Interest Rate Pegging, Fluctuations, and Fiscal Policy in China

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Abstract

This paper proves in a New Keynesian model that interest rate pegging can explain the unusual business cycle fluctuations in China. It is traditional wisdom that when the nominal interest rate is inflexible, there is no unique equilibrium in macroeconomic models. We prove that a unique equilibrium exists if the nominal rate is pegged for a limited period, after which it switches to a flexible rate regime. The peg alters the propagation of external shocks, magnifies volatility of endogenous variables, and leads to instability of the economy. Besides, the model becomes more unstable when the peg duration extends, and when the pegged rate deviates from steady state. At the same time, fiscal multiplier increases under the peg, indicating fiscal policy may be more effective in mitigating economic fluctuations when monetary policy is restricted by interest rate pegging.

Keyword: New Keynesian model, Chinese economy, Interest rate peg, Fiscal policy, Rational expectation

JEL: E31, E32, E43, E62

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1. Introduction

As the world's two largest economies, China and the United States are quite different in their business cycle fluctuations. Figure 1 shows that in most years, especially before 2000, the volatility of inflation is much larger in China than in the U.S. In terms of output fluctuation, it is also significantly larger in China. From 1987 to 2007, the standard deviations of annual and (seasonally adjusted) quarterly GDP growth rates in China are 2.8 and 2.2, compared with 1.8 and 1.2 in the U.S. during the same period.

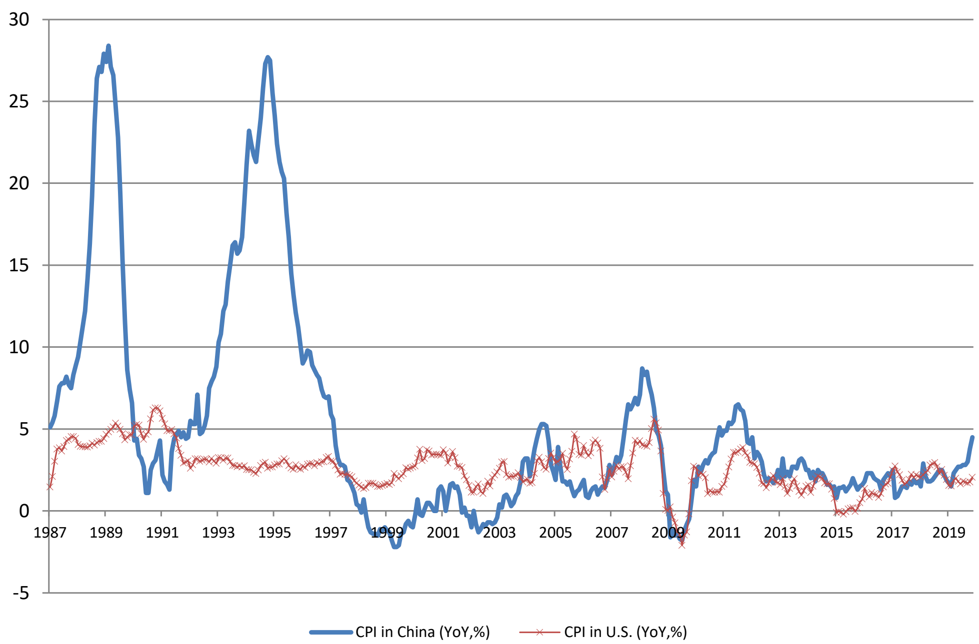


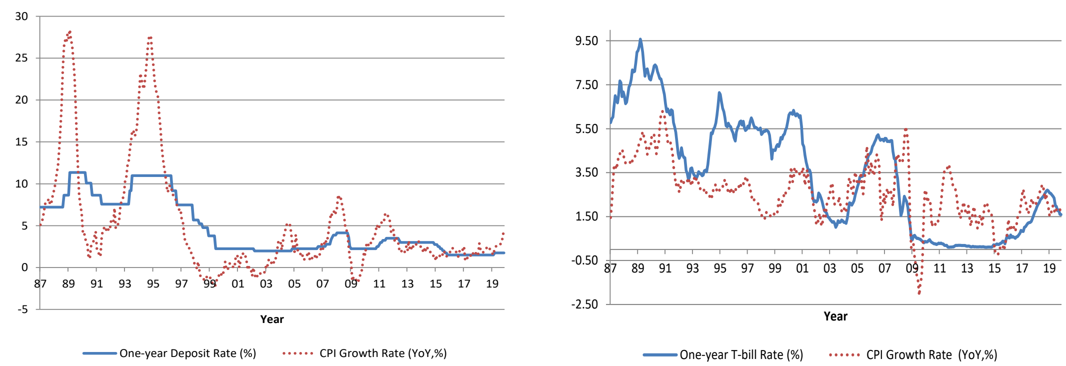
Figure 1: Inflation in China v.s. U.S.

Sources: CPI data in China come from the National Bureau of Statistics (NBS); CPI data in U.S. come from U.S. Bureau of Labor Statistics.

Notes: Since the NBS does not publish seasonally adjusted data of CPI, we use year-on-year growth rates of monthly CPI To remove seasonal factors.

Figure 2 reveals the reason for this difference: the nominal rate in the U.S.

is volatile, whereas that in China is relatively stable. Before the global financial crisis of 2007, the nominal rate in the U.S. generally co-moves with inflation, roughly conforming to the Taylor rule which guarantees the economic stability. In contrast, inflation in China changes drastically compared with the nominal rate, making the real rate move oppositely with inflation, hence the mirror-image relation between the real rate and output growth in Figure 3a. When GDP grows fast, the real rate decreases to make it grow faster; when GDP growth slows down, the real rate increases to the effect of aggravating recession and deflation. Therefore, the interest rate peg in Figure 2a brings about a positive feedback mechanism in Figure 3a which increases the volatility of the Chinese economy.

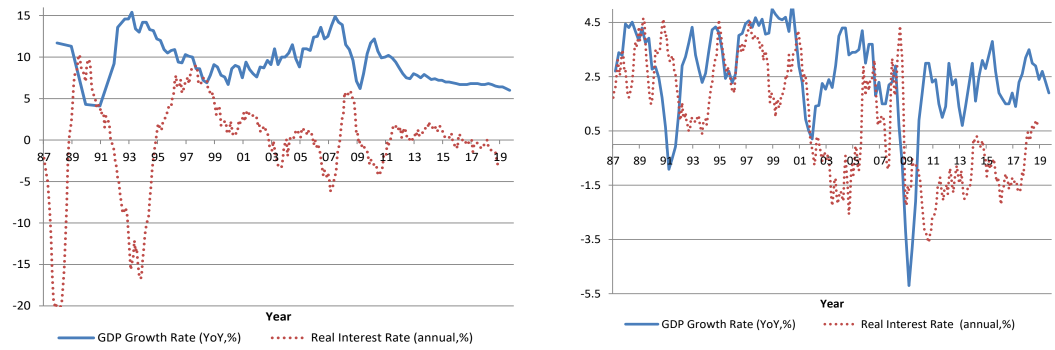


(a) China (b) U.S.

Figure 2: Nominal Rate and Inflation

Sources: One-year deposit rates in China are from the People's Bank of China (PBoC); one-year treasury bill rates in U.S. are retrieved from Federal Reserve Bank of St. Louis.

Figure 3b compares the real rate and output growth in the US. The two series generally co-move during the sample period except move oppositely between 2008 and 2015, when the nominal rate was pushed to the zero lower bound (ZLB) by the depressionary demand shock. The correlation between YoY output growth and the real interest rate in the U.S. changes from 0.1 $(1987-2008)$ to -0.56 $(2009-2015)$, roughly comparable to 0.54 $(1993-2015)$ in China before the interest rate liberalization. Correspondingly, the standard deviation of the YoY growth rate in quarterly GDP increases from 1.55 (1987 – 2008) to 1.94 (2009 – 2015), which reflects the magnifying effect of the interest rate peg.



(a) China

(b) U.S.

Figure 3: Real Rate and Output Growth

Sources: GDP data in China come from the NBS; GDP data in U.S. come from U.S. Bureau of Economic Analysis.

Notes: The (realized) real interest rate is constructed as the one-year nominal rate minus the YoY growth rate of CPI during the same period.

Both in China and the U.S., interest rate pegging leads to the negative relation between inflation and the real rate, which is the main cause of macroeconomic instability. In the U.S., when the nominal rate was pegged at the zero level from 2009 to 2015, conventional monetary policy lost its effects; the government relied on quantitative ease and fiscal expansion to stimulate the economy and stop deflation. In China, bank retail interest rates have been controlled by the government since the era of the planned economy; the Chinese government implements a variety of policy tools, such as direct control over loans and corporate investment, regulation over land use, environmental protection, industrial policies, but takes the benchmark deposit and lending rates as a last resort in its tool kit.

Recent years have seen an acceleration in financial reform: The interest rate liberalization in 2015 removed the ceilings and floors of retail interest rates.[1] Meanwhile, direct finance grows rapidly and a partially-regulated shadow banking system emerges. However, the banking sector still dominates the financial system, and commercial banks are restricted in adjusting retail rates. The PBoC remains a deparment of the government and does not enjoy the same independence as its western counterpart. One-year benchmark rates on deposits and loans, referred to as the "ballast stone" of China's interest rate system, stay unchanged from October 2015 until March 2019. The financial market is characterized as a dual-track system, composed of saving deposits and loans whose prices are administratively controlled, and the markets of currency, bond, and stock where asset prices have been liberalized. This is why the dual-track interest rate reform was launched in 2018, a new round of market-oriented reform which aims to integrate the two tracks of interest rates: regulated and market-determined interest rates.

The previous analysis establishes the following stylized facts of the Chinese business cycle: (1) Inflation and output fluctuate violently, especially before the year 2000; (2) The real interest rate moves in the opposite direction of inflation and output. We take the peg of the nominal interest rate as the primary explanation of these facts. But it is a challenge to study interest rate pegging in macroeconomic models, in which the interest rate is determined by market supply and demand. Traditional models (such as IS-LM) show that when the nominal rate is fixed exogenously, the economy enters a divergent process and no equilibrium exists. While in modern rational expectations models, multiple equilibria exist when the nominal rate is constant.

Our paper instead proves that when the duration of the interest rate peg is limited, a unique equilibrium exists in a rational expectation model. We then examine the properties of the model and prove that the peg magnifies shock

[1]See Liu et al. (2020) for the analysis on the effects of interest rate liberalization in China.

propagation and leads to economic instability. We also show that fiscal policy can mitigate the excessive fluctuation caused by interest rate pegging. This paper is organized as follows. Section 2 reviews the literature. Section 3 describes the model, and section 4 proves the existence and uniqueness of the equilibrium. In section 5, we carry out dynamic analyses and discuss model properties. In section 6, we calculate government expenditure multipliers and investigate the stabilizing effects of fiscal policy. Section 7 concludes.

2. Literature Review

It has been known since Sargent & Wallace (1975) that exogenous interest rate rules, including the interest rate peg as a special case, lead to indeterminacy of equilibrium price level. Subsequent literature on rational expectations models accordingly adopts a money growth rule, in which the nominal interest rate is endogenously determined. Examples include Lucas (1983), Chari et al. (2000), Christiano et al. (2003), Christiano et al. (2005), etc.

On the other hand, McCallum (1981) argues that prices can be determinate for interest rate rules that involve feedback from model variables to the nominal rate. Consequently, the New Keynesian literature emerging in the 1990s usually assumes an interest rate feedback rule proposed by Taylor (1993): The central bank adjusts the nominal rate based on the change of inflation and output. When the nominal rate moves more than one-for-one to inflation (the Taylor Principle), a unique equilibrium is guaranteed in the model.

There is also a debate over the monetary policy rule in China. On one hand, Xie & Luo (2002) asserts that China's monetary policy can be identified as an interest rate rule. Zhang (2009), Li & Liu (2017) prove that the interest rate rule is more effective in controlling inflation in China based on simulations in DSGE models. On the other hand, Taylor (2000) proposes money supply as a more reasonable instrument in policy rules for emerging countries. Chen et al. (2016) estimate a money growth rule for the Chinese economy that can be integrated into DSGE and SVAR models. However, both money growth and the interest rate rule imply a flexible interest rate. Although interest rate flexibility improves in China during recent years, the interest rate peg before the interest rate reform and its resulting economic instability should be separately modelled. Specifically, since the data in the estimation of Chinese DSGE models are often traced to the 1990s, ignoring interest rate pegging necessarily leads to model mis-specification and hence systematic errors in parameters.

The contribution of this paper is that we embed the interest rate peg of China in a dynamic stochastic general equilibrium (DSGE) model. To avoid the pathology of equilibrium, we follow a strategy of the recent ZLB literature, such as Krugman (1998), Eggertsson & Woodford (2003), Carlstrom et al. (2014), and Del Negro et al. (2015). The nominal interest rate is exogenously pegged, but only for a limited period, after which it switches to a flexible rate regime. This assumption guarantees the uniqueness of equilibrium in a rational expectations model. We differ from the ZLB literature in that the pegged value of the nominal rate is not necessarily zero, but can be any nonnegative value; and the nominal rate is not pushed by a depressionary shock, but subject to monetary news shocks, which offset the impact of other shocks on the nominal rate to maintain it at the fixed value. This approach is similar to Blake (2012), Laseen & Svensson (2011), and Gali (2009, 2011). But these authors focus on conditional forecasts of DSGE models, whereas we mainly concern how the peg changes the property and the propagation channel of the model.

As a substitution for traditional monetary policy, fiscal policy has been widely discussed in the ZLB literature. A partial list includes Eggertsson (2006, 2010), Cogan et al. (2010), Christiano et al. (2011), Woodford (2011), Drautzburg &

Uhlig (2015), Dupor & Li (2015). Our paper complements their results, but with special application to the Chinese economy.

3. The Model

3.1. Households

The basic structure of the model is similar to Clarida et al. (1999) and Woodford (2003). The utility function of the representative household is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - e^{\tau_t} \frac{{N_t}^{1+\gamma}}{1+\gamma} + \frac{{G_t}^{1-\chi}}{1-\chi} \right] \tag{1}$$

where E_0 is the conditional expectation operator, β is the subjective discount rate, C_t, N_t, and G_t are household consumption, labor supply, and unproductive government spending. Parameters γ, χ are respectively the elasticity coefficients related to labor supply and government consumption, and τ_t represents a shock on labor supply, satisfying $\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_t^\tau$, with $0 < \rho_\tau < 1$ and $\varepsilon_t^\tau \sim N(0, \sigma_\tau^2)$.

The intertemporal budget constraint is given by

$$P_t C_t + B_t \leq B_{t-1} R_{t-1} + W_t N_t + T_t \tag{2}$$

Here, P_t is price, W_t denotes nominal wage, B_t is the quantity of bonds purchased by households in period t and maturing in the next period, R_t is the one-period nominal rate of interest that pays off in period t, and T_t denotes a lump sum tax (transfer) from the government.

The representative household maximizes its utility function (1) subject to the budget constraint (2) and the non-Ponzi condition

$$E_0 \lim_{t \to \infty} \frac{B_{t+1}}{(1+R_0)(1+R_1)\cdots(1+R_t)} \geq 0$$

The equilibrium conditions associated with households are derived in Appendix A.

3.2. Firms

The final product (Y_t) is aggregated by a continuum of intermediary goods ($Y_t(i)$):

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where $i \in (0, 1)$, and $\varepsilon > 1$ is the substitution elasticity between intermediary goods. The final good producer maximizes its profit subject to equation (3):

$$\max P \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) di \tag{4}$$

which yields:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t \tag{5}$$

Equation (5) is the demand curve faced by intermediary goods producers. Substituting it into the zero profit condition of final good producers, we obtain the relationship between the prices of the final product and intermediary goods:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{6}$$

Differentiated intermediary goods are produced using the following technology:

$$Y_t(i) = e^{a_t} N_t(i) \tag{7}$$

where a_t is a technology shock satisfying $\Delta a_t = \rho_a \Delta a_{t-1} + \varepsilon_t^a$ with $0 < \rho_\tau < 1$ and $\varepsilon_t^a \sim N(0, \sigma_a^2)$. The marginal cost of the monopolistic producer is

$$s_t = (1 - v) \frac{W_t}{e^{a_t} P_t} \tag{8}$$

Here, $v = 1/\varepsilon$ denotes the employment subsidy financed by lump sum taxes, to correct the markup distortion at steady state.

We adopt a variant of Calvo sticky prices. In each period t, a fraction of intermediate goods producers, $1-\theta$, can reoptimize their prices. The *ith* firm that reoptimizes maxmizes the present discounted value of its future profits:

$$E_t \sum_{j=0}^{\infty} \mu_{t+j} \beta^{t+j} \left[P_{t+j}(i) Y_{t+j}(i) - (1-v) W_{t+k} N_{t+k}(i) \right] \tag{9}$$

where μ_{t+j} is the multiplier on firm profits in the household's budget constraint.

3.3. Aggregate Constraints

The resource constraint is

$$Y_t = C_t + G_t \tag{10}$$

Government consumption evolves according to

$$G_t = G_{t-1}^{\rho_g} e^{\varepsilon_t^g} \tag{11}$$

Here, G is the steady state level of government consumption, $0 < \rho_g < 1$, and $\varepsilon_t^g \sim N(0, \sigma_g^2)$. Government consumption is financed by lump sum taxes, so that Ricardian equivalence holds and the details of tax policy are irrelevant. After taking logarithm, equation (11) transforms to $g_t = \rho_g g_{t-1} + \varepsilon_t^g$.

Monetary policy follows the rule:

$$R_t = \begin{cases} Z_t, & t > k \\ \bar{d}, & t = 1, ..., k \end{cases} \tag{12}$$

Here, $Z_t = R\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} e^{\varepsilon_t^r}$, with $\phi_\pi > 1$, $0 < \phi_\pi < 1$, and $\varepsilon_t^r \sim N(0, \sigma_r^2)$. The variables R,Y,π are respectively the steady state values of the nominal interest rate, output, and inflation. Parameter k represents the duration of the interest rate peg, which measures the flexibility of monetary policy. In periods $t = 1, \ldots, k$, the government sets the interest rate at its level of $t = 0(\bar{d})$. When $t > k$, monetary policy switches to a flexible interest rate (Z_t), which is determined by the Taylor rule. This assumption—the peg lasts only a limited period, and then turns to a flexible interest rate—guarantees the uniqueness of the equilibrium.

4. Existence and Uniqueness of the Equilibrium

After log-linearization, the equilibrium conditions of the above model become (see Appendix A for details)

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) \tag{13}$$

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left\{[1+\gamma(1-S_g)]\,\widehat{c}_t + \widehat{\tau}_t - (1+\gamma)\widehat{a}_t + \gamma S_g\widehat{g}_t\right\} \tag{14}$$

$$\widehat{i}_t = \begin{cases} 0, & t = 1, ..., k \\ \phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t + \varepsilon_t^r, & t > k \end{cases} \tag{15}$$

where S_g denotes the fraction of government consumption in output at steady state. Equation (13) is the dynamic IS curve, equation (14) is the New Keynesian Philips curve. Monetary policy is represented by the change of the nominal rate in equation (15), which affects the real rate in equation (13) and thus determines the properties of the model. For example, the increase of inflation raises the real rate if the nominal rate follows the Taylor rule, while decreases the real rate if the nominal rate is pegged.

Substituting out for $\widehat{i}_t$ in equation (13), we obtain a system of difference equations of $\widehat{\pi}_t$ and $\widehat{c}_t$. Given that both variables are non-predetermined, the solution to the system is locally unique, if and only if, both eigenvalues of the coefficient matrix fall in the unit circle (Blanchard & Kahn, 1980).

4.1. Multiple Equilibria Under the Peg of Infinite Horizon

From the monetary policy rule (15), we have $\widehat{i}_t = 0$ under the interest rate peg. Substituting it into (13), we get

$$\widehat{c}_t = E_t\widehat{\pi}_{t+1} + E_t\widehat{c}_{t+1} \tag{16}$$

(14) can be simplified to

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \eta\widehat{c}_t + \widehat{X}_t \tag{17}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\eta = \lambda\left[1+\gamma(1-S_g)\right]$, and $\widehat{X}_t = \lambda\left(\widehat{\tau}_t - (1+\gamma)\widehat{a}_t + \gamma S_g \widehat{g}_t\right)$. (16) and (17) form the following system of difference equations:

$$\begin{bmatrix} \widehat{c}_t \\ \widehat{\pi}_t \end{bmatrix} = A_1 \begin{bmatrix} E_t\left(\widehat{c}_{t+1}\right) \\ E_t\left(\widehat{\pi}_{t+1}\right) \end{bmatrix} + \begin{bmatrix} 0 \\ \widehat{X}_t \end{bmatrix} \tag{18}$$

where $A_1 = \begin{bmatrix} 1 & 1 \\ \eta & \eta+\beta \end{bmatrix}$.

The above system corresponds to the case of the infinite period peg. Since $\widehat{c}_t$ and $\widehat{\pi}_t$ are both non-predetermined variables, the sufficient and necessary condition for the existence and uniqueness of (18) is the number of eigenvalues of A_1 within the unit circle equals 2 (the number of non-predetermined variables). On the contrary, if the number of eigenvalues in the unit circle is less than that of non-predetermined variables, multiple solutions exist.

Proposition 1: Let λ_1, λ_2 be eigenvalues of matrix A_1. The condition $-1 < \lambda_1, \lambda_2 < 1$ is not satisfied.

Proof. See Appendix B.

So multiple solutions exist in the system (18).

4.2. Unique Equilibrium Under the Peg of Finite Periods

A peg of finite periods implies that the nominal rate exits the peg after period k, and then turns to a flexible rate from period $t = k+1$. For simplicity, assume the response coefficient $\phi_y = 0$.

First, we prove that a unique equilibrium exists in the case of a flexible interest rate. Substituting $\widehat{i}_t = \phi_\pi \widehat{\pi}_t + \varepsilon_t^r$ into (13) and (14), after rearranging terms,

$$\begin{bmatrix} \widehat{c}_t \\ \widehat{\pi}_t \end{bmatrix} = A_2 \begin{bmatrix} E_t\left(\widehat{c}_{t+1}\right) \\ E_t\left(\widehat{\pi}_{t+1}\right) \end{bmatrix} - \frac{1}{1+\phi_\pi \eta} \begin{bmatrix} \varepsilon_t^r + \phi_\pi \widehat{X}_t \\ \eta \varepsilon_t^r - \widehat{X}_t \end{bmatrix} \tag{19}$$

where $A_2 = \frac{1}{1+\phi_\pi \eta} \begin{bmatrix} 1 & 1-\phi_\pi\beta \\ \eta & \eta+\beta \end{bmatrix}$. The solution to the difference equations is locally unique if and only if the eigenvalues of the coefficient matrix are both in the unit circle.

Proposition 2: Let λ_1, λ_2 be the eigenvalues of matrix A_2. Then λ_1 and λ_2 are real numbers and satisfy $-1 < \lambda_1, \lambda_2 < 1$.

Proof. See Appendix B.

Thus we proved that unique solution exists in the flexible rate system of (19). Equation (19) can be simplified as $\widehat{Z}_t = A_2 E_t \widehat{Z}_{t+1} + \widehat{V}_t$, where $\widehat{Z}_t = \begin{bmatrix} \widehat{c}_t \\ \widehat{\pi}_t \end{bmatrix}$, $\widehat{V}_t = -\frac{1}{1+\phi_\pi \eta} \begin{bmatrix} \varepsilon_t^r + \phi_\pi \widehat{X}_t \\ \eta \varepsilon_t^r - \widehat{X}_t \end{bmatrix}$. Using the method of undetermined coefficients, it is easy to find the solution to the flexible rate model after period k: $\widehat{Z}_t = a\widehat{Z}_{t-1} + b\widehat{V}_t$. In this way, we obtain the unique solution to the model when $t > k + 1$.

Second, we derive the solution for periods $t < k + 1$ by backward deduction. We take the solution of $t = k + 1$ as the terminal condition and solve the model backward. For example, in period $t = k$, the equilibrium condition (18) can be simplified as $\widehat{Z}_k = A_1 E_k \widehat{Z}_{k+1} + \widehat{U}_k$. Substituting the obtained solution $\widehat{Z}_{k+1} = a\widehat{Z}_k + b\widehat{V}_{k+1}$ into it, we have $\widehat{Z}_k = A_1 E_k \left(a\widehat{Z}_k + b\widehat{V}_{k+1} \right) + \widehat{U}_k = A_1 a \widehat{Z}_k + A_1 b E_k \widehat{V}_{k+1} + \widehat{U}_k$, that is, $\widehat{Z}_k = [I - A_1 a]^{-1} \left(A_1 b E_k \widehat{V}_{k+1} + \widehat{U}_k \right)$.

Likewise, in period $t = k - 1$, the evolution equation under the interest rate peg is $\widehat{Z}_{k-1} = A_1 E_{k-1} \widehat{Z}_k + \widehat{U}_{k-1}$. Substituting the obtained solution of $\widehat{Z}_k$ into it, we get the solution of $\widehat{Z}_{k-1}$. In the same manner, we can obtain the solutions of $t = 1, 2, \ldots, k - 2$.

Through the above steps, we obtain the unique equilibrium in the case of the finite period peg.

5. Dynamic Analysis

This section investigates the model property by simulation.[2] Parameter values are those commonly used in the literature. The parameter measuring price stickiness (θ) is 0.75, household discount rate (β) is 0.99, and the curvature of the disutility of labor (γ) is 1. The proportion of government consumption to output in steady state (S_g) is 0.1. The response coefficients of the nominal rate to inflation (ϕ_π) and output (ϕ_y) are 1.5 and 0. The autoregressive coefficient of external shocks is 0.5; the standard deviation of the preference (ε_t^τ) and technolog shock (ε_t^a) is 0.01; the standard deviation of fiscal policy (ε_t^g) and monetary policy shock (ε_t^r) are 0.1 and 0.0025. Thus in our quarterly model, a one standard deviation increase in the nominal interest rate amounts to a 1% increase in the annualized rate.

5.1. Impulse Response Analysis

We first discuss the impulse response functions of model variables, to show how the propagation of external shocks is affected by interest rate pegging. Assume the economy is at steady state in period 0. In period $t = 1$, the economy is subject to a one standard deviation shock. The nominal interest rate is pegged at steady state during periods 1 to k and turns to a flexible rate from period $k + 1$.

5.1.1. Responses to a Preference Shock

The shock ε_t^τ shifts marginal disutility of labor, i.e., the (un)willingness of the household providing labor supply. Figure 4 compares the reponses under a flexible rate, a 4-period peg, and an 8-period peg.

A positive preference shock reduces the willingness to supply labor, so that natural output (the output level of flexible prices) declines. The optimality condi-

[2] We thank Larry Christiano for providing the matlab codes of simulations.

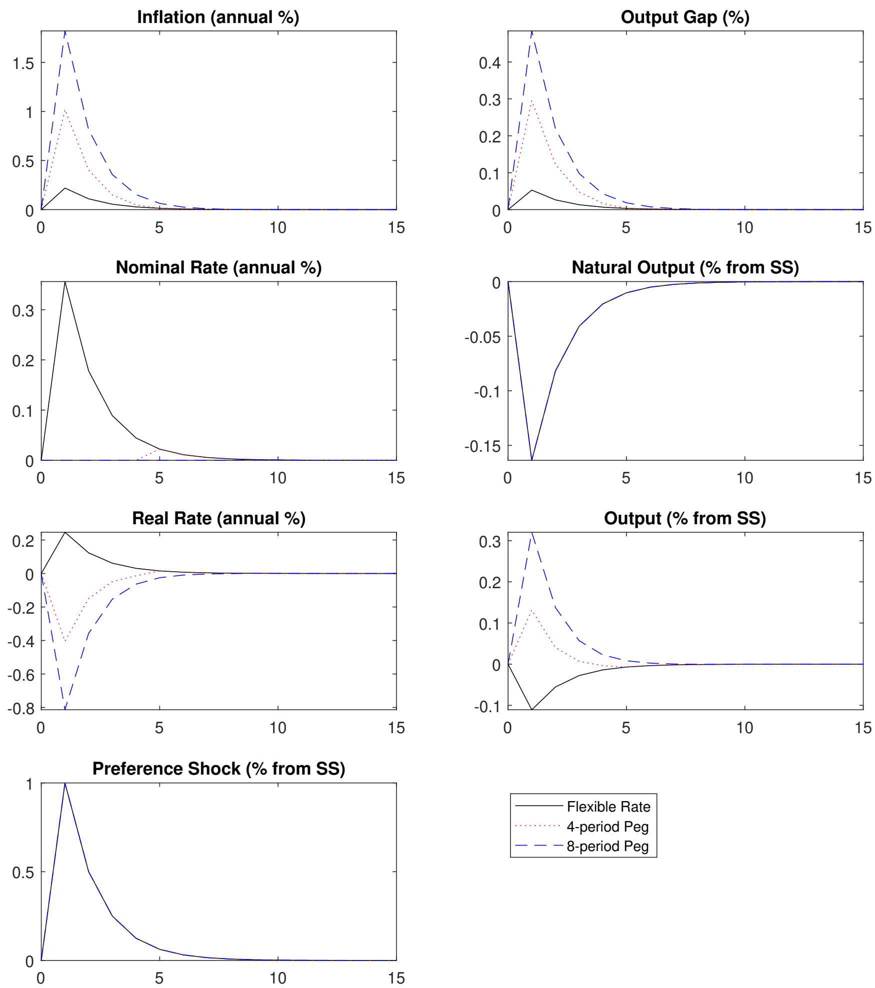


Figure 4: Responses to a Labor Preference Shock

Notes: This figure plots the impulse responses to a one S.D. increase in labor preference for different durations of an interest rate peg.

tion of the household requires that the marginal effects of labor and consumption change simultaneously, which makes consumption decline. Since natural output declines more than consumption demand, therefore inflation rises. And the output gap (defined as the difference between output and the natural level of output) also rises because it moves together with inflation.

The impact of interest rate pegging is reflected in the dynamic IS curve of equation (13). Under a flexible rate, the Taylor rule means that the nominal rate increases more than inflation, so that the real rate increases and thus restrains the rise of inflation and the output gap. Because output is determined by natural output and the output gap together, and natural output declines more than the output gap rises, therefore the output level decreases. On the contrary, under a pegged rate, the nominal rate remains unchanged, so the real rate increases and thus leads to higher inflation and a wider output gap. The increase of the output gap exceeds the decline in natural output, allowing output to increase rather than to decrease.

Under a flexible interest rate, the real rate moves in the same direction of inflation, producing a negative feedback mechanism to ensure economic stability. In contrast, under an interest rate peg, the real rate moves in the opposite direction of inflation, adding positive feedback to the transmission mechanism: The increase of inflation and expected inflation leads to a decrease in the real rate, which further increases inflation and expected inflation, thus decreasing the real rate even more ... The economy falls into a vicious circle and will eventually collapse. This is the "accumulation process" analyzed by Wicksell and Friedman. But if the interest rate peg lasts only a limited period, the expectation of switching to the flexible rate will constrain this divergent process. When the peg ends, higher inflation will lead to a higher nominal rate, which reduces inflation and output more. The representative agent with rational expectations will weigh these two factors when making decisions, and thus determine the optimal path of inflation and output. And the same logic applies in the case of deflation.

The impulse response analysis shows that model variables are more sensitive to external shocks under an interest rate peg, which enlarges the fluctuation of inflation and the output gap. In general, the interest rate peg magnifies model volatility and leads to instability of the economy. And the longer is the duration of the peg, the more volatile is the economy.

5.1.2. Responses to a Technology Shock

The effects of a temporary technology shock are similar to those of the labor preference shock, but in an opposite direction. So we focus on a permanent technology shock below. The change of technology (Δa_t) follows a first-order autoregressive process, meaning its level (a_t) is a unit root process. Thus a technology shock (ε_t^a) brings about a permanent change of a_t. Figure 5 compares the responses to a one standard deviation technology shock under a flexible rate, a 4-period pegged rate, and an 8-period peg.

A permanent increase in a_t leads to a permanent increase in output, so the household increases its consumption due to a positive wealth effect. Since natural output (production capacity) increases gradually with the technology level, it cannot catch up with the increase of current consumption. Consequently, current output exceeds natural output, leading to a positive output gap and inflation.

Under a flexible rate, the nominal rate increases more than inflation, thus restraining the rise of the output gap. Since output is determined by the output gap and natural output together, it gradually converges to the new steady state with the increase of technology and natural output, after a one-off jump caused by the output gap. In the case of the nominal rate peg, the real rate decreases to magnify the jump of the output gap, causing output overshoots its final steady state. In this case, the change of output is dominated by the output gap. Both fall

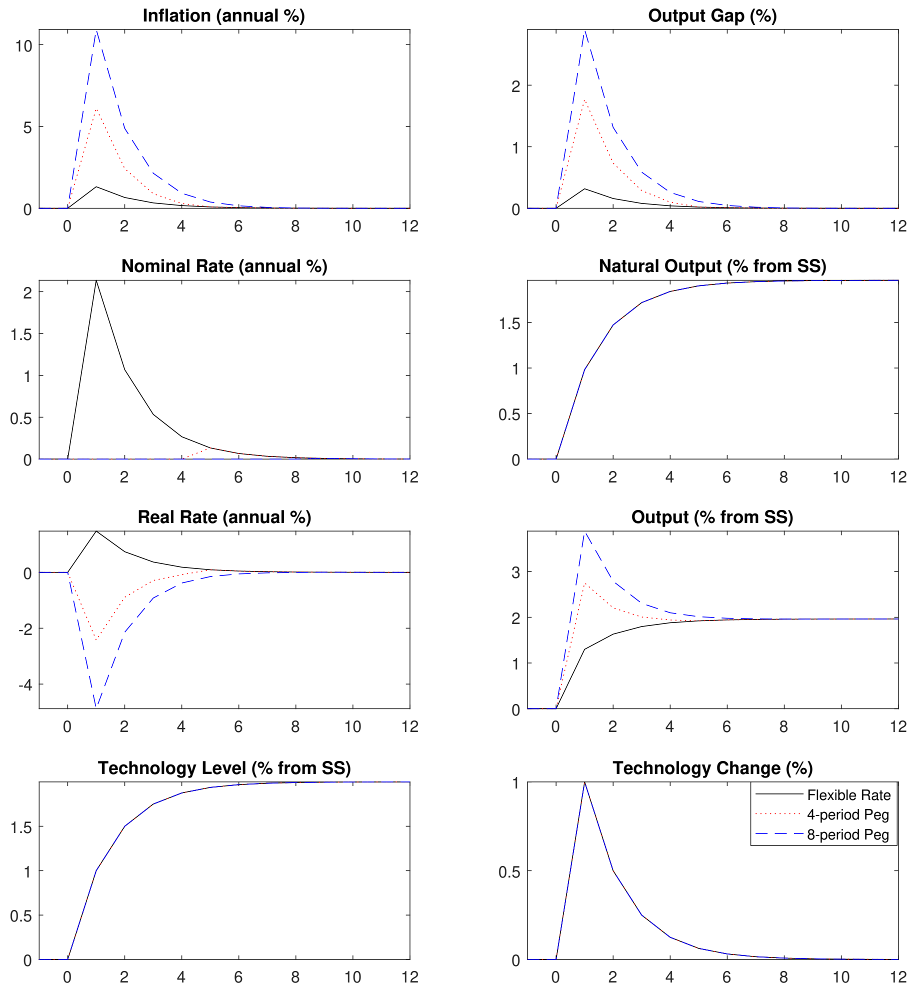


Figure 5: Responses to a Permanent Technology Shock

Notes: This figure plots the impulse responses to a one S.D. increase in technology growth rate for different durations of an interest rate peg.

gradually to steady state with the return of the real rate, though natural output rises with the technology level.

The responses to a permanent shock confirm the previous conclusion drawn from a temporary shock. The positive feedback from the interest rate peg magnifies the change of inflation and the output gap, thus increasing model volatility and leading to economic instability. Besides, the longer is the interest rate pegged, the more unstable is the economy, and specifically, the more over-reactive is the output level.

5.2. Simulation With all Shocks

The impulse response analysis above examines the properties of the model at steady state. In the real world, nominal interest rates are generally pegged at levels different from the steady state. Below we study this situation based on simulations with all the four shocks hitting the model. Assume that before $t = 0$, money policy follows the Taylor rule, and the economy fluctuates stochastically. From period 1 to k, the central bank pegs the nominal rate at its level of period 0. After period k, moentary policy returns to the flexible rate, and the model continue to fluctuate as before $t = 0$.

In Figure 6, we compare three scenarios when the nominal rate is pegged above, below, and at the steady state. The left column shows that when the interest rate is pegged above steady state, output and inflation fall below their steady state levels. The model enters a spiral of depression and deflation until the exit of the peg. The middle column shows the opposite situation of boom and inflation. We can see the difference between initial pegged levels lead to completely different paths of inflation and output. In both cases, the economy will deviate from steady state further and further with the extension of the peg duration, until it collapses in the infinite period. And the only constraint on this divergent process is the expectation of monetary policy returning to the flexible

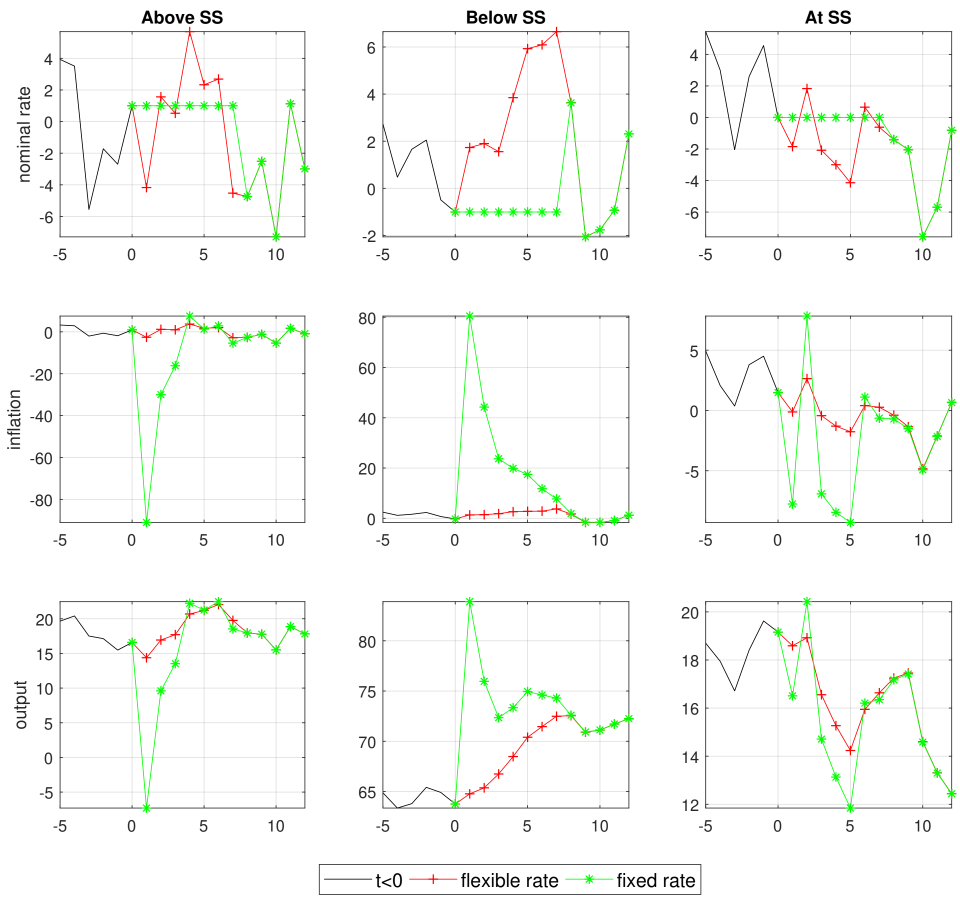


Figure 6: Simulation When the Interest Rate is Pegged Above, Below, and at the Steady State

Notes: Inflation and the interest rate are expressed in annualized percentage points, output and the output gap are the percent deviation from steady state.

rate in the future.

In the right column, the nominal rate is pegged at its steady state level, and the fluctuations of model variables are relatively mild with no tendency of divergence. But this situation is rare, because the real world is far more complex. First, the steady state of the economy may change. Once the pegged rate deviates from its steady state value, it returns to the previous situations. Second, even if the steady state of the nominal rate remains unchanged, it is difficult (if possible) for the government to find this value.

The simulations in Figure 6 confirm previous results that interest rate pegging magnifies economic volatility. And when nominal rates are pegged at different levels, the paths of model variables can be drastically different. Comparing the left and middle columns with the right column, we can also conclude that the further the pegged rate deviates from the steady state, the more volatile is the economy.

6. Fiscal Policy Under the Interest Rate Peg

A special case of interest rate pegging is the liquidity trap, when the nominal interest rate is pushed to a very low level and cannot be further reduced. Keynes (1936) studies this situation and asserts that monetary policy is invalid and fiscal expansion is needed to pull the economy out of the depression. In our previous analysis, monetary policy is restricted by interest rate pegging, so the only policy tool (in the model) is government spending. We thus examine its effects in this section.

6.1. Fiscal Multiplier

The fiscal multiplier is the ratio of the change in output (consumption) to the change in government spending that causes it. It is often used to measure the effectiveness of fiscal policy. Below we calculate and compare the fiscal multipliers under a flexible and a pegged rate.

First consider the limiting case when the nominal rate is pegged over an infinite period. We close the other shocks ($\widehat{\tau}_t = \widehat{a}_t = 0$ for all t), and keep only the government spending shock. We then solve the model using the method of undetermined coefficients. Substituting $\widehat{c}_t = A_c\widehat{g}_t$, $\widehat{\pi}_t = A_\pi\widehat{g}_t$ into equations (13)–(14) under the pegged rate, we can get the consumption multiplier under the infinitely pegged rate:

$$A_c = \frac{\rho_g \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\left[(1-S_g)(1+\gamma)+1\right]S_g\right)}{(1-\rho_g)(1-\beta\rho_g)-\rho_g\kappa} \tag{20}$$

It measures by how much consumption rises if government spending increases one unit. Based on the previously calibrated parameter values, we have $A_c > 0$.

Following the same strategy, we can obtain the consumption multiplier under the flexible rate (with $\phi_y = 0$) by substituting $\widehat{c}_t = B_c\widehat{g}_t$, $\widehat{\pi}_t = B_\pi\widehat{g}_t$ into equations (13)–(14):

$$B_c = \frac{\rho_g \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\left[(1-S_g)(1+\gamma)+1\right]S_g\right)(1-\phi_\pi\beta)}{(1-\rho_g+\phi_\pi\kappa)\left(1+\phi_\pi\kappa-(\kappa+\beta)\rho_g\right)-\rho_g\kappa\rho_g\left(1-\phi_\pi\beta\right)} \tag{21}$$

Based on the same parameter calibration, the numerator of B_c is smaller than that of A_c due to $\phi_\pi\beta > 0$, while the denominator of B_c is larger than that of A_c due to $\phi_\pi\kappa > 0$. Thus we have $A_c > B_c$.

From $\widehat{y}_t = \frac{C}{Y}\widehat{c}_t + \frac{G}{Y}\widehat{g}_t = (1-S_g)\widehat{c}_t + S_g\widehat{g}_t$, we have the output multiplier $\frac{dy_t}{dg_t} = \frac{1}{S_g}\frac{\widehat{y}_t}{\widehat{g}_t} = \frac{1-S_g}{S_g}\frac{\widehat{c}_t}{\widehat{g}_t} + 1$. In our model, the nominal rate is pegged for a limited period. Therefore, $\frac{\widehat{c}_t}{\widehat{g}_t}$ lies between A_c and B_c, which set the upper and lower bounds for the consumption multiplier. Obviously, with the extension of the peg duration, fiscal multiplier increases accordingly.

On the other hand, with the increase of ϕ_π, B_c decreases. When $\phi_\pi > 1/\beta$, B_c turns negative, meaning that fiscal expenditure crowds out private consumption. Thus we come to the conclusion that the more flexible is the interest rate, and the larger is the response of the nominal rate, the smaller is the fiscal multiplier.

6.2. Responses to a Fiscal Policy Shock

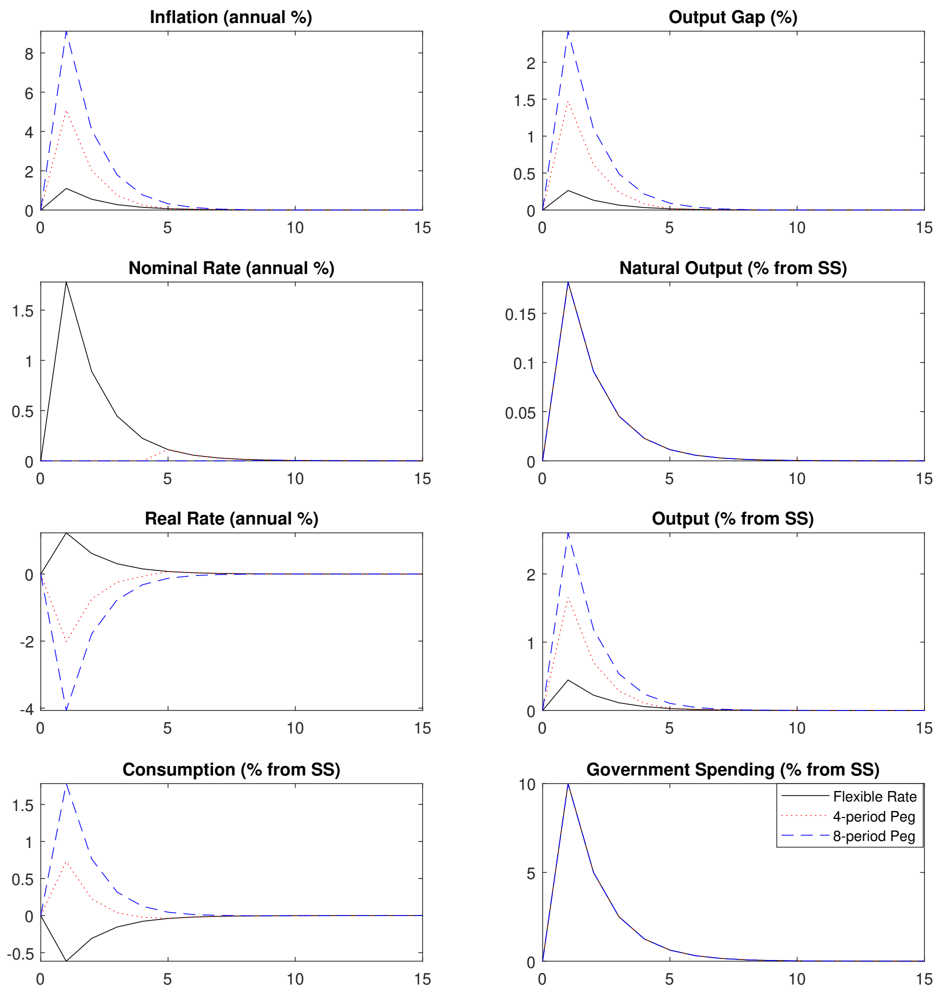


Figure 7: Responses to a Fiscal Policy Shock

Notes: This figure plots the impulse responses to a one S.D. increase in fiscal expenditure for different durations of an interest rate peg.

Assume that at period $t = 1$ government spending is subject to a one standard deviation shock (ε_t^g). Figure 7 compares the responses of model variables under a flexible rate, a pegged rate of 4-periods, and an 8-period peg.

A positive fiscal shock has two effects. On one hand, it expands demand. Since monopolistically competitive firms are constrained by demand, government spending expansion under sticky prices will lead to a simultaneous increase of output, marginal cost, and inflation. On the other hand, under the assumption of Ricardian equivalence, the tax increase accompanying fiscal expansion reduces households' lifetime income. Therefore, the household cuts its consumption on goods and leisure, leading to the increase of labor supply.

Under a flexible interest rate, the real rate increases with the rise of inflation and thus restrains consumption, reflecting the crowding-out effect of government spending. But the increase of government spending exceeds the decrease of private consumption, so the net effect is an increase of output. Quantitatively, a one standard deviation shock means fiscal expenditure increases 10% relative to its steady state value. The increase of output is only 0.45% in period $t = 1$, and the calculated output multiplier is 0.45; while consumption is reduced by 0.61%, and the corresponding consumption multiplier is -0.55.

Under a pegged rate, monetary policy does not respond to inflation or government spending. The real rate decreases with the increase of inflation and thus stimulates current consumption demand, offsetting the negative wealth effect and increasing output further. Dupor & Li (2015) term this interaction between inflation expectations and the real rate as the "expected inflation channel". Quantitatively, under the 4-period peg, output increases by 1.65% and consumption increases by 0.73% owing to a one standard deviation fiscal shock. The fiscal expenditure multiplier rises to 1.65, and the consumption multiplier turns from negative to 0.65. Under the interest rate peg, the crowding-out effect turns into a crowding-in effect. Obviously, the longer is the duration of the peg, the larger is the fiscal multiplier.

The conclusions here complement those in recent literature. The empirical study of Hall et al. (2009) finds that the output multiplier ranges from 0.5 to −1.0, and the consumption multiplier ranges from −0.5 to −0. Woodford (2011) and Christiano et al. (2011) prove in DSGE models that when the economy is at the zero lower bound, the fiscal expenditure multiplier reaches its maximum without the offsetting effect of monetary policy. Overall, the basic conclusion is that the size of the multiplier changes with monetary policy regimes and the state of the economy. And the inflexibility of the nominal interest rate leads to a higher fiscal multiplier.

6.3. Mitigating Fluctuations With Fiscal Policy

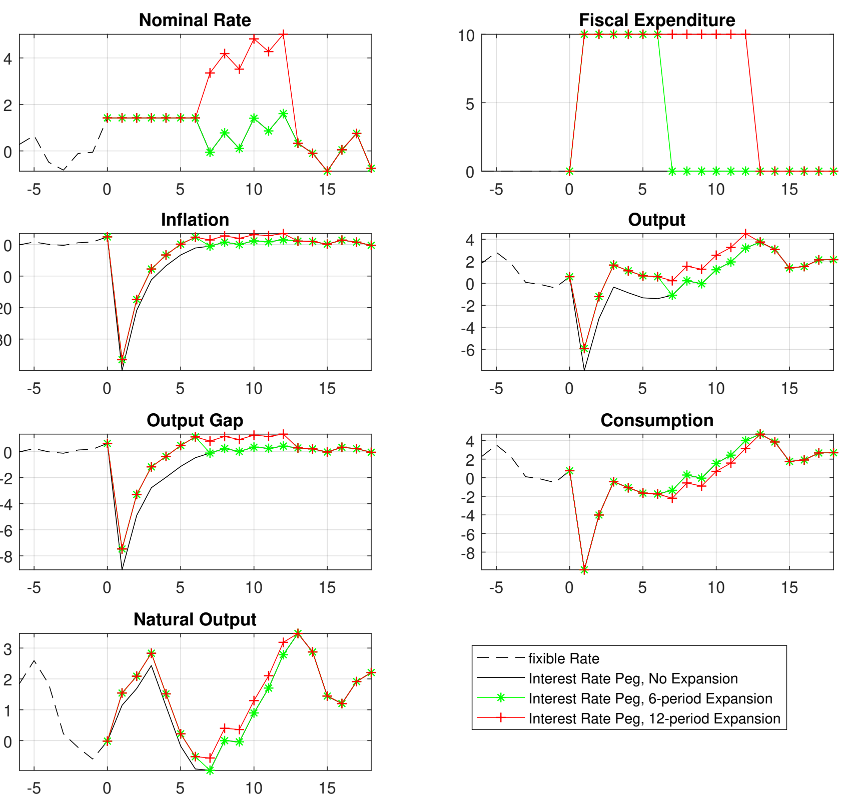


Figure 8: Mitigating Fluctuations with Fiscal Policy

Notes: Inflation and the interest rate are annualized percentage points, output and the output gap are the percent deviation from steady state.

Different from the shocks of technology and preference, fiscal expenditure is a variable controlled by the government. When the economy is subject to exogenous shocks, the government can counteract their effects and smooth out economic fluctuations through fiscal policy. The size of the fiscal multiplier reflects to some extent how effective the government can regulate the economy.

When the nominal rate is set higher than the steady state, inflation and output fall below the steady state. Figure 8 shows the effects of fiscal expansion in this situation. During the interest rate peg (periods $t = 1$ to 6), fiscal expenditure increases by 10% relative to the steady state. In Figure 8, the drop of inflation and output is mitigated.

We set the autocorrelation coefficient of fiscal expenditure to 0, so that it is easy to compare the multipliers. When fiscal expansion extends longer than the duration of the peg, the effects of fiscal expansion on output become smaller for periods $t = 6$ to 12. The reason is that under the flexible rate, the increase of inflation leads to an interest rate increase, which in turn depresses inflation and output, and crowds out consumption. Thus the multiplier of fiscal expenditure declines under the flexible rate. Christiano et al. (2011) also shows that when the economy gets out of the liquidity trap and the nominal rate leaves the zero lower bound, the effect of fiscal expansion will be greatly reduced.

Expansionary fiscal policy can be used by governments to stimulate the economy during a recession. For example, during $1998 - 2001$, the central government of China alleviated the recession and deflation by implementing active fiscal policies; the 4 trillion economic stimulus program at the end of 2008 also helps bring China's economy out of a sharp decline. By the same logic, contractionary fiscal policy (e.g., a decrease in government spending or an increase in taxes) can be used by governments to cool down an "overheating" economy.

Based on the above analysis, although economic fluctuation increases under the interest rate peg, the fiscal expenditure multiplier also increases. This means the government can manage and regulate the economy more effectively, and assuage economic fluctuations through fiscal policy.

7. Conclusion

The interest rate plays a key role in the macroeconomy. It is traditional wisdom when the nominal interest rate is constant, there is no unique equilibrium in macroeconomic models, including both IS-LM and modern rational expectation models.

We prove in a textbook New Keynesian model that a unique equilibrium exists when the nominal rate is pegged for a limited period, after which it switches to a flexible rate regime. The main conclusions can be summarized as follows: (1) Under the interest rate peg, model variables are more sensitive to external shocks. This explains the large fluctuations of the Chinese economy. (2) Besides, the model becomes more unstable when the peg duration extends, and when the pegged rate deviates from steady state. (3) Under the peg, the size of government spending multiplier increases, which means fiscal policy may be more effective in mitigating economic fluctuations.

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Appendix A. Computing the Equilibrium

Appendix A.1. Households

The representative household maximizes its utility (equation (1) in the text) subject to the intertemporal budget constraint (equation (2) in the text). We obtain the following first-order conditions:

$$\beta E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{R_t}{\pi_{t+1}} \right\} = 1 \tag{A.1}$$

$$\pi_{t+1} = \frac{P_{t+1}}{P_t} \tag{A.2}$$

$$e_t^{\tau} N_t^{\gamma} C_t^{\sigma} = \frac{W_t}{P_t} \tag{A.3}$$

Appendix A.2. Firms

Appendix A.2.1. Optimal Price Setting

From equation (9) in the text, the representative intermediary good producer chooses the optimal price $\widetilde{P}_t$ to maximize the discounted sum of its future profits when it can not reoptimize:

$$E_t \sum_{j=0}^{\infty} \mu_{t+j} \beta^j \theta^j \left[\widetilde{P}_t Y_{t+j}(i) - P_{t+j} s_{t+j} Y_{t+j}(i) \right]$$

Substituting out for $Y_t(i)$, we then have

$$E_t \sum_{j=0}^{\infty} \mu_{t+j} \beta^j \theta^j Y_{t+j} P_{t+j}^{\varepsilon} \left[\widetilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \widetilde{P}_t^{-\varepsilon} \right]$$

Taking its derivative with respect to $\widetilde{P}_t$ yields the following optimality condition:

$$E_t \sum_{j=0}^{\infty} \beta^j \theta^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{-\varepsilon} \left[\widetilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0$$

where $\widetilde{p}_t$ is the real price, and $X_{t,j} = \begin{cases} \frac{1}{\pi_{t+j}\pi_{t+j-1}\cdots\pi_{t+1}} & j \geq 1 \\ 1 & j = 0 \end{cases}$.

From the above condition we obtain

$$\widetilde{p}_t = \frac{E_t \sum\limits_{j=0}^{\infty} \beta^j \theta^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum\limits_{j=0}^{\infty} \beta^j \theta^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{1-\varepsilon}} = \frac{K_t}{F_t} \tag{A.4}$$

Here,

$$K_t = \frac{Y_t}{C_t} \frac{\varepsilon}{\varepsilon - 1} (1 - v) \frac{e^{\tau_t} N_t^{\gamma} C_t}{e^{a_t}} + \beta\theta E_t \left(\frac{1}{\pi_{t+1}} \right)^{-\varepsilon} K_{t+1} \tag{A.5}$$

$$F_t = E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} (X_{t,j})^{1-\varepsilon} = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\pi_{t+1}} \right)^{1-\varepsilon} F_{t+1} \tag{A.6}$$

Appendix A.2.2. Aggregation

The aggregate price index is

$$\begin{aligned} P_t &= \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} = \left[(1-\theta) \widetilde{P}_t^{1-\varepsilon} + \int P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \\ &= \left[(1-\theta) \widetilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \end{aligned}$$

Dividing both sides by P_t yields $\widetilde{p}_t = \left[\frac{1-\theta {\pi_t}^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$. Connecting it with (A.4), we have

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta {\pi_t}^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \tag{A.7}$$

We follow the strategy of Yun (1996) to derive the relationship between aggregate output (Y_t) and aggregate labor (N_t). Define $Y_t^* = \int_0^1 Y_{i,t} di = \int_0^1 A_t N_{i,t} di = A_t N_t$. Substituting out for $Y_{i,t}$ using $Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t$, we have

$$Y_t^* = \int_0^1 Y_{i,t} di = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 (P_{i,t})^{-\varepsilon} di = Y_t P_t^{\varepsilon} (P_t^*)^{-\varepsilon}$$

Rearranging terms,

$$Y_t = \left(\frac{P_t^*}{P} \right)^{\varepsilon} Y_t^* = p_t^* A_t N_t \tag{A.8}$$

Here,

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \pi_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \tag{A.9}$$

which represents the distortion of efficiency and satisfies $p_t^* \begin{cases} = 1 & P_{i,t} = P_{j,t} \\ \leq 1 & otherwise \end{cases}$.

Appendix A.2.3. Collecting the Equations

The above equations are listed as follows:

$$\beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{R_t}{\pi_{t+1}} \right\} = 1$$

$$\pi_{t+1} = \frac{P_{t+1}}{P_t}$$

$$e^{\tau_t} N_t^{\gamma} C_t = \frac{W_t}{P_t}$$

$$K_t = \frac{Y_t}{C_t} \frac{\varepsilon}{\varepsilon - 1}(1-v) \frac{e^{\tau_t} N_t^{\gamma} C_t}{e^{a_t}} + \beta\theta E_t {\pi_{t+1}}^{\varepsilon} K_{t+1}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t {\pi_{t+1}}^{(\varepsilon-1)} F_{t+1}$$

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta {\pi_t}^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$$

$$Y_t = p_t^* A_t N_t$$

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \pi_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}$$

Appendix A.2.4. Log-linearization

Assume the steady state inflation $\pi = 1$. Solving for steady state variables yields $R = \frac{1}{\beta} - 1$, $p^* = 1$, and $K = F = \frac{Y/C}{1-\beta\theta}$. Linearizing the Euler equation (A.1) around steady state yields

$$\widehat{c}_t = -\left[\widehat{i}_t - E_t \widehat{\pi}_{t+1} \right] + E_t \widehat{c}_{t+1} \tag{A.10}$$

Linearizing the firm's first-order conditions (A.5)–(A.7) around steady state yields

$$\frac{1}{1-\beta\theta} \widehat{K}_t = \tau_t + (1+\gamma)\widehat{n}_t + \frac{\beta\theta}{1-\beta\theta} E_t \left(\varepsilon \widehat{\pi}_{t+1} + \widehat{K}_{t+1} \right)$$

$$\frac{1}{1-\beta\theta} \widehat{F}_t = \widehat{y}_t - \widehat{c}_t + \frac{\beta\theta}{1-\beta\theta} E_t \left((\varepsilon - 1)\widehat{\pi}_{t+1} + \widehat{F}_{t+1} \right)$$

$$\widehat{K}_t - \widehat{F}_t = \frac{\theta}{1-\theta}\widehat{\pi}_t$$

Eliminating $\widehat{K}_t$ and $\widehat{F}_t$, we can obtain the New Keynesian Philips curve:

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \lambda\left(\widehat{\tau}_t + (1+\gamma)\widehat{n}_t - \widehat{y}_t + \widehat{c}_t\right) \tag{A.11}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

Evaluating the resource constraint $Y_t = C_t + G_t$ at steady state, we have $G/Y = S_g$ and $C/Y = 1 - S_g$. Log-linearizing the resource constraint yields

$$\widehat{y}_t = (1-S_g)\widehat{c}_t + S_g\widehat{g}_t \tag{A.12}$$

In steady state, price distortions are eliminated, hence $\log p_t^* = 0$. Log-linearizing (A.9), we get the law of motion for p_t^*: $\widehat{p}_t^* \approx \theta\widehat{p}_{t-1}^* + 0 \times \pi_t = \theta\widehat{p}_{t-1}^*$. Assume $\widehat{p}_0^* = 0$, then $\widehat{p}_t^* = 0$, which means $P_t^* = P_t$, i.e., $p_t^* = 1$. Substituting it into (A.8), we have $Y_t = A_t N_t$ around steady state. Log-linearizing it yields

$$\widehat{y}_t = \widehat{a}_t + \widehat{n}_t \tag{A.13}$$

Substituting (A.12) and (A.13) into (A.11),

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \lambda\left\{\left[1+\gamma(1-S_g)\right]\widehat{c}_t + \widehat{\tau}_t - (1+\gamma)\widehat{a}_t + \gamma S_g\widehat{g}_t\right\} \tag{A.14}$$

Log-linearizing monetary policy (equation (12) in the text) yields equation (15) in the text, i.e.,

$$\widehat{i}_t = \begin{cases} \phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t + \varepsilon_t^r, & t > k, \\ 0, & t = 1, \cdots, k. \end{cases} \tag{A.15}$$

Appendix A.2.5. Natural Output and Output Gap

Natural output is the level when price distortions are eliminated ($p_t^* = 1$). We have $N_{it} = N^n$, which is the (natural) employment level of flexible prices. Log-linearizing (A.8) yields $\widehat{y}_t^n = \widehat{a}_t + \widehat{n}_t^n$.

From (A.3) and the equilibrium condition of the labor market under flexible prices, $e^{\tau_t}(N_t^n)^\gamma C_t^n{=}A_t$. Log-linearizing it yields $\widehat{c}_t^n + \gamma\widehat{n}_t^n + \widehat{\tau}_t = \widehat{a}_t$.

Substituting out for $\widehat{n}_t^n$ and $\widehat{c}_t^n$ yields natural output:

$$\widehat{y}_t^n = \frac{(1-S_g)(1+\gamma)\widehat{a}_t - (1-S_g)\widehat{\tau}_t + S_g\widehat{g}_t}{1+(1-S_g)\gamma} \tag{A.16}$$

Further, we can obtain the output gap:

$$\begin{aligned}\widetilde{y}_t &= y_t - y_t^n = \widehat{y}_t - \widehat{y}_t^n \\ &= \tfrac{[1+(1-S_g)\gamma]\widehat{y}_t - (1-S_g)(1+\gamma)\widehat{a}_t + (1-S_g)\widehat{\tau}_t - S_g\widehat{g}_t}{1+(1-S_g)\gamma}\end{aligned} \tag{A.17}$$

Rewriting the dynamic IS curve (A.10) in terms of the output gap, we obtain:

$$\widetilde{y}_t = E_t\widetilde{y}_{t+1} - (1-S_g)\left(\widehat{i}_t - E_t\pi_{t+1}\right) + S_g\left(\widehat{g}_t - E_t\widehat{g}_{t+1}\right) - \left(\widehat{y}_t^n - \widehat{y}_{t+1}^n\right) \tag{A.18}$$

Rewriting the New Keynesian Philips curve (A.14) in terms of the output gap, we obtain:

$$\pi_t = \beta\pi_{t+1} + \kappa\widetilde{y}_t \tag{A.19}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1+(1-S_g)\gamma}{1-S_g}$.

We can also rewrite (A.10) and (A.14) in terms of log-deviation of output from its steady state, which yields

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - (1-S_g)\left(\widehat{i}_t - E_t\pi_{t+1}\right) + S_g\left(\widehat{g}_t - E_t\widehat{g}_{t+1}\right) \tag{A.20}$$

$$\begin{aligned}\pi_t &= \beta E_t\pi_{t+1} + \kappa\widehat{y}_t - \kappa\widehat{y_t^n} \\ &= \beta E_t\pi_{t+1} + \kappa\widehat{y}_t - \kappa\psi\left(\tfrac{(1-S_g)(1+\gamma)}{S_g}\widehat{a}_t - \tfrac{1-S_g}{S_g}\widehat{\tau}_t - \widehat{g}_t\right)\end{aligned} \tag{A.21}$$

where $\psi = \frac{S_g}{1+(1-S_g)\gamma}$.

Appendix B. Proofs

Appendix B.1. Proof of Proposition 1

The necessary condition of $-1 < \lambda_1, \lambda_2 < 1$ is $\begin{cases}\lambda_1 + \lambda_2 < 1 + \lambda_1\lambda_2 \\ |\lambda_1\lambda_2| < 1\end{cases}$. The trace of matrix A_1 satisfies $tr(A_1) = 1 + \eta + \beta = \lambda_1 + \lambda_2$, and the determinant satisfies $del(A_1) = \beta = \lambda_1\lambda_2$. The second inequality $|\lambda_1\lambda_2| < 1$ holds obviously. The first inequality $\lambda_1 + \lambda_2 < 1 + \lambda_1\lambda_2$ holds if and only if $\eta < 0$. From the calibration of model parameters, $0 < \theta, \beta, S_g < 1$, and $\gamma > 0$, we have $\eta > 0$. So the first inequality $\lambda_1 + \lambda_2 < 1 + \lambda_1\lambda_2$ does not hold. QED.

Appendix B.2. Proof of Proposition 1

The characteristic polynomial of matrix A_2 is $\lambda^2 - \frac{(1+\eta+\beta)}{(1+\phi_\pi\eta)}\lambda + \beta = 0$. For this equation, the sufficient and necessary condition of $-1 < \lambda_1, \lambda_2 < 1$ is $\begin{cases} \left|\frac{1+\eta+\beta}{1+\phi_\pi\eta}\right| < 1 + \beta \\ |\beta| < 1 \end{cases}$. The second inequality $|\beta| < 1$ holds obviously. The first inequality transforms to $1 + \eta + \beta < 1 + \beta + \phi_\pi\eta + \phi_\pi\eta\beta$, i.e., $\phi_\pi > \frac{1}{1+\beta}$, which holds when the Taylor Principle is satisfied. Thus the first inequality holds. QED.