Tariff Policy, Increasing Returns and Endogenous Fluctuations

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Abstract: We study the effects of government tariff policy in a one-sector small open economy RBC model with a productive externality that generates social increasing returns to scale. Various forms of endogenous fluctuations, including stable 2-, 4-, 8-, and 15-cycles, quasiperiodic orbits and chaos can be identified in this model if we introduce a constant tariff or subsidy (applied to the imported production factor) into the laissez-faire economy that exhibits local indeterminacy. In a somewhat different model, Guo and Lansing (2002) show that a constant capital tax or subsidy can give rise to similar dynamics in a closed-economy one sector model with a productive externality. From this perspective, factor income taxes and tariffs are equivalent to generate endogenous fluctuations in those economies with social increasing returns to scale. We further show that in our model, the local determinacy can coexist with the global indeterminacy for a plausible range of tariff rates, which brings our attention to the use of local steady state analysis to make conclusions about the global dynamics of the nonlinear models.

Key Words: Tariff Policy, Global Indeterminacy, Chaos.

JEL: E32, Q43

1. Introduction

It is well understood by now that those economies with incomplete markets, imperfect competition or externalities can be easily pushed into the instability, exhibiting endogenous cycles, indeterminacy or chaos[1]. The government usually uses monetary and fiscal policies to address those issues related to the aggregate instability. We show that in a standard one sector small open economy Ramsey model with a productive externality, if the government wants to use the tariff policy to close the gap between the social and private marginal products of imported energy (which is created by the productive externality), it can give rise to various forms of endogenous dynamics, such as bifurcations and chaos.

Our framework is a discrete-time version of the one-sector small open economy growth model, recently developed by Wen and Aguiar-Conraria (2005, 2006 and 2007 henceforth WAC) [based on Benhabib and Farmer (1994)]. WAC show that in the small open economy with the imported energy as a third production factor, as long as the positive production externality is strong enough, the model can exhibit "local indeterminacy" around the single interior steady state.

We start our project by solving for a benchmark tariff policy that closes the gap between the social and private marginal products of imported energy in the WAC structure. The benchmark policy involves constant subsidy (negative tariff) rates applied to imported energy incomes. We show that the tariff rate applied to imported energy incomes is a key bifurcation parameter for the model's dynamics. In a laissez-faire economy that exhibits local indeterminacy, the dynamical system will undergo a supercritical Hopf bifurcation as the tariff rate becomes sufficiently negative (representing a subsidy) and a supercritical flip bifurcation as the tariff rate becomes sufficiently positive. An attracting closed orbit or cycle emerges as the tariff rate passes those critical values. Pushing the tariff rate beyond the critical value (τ^{flip}) in either direction may give way to chaos.

For the numerical calibration, the flip bifurcation occurs at the tariff rate of 26.33 percent. As the tariff rate is further increased beyond the flip bifurcation value, the model exhibits a series of period-doubling bifurcations–a typical route to chaos. This means that stable 2-, 4-, and 8- cycles, even chaos may appear as τ varies. The economic explanation can be traced to the paper of Guo and Lansing (2002, page 635), "...In this region of parameter space, the substitution effect generated by expected movements in the after-tax interest rate overcomes the corresponding income effect by an amount that is sufficient to induce cycling in agents' optimal saving decisions...".

For tariff rates beyond the flip-bifurcation value, the equilibrium is saddle-path stable. However, the local determinacy of equilibrium near the steady state coexists with global indeterminacy, which means that in regions away from the steady state, a stable n-period cycle or a chaotic attractor can arise as the equilibrium paths.

The Hopf bifurcation occurs at the energy subsidy rate of 51.14 percent. If the government wants to encourage energy imports by setting the subsidy at or more than 51.14 percent, it will destabilize the steady state and allow for Hopf bifurcations and regular 15-cycles. As the subsidy rate is increased beyond the Hopf-bifurcation value of 51.14 percent, an attracting closed orbit will surround the steady state and quasi-periodic oscillations arise. Further increases in the subsidy rate may make the orbit break up into a regular 15-cycle. The economic explanation can also be found in the paper of Guo and Lansing (2002, page 635), "...The high-subsidy region is characterized by large intermittent spikes in hours worked which reflect a "bunching effect" in production as agents' decisions internalize more of the increasing returns...".

Before solving the model and doing the quantitative simulations, we briefly mention some other papers that are closely related to the contributions of ours. Some of those papers are Cazzavillan (1996), Guo and Lansing (2002) and Coury and Wen (2008). All of the above papers are concerned with indeterminacy issues near a single interior steady state. In the spirit, our paper is very close to Guo and Lansing (2002)'s work because both analyze the fiscal policy under the framework with social increasing returns to scale.

The remainder of the paper is organized as follows. Section 2 introduces tariff policy into the WAC model. Section 3 studies the model's dynamics with constant subsidy/tariff rates. Section 4 discusses some extensions. Section 5 concludes.

2. The Basic Model

The WAC (2005) model consists of three types of agents: firms, households, and the government. They describe two competitive decentralizations that make the social technology exhibit increasing returns-to-scale. We use the version of the model with the externality for the ease of interpretation.

2.1. Households

The infinitely-lived representative household, endowed with one unit of time, maximizes a discounted stream of utilities over her lifetime by choosing sequences of consumption $\{c_t\}_{t=0}^{\infty}$, hours to work $\{n_t\}_{t=0}^{\infty}$, and the stock of capital $\{k_{t+1}\}_{t=0}^{\infty}$:

$$\max \sum_{t=0}^{\infty} \beta^t (\log c_t - B\frac{n_t^{1+\gamma}}{1+\gamma}),\ B > 0, \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor and $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply. We assume that no intrinsic uncertainties are present in the economy.

The household's budget constraint is

$$c_t + i_t = r_t k_t + w_t n_t + T_t, \tag{2}$$

where i_t is investment, k_t is the household's stock of physical capital, r_t is the capital rental rate, w_t denotes the real wage and T_t is the lump-sum transfer/tax, i.e., T_t can be negative. The household receives income by supplying capital and labor services to firms. Fiscal policy parameters in our model include: (1) the variable T_t, which represents the lump-sum tariff transfer to the agent; and (2) the *implicit* fiscal policy parameter $\tau-$ the tariff or subsidy rate imposed on the imported energy (say oil). Under this framework, negative tariff rates represent energy subsidies and a negative value of T_t represents a lump-sum tax received by the government. The household views r_t, w_t and T_t as being exogenously given.

Investment follows the law of motion of capital,

$$k_{t+1} = (1 - \delta)\, k_t + i_t,\ k_0 \text{ given}, \tag{3}$$

where $\delta \in (0, 1)$ is the constant depreciation rate.

The first-order conditions for the household's optimization problem are given by

$$Bn_t^{\gamma} = \frac{w_t}{c_t}, \tag{4}$$

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}}(r_{t+1} + 1 - \delta), \tag{5}$$

and the transversality condition is

$$\lim_{t \to \infty} \beta^t \frac{k_{t+1}}{c_t} = 0. \tag{6}$$

Equation (4) requires that the household's marginal rate of substitution between consumption and leisure be equal. Equation (5) is the consumption Euler equation.

2.2. Firms

The representative firm produces a homogenous final good using the following constant returns to scale technology:

$$y_t = z_t k_t^{a_k} n_t^{a_n} o_t^{a_o}, \tag{7}$$

where y_t is the firm's output, o_t is the third factor, say imported oil and $a_k + a_n + a_o = 1$, i.e., constant returns to scale at the firm level. z_t is the state of technology or the production externality that the firm takes as given. Each firm chooses $\{k_t, n_t, o_t\}$ to maximize profits by solving

$$\pi = \max_{k_t, n_t, o_t} y_t - r_t k_t - w_t n_t - p^o (1 + \tau_t) o_t, \tag{8}$$

subject to equation (7), where τ_t is the tariff rate imposed on the imported oil. We assume that factor markets are perfectly competitive. The profit maximization implies

$$\begin{aligned} r_t &= a_k \frac{y_t}{k_t}, & (9) \\ w_t &= a_n \frac{y_t}{n_t}, & (10) \\ p^o(1 + \tau_t) &= a_o \frac{y_t}{o_t}. & (11) \end{aligned}$$

From equation (11), we have $p^o o_t = a_o y_t/(1+\tau_t)$.

In contrast to a standard RBC model, the state of technology or the production externality z_t, is given by

$$z_t = \left(K_t^{a_k} N_t^{a_n} O_t^{a_o}\right)^{\eta},\ \eta \geq 0, \tag{12}$$

where K_t, H_t and O_t are the economy-wide average input levels. In the (symmetric) equilibrium, all firms act in the same way such that $K_t = k_t$, $H_t = h_t$ and $O_t = o_t$. Hence, the social technology is given by

$$y_t = k_t^{\alpha_k} n_t^{\alpha_n} o_t^{\alpha_o}, \tag{13}$$

where $\alpha_k \equiv a_k(1+\eta)$, $\alpha_n \equiv a_n(1+\eta)$ and $\alpha_o \equiv a_o(1+\eta)$. The social technology exhibits increasing returns to scale $(\alpha_k + \alpha_n + \alpha_o > 1)$ for $\eta > 0$.[2]

Assuming that the foreign input is perfectly elastically supplied, then the factor price, p^o, is independent of the factor demand for o. Thus, we have $o_t = \frac{a_o y_t}{p^o(1+\tau_t)}$. Substituting this formula into the production function, we can obtain the following reduced-form production function:

$$y_t = A k_t^{\frac{\alpha_k}{1-\alpha_o}} n_t^{\frac{\alpha_n}{1-\alpha_o}}, \tag{14}$$

where $A = \left[\dfrac{a_o}{p^o(1+\tau_t)}\right]^{\frac{\alpha_o}{1-\alpha_o}}$ acts as the technology coefficient in a neoclassical growth model, which is inversely related to the foreign factor price. In the reduced-form production function, the effective returns to scale is measured by $\frac{\alpha_k+\alpha_n}{1-\alpha_o}$, which exceeds the real returns to scale,

$$\frac{\alpha_k + \alpha_n}{1-\alpha_o} > (\alpha_k + \alpha_n + \alpha_o)$$

2.3. Government

We assume that there is no government spending and the government transfers the tariff revenue to the households. The government balances the budget in each period:

$$T_t = \tau_t p^o o_t = \frac{a_o \tau_t}{1+\tau_t} y_t.$$

Using equations (2), (3), (9), (10) and $p^o o_t = a_o y_t/(1+\tau)$, we obtain the following aggregate resource constraint equation:

$$k_{t+1} = (1-\delta)\, k_t + y_t \left(1 - \frac{a_o}{1+\tau_t}\right) - c_t. \tag{15}$$

3. Dynamics with Constant Tariff/Subsidy Rates

As in Guo and Lansing (2002), the increasing-returns technology (13) introduces a nonconvexity into the constraint set of the social planner's problem, which makes the Kuhn-Tucker sufficiency theorem not applicable to our fiscal policy analysis[3]. As an alternative to computing the optimal tariff policy, we consider the following benchmark tariff policy that closes the gap between the social and private marginal products of the imported energy.

Proposition 1. *The wedge between the social and private marginal products of the imported energy is eliminated when* $\tau_t = \frac{1}{1+\eta} - 1$ *for all* t, $T_t = -a_o \eta y_t$ *for all* t.

Proof. The social marginal products from equation (13) is $\frac{\partial y_t}{\partial o_t} = \alpha_o y_t/o_t$. The after-tariff private marginal product is $(1+\tau_t)^{-1} a_o \frac{\partial y_t}{\partial o_t}$. With $\tau_t = \frac{1}{1+\eta} - 1 < 0$, $(1+\tau_t)^{-1} a_o \frac{\partial y_t}{\partial o_t} = \alpha_o y_t/o_t$. The lump-sum tariff revenue follows directly from equation $T_t = \tau_t p^o o_t = \frac{a_o \tau_t}{1+\tau_t} y_t = -a_o \eta y_t$. ■

The benchmark policy involves constant subsidy rates that are only governed by the externality parameter η. A similar result is also obtained in Guo and Lansing's model. In the following analysis, we assume the tariff rate is constant, which implies that the government income is endogenous.

3.1. Calibration (Quantitative Experiments)

According to the existing RBC literature, we calibrate the structural parameters for a quarterly model. Following WAC (2005, 2006 and 2007), table 1 summarizes the baseline parameter values.

Table 1: Baseline parameter values

Table 1: Parameter Values		
γ	0	Indivisible labor, see Hansen (1985).
β	0.99	Discount factor, see WAC (2005, 2006).
a_n	0.7	Labor's share.
a_o	0.16	Oil's share, see WAC (2005) for the country Canada.
a_k	$1 - a_n - a_o$	Capital's share.
δ	0.025	Depreciation rate.
B	2.984	Implies fraction of time spent working = 0.3.[4]
η	0.203	Externality parameter.

The baseline parameter values are commonly used in real business cycle models except the externality parameter η[5]. The degree of returns to scale in the model is $1+\eta$. WAC (2005) note that minimal returns to scale needed to generate local indeterminacy can vary dramatically depending on the degree of energy dependence of that specific country.

Given our baseline parameter values, it requires returns to scale at least 1.198 to exhibit local indeterminacy. We let $\eta = 0.203$ for our quantitative experiments, which implies returns to scale around 1.203 and the benchmark fiscal policy parameter $\tau^b = \frac{1}{1+\eta} - 1 = -0.16874$. This experiment makes the laissez-faire economy exhibit local indeterminacy, consistent with the range of indeterminacy region that WAC (2005) find. We should mention that a figure of 1.203 may be considered empirically implausible for the U.S, Canada or European countries. But "the quantitative experiments reported below should be viewed more from a methodological perspective as illustrating the pitfalls that can arise from focusing exclusively on log-linearized dynamics rather than considering the model's true nonlinear equilibrium conditions," as suggested by Guo and Lansing (2002, page 640).

3.2. Log-Linearized Dynamics

In the appendix, we show that the equilibrium conditions in our model can be described by the following log-linear system:

$$\begin{bmatrix} \ln\left(k_{t+1}/\overline{k}\right) \\ \ln\left(c_{t+1}/\overline{c}\right) \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & -\lambda_2 \\ \dfrac{\lambda_1\lambda_3}{\lambda_4} & \dfrac{1-\lambda_2\lambda_3}{\lambda_4} \end{bmatrix}}_{\mathbb{J}} \begin{bmatrix} \ln\left(k_t/\overline{k}\right) \\ \ln\left(c_t/\overline{c}\right) \end{bmatrix}, \; k_0 \text{ given,} \tag{16}$$

where $\overline{k}$ and $\overline{c}$ are steady-state values of capital and consumption and $\mathbb{J}$ denotes a 2×2 Jacobian matrix of partial derivatives evaluated at the steady-state solution for the original dynamic system. The elements of $\mathbb{J}$ can be represented by four constants, λ_i, $i = 1, 2, 3, 4$, which expressions can be found in the appendix. (λ_i can be represented by the model parameters B, a_k, a_n, a_o, δ, β, τ and γ.) The two eigenvalues of $\mathbb{J}$ will determine the stability of the log-linear system. The oil price p^o does not appear in $\mathbb{J}$ and thus will not affect the model's local stability properties. Equations (4) and (5) show that τ not only affects the tradeoff between consumption and leisure at a given date (it can be seen from (4)) but also affects the tradeoff between consumption goods at different dates (it can be seen from (5)). As Guo and Lansing (2002, page 641) pointed out, "...the intertemporal tradeoff is the crucial mechanism for generating multiple equilibria because agents' expectations of future returns must become self-fulfilling...".

Table 2 summarizes the model's local stability properties as we allow the tariff rate τ to vary from $-\infty$ to $+\infty$.

Tariff Rate	Eigenvalues of Jacobian Matrix		Steady State
$\tau < -0.5114$	complex	$\lvert\mu_1\rvert = \lvert\mu_2\rvert > 1$	Source
$\tau = -0.5114$ (Hopf Bifurcation)	complex	$\lvert\mu_1\rvert = \lvert\mu_2\rvert = 1$	Source changes to Sink
$-0.5114 < \tau < \tau_R$	complex	$\lvert\mu_1\rvert = \lvert\mu_2\rvert < 1$	Sink
$\tau_R < \tau < 0.2633$	Real	$\lvert\mu_1\rvert < 1$, $\lvert\mu_2\rvert < 1$	Sink
$\tau = 0.2633$ (flip Bifurcation)	Real	$\mu_1 = -1$, $\lvert\mu_2\rvert < 1$	Sink changes to Saddle
$\tau > 0.2633$	Real	$\mu_1 < -1$, $\lvert\mu_2\rvert < 1$	Saddle

Table 2: Stability Properties Near the Steady State

3.3. Local Indeterminacy

Figure 1 plots the combinations of η (the externality parameter) and τ (the tariff rate) that allow for different equilibrium dynamics. Local indeterminacy requires that both eigenvalues of $\mathbb{J}$ lie inside the unit circle. We know that the degree of returns to scale in the model is $1+\eta$. When $\eta = 0$ (CRS), the model is saddle point stable for all values of τ. The figure shows that $\eta > 0.1976$ is needed for the steady state to become locally indeterminate. Given $\eta > 0.1976$, decreases in τ eventually makes the steady state become a source while increases in τ eventually make the steady state become a saddle point. We set η to be 0.203 in the calibration and the local indeterminacy occurs for tariff rates in the range $-0.5114 < \tau < 0.2633$. When $\tau > 0.2633$, the model exhibits a locally unique equilibrium (a saddle point). Hence, if the government wants to stabilize the economy against sunspot fluctuations near the steady state by imposing a sufficiently high tariff rate on the imports, instead such a policy may make the economy susceptible to other forms of endogenous fluctuations, such as bifurcations and/or chaos.

Insert Figure 1 here

3.4. Flip Bifurcation

For the numerical experiment, the dynamical system undergoes a flip bifurcation as τ is increased past the value $\tau^{flip} = 0.2633$. We first prove that the flip bifurcation point is supercritical. This means that, within a small open neighborhood of τ^{flip} (in our case $\tau^{flip} + \varepsilon$), and as τ increases, the steady state goes from being a sink to being a saddle surrounded by an attracting period–2 cycle[6]. At the bifurcation point, we have $det(\mathbb{J}) + tr(\mathbb{J}) = -1$ (see Guo and Lansing (2002)). Using the expressions for $det(\mathbb{J})$ and $tr(\mathbb{J})$ derived in the appendix, we can get the following bifurcation value[7]:

$$\tau^{flip} = \frac{a_o H_1 (\rho + \delta)}{H_1 (\rho + \delta) + a_k (H_2 - \delta H_1)} - 1, \tag{17}$$

where $\rho \equiv 1/\beta - 1$ is the household's rate of time preference. In this model, we use numerical simulations to establish that the flip bifurcation is supercritical[8]. The supercritical flip bifurcation will make the model exhibit deterministic cycles that won't converge to the steady state. The two-cycle is an attractor and in this case, the global indeterminacy coexists with local determinacy because the steady state is a saddle point for $\tau > 0.2633$. If we set $\tau > 0.2633$ in oder to eliminate sunspot fluctuations near the steady state, it can make the economy susceptible to sunspots, cycles, or even chaos, in regions away from the steady state.

3.5. Hopf Bifurcation

For the numerical calibration, the dynamic system undergoes a Hopf bifurcation as τ is decreased past the value $\tau^{Hopf} = -0.5114$[9]. At the Hopf bifurcation point, we have $det(\mathbb{J}) = 1$ (see Guo and Lansing (2002)). Using the expression for $det(\mathbb{J})$ derived in the appendix, we get the following bifurcation value[10]:

$$\tau^{Hopf} = \frac{a_o M_4 (\rho + \delta)}{M_4 (\rho + \delta) - a_k [\delta + M_1 \beta (\rho + \delta)]} - 1. \tag{18}$$

We use numerical simulations to establish that the Hopf bifurcation in our model is supercritical. In the supercritical Hopf bifurcation, an *attracting* closed orbit emerges on the side of τ^{Hopf} where the steady state is unstable (in our case a source), that is, in the small neighborhood $\tau^{Hopf} - \varepsilon$.

The supercritical Hopf bifurcation makes our model exhibit deterministic, quasiperiodic oscillations that won't converge to the steady state. Because the invariant closed orbit is an attractor, there exists a continuum of equilibrium paths each leading to the closed orbit. This is a case of global indeterminacy.

3.6. Nonlinear Dynamics

The model's perfect foresight dynamics follow the nonlinear map: (see the appendix)

$$\frac{\beta}{c_{t+1}} \left[a_k A k_{t+1}^{\frac{\alpha_k + \alpha_o - 1}{1-\alpha_o}} \left(\frac{a_n A}{B} \frac{k_{t+1}^{\frac{\alpha_k}{1-\alpha_o}}}{c_{t+1}} \right)^{\frac{\alpha_n}{(1+\gamma)(1-\alpha_o)-\alpha_n}} + 1 - \delta \right] = \frac{1}{c_t}, \tag{19}$$

$$k_{t+1} = \left(1 - \frac{a_o}{1+\tau} \right) A k_t^{\frac{\alpha_k}{1-\alpha_o}} \left(\frac{a_n A}{B} \frac{k_t^{\frac{\alpha_k}{1-\alpha_o}}}{c_t} \right)^{\frac{\alpha_n}{(1+\gamma)(1-\alpha_o)-\alpha_n}} + (1-\delta) k_t - c_t. \tag{20}$$

To simulate the global dynamics, we iterate the above map for a range of values of τ. Following Guo and Lansing (2002), the iteration proceeds as follows. We disturb the steady state by an arbitrary amount and set our initial values (k_0, c_0), then we solve equation (20) for k_1. Substituting the value of k_1 into equation (19) yields a nonlinear equation that can be used to solve c_1. We can repeat the procedure to compute (k_2, c_2) and so on.

Figure 2 plots the bifurcation diagram and the largest Lyapunov exponent over the range $-0.51172 \leq \tau \leq 0.39780$. Figure 3 and 4 show the details near τ^{Hopf} and τ^{flip}. The bifurcation diagram gives us the long-run behavior of the model by plotting the last 250 points of a long simulation. The figures show that setting the tariff rate beyond τ^{flip} eventually leads to chaos. Since as τ increases, a significantly positive Lyapunov exponent occurs, which is an indicator of "sensitive dependence on initial conditions"—one of the characteristics of chaos[11]. The transition to chaos takes place via a "period-doubling" route in the high-tariff rate region ($\tau > \tau^{flip}$).

Insert Figures 2, 3 and 4 here

Figures 5 to 9 show us various forms of endogenous fluctuations as τ varies. Figures 5 and 6 show that the Hopf bifurcation is supercritical since the invariant closed orbit is attracting. As $\tau = \tau^{Hopf} - 2.1E - 4$, rational expectations equilibrium paths eventually converge to the invariant closed orbit for arbitrary starting points either inside or outside the circle. Figure 7 shows that the invariant closed orbit starts to break up into a regular 15-cycle when the tariff rate is decreased to some point in the left hand side of τ^{Hopf}. In the small neighborhood of the flip bifurcation point, the model exhibits stable 2- and 4- and 8-cycles for tariff rates in the range of $\tau^{flip} < \tau < 0.39780$. Figure 8 depicts these three kinds of cycles and their corresponding time-series simulated data. When we increase the value of the tariff rate to $\tau = 0.39775$, a type of chaotic attractors emerges as shown in figure 9.

Changes in τ have effect on the amplitude of the cycles or oscillations. In the high–subsidy region, the Hopf bifurcation as one kind of rational expectations equilibria is "characterized by large intermittent spikes in hours worked and output which reflect a “bunching effect” in production as agents' decisions internalize more of the increasing returns", as suggested by Guo and Lansing (2002, page 651). In the high-tariff region, the stable n-cycle as a kind of rational expectations equilibria can be explained in the similar way like Guo and Lansing (2002, page 652)"...the substitution effect generated by expected movements in the after-tariff interest rate overcomes the corresponding income effect by an amount that is sufficient to induce cycling in agents' optimal saving decisions...".

The time series plots in figures 5 through 9 give us a picture about the percentage changes in model output and consumption. These figures indicate that the consumption is quite smooth, while the output is quite volatile. The fluctuation amplitudes of output is much larger than those observed in the real Canada economy at business-cycle frequencies. It can be due to the presence of strong increasing returns.

Insert Figures 5, 6, 7, 8 and 9 here

4. Extension: Local Control—Adjustment Costs

In this section, similar to Guo and Lansing (2002), we describe some policy mechanisms which are used to eliminate sunspot fluctuations near the steady state.

It is well known that explicit adjustment costs for capital investment can be used to select a locally unique equilibrium. We then consider an economy where the household budget constraint (2) is described by the following new equation:

$$c_t + i_t \left[1 + \underbrace{\frac{\psi}{2} \left(\frac{k_{t+1}}{k_t} - 1 \right)^2}_{\tau_t(\cdot)} \right] = r_t k_t + w_t n_t, \tag{21}$$

the adjustment cost parameter ψ can be used as the bifurcation parameter. Given other parameter values in our baseline model, the dynamical system undergoes a supercritical flip bifurcation as ψ is increased past the value 0.1781.

This example shows that some local control methods can be used to select a locally unique equilibrium. However, they also have those problems that we encounter in this paper, i.e., when global indeterminacy coexists with local determinacy, equilibrium selection mechanisms designed using the log-linear approximation method may be misleading if we observe the true nonlinear dynamics in the model.

5. Conclusion

In this paper, we introduce a constant tariff or subsidy rate (applied to the imported energy) into the WAC (2005, 2006 and 2007) model. In this setting, our dynamical equilibrium is described by a two-dimensional difference equations system with one preditermined variable (capital). And we find that a rich set of endogenous fluctuations including bifurcations and/or chaos can arise in this setup. The dynamical system undergoes a supercritical Hopf bifurcation as the tariff rate becomes sufficiently negative, and a supercritical flip bifurcation as the tariff rate becomes sufficiently positive. The model's equilibrium dynamics exhibit stable 2-, 4-, 8-, and 15-cycles, quasi-periodic closed orbits, and chaos. In a somewhat different model, Guo and Lansing (2002) show that a constant capital tax or subsidy can give rise to similar dynamics in a closed-economy one sector model with a productive externality. From this perspective, factor income taxes and tariffs are equivalent to generate endogenous fluctuations.

Notes:

1. See Benhabib and Farmer (1999) for an excellent survey of the literature.

2. We only consider the case $\alpha_k < 1$, which says that the externality is not strong enough to generate sustained endogenous growth.

3. As Guo and Lansing (2002) pointed out, the nonconvexity will make it hard for us to compute the first-best allocations and find the optimal fiscal policy which implements the first-best as a competitive equilibrium.

4. Since our calibration exercise is a numerical experiment, we simply set the implied faction of time spent on working to be 0.3, which is used in Guo and Lansing (2002). Though 0.3 may not be exactly the one of Canada, we say our results will be robust for reasonable parameter selection. Usually, people work 8 hours per day.

5. We should mention that our results are robust to the parameter p^o. In our case, we set $p^o = 0.1$.

6. See Guckenheimer and Holmes (1983).

7. For H_1 and H_2, see our appendix.

8. For the analytical calculation, see Guckenheimer and Holmes (1983).

9. For the analytical calculation, see Guckenheimer and Holmes (1983).

10. For M_1 and M_4, see our appendix.

11. Guo and Lansing point this out (page 645), "We compute the Lyapunov ex-

ponents according to the procedure described by Alligood et. al. (1997). Since equation (19) cannot be solved explicitly for c_{t+1}, the required derivatives $\partial c_{t+1}/\partial k_t$ and $\partial c_{t+1}/\partial c_t$ are computed numerically by log-linearizing equation (19) around each successive point of the trajectory generated by the nonlinear map. This introduces some approximation error into our computation so that values of the Lyapunov exponent that are only slightly above zero are not reliable indicators of chaos."

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Appendix:

This part gives us the equations used to study the model's equilibrium dynamics in section 3.

The equilibrium conditions can be seen from the following four equations:

$$
\begin{aligned}
y_t &= A k_t^{\frac{\alpha_k}{1-\alpha_o}} n_t^{\frac{\alpha_n}{1-\alpha_o}}, \text{ where } A = \left[\frac{a_o}{p^o(1+\tau)}\right]^{\frac{\alpha_o}{1-\alpha_o}}, && \text{(E-1)} \\
B n_t^{1+\gamma} &= a_n \frac{y_t}{c_t}, && \text{(E-2)} \\
\frac{1}{c_t} &= \frac{\beta}{c_{t+1}} \left(a_k \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right), && \text{(E-3)} \\
c_t + k_{t+1} &= (1-\delta) k_t + y_t \left(1 - \frac{a_o}{1+\tau} \right). && \text{(E-4)}
\end{aligned}
$$

For the parameter values in Table 1, we can see that there is a unique interior steady state in this economy. Equations (E.1) and (E.2) imply:

$$
\begin{aligned}
n_t &= \left(\frac{a_n A}{B} \frac{k_t^{\frac{\alpha_k}{1-\alpha_o}}}{c_t} \right)^{\frac{1-\alpha_o}{(1+\gamma)(1-\alpha_o)-\alpha_n}} \\
&= \left(\frac{a_n A}{B} \right)^{\frac{1-\alpha_o}{(1+\gamma)(1-\alpha_o)-\alpha_n}} k_t^{\frac{\alpha_k}{(1+\gamma)(1-\alpha_o)-\alpha_n}} c_t^{-\frac{1-\alpha_o}{(1+\gamma)(1-\alpha_o)-\alpha_n}}.
\end{aligned}
$$

which can be used to substitute n_t in equations (E.3) and (E.4). Thus, we can imply equations (19) and (20).

In the small neighborhood of the steady state, equations (19) and (20) can be approximated by the log-linearization method and we have:

$$
\begin{bmatrix} \ln\left(k_{t+1}/\bar{k}\right) \\ \ln\left(c_{t+1}/\bar{c}\right) \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & -\lambda_2 \\ \frac{\lambda_1 \lambda_3}{\lambda_4} & \frac{1-\lambda_2\lambda_3}{\lambda_4} \end{bmatrix}}_{\mathbb{J}} \begin{bmatrix} \ln\left(k_t/\bar{k}\right) \\ \ln\left(c_t/\bar{c}\right) \end{bmatrix}, \ k_0 \text{ given,}
$$

where the four main elements are: $\lambda_1 = \delta M_4 + 1 - \delta + M_4 \frac{c}{k}$, $\lambda_2 = \delta M_1 + (1+M_1)\frac{c}{k}$, $\lambda_3 = M_3 \beta(\rho+\delta)$ and $\lambda_4 = 1 + M_1 \beta(\rho+\delta)$. $M_1 = \frac{\alpha_n}{(1+\gamma)(1-\alpha_o)-\alpha_n}$, $M_2 = \frac{(1+\gamma)(1-\alpha_o)}{(1+\gamma)(1-\alpha_o)-\alpha_n}$,

$M_3 = \frac{(1+\gamma)(\alpha_k+\alpha_o-1)+\alpha_n}{(1+\gamma)(1-\alpha_o)-\alpha_n}$ and $M_4 = \frac{\alpha_k(1+\gamma)}{(1+\gamma)(1-\alpha_o)-\alpha_n}$. The steady state capital/labor ratio $\dfrac{c}{k} = \frac{\rho+\delta}{a_k}\left(1-\frac{a_o}{1+\tau}\right)-\delta$. $\rho \equiv 1/\beta - 1$ is the household's rate of time preference.

The determinant and trace of J are:

$$\det(J) = \frac{\lambda_1}{\lambda_4}, \tag{E-5}$$

$$\text{trace}(J) = \lambda_1 + \frac{1-\lambda_2\lambda_3}{\lambda_4}. \tag{E-6}$$

At the Hopf bifurcation point, we have $\det(J) = 1$, and at the flip bifurcation point, we have $\det(J) + trace(J) = -1$.

At the Hopf bifurcation point, $\det(J) = 1$, say,

$$-M_4\left[\frac{\rho+\delta}{a_k}\left(1-\frac{a_o}{1+\tau}\right)-\delta\right] = \delta M_4 - \delta - M_1\beta\left(\rho+\delta\right),$$

$$\tau^{Hopf} = \frac{a_o M_4\left(\rho+\delta\right)}{M_4\left(\rho+\delta\right) - a_k\left[\delta + M_1\beta\left(\rho+\delta\right)\right]} - 1.$$

At the flip bifurcation point, $\det(J) + trace(J) = -1$, say,

$$\begin{aligned} & -2 - M_1\beta\left(\rho+\delta\right) \\ = \; & \left(\delta M_4 + 1 - \delta + M_4\frac{c}{k}\right)\left[2 + M_1\beta\left(\rho+\delta\right)\right] \\ & -\left[\delta M_1 + \left(1+M_1\right)\frac{c}{k}\right] M_3\beta\left(\rho+\delta\right). \end{aligned}$$

$$\begin{aligned} RHS \;=\; & \frac{c}{k}\left\{2M_4 + \beta\left(\rho+\delta\right)\left[M_1\left(M_4 - M_3\right) - M_3\right]\right\} + 2\delta M_4 + 2\left(1-\delta\right) \\ & +\beta\left(\rho+\delta\right)\left[M_1 M_4\delta + M_1 - M_1\delta - M_1 M_3\delta\right]; \end{aligned}$$

At the flip bifurcation point,

$$
\begin{aligned}
& -\frac{c}{k}\{2M_4+\beta(\rho+\delta)[M_1(M_4-M_3)-M_3]\} \\
= \;& 4-2\delta+2\delta M_4+\beta(\rho+\delta)[M_1M_4\delta+2M_1-M_1\delta-M_1M_3\delta] \\
= \;& 4-2\delta+2\delta M_4+\beta(\rho+\delta)[\delta M_1(M_4-M_3)+(2-\delta)M_1];
\end{aligned}
$$

Denote H_1, H_2 as

$$H_1=2M_4+\beta(\rho+\delta)[M_1(M_4-M_3)-M_3]$$

and $H_2=4-2\delta+2\delta M_4+\beta(\rho+\delta)[\delta M_1(M_4-M_3)+(2-\delta)M_1]$, then we have

$$-H_1\left[\frac{\rho+\delta}{a_k}\left(1-\frac{a_o}{1+\tau}\right)-\delta\right]=H_2,$$

$$\tau_{flip}=\frac{a_oH_1(\rho+\delta)}{H_1(\rho+\delta)+a_k(H_2-\delta H_1)}-1.$$

6. Figures

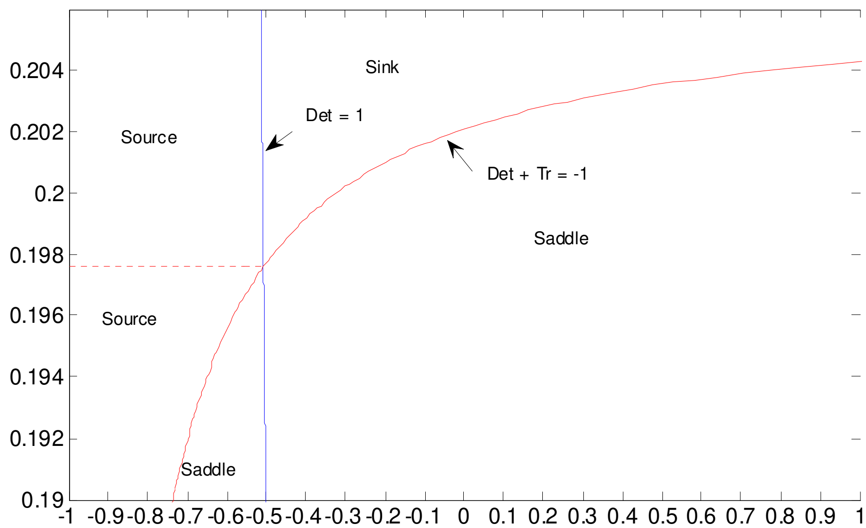


Figure 1. Regions of Stability Near Steady State

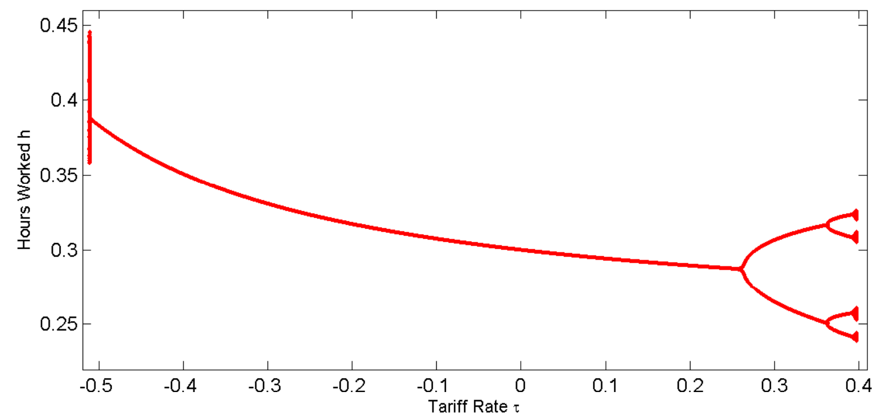


Figure 2A. Bifurcation Diagram

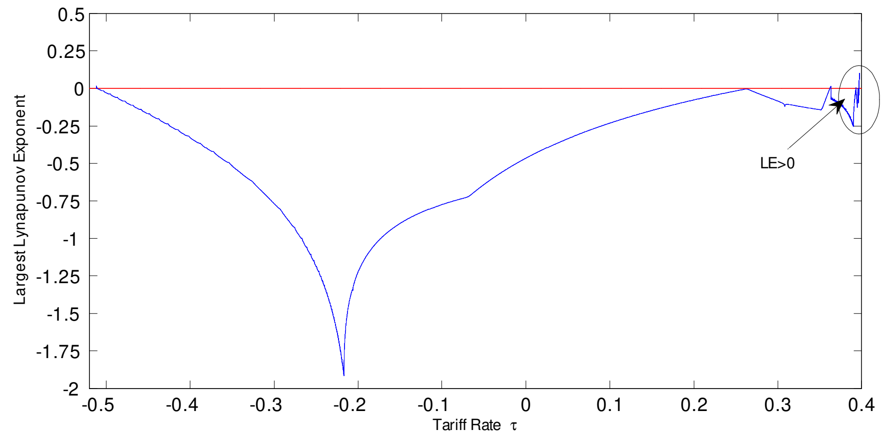


Figure 2B. Largest Lyapunov Exponent

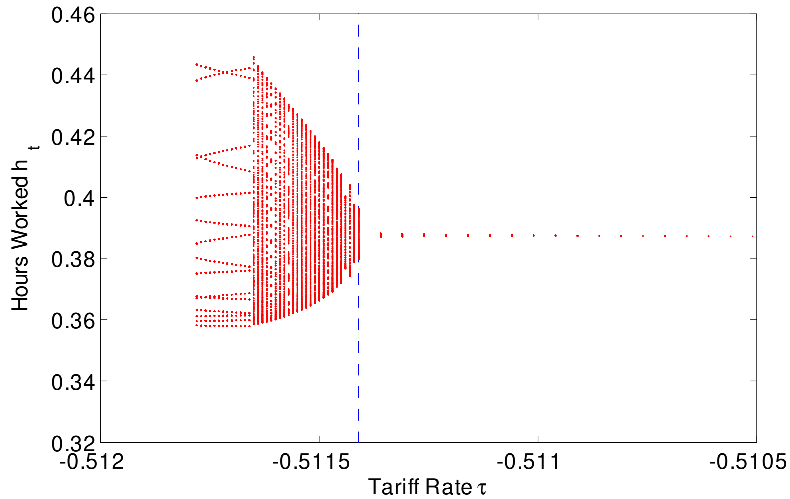


Figure 3A. Bifurcation Diagram (Details Near τ^{Hopf}).

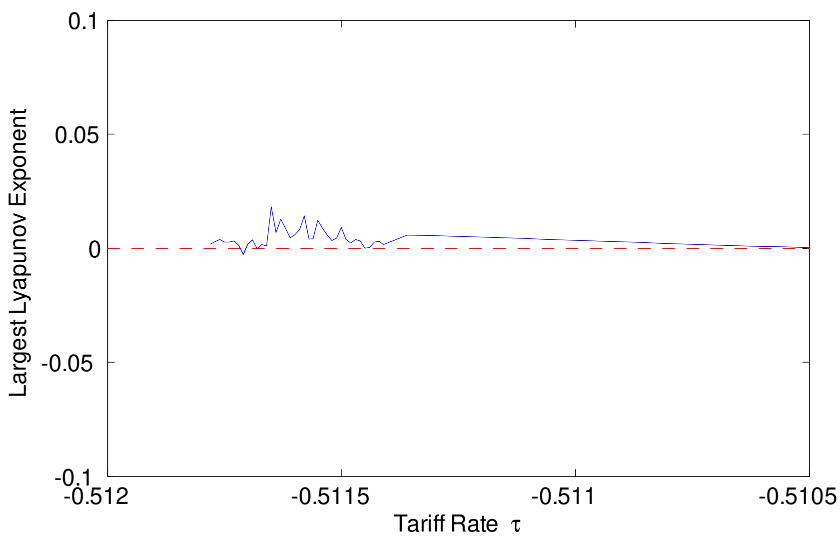


Figure 3B. Largest Lyapunov Exponent (Detail Near τ^{Hopf}).

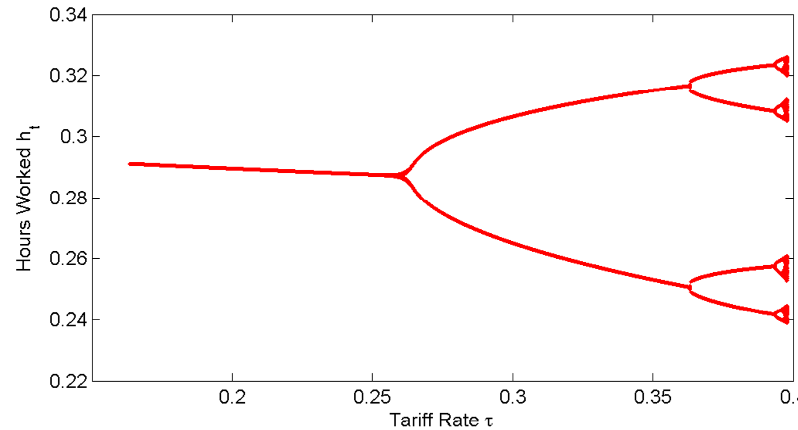
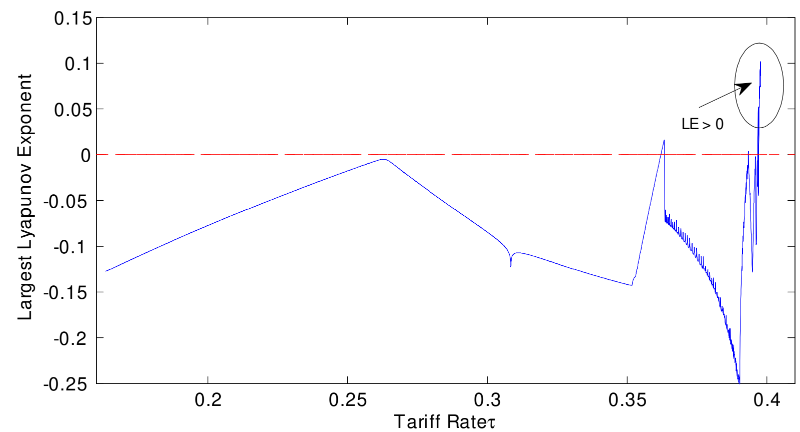


Figure 4A. Bifurcation Diagram (Detail Near τ^{flip})



Figrure 4B. Largest Lyapunov Exponent (Detail Near τ^{flip})

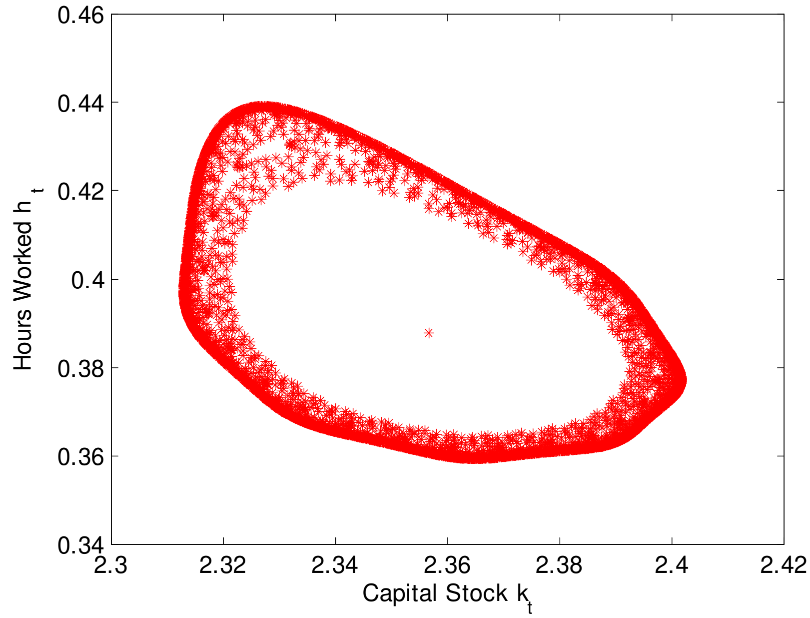


Figure 5A. Phase Diagram (Attracting Circle—Start Inside): $\tau = \tau^{Hopf} - 2.1E-4$

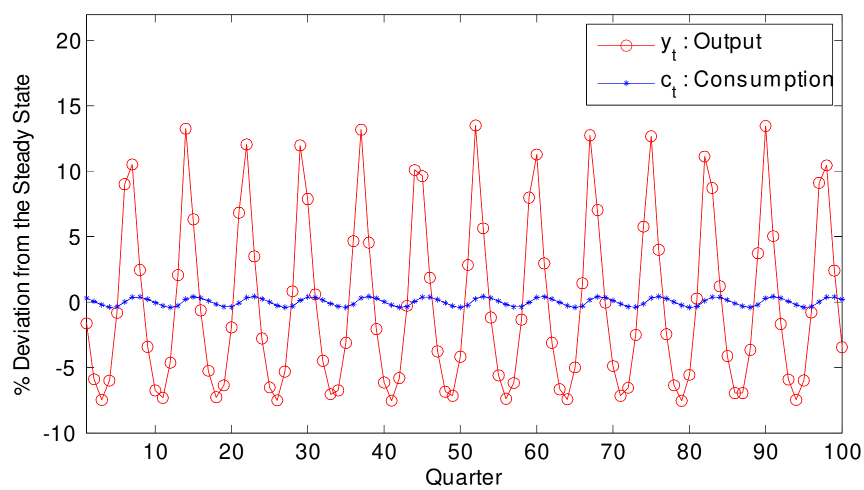


Figure 5B. Time-Series Plot (Attracting Circle—Start Inside).

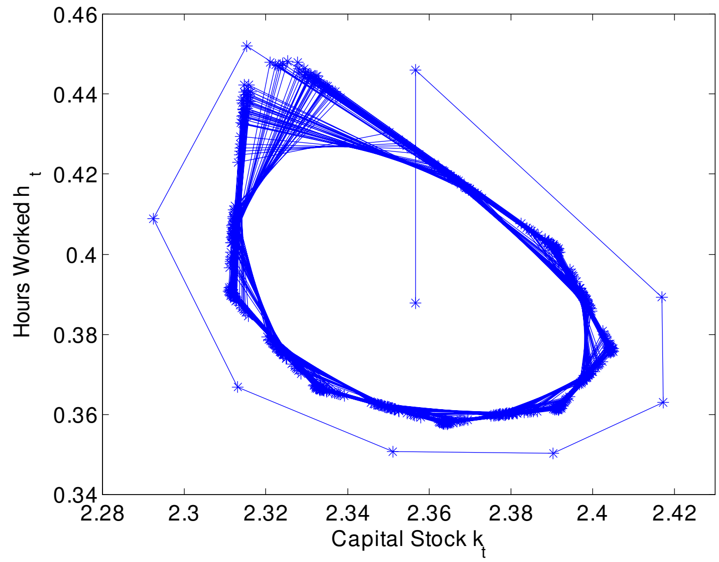


Figure 6A. Phase Diagram (Attracting Circle—Start Outside): $\tau = \tau^{Hopf} - 2.1E-4$

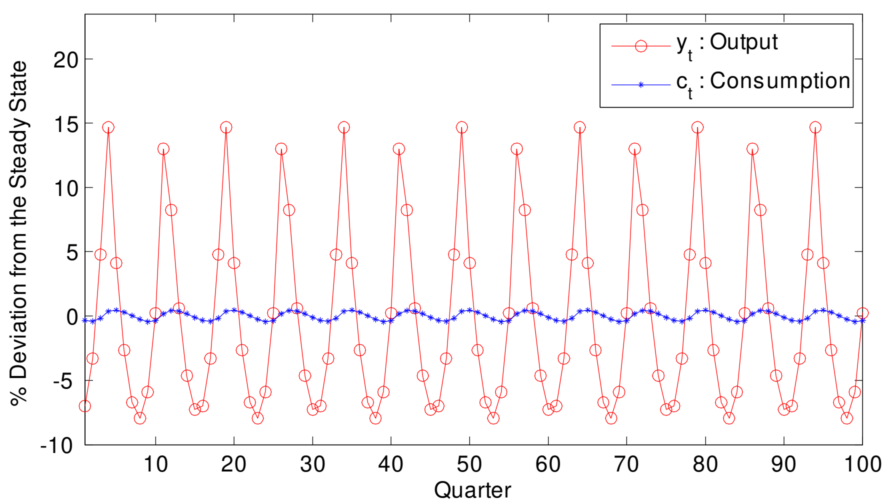


Figure 6B. Time-Series Plot (Attracting Circle—Start Outside).

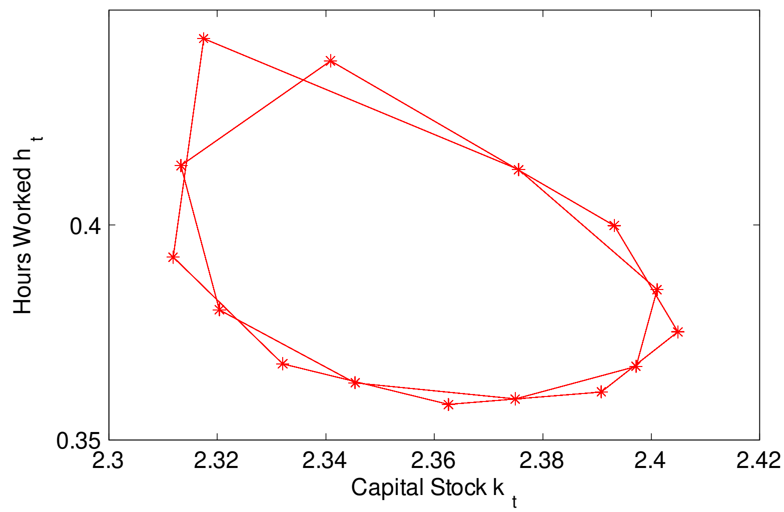


Figure 7A. Long–Run Phase Diagram (Attracting 15-Cycle): $\tau = \tau^{Hopf} - 3.7E - 4$

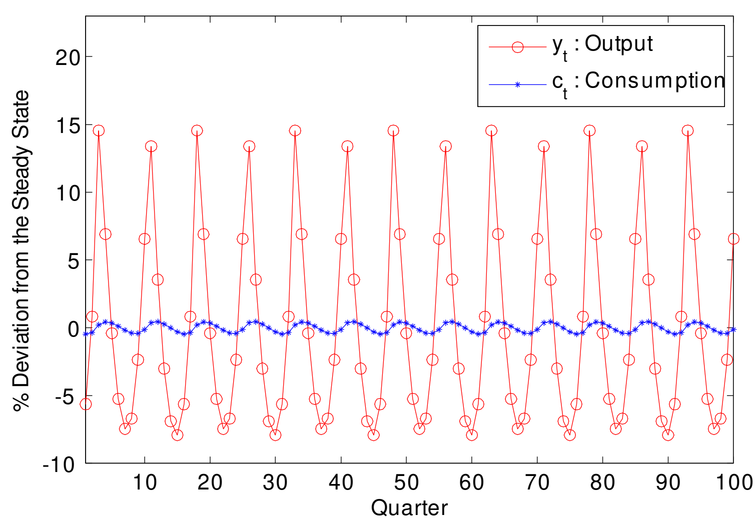


Figure 7B. Time-Series Plot (Attracting 15-Cycle).

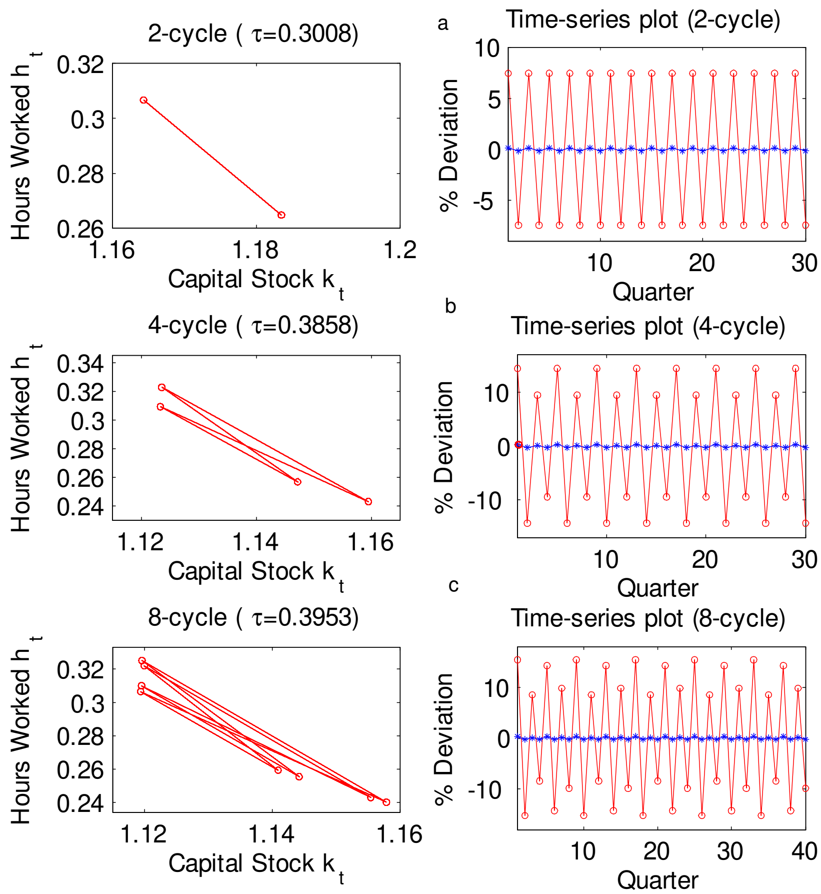


Figure 8. 2, 4, 8-Cycles in the Vicinity of Flip Bifurcation Point

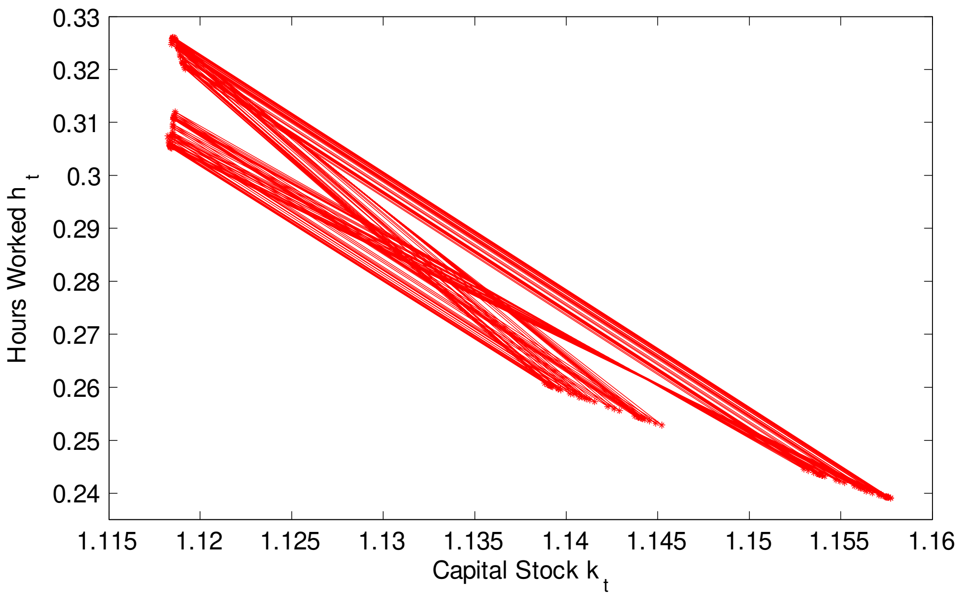


Figure 9A. Phase Diagram (Chaotic Attractor): $\tau = 0.39775$.

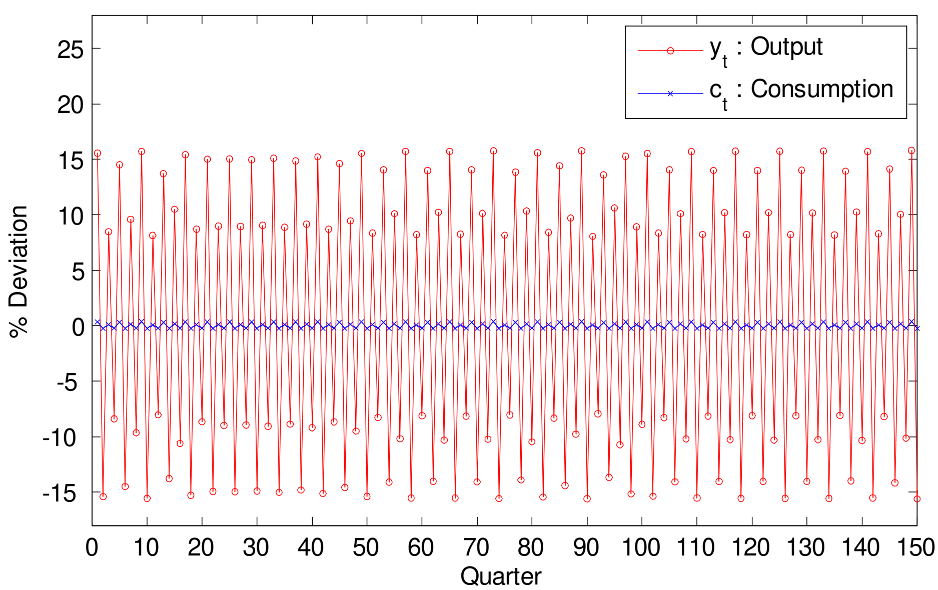


Figure 9B. Time-Series Plot (Chaotic Attractor).