{\partial q^*}\frac{dq^*}{dt}+\frac{\partial d}{\partial w^*}\frac{dw^*}{dt}} < 0$ which implies that $\ddot{t}^* < \frac{\partial D^*}{\partial d}$ and so the optimal tax rates are less than marginal emissions damage. Hence, we can further infer the following results:

Corollary 1 *Given the Pigouvian tax rule obtained for a CSR firm that only cares about consumers and not about the environment, (10), we conclude the following:*

a. *Whenever all consumer's welfare is of interest to the CSR firm, we recover the Pigouvian tax for perfect competition in which* optimal tax rates equal marginal emissions damage

b. *Whenever the demand is perfectly elastic, we also recover the Pigouvian tax for perfect competition in which* optimal tax rates equal marginal emissions damage

c. *Whenever only a portion of the consumer's welfare is of interest to the CSR firm, we obtain a Pigouvian tax rule in which* optimal tax rates are less than marginal emissions damage.

The result highlighted in Corollary 1 a) can be explained by the fact that as the CSR firm cares for all consumer's welfare, its production level will be higher than the one under the private duopoly studied in Proposition 1. This will compensate for the welfare loss associated with the private duopoly restricted output and so the optimal tax rule in this case will equal environmental damage, recovering the competitive market Pigouvian tax. Similarly, Corollary 1 b) captures the result first put forward by Barnett (1980) that as demand approaches the perfectly elastic state the value of the optimal tax rate approaches marginal external damages. By contrast, Corollary 1 c) reflects the fact that whenever the CSR firm cares for only a part of the consumer's surplus and hence its relatively greater output than the one obtained in the previous case is not enough to compensate for the welfare loss associated with the private firm restricted output. This in turn implies an optimal tax rate lower than the marginal emissions.

Third, we analyze the case of a CSR-firm that maximizes its material profit minus the environmental damage produced by the duopoly, that is: $v_0 = \pi_0 - \gamma D$.

Proposition 3 *The equilibrium welfare-maximizing tax for the duopoly when the CSR firm only cares about the environment, and not about consumers (i.e.* $\gamma > 0$ *and* $\theta = 0$*), becomes:*

$$t^* = \frac{(2-\gamma)}{2}\frac{\partial D^*}{\partial d} + \frac{q^*\frac{\partial f(Q^*)}{\partial q^*}\frac{dq^*}{dt}}{\frac{\partial d^*}{\partial q^*}\frac{dq^*}{dt}+\frac{\partial d^*}{\partial w^*}\frac{dw^*}{dt}} \tag{11}$$

To ease the interpretation of (11), we get the Pigouvian tax rule as a function of the demand price elasticity η, that is: $t^* = \frac{(2-\gamma)}{2}\frac{\partial D^*}{\partial d} - \frac{\frac{f(Q^*)}{\eta}\frac{dq^*}{dt}}{\frac{\partial d^*}{\partial q^*}\frac{dq^*}{dt}+\frac{\partial d^*}{\partial w^*}\frac{dw^*}{dt}}$. As $\gamma > 0$, we have in this setting that even if $\gamma = 1$, that is, all environmental damage is of interest to the CSR firm we would obtain: $t^* = \frac{1}{2}\frac{\partial D^*}{\partial d} - \frac{\frac{f(Q^*)}{\eta}\frac{dq^*}{dt}}{\frac{\partial d^*}{\partial q^*}\frac{dq^*}{dt}+\frac{\partial d^*}{\partial w^*}\frac{dw^*}{dt}}$, which assuming $\frac{dq^*}{dt} < 0, \frac{dw^*}{dt} > 0$ and $\frac{\partial d^*}{\partial t} < 0$, implies that $\ddot{t}^* < \frac{\partial D^*}{\partial d}$, i.e. optimal tax rates will always be less than marginal emissions damage. In fact, even if the demand is perfectly elastic, $\eta \to \infty$, and $\gamma = 1$ we would obtain: $t^* = \frac{1}{2}\frac{\partial D^*}{\partial d}$, that is: the optimal tax rate is half the marginal emissions damage.

We state these results in the following Corollary.

Corollary 2 *Given the Pigouvian tax rule obtained for a CSR firm that only cares about the environment and not about consumers (11), we conclude the following:*

a. *Whenever the CSR firm is fully committed to the environment, i.e. all environmental damage is of interest to the CSR firm, we obtain that* optimal tax rates will always be less than marginal emissions damage.

b. *Whenever the demand is perfectly elastic, and the CSR firm is fully committed to the environment the* optimal tax rate is half the marginal emissions damage.

The rationale behind this result is that when the CSR firm cares only for the environment, its production level will be too low and therefore the optimal trade-off between the environmental negative externality and the welfare loss associated with restricted output will necessarily require a tax rate less than marginal external damages. In other words, in this case the low production level chosen by the CSR firm due to its emphasis on the environment has to be compensated in equilibrium by the regulator setting a lower tax rate.

Finally, we consider the case of a consumer-environment friendly CSR firm, where $v_0 = \pi_0 + \theta CS - \gamma D$.

Proposition 4 *The equilibrium welfare-maximizing tax for the duopoly when the CSR firm cares about consumers and the environment, (i.e. $\theta > 0$ and $\gamma > 0$), becomes:*

$$t^* = \frac{(2-\gamma)}{2}\frac{\partial D^*}{\partial d} + \frac{(1-\theta)\, q^* \frac{\partial f(Q^*)}{\partial q^*}\frac{dq^*}{dt}}{\frac{\partial d^*}{\partial q^*}\frac{dq^*}{dt}+\frac{\partial d^*}{\partial w^*}\frac{dw^*}{dt}} \tag{12}$$

From (12), we get the following Pigouvian tax rule as a function of the demand price elasticity η, that is: $t^* = \frac{(2-\gamma)}{2}\frac{\partial D^*}{\partial d} - \frac{(1-\theta)\frac{f(Q^*)}{\eta}\frac{dq^*}{dt}}{\frac{\partial d}{\partial q^*}\frac{dq^*}{dt}+\frac{\partial d}{\partial w^*}\frac{dw^*}{dt}}$. From this expression, it is easily seen that when the CSR firm cares for all consumer's welfare, i.e. $\theta = 1$, and for all environmental damage, i.e. $\gamma = 1$, we obtain: $t^* = \frac{1}{2}\frac{\partial D}{\partial d^*}$, which implies that the *optimal tax rate is half the marginal emissions damage.* Interestingly, in stark contrast with Barnett (1980), even if $\eta \to \infty$, that is the demand is perfectly elastic, we obtain that: $t^* = \frac{(2-\gamma)}{2}\frac{\partial D^*}{\partial d}$, which given that $\gamma \in [0, 1]$ implies an *optimal tax rate lower than marginal emissions damage.* In fact, assuming also that $\gamma = 1$ we get that the *optimal tax rate is half the marginal emissions damage.*

For this case, we put forward the following additional remarks:

Corollary 3 *Given the Pigouvian tax rule obtained for a CSR firm that cares about consumers and the environment (12), we can infer the following results:*

a. *Whenever the CSR firm is fully committed to consumers and to the environment, we obtain that the* optimal tax rate is half the marginal emissions damage.

b. *Whenever the demand is perfectly elastic, we obtain an* optimal tax rate lower than marginal emissions damage*. This result still holds even when we also assume that the CSR firm is fully committed to the environment, i.e. all environmental damage is of interest to the CSR firm, here we obtain the result from (a) in which the* optimal tax rate is half the marginal emissions damage.

From the results put forward in Corollary 3 we gather that when the CSR firm is fully committed to consumers and the environment, even though the demand is perfectly elastic the optimal tax rate will not be equal to the marginal external damages.

4 Conclusions

We formally study Pigouvian taxation in a duopoly market in which a CSR firm interacts with a profit maximizing firm. We obtained the following results.

First, for a CSR firm that only cares about consumers and not about the environment optimal tax rates equals marginal emissions damage whenever all consumer's welfare is of interest to the CSR firm. By contrast, when only a portion of the consumer's welfare is of interest to the CSR firm, optimal tax rates are less than marginal emissions damage.

Second, for a CSR firm that only cares about the environment and not about consumers, optimal tax rates are less than marginal emissions damage whenever the CSR firm is fully committed to the environment. Besides, when the demand is perfectly elastic, and the CSR firm is also fully committed to the environment the optimal tax rate is half the marginal emissions damage.

Finally, for a CSR firm that cares about consumers and the environment the optimal tax rate is half the marginal emissions damage when the CSR firm is fully committed to consumers and to the environment. Moreover, we obtain an optimal tax rate lower than marginal emissions damage when the demand is perfectly elastic. This result still holds even when we also assume that the CSR firm is fully committed to the environment here the optimal tax rate is half the marginal emissions damage.

In terms of policy recommendations, this analysis calls for discriminatory taxes depending on the motivations of the CSR firms. A potential way to implement this policy would be through reporting and certification of CSR practices. This provides an avenue for future research on the subject.

References

Barman, E. (2018). Doing Well by Doing Good: A Comparative Analysis of ESG Standards for Responsible Investment', Sustainability, Stakeholder Governance, and Corporate Social Responsibility. *Advances in Strategic Management*, Volume 38: 289-311.

Barnett, A. H. (1980). The Pigouvian tax rule under monopoly. *The American Economic Review*, 70(5): 1037-1041.

Baumol, W. J. (1972). On Taxation and the Control of Externalities. *The American Economic Review*, 62 (3): 307–322.

Brand, B., Grothe, M. (2015), "Social Responsibility in a Bilateral Monopoly," *Journal of Economics* 115, pp.275-289.

Garcia, A., Leal, M., & Lee, S. H. (2018). Time-inconsistent environmental policies with a consumer-friendly firm: tradable permits versus emission tax. *International Review of Economics & Finance*, 58, 523-537.

Goering, G. E. (2014), "The Profit-Maximizing Case for Corporate Social Responsibility in a Bilateral Monopoly," *Managerial and Decision Economics* 35, pp.493-499.

Kopel, M. and B. Brand (2012), "Socially Responsible Firms and Endogenous Choice of Strategic Incentives," *Economic Modelling* 29, pp.982-989.

Leal, M., A. Garcia and S.H. Lee (2018), "The Timing of Environmental Tax Policy with a Consumer-friendly Firm," *Hitotsubashi Journal of Economics* 59, pp.25-43.

Liu, C.C., L. Wang and S.H. Lee (2015), "Strategic Environmental Corporate Social Responsibility in a Differentiated Duopoly Market," *Economics Letters* 129, pp.108-111.

Liu, Q., L. Wang and C.L. Chen (2018), "CSR in an Oligopoly with Foreign Competition: Policy and Welfare implications," *Economics Modelling* 72, pp.1-7.

Matsumura, T. and A. Ogawa (2014), "Corporate Social Responsibility or Payoff Asymmetry? A Study of an Endogenous Timing Game," *Southern Economic Journal* 81, pp.457-473.

Villena, M.G. (2019). What Corporate Social Responsibility Motivations are better for The Environment?. MPRA Paper No. 100281. https://mpra.ub.uni-muenchen.de/100281/8/MPRA_paper_100281.pdf

Wang, L.F.S., Y.C. Wang and L. Zhao (2012), "Tariff Policy and Welfare in an International Duopoly with Consumer-Friendly Initiative," *Bulletin of Economic Research* 64, pp.56-64.

Xu, L and S.H.Lee (2018) "Corporate Social Responsibility And Environmental Taxation With Endogenous Entry", *Hitotsubashi Journal of Economics* 59, pp.61-82.