Trade, Growth, and the International Transmission of Financial Shocks

Ohdoi, Ryoji

Tokyo Institute of Technology

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Ryoji Ohdoi†

Tokyo Institute of Technology

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Abstract

This study develops a two-country model to explore how financial shocks in one country affect its partner country's business cycles through international trade. Unlike existing studies, I introduce the mechanism of endogenous trade patterns, by which a shock can affect both the intensive and extensive margins of trade. I also embed the mechanism of endogenous growth into the model to indicate the potential for prolonged recessions, even for a transitory shock. I obtain the following four main findings. First, an adverse financial shock in one country induces a global recession, even in the absence of international financial transactions. Second, although the downward shift of real GDP in the partner country is not so large, it can be very prolonged. Third, the real value of exports in the partner drops more seriously than its real GDP. Finally, this drop is caused mainly by a change at the intensive margin rather than the extensive margin.

JEL classification: E22; E32; E44; F11; F44

Keywords: Eaton–Kortum model; Endogenous growth; Financial frictions; Financial shocks; International business cycles; Margins of trade

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†Department of Industrial Engineering and Economics, Tokyo Institute of Technology, 2-12-1, O-okayama, Meguro-ku, Tokyo 152-8552, Japan; E-mail: ohdoi.r.aa@m.titech.ac.jp

1 Introduction

The recent global recession during the 2008–2009 financial crisis drew more attention to the important interdependence among countries. In Figure 1, I plot the level and growth rate of GDP for the United States (U.S.) and an aggregate of the other 35 OECD member countries, where the shaded gray bar denotes the 2008—2009 period.[1] In the first panel, the GDP level in 2008:Q1 is normalized to one in each economy. Without much difficulty, this figure illustrates three trends. First, before the crisis, the economies have similar rates of trend growth. Second, during the crisis, they experienced a synchronized economic downturn. Third, after the crisis, the *growth rate* recovered, and thereby the *level* of GDP in both the U.S. and the other group of economies continued to move along the trend below the potential trend without the crisis. Before the crisis, macroeconomic models with financial frictions were major workhorses in business cycle studies. After the crisis, newer studies shed light on the shocks to the financial constraint itself as a key influence on business cycles in a closed economy.[2]

Then, how do such financial shocks in one country affect business cycles in other countries? As is well known, world trade also suddenly and severely contracted during the 2008–2009 global crisis. According to WTO (2009), international trade in manufactured goods declined by about 30%, which is more serious than the drop in the world GDP. This so-called "Great Trade Collapse" led researchers to investigate the role of financial frictions in the trade decline. However, the existing studies on international macroeconomics did not pay sufficient attention to international trade as the transmission channel. For example, Devereux and Yetman (2010), Devereux and Sutherland (2011), Kollman et al. (2011), and Perri and Quadrini (2018) assume an economy with a single traded good. Hence, these studies overlook the possible transmission channel of the intratemporal trade of multiple goods. Dedola and Lombardo (2012), Imura and Thomas (2016), and Yao (2019) employ a two-tradable-good framework and examine the role of the terms of trade in the transmission of financial shocks across countries.[3] However, all of these works build on the Armington model, in which both the number of traded goods and trade pattern are fixed.

Some empirical studies investigate how the financial crisis affected the intensive and extensive margins of trade.[4] Using a dataset of French firms, Bricongne et al. (2012) show that while the crisis affected all

[1] The data were obtained before Columbia became a member of the OECD.

[2] Examples of such studies include those by Jermann and Quadrini (2012), Kahn and Thomas (2013), Buera and Moll (2015), and Shi (2015).

[3] In Dedola and Lombardo (2012), home and foreign products consist of a continuum of horizontally differentiated goods, but its size is normalized to one.

[4] Generally, the extensive margin of trade is the number of traded goods, trading firms, or trading partners, whereas the intensive margin of trade is the value for existing firms or goods. The margins of exports and imports are defined similarly.

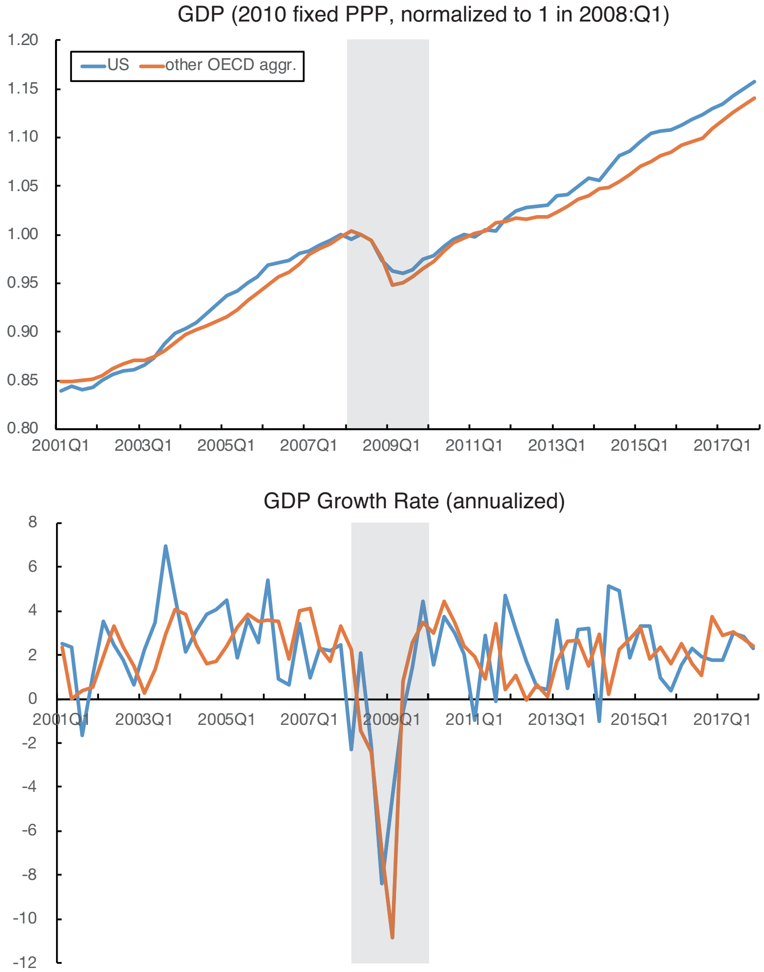

Figure 1: Internationally synchronized recession (Source: OECD Statistics)

firms, the decline in trade volume occured mainly at the intensive margin rather than at the extensive margin. Similarly, Behrens et al. (2013) report a total fall in Belgian exports of 26.23% between 2008:Q1 and 2009:Q1, and the decline at the intensive margin contributed 97.32% of this decrease (that is, the value of already exported goods declined by 25.63%). These results may therefore seem to justify the assumptions of a fixed number of traded goods and fixed trade patterns in these prior studies. However, because such studies start by fixing these elements, explaining these facts in theory is still an open issue. Against this background, we need a framework to investigate how financial shocks affect both the extensive and intensive margins of trade.

In this study, I construct a simple two-country dynamic general equilibrium with just three twists: financial frictions, endogenous growth, and international trade with endogenous trade patterns. To embed financial frictions simply, I apply the heterogeneous-agent framework of Buera and Moll (2015) to the two-country model. More precisely, I assume that investors have heterogeneous capital investment technologies and face credit constraints. Thanks to endogenous growth with learning-by-doing externalities, it is possible to explain the persistent downward shift in the level of world GDP theoretically, even though a shock itself is transitory, as we actually observed after the financial crisis.[5] To make financial shocks affect both the intensive and extensive margins of trade, I use the recent Ricardian trade model developed by Eaton and Kortum (2002) and refined by Alvarez and Lucas (2007).[6] The model assumes a continuum of a variety of intermediate goods, the trade in which is subject to iceberg costs. A country's efficiency in producing each variety is subject to a realization of a random draw from a country-specific distribution.[7] Within this framework, I first analytically characterize the equilibrium where two countries are asymmetric; that is, the trade costs, degree of financial frictions, and other key parameters can vary across countries.

Then, I use the model to quantify the international transmissions of temporal financial shocks in one country. I calibrate the two-country model so that one country is the U.S., which experiences an adverse financial shock, and the other is the aggregate of the other OECD member countries. Following Jermann and Quadrini (2012) and Buera and Moll (2015), I consider a credit crunch as the adverse financial shock; that is, a negative shock to the investors' credit constraints, which decreases their borrowing capacity. The numerical analysis leads to four main findings. First, an adverse financial shock in one country induces a recession not only in that country, but also in its partner country, even when only the goods are traded. Second, the downward shift in the real GDP of the partner country that does not directly experience the financial shock is not so large. However, the shift can be more sustained than in the country that experiences the shock. Third, the real value of exports in the partner drops more seriously than its real GDP. Fourth, this drop is caused mainly by the change at the intensive margin (the real value of already exported goods) rather than at the extensive margin (the number of exported goods here). The first and second findings imply that a recession in a country due to a domestic financial crisis can propagate to other countries and have a sustained impact, even if policy makers restrict international

[5] In this sense, the model in this study shares some qualitative characteristics with those of existing studies incorporating both financial frictions and endogenous growth. Mino (2015, 2016), Kunieda and Shibata (2016, 2017), and Hirano and Yanagawa (2017) employ a simple AK framework. Guerron-Quintana and Jinnai (2019) quantitatively examine how liquidity shocks are responsible for the recent recession using a class of R&D-based endogenous growth models with expanding variety. Kobayashi and Shirai (2018) also use this class to examine a financial shock that redistributes wealth from firms to households suddenly. All of these models assume a closed economy.

[6] Naito (2017) extends the Eaton–Kortum model to a three-country endogenous growth model of AK technology, and investigates the growth effect of a permanent decline in one country's iceberg trade costs. He does not consider any financial frictions.

[7] Ohdoi (2018) introduces financial frictions into a two-country dynamic Ricardian model of Dornbusch–Fischer–Samuelson type, and shows that a credit crunch in one country reduces the level of GDP in both countries. However, the model is highly stylized, and thus not suited to obtain quantitative implications sufficiently.

financial transactions using capital control policies. The third and fourth results are partly consistent with the actual phenomenon of trade collapse. In addition, the fourth result means that even when the extensive margin of exports is endogenous, the change at the intensive margin of exports has the dominant role. In this sense, the results I obtain here complement the existing theoretical studies so far.

The mechanism is simple and explained as follows. Suppose that country 1 experiences a temporal credit crunch and capital accumulation slows down. This leads to a lower level of capital in country 1 relative to that in country 2, and hence the capital price in country 2 declines relative to country 1. Because this outcome makes the prices of tradable goods relatively cheaper in country 2, on the one hand, the number of exported goods decreases in country 1, whereas it increases in country 2. That is, the extensive margin of exports increases in country 2. On the other hand, the decline in the capital price makes the investors' real income in country 2, which in turn also harms capital investment in country 2. Owing to the learning-by-doing mechanism, this negative effect on capital accumulation is amplified and sustained, and generates a permanent downward shift in the GDP level in both countries, even though the credit crunch is a transitory phenomenon. In addition, because both countries experience an economic slowdown, demand for all tradable goods shrinks accordingly. Then, the intensive margin of both exports and imports decreases in both countries. The numerical analysis under some sets of calibrated parameter values here show that in country 2, the decrease in exports at the intensive margin outweighs the increase in that at the extensive margin.

The results have some empirical relevances. After observing the trade collapse during the 2008-2009 crisis, some trade economists first identified the trade credit channel as its possible cause (e.g., Amiti and Weinstein, 2011; Chor and Manova, 2012). Basically, exporters are likely to require more operating funds than firms supplying products only to domestic markets because there is a longer time lag from production to distribution when exporting. Thus, exporters often use letter of credit transactions. Some researchers claim that a decrease in such transactions due to financial shocks induces the collapse of trade. However, the empirical evidence on this hypothesis is not so robust. For example, Bricongne et al. (2012) report that the number of credit constrained firms in France did not increased drastically during the crisis, and they conclude that financial constraints on French exporters played little role in explaining the decline in overall French exports. Indeed, the results I obtain in this study do not rely on the credit constraints of firms in tradable sectors, but to investors who conduct capital investment. In addition, Behrens et al. (2013) claim that one of the most important factors explaining the decline in Belgian exports is the destination country's GDP growth rate. The result on the trade collapse in the present study is also consistent with this finding. In summary, the results in this study suggest that the connection with endogenous productivity growth and financial shocks can be a key to explain the mechanism of global recession and trade collapse theoretically. To the best of my knowledge, only

Feng and Lin (2013) examine the international transmission of financial shocks in a two-country model with endogenous extensive margins of trade. However, their model differs significantly as they build on monopolistic competition with exporters' fixed costs. In addition, they do not embed any mechanisms of endogenous productivity growth.

The rest of this paper proceeds as follows. Section 2 sets up the model. Section 3 characterizes the equilibrium analytically. Section 4 then calibrates the model and provides the quantitative results. Section 5 concludes. The derivations of key equations are given in the Appendix.

2 Model

Time is discrete and indexed by $t = 0, 1, 2, \ldots$ The world consists of two countries, indexed by j or $n \in \{1, 2\}$. The structure of international trade is based on Eaton and Kortum (2002) and Alvarez and Lucas (2007). I extend their framework to a dynamic environment with capital accumulation along the same line as Mutreja et al. (2014, 2018), Eaton et al. (2016), Alvarez (2017), and Ravikumar et al. (2018). Following Mutreja et al. (2014, 2018) and Alvarez (2017), I assume no international financial transactions and a trade balance is obtained.[8] Capital and labor are also immobile between the two countries.

In this model, I classify households into two types of agents: a mass one of heterogeneous investors and a mass L_j of homogeneous workers. Only investors have access to the ownership of domestic capital, whereas workers do not have investment opportunities and cannot borrow/save: they are hand-to-mouth consumers.

2.1 Firms

Intermediate composite firms: The world has a continuum of various intermediate goods. Each variety, indexed by $\omega \in [0, 1]$, is tradable but is subject to iceberg trade costs. A representative assembling firm in each country combines the intermediate goods to produce the intermediate composite according to the following CES function:

$$M_{j,t} = \left(\int_0^1 (y^m_{j,t}(\omega))^{1-1/\sigma} d\omega \right)^{\sigma/(\sigma-1)},$$

where $M_{j,t}$ is the output of the intermediate composite, $y^m_{j,t}(\omega)$ is the demand for variety ω, and σ is the elasticity of substitution. As Eaton and Kortum (2002) show, the value of σ is not important in this model. Let $p^m_{j,t}(\omega)$ denote the domestic price of variety ω and $p^m_{j,t}$ denote the price of the intermediate

[8] This financial autarky assumption is sometimes used, especially in the studies that pursue international trade as a potential source of international co-movement. See, for instance, Heathcote and Perri (2002, 2004), Kose and Yi (2006), and Arkolakis and Ramanarayanan (2009).

composite. The firm chooses the demand for varieties to maximize its profit, taking the prices as given. This yields the following demand function for each variety:

$$y^m_{j,t}(\omega) = M_{j,t} \left(\frac{p^m_{j,t}(\omega)}{p^m_{j,t}} \right)^{-\sigma},$$

and the zero-profit condition:

$$p^m_{j,t} = \left(\int_0^1 (p^m_{j,t}(\omega))^{1-\sigma} d\omega \right)^{1/(1-\sigma)}.$$

Intermediate goods firms: Intermediate goods firms produce each variety using capital, labor, and the intermediate composite. The production function of variety ω in country j is

$$Y^m_{j,t}(\omega) = Z^m_{j,t}(\omega) \left[\frac{1}{\nu^m} \left(\frac{K^m_{j,t}(\omega)}{\alpha} \right)^{\alpha} \left(\frac{L^m_{j,t}(\omega)}{1-\alpha} \right)^{1-\alpha} \right]^{\nu^m} \left(\frac{M^m_{j,t}(\omega)}{1-\nu^m} \right)^{1-\nu^m},$$

where $Y^m_{j,t}(\omega)$ denotes the output, while $K^m_{j,t}(\omega)$, $L^m_{j,t}(\omega)$, and $M^m_{j,t}(\omega)$ are demand for capital, labor, and the intermediate composite, respectively. The parameter $\nu^m \in (0,1)$ denotes the share of value added in total output and $\alpha \in (0,1)$ denotes capital's share of value added.

The term $Z^m_{j,t}(\omega)$ represents the technology level to produce variety ω in country j. I specify $Z^m_{j,t}(\omega)$ as

$$Z^m_{j,t}(\omega) = z_{j,t}(\omega) \left(\frac{K_{j,t}}{L_j} \right)^{(1-\alpha)\nu^m}.$$

In this formulation, $K_{j,t}$ and L_j are the aggregate capital and worker population in country j, respectively. The dependency of the technology level on $K_{j,t}$ captures the learning-by-doing externality in a spirit of Arrow (1962) and Romer (1986). To eliminate the "scale effect" property induced by the externality simply, I assume that the capital per worker affects productivity.

The component $z_{j,t}(\omega)$ denotes the sector-specific productivity in the production of variety ω. The productivity draw comes from an independent Fréchet distribution with shape parameter $\theta > 1$ and country-specific productivity $T_j > 0$:

$$\text{Prob}(z_{j,t}(\omega) \leq z | j,t) = \exp\left(-T_j z^{-\theta} \right).$$

The unit cost to produce variety ω is therefore $b_{j,t}/z_{j,t}(\omega)$, where $b_{j,t}$ is given by

$$b_{j,t} \equiv \left(\frac{L_j}{K_{j,t}} \right)^{(1-\alpha)\nu^m} \left(r^{\alpha}_{j,t} w^{1-\alpha}_{j,t} \right)^{\nu^m} \left(p^m_{j,t} \right)^{1-\nu^m}. \tag{1}$$

In (1), $r_{j,t}$ is the rental price of capital and $w_{j,t}$ is the wage rate.

Delivering one unit of intermediate good from country j to country n requires τ_{nj} units of this good, where $\tau_{jj} = 1$ and $\tau_{nj} > 1$ for $n \neq j$. As in Eaton and Kortum (2002), the first subscript is the destination country, while the second one is the origin country. The demand price of variety ω in country n is therefore

$$p^m_{n,t}(\omega) = \min_{j\in\{1,2\}} \{\tau_{nj} b_{j,t}/z_{j,t}(\omega)\}.$$

Following the same calculation procedure as Eaton and Kortum (2002), the probability that country n buys the product from country j is $\pi_{nj,t} \in (0,1)$ for any variety:

$$\pi_{nj,t} = \frac{T_{j,t}\left(\tau_{nj} b_{j,t}\right)^{-\theta}}{\sum_{j'\in\{1,2\}} T_{j',t}\left(\tau_{nj'} b_{j',t}\right)^{-\theta}}, \tag{2}$$

where $\sum_j \pi_{nj,t} = 1$. Then, $p^m_{j,t}$ given above is rewritten as

$$p^m_{j,t} = \gamma \left[\sum_{n\in\{1,2\}} T_{n,t}\left(\tau_{jn} b_{n,t}\right)^{-\theta}\right]^{-1/\theta}. \tag{3}$$

In (3), $\gamma \equiv \{\Gamma(1-(\sigma-1)/\theta)\}^{1/(1-\sigma)}$ where $\Gamma(\cdot)$ is the gamma function: $\Gamma(h) = \int_0^\infty u^{h-1}\exp(-u)du$. I assume $\theta > \sigma - 1$ such that $\Gamma(\cdot)$ is well defined.

Final good firms: Each country has a single non-traded final good used for domestic consumption and investment. The final good firm produces output $Y^f_{j,t}$ using capital $K^f_{j,t}$, labor $L^f_{j,t}$, and the intermediate composite $M^f_{j,t}$ according to

$$Y^f_{j,t} = Z^f_{j,t}\left[\frac{1}{\nu^f}\left(\frac{K^f_{j,t}}{\alpha}\right)^{\alpha}\left(\frac{L^f_{j,t}}{1-\alpha}\right)^{1-\alpha}\right]^{\nu^f}\left(\frac{M^f_{j,t}}{1-\nu^f}\right)^{1-\nu^f},$$

where $\nu^f \in (0,1)$. Letting $p^f_{j,t}$ denote the price of the final good, the zero-profit condition resulting from profit maximization is

$$p^f_{j,t} = \frac{1}{Z^f_{j,t}}\left(r^{\alpha}_{j,t} w^{1-\alpha}_{j,t}\right)^{\nu^f}\left(p^m_{j,t}\right)^{1-\nu^f}. \tag{4}$$

The technology $Z^f_{j,t}$ is specified such that it also exhibits the learning-by-doing externality.

$$Z^f_{j,t} = \left(\frac{K_{j,t}}{L_j}\right)^{(1-\alpha)\nu^f}.$$

2.2 Households

Investors: Investors are heterogeneous and indexed by $i \in [0,1]$. The following expected expected utility function expresses the preferences of investor i in country j:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \ln c^i_{j,t}\right],$$

where $c^i_{j,t}$ is consumption and $\beta \in (0,1)$ is the discount factor. The budget constraint evaluated in terms of the domestic final good is

$$\frac{r_{j,t}}{p^f_{j,t}} k^i_{j,t} - (1 + r^d_{j,t}) d^i_{j,t-1} + d^i_{j,t} = c^i_{j,t} + \iota^i_{j,t}, \tag{5}$$

where $k^i_{j,t}$ is the investor's capital, $d^i_{j,t}$ is the end-of-period stock of the one-period real bond (i.e., the investor's debt evaluated in terms of the final good), $r^d_{j,t}$ is the real interest rate, and $\iota^i_{j,t}$ is the gross investment.

Investors receive an idiosyncratic capital quality shock. Let $x^i_{j,t} = \iota^i_{j,t} + (1-\delta) k^i_{j,t}$ denote the amount of capital before the shock. Then, the capital in the next period is

$$k^i_{j,t+1} = s^i_{j,t} x^i_{j,t}, \tag{6}$$

where $s^i_{j,t}$ is continuous, included in $[s_{\min}, s_{\max}]$, and i.i.d. across not only investors but also periods. Hereafter, I refer to $s^i_{j,t}$ as investor i's investment productivity. Each investor draws his/her productivity from the time-invariant distribution $G(s) \equiv \text{Prob}\left(s^i_{j,t} \leq s \mid j\right)$. As in Buera and Moll (2015), I assume that investors can make decisions in a period with knowledge of their investment productivity in this period, whereas their productivity in future periods is not observable.

Let $a^i_{j,t}$ denote the investor's net worth at the end of period t: $a^i_{j,t} = x^i_{j,t} - d^i_{j,t}$. In this model, each investor faces the following credit constraint:

$$x^i_{j,t} \leq (1 + \lambda_{j,t}) a^i_{j,t}, \tag{7}$$

where $\lambda_{j,t} > 0$ captures the financial frictions in country j. The term $\lambda_{j,t}$ creates the upper bound of the investors' leverage ratios and all investors are subject to this constraint. If constraint (7) is binding, then $\lambda_{j,t}$ is exactly the investors' leverage ratio. This constraint also means that at most, investors can finance a fraction $\lambda_{j,t}/(1+\lambda_{j,t})$ of investment externally:

$$\frac{d^i_{j,t}}{x^i_{j,t}} \leq \frac{\lambda_{j,t}}{1+\lambda_{j,t}} \in [0,1].$$

Following Jermann and Quadrini (2012), Buera and Moll (2015), and other prior studies, I assume that the credit constraint is subject to a financial shock:

$$\ln(\lambda_{j,t+1}/\lambda_j) = \rho_\lambda (\ln \lambda_{j,t}/\lambda_j) + \varepsilon_{j,t+1},$$

where λ_j is the baseline value of $\lambda_{j,t}$ and $\varepsilon_{j,t}$ is an i.i.d. shock. This shock is common to all investors in a country. The value of $\lambda_{j,t}$ is realized at the beginning of a period. The parameter $\rho_\lambda \in (0,1)$ captures the persistence of the financial shock.

An investor maximizes the expected utility function subject to (5)–(7). This optimization problem is solved as

$$a^i_{j,t} = \beta\left[\left(r_{j,t}/p^f_{j,t} + 1 - \delta\right)k^i_{j,t} - (1 + r^d_{j,t})d^i_{j,t-1}\right],$$

$$\left(x^i_{j,t}, d^i_{j,t}\right) = \begin{cases} \left(0,\ -a^i_{j,t}\right) & \text{if } s_{\min} < s^i_{j,t} < s^c_{j,t}, \\ \left((1+\lambda_{j,t})a^i_{j,t},\ \lambda_{j,t}a^i_{j,t}\right) & \text{if } s^c_{j,t} \leq s^i_{j,t} < s_{\max}, \end{cases}$$

the derivation of which is in the Appendix. In the second equation, $s^c_{j,t}$ is the cutoff productivity of investment, defined as

$$s^c_{j,t} \equiv \frac{1 + r^d_{j,t+1}}{r_{j,t+1}/p^f_{j,t+1} + 1 - \delta}.$$

In the next section, it will be shown that the equilibrium value of $s^c_{j,t}$ is determined within the interval of $[s_{\min}, s_{\max}]$. If the investors' investment productivity is below the cutoff in a period, then they are not active as investors in this period, but lend all financial funds to other active investors. If they are productive enough, then they want to leverage their investment by additional funds from others as much as they can. The resulting leverage ratio of these investors is $\lambda_{j,t}$. Thus, both financial frictions and the idiosyncratic shocks are important to induce the spread of the rate of returns between purchasing capital and lending to other agents.

Let $A_{j,t} = \int_0^1 a^i_{j,t}di$ denote the investors' net worth:

$$A_{j,t} = \beta\Big[(r_{j,t}/p^f_{j,t} + 1 - \delta)K_{j,t} - (1 + r^d_{j,t})D_{j,t-1}\Big]. \tag{8}$$

where $D_{j,t-1}$ is the aggregate net supply of the one-period bonds in period $t-1$. Accordingly, the consumption aggregated over all investors is $C^E_{j,t} \equiv (1-\beta)A_{j,t}/\beta$. Then, $x^i_{j,t}$ is aggregated over all investors in country j:

$$X_{j,t} = (1 + \lambda_{j,t})(1 - G(s^c_{j,t}))A_{j,t}. \tag{9}$$

The aggregate capital in the next period $K_{j,t+1}$ is

$$K_{j,t+1} = \frac{1}{1 - G(s^c_{j,t})}\int_{s^c_{j,t}}^{s_{\max}} sdG(s)X_{j,t}. \tag{10}$$

Workers: Each worker is endowed with one unit of time in each period and inelastically supplies to the domestic labor market, meaning that the population of workers L_j is also the aggregate labor supply.[9] The workers' aggregate consumption is thus $C^W_{j,t} = w_{j,t}L_j/p^f_{j,t}$.

[9] Even if each worker endogenously determines his/her labor supply, one can obtain the same result as long as the balanced growth equilibrium occurs. For example, suppose that each worker maximizes his/her utility function $\ln c^W_{j,t} + \zeta\ln(1 - h_{j,t})$ subject to the budget constraint $p^f_{j,t}c^W_{j,t} = w_{j,t}h_{j,t}$, where $c^W_{j,t}$ and $h_{j,t}$ are the consumption and labor supply per worker, respectively. The aggregate labor supply is thus $L_jh_j = L_j/(1+\zeta)$.

2.3 Market-Clearing Conditions

The market-clearing conditions of capital, labor, and the intermediate composite in country j are, respectively,

$$
\begin{aligned}
K_{j,t} &= \int_{\omega \in \Omega_{j,t}} K^m_{j,t}(\omega) d\omega + K^f_{j,t}, \\
L_j &= \int_{\omega \in \Omega_{j,t}} L^m_{j,t}(\omega) d\omega + L^f_{j,t}, \\
M_{j,t} &= \int_{\omega \in \Omega_{j,t}} M^m_{j,t}(\omega) d\omega + M^f_{j,t},
\end{aligned}
$$

where $\Omega_{j,t} \subset [0,1]$ denote the set of varieties produced in country j. Let $\Omega_{nj,t} \subset \Omega_{j,t}$ denote the set of varieties that country j exports to country n. By the law of large numbers, their measures are $\pi_{nj,t}$. Using this definition, the market-clearing condition of each variety produced in country j is

$$
\forall \omega \in \Omega_{j,t},\ Y^m_{j,t}(\omega) = y^m_{j,t}(\omega) + \begin{cases} \tau_{nj} y^m_{n,t}(\omega) & \text{if } \omega \in \Omega_{nj,t}, \\ 0 & \text{otherwise.} \end{cases}
$$

The final good market clears in each country:

$$
Y^f_{j,t} = C^E_{j,t} + C^W_{j,t} + X_{j,t} - (1-\delta) K_{j,t}.
$$

Since there is no international lending/borrowing, the following trade balance condition applies:

$$
\int_{\omega \in \Omega_{jn,t}} p^m_{j,t}(\omega) y^m_{j,t}(\omega) d\omega = \int_{\omega \in \Omega_{nj,t}} p^m_{n,t}(\omega) y^m_{n,t}(\omega) d\omega,
$$

where the left-hand-side is the imports of country j and the right-hand-side is that of country n. From these market-clearing conditions, the firms' zero-profit conditions, and Walras' law, the following budget constraint automatically holds:

$$
A_{j,t} = (r_{j,t}/p^f_{j,t} + 1 - \delta) K_{j,t} - (1 + r^d_{j,t}) D_{j,t-1} + (w_{j,t}/p^f_{j,t}) L_j - (C^E_{j,t} + C^W_{j,t}).
$$

3 Equilibrium

3.1 Equilibrium conditions

In this section, I provide the equilibrium conditions, with a focus on the derivation of key equations. I provide the derivation of supplemental equations in the Appendix.

The wage income $w_{j,t} L_j$ is

$$
w_{j,t} L_j = \frac{1-\alpha}{\alpha} r_{j,t} K_{j,t},
$$

which I derive in the Appendix. The learning-by-doing externalities in the final and intermediate goods sectors are therefore expressed as

$$\left(\frac{K_{j,t}}{L_j}\right)^{(1-\alpha)\nu} = \left(\frac{\alpha}{1-\alpha}\frac{w_{j,t}}{r_{j,t}}\right)^{(1-\alpha)\nu} \quad (\nu = \nu^m, \nu^f).$$

Substituting this result into equations (1) and (4) yields $b_{j,t}$ and $p^f_{j,t}$, respectively, as

$$b_{j,t} = \psi^m (r_{j,t})^{\nu^m} (p^m_{j,t})^{1-\nu^m},$$

and

$$p^f_{j,t} = \psi^f (r_{j,t})^{\nu^m} (p^m_{j,t})^{1-\nu^m}, \tag{11}$$

where $\psi^m \equiv [(1-\alpha)/\alpha]^{(1-\alpha)\nu^m} > 0$ and $\psi^f \equiv [(1-\alpha)/\alpha]^{(1-\alpha)\nu^f} > 0$. I define

$$B_j \equiv T_j^{1/\theta}/\gamma,$$

which represents the parameter that reflects country j's overall productivity in the intermediate goods sector. Substituting the obtained $b_{j,t}$ into equation (3),

$$p^m_{j,t} = \left\{ \sum_{n\in\{1,2\}} \left[\frac{\tau_{jn}\psi^m (r_{n,t})^{\nu^m} (p^m_{n,t})^{1-\nu^m}}{B_n} \right]^{-\theta} \right\}^{-1/\theta}. \tag{12}$$

Since the denominator of the right-hand side of (2) is $(p^m_{n,t}/\gamma)^{-\theta}$, equation (2) becomes

$$\pi_{nj,t} = \left(\frac{1}{p^m_{n,t}} \frac{\tau_{nj}\psi^m (r_{j,t})^{\nu^m} (p^m_{j,t})^{1-\nu^m}}{B_j} \right)^{-\theta}. \tag{13}$$

In this model, the condition for the trade balance is rewritten as

$$\pi_{12,t} r_{1,t} K_{1,t} = \pi_{21,t} r_{2,t} K_{2,t}, \tag{14}$$

the derivation of which is in the Appendix. In this study, capital in country 1 is chosen as the numeraire:

$$r_{1,t} = 1.$$

Thus, given capital stock $K_{j,t}$, the variables $p^f_{j,t}$, $p^m_{j,t}$, $\pi_{nj,t}$, and $r_{2,t}$ are determined from equations (11)–(14).

Turn to the dynamic behavior of the model. Without international financial transactions, the bonds are in zero net supply in each country. Since this means $D_{j,t} = X_{j,t} - A_{j,t} = 0$ for all t, equation (8) determines the net worth of the investors at the end of period t:

$$A_{j,t} = \beta \left(r_{j,t}/p^f_{j,t} + 1 - \delta \right) K_{j,t}. \tag{15}$$

In addition, equation (9) implies

$$1 - G(s_{j,t}^c) = \frac{1}{1+\lambda_{j,t}}. \tag{16}$$

From this equation, the equilibrium value of the cutoff $s_{j,t}^c$ is uniquely determined within the interval of $[s_{\min}, s_{\max}]$:

$$s_{j,t}^c = s^c(\lambda_{j,t}), \quad ds^c(\lambda_j)/d\lambda_j = \frac{1-G}{(1+\lambda)G'} > 0.$$

Then, using equations (10), (16), and $X_{j,t} = A_{j,t}$, the capital in the next period $K_{j,t+1}$ is

$$K_{j,t+1} = f(\lambda_{j,t})A_{j,t}, \tag{17}$$

where function $f(\lambda_{j,t})$ is given by

$$f(\lambda_{j,t}) \equiv (1+\lambda_{j,t})\int_{s^c(\lambda_{j,t})}^{s_{\max}} sdG(s), \quad df(\lambda_j)/d\lambda_j = \frac{f(\lambda_j) - s^c(\lambda_j)}{1+\lambda_j} > 0,$$

where the sign of $df(\lambda_j)/d\lambda_j$ comes from the fact that $f(\lambda_j)$ corresponds to the tail-conditional average of s.[10]

Given $K_{j,t}$ and $\lambda_{j,t}$, the variables $p_{j,t}^f$, $p_{j,t}^m$, $\pi_{nj,t}$, $r_{2,t}$, $s_{j,t}^c$, $A_{j,t}$, and $K_{j,t+1}$ are determined from the system of equations (11)–(17). Equations (11)–(14) constitute the bloc of the Eaton–Kortum model, which is affected by capital stock in each country. Given $K_{j,t}$, these equations jointly determine the patterns of trade, prices of tradable intermediate goods, price of the domestic final good, and the rental price of domestic capital. Equations (15)–(17) constitute the bloc of capital accumulation, which is affected by both international trade and financial frictions. The prices determined in the Eaton–Kortum bloc affect the investors' net worth $A_{j,t}$ through equation (15). After the aggregate financial shock $\lambda_{j,t}$ and the idiosyncratic shock $s_{j,t}^i$ are realized, the financial market equilibrium in each country (16) determines the cutoff of investors' productivity to actively invest, $s_{j,t}^c$. The financial shock, through its impact on financial markets to sort productive investors, is a key determinant of the average productivity of investment, captured by $f(\lambda_{j,t})$. Then, as equation (17) shows, the capital stock in period $t+1$ depends significantly on international trade (via $A_{j,t}$) as well as the financial shock (via $f(\lambda_{j,t})$). In the next period, capital stock $K_{j,t+1}$ in turn influences the trade equilibrium in the Eaton–Kortum bloc through equation (14). Because of the learning-by-doing externalities, this interplay also governs the endogenous growth rate of productivity in each country.

3.2 Equilibrium prices

Because of the learning-by-doing externalities, the wage rate disappears from the equilibrium conditions (11)–(17), and one can treat the model as a simple one-factor model. Therefore, the theorem on the

[10]Since $(1+\lambda_j) = 1/(1-G(s_j^c))$, $f(\lambda_j)$ is expressed as $f(\lambda_j) = \int_{s^c(\lambda_{j,t})}^{s_{\max}} sdG(s)/(1-G(s^c(\lambda_j)))$.

existence of equilibrium established by Alvarez and Lucas (2007) is applied to the static equilibrium in this model. However, the following two lemmas are useful to characterize the balanced growth equilibrium in this model.

Equation (12) implies

$$\left(p^m_{1,t}\right)^{-\theta} = (\psi^m)^{-\theta}\left\{B_1^\theta (p^m_{1,t})^{-\theta(1-\nu^m)} + B_2^\theta \tau_{12}^{-\theta}\left[(r_{2,t})^{\nu^m}(p^m_{2,t})^{1-\nu^m}\right]^{-\theta}\right\}, \tag{18}$$

$$\left(p^m_{2,t}\right)^{-\theta} = (\psi^m)^{-\theta}\left\{B_1^\theta \tau_{21}^{-\theta}(p^m_{1,t})^{-\theta(1-\nu^m)} + B_2^\theta \left[(r_{2,t})^{\nu^m}(p^m_{2,t})^{1-\nu^m}\right]^{-\theta}\right\}. \tag{19}$$

I introduce a new variable: $\widetilde{P}^m_t \equiv \left(p^m_{2,t}/p^m_{1,t}\right)^\theta$. The above two equations imply

$$\begin{aligned}\widetilde{P}^m_t &= \mathcal{H}(\widetilde{P}^m_t; r_{2,t})\\ &\equiv \frac{(\widetilde{P}^m_t)^{1-\nu^m} + \tau_{12}^{-\theta}(B_2/B_1)^\theta (r_{2,t})^{-\theta\nu^m}}{\tau_{21}^{-\theta}(\widetilde{P}^m_t)^{1-\nu^m} + (B_2/B_1)^\theta (r_{2,t})^{-\theta\nu^m}}.\end{aligned}$$

Lemma 1.

(i) Given $r_{2,t} > 0$, there uniquely exists the solution to equation $\widetilde{P}^m = \mathcal{H}(\widetilde{P}^m; r_{2,t})$.

(ii) Let $\widetilde{P}^m(r_{2,t})$ denote the solution. $\widetilde{P}^m(r_{2,t})$ is an increasing function of $r_{2,t}$ and

$$\lim_{r_{2,t}\to 0}\widetilde{P}^m(r_{2,t}) = \tau_{12}^{-\theta},$$

$$\lim_{r_{2,t}\to \infty}\widetilde{P}^m(r_{2,t}) = \tau_{21}^{\theta}.$$

Proof. See the Appendix. □

Then, equations (18) and (19) uniquely provide the price of the intermediate composite as a function of $r_{2,t}$: $p^m_{j,t} = P^m_j(r_{2,t})$.

Lemma 2. *$P^m_1(r_{2,t})$ and $r_{2,t}/P^m_2(r_{2,t})$ are strictly increasing functions of $r_{2,t}$ and*

$$\lim_{r_{2,t}\to 0}\frac{1}{P^m_1(r_{2,t})} = \infty, \quad \lim_{r_{2,t}\to \infty}\frac{1}{P^m_1(r_{2,t})} = (B_1/\psi^m)^{1/\nu^m},$$

$$\lim_{r_{2,t}\to 0}\frac{r_{2,t}}{P^m_2(r_{2,t})} = (B_2/\psi^m)^{1/\nu^m}, \quad \lim_{r_{2,t}\to \infty}\frac{r_{2,t}}{P^m_2(r_{2,t})} = \infty.$$

Proof. From (18) and (19), $P^m_1(r_{2,t})$ and $r_{2,t}/P^m_2(r_{2,t})$ are given by

$$P^m_1(r_{2,t}) = (\psi^m)^{\nu^m}\left\{B_1^\theta + B_2^\theta\tau_{12}^{-\theta}(r_{2,t})^{-\theta\nu^m}\left(\widetilde{P}^m(r_{2,t})\right)^{-(1-\nu^m)}\right\}^{-1/(\theta\nu^m)},$$

$$r_{2,t}/P^m_2(r_{2,t}) = (\psi^m)^{-1/\nu^m}\left\{B_1^\theta\tau_{21}^{-\theta}(r_{2,t})^{\theta\nu^m}\left(\widetilde{P}^m(r_{2,t})\right)^{1-\nu^m} + B_2^\theta\right\}^{1/(\theta\nu^m)}.$$

From these equations and Lemma 1, it is apparent that this lemma is true. □

Substituting $p^m_{j,t} = P^m_j(r_{2,t})$ into (11) yields the real rate of return from capital investment as follows:

$$\frac{1}{p^f_{1,t}} = \frac{1}{\psi^f}\frac{1}{(P^m_1(r_{2,t}))^{1-\nu^f}}, \quad \frac{r_{2,t}}{p^f_{2,t}} = \frac{1}{\psi^f}\left(\frac{r_{2,t}}{P^m_2(r_{2,t})}\right)^{1-\nu^f}. \tag{20}$$

3.3 Balanced growth equilibrium

In this section, I consider the case where $\lambda_{j,t} = \lambda_j$ by assuming $\varepsilon_{j,t} = 0$. The balanced growth equilibrium is the equilibrium at which $K_{1,t}$ and $K_{2,t}$ grow at the same, constant rate. From equations (15) and (17),

$$1 + g^* = \beta f(\lambda_1)(1/p^f_{1,t} + 1 - \delta) = \beta f(\lambda_2)(r_{2,t}/p^f_{2,t} + 1 - \delta), \tag{21}$$

where g^* is the growth rate in the balanced growth equilibrium, hereafter referred to simply as the balanced growth rate. An asterisk over a variable indicates the balanced growth equilibrium. Since $p^f_{1,t}$ and $r_{2,t}/p^f_{2,t}$ are functions of $r_{2,t}$, equation (21) shows that $r_{2,t}$ must be constant. Therefore, I omit the time subscript. Substituting (20) into (21) gives the following equation to determine r_2:

$$\mathcal{R}_1(r_2) = \mathcal{R}_2(r_2),$$

where

$$\mathcal{R}_j(r_2) \equiv f(\lambda_j)\left[\frac{1}{\psi^f}\left(\frac{r_j}{P^m_j(r_2)}\right)^{1-\nu^f} + 1 - \delta\right].$$

From Lemma 2, $\mathcal{R}_1(r_2)$ is a strictly decreasing function, while $\mathcal{R}_2(r_2)$ is a strictly increasing function with respect to r_2. In addition,

$$\lim_{r_2\to 0}[\mathcal{R}_1(r_2) - \mathcal{R}_2(r_2)] > 0, \quad \lim_{r_2\to\infty}[\mathcal{R}_1(r_1) - \mathcal{R}_2(r_2)] < 0,$$

which means that there uniquely exists $r_2^* > 0$ that solves $\mathcal{R}_1(r_2) = \mathcal{R}_2(r_2)$. Once r_2^* is determined, the price of the intermediate composite is given by $p^{m*}_j \equiv P^m_j(r_2^*)$. Substituting r_2^* and p^{m*}_j into (11) and (13) yields $p^{f^*}_j$ and π^*_{nj}, respectively. Since the prices become constant, all quantity variables grow at the same rate of g^*. Let $k_{2,t} \equiv K_{2,t}/K_{1,t}$ denote the level of capital in country 2 relative to that in country 1. In the balanced growth equilibrium, it is constant and given by $k^*_2 \equiv \pi^*_{12}/(r^*_2\pi^*_{21})$ from (14).

Proposition 1. *The model has the balanced growth equilibrium in which the two countries grow at the balanced growth rate of g^*.*

In the rest of this section, I examine the comparative statics of the balanced growth equilibrium to gain insight into the inner workings of the model. Figure 2 depicts how financial frictions in each country influence the determination of g^* and r^*_2. A lower λ_j moves the location of the $\beta\mathcal{R}_j(r_2)$ curve downward. Without loss of generality, suppose that λ_1 becomes small. This decreases the borrowing capacity of the investors in this country, which in turn induces the entry of less productive investors who otherwise become inactive. This inefficient reallocation of financial resources lowers the average productivity of the aggregate investment in country 1. Then, capital accumulation slows down in country 1, making the level

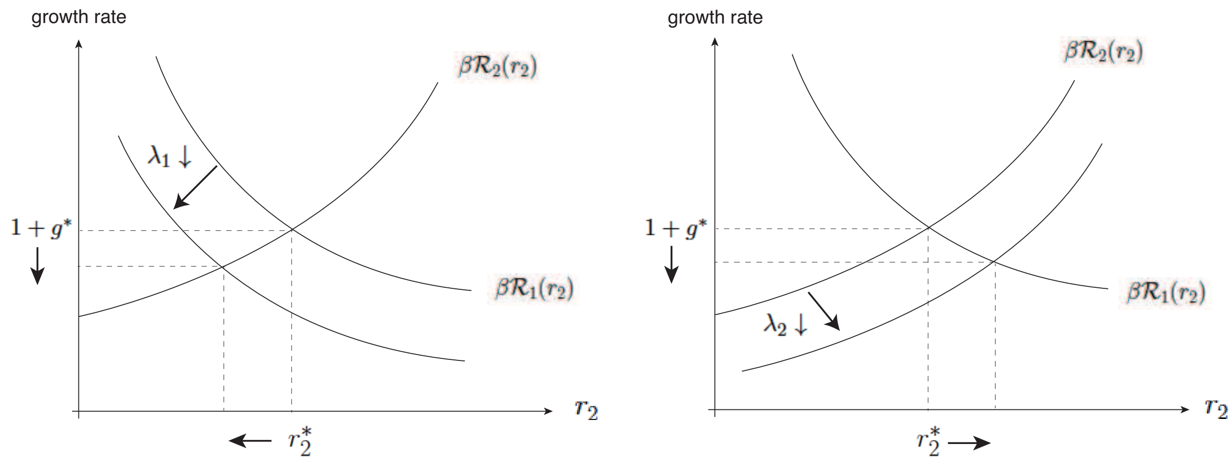

Figure 2: Comparative statics of the balanced growth equilibrium

of capital in country 1 small relative to that in country 2. In response to this, r_2^* becomes low. This also slows down the wealth accumulation in country 2. Through the learning-by-doing mechanism, this negative effect on capital accumulation is amplified and sustained, leading both countries to settle at a lower balanced growth rate.

The relative price of capital in turn affects the patterns of international trade. From equation (13),

$$
\begin{aligned}
\pi_{12}^* &= \left(\frac{\tau_{12}\psi^m}{B_2}\right)^{-\theta}\left(\frac{r_2^*}{P_2^m(r_2^*)}\right)^{-\theta\nu^m}\frac{1}{\widetilde{P}^m(r_2^*)},\\
\pi_{21}^* &= \left(\frac{\tau_{21}\psi^m}{B_1}\right)^{-\theta}(P_1^m(r_2^*))^{\theta\nu^m}\widetilde{P}^m(r_2^*).
\end{aligned}
$$

From Figure 2, r_2^* is low (high) when λ_1 (λ_2) is low. From this result and Lemmas 1 and 2, π_{12}^* is accordingly high (low), whereas π_{21}^* is low (high). For country j, $\pi_{jn,t}$ and $\pi_{nj,t}$ are numbers of importing and exporting varieties, respectively. Suppose that λ_1 is low, and hence r_2^* is low. This provides downward pressure to the price of the intermediate goods produced in country 2. Thus, country 1's importing varieties increase while its exporting varieties decrease.

4 Numerical analysis of financial shocks

4.1 Calibration

A period in the model corresponds to a quarter of a year. I identify country 1 as the U.S. and country 2 as an aggregate of the other 35 OECD member countries as a proxy for the rest of the world. I assume that the world economy is on a balanced growth path before 2008:Q3.

The model has 15 parameters and Table 1 reports calibration results. I set the discount factor, capital share, and depreciation rate to $\beta = 0.98$, $\alpha = 0.35$, and $\delta = 0.025$, respectively, which are standard in the

Table 1: Parameters

Parameter	Value	Source/Targets
β	0.98	Exogenously chosen
α	0.35	Exogenously chosen
δ	0.025	Exogenously chosen
σ	2	Exogenously chosen
ν^m	0.5	Alvarez and Lucas (2007)
ν^f	0.75	Alvarez and Lucas (2007)
θ	(i)7.9, (ii)6.7, (iii)4, (iv)2	Four existing studies
λ_1	2.226	$D^{E*}_{1,t-1}/K^*_{1,t} = 0.69$
λ_2	2.226	$r^{d*}_1 = 1.04^{1/4} - 1$
$s_{\min}$	0.913	$r^{d*}_1 = r^{d*}_2$
$s_{\max}$	1.016	$f(\lambda_1) = 1$
τ_{12}	(i)1.196, (ii)1.236, (iii)1.422, (iv)2.024	$g^* = 5.8 \times 10^{-3}$
τ_{21}	(i)1.196, (ii)1.236, (iii)1.422, (iv)2.024	$\tau_{12} = \tau_{21}$
B_1	(i)5.66×10^{-3} (ii)5.62×10^{-3}, (iii)5.45×10^{-3}, (iv)5.03×10^{-3}	$(1-\nu^f)\pi^*_{12}/\nu^m = 0.135$
B_2	(i)5.78×10^{-3} (ii)5.76×10^{-3}, (iii)5.68×10^{-3}, (iv)5.47×10^{-3}	$r^*_2 k^*_2 = 1.947$

literature. In the Eaton–Kortum model, the elasticity of substitution across intermediate varieties σ is not quantitatively important. I therefore set this at $\sigma = 2$ to satisfy the technical restriction $1+(1-\sigma)/\theta > 0$. As for the shares of value added (ν^f, ν^m), I follow Alvarez and Lucas (2007) to set these at $\nu^f = 0.75$ and $\nu^m = 0.5$. The dispersion in the productivity levels in the intermediate goods sector θ varies in the existing studies. I therefore consider four cases: (i) $\theta = 7.9$ (the estimate for OECD countries by Waugh 2010); (ii) $\theta = 1/0.15 \simeq 6.66\ldots$ (Alvarez and Lucas, 2007; Alvarez, 2017); (iii) $\theta = 4$ (Mutreja et al., 2014, 2018; Ravikumar et al., 2018); and (iv) $\theta = 2$ (Eaton et al., 2016).[11]

There are 8 parameters to be calibrated: λ_1, λ_2, $s_{\min}$, $s_{\max}$, τ_{12}, τ_{21}, B_1, and B_2. In each case of θ, these parameters are chosen such that the endogenous variables achieve their target values in the balanced growth equilibrium. The distribution of $s^i_{j,t}$ is specified as the uniform distribution: $G(s) = (s - s_{\min})/(s_{\max} - s_{\min})$. The details of the calibration is given in the Appendix. The average growth rate of total GDP for the OECD countries from 2001:Q1 to 2008:Q2 is 0.58%, or about 2.34% per year.[12] I therefore set the growth rate at $g^* = 0.0058$. I assume that the countries have equal real interest rates

[11] Note I use θ as in Eaton and Kortum (2002), Eaton et al. (2016), and Mutreja et al. (2018), which correspond to $1/\theta$ in Waugh (2010), Alvarez and Lucas (2007), Mutreja et al. (2014), and Alvarez (2017).

[12] The data are from the OECD Quarterly National Accounts. The GDP is in international PPP dollars with 2015 as the reference year in all countries. During this period, the average growth rate of GDP was 0.61% ($\simeq$ 2.44% annually) in the U.S. and 0.57% ($\simeq$ 2.29% annually) in the other OECD countries.

and set the common rate at $r_1^{d*} = r_2^{d*} = (1.04)^{1/4} - 1$. As I show in the Appendix, $r_1^{d*} = r_2^{d*}$ implies $\lambda_1 = \lambda_2$. Let $D^E_{j,t-1} = \lambda_{j,t}(1 - G(s^c(\lambda_{j,t})))A_{j,t-1}$ denote the debt of the active investors in the balanced growth equilibrium. I set the long-run ratio of debt to capital to

$$\frac{D^E_{j,t-1}}{K_{j,t}} = \frac{\lambda_1(1 - G(s^c(\lambda_1)))}{f(\lambda_1)} = 0.69,$$

following Buera and Nicolini (2017), who report that the average ratio of liabilities to non-financial assets for the U.S. non-financial business sector between 1997:Q3 and 2007:Q3 is 0.69. I assume $f(\lambda_1) = 1$; that is, in the balanced growth equilibrium, the final good is transformed into capital on a one-to-one basis on average, as in standard macroeconomic models.

In this model, I define the real GDP in country j as

$$\begin{aligned} GDP_{j,t} &\equiv \frac{r_{j,t}K_{j,t} + w_{j,t}L_j}{p^f_{j,t}} \\ &= \frac{r_{j,t}K_{j,t}}{\alpha p^f_{j,t}}, \end{aligned}$$

and the real value of this country's imports is

$$\begin{aligned} Imports_{j,t} &\equiv \frac{\pi_{jn,t}p^m_{j,t}M_{j,t}}{p^f_{j,t}} \\ &= \frac{1 - \nu^f}{\nu^m}\pi_{jn,t}GDP_{j,t}. \end{aligned}$$

I set the share of imports to GDP in country 1 to $(1 - \nu^f)\pi^*_{12}/\nu^m = 0.135$, which is the average ratio of imports to GDP for the U.S. obtained from the quarterly data of 2001:Q1–2008:Q2. The GDP ratio of country 2 to country 1 is

$$\frac{p^f_{2,t}GDP_{2,t}}{p^f_{1,t}GDP_{1,t}} \equiv r_{2,t}k_{2,t}.$$

I then set this value to $r^*_2 k^*_2 = 1.947$, which is the average GDP ratio from 2001:Q1 to 2008:Q2.

Finally, I assume $\tau_{12} = \tau_{21} = \tau$ when I calibrate the trade costs τ_{jn} and B_j. Table 1 shows that the value of τ gets larger as the value of θ becomes smaller. This is because a smaller θ induces the trade of more varieties by increasing the variability in the production technologies. The calibrated values of trade costs increase accordingly, such that the economy in each case achieves the same balanced growth equilibrium.

4.2 Impulse responses

In period 0, the economy is on the balanced growth path. At the beginning of period 1, $\lambda_{1,t}$ drops unexpectedly by 20% relative to its baseline value. Thereafter, the economy experiences no exogenous shocks. The leverage ratio deterministically recovers to its baseline according to $\ln(\lambda_{1,t}/\lambda_1) = \rho_\lambda \ln(\lambda_{1,t-1}/\lambda_1)$.

Table 2: Convergence coefficient ($\rho_\lambda = 0.9$ for all cases)

θ	(i)7.9	(ii)6.7	(iii)4	(iv)2
ρ_k^*	0.9993	0.9991	0.9987	0.9978

I set the parameter for the persistence of shocks to $\rho_\lambda = 0.9$, which is a standard choice in the literature. The model enables us to numerically examine the outcome when both countries experience financial shocks. However, in this study, I consider only the case in which country 1 experiences the shocks; that is, $\lambda_{2,t} = \lambda_2$ for all t, because I focus on the quantification of the international transmission of country-specific shocks in one country to another via trade.

From the definition of capital in country 2 relative to that in country 1, $k_{2,t} \equiv K_{2,t}/K_{1,t}$, equations (14) and (17) are rewritten as

$$\pi_{12,t} = \pi_{21,t} r_{2,t} k_{2,t}, \tag{22}$$

$$k_{2,t+1} = \frac{f(\lambda_2)(r_{2,t}/p^f_{2,t} + 1 - \delta)}{f(\lambda_{1,t})(1/p^f_{1,t} + 1 - \delta)}. \tag{23}$$

In the balanced growth equilibrium with $\lambda_{j,t} = \lambda_j$, the variables $p^m_{j,t}$, $p^f_{j,t}$, $\pi_{nj,t}$, $r_{2,t}$, and $k_{2,t}$ are stationary at $(p_j^{m*}, p_j^{f*}, \pi^*_{nj}, r^*_2, k^*_2)$ based on the system of equations (11), (12), (13), (22), and (23). I log-linearize the system around them and compute the impulse response functions for these variables. The log-linear approximation of the system is given in the Appendix.

I then numerically derive the time series of all variables. In doing so, I calculate the percentage deviations from their baseline values with no shock to the economy. For example, let

$$GDP^*_{j,t} \equiv \frac{r^*_j K^*_{j,t}}{\alpha p^{f*}_j} = \frac{r^*_j K^*_{j,0}(1+g^*)^t}{\alpha p^{f*}_j},$$

denote the real GDP level under the alternative scenario without a shock. I calculate $100 \times \ln(GDP_{j,t}/GDP^*_{j,t})$ with the initial condition $K_{j,0} = K^*_{j,0}$.[13]

Since only equation (23) is the difference equation in the system (11), (12), (13), (22), and (23), from their log-linearization, the following reduced dynamics of $k_{2,t}$ is obtained:

$$\ln(k_{2,t+1}/k^*_2) = \rho^*_k \ln(k_{2,t}/k^*_2) - \xi \ln(\lambda_{1,t}/\lambda_1), \quad \xi \equiv \frac{\lambda_1 f'(\lambda_1)}{f(\lambda_1)} > 0,$$

the derivation of which is given in the Appendix. Table 2 reports the convergence coefficient ρ^*_k in the four cases. In all cases, the coefficient is very close to one. A transitory shock can have a prolonged impact on the world economy.

[13] In the numerical analysis, I set these variables to $K^*_{1,0} = 1$ and $K^*_{2,0} = k^*_2$.

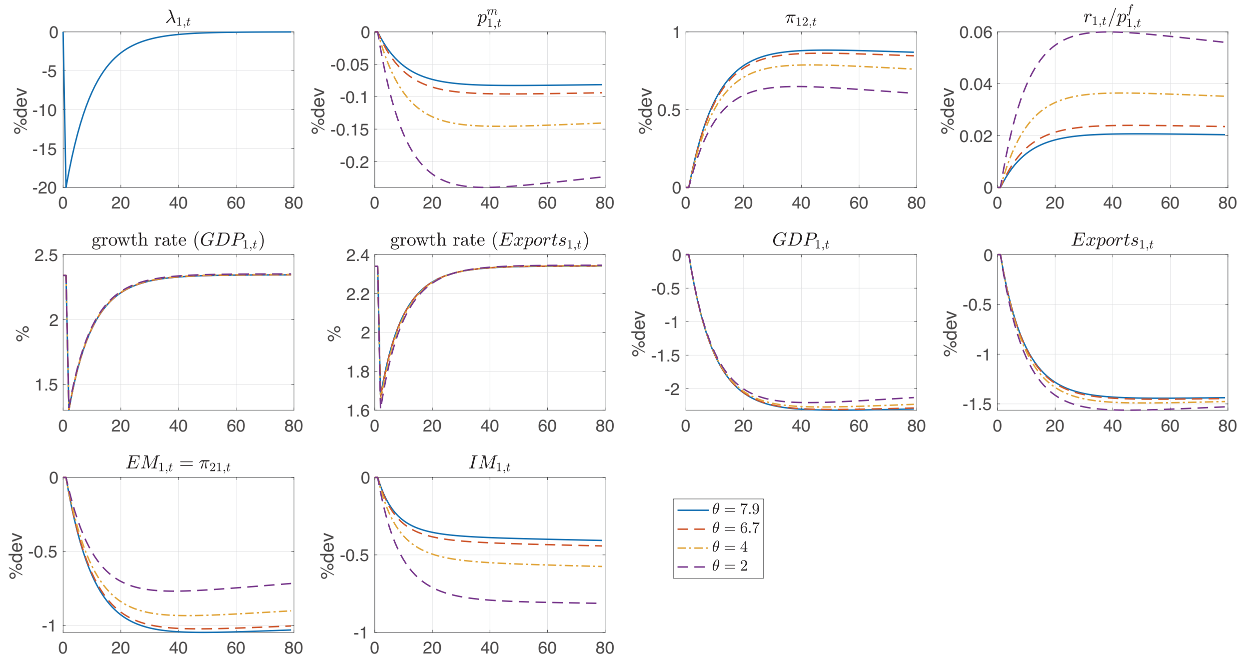

Figure 3: Impulse responses in country 1 to a domestic financial shock

Note: Except for the growth rates, the variables are expressed in their percentage deviation from the balanced growth equilibrium without shocks.

Figures 3 and 4 report the impulse response functions of the major variables in countries 1 and 2, respectively. The horizontal axis is the number of quarters. Except for the growth rates, I plot the percentage deviations in the levels of the variables from the trends when the economy experiences no shocks. The first panel shows the movement of the leverage ratio in each country. The second and third panels in each figure report how such a country-specific shock affects the outcome of international trade between the two countries. As the second panel shows, the price of intermediate composite declines in both countries. As the third panel shows, the number of imported varieties increases in country 1, while decreases in country 2. This comes from the mechanism similar to the comparative statics in Section 3.3. A financial crisis in country 1 decelerates domestic capital accumulation, which leads to a lower level of capital in country 1 relative to that in country 2. The rental price of capital in country 2, $r_{2,t}$, then receives downward pressure in response. Such a change in the capital price uniformly decreases the prices to produce country 2's intermediate goods, which has the following two effects. First, this lowers the price of the intermediate composite goods lower in both countries, as the second panel shows. Second, the third panel shows, this changes the patterns of trade such that the imports of intermediate goods (i.e., the extensive margin) increases in country 1 and decreases in country 2, which is the result of international competition.

The change in the price of the intermediate composite goods affects the price of the final good used

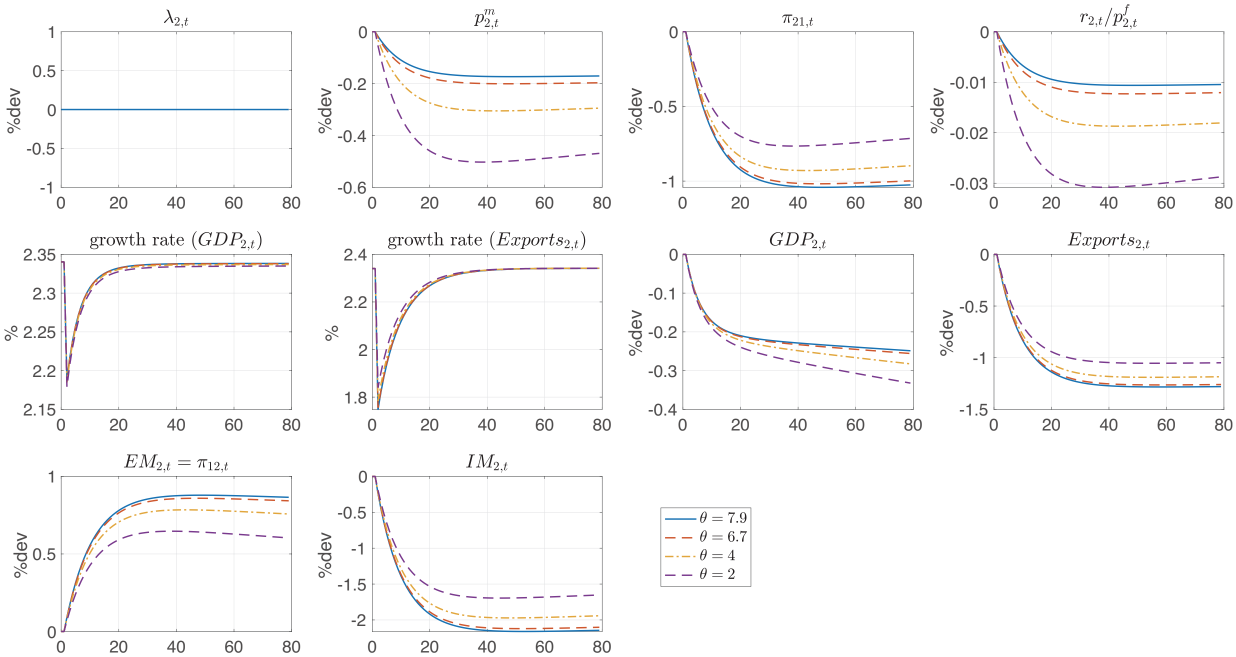

Figure 4: Impulse responses in country 2 to a foreign financial shock

Note: Except for the growth rates, the variables are expressed in their percentage deviation from the balanced growth equilibrium without shocks.

for investment, which in turn affects the real rate of return from holding capital stock. The fourth panel in each figure shows this response. In both countries, the price of the investment good becomes low. In country 1, the real rate of return on investment accordingly increases. However, this rise is not so large relative to the direct effect of the negative financial shock. Consequently, capital accumulation continues to slow down in this country. In country 2, the decline in the rental price of capital offsets the decline in the final good price. Then, the rate of return declines and capital accumulation also slows down in this country, even though it does not experience the financial shock directly.

The panels in the second row in each figure report the size and length of the impact of this financial shock on real GDP and real exports.[14] The fifth and sixth panels show their growth rates on an annual basis. After the shock, the growth rates recover to the common balanced growth rate. The seventh and eighth panels report the responses of the levels of real GDP and exports. Because of the endogenous growth mechanism, even a transitory shock has a sustained effect on these levels. As the seventh panel in these figures shows, compared to country 1, which experiences the trigger event, the GDP in country 2 drops less seriously. However, the result also suggests that the drop in GDP does not hit the bottom, even after 80 quarters (that is, 20 years) passes. Thus, the drop in the country 2's GDP is very prolonged.

[14]In this model, the real value of exports is always equal to that of imports ($Exports_{j,t} = Imports_{j,t}$).

In addition, the eighth panel in Figure 4 shows that the degree of decline in exports is larger than that of GDP. This is partly consistent with the Great Trade Collapse observed during the 2008-09 financial crisis.

Finally, the ninth and tenth panels in the third row report the decomposition of the change in real exports into the change in the extensive and intensive margins. In this model, the extensive margin of exports in country j, denoted by $EM_{j,t}$, is equal to the number of varieties that country n buys from country j, $\pi_{nj,t}$. Then, the intensive margin of exports is

$$IM_{j,t} = \frac{Exports_{j,t}}{EM_{j,t}}.$$

The ninth panel in Figure 4 suggests that the extensive margin of exports increases in country 2. This makes sense given the aforementioned reason that the financial shock in country 1 increases the amount of country 2's exported varieties. The tenth panel in this figure shows that the intensive margin of exports decreases sharply in country 2, such that it offsets the increase in the extensive margin to decrease its exports. This occurs because the intensive margin of exports in country 2 depends crucially on the GDP level in country 1.

The numerical results here are summarized as follows. First, the credit crunch to the capital investors in country 1 induces recessions in both countries, even in the absence of cross-border financial transactions. Since country 2 does not experience the financial shock directly, its real GDP does not drop so much. Second, however, the shift may be more sustained than in country 1. Third, the real value of exports in country 2 declines more seriously than its real GDP. Fourth, this drop is caused mainly by the change at the intensive margin rather than at the extensive margin. The connection between financial shocks and endogenous productivity growth has an important role in internationally synchronized recessions and the collapse of trade.

5 Concluding remarks

The existing studies that examine the international transmission of financial shocks paid little attention to international trade. In particular, they assume a single good economy or assume a fixed extensive margin of trade. This study constructs a simple two-country model featuring financial frictions, endogenous growth, and endogenous trade status. I explore how financial shocks in one country affect the potential for a recession in another country via both the extensive and intensive margins of trade. The connection of financial shocks and endogenous productivity growth is the key to generate a prolonged global recession and determine the contribution of the intensive margin to the trade collapse. These results complement findings in prior studies.

The tractability of the model here enables further research in some different directions. Among others, it would be interesting to relax the assumption that the iceberg trade costs are exogenous. As already stated, after the trade collapse during the 2008-2009 crisis, some trade economists identified trade credit or trade financing as its possible cause. Although its empirical validity is mixed according to previous studies, it might be important to examine its role as the transmission channel in both the qualitative and quantitative sense. Therefore, relating the trade costs to the degree of financial frictions would be a promising extension.

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Appendix to "Trade, Growth, and the International Transmission of Financial Shocks"

Ryoji Ohdoi

Tokyo Institute of Technology

An investor's optimization and aggregation

Let $m^i_{j,t} \equiv \left(r_{j,t}/p^f_{j,t} + 1 - \delta\right) k^i_{j,t} - (1 + r_{j,t})d^i_{j,t-1}$ denote investor i's net income in real terms. As noted in the main body, each investor can decide $a^i_{j,t}$, $x^i_{j,t}$, and $d^i_{j,t}$ after observing his/her investment efficiency $s^i_{j,t}$, which implies that both $m^i_{j,t}$ and $s^i_{j,t}$ are state variables in period t. Then, the optimization problem is formulated as the following Bellman equation:

$$V_{j,t}(m, s) = \max_a \left\{\ln(m - a) + \beta E_t\Big[V_{j,t+1}\big(m_{j,t+1}(a, s), s'\big)\Big]\right\},$$

where $m_{j,t+1}(a, s)$ is the net income in period $t + 1$, which is given by the following static optimization problem:

$$m_{j,t+1}(a, s) = \max_x \left\{ \left(r_{j,t+1}/p^f_{j,t+1} + 1 - \delta\right) sx - (1 + r_{j,t+1})(x - a) \;\middle|\; 0 \le x \le (1 + \lambda_{j,t})a\right\}.$$

By solving the static problem, the optimal investment and borrowing are

$$(x_{j,t}(a, s), d_{j,t}(a, s)) = \begin{cases} (0,\ -a) & \text{if } \ s_{\min} \le s < s^c_{j,t}, \\ ((1 + \lambda_{j,t})a,\ \lambda_{j,t}a) & \text{if } \ s^c_{j,t} \le s \le s_{\max}, \end{cases} \tag{24}$$

where $s^c_{j,t}$ is defined in the main body of the manuscript. Then, $m_{j,t+1}(a, s)$ is given by $m_{j,t+1}(a, s) = R_{j,t+1}(s)a$, where

$$R_{j,t+1}(s) \equiv \begin{cases} 1 + r^d_{j,t+1} & \text{if } \ s_{\min} \le s < s^c_{j,t}, \\ (1 + \lambda_{j,t}) \left(r_{j,t+1}/p^f_{j,t+1} + 1 - \delta\right) s - \lambda_{j,t}(1 + r^d_{j,t+1}) & \text{if } \ s^c_{j,t} \le s \le s_{\max}. \end{cases}$$

Using this result, the first-order condition of the first problem is

$$\frac{1}{m - a} = \beta E_t \left[R_{j,t+1}(s)\frac{\partial V_{j,t+1}(m', s')}{\partial m'}\right].$$

Suppose that the value function takes the form $V_{j,t}(m, s) = v_{j,t}(s) + \kappa \ln m$, where $v_{j,t}(s)$ is a time-varying parameter and κ is a time-invariant parameter. Since $m' = R_{j,t+1}(a, s)$, the above condition implies $a = \beta\kappa m/(1 + \beta\kappa)$ and the Bellman equation therefore becomes

$$v_{j,t} + \kappa \ln m = \log\left(\frac{1}{1 + \beta\kappa}m\right) + \beta E_t \left[v_{j,t+1}(s) + \kappa \ln R_{j,t+1}(s) + \kappa \ln\left(\frac{\beta\kappa}{1 + \beta\kappa}m\right)\right],$$

which gives $\kappa = 1/(1-\beta)$. Then, one can obtain

$$a^i_{j,t} = \beta m^i_{j,t} = \beta \left[\left(r_{j,t}/p^f_{j,t} + 1 - \delta \right) k^i_{j,t} - (1 + r^d_{j,t}) d^i_{j,t-1} \right].$$

Since net income $m^i_{j,t}$ is determined when the value of $s^i_{j,t}$ is realized, $a^i_{j,t} (= \beta m^i_{j,t})$ is independent of $s^i_{j,t}$. Therefore, the aggregate wealth $A_{j,t}$ is expressed as $\int a dF_{j,t}(a)$, where $F_{j,t}$ is the resulting distribution of $a^i_{j,t}$. Furthermore, since s is i.i.d. across agents, one needs no information on $F_{j,t}$ to obtain the aggregate values. Equation (24) provides the aggregate investment, $X_{j,t} = \int \int_{s_{\min}}^{s_{\max}} x_{j,t}(a,s) dG(s) dF_{j,t}(a) = (1+\lambda_{j,t}) A_{j,t} (1 - G(s^c_{j,t}))$, which is (9) in the main body of the manuscript. In the same way, the resulting amount of aggregate capital $K_{j,t+1}$ is $K_{j,t+1} \equiv \int \int_{s_{\min}}^{s_{\max}} s x_{j,t}(a,s) dG(s) dF_{j,t}(a) = (1+\lambda_{j,t}) A_{j,t} \int_{s^c_{j,t}}^{s_{\max}} s dG(s)$, which is equivalent to (10).

Supplement to Section 3.1

Applying Shepherd's lemma to the unit cost functions of the final and intermediate goods sectors,

$$\begin{aligned}
r_{j,t} K^m_{j,t}(\omega) &= \alpha \nu^m p^m_{j,t}(\omega) Y^m_{j,t}(\omega), \\
w_{j,t} L^m_{j,t}(\omega) &= (1-\alpha) \nu^m p^m_{j,t}(\omega) Y^m_{j,t}(\omega), \\
p^m_{j,t} M^m_{j,t}(\omega) &= (1-\nu^m) p^m_{j,t}(\omega) Y^m_{j,t}(\omega), \\
r_{j,t} K^f_{j,t} &= \alpha \nu^f p^f_{j,t} Y^f_{j,t}, \\
w_{j,t} L^f_{j,t} &= (1-\alpha) \nu^f p^f_{j,t} Y^f_{j,t}, \\
p^f_{j,t} M^f_{j,t} &= (1-\nu^f) p^f_{j,t} Y^f_{j,t}.
\end{aligned}$$

Using these equations, one can rewrite the market-clearing conditions of capital, labor, and the intermediate composite in the main body of the manuscript as follows:

$$r_{j,t} K_{j,t} = \alpha \nu^m \int_{\omega \in \Omega_{j,t}} p^m_{j,t}(\omega) Y^m_{j,t}(\omega) d\omega + \alpha \nu^f p^f_{j,t} Y^f_{j,t}, \tag{25}$$

$$w_{j,t} L_j = (1-\alpha) \nu^m \int_{\omega \in \Omega_{j,t}} p^m_{j,t}(\omega) Y^m_{j,t}(\omega) d\omega + (1-\alpha) \nu^f p^f_{j,t} Y^f_{j,t}, \tag{26}$$

$$p^m_{j,t} M_{j,t} = (1-\nu^m) \int_{\omega \in \Omega_{j,t}} p^m_{j,t}(\omega) Y^m_{j,t}(\omega) d\omega + (1-\nu^f) p^f_{j,t} Y^f_{j,t}. \tag{27}$$

Derivation of $w_{j,t} L_j = \frac{1-\alpha}{\alpha} r_{j,t} K_{j,t}$: Equations (25) and (26) immediately imply this equation.

Derivation of equation (13): The condition for the trade balance is

$$\int_{\omega \in \Omega_{jn,t}} p^m_{j,t}(\omega) y^m_{j,t}(\omega) d\omega = \int_{\omega \in \Omega_{nj,t}} p^m_{n,t}(\omega) y^m_{n,t}(\omega) d\omega. \tag{28}$$

Now I show that this equation reduces to (13).

Using (25), one can rewrite equation (27) as

$$p^m_{j,t}M_{j,t} = p^f_{j,t}Y^f_{j,t} + \int_{\omega\in\Omega_{j,t}} p^m_{j,t}(\omega)Y^m_{j,t}(\omega)d\omega - \frac{1}{\alpha}r_{j,t}K_{j,t}. \tag{29}$$

Equation (28) can be expressed as[15]

$$\int_{\omega\in\Omega_{j,t}} p^m_{j,t}(\omega)Y^m_{j,t}(\omega)d\omega = p^m_{j,t}M_{j,t}.$$

Therefore, equation (29) shows that the term $p^f_{j,t}Y^f_{j,t}$ is equal to $r_{j,t}K_{j,t}/\alpha$. From this result and equation (27), one obtains

$$p^m_{j,t}M_{j,t} = \frac{1-\nu^f}{\nu^m}\frac{r_{j,t}K_{j,t}}{\alpha}. \tag{30}$$

Since demand for ω is $y^m_{j,t}(\omega) = M_{j,t}\left(p^m_{j,t}(\omega)/p^m_{j,t}\right)^{-\sigma}$, the total expenditure in country j for country n's product is expressed as

$$\int_{\omega\in\Omega_{jn,t}} p^m_{j,t}(\omega)y^m_{j,t}(\omega)d\omega = M_{j,t}(p^m_{j,t})^{\sigma}\int_{\omega\in\Omega_{jn,t}} \left(p^m_{j,t}(\omega)\right)^{1-\sigma} d\omega. \tag{31}$$

Let $p^m_{jn,t}(\omega) = \tau_{jn}b_{n,t}/z_{n,t}(\omega)$ denote the demand price of variety ω if country j buys it from country n. The distribution of $p^m_{jn,t}(\omega)$ is given by

$$\text{Prob}(p_{jn,t}(\omega) \le p) = 1 - \exp\left[-T_n(\tau_{jn}b_{n,t})^{-\theta}p^{\theta}\right].$$

Since $p^m_{j,t}(\omega) = p^m_{jn,t}(\omega)$ for $\omega \in \Omega_{jn,t}$,

$$\int_{\omega\in\Omega_{jn,t}} \left(p^m_{j,t}(\omega)\right)^{1-\sigma} d\omega = T_n(\tau_{jn}b_{n,t})^{-\theta}\theta\int_0^\infty p^{\theta-\sigma}\exp\left(-\Phi_{j,t}p^{\theta}\right)dp, \tag{32}$$

where $\Phi_{j,t} \equiv \sum_{j'\in\{1,2\}} T_{j'}(\tau_{jj'}b_{j',t})^{-\theta}$. Using the definition of the gamma function $\Gamma(v) = \int_0^\infty u^{v-1}\exp(-u)du$,

$$\int_0^\infty p^{\theta-\sigma}\exp\left(\Phi_{j,t}p^{\theta}\right)dp = \frac{1}{\theta}\left(\Phi_{j,t}\right)^{\frac{\sigma-1}{\theta}-1}\Gamma\left(1+(1-\sigma)/\theta\right).$$

[15] The zero-profit condition of the intermediate composite firm is

$$p^m_{j,t}M_{j,t} = \int_0^1 p^m_{j,t}(\omega)y^m_{j,t}(\omega)d\omega = \int_{\omega\in\Omega_{jj,t}} p^m_{j,t}(\omega)y^m_{j,t}(\omega)d\omega + \int_{\omega\in\Omega_{jn,t}} p^m_{j,t}y^m_{j,t}(\omega)d\omega.$$

From this result, the trade balance (28) is rewritten as

$$p^m_{j,t}M_{j,t} = \int_{\omega\in\Omega_{jj,t}} p^m_{j,t}(\omega)y^m_{j,t}(\omega)d\omega + \int_{\omega\in\Omega_{nj,t}} p^m_{n,t}(\omega)y^m_{n,t}(\omega)d\omega.$$

In the above equation, the right-hand-side is the total sales of country j's intermediate goods firms, where the first term is the domestic sales and the second term is the export sales. Therefore, using the market clearing condition of the intermediate goods, the right-hand-side is expressed as $\int_{\omega\in\Omega_{j,t}} p^m_{j,t}(\omega)Y^m_{j,t}(\omega)d\omega$. Then, it follows that $p^m_{j,t}M_{j,t} = \int_{\omega\in\Omega_{j,t}} p^m_{j,t}(\omega)Y^m_{j,t}(\omega)d\omega$.

Substituting this result into (32) and using the fact that $p^m_{j,t}$ and $\pi_{jn,t}$ are respectively given by $p^m_{j,t} = \Phi_{j,t}^{-1/\theta}\left(\Gamma(1+(1-\sigma)/\theta)\right)^{1/(1-\sigma)}$ and $\pi_{jn,t} = T_n(\tau_{jn}b_{n,t})^{-\theta}/\Phi_{j,t}$, one obtains

$$\int_{\omega\in\Omega_{jn,t}} \left(p^m_{j,t}(\omega)\right)^{1-\sigma} d\omega = \pi_{jn,t}(p^m_{j,t})^{1-\sigma}.$$

Substituting the above equation to the right-hand-side of equation (31) yields

$$\int_{\omega\in\Omega_{jn,t}} p^m_{j,t}(\omega)y^m_{j,t}(\omega)d\omega = \pi_{jn,t}p^m_{j,t}M_{j,t}.$$

Then, the trade balance (28) is rewritten as

$$\pi_{jn,t}p^m_{j,t}M_{j,t} = \pi_{nj,t}p^m_{n,t}M_{n,t}.$$

Using equation (30), this further reduces to

$$\pi_{12,t}r_{1,t}K_{1,t} = \pi_{21,t}r_{2,t}K_{2,t},$$

which is equation (13).

Proof of Lemma 1

Proof of claim (i): From the definition of $\mathcal{H}(\widetilde{P}^m; r_{2,t})$, one can verify that $\lim_{\widetilde{P}^m\to 0}\mathcal{H}(\widetilde{P}^m; r_{2,t}) = \tau_{12}^{-\theta} \in (0,1)$ and $\lim_{\widetilde{P}^m\to\infty}\mathcal{H}(\widetilde{P}^m; r_{2,t}) = \tau_{21}^{\theta} > 1$. Differentiating $\mathcal{H}(\widetilde{P}^m; r_{2,t})$ with respect to P^m yields

$$\begin{aligned}
\frac{\partial\mathcal{H}(\widetilde{P}^m; r_{2,t})}{\partial\widetilde{P}^m} &= \frac{(1-\nu^m)Q_t[1-(\tau_{12}\tau_{21})^{-\theta}](\widetilde{P}^m)^{-\nu^m}}{\left[\tau_{21}^{-\theta}(\widetilde{P}^m)^{1-\nu^m}+Q_t\right]^2} > 0,\\
\frac{\partial^2\mathcal{H}(\widetilde{P}^m;\cdot)}{(\partial\widetilde{P}^m)^2} &= \frac{(1-\nu^m)Q_t[1-(\tau_{12}\tau_{21})^{-\theta}]}{\left[\tau_{21}^{-\theta}(\widetilde{P}^m)^{1-\nu^m}+Q_t\right]^4}\Delta,
\end{aligned}$$

where $Q_t \equiv (B_2/B_1)^{\theta}(r_{2,t})^{-\theta\nu^m}$ and

$$\Delta \equiv -\nu^m(\widetilde{P}^m)^{-\nu^m-1}\left[\tau_{21}^{-\theta}(\widetilde{P}^m)^{1-\nu^m}+Q_t\right]^2 - 2\left[\tau_{21}^{-\theta}(\widetilde{P}^m)^{1-\nu^m}+Q_t\right]\tau_{21}^{-\theta}(1-\nu^m)(\widetilde{P}^m)^{-2\nu^m} < 0.$$

Then, $\mathcal{H}(\widetilde{P}^m; r_{2,t})$ is strictly increasing and strictly concave. These results show that there uniquely exists the solution to equation $\widetilde{P}^m = \mathcal{H}(\widetilde{P}^m; r_{2,t})$ within the interval of $(\tau_{12}^{-\theta}, \tau_{21}^{\theta})$.

Proof of claim (ii): One can find that

$$\frac{\partial\mathcal{H}(\widetilde{P}^m; r_2)}{\partial r_2} = \frac{[1-(\tau_{12}\tau_{21})^{-\theta}](\widetilde{P}^m)^{1-\nu^m}\theta\nu^m}{\left[\tau_{21}^{-\theta}(\widetilde{P}^m)^{1-\nu^m}+Q_t\right]^2}\frac{Q_t}{r_{2,t}} > 0,$$

and

$$\lim_{r_2 \to 0} \mathcal{H}(\widetilde{P}^m; r_2) = \tau_{12}^{-\theta}, \lim_{r_2 \to \infty} \mathcal{H}(\widetilde{P}^m; r_2) = \tau_{21}^{\theta},$$

which imply that $\widetilde{P}^m(r_{2,t})$ is an increasing function of $r_{2,t}$, and

$$\lim_{r_{2,t} \to 0} \widetilde{P}^m(r_{2,t}) = \tau_{12}^{-\theta}, \quad \lim_{r_{2,t} \to \infty} \widetilde{P}^m(r_{2,t}) = \tau_{21}^{\theta}.$$

Calibration details

Determination of $\lambda_1, \lambda_2, s_{\max}$, and $s_{\min}$: As stated in the main body of the manuscript, I specify investment productivity as a uniform distribution: $G(s) = (s - s_{\min})/(s_{\max} - s_{\min})$. Then, it follows that

$$\begin{aligned}
s^c(\lambda_j) &= \frac{\lambda_j s_{\max} + s_{\min}}{1 + \lambda_j}, \\
f(\lambda_j) &= s_{\max} - \frac{s_{\max} - s_{\min}}{2(1 + \lambda_j)}, \\
1 - G(s^c(\lambda_j)) &= \frac{s_{\max} - s^c(\lambda_j)}{s_{\max} - s_{\min}} = \frac{1}{1 + \lambda_j}.
\end{aligned}$$

Since $f(\lambda_1) = 1$ is assumed, the ratio of debt to capital of active investors is

$$\frac{\lambda_1(1 - G(s^c(\lambda_1)))}{f(\lambda_1)} = \frac{\lambda_1}{1 + \lambda_1} = 0.69,$$

from which one obtains λ_1. In addition, since $f(\lambda_1) = 1$, the balanced growth rate g^* satisfies

$$1 + g^* = \beta(1/p_1^{f*} + 1 - \delta).$$

From the definition of the cutoff $s^c(\lambda_1)$, it follows that $s^c(\lambda_1)(1/p_1^{f*} + 1 - \delta) = 1 + r_1^{d*}$. The balanced growth rate g^* and the real interest rate r_1^{d*} thus satisfy

$$1 + g^* = \beta \frac{1 + r_1^{d*}}{s^c(\lambda_1)}.$$

Therefore, by substituting $s^c(\lambda_1)$ given above into this equation, one obtains

$$\frac{\lambda_1 s_{\max} + s_{\min}}{1 + \lambda_1} = \frac{1 + r_1^{d*}}{\underbrace{\beta(1 + g^*)}_{\text{already known}}}. \tag{33}$$

In addition, the assumption $f(\lambda_1) = 1$ requires

$$s_{\max} - \frac{s_{\max} - s_{\min}}{2(1 + \lambda_1)} = 1. \tag{34}$$

Equations (33) and (34) determine $s_{\max}$ and $s_{\min}$. Once r_2^{d*} is given, λ_2 is determined by

$$1 + g^* = \beta \frac{1 + r_2^{d*}}{s^c(\lambda_2)}.$$

Thus, $r_1^{d*} = r_2^{d*}$ implies $\lambda_1 = \lambda_2$.

Determination of τ_{12}, τ_{21}, B_1, and B_2: Since $f(\lambda_j) = 1$, the balanced growth condition (21) is

$$1 + g^* = \beta(1/p_1^{f*} + 1 - \delta) = \beta(r_2^*/p_2^{f*} + 1 - \delta),$$

which gives p_1^{f*} and p_2^{f*}/r_2^* as

$$p_1^{f*} = p_2^{f*}/r_2^* = q^f \equiv \beta/[1 + g^* - \beta(1 - \delta)].$$

From equation (11), one obtains both of p_1^{m*} and p_2^{m*}/r_2^* as $q^m \equiv (q^f/\psi^f)^{1/(1-\nu^f)}$. By use of q^m, equations (18) and (19) are respectively rewritten as

$$1 = (\psi^m)^{-\theta} (q^m)^{\theta\nu^m} \left[B_1^\theta + B_2^\theta \tau_{12}^{-\theta} (\widetilde{P}^m)^{-1}\right], \tag{35}$$

$$1 = (\psi^m)^{-\theta} (q^m)^{\theta\nu^m} \left[B_1^\theta \tau_{21}^{-\theta} \widetilde{P}^m + B_2^\theta\right]. \tag{36}$$

Using equation (13), one can express π_{12}^* and π_{21}^* as

$$\pi_{12}^* = B_2^\theta \tau_{12}^{-\theta} (\psi^m)^{-\theta} (q^m)^{\theta\nu^m} (\widetilde{P}^m)^{-1}, \tag{37}$$

$$\pi_{21}^* = B_1^\theta \tau_{21}^{-\theta} (\psi^m)^{-\theta} (q^m)^{\theta\nu^m} \widetilde{P}^m. \tag{38}$$

Since $\psi^m = [(1-\alpha)/\alpha]^{(1-\alpha)\nu^m}$ is known, equations (35)–(38) include 6 unknowns, τ_{12}, τ_{21}, B_1, B_2 π_{12}^*, π_{21}^*, and $\widetilde{P}^m$. As stated in the main body of the manuscript, π_{12}^* is determined by

$$\frac{1 - \nu^f}{\nu^m} \pi_{12}^* = 0.135,$$

where the value of the right-hand side is the target value of U.S. imports to its GDP share from the data. Then, I determine π_{21}^* using (14):

$$\pi_{21}^* = \underbrace{\pi_{12}^* \times \frac{1}{r_2^* k_2^*}}_{\text{already known}}.$$

I assume $\tau_{12} = \tau_{21} = \tau$. Then, equations (35)–(38) provide τ, B_1, and B_2, together with $\widetilde{P}^m$.

Log-linear approximation

Let a hat over a variable indicate the log-deviation of the variable from its stationary value. For example, $\widehat{k}_{2,t} = \ln k_{2,t} - \ln k_2^* \simeq (k_{2,t} - k_2^*)/k_2^*$. The log-linear approximation of the system around

$(p_1^{f*}, p_2^{f*}, p_1^{m*}, p_2^{m*}, \pi_{12}^*, \pi_{21}^*, r_2^*, k_2^*, \lambda_1)$ is

$$\widehat{p}_{1,t}^f = (1-\nu^f)\widehat{p}_{1,t}^m, \tag{39}$$

$$\widehat{p}_{2,t}^f = \nu^f \widehat{r}_{2,t} + (1-\nu^f)\widehat{p}_{2,t}^m, \tag{40}$$

$$\widehat{p}_{1,t}^m = \pi_{11}^* \left[(1-\nu^m)\widehat{p}_{1,t}^m\right] + \pi_{12}^* \left[\nu^m \widehat{r}_{2,t} + (1-\nu^m)\widehat{p}_{2,t}^m\right], \tag{41}$$

$$\widehat{p}_{2,t}^m = \pi_{21}^* \left[(1-\nu^m)\widehat{p}_{1,t}^m\right] + \pi_{22}^* \left[\nu^m \widehat{r}_{2,t} + (1-\nu^m)\widehat{p}_{2,t}^m\right], \tag{42}$$

$$\widehat{\pi}_{12,t} = -\theta \left[\nu^m \widehat{r}_{2,t} + (1-\nu^m)\widehat{p}_{2,t}^m - \widehat{p}_{1,t}^m\right], \tag{43}$$

$$\widehat{\pi}_{21,t} = -\theta \left[(1-\nu^m)\widehat{p}_{1,t}^m - \widehat{p}_{2,t}^m\right], \tag{44}$$

$$\widehat{\pi}_{12,t} = \widehat{\pi}_{21,t} + \widehat{r}_{2,t} + \widehat{k}_{2,t}, \tag{45}$$

$$\widehat{k}_{2,t+1} = \widehat{k}_{2,t} + \mu_1^* \widehat{p}_{1,t}^f + \mu_2^*(\widehat{r}_{2,t} - \widehat{p}_{2,t}^f) - \xi \widehat{\lambda}_{1,t}, \tag{46}$$

$$\widehat{\lambda}_{1,t+1} = \rho_\lambda \widehat{\lambda}_{1,t}, \tag{47}$$

where $\mu_1^* \equiv 1/[1+(1-\delta)p_1^{f*}] \in (0,1)$, $\mu_2^* \equiv r_2^*/[r_2^* + (1-\delta)p_2^{f*}] \in (0,1)$, and $\xi \equiv \lambda_1 f'(\lambda_1)/f(\lambda_1) > 0$. Equations (41) and (42) reduce to

$$\underbrace{\begin{pmatrix} 1-\pi_{11}^*(1-\nu^m) & -\pi_{12}^*(1-\nu^m) \\ -\pi_{21}^*(1-\nu^m) & 1-\pi_{22}^*(1-\nu^m) \end{pmatrix}}_{\equiv\, \mathbf{J}} \begin{pmatrix} \widehat{p}_{1,t}^m \\ \widehat{p}_{2,t}^m \end{pmatrix} = \begin{pmatrix} \pi_{12}^* \nu^m \widehat{r}_{2,t} \\ \pi_{22}^* \nu^m \widehat{r}_{2,t} \end{pmatrix},$$

where the determinant of matrix $\mathbf{J}$ is

$$\begin{aligned}
\det \mathbf{J} &= [1-\pi_{11}^*(1-\nu^m)][1-\pi_{22}^*(1-\nu^m)] - \pi_{12}^*\pi_{21}^*(1-\nu^m)^2 \\
&= 1-(\pi_{11}^*+\pi_{22}^*)(1-\nu^m) + \pi_{11}^*\pi_{22}^*(1-\nu^m)^2 - \pi_{12}^*\pi_{21}^*(1-\nu^m)^2 \\
&= 1-(2-\pi_{12}^*-\pi_{21}^*)(1-\nu^m) + (1-\pi_{12}^*)(1-\pi_{21}^*)(1-\nu^m)^2 - \pi_{12}^*\pi_{21}^*(1-\nu^m)^2 \;\; (\because \pi_{jj} = 1-\pi_{jn}) \\
&= 1-(2-\pi_{12}^*-\pi_{21}^*)(1-\nu^m) + (1-\pi_{12}^*-\pi_{21}^*)(1-\nu^m)^2 \\
&= 1-2(1-\nu^m) + (1-\nu^m)^2 + (\pi_{12}^*+\pi_{21}^*)(1-\nu^m) - (\pi_{12}^*+\pi_{21}^*)(1-\nu^m)^2 \\
&= (\nu^m)^2 + (\pi_{12}^*+\pi_{21}^*)(1-\nu^m)\nu^m \\
&= \nu^m\left[\nu^m + (1-\nu^m)(\pi_{12}^*+\pi_{21}^*)\right] > 0.
\end{aligned}$$

Then,

$$\begin{aligned}
\widehat{p}_{1,t}^m &= \frac{1}{\nu^m\left[\nu^m + (1-\nu^m)(\pi_{12}^*+\pi_{21}^*)\right]} \begin{vmatrix} \pi_{12}^*\nu^m \widehat{r}_{2,t} & -\pi_{12}^*(1-\nu^m) \\ \pi_{22}^*\nu^m \widehat{r}_{2,t} & 1-\pi_{22}^*(1-\nu^m) \end{vmatrix} \\
&= \eta_1^* \widehat{r}_{2,t},
\end{aligned} \tag{48}$$

and

$$
\begin{aligned}
\widehat{p}^m_{2,t} &= \frac{1}{\nu^m \left[\nu^m + (1-\nu^m)(\pi^*_{12}+\pi^*_{21})\right]} \begin{vmatrix} 1-\pi^*_{11}(1-\nu^m) & \pi^*_{12}\nu^m \widehat{r}_{2,t} \\ \pi^*_{21}(1-\nu^m) & \pi^*_{22}\nu^m \widehat{r}_{2,t} \end{vmatrix} \\
&= \eta^*_2 \widehat{r}_{2,t}, \qquad (49)
\end{aligned}
$$

where η^*_1 and η^*_2 are respectively defined as

$$
\begin{aligned}
\eta^*_1 &\equiv \frac{\pi^*_{12}}{\nu^m + (1-\nu^m)(\pi^*_{12}+\pi^*_{21})} \in (0,1), \\
\eta^*_2 &\equiv \frac{\pi^*_{12} + \nu^m[1-(\pi^*_{12}+\pi^*_{21})]}{\nu^m + (1-\nu^m)(\pi^*_{12}+\pi^*_{21})} \in (\eta^*_1, 1).
\end{aligned}
$$

Substituting (48) and (49) into (43) and (44), $\widehat{\pi}_{12,t}$ and $\widehat{\pi}_{21,t}$ are respectively given by

$$
\begin{aligned}
\widehat{\pi}_{12,t} &= -\theta\left[\nu^m(\widehat{r}_{2,t} - \widehat{p}^m_{2,t}) + \widehat{p}^m_{2,t} - \widehat{p}^m_{1,t}\right] \\
&= -\theta\left[\nu^m(1-\eta^*_2) + \eta^*_2 - \eta^*_1\right]\widehat{r}_{2,t} \\
&= -\zeta^* \pi^*_{11} \nu^m \widehat{r}_{2,t}, \qquad (50)
\end{aligned}
$$

and

$$
\begin{aligned}
\widehat{\pi}_{21,t} &= -\theta\left[(1-\nu^m)\eta^*_1 \widehat{r}_{2,t} - \eta^*_2 \widehat{r}_{2,t}\right] \\
&= -\theta\left[(1-\nu^m)\eta^*_1 - \eta^*_2\right]\widehat{r}_{2,t} \\
&= \zeta^* \pi^*_{22} \nu^m \widehat{r}_{2,t}, \qquad (51)
\end{aligned}
$$

where $\zeta^* \equiv \theta/[\nu^m + (1-\nu^m)(\pi^*_{12}+\pi^*_{21})] > 0$. Substituting (50) and (51) into (45),

$$
\widehat{r}_{2,t} = -h^* \widehat{k}_{2,t}, \qquad (52)
$$

where

$$
h^* \equiv \frac{1}{1 + \nu^m \zeta^* (\pi^*_{11} + \pi^*_{22})} \in (0,1).
$$

Using (39) and (40), one can rewrite equation (46) as

$$
\widehat{k}_{2,t+1} = \widehat{k}_{2,t} + (1-\nu^f)\left[\mu^*_1 \widehat{p}^m_{1,t} + \mu^*_2\left(\widehat{r}_{2,t} - \widehat{p}^m_{2,t}\right)\right]. \qquad (53)
$$

Substituting (52) into (48) and (49),

$$
\begin{aligned}
\widehat{p}^m_{1,t} &= -\eta^*_1 h^* \widehat{k}_{2,t}, \qquad (54) \\
\widehat{r}_{2,t} - \widehat{p}^m_{2,t} &= -(1-\eta^*_2) h^* \widehat{k}_{2,t}. \qquad (55)
\end{aligned}
$$

Then, substituting (54) and (55) into (53) gives

$$
\widehat{k}_{2,t+1} = \rho^*_k \widehat{k}_{2,t} - \xi \widehat{\lambda}_{1,t},
$$

where

$$\rho_k^* \equiv 1 - (1-\nu^f)h^* \left[\mu_1^*\eta_1^* + \mu_2^*(1-\eta_2^*)\right].$$

When $\lambda_1 = \lambda_2$, it follows that $1/p_1^{f*} = r_2^*/p_2^{f*}$ is satisfied and hence $\mu_1^* = \mu_2^* = \mu^*$ holds. Then, ρ_k^* is simplified to

$$\rho_k^* = 1 - (1-\nu^f)h^*\mu^* \left(1 + \eta_1^* - \eta_2^*\right) \in (0,1).$$