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Abstract

We study an equilibrium risk and return model to explore the effects of the coronavirus crisis and associated skewness. We derive the moment and equilibrium equations, specifying skewness price of risk as an additive component of the effect of variance on mean expected return. We estimate our model using the flexible skewed generalized error distribution, for which we derive the distribution of returns and the likelihood function. Using S&P 500 Index returns from January 1990 to mid-May 2020, our results show that the coronavirus crisis generated the most negative reaction in the skewness price of risk, more negative even than the subprime crisis.

Keywords: Asset pricing; Risk and return; Skewness; Coronavirus crisis; Subprime crisis

JEL classification: G01; G11; G12; C32; C51

1. Introduction

The severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) and the disease it causes (COVID-19) were first identified after a cluster of cases in December 2019 in Wuhan, China. In response to reports of new cases surging daily in China and subsequently in Europe, especially Italy, the market's negative reaction to the coronavirus' spread began in mid-February 2020, with the first large crash on February 21. The U.S. equity markets hit a low on March 23 following the World Health Organization's declaration of a pandemic on March 11, the exponential increase of daily new COVID-19 cases worldwide, and government decisions to enact lockdowns.

In this paper, we include the coronavirus crisis in a dynamic model of risk and return, in order to illustrate the importance of the skewness price of risk for modeling the risk–return relation. Our results have important implications for estimating the depth of the coronavirus financial crisis and its comparison with the 2008 subprime mortgage lending crisis. More generally, our analysis highlights the role of skewness in measuring the depth of financial crises and investors' reaction to bad news.

The economic case for including skewness in a model of risk and return is that mean and variance are not the only factors driving the returns distribution (unconditional in the cross-section or conditional in the time series). This is because, *ceteris paribus*, risk-averse investors must prefer right-skewed portfolios to left-skewed ones. For example, adding assets that decrease a portfolio's skewness (and thus making them more left-skewed) should have higher expected returns. Skewness is thus a key element in models of risk and return, especially for short- and medium-term investors. During periods of excess volatility, investors realize that their expected returns will not materialize and switch to less risky assets, thereby introducing increased negative skewness to the price of risk.

Because of the downturn's exogenous nature, the coronavirus crisis presents a natural experiment to examine the role of skewness in models of risk and return. Chan and Marsh (2020) show that the market's trajectory in the coronavirus crisis resembles that of the Great Depression and the Lehman Brothers collapse, but newer data for the coronavirus era shows a quicker market rebound since the U.S. market low on March 23, 2020. This phenomenon might be the result of the different and exogenous nature of the coronavirus crisis, with markets perceiving that the economy will rebound more easily or because of the quickly enhanced liquidity by policy makers. We note, however, that the initial (one-month) coronavirus market crash was much deeper than the equivalent 2008 crash, reflecting a very strong investor reaction as well as associated changes in risk and investment horizons.

On this basis, our paper answers two interrelated questions. First, we examine whether skewness is significantly more negative during crisis periods while maintaining the prediction of a positive risk–return nexus. The latter is important for consistency with key relevant theory in financial economics (e.g., French et al., 1987; Theodossiou and Savva, 2016). Second, we examine whether the sharpness and exogeneity of the coronavirus crisis implies that skewness was more negative during this period compared with the subprime crisis.

We incorporate skewness into a dynamic equilibrium model of risk and return. For a positive risk–return relation, investors require a higher (lower) risk premium during periods that are more (less) volatile. Technically, this dynamic implies that a portfolio's excess return is a positive function of the conditional variance of returns (e.g., Engle et al., 1987). In this respect, we build on Theodossiou and Savva (2016), who explicitly model skewness in the price of risk. Specifically, we examine the joint effect of price shocks reflected in the subprime and coronavirus crises on the distribution of returns using a skewed extension of the generalized autoregressive conditional heteroskedasticity in the mean (GARCH-M) model of Glosten et al. (1993). We derive the conditional variance as a positive function of financial crises, and we estimate skewness in the price of risk as an additive component of the effect of variance (the pure price of risk) on the mean expected returns. The skewness equation depends on the conditional asymmetry parameter, and together these factors determine how the pure and skewness prices of risk affect expected returns. Importantly, we derive the risk-neutral equation of returns, which shows the effect of crises, skewness, and other parameters on equilibrium returns. Our model's key prediction is that a negative value on the crisis parameter implies a negative unconditional asymmetry parameter and yields a negatively skewed distribution for returns.

We estimate our model using data from the S&P 500 Index for the period January 2, 1990 to May 15, 2020 (7,652 daily continuously compounded returns). We use the skewed generalized error distribution, which has the appealing property of allowing us to develop exponential asset pricing equations, thereby significantly improving flexibility. We use maximum likelihood estimation with robust standard errors and obtain confidence intervals using the procedure of Rapach and Wohar (2009).

Our main results show the importance of modeling crisis periods and associated skewness in asset pricing models. Aside from finding large volatility increases in the crisis periods, and consistent with our theoretical priors, we show that the conditional skewness price of risk is indeed negative in our full sample. Importantly, the conditional skewness price of risk drops (becomes more negative) by approximately 74% during the subprime crisis compared with the "regular" period, whereas the equivalent decrease during the coronavirus crisis is much larger, reaching 149% compared with the regular period. These decreases show the immense reaction of investors during crisis periods, as well as the quickly altering sentiments regarding portfolio risk and investment horizon. Evidently, the negative reactions were deeper during the coronavirus crisis compared with the subprime crisis. Finally, we show that accounting for skewness in our model allows maintaining the theoretically positive risk–return relation.

The rest of the paper proceeds as follows. The next section develops the moment equations for prices of financial assets, derives the risk-neutral equilibrium equation for their returns, and explores the basic properties of their distributions. Section 3 specifies the key equations for the conditional moments in our model, as well as the risk-neutral equilibrium equation for returns. Sections 4 discusses the estimation method and the empirical findings. Section 5 concludes the paper.

2. Asset Price Moments and Equilibrium Returns

2.1. Asset Price Moments

In a discrete time setup, the price of an asset at time $t + 1$ is

$$S_{t+1} = S_t e^{r_{t+1}} = S_t e^{\mu_{t+1|t} + \sigma_{t+1|t} z_{t+1}}, \tag{1}$$

where S_t is the price at time t (spot price), r_{t+1} is a continuously compounded return for the period t to $t + 1$, z_{t+1} is the standardized value of r_{t+1},

$$\mu_{t+1|t} = E\left(r_{t+1} | \Phi_t\right)$$

and

$$\sigma^2_{t+1|t} = \operatorname{var}\left(r_{t+1} | \Phi_t\right)$$

are respectively the mean and variance of r_{t+1} conditional on the information set Φ_t, which includes information available at time t used in the formation of price expectations.

The conditional probability mass function (pmf) for standardized returns is

$$dF_{z|t} = f_{z|t}\left(z_{t+1}\middle|\Phi_t\right)dz_{t+1}, \tag{2}$$

where $f_{z|t}$ is the conditional and probability density function (pdf) for z_{t+1}, which is assumed to be continuous and unimodal with a moment generating function. The latter is necessary for the existence of price expectations.

The j^{th} conditional moment for asset prices is

$$\begin{aligned} E\left(S_{t+1}^{j}\middle|\Phi_t\right) &= S_t^j e^{j\mu_{t+1|t}}\int_{-\infty}^{\infty} e^{j\sigma_{t+1|t}z_{t+1}}dF_{z|t} \\ &= S_t^j e^{j\mu_{t+1|t}+\ln E_{j,z|t}}, \end{aligned} \tag{3}$$

where

$$E_{j,z|t} = \int_{-\infty}^{\infty} e^{j\sigma_{t+1|t}z_{t+1}}dF_{z|t} \tag{4}$$

for $j = 1, 2, \ldots$ The conditional mean and conditional variance of S_{t+1} are respectively

$$E\left(S_{t+1}\middle|\Phi_t\right) = S_t e^{\mu_{t+1|t}+\ln E_{1,z|t}} \tag{5}$$

and

$$\operatorname{var}\left(S_{t+1}\middle|\Phi_t\right) = S_t^2 e^{2\mu_{t+1|t}}\left(e^{\ln E_{2,z|t}} - e^{2\ln E_{1,z|t}}\right). \tag{6}$$

2.2. Equilibrium Prices and Rates

The asset's accrued dividends at time $t + 1$, paid continuously over the period t to $t + 1$, are

$$D_{t+1} = S_{t+1}\left(e^{q_{t+1}} - 1\right),$$

where q_{t+1} is a dividend payment rate. The sum of the price and accrued dividends is

$$S_{t+1} + D_{t+1} = S_t e^{q_{t+1}+\mu_{t+1|t}+\sigma_{t+1|t}z_{t+1}}. \tag{7}$$

In liquid and frictionless markets, the absence of arbitrage opportunities requires that the expected value of the price and accrued dividends discounted at the asset's required rate of return to be equal to its price at the beginning of the period. That is,

$$E\left(S_{t+1} + D_{t+1}\middle|\Phi_t\right)e^{-r_{f,t+1}-\rho_{t+1|t}} = S_t e^{-\rho_{t+1|t}-r_{f,t+1}+\mu_{t+1|t}+q_{t+1}+\ln E_{1,z|t}} = S_t,$$

where $r_{f,t+1}$ and ρ_{t+1} are respectively the risk-free rate of interest and the asset's risk premium over the period t to $t+1$. The risk premium ρ_{t+1} represents the minimum return in excess of the risk-free rate r_{t+1} that induces investors to buy or hold the risky asset. A deviation of the above pricing kernel from S_t constitutes a violation of the martingale property for prices and is indicative of arbitrage opportunities. The arbitrage-free equality above gives the following equilibrium rate of return

$$\hat{K}_{t+1|t} = r_{f,t+1} + \rho_{t+1|t} = \mu_{t+1|t} + q_{t+1} + \ln E_{1,z|t}\,, \tag{8}$$

where $\mu_{t+1|t}$, q_{t+1} and $\ln E_{1,z|t}$ are as defined previously.

2.3. Distribution of Returns

We examine the impact of distributional asymmetry on equilibrium returns using the following decomposition for standardized returns:

$$z_{t+1} = -\delta_{t+1|t} + \left(1 + sgn\left(w_{t+1}\right)\lambda_{t+1|t}\right)\theta_{t+1|t} w_{t+1}\,, \tag{9}$$

where $-\delta_{t+1|t}$ is by construction the mode of z_{t+1}, $\theta_{t+1|t}$ is a scaling constant for the tails of the distribution of z_{t+1}, $\lambda_{t+1|t}$ is a conditional asymmetry parameter with its values constrained in the closed interval [–1, 1] and w_{t+1} is a symmetric random variable distributed as $dF_{w|t}$. The asymmetry parameter $\lambda_{t+1|t}$ controls the shape of the distribution to the left and right of the mode of $dF_{z|t}$. Negative values of $\lambda_{t+1|t}$ trigger a negatively skewed distribution and vice versa. Zero values are associated with a symmetric distribution.

The assumption that $\text{mode}(z_{t+1}) = -\delta_{t+1|t}$ implies that the mode for returns is

$$\begin{aligned} \text{mode}\left(r_{t+1}\right) &= \mu_{t+1|t} + \sigma_{t+1|t}\text{mode}\left(z_{t+1}\right) \\ &= \mu_{t+1|t} - \delta_{t+1|t}\sigma_{t+1|t} \equiv m_{t+1|t}\,. \end{aligned} \tag{10}$$

Under the two-sided distribution framework used in the modeling of the distribution asset returns in downside and upside markets, the asymmetry parameter can be factored out of the distribution of standardized returns as follows (for details, see Savva and Theodossiou, 2018):

$$dF_{z|t} = \left(1 + sgn\left(w_{t+1}\right)\lambda_{t+1|t}\right)dF_{w|t}\,, \tag{11}$$

where

$$dF_{w|t} = f_{w|t} dw_{t+1}.$$

and $f_{w|t} = \theta_{t+1|t} f_{z|t}\left(z\left(w_{t+1}\right)\right)$. The probability function $f_{z|t}(w_{t+1})$ above is obtained via direct substitution of $z_{t+1}(w_{t+1})$ into $f_{z|t}$. By construction, the maximum likelihood point occurs at $f_{z|t}(-\delta_{t+1|t})$, which coincides with $f_{w|t}(0)$, thus $f_{w|t}$ has a zero mode.

Potential candidates for $dF_{w|t}$ include symmetric unimodal distributions with moment generating functions, such as the Generalized Error Distribution (GED), Laplace, normal, the symmetric exponential generalized beta of the second kind (EGB2) of McDonald and Xu (1995), the Generalized type III logistic and the logistic distributions.

The random variable decomposition and pdf factorization, given respectively by equations (9) and (10), yield the following functional forms for the scaling constants $\delta_{t+1|t}$ and $\theta_{t+1|t}$

$$\delta_{t+1|t} = 2\lambda_{t+1|t}\theta_{t+1|t}G_{1|t} \,, \tag{12}$$

$$\theta_{t+1|t} = 1\Big/\sqrt{\left(1+3\lambda_{t+1|t}^{2}\right)G_{2|t} - 4\lambda_{t+1|t}^{2}G_{1|t}^{2}} \tag{13}$$

and

$$G_{s|t} = E\left(\left|w_{t+1}\right|^{s}\right) = 2\int_{0}^{\infty} w_{t+1}^{s}\, dF_{w|t} \,, \tag{14}$$

for s = 1, 2, … See Equations (A3) and (A4) in the Appendix for the derivations. Note that these equations are general and do not depend on specific parametric distributions.

2.4. Generalized Error Distribution

For computational purpose, we use the flexible type generalized error distribution (GED) to model the distribution of the "stripped" random variable w_{t+1}

$$f_{w|t}\left(w_{t+1}\right) = \frac{1}{2}k_{t+1|t}^{\,1-\frac{1}{k_{t+1|t}}}\Gamma\left(\frac{1}{k_{t+1|t}}\right)^{-1}\exp\left(-\frac{1}{k_{t+1|t}}\left|w_{t+1}\right|^{k_{t+1|ti}}\right) \,, \tag{15}$$

where $k_{t+1|t}$ is a shape parameter that controls its tails and peakness around the zero mode of $f_{w|t}$ and $\Gamma(\cdot)$ is the gamma function. The above pdf is unimodal and symmetric with a moment generating function. The analytical equations for its absolute moments, derived in the appendix, are

$$G_{s|t} = E\left|w_{t+1}\right|^{s} = k_{t+1|t}^{\frac{s}{k_{t+1|t}}}\Gamma\left(\frac{s+1}{k_{t+1|t}}\right)\Gamma\left(\frac{1}{k_{t+1|t}}\right)^{-1}, \text{ for } s = 1, 2 \ldots \tag{16}$$

We choose not to use the Student's or the generalized t distribution of McDonald and Newey (1988), because of lack of moment generating functions. Their skewed extensions along the lines in this paper yield the skewed t of Hansen (1994) and the skewed generalized t of Theodossiou (1998). Both distributions are known to provide a good fit to the empirical distributions of financial returns.

3. Specification of Conditional Moments

We examine the joint impact of price shocks on the distribution of financial returns using a skewed extension of the generalized autoregressive conditional heteroskedasticity in the mean (GARCH-M) model of Glosten et al. (1993). We extend the model's structure to account for the impact of the subprime and coronavirus crises.

3.1. Conditional Variance

The conditional variance of returns is

$$\sigma_{t+1|t}^2 = \operatorname{var}\left(r_{t+1} \middle| \Phi_t\right) = \sum_{j=1}^{J} v_{D,j} D_{j,t} + v_0 + \left(\alpha + \alpha_N N_t\right) \varepsilon_t^2 + \beta\, \sigma_{t|t-1}^2 , \tag{17}$$

where $D_{j,t}$ is an indicator variable that takes the value of one during the period of crisis j and zero otherwise, $j = 1, 2, \ldots, J$, $\varepsilon_t = r_t - \mu_{t|t-1}$ is a deviation of return r_t from its conditional mean, used as a proxy for shocks in period $t - 1$ to t, and $N_t = 1$ for $\varepsilon_t < 0$ and $N_t = 0$ for $\varepsilon_t > 0$. The parameter $v_{D,j}$ measures the impact of crisis j on the conditional variance of returns. The second and third terms account for asymmetric volatility and volatility clustering. Larger positive values of a and β are indicative of higher volatility persistence. The parameter a_N measures the impact, if any, of past negative shocks on current market volatility (asymmetric volatility). In fact, the persistence of volatility is

$$VP = \alpha + a_N \cdot\ P\left(\varepsilon_t < 0\right) + \beta < 1 ,$$

where $P(\varepsilon_t < 0)$ is the probability of negative shocks, which is 0.5 in case of a symmetric distribution for ε_t. Stationarity of volatility requires that $VP < 1$.

3.2. Conditional Mode and Mean

The equation for the conditional mode of r_{t+1} is

$$m_{t+1|t} = \operatorname{mode}\left(r_{t+1} \middle| \Phi_t\right) = \sum_{j=1}^{J} m_{D,j} D_{j,t} + m_0 + b\, r_t + c\, \sigma_{t+1|t} , \tag{18}$$

where $D_{j,t}$, for $j = 1, 2, \ldots, J$, are as defined previously, r_t is a past return, $\sigma_{t+1|t} = \mathrm{var}(r_{t+1}|\Phi_t)$, and c is the GARCH-in-mean coefficient (Engle et al., 1987), or the "pure price of risk" (Theodossiou and Savva, 2016). It follows from equation (10) that

$$\mu_{t+1|t} = E\left(r_{t+1}\middle|\Phi_t\right) = \sum_{j=1}^{J} m_{D,j} D_{j,t} + m_0 + br_t + \left(c + \delta_{t+1|t}\right)\sigma_{t+1|t}\,, \tag{19}$$

where $\delta_{t+1|t}$ is skewness price of risk. It also follows from equations (12) and (13) that

$$\delta_{t+1|t} = \frac{2\lambda_{t+1|t} G_{1|t}}{\sqrt{\left(3\lambda_{t+1|t}^2 + 1\right) G_{2|t} - 4\lambda_{t+1|t}^2 G_{1|t}^2}} \tag{20}$$

depends directly on the conditional asymmetry parameter $\lambda_{t+1|t}$ and the conditional absolute moments $G_{1|t}$ and $G_{2|t}$, which are functions of the shape parameter $k_{t+1|t}$. Note that $\delta_{t+1|t}$ and $\lambda_{t+1|t}$ have the same sign. Both parameters are of primary interest because they determine how the total, pure, and skewness price of risk affect expected returns.

The substitution of $\mu_{t+1|t}$ into equation (8) gives

$$\hat{K}_{t+1|t} = \sum_{j=1}^{J} m_{D,j} D_{j,t} + m_0 + br_t + \left(c + \delta_{t+1|t}\right)\sigma_{t+1|t} + q_{t+1} + \ln E_{1,z|t}\,, \tag{21}$$

which is a type risk-neutral equilibrium equation for returns. This equation shows how crises, volatility, and other distributional parameters affect equilibrium returns.

3.3. Conditional Asymmetry

The conditional asymmetry parameter, which controls the shape of the distribution of returns to the left and right of its conditional mode, is

$$\lambda_{t+1|t} = asym\left(r_{t+1}\middle|\Phi_t\right) = 1 - \frac{2}{1 + e^{h_{t+1|t}}} \tag{22}$$

where

$$h_{t+1|t} = \sum_{j=1}^{J} \gamma_{D,j} D_{j,t} + \gamma_0 + \gamma_N u_t^- + \gamma_P u_t^+ + \gamma_h h_{t|t-1} \tag{23}$$

and

$$u_t = \frac{r_t - m_{t|t-1}}{\sigma_{t|t-1}}\,, \tag{24}$$

are standardised returns in excess of their conditional mode, with $u_t^- = |u_t|$ for $u_t < 0$ and zero otherwise, and $u_t^+ = |u_t|$ for $u_t > 0$ and zero otherwise. These are used respectively as measures of downside and upside shocks (e.g., Feunou et al., 2012).

The intercept γ_0 can be negative, zero, or positive. A negative value is associated with a negative unconditional asymmetry parameter and yields a negatively skewed distribution for returns (and vice versa for a positive value). The crisis parameter $\gamma_{D,j}$ measures the impact of crises on the intercept of $h_{t+1|t}$. The coefficient γ_N measures the marginal impact of downside price shocks on the asymmetry index $h_{t+1|t}$ and the asymmetry parameter $\lambda_{t+1|t}$. A positive value indicates that past downside price shocks have a positive impact on both $h_{t+1|t}$ and $\lambda_{t+1|t}$ (and vice versa). On the other hand, the coefficient γ_P measures the marginal impact of past price shocks on $h_{t+1|t}$ and $\lambda_{t+1|t}$. Positive values indicate that past upside price shocks have a positive impact on the parameters (and vice versa). The coefficient γ_h measures the persistence of past upside and downside shocks on the conditional values of $h_{t+1|t}$ and $\lambda_{t+1|t}$.

3.4. Conditional Shape Parameter

As in Mazur and Pipień (2018), we examine the dynamic behavior of the shape parameter $k_{t+1|t}$ using

$$k_{t+1|t} = k_U - \frac{k_U - k_L}{1 + e^{g_{t+1|t}}}, \tag{25}$$

where

$$g_{t+1|t} = \sum_{j=1}^{J} d_{D,j} D_{j,t} + d_0 + d_N u_t^- + d_P u_t^+ + d_h g_{t|t-1}, \tag{26}$$

u_t^- and u_t^+ are as defined previously, and k_L and k_U are predetermined lower and upper limits for the time varying shape parameter $k_{t+1|t}$. Typical values in financial series are $k_L = 1$ (Laplace) and $k_U = 2$ (normal). The parameters d_N and d_P control the shape of the distribution to the left and right of the conditional mode $m_{t|t-1}$. Zero values for d_N and d_P indicate a time invariant shape parameter $k_{t+1|t}$.

3.5. Upside and Downside Probabilities

The equations for the conditional probabilities for downside and upside markets are

$$P\left(r_{t+1} \le m_{t+1|t}\right) = \frac{1}{2}\left(1 - \lambda_{t+1|t}\right) = \frac{1}{1 + e^{h_{t+1|t}}} \tag{27}$$

and

$$P\left(r_{t+1} > m_{t+1|t}\right) = \frac{1}{2}\left(1 + \lambda_{t+1|t}\right) = \frac{1}{1 + e^{-h_{t+1|t}}}. \tag{28}$$

These equations are obtained from the substitution of the conditional asymmetry parameter of equation (14) into the upside and downside probability equations (A1) and (A2) in the Appendix.

4. Sample Likelihood Specification and Estimation

This section presents the specification of the sample log-likelihood function and estimation of the parameters, and discusses the empirical findings.

4.1 Skewed Generalized Error Distribution

The distribution of returns is

$$f_r\left(r_{t+1}\middle|\Phi_t\right)=\frac{1}{2\theta_{t+1|t}\sigma_{t+1|t}}k_{t+1|t}^{1-\frac{1}{k_{t+1|t}}}\Gamma\left(\frac{1}{k_{t+1|t}}\right)^{-1}$$

$$\exp\left(-\frac{1}{k_{t+1|t}}\left|\frac{r_{t+1}-\mu_{t+1|t}+\delta_{t+1|t}\sigma_{t+1|t}}{\left(1+sgn(r_{t+1}-\mu_{t+1|t}+\delta_{t+1|t}\sigma_{t+1|t})\lambda_{t+1|t}\right)\theta_{t+1|t}\sigma_{t+1|t}}\right|^{k_{t+1|t}}\right), \tag{29}$$

where $\mu_{t+1|t}$, $\sigma_{t+1|t}$, $\delta_{t+1|t}$, $\theta_{t+1|t}$, $\lambda_{t+1|t}$, $k_{t+1|t}$ are as defined previously. This is the conditional version of the skewed generalized error distribution (SGED) of Theodossiou (2015). It follows easily from equations (9), (10), (11) and (15). Since its inception, the SGED has been employed in the literature for the measurement of risk, pricing of options, and modeling of the time-series behavior of returns of stock indices, currencies, oil and precious metals, etc. Unlike the skewed generalized t that is often used in empirical work, the SGED enables the development of asset exponential asset pricing equations. It gives for $k_{t+1|t}$ = 1 the skewed Laplace or double exponential distribution, for $k_{t+1|t}$ = 2 the skewed normal distribution used in Feunou et al. (2012) and Roon and Karehnke (2017), and for $k_{t+1|t} = \infty$ the uniform distribution.

4.2. Maximum Likelihood Estimation

We obtain maximum likelihood estimates (MLE) for the parameters of the conditional mean, variance, asymmetry, and shape equations of the distribution of returns via the Berndt et al. (1974) optimization procedure of the sample log-likelihood

$$L(\mathbf{\theta})=\sum_{t=1}^{T}\log f_r\left(\mathbf{\theta}\middle|r_{t+1},\Phi_t\right)=\sum_{t=1}^{T}L_{t+1}(\mathbf{\theta}), \tag{30}$$

where $f_y(\mathbf{\theta}\,|\,r_{t+1},\Phi_t)$ is the conditional likelihood function of returns given by equation (29) and $\mathbf{\theta}$ is a column vector of parameters for the conditional mean, variance, asymmetry, and shape equations specified previously. We obtain estimates for the time-varying skewness price of risk $\delta_{t+1|t}$ via the substitution of the MLE for $k_{t+1|t}$ and $\lambda_{t+1|t}$ into equation (20). Last, we obtain robust standard errors for the MLE estimates denoted by $\tilde{\boldsymbol{\theta}}$ from the equation

$$\operatorname{var}\left(\tilde{\boldsymbol{\theta}}\right)=\left(\sum_{t=1}^{T}\frac{\partial^2 L_{t+1}\left(\tilde{\boldsymbol{\theta}}\right)}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\right)^{-1}\sum_{t=1}^{T}\frac{\partial L_{t+1}\left(\tilde{\boldsymbol{\theta}}\right)}{\partial\boldsymbol{\theta}}\frac{\partial L_{t+1}\left(\tilde{\boldsymbol{\theta}}\right)}{\partial\boldsymbol{\theta}'}\left(\sum_{t=1}^{T}\frac{\partial^2 L_{t+1}\left(\tilde{\boldsymbol{\theta}}\right)}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\right)^{-1}. \tag{31}$$

These are more appropriate in the case of missspecified sample likelihood functions (e.g., Engle and Gonzalez-Rivera, 1991; Bollerslev and Wooldridge, 1992).

Moreover, we calculate confidence intervals based on the bootstrap procedure of Rapach and Wohar (2009). Specifically, we generate a series of innovations to construct a pseudo-sample of observations making T + 100 independent draws from the SGED distribution. Using the randomly drawn innovations, we create the series according to the equations of the model (where the parameters of the model are set to their maximum-likelihood estimates). To randomize the initial observations of the pseudo-series, we drop the first 100 transient start-up, leaving us with a pseudo-sample of T observations matching the original sample. Then, we estimate the model using the pseudo-sample and repeat the process 1,000 times. We construct the 90% confidence intervals for each parameter using the percentile method in Davidson and MacKinnon (1993).

4.3. Empirical Findings

Our dataset covers the period January 2, 1990 to May 15, 2020, corresponding to 7,652 daily continuously compounded returns for the S&P 500. We compute the returns using the equation

$$r_{t+1}=100\cdot\ \ln\left(S_{t+1}/S_t\right),$$

where S_t and S_{t+1} are the values of the index on two consecutive trading days. We differentiate among three periods: the subprime crisis (June 1, 2008 to February 1, 2010), the coronavirus crisis (February 21, 2020 to the end of our sample), and the "regular" period (the rest of our sample).[1]

[1] We list several robustness tests in Appendix 2, the results of which are available on request. Importantly, the results are robust to the use of an earlier starting period for the subprime crisis (e.g., April 2008, when the first market downturn occurred) and to different definitions for the end of the subprime crisis. Moreover, reference to a "regular" period does not imply lack of turmoil in specific subperiods; such turmoil, however (as occurred, e.g., in the early and late 1990s or during the European sovereign debt crisis), was not nearly as deep as the subprime and coronavirus crises. As shown in the following, our model captures these spikes.

Panel A of Table 2 reports the estimates for the conditional variance of daily returns, along with its structural terms for the two crisis periods. The results indicate intense volatility clustering throughout the period. Volatility persistence

$$VP = a + a_N P(\varepsilon_t < 0) + \beta$$

$$= 0.0091 + 0.1641 \cdot\ 0.485 + 0.8873 = 0.976$$

is very high but stationary (given that $VP < 1$). The asymmetry coefficient is positive, supporting previous findings that negative stock market shocks have a larger impact on future volatility than do positive shocks. In general, these results are typical in studies of financial markets and in particular the S&P 500 (e.g., Sun and Yu, 2020).

Panel A of Table 3 reports the monthly means of the conditional standard deviations, $\sigma_{t+1|t}$, in the regular, subprime crisis, and coronavirus crisis periods. Notably, during the coronavirus crisis period, the mean of $\sigma_{t+1|t}$ is about 3.5 times larger relative to that of the regular period. For the subprime crisis, the mean is about two times larger. The pairwise *t*-test statistics and Figure 1 confirm these large differences, with the figure showing large spikes in the two crisis periods (the largest spike in the coronavirus crisis).

Panel B of Table 2 presents the estimates of the conditional mode equation. Negative serial correlation is present in the return series. The pure market price of risk during the regular period is 0.228 and statistically significant. The bootstrap intervals also confirm statistically significant positive deviations of the pure market price of risk during the subprime and coronavirus crisis periods. Specifically, the pure market price of risk is 0.3281 (= 0.228 + 0.1001) during the subprime crisis and 0.391 during the coronavirus crisis. Also, Panel B of Table 3 reports the means of the conditional mode of daily returns, $m_{t+1|t}$, for the subprime, coronavirus, and regular periods. The means are approximately double during the subprime period and triple during the coronavirus period. The *t*-test statistics for testing these differences are highly statistically significant, a fact illustrated in Figure 2.

Panel C of Table 2 reports the estimates for the conditional asymmetry index $h_{t+1|t}$ and parameter $\lambda_{t+1|t}$ of the distribution of daily returns. The intercept of the asymmetry index is statistically significant and negative, thus reflecting a negatively skewed distribution of returns. The bootstrap intervals for its deviations in the subprime and coronavirus periods are statistically significant and negative, implying larger negative values during the two periods and thus more-negatively skewed distribution for returns. The coefficient for past downside market shocks is also negative, indicating a negative impact of such shocks on current values of the asymmetry index and parameter. On the other hand, the coefficient for past upside market shocks is positive, indicating a positive impact on current values of the asymmetry index and parameter. Finally, the coefficient for past values of the asymmetry index is negative, indicating strong mean reversion of the asymmetry index and parameter over time.

Panel C of Table 3 reports the equivalent means of the conditional asymmetry parameter, $\lambda_{t+1|t}$, for the subprime, coronavirus, and regular periods. The mean during the regular period is –0.0907, showing that on average, there are 9.07% more returns in the sample positioned to the left of the conditional mode of the distribution of returns (negative skewness). This number increases significantly to 15.91% and 22.65% during the subprime and coronavirus crisis periods, respectively. Figure 3 confirms the significant decrease of the asymmetry parameter during the subprime and coronavirus crisis periods. The dive in the monthly average of $\lambda_{t+1|t}$ is indeed considerably deeper for the coronavirus crisis, showing more negative skewness during this period compared with the subprime crisis.

Panel D of Table 2 reports estimates for the conditional shape index $g_{t+1|t}$ and parameter $k_{t+1|t}$ of the distribution of daily returns. As expected, the intercept of the shape index is statistically significant and positive. The bootstrap intervals for its deviations in the subprime and coronavirus periods are also statistically significant and positive. The coefficient for past downside market shocks is also positive, indicating a positive impact of downside shocks on current values of the shape parameter. On the other hand, the coefficient for past upside market shocks is negative, indicating a negative impact on current values of the shape parameter. Finally, the coefficient for past values of the shape index is positive, indicating slow mean reversion of the shape index and parameter over time.

Panel D of Table 3 reports the respective means of the conditional shape parameter, $k_{t+1|t}$, for the regular, subprime, and coronavirus periods. The mean during the regular period is 1.36, showing a large deviation of the distribution of returns from normality (the shape parameter for the normal distribution is 2). The mean values of the shape parameters increase mildly to 1.43 during the subprime crisis and to 1.56 during the coronavirus crisis. Figure 4 confirms the latter increase of the shape parameter during the subprime and coronavirus periods.

Most importantly, Panels E and F of Table 3 show the behavior of the skewness price of risk and the conditional mean of daily returns given by equations (20) and (19), respectively. The conditional skewness price of risk $\delta_{t+1|t}$ is considerably more negative during the subprime crisis, decreasing by approximately 74% compared with the regular period. The equivalent decrease is even larger for the coronavirus crisis—we document a decrease of 149% compared with the regular period. In fact, the difference between the coronavirus crisis and the regular period is double that of the difference between the subprime crisis and the regular period (–0.205 versus –0.103). Figures 5 and 6 illustrate the relevant distributions (also drawing lines for the respective normal distributions) and show the shift of the distributions to the left of that of regular periods. Most notably, Figure 7 illustrates the immense decreases in $\delta_{t+1|t}$ during the crisis periods, with the decrease during the coronavirus crisis being considerably deeper compared with the equivalent decrease during the subprime crisis.

Finally, in Panels G and H of Table 3, we report the means of the risk-neutral equilibrium returns along with log of the exponential component given by equation (21). Equilibrium returns are about 5 and 10 times higher in the subprime and coronavirus crisis periods, respectively, than during the regular period. Figures 7 and 8 illustrate the time series behavior of their monthly averages. The large spikes observed during the two periods indicate excessive risk premia required by investors.

5. Conclusions

We model the coronavirus crisis in an equilibrium model of asset pricing with skewness in the price of risk. We define the conditional moment equations as functions of the subprime and coronavirus crises, and we show theoretically that skewness becomes more negative in crisis periods. We further define the risk-neutral equilibrium equation for returns as a function of crisis periods. Notably, in equilibrium, skewness enters as an additive parameter to the effect of the variance on returns (the pure price of risk). This result is important theoretically, because the model allows the effect of the pure price of risk on returns to be positive (consistent with a positive risk and return relation).

We estimate our model using S&P 500 data for January 2, 1990 to May 15, 2020. We use the skewed generalized error distribution, for which we derive the distribution of returns, the likelihood function, robust standard errors, and bootstrapped confidence intervals. Our results show extreme skewness during the subprime and coronavirus crises, whereas skewness during other turmoil periods was significantly milder. In fact, skewness during the coronavirus crisis overshadows its counterpart during the subprime crisis. Our findings open up new research pathways on understanding the determinants of skewness in crisis periods.

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Table 1: Detailed summary statistics of returns

The table reports detailed summary statistics (number of observations, mean, standard deviation, minimum, maximum, skewness, and kurtosis) for returns for the full sample, the sub-sample corresponding to the subprime crisis, and the subsample corresponding to the coronavirus crisis.

	Obs.	Mean	St. dev.	Min.	Max.	Skewness	Kurtosis
<u>Returns</u>							
Full sample	7,630	0.016	1.146	-12.771	10.953	-0.384	14.653
Subprime crisis	716	-0.014	1.938	-9.474	10.953	-0.167	8.677
Coronavirus crisis	92	-0.138	3.162	-12.771	8.962	-0.495	6.866

Table 2. Dynamics of Equilibrium Returns: Subprime and Coronavirus Crises

The table reports estimation results from our asset pricing model. Our sample covers January 2, 1990 to May 15, 2020. A * indicates statistical significance at the five percent level. The conditional variance, mode, asymmetry and shape parameters for daily returns are respectively given by equations (17), (18), (22) and (25). The coefficients with subscripts SubP and Covid represent deviations in subprime and coronavirus crisis periods. The subprime crisis covers June 1, 2008 to February 1, 2010 and the coronavirus crisis February 21, 2010 to May 15, 2020. The coefficient c measures the impact of conditional standard deviation of returns on their conditional mode and is the "pure price of risk". The conditional mean of returns $\mu_{t+1|t}$ is given by equation (19), where $\delta_{t+1|t}$ is the skewness price of risk. The total price of risk is $\xi_{t+1|t} = c + \delta_{t+1|t}$. The lower part of the table reports the arithmetic averages of the conditional Pearson's conditional coefficients of skewness and kurtosis, computed using the equations (A5) and (A6) in Appendix 1.

	Estimates	Std. Error	t-stat	Lower	Upper
A. Variance - $\sigma^2_{t+1\|t} = \mathrm{var}\left(r_{t+1}\|\Phi_t\right)$					
v_0	0.0202	0.0023	8.62*	–0.0085	0.0509
v_{SubP}	0.0240	0.0173	1.39	–0.0093	0.0499
v_{Covid}	0.0471	0.0658	0.72	0.0164	0.0759
α	0.0091	0.0066	1.37	–0.0173	0.0419
α_N	0.1641	0.0123	13.34*	0.1368	0.1970
β	0.8873	0.0079	11.58*	0.8627	0.9205
B. Mode - $m_{t+1\|t} = \mathrm{mode}\left(r_{t+1}\|\Phi_t\right)$ and Mean - $\mu_{t+1\|t} = m_{t+1\|t} + \delta_{t+1\|t}\sigma_{t+1\|t}$					
m_0	–0.0421	0.0257	–1.64	–0.0749	–0.0156
m_{SubP}	–0.0469	0.1315	–0.36	–0.0775	–0.0183
m_{Covid}	–0.0700	0.4758	–0.15	–0.1033	–0.0448
b	–0.1402	–0.0133	10.54*	–0.1709	–0.1117
c	0.2280	0.0378	6.04*	0.1947	0.2539
c_{SubP}	0.1001	0.1266	0.79	0.0668	0.1270
c_{Covid}	0.1630	0.3407	0.48	0.1334	0.1927
C. Asymmetry parameter - $\lambda_{t+1\|t} = asym\left(r_{t+1}\|\Phi_t\right)$					
γ_0	–0.2160	0.0413	–5.24*	–0.2448	–0.1850
γ_{SubP}	–0.1390	0.1247	–1.11	–0.1720	–0.1122
γ_{Covid}	–0.2560	0.4892	–0.52	–0.2863	–0.2272
γ_N	–0.1350	0.0272	–4.97*	–0.1645	–0.1052
γ_P	0.2190	0.0382	5.73*	0.1916	0.2512
γ_h	–0.1470	0.0875	–1.68*	–0.1764	–0.1173
D. Shape parameter - $k_{t+1\|t} = shape\left(r_{t+1}\|\Phi_t\right)$					
d_0	0.4570	0.1190	3.84*	0.4285	0.4877
d_{SubP}	0.0520	0.2287	0.23	0.0205	0.0798
d_{Covid}	0.1850	1.0259	0.18	0.1567	0.2159
d_N	0.3500	0.1725	2.03*	0.3203	0.3797
d_P	–0.6220	0.1206	–5.16*	–0.6531	–0.5941
d_h	0.5530	0.1005	5.50*	0.5205	0.5793
SK - Average	–0.2261	0.0028	–80.58*		
KU - Average	4.3458	0.0123	352.37*		
Log-L	–9,717.2				
Observations	7,652				

Table 3. Testing Differences in the Means of Conditional Parameters of the Distribution of Daily Returns in the Regular, Subprime, and Coronavirus Periods

A. Conditional standard deviation of returns, $\sigma_{t+1|t}$

Period	Mean	St. Dev.
Regular	0.9172	0.4110
Subprime	1.9078	1.1714
COVID-19	3.4147	1.7826
Difference in the means tests	Diff	T-Value
Subprime vs. regular	0.9906	18.10
COVID-19 vs. regular	2.4975	11.03

B. Conditional mode of returns, $m_{t+1|t}$

Period	Mean	St. Dev.
Regular	0.1637	0.1761
Subprime	0.5423	0.5298
COVID-19	1.2613	0.9221
Difference in the means tests	Diff	T-Value
Subprime vs. regular	0.3786	15.31
COVID-19 vs. regular	1.0975	9.37

C. Conditional asymmetry parameter for the distribution of returns, $\lambda_{t+1|t}$

Period	Mean	St. Dev.
Regular	–0.0907	0.0833
Subprime	–0.1591	0.0807
COVID-19	–0.2265	0.0813
Difference in the means tests	Diff	T-Value
Subprime vs. regular	–0.0685	–17.63
COVID-19 vs. regular	–0.1358	–13.10

D. Conditional shape parameter for the distribution of returns, $k_{t+1|t}$

Period	Mean	St. Dev.
Regular	1.3618	0.1753
Subprime	1.4331	0.1441
COVID-19	1.5576	0.0956
Difference in the means tests	Diff	T-Value
Subprime vs. regular	0.0713	10.16
COVID-19 vs. regular	0.1958	15.90

E. Conditional skewness price of risk, $\delta_{t+1|t}$

Period	Mean	St. Dev.
Regular	–0.1379	0.1241
Subprime	–0.2404	0.1220
COVID-19	–0.3433	0.1221
Difference in the means tests	Diff	T-Value
Subprime vs. regular	–0.1025	–17.45
COVID-19 vs. regular	–0.2054	–13.19

F. Conditional mean of daily returns, $\mu_{t+1|t}$

Period	Mean	St. Dev.
Regular	0.0300	0.0370
Subprime	0.0631	0.1005
COVID-19	0.0916	0.1303
Difference in the means tests	Diff	T-Value
Subprime vs. regular	0.0332	7.07
COVID-19 vs. regular	0.0616	3.72

G. Log of Exponential Integral, $\ln E_{z|t}$

Period	Mean	St. Dev.
Regular	–0.0139	0.1176
Subprime	0.0212	0.0376
COVID-19	0.0732	0.0722
Difference in the means tests	Diff	T-Value
Subprime vs. Regular	0.0351	15.71
COVID-19 vs. Regular	0.0871	9.39

H. Ex-dividend required returns, $\hat{K}_{t+1|t} = \mu_{t+1|t} + \ln E_{z|t}$

Period	Mean	St. Dev.
Regular	0.0160	0.1267
Subprime	0.0843	0.1328
COVID-19	0.1648	0.1924
Difference in the means tests	Diff	T-Value
Subprime vs. regular	0.0683	10.74
COVID-19 vs. Regular	0.1488	6.08

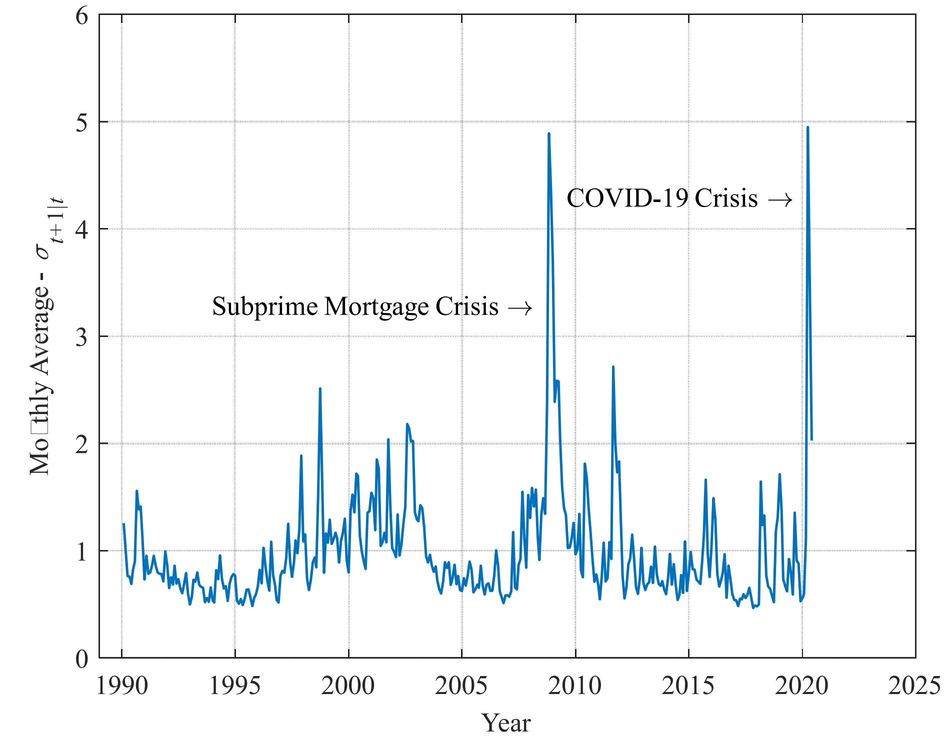

Figure 1. Conditional Standard Deviation of Daily Returns, Monthly Averages

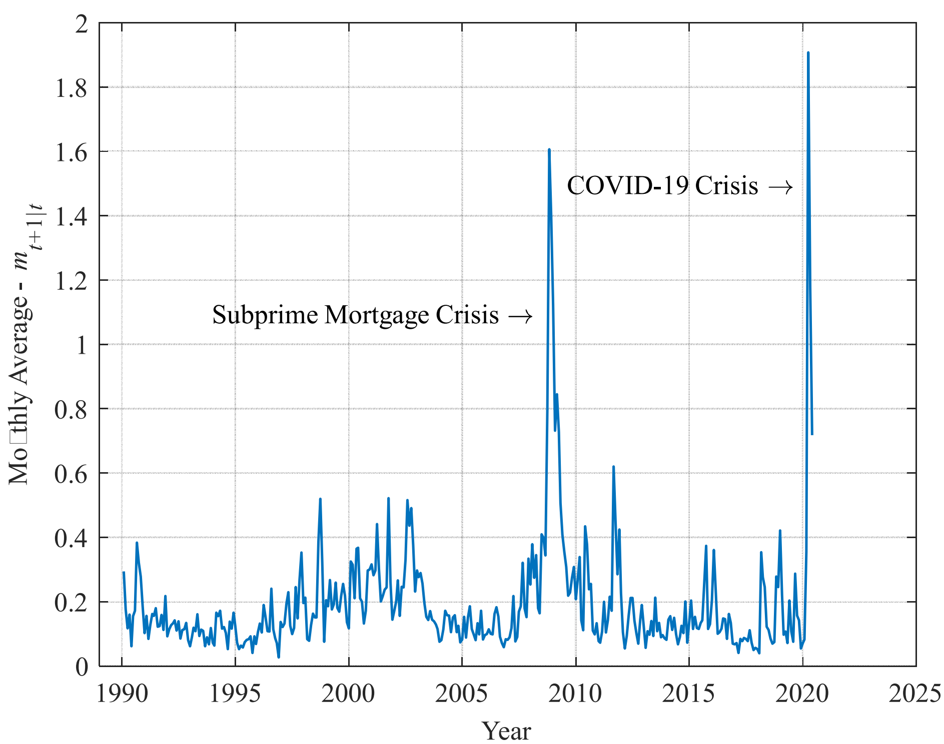

Figure 2. Conditional Mode for Daily, Monthly Averages

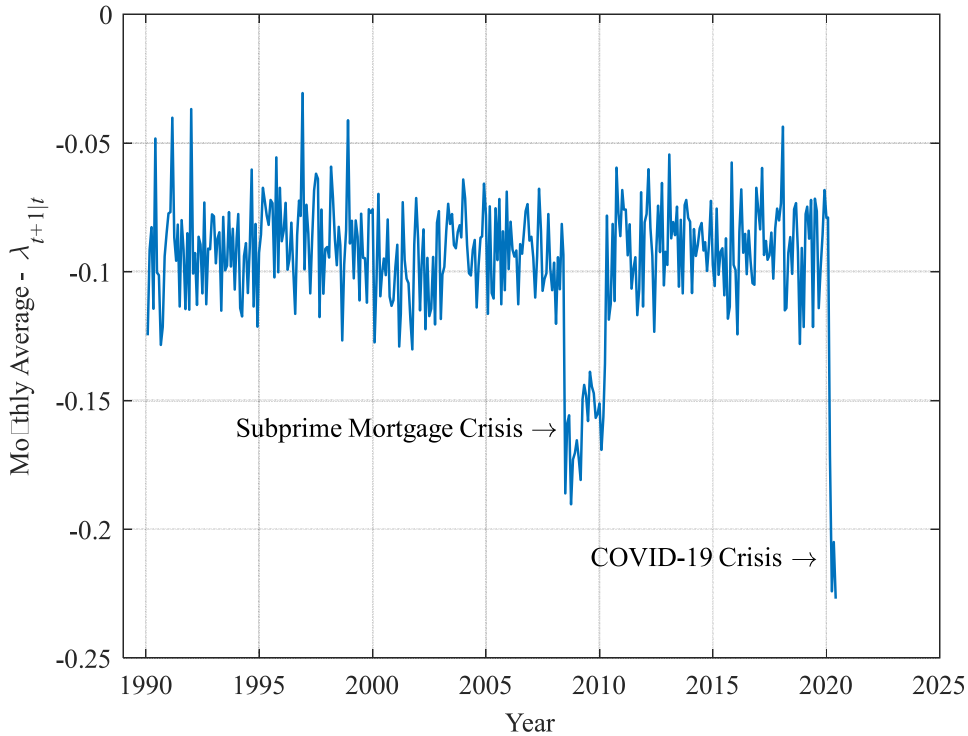

Figure 3. Conditional Asymmetry Parameter for Distribution of Daily Returns

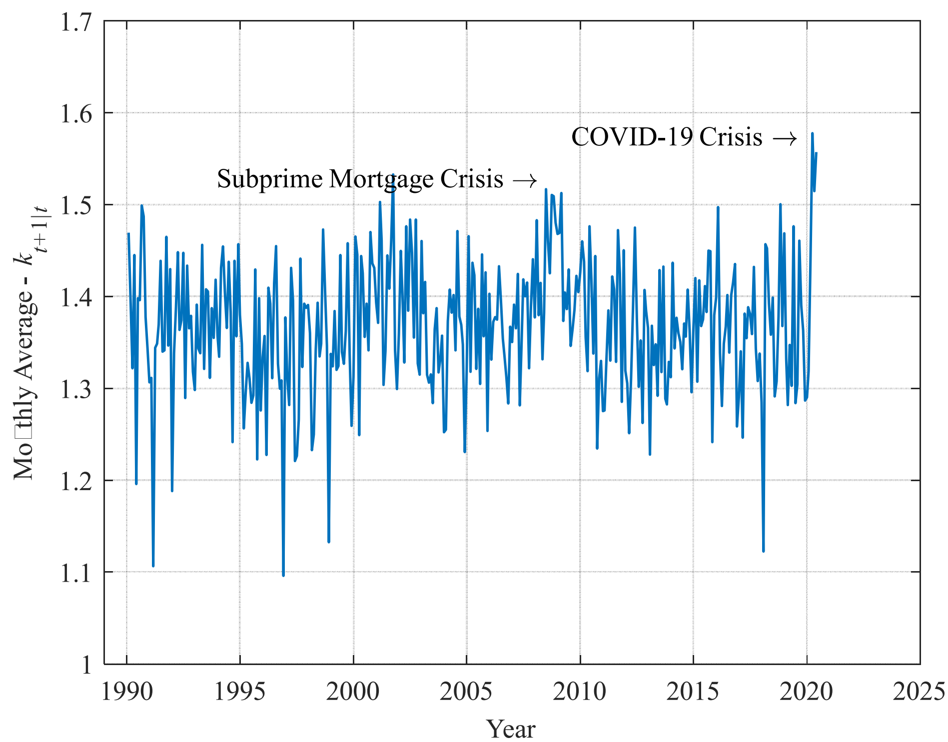

Figure 4. Conditional Shape Parameter for Distribution of Daily Returns

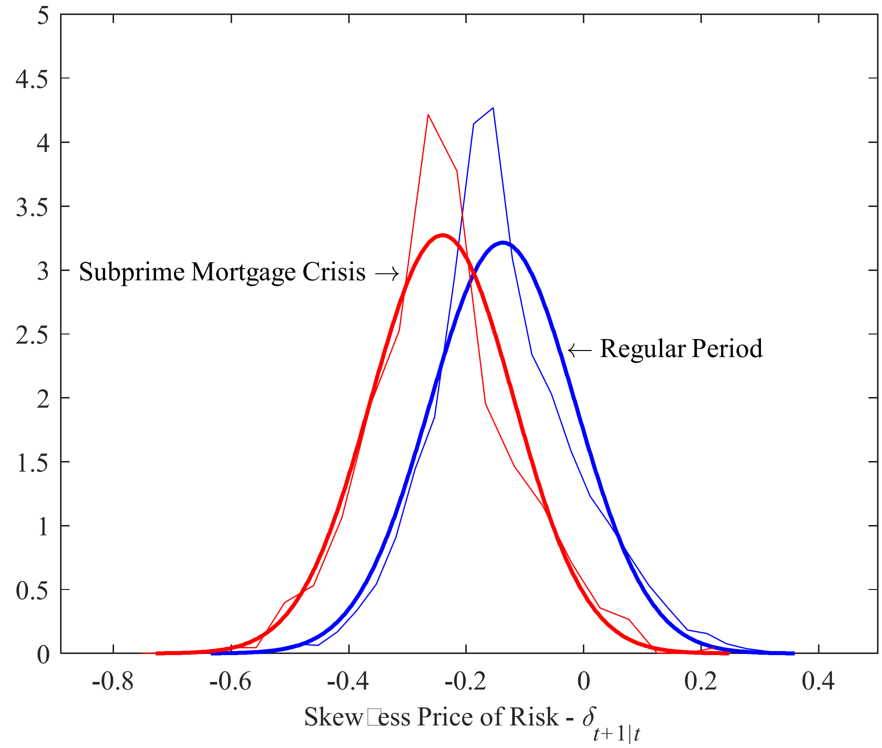

Figure 5. Distributions of Conditional Skewness – Subprime vs. Regular Period

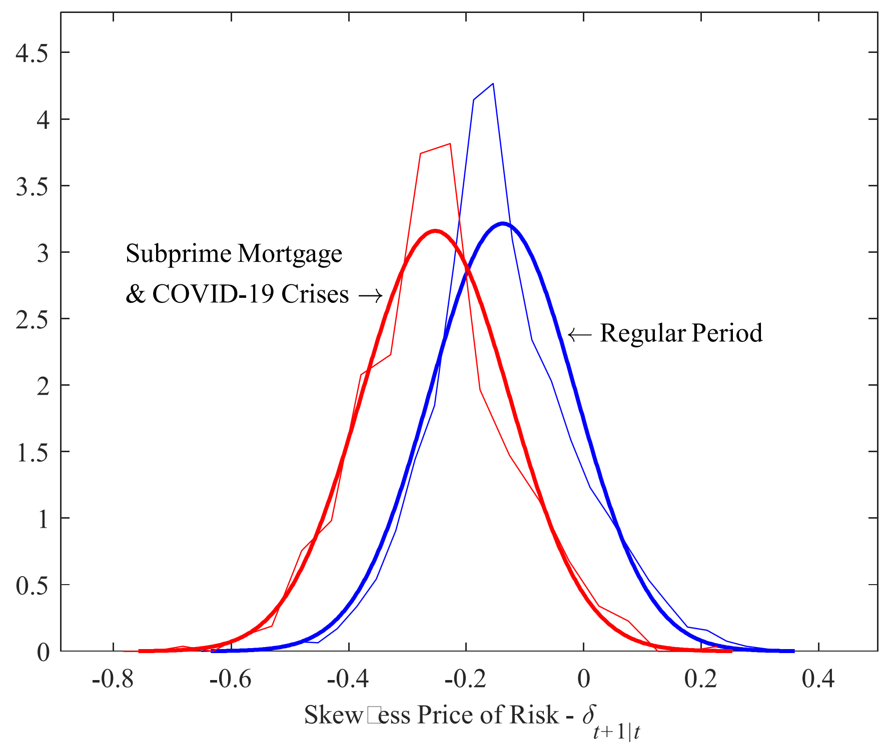

Figure 6. Distributions of Conditional Skewness - Coronavirus vs. Regular Period

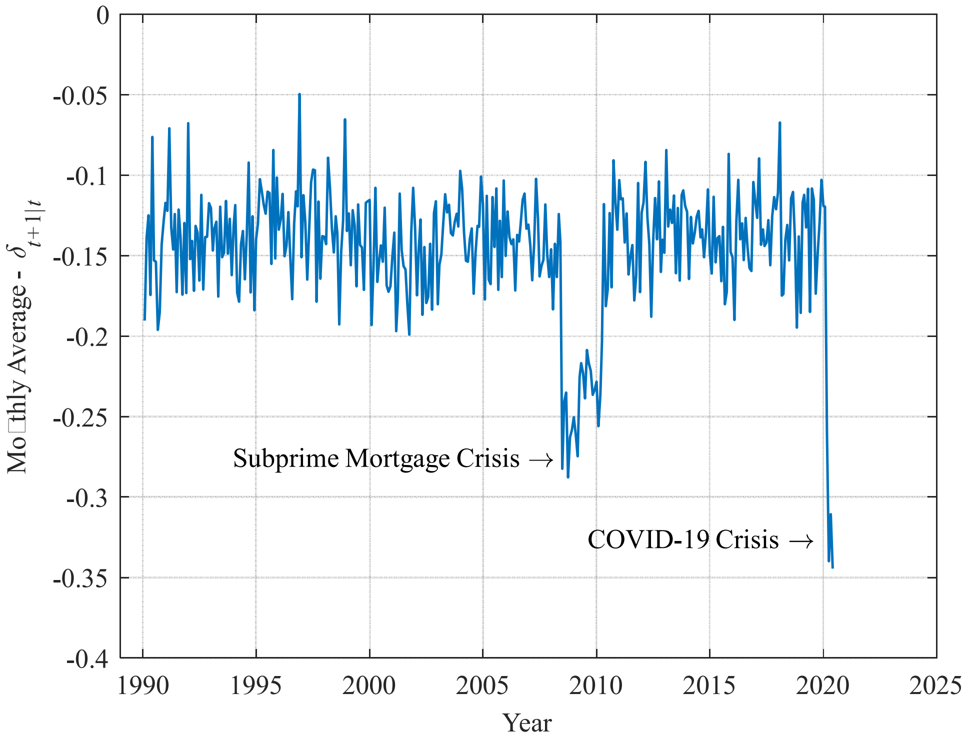

Figure 7. Conditional Skewness Price of Risk, Monthly Averages

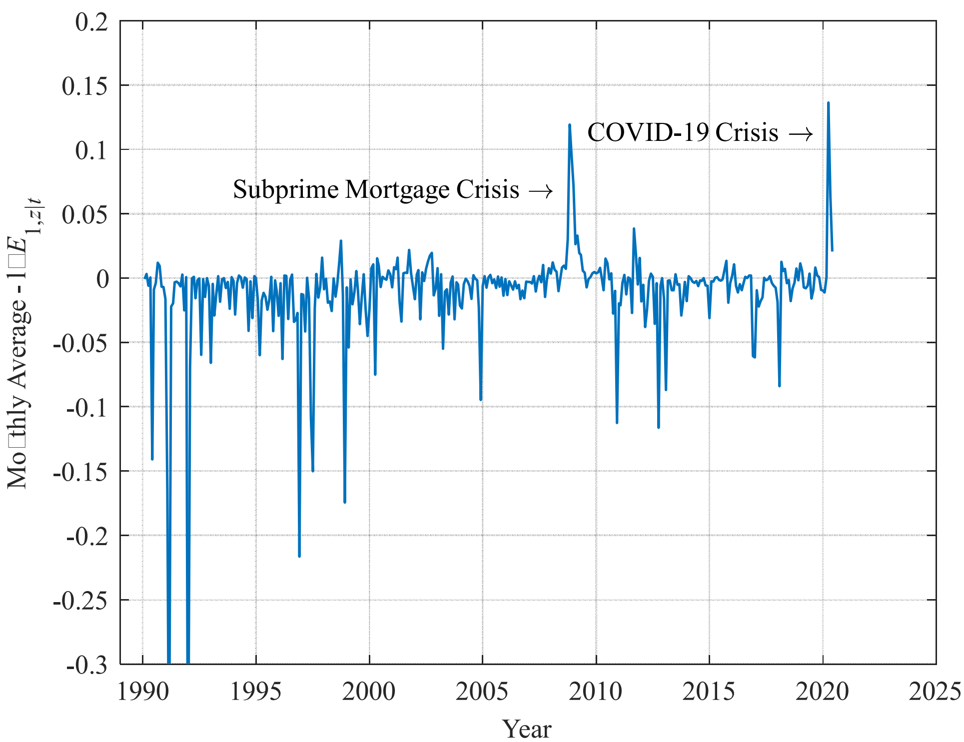

Figure 8. Log of Exponential Integral, Monthly Averages

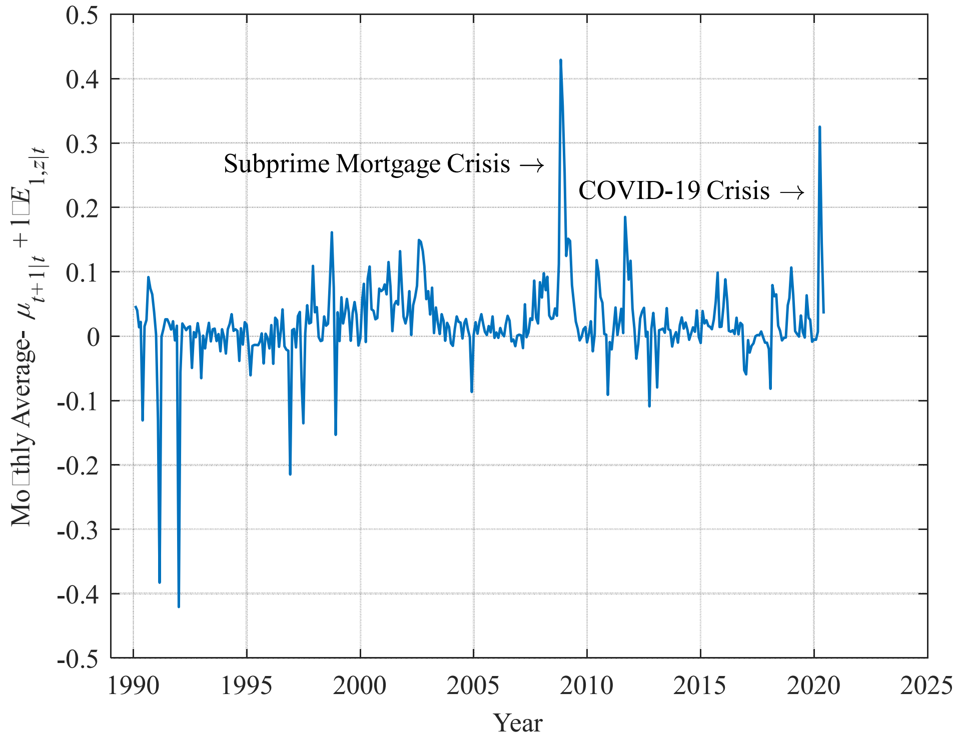

Figure 9. Ex-Dividend Required Rate of Return

Online Appendix 1. Distribution of Standardized Returns: Derivation of Parameters

To simplify the exposition and derivation of the results the time subscripts in Equations (9) and (11) are dropped from the random variables and parameters. That is,

$$z = -\delta + \left(1 + sgn(w)\lambda\right)\theta w$$

and

$$dF_z = \left(1 + sgn(w)\lambda\right) dF_w .$$

Upside and Downside Probabilities

The probability for downside markets is

$$P(r \le m) = P(z \le -\delta) = \int_{-\infty}^{\delta} dF_z$$

$$= (1-\lambda)\int_{-\infty}^{0} dF_w = \frac{1-\lambda}{2} . \qquad \text{(A1)}$$

Similarly, the probability for upside markets

$$P(r > m) = P(z > -\delta) = \int_{-\delta}^{\infty} dF_z$$

$$= (1+\lambda)\int_{0}^{\infty} dF_w = \frac{1+\lambda}{2} . \qquad \text{(A2)}$$

Parameter *δ*

By construction $Ez = 0$, thus

$$E(z+\delta) = Ez + \delta = \delta.$$

Also, because

$$z = -\delta + \left(1 + sgn(w)\lambda\right)\theta w ,$$

$$E(z+\delta) = \int_{-\infty}^{\infty} (z+\delta)\, dF_z$$

$$= \theta\left((1-\lambda)^2 \int_{-\infty}^{0} w dF_w + (1+\lambda)^2 \int_{0}^{\infty} w dF_w \right)$$

$$= \theta\left[-(1-\lambda)^2 + (1+\lambda)^2 \right] \int_{0}^{\infty} w dF_w$$

$$= 4\lambda\theta \int_{0}^{\infty} w dF_w = 2\lambda G_1 \theta ,$$

where

$$G_1 = E|w| = 2\int_0^{\infty} w\, dF_w.$$

It follows easily from the above equations that

$$\delta = 2\lambda G_1 \theta \,. \tag{A3}$$

Parameter *θ*

By construction $Ez = 0$ and $Ez^2 = 1$, thus

$$E(z+\delta)^2 = E(z^2) + 2\delta E(z) + \delta^2$$

$$= 1 + \delta^2 = 1 + 4\lambda^2 G_1^2 \theta^2 \,.$$

Also,

$$E(z+\delta)^2 = \theta^2 \int_{-\infty}^{\infty} (1 + sgn(w)\lambda)^3 \, w^2 dF_w$$

$$= \left[(1-\lambda)^3 + (1+\lambda)^3 \right] \theta^2 \int_0^{\infty} w^2 dF_w$$

$$= (1+3\lambda^2)\theta^2 2\int_0^{\infty} w^2 dF_w = (1+3\lambda^2) G_2 \theta^2 .$$

where

$$G_2 = E|w|^2 = 2\int_0^{\infty} w^2 \, dF_w.$$

It follows from above

$$1 + 4\lambda^2 G_1^2 \theta^2 = (1+3\lambda^2) G_2 \theta^2$$

or

$$1 = \left((1+3\lambda^2) G_2 - 4\lambda^2 G_1^2 \right) \theta^2 \,.$$

Thus,

$$\theta = 1 \Big/ \sqrt{(1+3\lambda^2) G_2 - 4\lambda^2 G_1^2} \,. \tag{A4}$$

Pearson's Moment Coefficient of Skewness

$$E(z+\delta)^3 = \theta^2 \int_{-\infty}^{\infty} (1 + sgn(w)\lambda)^4 \, dF_w$$

$$= \left[(-1)^3 (1-\lambda)^4 + (1+\lambda)^4 \right] \theta^3 \int_0^{\infty} w^3 dF_w$$

$$= 4\lambda (1+\lambda^2) G_3 \theta^3 ,$$

where $G_3 = 2\int_0^{\infty} w^3 dF_w$. The latter measure is computed numerically. Moreover,

$$E(z+\delta)^3 = E(z^3) + 3\delta E(z^2) + 3\delta^2 E(z) + \delta^3$$

$$= E(z^3) + 3\delta + \delta^3 .$$

The above results imply that

$$E\left(z^3\right)+3\delta+\delta^3=4\lambda\left(1+\lambda^2\right)G_3\theta^3,$$

thus the Pearson's conditional moment coefficient of skewness is

$$SK=E\left(z^3\right)=4\lambda\left(1+\lambda^2\right)\theta^3G_3-3\delta-\delta^3. \tag{A5}$$

Pearson's moment coefficient of kurtosis

$$E\left(z+\delta\right)^4=\theta^4\int_{-\infty}^{\infty}\left(1+sgn\left(w\right)\lambda\right)^5 dF_w$$

$$=\left[\,(-1)^4\left(1-\lambda\right)^5+\left(1+\lambda\right)^5\right]\theta^4\int_0^{\infty}w^4dF_w$$

$$=\left(1+10\lambda^2+5\lambda^4\right)G_4\theta^4,$$

where

$$G_4=2\int_0^{\infty}w^4dF_w.$$

Moreover,

$$E\left(z+\delta\right)^4=E\left(z^4\right)+4\delta E\left(z^3\right)+6\delta^2E\left(z^2\right)+4\delta^3E\left(z\right)+\delta^4$$

$$=E\left(z^4\right)+4\delta SK+6\delta^2+\delta^4.$$

The above imply

$$E\left(z^4\right)+4\delta SK+6\delta^3+\delta^4=\left(1+10\lambda^2+5\lambda^4\right)G_4\theta^4,$$

thus the Pearson's moment coefficient of kurtosis is

$$KU=E\left(z^4\right)=\left(1+10\lambda^2+5\lambda^4\right)G_4\theta^4-4\delta SK-6\delta^2-\delta^4. \tag{A6}$$

Generalized Error Distribution – Absolute Moments

Under the skewed generalized error distribution, the moment function for absolute values is

$$G_s\equiv E\left|w\right|^s=2\int_0^{\infty}w^s dF_w=k^{1-\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)^{-1}\int_0^{\infty}\exp\left(-\frac{1}{k}\left|w\right|^k\right)\,dw$$

$$=k^{1-\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)^{-1}\frac{1}{k}k^{\frac{s}{k}+\frac{1}{k}}\int_0^{\infty}x^{\frac{s}{k}}e^{-x}\,dx$$

$$=k^{\frac{s}{k}}\Gamma\left(\frac{s+1}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-1}. \tag{A7}$$

Online Appendix 2. Robustness Tests in Our Empirical Analysis

We conduct the following robustness tests, the results of which are very similar to our baseline and are available on request:

1. Use several different dates for the beginning and ending of the subprime and coronavirus crises.

2. Use additional crisis dummies for the early 1990s recession, the dot-com bubble, and the 9/11 attacks (and combinations of these).

3. Use additional crisis dummies for the periods of the SARS epidemic and the H1N1 pandemic (and combinations of these).

4. Use any combination of the different choices concerning points 1 to 3 above.