Classical labour values – properties of economic reproduction

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Abstract

We attempt to clarify and generalize the meaning of economic value as conceptualized in classical political economy. Using a modern formalism, we show that value can be derived as a basic property of systems of economic reproduction. The applicability of the concept is discussed and its relation to inequality, productivity, employment, and unproductive activities are demonstrated.

1 Introduction

What is the basis of economic value? This question repeatedly crops up in practical political discourse, cf. [Bacon and Eltis, 1978, Mazzucato, 2018]. In this paper, we show that the process of economic reproduction gives rise to a characteristic accounting structure in which value is assigned uniquely to goods and services, using the formalism of [Schwartz, 1961, Pasinetti, 1979]. After deriving value from this structure, we show that it corresponds to the conception found in the classical approach to political economy, as well as in the early labour movement, namely, social labour requirements, see [Smith, 1776, Ricardo, 1817, Marx, 1867].

It has often been assumed that the notion of value is only applicable to market-based economies. Conventional economic theory denies valuation is possible outside the relations of commodity-exchanging agents.[1] But also some heterodox economic theories claim that value and hence rational comparison of economic alternatives can only exist through market relations.[2] Our conceptualization, however, shows that value can be derived as a property of a self-reproducing economic system that can redeploy labour across a range of production processes. This includes capitalist market economies, planned economies and mixed state-regulated economies. We proceed to show that this generalized conception addresses central questions that concerned classical political economy and the early labour movement: the organization of production, productivity of an economy, extraction of an economic surplus, real-economic class inequalities, unproductive activities, and so on.

The derivation of all results are provided in the footnotes.

2 Economic reproduction and value

> The real price of everything, *what everything really costs* to the man who wants to acquire it, is the toil and trouble of acquiring it.
> [Smith, 1776, book 1, ch. 5, emph. added]

We consider an interconnected economic system that is capable of reproducing itself. It produces distinct types of outputs for use[3], that can be represented by an ordered list of names, such as (iron, corn, sugar, ...). Since the list is ordered, we can equivalently represent each output-type by a number[4], which leads to an efficient representation to describe d distinct kinds of outputs, numbered as $1, 2, \ldots, d$. Associated with output-types there are socially defined units of measure: metric tons of steel, bushels of corn, kilograms of sugar, etc. Once the lists of output-types and their units of measure are fixed, they permit representing quantities of products in terms vectors.

Example 2.1 (Bundles of products). Consider a simple economy with $d = 3$ outputs-types: iron, corn and sugar, and two different bundles:

$$\mathbf{b} = \begin{bmatrix} b_{\text{iron}} \\ b_{\text{corn}} \\ b_{\text{sugar}} \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b}' = \begin{bmatrix} b'_{\text{iron}} \\ b'_{\text{corn}} \\ b'_{\text{sugar}} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

The set of product bundles is a vector space that allows operations such as addition, $\mathbf{b} + \mathbf{b}'$, and multiplication by a scalar, $2 \cdot \mathbf{b}$. The most basic point about *economic value* is that it permits also the ordering of product bundles, $\mathbf{b} \succ \mathbf{b}'$, as one being greater than the other. That is, value renders heterogeneous bundles of products into commensurable units.

But this raises a series of questions: When and why do distinct products have any value? What indeed does it mean to have economic value? Or to put it another way, what are the formal properties of value? We will be arguing in what follows that both its formal properties – the value form if you will – and its quantitative properties are derived from something more fundamental than commodity exchange, namely, the technical and social structure of reproduction.

Value maps bundles of products to scalar numbers that enable relative comparisons between different bundles. Monetary prices may seem to provide such a valuation, since they assign a quantity of money to each unit of a product. Market prices observed in commodity exchange, however, randomly fluctuate from one transaction to the next and, as the classical economists understood well, the very notion of goods being over- and underpriced implies that value is more fundamental than prices. Suppose there exists some nonnegative row vector $\mathbf{v}$ that contains values for each unit of output.

Example 2.2 (Valuation). In the simple economy above, the vector

$$\mathbf{v} = \begin{bmatrix} v_{\text{iron}} & v_{\text{corn}} & v_{\text{sugar}} \end{bmatrix}$$

encodes the (yet unspecified) amount of value per unit of iron, corn and sugar, respectively. Then the value of bundles $\mathbf{b}$ and $\mathbf{b}'$ above equal

$$\mathbf{v}\mathbf{b} = 5 \cdot v_{\text{iron}} \quad \text{and} \quad \mathbf{v}\mathbf{b}' = 2 \cdot v_{\text{corn}} + 1 \cdot v_{\text{sugar}},$$

respectively.

Below we will *deduce* $\mathbf{v}$ using two basic assumptions:

(i) Value is a real cost that only changes with the structure of economic system.

(ii) Labour can be trained and redeployed across economic activities.

2.1 Production and consumption by workforce

After deducting the intermediate inputs consumed over a given period in the overall production process, the economy produces a *net product* of goods and services that are consumed, invested or hoarded. We shall denote this net product bundle as $\mathbf{n}$.

Example 2.3 (Net product). Suppose the net output of the economic system is

$$\mathbf{n} = \begin{bmatrix} n_{\text{iron}} \\ n_{\text{corn}} \\ n_{\text{sugar}} \end{bmatrix} = \begin{bmatrix} 10 \\ 100 \\ 50 \end{bmatrix}, \quad \text{then} \quad \mathbf{vn} = 10 \cdot v_{\text{iron}} + 100 \cdot v_{\text{corn}} + 50 \cdot v_{\text{sugar}}$$

is the value of the net product, or total value added, yet to be defined.

Economic reproduction requires work so that one part of $\mathbf{n}$ is necessarily consumed by the workforce and its dependents. The remainder is a surplus product consisting of investment goods, luxuries, and so on. That is,

$$\text{surplus product} = \text{net product} - \text{consumption by workforce} \tag{1}$$

We now turn to specifying consumption of the workforce, which occurs in conjunction with production. For a given period let $\boldsymbol{\kappa}$ denote the consumption rate vector, which records the total amount of each output consumed by the workforce divided by the number of units of labour deployed.[5]

Example 2.4 (Real-consumption rates). During a given period, suppose the average consumption rate in the simple economy is one unit of corn per person week deployed. Then we can write

$$\boldsymbol{\kappa} = \begin{bmatrix} \kappa_{\text{iron}} \\ \kappa_{\text{corn}} \\ \kappa_{\text{sugar}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

The average technical conditions of production can be described by the direct labour requirements per unit of output in each sector, denoted by the row vector $\boldsymbol{\ell}$, and a square matrix $\mathbf{A} = \{a_{ij}\}$, which records the amount of output j directly required to produce a unit of output i. Both quantities can be estimated in real economies using data from national accounts.[6]

Definition 2.1 (Consumption requirement matrix)**.** The total workforce consumption requirement equals $\boldsymbol{\kappa}$ multiplied by the total person weeks of labour deployed to reproduce $\mathbf{n}$. The total consumption can be expressed as $\mathbf{Rn}$, where

$$\mathbf{R} = \boldsymbol{\kappa}\boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1} \tag{2}$$

is a matrix of workforce consumption requirements.[7] Thus $\mathbf{vRn}$ equals the value of the total consumption requirement in the economy.

Example 2.5 (Reproduction of simple economy). Let iron, corn and sugar be indexed as $\{1, 2, 3\}$.[8] Then the average input requirements to reproduce output i is given by the ith columns of

$$\boldsymbol{\ell} = \begin{bmatrix} 0.6 & 0.2 & 0.3 \end{bmatrix} \quad \text{and} \quad \mathbf{A} = \begin{bmatrix} 0 & 0.20 & 0.30 \\ 0 & 0.02 & 0.10 \\ 0 & 0 & 0 \end{bmatrix},$$

so that to reproduce one unit of iron requires 0.6 units of labour, and 0 units of corn or sugar. Similarly, to reproduce one unit of corn requires 0.2 units of labour, 0.2 units of steel, 0.02 units of corn and 0 units of sugar. As a consequence, the net product $\mathbf{n}$ in Example 2.3 requires a total worker consumption of

$$\mathbf{Rn} = \begin{bmatrix} 0 & 0 & 0 \\ 0.60 & 0.33 & 0.51 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 0 \\ 64 \\ 0 \end{bmatrix}, \tag{3}$$

where $\mathbf{R}$ is computed using (2) and $\boldsymbol{\kappa}$ in Example 2.4. Thus the workforce consumes a total of 64 units of corn, with an unknown value of $\mathbf{vRn} = 64 \cdot v_{\text{corn}}$.

2.2 Deriving economic value

We now study the net output of the economic system in terms of value. The net product $\mathbf{n}$ can be decomposed into the total consumption by workforce and a surplus product (1). The value of the surplus is then the difference between $\mathbf{vn}$ and $\mathbf{vRn}$.

Definition 2.2 (Share of surplus value)**.** In value terms, the surplus (1) forms a share of the total value of the net product $\mathbf{vn}$, which we denote

$$\boxed{\sigma = \frac{\mathbf{vn} - \mathbf{vRn}}{\mathbf{vn}},} \tag{4}$$

and is bounded between 0% and 100%.[9]

At first sight, (4) is a mere accounting identity that is applicable to any reasonable choice of valuation vector $\mathbf{v}$. However, the following result shows that this fundamental identity is deeper than that.

Result 2.1 (Determination of value)**.** *Economic value for each of output-type,* $\mathbf{v}$*, is determined uniquely by* (4)*, up to a unit of choice. Specifically,* $\mathbf{v}$ *is a left-eigenvector of* $\mathbf{R}$ *in* (2).[10] *Thus value naturally arises when production requires a workforce that consumes part of the net product.*

Example 2.6 (Values). In the simple economy considered above, the relative values of iron, corn and sugar are determined by computing the nonnegative left eigenvector of $\mathbf{R}$. Using the consumption requirement matrix in (3), we obtain

$$\begin{aligned} \mathbf{v} &= \begin{bmatrix} v_{\text{iron}} & v_{\text{corn}} & v_{\text{sugar}} \end{bmatrix} \\ &= \begin{bmatrix} 0.60 & 0.33 & 0.51 \end{bmatrix} \end{aligned}$$

Thus a unit of iron is roughly twice as valueable as a unit of corn.

> The value of a commodity [...] depends on the relative quantity of labour which is necessary for its production, and *not* on the greater or less compensation which is paid for that labour.
> [Ricardo, 1817, ch. 1, emph. added]

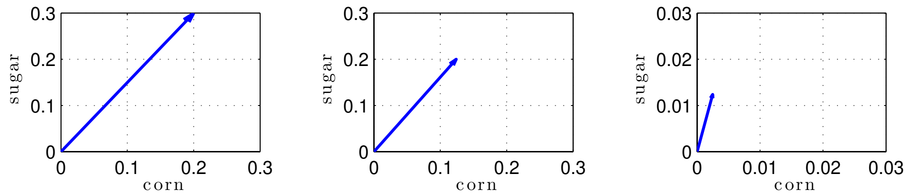
Result 2.2 (Field property)**.** *Economic value* $\mathbf{v}$ *derived in Result 2.1 can be obtained by integrating all coexisting labour requirements in production,*

$$\boxed{\mathbf{v} \propto \boldsymbol{\lambda}(0) + \boldsymbol{\lambda}(1) + \boldsymbol{\lambda}(2) + \cdots,} \tag{5}$$

where $\boldsymbol{\lambda}(k) = \boldsymbol{\ell}\mathbf{A}^k$ *is the vector of labour requirements of the* k*th intermediary inputs in production.*[11] *Value is therefore invariant to changes in workers' consumption or the distribution of the net product.*[12] *The sum in* (5) *converges for an economy capable of reproducing itself.*

Value is not an intrinsic property of products, but is rather a field property that reproducible goods and services acquire from the economic system. That is, the value of a product bundle $\mathbf{b}$ is obtained by an integration over the field, $(\boldsymbol{\lambda}(0) + \boldsymbol{\lambda}(1) + \boldsymbol{\lambda}(2) + \cdots)\mathbf{b} = \mathbf{v}\mathbf{b}$.[13]

Example 2.7 (Vector field in simple economy). The coexisting labour requirements for corn and sugar are illustrated in Figure 1.



(a) $\boldsymbol{\lambda}(0)$ (b) $\boldsymbol{\lambda}(1)$ (c) $\boldsymbol{\lambda}(2)$

Figure 1: Illustration of coexisting labour requirements for corn and sugar, described as a vector field. (a) Direct labour requirements. (b) Indirect labour requirements via the inputs to corn and sugar. (c) Indirect labour requirements via the subsequent set of inputs. Note that the magnitudes decrease for each set of inputs $k = 0, 1, 2, \ldots$ and their sums are proportional to value (5). The value of a bundle of corn and sugar $\mathbf{b}'$ in Example 2.2 equals $\mathbf{v}\mathbf{b}' = 0.700 + 0.448 + 0.017 + \cdots = 1.165$.

The production of value is therefore inseparable from socially organized production of real goods and services. It follows from the derivation above that value production is distinct from monetary income generation; money and prices are symbolic means by which value is claimed and distributed in market economies. The natural unit of value is worker-time and using such units we shall call $\mathbf{v}\mathbf{b}$ the 'labour value' of product bundle $\mathbf{b}$.[14]

3 Applicability of concept

When deriving $\mathbf{v}$ from (4), it appears that value is an economic property that is invariant to the social institutions under which production takes place. We present here a few remarks on the applicability of the concept.

> Every child knows [...] that the masses of products corresponding to the different needs required different and *quantitatively determined masses of the total labour of society*. That this necessity of the distribution of social labor in definite proportions *cannot* possibly be done away with by a particular form of social production but can only change the mode of its appearance, is self-evident. No natural laws can be done away with. What can change in historically different circumstances is *only the form* in which these laws assert themselves.
> [Marx, 1868, emph. added]

Using our notation, $\mathbf{n}$ represents the 'masses' of different products and the corresponding elements of $\mathbf{v}$ quantitatively determine the masses of total labour of society required. The derivation of $\mathbf{v}$ assumes that (i) value only changes with the structure of a viable interconnected economic system that (ii) is capable of training and redeploying its finite amount of available labour time across different production processes. This latter assumption implies the existence of a vector $\boldsymbol{\ell}$ representing labour inputs across different production processes in commensurable units of time, while the former assumption implies to a matrix $\mathbf{A}$ with a maximum eigenvalue less than unity.

It is in the continual process of training and redeployment of labouring capacity across production that an economic system renders concrete work tasks as an expenditure of a commensurable abstract labour resource, quantified in units of time.[15] This would include a range of self-reproducing economic systems. Did, for instance, value and the disposition of labour time matter to the slave lords of antiquity? According to Cato, it appears that they did:

> When [the master of a farmstead] has learned the condition of the farm, what work has been accomplished and what remains to be done, let him call in his overseer the next day and inquire of him what part of the work has been completed, what has been left undone; whether what has been finished was done betimes, and whether it is possible to complete the rest; and what was the yield of wine, grain, and all other products. Having gone into this, he should make a calculation of the labourers and the time consumed. [Hooper and Ash, 1935, p.9].

In the slave plantations described above by Cato, the disposition of labour is self evident and 'natural', it is not obscured by monetary indirection. But it is still labour in the abstract, albeit of a given group of slaves, being distributed between concrete tasks: meadow clearing, faggot bundling, road-work, etc.

The necessity to take into account the usage of labour time, whether that be the time of slaves, wage labourers, citizens of a socialist commonwealth, is a natural necessity that could not be abolished, only change its historical form. By contrast, in economies with institutions that prevent the redeployment of workers across tasks, e.g. rigid forms of caste hierarchies, there can be no general labour resource quantifiable in commensurable units.

3.1 Capitalist market economies

In a capitalist economy, the necessity to distribute labour appears as simply expenditures of money on wages to top-level managers in decentralized firms. So the wage budget allocated to different branches of a firm provides an indirect representation of the needed allocation of labour.

As one descends the management hierarchy, the simple monetary view of things becomes insufficient. The subsidiary managers have to allocate specific people to specific tasks just as the slave overseer had to. By contrast, as one moves further away from the production process, the representation of labour becomes increasingly obscure and monetary. Indeed, when the products of the economy are allocated between agents as commodities, the monetary calculations are based on market prices which randomly fluctuate from one transaction to the other. The relation between market prices of commodities and their labour values is necessarily a statistical one, see [Farjoun and Machover, 1983].[16] To an individual, money appears to be freely disposable between different products, but in reality such choices are limited by macroeconomic constraints set by $\mathbf{v}$, which represent real costs irrespective of random market prices.

Nevertheless, firms in a capitalist market economy do solve labour allocation problems via decentralized monetary calculations. The feasibility of this monetary accounting mechanism rests on the fact that human labour is flexible and can be redirected, either within the firm or on the employment market, between activities. In capitalism, the redeployment of labour between concrete tasks across the production system occurs through the transfer, hiring and firing of workers within and across decentralized firms. This allows single scalar measure like money to function as a system of social accounting.

3.2 Planned economies

Planned economies too have to grapple with the finite nature of their labour supplies, and the need to expend effort for any worthwhile effect. This implies that they too will have to have social forms in which this necessity will be expressed. The necessity for the labour force to be allocated in a manner determined by technical conditions took in the planned Soviet-socialist economies the form of a directive plan of $\mathbf{n}$. This plan involved drawing up material and labour balances for the overall economy. We know that Soviet-socialist economies continued to use monetary calculations, which, to a greater or lesser degree of adequacy, allowed indirect calculations to be done on social labour requirements. While monetary calculation and allocation in capitalist market economies redeploys a certain amount of labour via the recreation of a pool of unemployed, the Soviet-socialist economies did not develop the kind of labour time accounting, planning and regulation that would be required to carry out reallocations of labour within a fully employed workforce.

In capitalist war economies, production, by and large, still took place in privately owned firms. There were state munitions factories like the Royal Arsenal or the Oak Ridge and Los Alamos atomic weapons plants, but these were exceptions. The state directed labour, by conscripting it into the army, and by conscripting women and men in key trades into essential war work. It also rationed the supply of key materials, fuels, and foodstuffs. Firms were subject to negotiated direction to produce only munitions, or restricted ranges of 'utility' products [Edgerton, 2011]. Money was still used to pay for the munitions delivered, and to pay workers. Buying food required both money and ration cards. Money alone was not enough either for the consumer or for firms. In peace, money as the universal ration constrains everything. Shortage of it constrains the working class consumers and uncertainty about future revenue constrains even those firms who have good cash reserves. Because the constraint on production comes via market exchange in price units rather than directly in units of products, peace-time capitalist market economies typically operate somewhat below full capacity. In war, national survival dictates that every available resource be put to use. The economy operates at the limits of its physical resources in materials, people and machines.

The state as primary purchaser has to look not just at the projected costs of ships, aircraft, etc., that it is ordering, but at all sorts of material constraints. In deciding what type of destroyers to order, the Navy first took into account the requirements of their admirals for the ships to carry guns of different types, torpedoes and anti-submarine weapons: all technical not financial issues. They then had to take into account the number of ship yards in the country able to build ships of different sizes, the delivery schedules for different kinds of projected weapons and ships machinery, the availability of metals and alloys of different weights and strengths. They then had to ask whether the demands on skilled labour would require the cancellation or postponement of other orders.[17] Money was a relatively secondary concern. The availability of state credit, that, at least within the domestic economy was effectively unlimited, removed money as a constraining resource [Keynes, 2010]. The same point about money applied *a fortiori* to the Soviet-socialist economies. Money was never a constraint for them. Labour, plus available plant and equipment, however, were.

If we imagine a planned economy that does away with money altogether, then it would still need value. Marx presented a version of this [Marx, 1970] where labour time was to be used to allocate consumer goods. But value accounting would also be needed to set budgets for public projects. Research programs to develop vaccines, or to explore the moons of Jupiter would need some limits posed on the amount of social labour that they could use. The same applies to general democratic decisions about long-term structural investment. Society as a whole could not meaningfully decide what portion of its output should be devoted to investment and research, that is to say σ in our notation, unless the surplus was measurable. By Equation (4) any consistent measure of this surplus implies labour value.

3.3 Fully automated economies?

Choosing to evaluate the replacement cost of products in terms of labour time reflects not merely a concern for human beings but also the fact that humans possess a capacity to be allocated across a wide range of concrete tasks. This general capacity is then realized in economies that redeploy labour and gives rise to the abstract representation $\boldsymbol{\ell}$ of direct labour requirements. However, it may be objected that some future society may have at its disposal a race of robots, so skilled and dexterous, so intelligent and adaptable, that these beings may come to supplant us in our toils. Would that not invalidate our system of calculation of values?

Not at all, it would merely substitute the time of these general purpose robots for our own time. The equations of value would still apply, but with this simple proviso, that the labour input time is to be understood as redeployable general robot time. The consumption requirement matrix $\mathbf{R}$ would at that point be translated into the 'robot construction and maintainence' requirement matrix, for these robots too will need energy, will require repair and will absorb the effort of other robots in their initial construction.

Humans, in this hypothetical society, would be in the position of slave-owning ancients: idlers depending on the surplus labour of others.

4 Implications for labour

We now proceed to apply the generalized conception of value to central questions that concerned classical political economy and the early labour movement: Economic inequality, productivity, employment and the utilization of surplus economic capacity.

4.1 Extraction of surplus labour

> The specific economic form, in which unpaid surplus-labour is pumped out of direct producers, determines the relationship of rulers and ruled, as it *grows directly out of production itself* and, in turn, reacts upon it as a determining element.
> [Marx, 1894, ch. 47, emph. added]

Result 4.1 (Surplus labour-time)**.** *The share of surplus value σ in* (4) *equals the fraction of work in the economy to replace the surplus product. The share is given directly by the structure of* $\mathbf{R}$*, which has a unique nonzero eigenvalue* $1-\sigma$*.*

Example 4.1 (Share of surplus)*.* Consider the real-consumption requirement matrix $\mathbf{R}$ in (3). Its eigenvalue is readily computed, and yields the following share of surplus value $\sigma = 67\%$. Thus 67% of the work in the economy is materialized in the form of surplus outputs.

In economic systems in which the composition and distribution of the net product $\mathbf{n}$ is not determined by the workforce, their surplus labour is extracted, controlled and consumed by a distinct economic class.

4.2 Total productivity and employment

> The values of commodities are directly as the times of labour employed in their production, and are *inversely* as the productive powers of the labour employed.
> [Marx, 1865, sec. IV, emph. added]

Technical and organizational changes in the economy alter the average production requirements, and therefore the vector field in (5).[18] Let $\dot{\mathbf{v}} = \frac{d}{dt}\mathbf{v}$ denote the change in labour values per unit of time. This quantity has profound effects on both production and employment.

Result 4.2 (Productivity)**.** *Suppose the labour value of output-type i is reduced at the relative rate $\rho_i \equiv -\dot{v}_i/v_i$. Then, for a fixed level of employment, the net output of i can grow at the relative rate ρ_i. Thus labour values are (inverse) measures of total productivity in the economy.*

Example 4.2 (Labour value and total productivity growth)*.* Suppose the simple economy produces a net output of 100 units of corn. The labour value of corn can be lowered by decreasing the direct labour input and/or by decreasing the amount of coexisting inputs required. Thus technical improvements in the production of iron affect the labour value of corn. Suppose its unit value v_{corn} decrease by the rate $\rho_{\text{corn}} = 5\%$ per annum. Then the net output of corn can increase exponentially as shown in Figure 2.

Result 4.3 (Employment)**.** *Suppose the final demand for output-type i grows at the relative growth rate g_i. Then the total demand for labour changes by the rate $g_i - \rho_i$. Thus labour values are employment multipliers in the economy.*[19]

We see that economies with institutions that progressively lower the labour values of the outputs are capable of increasing material living standards and/or leisure time exponentially. At the same time, economies that lack coordination between technical change and changes in consumption

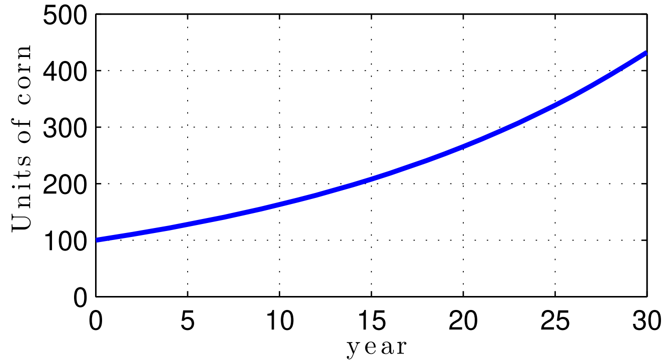


Figure 2: Growth of capacity to produce corn as labour value of corn decreases by $\rho_{\text{corn}} = 5\%$ per annum. Starting with an annual output of 100 units of corn, the amount increases fourfold within 30 years.

and investment demands can give rise to both persistent unemployment and chronic labour shortages. When $g_i < \rho_i$, the total demand for labour declines exponentially and must be compensated by increased demand among other outputs to prevent the rise of unemployment.

4.3 Productive and unproductive activities

> A man *grows rich* by employing a multitude of manufacturers: he *grows poor* by maintaining a multitude of menial servants.
> [Smith, 1776, book II, ch. III, emph. added]

This remark may merely seem to apply to an individual employer but in fact generalizes into a macroeconomic property: The surplus product (1) is by definition the residual of the net product after deducting the outputs consumed by the workforce, i.e., $\mathbf{s} = \mathbf{n} - \mathbf{R}\mathbf{n}$. The relations between the products of the economy are in general not symmetric: some outputs may enter directly or indirectly as inputs to all goods and services, while other outputs may not. This implies certain consequences in the production of $\mathbf{s}$ which we deduce below.

Definition 4.1 (Basic and nonbasic outputs)**.** Basic outputs are directly and indirectly required in the production of all outputs, while the nonbasic outputs are not. More formally, let $\widetilde{\mathbf{A}} = \mathbf{A} + \boldsymbol{\kappa}\boldsymbol{\ell}$ denote the augmented input-output matrix, where the order of the outputs is arranged to form a upper-block triangular structure:

$$\widetilde{\mathbf{A}} = \begin{bmatrix} \widetilde{\mathbf{A}}_b & \widetilde{\mathbf{A}}_{bu} \\ \mathbf{0} & \widetilde{\mathbf{A}}_u \end{bmatrix} \tag{6}$$

The upper-left block corresponds to outputs indexed $i = 1, \dots, b$, which we denote as 'basic'. The remaining $u = d - b$ outputs are 'nonbasic'.[20]

Example 4.3 (Basics and nonbasics). For the simple economy we have

$$\begin{aligned}\widetilde{\mathbf{A}} &= \begin{bmatrix}0 & 0.20 & 0.30\\ 0 & 0.02 & 0.10\\ 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}0.6 & 0.2 & 0.3\end{bmatrix}\\ &= \begin{bmatrix}0 & 0.20 & 0.30\\ 0.60 & 0.22 & 0.40\\ 0 & 0 & 0\end{bmatrix}\end{aligned} \tag{7}$$

which is upper block-triangular as in (6). Therefore corn and iron ($i = 1, 2$) are basic outputs, while sugar is a nonbasic output.

The production of basic outputs forms a self-reproducing sector of the economy which is critical in determining the share of surplus labour σ.

Result 4.4 (Determinants of the surplus)**.** *The share of surplus labour is determined by productivity in and the workers' consumption from the basic sectors of the economy. That is,*

$$\sigma = 1 - \mathbf{v}_b \boldsymbol{\kappa}_b,$$

where $\mathbf{v}_b$ *and* $\boldsymbol{\kappa}_b$ *are the values of basic outputs and corresponding worker's consumption rates, respectively. Luxuries and other nonbasic outputs do not affect* σ.[21]

In other words, the share of surplus labour σ increases when the workers' consumption rates decrease and/or the labour values of basic outputs decrease. The consumption rates $\boldsymbol{\kappa}_b$ can be reduced by extending the number of working hours without compensation, while the lowering of labour values $\mathbf{v}_b$ require technical changes in the basic sector.[22]

Result 4.5 (Dependence on surplus labour)**.** *Production of nonbasic outputs is predicated on the extraction of surplus labour. More formally, if the share of surplus labour is* $\sigma = 0$ *then the production of nonbasics outputs equals* $\mathbf{0}$.[23]

Production of luxuries and other nonbasic outputs drains the surplus in the basic sector. Activities involved such production impedes the expansion of the basic sectors and are 'unproductive' in the sense of classical political economy. In modern capitalist economies, this includes the arms industry and finance sector. Conversely, many socialized goods and services, such as public health care and education, are basic outputs and thus 'productive'.

Result 4.6 (Drain on the basic sectors)**.** *The surplus of basic outputs is impeded by the production of nonbasic outputs. More formally, let* $\mathbf{b}$ *and* $\mathbf{b}'$ *denote the net production of basic and nonbasic outputs, respectively, so that* $\mathbf{n} = \mathbf{b} + \mathbf{b}'$. *By redeploying labour from nonbasic to basic sectors, the surplus product in the latter sectors can be increased by*

$$\mathbf{R}\mathbf{b}' + \frac{\mathbf{v}\mathbf{b}'}{\mathbf{v}\mathbf{b}}(\mathbf{I} - \mathbf{R})\mathbf{b} \;\geq\; \mathbf{0}, \tag{8}$$

where the first term is the workers' consumption freed up from the nonbasic sectors and the second term is due to the increased production in the basic sectors.[24] *Thus* (8) *represents a drain on the surplus capacity of the basic sectors incurred through the production of nonbasic outputs.*[25]

Example 4.4 (Redeployment to basic sectors). Consider the net product in Example 2.3, where 50 units of sugar constitutes the nonbasic outputs. That is,

$$\mathbf{n} = \mathbf{b} + \mathbf{b}' = \begin{bmatrix} 10 \\ 100 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}$$

Then the surplus product equals

$$\mathbf{s} = \mathbf{n} - \mathbf{R}\mathbf{n} = \begin{bmatrix} 10 \\ 100 \\ 50 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0.60 & 0.33 & 0.51 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 10.00 \\ 35.71 \\ 50.00 \end{bmatrix}.$$

Suppose the total labour devoted to sustain the nonbasic sugar is redeployed to expand net output of basic iron and corn uniformly. Using (8), the form of the surplus product then changes into

$$\mathbf{s}' = \begin{bmatrix} 16.63 \\ 102.03 \\ 0 \end{bmatrix}.$$

That is, an increase of surplus iron and corn by +66% and +185%, respectively.

5 The form of value

The analysis above is based on the premise that the value of product bundles $\mathbf{b}$ has the form of a linear function $\mathbf{v}\mathbf{b}$. This function is encoded by the valuation vector $\mathbf{v}$, where the ith element has the dimension value per unit of output type i. This form of value gives rise to an ordering among product bundles, $\mathbf{b} \preceq \mathbf{b}'$, defined as $\mathbf{v}\mathbf{b} \leq \mathbf{v}\mathbf{b}'$.[26] The value form enables therefore rational comparisons between economic alternatives. In market-based economies, this form is a necessity or else any agent could through a sequence of exchanges with product bundles end up with greater quantities of each output-type than they started out with, see also [Cockshott, 2009, Cockshott et al., 2004].

The linear value form appears economically sound and intuitive, since we are used to prices working this way. But in more general terms of product bundles, one could conceive of other, nonlinear value forms that enable an ordering of bundles. Indeed, it is not clear whether the linear form is *necessary* in a non-commodity producing economic system. We leave it as a challenge to the readers to either prove that this form of value arises as a necessary property of social reproduction, or conversely to demonstrate that other, nonlinear forms would be appropriate in post-capitalist economy.

6 Conclusion

We have shown that when production requires a workforce that consumes a part of the net product, economic value can be derived as a fundamental property of a self-producing economic system. This derivation led us back to the classical conception of value, with implications for inequality, productivity, employment and distribution of surplus.

The classical economists assumed social labour as the basis of value on the ground that it was the original social cost of all produced commodities, see [Ricardo, 1817]. An early attempt to instead deduce this relation can be found in [Marx, 1867, ch. 1] and was based on the observation that labour is a universal input that enters directly or indirectly into the production of every output-type. This line of reasoning was, however, necessarily incomplete since there are several other inputs that are universal in the same sense [Sraffa, 1960, ch. 2].

By contrast, [Marx, 1885, pt. III] pioneered an analysis of economic reproduction that forms the foundation of this paper. Embedded in this analysis is the necessary consumption of the workforce and, as we have shown, consequently the classical conception of value in its general form.

Notes

[1][Steele, 1981] provides a review of criticisms against the idea that economic value could exist outside markets.

[2]In the analysis of the Russian Marxist economist Rubin,

> We moved from physiologically equal labour to socially equated labour, and from socially equated to abstract universal labour. We enriched our definition of labour by new characteristics in the three stages of our investigation and only when we moved on to the third stage and defined labour as abstract universal, from which the category of value must necessarily follow, was it possible for us to move from labour to value. We could define abstract labour approximately as follows: Abstract labour is the designation for that part of the total social labour which was equalised in the process of social division of labour through the equation of the products of labour on the market. [Rubin, 1978]

This line of thought has been advanced by the so called value-form school [Heinrich, 2012].

[3]What Marx called 'use values' [Marx, 1867].

[4]This is systematised in the international system of bar codes which associates a 12 digit number with each product kind.

[5]The consumption rate vector $\boldsymbol{\kappa}$ can be estimated from national accounts data using the inputs to the household sector and the total wage bill.

[6]See product-product input-output matrices from national statistics agencies.

[7]The definition can be extended to the case of joint production with d distinct sectors operating with activity levels $\mathbf{q}$, in which the net product equals $\mathbf{n} = (\mathbf{B} - \mathbf{A})\mathbf{q}$, where $\mathbf{A}$ and $\mathbf{B}$ are input and output matrices. We assume that each output i is seperately reproducible so that the exists a set of activity levels $\mathbf{q}_i$, for which $n_i\mathbf{e}_i = (\mathbf{B} - \mathbf{A})\mathbf{q}_i$. Therefore the inverse of $(\mathbf{B} - \mathbf{A})$ exists and $\mathbf{n}$ requires $\boldsymbol{\ell}\mathbf{q} = \boldsymbol{\ell}(\mathbf{B} - \mathbf{A})^{-1}\mathbf{n}$ units of labour, which consumes a total bundle $\boldsymbol{\kappa}(\boldsymbol{\ell}\mathbf{q}) = \boldsymbol{\kappa}\boldsymbol{\ell}(\mathbf{B} - \mathbf{A})^{-1}\mathbf{n} = \mathbf{R}\mathbf{n}$, where $\mathbf{R} = \boldsymbol{\kappa}\boldsymbol{\ell}(\mathbf{B} - \mathbf{A})^{-1}$. In the case of single outputs, considered in the text, we have $\mathbf{B} = \mathbf{I}$.

[8]The example is adapted from [Wright, 2017] but is designed to resemble the structure of reproduction schemes considered in [Marx, 1885] where Departments I, IIa and IIb correspond to 'iron', 'corn' and 'sugar', resepectively.

[9]More specifically, $\sigma \in [0, 1)$. Note that Marx's unbounded 'rate of surplus value' $\frac{\mathbf{vn}-\mathbf{vRn}}{\mathbf{vRn}} = \frac{\sigma}{1-\sigma} \in [0, \infty)$ is a mere transformation of σ.

[10]Eq. (4) can be rearranged into $[(1 - \sigma)\mathbf{v} - \mathbf{vR}]\mathbf{n} = \mathbf{0}$, which holds irrespective of the composition of $\mathbf{n}$. This corresponds to an eigenequation $\lambda\mathbf{v} = \mathbf{vR}$, where $\lambda \equiv 1 - \sigma$ is an eigenvalue that is obtained as the solution to $\det(\lambda\mathbf{I} - \mathbf{R}) = 0$. Using the matrix determinant lemma, this is equivalent to $(1 - \boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1}\boldsymbol{\kappa}\lambda^{-1})\lambda^d = 0$. Since $\sigma = 1$ corresponds to a workforce that does not consume anything, $\lambda = 1 - \sigma = 0$ is an economically meaningless eigenvalue and only $\lambda = \boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1}\boldsymbol{\kappa} > 0$ is a meaningful solution. Next, after inserting (2) into the eigenequation, we have that

$$\lambda\mathbf{v} = (\mathbf{v}\boldsymbol{\kappa})\boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1} \tag{9}$$

so that $\mathbf{v} \propto \boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1}$ is a nontrivial solution after dividing (9) by $\lambda > 0$. Note that the solution is invariant to the consumption-rate vector $\boldsymbol{\kappa}$, which may vary with the distribution of net production. This is of course the classical unit labour values as defined in the standard literature [Pasinetti, 1979].

[11]From the eigenequation (9) we derived the eigenvector $\mathbf{v} = \boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1}$. Using the series expansion $(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k$, it follows that $\mathbf{v} = \sum_{k=0}^{\infty} \boldsymbol{\lambda}(k)$ and proves (5).

[12]The derivation of $\mathbf{v}$ from (4) is based on the decomposition of the net product, and is not interpreted by evaluating 'inputs and outputs' in production (whether 'simultaneous' or 'sequential' evaluation), see Section 9 in the review by [Foley, 2000].

[13]Thus classical labour values can be understood as a field theory of value rather than a substance theory of value, contrary to the characterization in [Mirowski, 1989].

[14]That is, a standard choice of numeraire is $\mathbf{vn} = L$, where $L = (\boldsymbol{\lambda}(0) + \boldsymbol{\lambda}(1) + \boldsymbol{\lambda}(2) + \cdots)\mathbf{n}$ is the total units of labour required.

[15]Classical labour values therefore differ radically from the concept of 'value' developed by the so-called value-form school. In the latter conception, there can be no abstract labour measured in hours nor can it be measured before the act of market exchange [Heinrich, 2012, pp.50, 55, 65].

[16]Since market prices are randomly fluctuating quantities they do not form a 'dual system' with repect to labour values.

[17][Friedman and Baker, 2009] gives several examples of scheduling constraints on new gun mountings, and slip sizes affecting UK destroyer construction plans in WWII. [Friedman, 2015] gives the example of construction of the Admiral class capital ships being postponed due to there not being enough shipbuilding labour to both build them and destroyers in 1917. For large scale shipbuilding programmes, even in peace, similar forward planning of physical constraints has to be done by the state [Arena et al., 2005].

[18]"The value of a commodity would therefore remain constant, if the labour time required for its production also remained constant. But the latter changes with every variation in the productiveness of labour. This productiveness is determined by various circumstances, amongst others, by the average amount of skill of the workmen, the state of science, and the degree of its practical application, the social organisation of production, the extent and capabilities of the means of production, and by physical conditions." [Marx, 1867, ch. 1]

[19]The total employment requirement for producing n_i units of output i is $L_i = v_i n_i$. Therefore the relative change of employment is given by the identity $\dot{L}_i/L_i = -\rho_i + g_i$, where $g_i = \dot{n}_i/n_i$. If the actual employment is fixed, then the left-hand side is 0 and correspondingly $g_i = \rho_i$.

[20]An equivalent definition, which does not require rearranging the sectors, is that output i is basic if $\mathbf{e}_i^\top(\widetilde{\mathbf{A}}^1 + \widetilde{\mathbf{A}}^2 + \cdots + \widetilde{\mathbf{A}}^d) > \mathbf{0}$. We are naturally assuming that all consumption goods require some amount of direct labor. The concept is a slight generalization of Sraffas 'basic goods' and includes the production of the workers' consumption bundle. Note that the outputs that are basic and nonbasic may change over time as the structure of the economy changes, see [Cockshott and Zachariah, 2006].

[21]Using the inverse of the upper block triangular matrix $(\mathbf{I} - \mathbf{A})$, we have that

$$\begin{aligned}
\mathbf{v} &= \boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1} \\
&= \begin{bmatrix} \boldsymbol{\ell}_b & \boldsymbol{\ell}_u \end{bmatrix} \begin{bmatrix} (\mathbf{I} - \mathbf{A}_b)^{-1} & (\mathbf{I} - \mathbf{A}_b)^{-1}\mathbf{A}_{bu}(\mathbf{I} - \mathbf{A}_u)^{-1} \\ \mathbf{0} & (\mathbf{I} - \mathbf{A}_u)^{-1} \end{bmatrix} \\
&= \begin{bmatrix} \boldsymbol{\ell}_b(\mathbf{I} - \mathbf{A}_b)^{-1} & \boldsymbol{\ell}_b(\mathbf{I} - \mathbf{A}_b)^{-1}\mathbf{A}_{bu}(\mathbf{I} - \mathbf{A}_u)^{-1} + \boldsymbol{\ell}_u(\mathbf{I} - \mathbf{A}_u)^{-1} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{v}_b & \mathbf{v}_u \end{bmatrix}.
\end{aligned} \tag{10}$$

Then it follows that $\sigma = 1 - \boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1}\boldsymbol{\kappa} = 1 - \mathbf{v}_b\boldsymbol{\kappa}_b$. Note that we also have

$$\begin{aligned}
\mathbf{vR} &= \boldsymbol{\kappa}\boldsymbol{\ell}(\mathbf{I} - \mathbf{A})^{-1} \\
&= \mathbf{v}\begin{bmatrix} \boldsymbol{\kappa}_b \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\ell}_b(\mathbf{I} - \mathbf{A}_b)^{-1} & \boldsymbol{\ell}_b(\mathbf{I} - \mathbf{A}_b)^{-1}\mathbf{A}_{bu}(\mathbf{I} - \mathbf{A}_u)^{-1} + \boldsymbol{\ell}_u(\mathbf{I} - \mathbf{A}_u)^{-1} \end{bmatrix} \\
&= \mathbf{v}\begin{bmatrix} \mathbf{R}_b & \mathbf{R}_{bu} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{v}_b\mathbf{R}_b & \mathbf{v}_b\mathbf{R}_{bu} \end{bmatrix}
\end{aligned} \tag{11}$$

Then the left eigenequation $\lambda\mathbf{v} = \mathbf{vR}$ yields two equations: $\lambda\mathbf{v}_b = \mathbf{v}_b\mathbf{R}_b$ and $\lambda\mathbf{v}_u = \mathbf{v}_b\mathbf{R}_{bu}$, and consequently $\sigma = 1-\lambda$ is determined by the basic outputs $\mathbf{R}_b$. It is seen that the theory of surplus value is only completed in the analysis of input-output relations [Marx, 1885, pt. III] rather than the presentation in [Marx, 1867].

[22]This corresponds to distinction between 'absolute' and 'relative' surplus value described in [Marx, 1867]. Note that the nonbasic sector therefore cannot produce 'relative' surplus value, see [Cockshott and Zachariah, 2006].

[23]Using (4), we have $\mathbf{vs} = \mathbf{vn} - \mathbf{vRn} = \sigma \mathbf{vn} = 0$, when $\sigma = 0$. Since $\mathbf{v} > \mathbf{0}$ and $\mathbf{s} \geq \mathbf{0}$ it follows that $\mathbf{s} = \mathbf{0}$. By definition, $\mathbf{s} = \mathbf{n} - \mathbf{Rn} = (\mathbf{I} - \mathbf{R})\mathbf{n}$. Using the partitioning of $\mathbf{R}$ in (11), it follows that the net production of nonbasic outputs is a surplus product, that is, $\mathbf{n}_u = \mathbf{s}_u$. Using $\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{n}$, we have that the activity levels in the nonbasic sectors equal $\mathbf{q}_u = (\mathbf{I} - \mathbf{A}_u)^{-1}\mathbf{n}_u = (\mathbf{I} - \mathbf{A}_u)^{-1}\mathbf{s}_u = \mathbf{0}$.

[24]Consider redeploying the resources devoted to support the nonbasic sector to the basic sector alone. Let the net product before and after the change be $\mathbf{n}$ and $\mathbf{n}'$, respectively, where total employment remains the same, i.e. $\mathbf{vn}' = \mathbf{vn}$. Suppose the redeployment is such that the net product in the basic sector is increased uniformly by a factor α, i.e.,

$$\mathbf{n}' = \begin{bmatrix} (1+\alpha)\mathbf{n}_b \\ \mathbf{0} \end{bmatrix}.$$

Then it follows that the factor is $\alpha = \frac{\mathbf{v}_u \mathbf{n}_u}{\mathbf{v}_b \mathbf{n}_b}$. The resulting change in the surplus product of the economy is

$$\begin{aligned} \boldsymbol{\Delta} &= \mathbf{s}' - \mathbf{s} \\ &= (\mathbf{I} - \mathbf{R})\mathbf{n}' - (\mathbf{I} - \mathbf{R})\mathbf{n} \\ &= (\mathbf{I} - \mathbf{R}) \begin{bmatrix} \alpha \mathbf{n}_b \\ -\mathbf{n}_u \end{bmatrix} \\ &= \begin{bmatrix} \alpha(\mathbf{I} - \mathbf{R}_b)\mathbf{n}_b + \mathbf{R}_{bu}\mathbf{n}_u \\ -\mathbf{n}_u \end{bmatrix}, \end{aligned} \tag{12}$$

where the top rows correspond to the basic sectors.

[25]A early step towards such an analysis can be found in [Marx, 1885, ch. 20].

[26]That is, the relation satisfies i) reflexivity: $\mathbf{b} \preceq \mathbf{b}$ and ii) transitivity: $\mathbf{b} \preceq \mathbf{b}'$ and $\mathbf{b}' \preceq \mathbf{b}''$ imply $\mathbf{b} \preceq \mathbf{b}''$. The value form also induces an equivalence relation $\mathbf{b} \sim \mathbf{b}'$, for all pairs $(\mathbf{b}, \mathbf{b}')$ that satisfy $\mathbf{b} \preceq \mathbf{b}'$ and $\mathbf{b}' \preceq \mathbf{b}$.

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