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# Affirmative Action Subcontracting Regulations in Dynamic Procurement Auctions

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## Abstract

I study affirmative action subcontracting regulations in a model where governments use auctions to repeatedly procure goods and services at the lowest possible price. Through using disadvantaged subcontractors, prime contractors build relationships over time, resulting in lower subcontracting costs in future periods. I find that regulation in the form of a minimum subcontracting requirement expands bidder asymmetries, favoring prime contractors with stronger relationships over those with weaker ones. Simulations show that the manner in which relationships evolve determines not only the utilization of disadvantaged subcontractors but also the procurement costs attained under affirmative action.

## 1 Introduction

Public procurement is a substantial part of government spending. In 2015, government procurement accounted for 29.1 percent of all government spending and 11.9 percent of GDP in OECD countries.<sup>1</sup> Embedded within many of these procurement markets are affirmative action regulations, which are implemented to facilitate the participation of disadvantaged<sup>2</sup> firms in government contracting. Although affirmative action can take on many different forms, a common brand of policies in procurement are mandatory subcontracting goals. Under these policies, a prime contractor (or prime) must set aside a percentage share of a contract for subcontractors (or subs) designated as disadvantaged.

A key factor in a prime's disadvantaged subcontractor selection and associated subcontracting cost is their relationship with their pool of disadvantaged firms. In a needs assessment report by the Minnesota

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<sup>1</sup>See Government at a Glance 2017, which is available at [http://dx.doi.org/10.1787/gov\\_glance-2017-en](http://dx.doi.org/10.1787/gov_glance-2017-en).

<sup>2</sup>In U.S. procurement, firms that benefit from these policies tend to be small firms owned and controlled by ethnic minorities or women, but veteran-owned small businesses and small businesses in general can also benefit from affirmative action policies.

Department of Transportation,<sup>3</sup> surveys revealed that prime contractors rely on relationships to identify and hire disadvantaged subcontractors and that primes prefer to hire disadvantaged firms with whom they have existing relationships. The report concluded that relationships and credibility were integral to primes meeting their affirmative action requirements. The economics literature indicates that relationships can serve as a mechanism to lower coordination costs, promote learning-by-doing (Kellogg (2011)), and establish reputations (Banerjee and Duflo (2000)). Gil and Marion (2013) find that prior subcontracting interactions reduce bids on California highway procurement contracts, which is suggestive of lower subcontracting costs. Thus, the goal of expanding disadvantaged subcontractor utilization and the cost of affirmative action are both tied to the relationships primes build with their disadvantaged subs.

A shortcoming of the literature is that it does not directly address this relationship dynamic. Instead, the literature often relies on a static framework or uses proxies for future demand to approximate continuation values.<sup>4</sup> In this paper, I seek to fill that gap in the literature by investigating how dynamic relationship formation impacts procurement auctions in a model with affirmative action. To do so, I numerically solve for the Markov-perfect equilibrium of a repeated auction game with relationship dynamics and contrast the equilibria obtained with and without affirmative action subcontracting quotas. Primes in my model stochastically improve relationships through the continued utilization of disadvantaged subs, leading to an expectation of lower disadvantaged subcontracting costs in future periods. This relationship-building dynamic endogenously creates asymmetries between bidders, where primes with stronger relationships have a cost advantage over primes with weaker ones. By requiring primes to use disadvantaged firms, affirmative action expands this asymmetry and increases the marginal value from attaining a better state of relationships in the future. As a result, farsighted primes, which are primes that have a positive discount factor, have more of an incentive to subcontract with disadvantaged firms relative to myopic primes, which are primes with a discount factor of zero, and this incentive is amplified in markets with affirmative action.

The dynamic framework in this paper can answer several questions that a static framework cannot. Given that an objective of these programs is to remove barriers to the participation of disadvantaged firms in contracting,<sup>5</sup> and in the subcontracting case, one of those barriers is a lack of established relationships – a dynamic analysis can explore how relationships evolve and how much affirmative action contributes to that evolution. Equally relevant is the long-run impact of removing affirmative action programs, which has

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<sup>3</sup>The full name of the report is the MnDOT DBE and OJT Program Needs Assessment.

<sup>4</sup>Rosa (2018) and De Silva et al. (2012) use a static framework and Marion (2009) proxies for future demand using upcoming opportunities for disadvantaged firms.

<sup>5</sup>For U.S. Department of Transportation contracting, see 64 FR 5126, Feb. 2, 1999, as amended at 79 FR 59592, Oct. 2, 2014.

been implemented through laws such as California’s Proposition 209.<sup>6</sup> In this case, a dynamic structure can provide insights on how relationships – and therefore, disadvantaged subcontracting – will adjust when the quotas are removed.

I explore these questions through a model simulated under a range of different parameter and starting values. I find that the manner in which relationships transition between periods has implications for how affirmative action affects a given market. When relationships are long-lasting, affirmative action has negligible effects on bids yet leads to marked increases in disadvantaged subcontracting. When relationships deteriorate, affirmative action still improves subcontracting but at the cost of higher bids. These simulations highlight the importance of accounting for relationships in evaluating affirmative action programs.

My paper is closely related to the literature on mandatory subcontracting goals in government procurement contracts. Rosa (2018) studies Disadvantaged Business Enterprise (DBE) subcontracting goals in New Mexico using an estimated model of bidding and subcontracting that is similar to my paper. He finds that subcontracting goals may not lead to significant changes in bids because primes have to use a common pool of disadvantaged subs, leading to lower markups in equilibrium. I extend his model by including relationship dynamics and allowing for asymmetries in the cost of using disadvantaged firms. Other empirical papers on DBE subcontracting goals include Marion (2009) who finds that subcontracting goals significantly increased subcontracting and the winning bids and De Silva et al. (2012) who use a structural model to compare costs across contracts with and without subcontracting goals in Texas, finding negligible differences in project costs. Marion (2017) studies DBE subcontracting goals in Iowa comparable to the ones I investigate in my paper, focusing on how exemption policies impact DBE utilization.

My model borrows methods from the dynamic auction literature. In that literature, the model that is closest to mine is Jeziorski and Krasnokutskaya (2016). They use a dynamic model of subcontracting and bidding to explore how subcontracting affects capacity-constrained bidders, finding that subcontracting reduces bidder asymmetries by allowing primes to modify current costs and control future costs via backlog accumulation. Although I borrow their subcontracting model, our papers differ in dynamics; their paper has dynamic capacity, while my paper has dynamic relationship formation. This distinction fundamentally changes the role of subcontracting. In my model, primes subcontract with disadvantaged firms to gain a cost advantage in future periods through relationship formation; therefore, subcontracting *increases* bidder asymmetry in future periods. My dynamics also differ from Saini (2012) – which is an earlier paper that proposed a framework for investigating equilibrium bidding with capacity dynamics, except without subcontracting.

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<sup>6</sup>See Marion (2009) for an empirical analysis of Proposition 209’s effect on bidding and subcontracting and Holzer and Neumark (2000) for a general overview of affirmative action laws and literature.

Our papers relate in their study of equilibrium behavior in dynamic auctions.

My simulation results and the asymmetries generated through relationship formation connect to the literature on asymmetric auctions and how auction formats rank in different settings. Kirkegaard (2012) and Kirkegaard (2014) use a mechanism design approach to rank revenues from the first-price and second-price auction formats when there are two bidders, finding that the first-price auction is generally more profitable under certain value distributions. Maskin and Riley (2000) consider three separate cases where it is possible to rank the revenue from first- and second-price auctions. In contrast to this literature, I explore auctions with varying degrees of affirmative action and relationship formation instead of auctions with different formats, although the model’s complexity limits me to simulations. Additionally, I consider how these different environments affect yet another outcome – disadvantaged subcontracting – along with the procurement analog of revenues (i.e., procurement costs). This additional consideration is an integral part of evaluating the effectiveness of affirmative action regulations.

The remainder of the paper has the following structure. Section 2 outlines the model, and section 3 characterizes the model’s equilibrium. Section 4 contains solved examples of the model, which I use to show how affirmative action subcontracting regulations affect bidding and disadvantaged subcontracting and to study the long-run implications of subcontracting regulations. Section 5 concludes.

## 2 Model

In this section, I describe the dynamic procurement auction model with subcontracting regulation. Although I make references to a repeated construction project, the model applies to many repeated procurement settings with subcontracting possibilities.

### Environment

In my environment, time is discrete, and the horizon is infinite. Each period, two infinitely lived prime contractors bid for the rights to complete a homogeneous construction project. Prime contractors can either complete the entire project in house or award part of it to subcontractors, some of which belong to the disadvantaged group of subcontractors. For simplicity, I group in-house costs and costs from non-disadvantaged subcontractors for prime contractor  $i$  in period  $t$  into one cost,  $c_{i,t}$ , which I refer to as a prime contractor’s unregulated cost. Unregulated costs represent a prime contractor’s cost of completing the entire project without using any disadvantaged subcontractors, and I assume that they are private information and are

independently and identically distributed according to the CDF  $F_c$  with support  $[\underline{c}, \bar{c}]$ . In contrast, the disadvantaged subcontracting market determines the cost of using disadvantaged subcontractors.

Mirroring how the U.S. government awards its sealed-bid contracts, I assume that the government uses a first-price sealed-bid format to select the winning firm. In many procurement settings, government agencies have the right to reject any or all bids if they are irregularly high. Following the empirical literature,<sup>7</sup> I model that right as a secret reserve price with realization  $r_t$  that, for simplicity, is distributed independently across time according to the CDF  $F_R$  with support  $[\underline{r}, \bar{r}]$ . The winning prime contractor is then the one that submits the lowest bid, provided that it is below the reserve price. Observe that this bid-rejection power can come, in part, through smaller irregular (or fringe) bidders that bid myopically.

The government can also impose regulations on the amount of a project that primes must award disadvantaged subcontractors. Within the context of the model, the government requires that primes allocate a fraction of the project  $\bar{s} \in [0, 1]$  to disadvantaged subs in each period; if  $\bar{s} = 0$ , then the market is unregulated. This feature of the model resembles the subcontracting regulations in the U.S. Department of Transportation’s (USDOT’s) DBE Program.<sup>8</sup>

## Disadvantaged Subcontracting Market

I assume that each prime has their own pool of disadvantaged subcontractors, which I model as a single representative agent. In this framework, a prime’s cost of using this agent corresponds to the cost of using its pool. Therefore, I will refer to the agent as if it were the pool of disadvantaged subs for the remainder of this paper. Disadvantaged subcontractors myopically supply services according to their relationship with prime contractor  $i$  at time  $t$ . Suppose that this relationship can be summarized by a single state variable,  $\omega_{i,t}$ . Disadvantaged subcontractors then supply a share of total services,  $s_{i,t} \in [0, 1]$  according to disadvantaged cost function,  $P(s_{i,t}, \omega_{i,t})$ , which maps shares and relationships into a cost of using disadvantaged firms. Note that a disadvantaged firm can appear in both pools, meaning that pools can partially or completely overlap.<sup>9</sup>

In this setup, the disadvantaged cost function is central to a prime contractor’s disadvantaged subcontracting decision. In particular, it generates a trade-off between using disadvantaged subcontractors and their alternatives. This trade-off is affected by relationships in the sense that a prime with better relationships

<sup>7</sup>See Jofre-Bonet and Pesendorfer (2003).

<sup>8</sup>Although it is currently illegal to use explicit quotas in the U.S., states will regularly set DBE goals on their procurement projects, which require prime contractors to award a pre-specified percentage share of a project to subcontractors that qualify as disadvantaged. Prime contractors typically meet this requirement; see Marion (2017) and Rosa (2018).

<sup>9</sup>The use of a common pool of subcontractors can also imply affiliation, or dependence, between prime contractors’ costs of completing a project. I abstract away from affiliation in this paper, as it would add even more complexity to an already involved model. Hubbard et al. (2012) provide a methodology to account for affiliation in general auction settings.

may face lower disadvantaged subcontracting costs for the same share of the project completed. In real-world procurement, part of the cost of using disadvantaged subs comes from monitoring. Therefore, it is plausible that primes with better relationships would have lower disadvantaged subcontracting costs since, through repeat usage, disadvantaged subs may become more trustworthy. Primes may also have access to a more extensive network of disadvantaged subcontractors when their relationships are better, which can lead to lower quotes on subcontracting services. These examples serve as motivation for my relationship dynamics.

Observe that my subcontracting model is stylized and rules out some potentially complex subcontracting and capacity issues. In particular, I do not allow the prime contractors or the disadvantaged subcontractors to be capacity constrained, and I abstract away from task heterogeneity. My assumptions do, however, lead to a tractable model that will be shown to illustrate some of the primary facets of a prime contractor's disadvantaged subcontracting decisions. Namely, that more efficient primes are less inclined to use disadvantaged subcontractors and that subcontracting costs increase as disadvantaged subcontractors complete more of the project. I explore these and other issues further in section 3.1.

I make the following regularity assumptions on the disadvantaged cost function:

*Assumptions on  $P$ :*

1.  $P$  is increasing and convex in its first argument and decreasing in its second argument.
2.  $P(0, \omega_{i,t}) = 0$  for every  $\omega_{i,t}$ .
3.  $\lim_{s_{i,t} \rightarrow 1} P'(s_{i,t}, \omega_{i,t}) = +\infty$ .
4.  $P_{12}(s_{i,t}, \omega_{i,t}) < 0$ .

Assumption 1 requires that disadvantaged subcontracting costs increase at an increasing rate as primes use more disadvantaged subcontractors and decrease with better relationships. This assumption helps generate a unique subcontracting solution. Assumption 2 ensures that prime contractors that use no disadvantaged subcontractors pay nothing to disadvantaged subcontractors, and assumption 3 makes subcontracting large portions of a project prohibitively costly. Governments typically have a maximum subcontracting threshold for firms to qualify as prime contractors. The USDOT, for example, has a subcontracting maximum of around 60 percent for its highway procurement projects. Assumption 3 is meant to approximate that rule. My fourth and final assumption means that the marginal price increase decreases with better relationships. That is to say, the disadvantaged cost function is flatter with better relationships.

## Relationship Dynamics

I assume that the state variable that describes relationships evolves according to how often primes use disadvantaged subcontractors. In particular, I assume that the state variable transitions stochastically according to

$$\sigma_i(\omega_{i,t}, s_{i,t}) = \psi\omega_{i,t} + s_{i,t} + \epsilon_{i,t}, \quad (1)$$

where  $\psi$  measures how easily past relationships carry over into future periods,  $s_{i,t}$  is prime contractor  $i$ 's disadvantaged subcontracting share, and  $\epsilon_{i,t}$  is a random shock that is drawn from a time-invariant distribution  $F_\epsilon$  with  $\mathbb{E}[\epsilon_{i,t}] = 0$ . If  $\psi = 1$  in equation (1), for example, then past relationships are expected to persist into future periods, whereas  $\psi < 1$  means that past relationships tend to diminish over time. This parameter's value plays a major role in a prime contractor's dynamic incentive to use disadvantaged subs since more persistent relationships lead to longer stretches of time with lower disadvantaged subcontracting costs in the future. In real-life procurement,  $\psi$  might be low if there is high turnover in a prime's pool of disadvantaged subcontractors from firms exiting the market. The shock,  $\epsilon_{i,t}$ , captures any randomness that affects the stock of relationships next period.<sup>10</sup>

To ensure that prime contractors will not face implausibly high or low disadvantaged subcontracting costs, I bound each state variable from above by a constant  $\overline{M}$  and below by a constant  $\underline{M}$  so that  $\omega_{i,t} \in [\underline{M}, \overline{M}]$ . The state space is then given by  $\Omega \in [\underline{M}, \overline{M}] \times [\underline{M}, \overline{M}]$ , and from now on, I will define  $\boldsymbol{\omega}_t = (\omega_{i,t}, \omega_{-i,t})$  as the state vector for prime  $i$  at time  $t$ .

## Timing and Equilibrium Description

At the beginning of each period, primes observe their unregulated cost and the public history of states, disadvantaged shares, and bids by them and their competitor. For tractability, I assume that primes use Markov strategies and focus attention on Markov-perfect equilibria, where strategies only depend on payoff-relevant information. In my model, that information is the state vector of prime relationships, as it summarizes the current period's disadvantaged cost while its transition gives disadvantaged costs in the future. Furthermore, payoffs do not depend on time given the state, so I can focus on stationary strategies.

Given my equilibrium assumptions, the relevant information that primes observe at the start of each period is their unregulated cost,  $c_{i,t}$  and the state,  $\boldsymbol{\omega}_t$ . Primes use these two observations and their knowledge of

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<sup>10</sup>Note that if a prime uses no disadvantaged subs in a period, then equation (1) implies that  $\sigma_i(\omega_{i,t}, 0) = \omega_{i,t+1} = \psi\omega_{i,t} + \epsilon_{i,t}$ . That is to say, relationships will not necessarily reset to the lowest value following a loss or a choice not to subcontract.



the reserve price and unregulated cost distributions to choose their disadvantaged subcontracting shares,  $s_{i,t} \in [\bar{s}, 1]$ , and their bids on the project,  $b_{i,t} \in \mathbb{R}_+$ . A stationary Markov strategy is therefore a mapping between a prime's unregulated costs and the state, given the current regulatory regime, into disadvantaged subcontracting shares and bids. I define the disadvantaged subcontracting strategy for prime contractor  $i$  given regulation  $\bar{s}$  as  $S_i(c_i, \omega; \bar{s})$  and the bidding strategy by  $B_i(c_i, \omega; \bar{s})$ , where stationarity allows me to drop the time subscript. The stationary strategy profile is then  $\{B_i(c_i, \omega; \bar{s}), S_i(c_i, \omega; \bar{s})\}_{i=1,2; \omega \in \Omega; c_i \in [\underline{c}, \bar{c}]}$ .

Implicit in the timing is the assumption that prime contractors must commit to their disadvantaged subcontracting, which prevents primes from revising their subcontracting plans after letting. Many states have laws in place to avoid just that; in New Mexico, for example, prime contractors can be fined if their disadvantaged shares ex-post do not align with their planned shares. In the model, I assume that these penalties are large enough to deter firms from changing their plans, although models without commitment may also be of interest.

### 3 Equilibrium

#### Bellman Equation

I now construct a prime contractor's Bellman equation. Let  $V_i(\omega)$  be prime contractor  $i$ 's value function given state  $\omega$  prior to observing its unregulated cost. For ease of notation, I drop the reliance of optimal disadvantaged shares and bids on the state and subcontracting regulation where it does not cause confusion. I then build the Bellman equation from the possible outcomes of each period's procurement.

In each period, a prime can either outbid their competitor or not and either outbid the reserve price or not. Therefore, there are four possible outcomes for prime contractor  $i$ :

1. Prime  $i$  bids below their opponent and reserve price.
2. Prime  $i$  bids below their opponent but above the reserve price.
3. Prime  $i$  bids above their opponent and their opponent bids below the reserve price.
4. Prime  $i$  bids above their opponent, but their opponent bids above the reserve price.

Each of these cases yields different payoffs for the prime.

**Case 1.** If the prime outbids its competitor with an unregulated cost of  $c_i$ , a bid of  $b_i$ , and a disadvantaged subcontracting share of  $s_i$  while bidding below the reserve price, then its payoff is

$$b_i - (1 - s_i) c_i - P(s_i) + \delta \mathbb{E}_\epsilon V_i(\sigma_i(s_i), \sigma_{-i}(0)).$$

Since the prime wins the contract in this case, it receives the static profit of  $b_i - (1 - s_i) c_i - P(s_i)$ , which consists of the bid less the fraction of the unregulated cost used on the project,  $(1 - s_i) c_i$ , and the disadvantaged subcontracting cost,  $P(s_i)$ . The prime contractor also receives a dynamic payoff of  $\delta \mathbb{E}_\epsilon V_i(\sigma_i(s_i), \sigma_{-i}(0))$ , where  $\delta \in [0, 1)$  is the common discount factor. This portion of the payoff accounts for the future value of using the disadvantaged subcontracting share  $s_i$ .

**Case 2.** In the event that the prime contractor outbids its opponent but bids above the reserve price, its payoff is

$$\delta \mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)).$$

Indeed, the prime contractor loses in this case. However, since there is no winner, no primes use any disadvantaged subcontractors, and the next period's payoff is the corresponding discounted value.

**Case 3.** When prime  $i$  is outbid by its competitor and the competitor beats the reserve price, prime  $i$ 's expected payoff is

$$\delta \mathbb{E}_{c_{-i}} [\mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(S_{-i}(c_{-i}))) | B_{-i} < b_i].$$

In words, prime  $i$  receives the expected future value of its competitor using equilibrium share  $S_{-i}(c_{-i})$ . Since the prime must be outbid for this case to happen, the expectation is conditional on the competitor's equilibrium bid,  $B_{-i}$ , being less than their bid,  $b_i$ . Observe that the competitor's unregulated cost does not appear in the first two cases, which is why there is no conditioning argument in those expressions.

**Case 4.** The final case occurs when prime  $i$  bids above its competitor, but the competing firm bids above the reserve price. In this situation, prime  $i$ 's expected payoff is

$$\delta \mathbb{E}_{c_{-i}} [\mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) | B_{-i} < b_i].$$

**The Value Function and Equilibrium.** Combining all of the cases together and weighting them by their appropriate probabilities yields the value function. Let  $W_i(b_i) = 1 - F_c((B_{-i})^{-1}(b_i))$  be the probability of

winning given a bid of  $b_i$ . The value function is then

$$\begin{aligned}
V_i(\boldsymbol{\omega}) = & \int_{c_i} \max_{b_i, s_i} \left\{ W_i(b_i) \left[ (1 - F_R(b_i)) (b_i - (1 - s_i) c_i - P(s_i)) \right. \right. \\
& + \delta \mathbb{E}_\epsilon V_i(\sigma_i(s_i), \sigma_{-i}(0)) + F_R(b_i) \delta \mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) \left. \right] \\
& + (1 - W_i(b_i)) \delta \mathbb{E}_{c_{-i}} \left[ (1 - F_R(B_{-i}(c_{-i}))) \mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(S_{-i}(c_{-i}))) \right. \\
& \left. \left. + F_R(B_{-i}(c_{-i})) \mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) \mid B_{-i} < b_i \right] \right\} dF_c(c_i). \tag{2}
\end{aligned}$$

A stationary Markov-perfect equilibrium in this game consists of strategy profile  $\{B_i(c_i, \boldsymbol{\omega}), S_i(c_i, \boldsymbol{\omega})\}_{i=1,2; \boldsymbol{\omega} \in \Omega; c \in [\underline{c}, \bar{c}]}$  and value functions  $\{V_i(\boldsymbol{\omega})\}_{i=1,2; \boldsymbol{\omega} \in \Omega}$  such that (i) given  $B_{-i}$  and  $S_{-i}$ ,  $V_i(\boldsymbol{\omega})$  solves the Bellman equation in (2) for every  $i$  and (ii) given  $B_{-i}$  and  $S_{-i}$ ,  $B_i$  and  $S_i$  solve the optimization on the right-hand side of equation (2) for all  $\boldsymbol{\omega}$  and all  $c_i$  given correct beliefs about the unregulated cost distributions and the reserve price distributions.

I follow the literature in assuming properties on how the value function changes in state.<sup>11</sup> In particular, I assume that the value function is weakly increasing in  $\omega_i$ ,  $V_{i,1}(\omega_i, \omega_{-i}) \geq 0$ , and weakly decreasing in  $\omega_{-i}$ ,  $V_{i,2}(\omega_i, \omega_{-i}) \leq 0$ . The intuition behind these assumptions is that primes with better relationships receive lower disadvantaged subcontracting prices, which lowers their costs and increases their profits. Similarly, when a prime's opponent has a better state, the prime must bid more aggressively to compete with its lower-cost competitor, leading to reduced profits. I verify these properties in my simulations.

### Subcontracting Strategies

I now turn to the prime contractor's optimal subcontracting strategy. Primes choose their disadvantaged subcontracting shares such that

$$S_i(c_i) \in \arg \max_{s_i \in [\bar{s}, 1]} b_i - (1 - s_i)c_i - P(s_i) + \delta \mathbb{E}_\epsilon V_i(\sigma_i(s_i), \sigma_{-i}(0)).$$

Let  $V_{i,1}(\cdot, \cdot)$  be the partial derivative of the value function with respect to its first argument and  $V_{i,11}(\cdot, \cdot)$  be the second partial derivative, again with respect to the first argument. To have a unique maximum, the

<sup>11</sup>See Jeziorski and Krasnokutskaya (2016), who make similar kinds of assumptions on the shape of the value function when firms are capacity constrained. Saini (2012) finds that those assumptions hold for a wide range of parameter values.

prime's objective must be concave, or

$$-P''(s_i) + \delta [\mathbb{E}_\epsilon V_{i,11}(\sigma_i(s_i), \sigma_{-i}(0))] < 0.$$

Given my first assumption on the convexity of  $P$ , this condition would be satisfied if  $V_i$  is concave in its first argument. However, proving the concavity of  $V_i$  analytically is challenging in this setting; instead, I follow the literature in verifying that this condition holds via simulation.<sup>12</sup>

Assuming the second-order conditions hold, the prime contractor's optimal share is defined implicitly as the solution to

$$P'(s_i) = c_i + \delta \mathbb{E}_\epsilon V_{i,1}(\sigma_i(s_i), \sigma_{-i}(0)) \quad (3)$$

but takes on a corner value of  $\bar{s}$  if

$$P'(\bar{s}) > c_i + \delta \mathbb{E}_\epsilon V_{i,1}(\sigma_i(\bar{s}), \sigma_{-i}(0)).$$

My third assumption on  $P$  prevents primes from subcontracting the entire project.

Intuitively, the left-hand side of equation (3) is a prime's marginal cost of increasing the share; the right-hand is the marginal benefit – which consists of the marginal savings on unregulated costs,  $c_i$ , and the discounted marginal change in future value,  $\delta \mathbb{E}_\epsilon V_{i,1}(\sigma_i(s_i), \sigma_{-i}(0))$ . With no regulations, prime contractors choose a share that equates their marginal benefit and marginal cost, meaning that they cost minimize in the event of a win. Regulations distort these shares upwards if the cost-minimizing share falls below the required share.

There are a few properties that arise from a prime's optimal subcontracting behavior. I summarize those properties below and include their proofs in the appendix.

**Proposition 1.** *Disadvantaged subcontracting shares are weakly increasing in unregulated costs,  $c_i$ .*

**Proposition 2.** *If  $P_{12}$  is sufficiently negative, then disadvantaged shares are weakly increasing in own relationship,  $\omega_{i,t}$ .*

These properties follow from the implicit function theorem applied to the first-order conditions on subcontracting and mean that more efficient primes and primes with weaker relationships will subcontract less with

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<sup>12</sup>Jeziorski and Krasnokutskaya (2016) use simulation to verify a similar assumption, finding that it holds in their analysis.

disadvantaged firms than less efficient primes and primes with stronger relationships. Intuitively, prime contractors with higher unregulated costs have a higher marginal benefit from subcontracting and, therefore, are more inclined to subcontract. Since primes likely derive higher value from having stronger relationships with disadvantaged firms, the dynamic component increases the marginal benefit – thus increasing the incentive to subcontract for farsighted primes.

Regulations also play an essential role. As the government requires higher disadvantaged shares, primes with strong relationships become more advantaged relative to primes with weak relationships since every firm must subcontract. Regulations then lead to more pronounced differences in costs between primes with strong and weak ties.

For ease of exposition, I will follow the literature in assuming that a prime’s disadvantaged subcontracting function,  $S(c_i)$ , is smooth so that its derivative exists everywhere.<sup>13</sup> This assumption is not too restrictive since a kink can only occur at the highest  $c_i$  such that  $S(c_i) = \bar{s}$ , which means that the derivative exists almost everywhere. Moreover, a kinked function can be approximated well with a smooth function.

## Bidding Strategies

Next, I derive the bidding strategies. In this environment, a prime’s bid must account for both static costs and dynamic changes in state that come from winning. To this end, the lowest possible value from losing for prime  $i$  in state  $\omega$  is

$$\underline{V}_i(\omega) = \mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(S(\bar{c}_{-i}))),$$

which follows from my assumptions on  $V$  and the properties of  $S$ . Dropping the reliance of  $\underline{V}_i$  on  $\omega$ , I define the effective cost as

$$\phi(c_i; \omega) = \underbrace{(1 - S(c_i))c_i + P(S(c_i))}_{\text{static}} - \underbrace{\delta [\mathbb{E}_\epsilon V_i(\sigma_i(S(c_i)), \sigma_{-i}(0)) - \underline{V}_i]}_{\text{dynamic}}, \quad (4)$$

which is the cost relevant for bidding. Notice that the effective cost has two components: the static cost of completing the project and the dynamic opportunity cost of winning against an opponent with the highest unregulated cost. In contrast to Jeziorski and Krasnokutskaya (2016), the dynamic part is positive, so a forward-looking bidder has a lower effective cost than a myopic bidder.

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<sup>13</sup>Jeziorski and Krasnokutskaya (2016) make a similar assumption by assuming that the primes choose to subcontract at all cost realizations.

Observe that

$$\phi'(c_i; \boldsymbol{\omega}) = (1 - S(c_i)) + \{P'(S(c_i)) - \delta \mathbb{E}_\epsilon V_{i,1}(\sigma_i(S(c_i)), \sigma_{-i}(0)) - c_i\} S'(c_i) > 0.$$

This property holds because the term inside of the brackets is zero by the first-order conditions, and  $S'(c_i)$  is zero at the corner and positive otherwise. Effective costs are, therefore, monotone in  $c_i$  and can be inverted.

A prime's bid can now be expressed as a function that maps effective costs into bids given the state,  $B_i(\cdot, \boldsymbol{\omega}) : [\underline{\phi}_i, \bar{\phi}_i] \rightarrow [\underline{B}(\boldsymbol{\omega}), \bar{B}(\boldsymbol{\omega})]$ , instead of unregulated costs into bids. The inverse bid function,  $\xi_i$ , then maps bids into effective costs,  $\xi_i(\cdot, \boldsymbol{\omega}) : [\underline{B}(\boldsymbol{\omega}), \bar{B}(\boldsymbol{\omega})] \rightarrow [\underline{\phi}_i(\boldsymbol{\omega}), \bar{\phi}_i(\boldsymbol{\omega})]$ . A prime chooses bid  $b_i$  to maximize expected profits:

$$\begin{aligned} \max_{b_i} & \left[ (b_i - \phi_i) (1 - F_R(b_i)) + \delta (\mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) - \underline{V}_i) F_R(b_i) \right] (1 - F_{\phi, -i}(\xi_{-i}(b_i))) \\ & + \delta \int_{\underline{\phi}_{-i}}^{\xi_{-i}(b_i)} \left[ (1 - F_R(B_{-i}(\phi))) (\mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(S_{-i}(\phi)))) \right. \\ & \left. + F_R(B_{-i}(\phi)) \mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) - \underline{V}_i \right] f_{\phi, -i}(\phi) d\phi. \end{aligned}$$

Taking the first-order conditions leads to the following system of differential equations:

$$\begin{aligned} (1 - F_{\phi, -i}(\xi_{-i}(b_i))) [(1 - F_R(b_i)) - (b_i - \phi_i) f_R(b_i) + \delta (\mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) - \underline{V}_i) f_R(b_i)] & \quad (5) \\ - f_{\phi, -i}(\xi_{-i}(b_i)) \xi'_{-i}(b_i) (1 - F_R(b_i)) [b_i - \phi_i - \delta (\mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(S_{-i}(\xi_{-i}(b_i)))) - \underline{V}_i)] & = 0. \end{aligned}$$

Note that by making primes myopic (i.e., setting  $\delta = 0$ ) and removing the reserve price, one arrives at the standard first-order conditions for an auction with two bidders asymmetric in their effective costs. Furthermore, notice that the support of a prime's effective cost distribution can change with the state. Indeed, a prime with the highest unregulated cost can use disadvantaged subcontractors to lower its effective cost and can achieve even lower effective costs with better relationships. Therefore, the auction is asymmetric as in Maskin and Riley (2000) but with different supports like in Kaplan and Zamir (2012). In these environments, a bidder with no probability of winning – which occurs either because their effective cost is above the highest equilibrium bid or the reserve price – is indifferent between any bid at or above its reserve price. I follow Kaplan and Zamir (2012) in assuming that primes that find themselves in this situation bid their effective

cost.

Let  $[\underline{b}, \bar{b}]$  be the interval of equilibrium bids where a prime has a positive probability of winning. Assuming, without loss of generality, that  $\bar{\phi}_1 \leq \bar{\phi}_2$ , the boundary conditions are

$$\begin{aligned}\xi_1(\bar{b}, \omega) &= \bar{\phi}_1 \\ \xi_2(\bar{b}, \omega) &= \bar{b} \\ \xi_1(\underline{b}, \omega) &= \underline{\phi}_1 \\ \xi_2(\underline{b}, \omega) &= \underline{\phi}_2,\end{aligned}\tag{6}$$

where  $\bar{b} = \min\{b_0, \bar{\phi}_2, \bar{r}\}$  and, following Jeziorski and Krasnokutskaya (2016),  $b_0$  is defined implicitly by the equation

$$\begin{aligned}(1 - F_{\phi,2}(b_0)) \left[ (1 - F_R(b_0)) - (b_0 - \bar{\phi}_1) f_R(b_0) + \delta (\mathbb{E}_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_1) f_R(b_0) \right] \\ - f_{\phi,2}(b_0) (1 - F_R(b_0)) \left[ b_0 - \bar{\phi}_1 - \delta (\mathbb{E}_\epsilon V_1(\sigma_1(0), \sigma_2(S_2(b_0))) - \underline{V}_1) \right] = 0.\end{aligned}\tag{7}$$

The intuition behind the boundary conditions is that both primes must submit the same low bid in equilibrium. Otherwise, the low prime could increase its bid without changing its probability of winning. The high bid is slightly more involved. When prime 1 has the lowest effective cost, it can use its first-order conditions to find its high bid, which is equivalent to finding  $b_0$ . In equilibrium, the least efficient prime cannot make a profit, so  $\bar{b}$  must be the minimum of  $b_0$  and  $\bar{\phi}_2$ . Moreover,  $\bar{b}$  must be at or below the upper bound of the reserve price distribution since bids above the reserve price are rejected outright.

### 3.1 Discussion of Modeling Choices and Extensions

My analysis contains several abstractions used to produce a model with tractable results. In real procurement markets, projects can be more complex and can require primes to make more involved bidding and subcontracting decisions. In what follows, I discuss my modeling choices and consider additional factors that may influence how primes use disadvantaged subcontractors.

**Task Heterogeneity.** Procurement projects are rarely homogeneous and can consist of many different tasks. If tasks are sufficiently distinct, then primes may have to make subcontracting decisions in different markets, which will affect their disadvantaged subcontracting. Likewise, primes may have specialties in

specific tasks, which again will perturb their disadvantaged subcontracting decisions. Since accounting for heterogeneous tasks in my model is infeasible because it would require separate states for each task and each prime, I abstract away from this feature of the disadvantaged subcontracting market.

**Entry.** For simplicity, my model assumes that the two competing primes invariably enter each period’s auction, but in reality, primes can choose whether to participate. If entry is based on the expected profitability of a project, then subcontracting regulations can potentially deter entrants, resulting in higher procurement costs than my model predicts. Less likely to be impacted, though, is a prime’s disadvantaged subcontracting decisions since they are based on cost minimization rather than the number of entrants. In fact, Jeziorski and Krasnokutskaya (2016) show that subcontracting shares do not depend on the number of entrants in a closely related model, and empirically, studies such as Rosa (2018) find that disadvantaged subcontracting is uncorrelated with the number of participating firms. As a result, my findings on relationships formed through disadvantaged subcontracting are unlikely to change significantly with entry unless there is a substantial deterrence effect.

**Capacity Constraints.** The dynamic auction literature typically has a firm’s capacity as the dynamic variable.<sup>14</sup> In these environments, a firm that wins a project now will have higher costs in the next period because they are operating closer to their capacity. In reality, disadvantaged firms are usually a small part of total subcontracting and are less likely to be impacted by a prime’s capacity constraints. Moreover, some studies show that capacity constraints are not a major factor in a prime’s overall subcontracting decisions; in the case of construction, see González et al. (2000). As such, my analysis focuses on dynamic relationship formation, which is one of the primary arguments for using affirmative action regulations in subcontracting.

Nevertheless, one could consider the possible implications of including capacity dynamics in the model. Doing so would be challenging since accounting for capacity requires additional state variables and would be subject to the curse of dimensionality; however, the literature provides some intuition. As indicated by Jeziorski and Krasnokutskaya (2016), subcontracting allows primes to mitigate unfavorable within-period and future cost draws by shifting more of the work to their subcontracting firms. Thus, firms closer to their capacity are more likely to use subcontractors. When applied to my setting, this intuition means that capacity constraints would provide yet another incentive for primes to use disadvantaged subs, possibly resulting in higher disadvantaged shares than predicted by my model without capacity constraints.

Related to capacity issues for prime contractors is the possibility that the disadvantaged firms are capacity

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<sup>14</sup>See Saini (2012), Jeziorski and Krasnokutskaya (2016), Jofre-Bonet and Pesendorfer (2003) and Groeger (2014).



constrained. Incorporating subcontractor capacity constraints into the model would again require additional state variables for each disadvantaged subcontractor, leading to a high-dimensional state space that quickly becomes unmanageable. Additionally, any capacity-related subcontracting concerns are likely short-lived because the work subcontracted to each firm tends to be small. For these reasons, I abstract away from capacity constraints in the disadvantaged subcontracting market.

**The Reserve Price Distribution.** I assume a time and state invariant reserve price distribution, but governments may wish to condition their reserve price on the state to allow for more leniency in the range of accepted bids when relationships are low. In practice, bids tend to be rejected for exceeding estimated costs, and cost estimates rarely factor in subcontracting regulations.<sup>15</sup> To keep the model simple, I abstract away from state-dependent reserve price distributions.

A somewhat related issue is that contracts rarely have only two bidders. There may be other less-frequent bidders that often bid, which the literature refers to as fringe bidders. Although the reserve price partially accounts for fringe bidders in my framework, an immediate extension would be to have the reserve price distribution depend on the number of fringe bidders. This extension complicates the model by adding another state variable, so I do not include it in this paper.

**Relationships with Other Subcontractors.** My analysis centers around the relationships built between prime contractors and disadvantaged subcontractors to investigate the long-run effects of affirmative action policies. In real procurement settings, similar relationships can form between prime contractors and other subcontractors not considered disadvantaged. Accounting for these relationships in the model would be challenging, though, as a prime contractor would need to consider their disadvantaged and non-disadvantaged subcontracting costs as well as the corresponding state of relationships for both types of subcontractors for them and their competitor. Dimensionality issues complicate an analysis of those considerations. My intuition for these settings is that those relationships would lower unregulated costs just as relationships with disadvantaged subcontractors lower effective costs. Primes with strong connections to other subcontractors would, therefore, have less of an incentive to subcontract with disadvantaged firms since it would be relatively more costly to do so. I explore the implications of different unregulated cost distributions in section (4.5).

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<sup>15</sup>In the state of Iowa, cost estimates and subcontracting regulations are handled by two different agencies.

## 4 Simulation

To illustrate the impact of affirmative action subcontracting regulations, I simulate bids, subcontracting, and industry outcomes in two situations: one in which there is no regulation and one in which there is regulation. This section contains those simulations and a brief description of my simulation methodology.

### 4.1 Simulation Method

To simulate the Markov-perfect equilibrium of the model, I must approximate the strategy profile  $\{B_i(c_i, \omega), S_i(c_i, \omega)\}_{i=1,2; \omega \in \Omega; c \in [c, \bar{c}]}$  and value functions  $\{V_i(\omega)\}_{i=1,2; \omega \in \Omega}$ . Implicit in calculating the bids, I must also approximate the PDF and CDF of the effective cost distribution. For the bids, value function, and effective cost distribution, I use Chebyshev polynomial approximations; for subcontracting, I use Hermite splines, which perform well in approximating the flat area that can occur at the corner.

My algorithm consists of an inner loop that calculates the strategy profiles on a select grid of states and an outer loop that iterates on the value function. In the inner loop, I solve equation (3) to get the shares and use the shares to approximate effective costs. In approximating the bids, I use projection methods with a mathematical programming with equilibrium constraints (or MPEC) optimization routine proposed by Hubbard and Paarsch (2009). To extend my strategy profile approximations to other states, I use linear interpolations. These equilibrium objects allow me to find the following period's value function, and I iterate until the difference between the current and next value functions is small.<sup>16</sup> My approach's structure is similar to that of the Rust (1987) nested fixed point approach used in estimating the parameters underlying a dynamic discrete choice problem. Both methods have a nesting structure – except mine nests the bid and subcontracting solutions inside of a dynamic programming problem, whereas the nested fixed point approach nests a dynamic programming problem inside of a maximum likelihood problem.

A concern with my approach, however, is that there can potentially be multiple equilibria. To ease that concern, I use the robust equilibrium selection rule proposed by Chen et al. (2009). This rule selects the equilibrium that corresponds to the limit of a finite game, and the intuition behind this rule is that one can induct backward in time for an arbitrary number of periods in a finite game by starting from a continuation value of zero. My algorithm emulates this backwards-induction process. Appendix B contains a more detailed description of my numerical routine.

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<sup>16</sup>To put the procedure's computational complexity into perspective, note that finding the effective costs and bidding strategies for all of the necessary grid points takes about 12 minutes when not run in parallel. This process is usually repeated 11 times in the outer loop for the value function to converge, meaning that a single parameter configuration can take over 2 hours to solve.

## 4.2 Simulation Parameters

Simulating outcomes from the theoretical model requires parametric assumptions on the disadvantaged cost function, the unregulated cost distribution, the secret reserve price distribution, and the states. I begin by assuming that the disadvantaged cost function takes the form

$$P(s_i, \omega_i) = \frac{s_i}{\omega_i(1 - s_i)}.$$

Observe that this specification adheres to my theoretical disadvantaged cost function assumptions for  $s_i \in [0, 1]$ .

For simplicity, I assume that unregulated costs are distributed uniformly on an interval bounded below by 0 and above by 1.5. In real procurement environments, the government often provides a range of estimated project cost values to primes prior to bidding. I envision a realization of 1 equivalent to the midpoint of any cost range; a value of 1.5 would then be the upper bound, while a value of 0.5 would be the lower bound. Consequently, I assume that the reserve price distribution is also uniform, but bounded below by the lower cost estimate of 0.5 and above by the upper cost estimate of 1.5. In other words, bids can only be rejected if they are within or above the established cost range.

To simulate plausible disadvantaged shares, I set the lower bound for the states to 0.5 and the upper bound to 2. Given my previous assumptions, a prime that draws the average unregulated cost will not use any disadvantaged subcontractors in the lowest state absent regulations. In contrast, an equivalent draw in the highest state leads to a disadvantaged share of about 18 percent. I set the distribution of the stochastic part of the relationship formation process,  $\epsilon$ , equal to a uniform distribution on the interval ranging from -0.1 to 0.1. This parameterization means that primes expect relationships to remain the same, but the highest random draw is equivalent to the relationships formed under a disadvantaged subcontracting share of 10 percent in the current period.

I investigate two different sets of values for the quota level,  $\bar{s}$ , and the relationship persistence parameter,  $\psi$ . Subcontracting regulations typically range from 1 to 15 percent. In Iowa, for example, the DOT can recommend a DBE goal anywhere from 1 to 10 percent, and in states such as Texas, DBE goals can reach as high as 15 percent.<sup>17</sup> To highlight the distinction between projects with and without subcontracting regulations, I contrast the more extreme  $\bar{s} = 0.15$  case with the case where  $\bar{s} = 0$ . I consider persistence parameter values of  $\psi \in \{0.95, 1\}$ , which covers cases where relationships are expected to persist and where there is

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<sup>17</sup>See the 2018 Iowa DBE Program Plan for Iowa and De Silva et al. (2012) for Texas.

some expected deterioration. Finally, I hold the common discount factor fixed at  $\delta = 0.95$ , corresponding to an interest rate of about 5 percent. Table 1 summarizes the elements of the model, their meanings, my assumed values, and the motivation for those values.

Table 1: Parameter Values

Model Element	Meaning	Value	Motivation
$F_c$	Unregulated cost dist.	$\mathcal{U}[0, 1.5]$	Cost relative to govt. estimate midpoint
$F_R$	Reserve pride dist.	$\mathcal{U}[0.5, 1.5]$	Govt. estimated cost range
$\underline{M}$	Relationship lower bound	0.5	Plausible shares
$\overline{M}$	Relationship upper bound	2	Plausible shares
$F_\epsilon$	Relationship shock dist.	$\mathcal{U}[-0.1, 0.1]$	Relationships expected to be the same
$\bar{s}$	Affirmative action quota	$\{0, 0.15\}$	State DBE goals
$\psi$	Relationship persistence	$\{0.95, 1\}$	Different possible relationship persistence
$\delta$	Discount factor	0.95	$\approx 5\%$ interest rate

*Note:*  $\mathcal{U}[a, b]$  corresponds to a uniform distribution bounded below by  $a$  and above by  $b$ .

### 4.3 Simulation Results: The Role of Subcontracting Regulations

My simulation analysis begins by analyzing the role of affirmative action subcontracting regulations. To isolate this effect, I fix  $\psi$  at 1 so that relationships persist over time in expectation, and I vary the quota from 0 to 15 percent. I first study bidding, disadvantaged subcontracting, and effective costs at select states and then move on to the value function.

Figure 1 illustrates how primes select their disadvantaged shares at different states and under various regulations. The two left panels depict optimal subcontracting in markets without affirmative action and the right two in markets with affirmative action. In the poor state ( $\omega = (0.5, 0.5)$  in the top two panels), primes use no disadvantaged subcontractors at every unregulated cost when there is no affirmative action. As a result, primes in this state would never build any relationships with disadvantaged subcontractors without relationship shocks, even if those relationships are expected to persist over time. Once the government imposes affirmative action, all primes in the poor state subcontract exactly at the quota.

Introducing asymmetries between prime contractors ( $\omega = (1.25, 0.5)$  in the bottom two panels) lead to different disadvantaged subcontracting behavior. Based on the state transitions characterized by equation (1), these asymmetries can arise by either random relationship changes or through differences in past disadvantaged subcontracting. With a stronger relationship than its competitor, prime 1 will subcontract at higher unregulated costs absent affirmative action, while prime 2 will still not subcontract at any unregulated cost. When the government introduces affirmative action, primes follow a similar pattern, except the lower bound on subcontracting increases to the quota level. These actions trace back to the optimal disadvantaged

subcontracting conditions in equation (3): primes with better relationships have a lower marginal cost of subcontracting and will, therefore, subcontract more often.

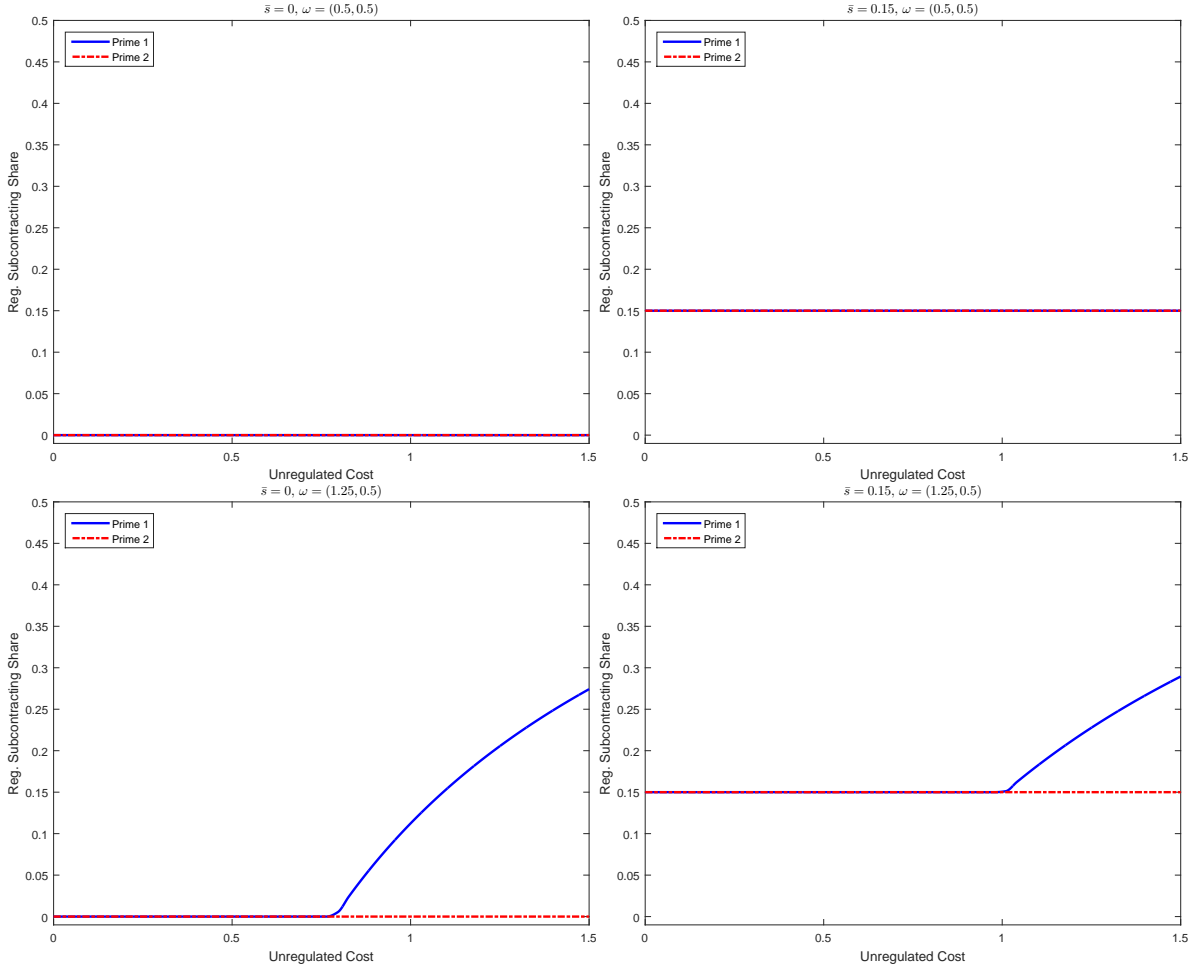


Figure 1: Disadvantaged Subcontracting

Next, I turn to the implied effective costs – which I display in Figure 2 in the same format as the disadvantaged shares. I include a 45-degree line to illustrate when effective costs are above or below unregulated costs. Because primes are symmetric in their disadvantaged subcontracting, their effective costs overlap in the poor state. Effective costs become more distinct when prime 1 builds stronger relationships. Without affirmative action, effective costs are similar at lower unregulated costs but diverge at higher unregulated costs. This result arises because prime 1 can use disadvantaged subcontractors to achieve lower effective costs at higher unregulated costs. Effective costs are not exactly the same at lower unregulated cost levels because the option value from losing against an opponent is different.<sup>18</sup> The disparity between effective costs

<sup>18</sup>In particular, the option value for prime 1 is zero since prime 2 will not subcontract even with the least efficient draw. The

is magnified when the government adds on affirmative action regulations, which indicates that regulations make bidders more asymmetric.

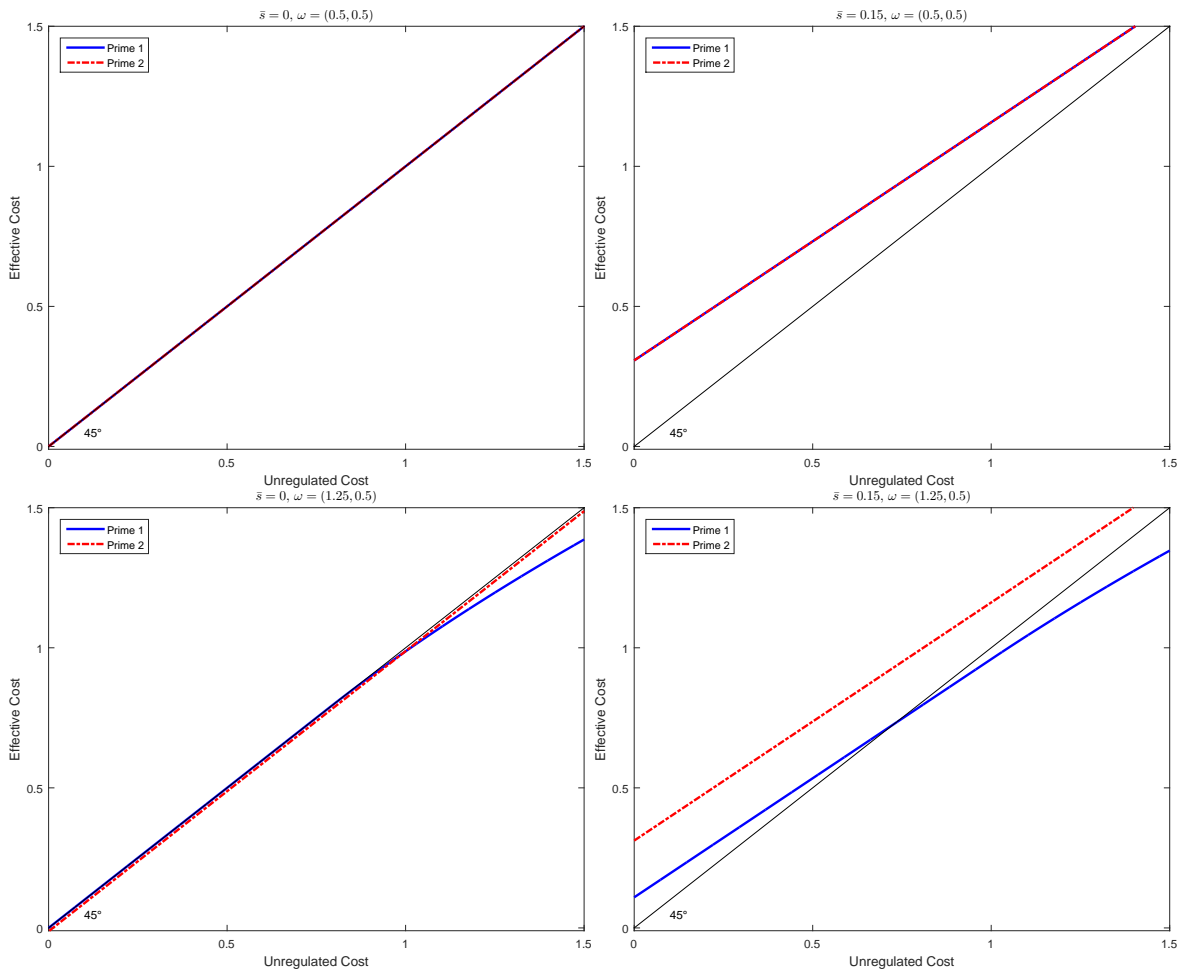


Figure 2: Effective Costs

Figure 3 illustrates the equilibrium bids that have a positive probability of winning. As is implied by the effective costs, symmetric bidders in the poor state bid the same irrespective of whether there is affirmative action. Once bidders become asymmetric, the bid functions become more distinct in ways that mirror static asymmetric auctions. In particular, the bids have a “weakness-leads-to-aggression” property,<sup>19</sup> whereby primes with higher effective costs (or weak bidders) bid more aggressively (or closer to their effective costs) than strong bidders. As a result, asymmetry softens competition between firms, leading to a reduced impact of relationship-related cost savings on procurement costs.

option value for prime 2 lowers effective costs since its competitor will have a higher state if it draws the highest unregulated cost.

<sup>19</sup>See Maskin and Riley (2000) and Krishna (2002) for the static case and Saini (2012) for the dynamic case with capacity constraints.

Without affirmative action, the weakness-leads-to-aggression property causes the bid functions to cross because the identity of the weak and strong bidder changes at different effective costs. With affirmative action, there is a pronounced separation in effective costs at every unregulated cost, leading to a similar separation in bids. Because the state generates asymmetries and arises as a result of each prime's past actions, bidder asymmetry is endogenous. The key observation in this environment is that affirmative action subcontracting regulations intensify that asymmetry.

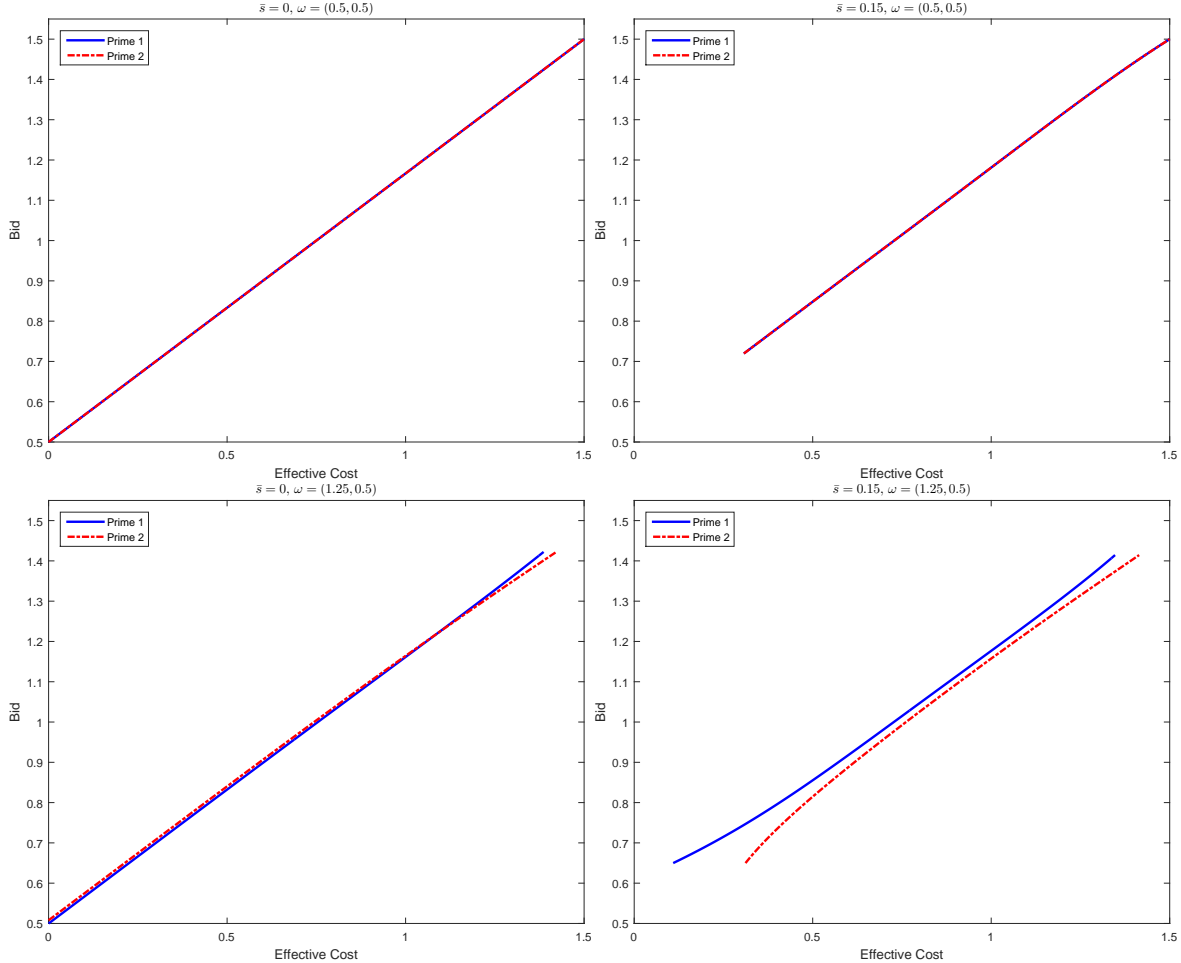


Figure 3: Bidding

The final equilibrium object is the value function, which I show in Figures 4 and 5. My assumptions imply that the value function is symmetric for each prime. To illustrate how symmetry works, let  $\omega_{own} \in [\underline{M}, \overline{M}]$  be an arbitrary state draw for a prime and  $\omega_{rival} \in [\underline{M}, \overline{M}]$  be an arbitrary state draw for the prime's competitor. Symmetry implies that  $V_1(\omega_{own}, \omega_{rival}) = V_2(\omega_{own}, \omega_{rival}) = V(\omega_{own}, \omega_{rival})$ , which obviates the need for computing multiple value functions for a given parameter configuration.

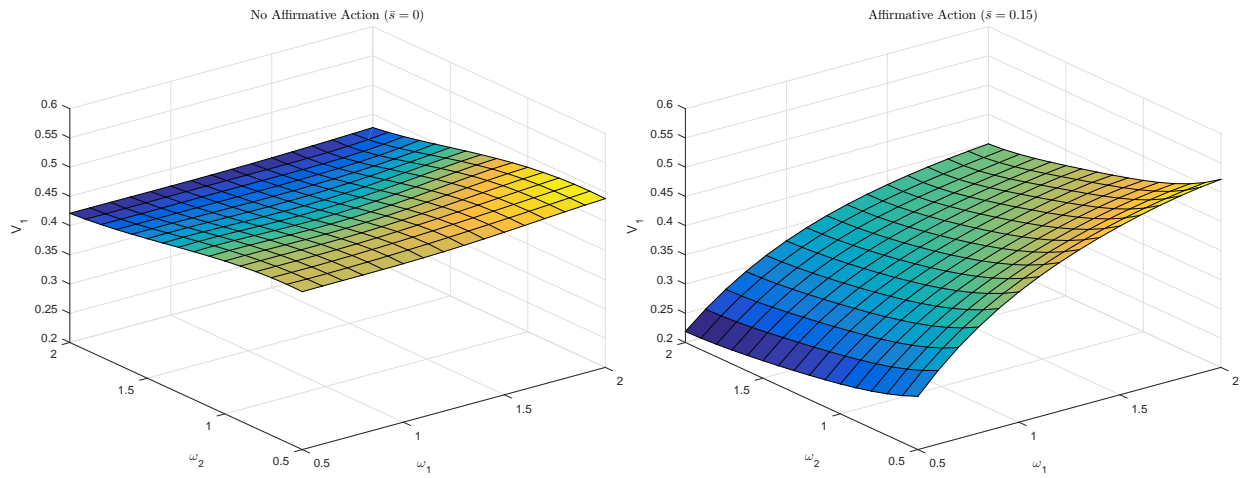


Figure 4: Three-Dimensional Value Functions

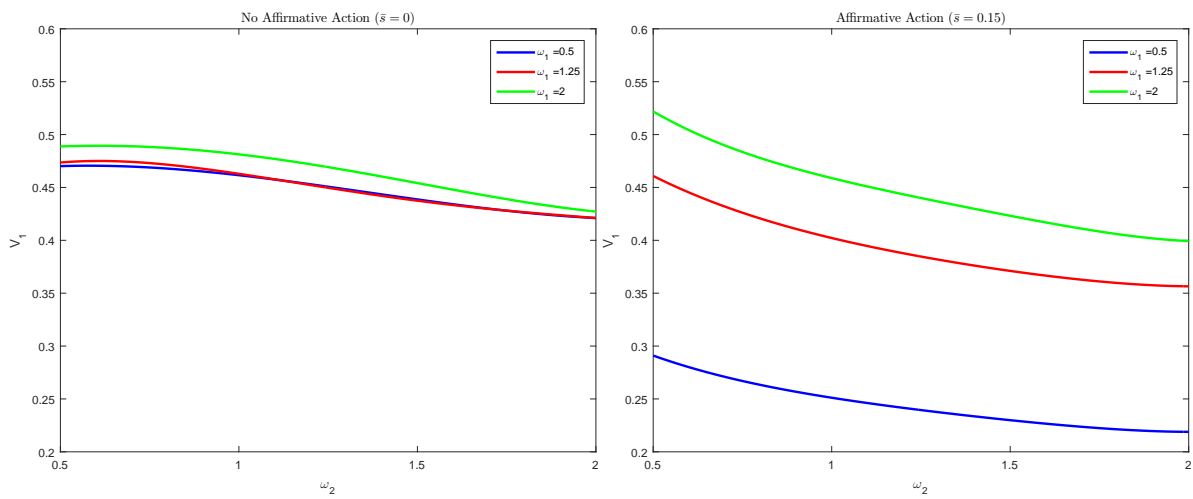


Figure 5: Value Function Level Sets



The value functions, which differ in whether there is affirmative action, have all of my conjectured properties: the value functions increase in own state and decrease in the competitor’s state. Affirmative action affects how quickly the value functions increase in own state, which I will refer to as the value function’s steepness. Indeed, primes only realize the gain in marginal value from attaining a higher state when unregulated costs are high in a market without affirmative action. On the other hand, primes realize the gain in marginal value for every unregulated cost when there is affirmative action because they are required to subcontract. Regulations lead to a steeper value function as a result. Furthermore, note that by increasing the value function’s steepness, affirmative action increases the dynamic part of the marginal benefit of subcontracting (i.e.,  $\mathbb{E}_\epsilon V_{i,1}(\sigma_i(s_i), \sigma_{-i}(0))$  in equation (3)). Therefore, primes will subcontract more with disadvantaged firms under affirmative action at every unregulated cost – even ones that would lead to subcontracting over the quota absent affirmative action.

#### 4.4 Simulation Results: Dynamic Equilibrium

I now analyze the evolution of the relationship state variable over time, which is dictated by equation (1). Central to this analysis is the initial state,  $\omega_0$ , and the relationship persistence parameter,  $\psi$ ; the values that I consider for these variables are  $\omega_0 \in \{(0.5, 0.5), (2, 2)\}$  and  $\psi \in \{1, 0.95\}$ . My initial state values account for two extreme cases: one where the market begins at the lowest possible state and the other where the market starts at the highest possible state. My  $\psi$  values cover the cases of when relationships persist and when there is some deterioration. Since markets with different relationship persistence will have different equilibria, I re-solve the model every time there is a change in  $\psi$ .

Figure 6 illustrates the evolution of different markets. In creating this figure, I simulated 100 procurement auctions for the first 100 periods, starting at various initial states. Figure 6 shows the average state for prime 1 across simulations in each period; the plot for prime 2 is similar and is, therefore, omitted. Note that this figure (and later, the distributions in Figure 7) internalizes whether prime 1 loses the auction based on the equilibrium bidding of the two primes.

The top left panel depicts a market without affirmative action that starts in the lowest possible state. When relationships deteriorate, primes never develop any relationships because disadvantaged subcontractors are too expensive. This market is reminiscent of the lack of disadvantaged subcontractor utilization before affirmative action laws – as was the case in Atlanta in 1973, where one-tenth of one percent of the city’s procurement business went to African American firms despite Atlanta’s majority African American popula-

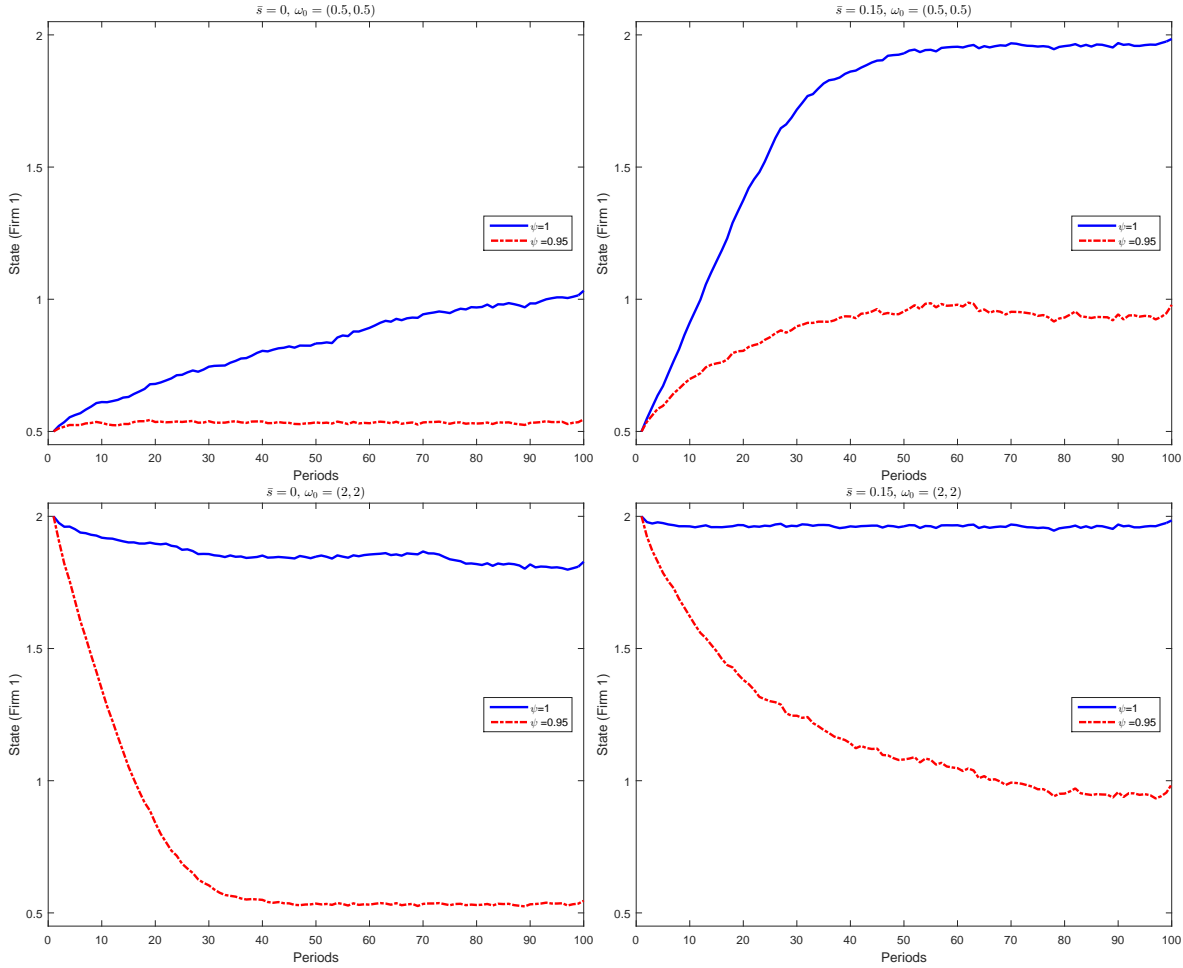


Figure 6: Time Paths

tion.<sup>20</sup> When relationships persist, random forces cause primes to slowly build relationships to a point where they are willing to use disadvantaged subs. This scenario illustrates a case where long-term relationship formation does not require affirmative action.

When the government uses affirmative action quotas – which is displayed in the top right panel – relationships no longer remain stagnant at the lowest state when there is relationship deterioration between periods. The policy forces primes out of the lowest state through mandatory subcontracting, leading to a higher average state than would be obtained absent affirmative action. When relationships persist between periods, the average state increases more rapidly than without affirmative action and is not driven by random forces. These results justify the early use of affirmative action quotas in procurement: when primes develop long-lasting relationships, quotas can help markets reach states characterized by high disadvantaged

<sup>20</sup>This example comes from Holzer and Neumark (2000).

subcontracting quicker; when relationships are more short-lived, the state still improves, but not as much as when relationships are long-lasting.

Now consider the market's evolution when it begins at the highest state, which is shown in the bottom two panels of Figure 6. Even when relationships are long-lasting, random forces lead to a slight downward trend in the average state. That trend is more extreme when relationships deteriorate between periods because more factors lead to a reduction in the next period's state. The mandatory subcontracting required by affirmative action policies prevents the state from trending downward when relationships persist by correcting the random shocks that lower relationships between periods. With less persistent relationships, affirmative action is not strong enough to prevent a downward trend but does prevent primes from reaching the lowest state.

Figure 7 extends my analysis to the limiting (or ergodic) state distribution. I construct these figures by first simulating 1,000 periods and then treating the next 100 periods as draws from the ergodic distribution. I repeat this process 100 times, leading to a total of 10,000 draws. Since the limiting distributions look the same irrespective of the starting value, I only present the cases with differing levels of affirmative action and relationship persistence.

The limit distributions are a natural consequence of extending the number of periods in Figure 6. With long-lasting relationships, the limit distribution has more density at the higher state for both primes. This pattern emerges because past relationships tend to accumulate over time, culminating in a situation where primes eventually reach the higher states. With the mandatory subcontracting that affirmative action entails, the ergodic distribution becomes more concentrated in the higher states since winning primes are required to keep adding to their stock of existing relationships.

The limit distributions look much different when relationships tend to deteriorate, which occurs in the two right-most graphs in Figure 7. With deteriorating relationships and no affirmative action, the ergodic distribution concentrates at the lower states, with variation driven by random forces. This distribution arises as a consequence of the declining nature of relationships: a prime can only increase its stock conditional on winning, yet its relationships deteriorate regardless of a win. Consequently, relationships eventually hover around the lowest state, where primes avoid disadvantaged firms altogether. Affirmative action can work against deteriorating relationships by requiring a minimum level of subcontracting, leading to an ergodic distribution that is more dispersed across states.

I conclude my simulation analysis by exploring industry statistics averaged over all simulations and time periods. I consider the following outcome variables. *Bid* is the average equilibrium bid, and *Subcon* is the average share of the project awarded to disadvantaged subcontractors. *Perc. Bind* is the percent of all

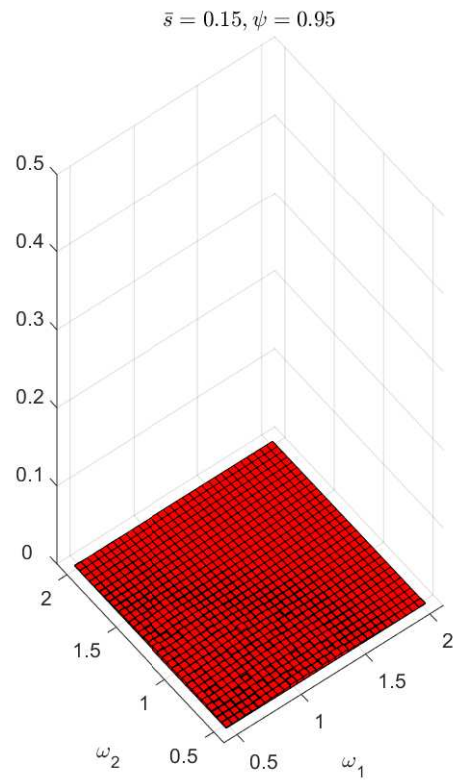
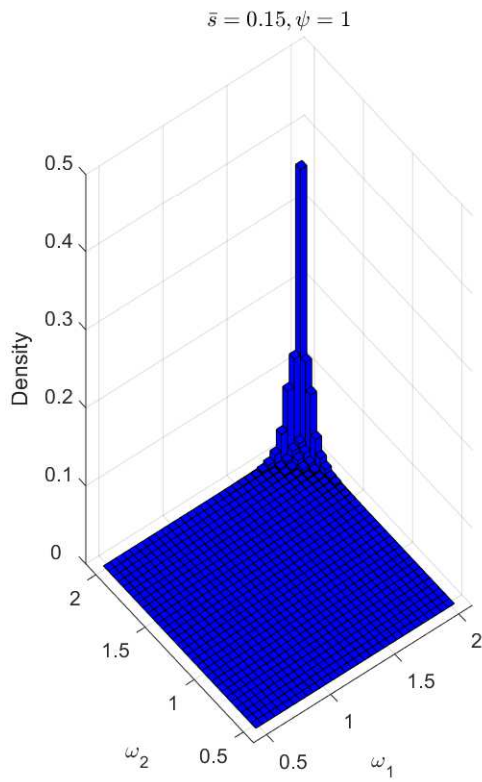
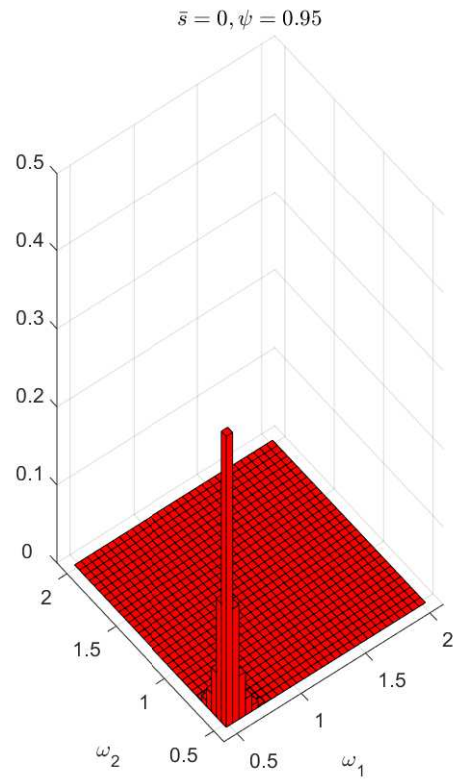
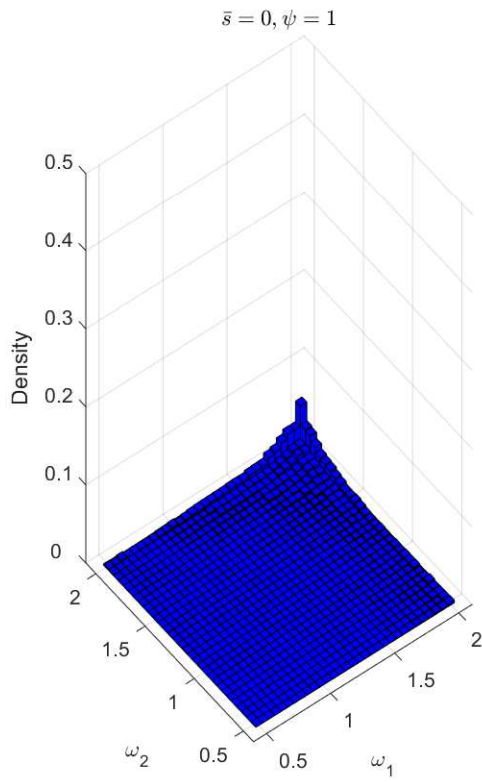


Figure 7: Ergodic State Distributions

projects where the quota binds, which I use as a measure of how constraining affirmative action regulations are. *Proc. Cost* is the average procurement cost conditional on the lowest bidder beating the reserve price, and *Perc. Allocated* is the percent of all projects awarded to one of the two primes. I remark here that a project awarded to neither prime does not necessarily go uncompleted; the project may be awarded to a fringe bidder, which is embedded in the reserve price distribution.

Table 2: Simulation Results

$\psi$	$\bar{s}$	$\omega_0$	Bid	Subcon.	Perc. Bind	Proc. Cost	Perc. Allocated
1.00	0.00	(0.5,0.5)	0.95	0.12		0.75	69.22
1.00	0.00	(2,2)	0.93	0.15		0.74	69.99
0.95	0.00	(0.5,0.5)	1.00	0.00		0.75	66.76
0.95	0.00	(2,2)	1.00	0.00		0.75	66.79
1.00	0.15	(0.5,0.5)	0.93	0.24	43.55	0.76	69.41
1.00	0.15	(2,2)	0.92	0.24	42.81	0.76	69.65
0.95	0.15	(0.5,0.5)	1.03	0.16	74.52	0.83	60.70
0.95	0.15	(2,2)	1.03	0.17	73.22	0.83	61.09

*Note:* Table shows the simulated market statistics.

I summarize the industry statistics in Table 2. I begin by analyzing the first four rows, which summarize markets without affirmative action regulations. A feature of these markets is that the average bid is lower and the average level of subcontracting is higher when relationships are long-lasting relative to when relationships deteriorate. These results are a consequence of disadvantaged subcontractors becoming a viable input in completing a project under the high stocks of relationships that accumulate with long-lasting relationships: primes willingly use disadvantaged subs to lower their project costs with sufficiently strong relationships, resulting in lower bids. Although the bids are lower, procurement costs remain relatively similar across the different relationship persistence parameters because the winning prime is usually the more efficient one (in that it has a lower unregulated cost) and therefore is less likely to use disadvantaged subs. The two primes also win more projects when relationships last longer since disadvantaged subcontractors can work to reduce costs.

Next, I turn to the markets with affirmative action quotas, which are contained in the bottom four rows of Table 2. Remarkably, bids in these markets are comparable to the bids received without affirmative action, but procurement costs are generally higher. These results arise because subcontracting quotas are more likely to distort the winning prime’s costs upwards, and the better states obtained through affirmative action allow less efficient primes to lower their costs – and therefore their bids – through subcontracting. Subcontracting under affirmative action is higher across the board, with the average subcontracting share exceeding the required quota. The values for the percent of times the quota binds, which is my measurement

for how constraining affirmative action is, reveals that affirmative action is more binding when relationships are deteriorating because the lower stocks of relationships mean that primes would want to subcontract less relative when relationships are long-lasting.

When taken together, my industry statistics suggest that the impact of affirmative action programs depends crucially on how relationships between prime contractors and disadvantaged subcontractors evolve. With persistent relationships, affirmative action does not have much of an impact on the bids and procurement costs but leads to substantially higher subcontracting shares. This result is consistent with cross-project analyses of Rosa (2018) for highway construction in New Mexico and De Silva et al. (2012) for asphalt projects in Texas. On the other hand, deteriorating relationships lead to higher bids and procurement costs, although disadvantaged subcontracting is noticeably higher. These results are consistent with Marion (2009) for California highway procurement projects. From a policymaker's point of view, it is crucial to assess contractor-subcontractor relationships when setting or removing affirmative action regulations.

## 4.5 Robustness

My analysis explores the impact of affirmative action policies for a uniform unregulated cost distribution, a particular disadvantaged cost function, and a fixed discount factor. In practice, these objects can take on a range of different values outside of the ones presented in this paper. To explore the robustness of my simulations to these other possibilities, I use my numerical routine to repeat my analysis, varying each of these objects individually.

**Alternative Discount Factors.** In my first set of robustness simulations, I study how a prime's patience affects affirmative action outcomes by re-simulating the model with various discount factors. As the discount factor decreases, primes derive less value in the current period from better relationships in the future, resulting in a reduced impact of the dynamic elements of the model. One place where that effect appears is in disadvantaged subcontracting: a lower discount factor reduces the dynamic incentive to subcontract, resulting in less disadvantaged subcontracting across the board. Another place where that effect appears is in bidding: lower discount factors reduce the dynamic part of the effective cost – which leads to higher effective costs and higher bids than the ones presented earlier in Section 4. These differences in outcomes are more pronounced when there is affirmative action because dynamic incentives are strongest when primes are required to subcontract.

**Alternative Unregulated Cost Distributions.** My second set of robustness simulations investigate alternative unregulated cost distributions. When these distributions are denser at the lower end of their supports, it is more likely to be cheaper for primes to complete projects without disadvantaged subs. As a result, primes are less inclined to use these subcontractors, and the lower unregulated costs result in lower procurement costs than the ones presented in the paper. Moreover, the dynamic incentive to form relationships with disadvantaged subs is weakened because primes understand that they are more likely to be of relatively higher cost than their unregulated alternatives. Given that relationships are less likely to form absent any regulation in these settings, affirmative action tends to have a larger impact on disadvantaged subcontracting. Altering relationship persistence yields results similar to the ones presented earlier.

**Alternative Disadvantaged Cost Functions.** In my final set of robustness simulations, I vary the steepness of the disadvantaged cost function, which changes the relative cost of using disadvantaged subcontractors. As the disadvantaged cost function becomes flatter, disadvantaged subs become relatively less costly, and primes utilize them more often. Increased utilization creates stronger relationships over time and, when combined with the already lower subcontracting costs, generates lower procurement costs than the ones presented in my main analysis.

With long-lasting relationships, there is less scope for affirmative action to increase subcontracting in these environments – as primes tend to subcontract over the requirement, and relationships will form naturally for sufficiently flat disadvantaged costs. With deteriorating relationships, the results can differ considerably. If the disadvantaged cost function is not flat enough, primes will hover around the low state and not subcontract without affirmative action. With affirmative action, primes will subcontract enough to form even better relationships than those in my main analysis under a similar situation. As a result, average procurement costs can be lower with affirmative action in the long run than without affirmative action.

## 5 Conclusion

This paper explores the impact of affirmative action subcontracting regulations in a procurement environment with dynamic relationship formation. Although evidence from the literature and actual procurement agencies suggest that relationships are vital in a firm’s cost of and ability to comply with affirmative action, much of the literature does not directly account for relationships. In this paper, I fill that gap by numerically solving for the Markov-perfect equilibria of an infinitely repeated procurement auction, where relationships evolve endogenously.

I first explore the role of affirmative action on farsighted bidders with relationship formation fixed. I find that affirmative action accentuates bidder asymmetries, favoring primes with stronger relationships over those with weaker ones. This distortion leads to a value function that increases more rapidly in a prime's own state when there are regulations, which translates into a stronger dynamic incentive for farsighted primes to subcontract with disadvantaged firms relative to the no-regulation case.

I then use the solved model to study how relationships evolve, allowing for the possibility that relationships can deteriorate. I find that the impact of affirmative action quotas on a procurement market depends crucially on the relationship formation process. When relationships are long-lasting, affirmative action rapidly improves the state of relationships, increases the average share of a project allocated to disadvantaged subs, and creates negligible differences in the cost of procurement. Conversely, a market characterized by higher relationship deterioration has more substantial differences in procurement costs with and without affirmative action, although affirmative action still markedly increases the utilization of disadvantaged firms. These results can explain a variety of patterns observed in real procurement data.

My analysis highlights the importance of relationship formation in affirmative action subcontracting regulations. Due to the complexities that can arise from having too many strategic agents, the model treats the pool of disadvantaged subs as non-strategic. An area open to future research would be to consider how relationships influence environments where the disadvantaged subcontracting market is strategic. Another potential avenue for future research would be to evaluate the long-run effectiveness of programs aimed at directly improving relationships and how they interact with markets with affirmative action. Examples of these types of programs include the Mentor-Protege Program sponsored by the USDOT – which seeks to match prime contractors with new disadvantaged firms – and the Learning, Information, Networking, Collaboration training program in Texas highway procurement – which provides disadvantaged firms with networking opportunities as part of its training program (see De Silva et al. (2019) for more details). I leave the evaluation of such joint policies for future research.

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## A Proofs

This section contains the proofs of my two propositions on optimal disadvantaged subcontracting. Because this section aims to recover comparative statics, I include the state in the optimal subcontracting function.

## A.1 Proof of Proposition 1

**Proposition.** *Disadvantaged subcontracting shares are weakly increasing in  $c_i$ .*

*Proof.* There are two cases for  $S(c_i, \boldsymbol{\omega})$ :  $S(c_i, \boldsymbol{\omega}) = \bar{s}$  or  $S(c_i, \boldsymbol{\omega}) \in (\bar{s}, 1)$ . If  $S(c_i, \boldsymbol{\omega}) \in (\bar{s}, 1)$ , then differentiating the first-order conditions with respect to  $c_i$  gives

$$S_1(c_i, \boldsymbol{\omega}) = -\frac{1}{\delta [\mathbb{E}_\epsilon V_{i,11}(\sigma_i(S(c_i, \boldsymbol{\omega}), \omega_i), \sigma_{-i}(0, \omega_{-i}))] - P_{11}(S(c_i, \boldsymbol{\omega}), \omega_i)},$$

Since the denominator is negative by the second-order conditions,  $S_1(c_i, \boldsymbol{\omega}) > 0$ . If  $S(c_i, \boldsymbol{\omega}) = \bar{s}$ , then  $S(c_i, \boldsymbol{\omega})$  does not change in  $c_i$ , implying that  $S_1(c_i, \boldsymbol{\omega}) = 0$ .<sup>21</sup>  $\square$

## A.2 Proof of Proposition 2

**Proposition.** *If  $P_{12}$  is sufficiently negative, then disadvantaged shares are weakly increasing in own relationship,  $\omega_{i,t}$ .*

*Proof.* For interior shares, differentiating the first-order conditions with respect to  $\omega_i$  gives

$$S_2(c_i, \boldsymbol{\omega}) = \frac{-P_{12}(S(c_i, \boldsymbol{\omega}), \omega_i) + \delta [\mathbb{E}_\epsilon V_{i,11}(\sigma_i(S(c_i, \boldsymbol{\omega}), \omega_i), \sigma_{-i}(0, \omega_{-i})) \psi]}{P_{11}(S(c, \boldsymbol{\omega}), \omega_i) - \delta [\mathbb{E}_\epsilon V_{i,11}(\sigma_i(S(c_i, \boldsymbol{\omega}), \omega_i), \sigma_{-i}(0, \omega_{-i}))]}.$$

The denominator is positive by the second-order conditions, so if

$$-P_{12}(S(c_i, \boldsymbol{\omega}), \omega_i) + \delta [\mathbb{E}_\epsilon V_{i,11}(\sigma_i(S(c_i, \boldsymbol{\omega}), \omega_i), \sigma_{-i}(0, \omega_{-i})) \psi] > 0$$

(or if  $P_{12}$  is sufficiently negative), then  $S_2(c_i, \boldsymbol{\omega}) > 0$ . For corner shares, take two arbitrary states  $\omega'$  and  $\omega''$ , where  $\omega' < \omega''$  and  $S(c_i, \omega', \omega_{-i}) = \bar{s}$ . Then, either  $S(c_i, \omega', \omega_{-i}) = S(c_i, \omega'', \omega_{-i}) = \bar{s}$  or  $S(c_i, \omega', \omega_{-i}) < S(c_i, \omega'', \omega_{-i})$ , so the share is weakly increasing in the state.  $\square$

## B Numerical Details

My numerical algorithm is similar to the one used by Jeziorski and Krasnokutskaya (2016). Ultimately, I aim to approximate the solution to a dynamic auction game with subcontracting and asymmetric bidders.

<sup>21</sup>Observe that when there are unregulated costs that yield corner solutions, then there are unregulated cost draws where the optimal disadvantaged subcontracting function is non-differentiable. In particular, define  $\underline{c}$  as the marginal unregulated cost such that  $S(\underline{c}, \boldsymbol{\omega}) = \bar{s}$  and  $S(\underline{c} + \tilde{\epsilon}, \boldsymbol{\omega}) > \bar{s}$  for an arbitrarily small  $\tilde{\epsilon}$ . Since  $S(\underline{c} + \tilde{\epsilon}, \boldsymbol{\omega}) > S(\underline{c}, \boldsymbol{\omega})$ , the increasing property is preserved when  $S(\cdot, \cdot)$  is non-differentiable.

To this end, I need to approximate the value function for the prime contractors as well as bidding and subcontracting in each state. In this section, I will work backward, starting from bidding and subcontracting and then moving into the value function computation. Finally, I will outline my methods for simulation.

## B.1 Bidding and Subcontracting

Given a state,  $\omega$ , and the current value function guess,  $\hat{V}(\cdot)$ , each prime contractor uses their first-order conditions on subcontracting and bidding to determine their optimal disadvantaged subcontracting share and bids. I use Monte Carlo integration techniques to get an approximation of the expected future value,  $E_\epsilon \hat{V}$ . To obtain the optimal subcontracting shares, I solve the first-order conditions in equation (3) on a grid of 50 uniformly spaced unregulated costs. I then interpolate that grid to a grid of 1,000 unregulated costs using a Hermite polynomial, which is known to have fewer undulations than cubic splines. This property of Hermite polynomials allows me to capture the flat spot in subcontracting that occurs at the left corner solution.

Next, I calculate the effective costs on the fine grid of unregulated costs using equation (4). I then run a Chebyshev regression through the effective cost grid and CDF values of the unregulated costs to get a Chebyshev polynomial estimate of the effective cost CDF,  $\hat{F}_{\phi,i}(\phi)$ . I approximate the CDF with a Chebyshev polynomial of degree 50; the PDF approximation for effective costs is then the derivative of the Chebyshev approximation of the CDF.

The estimates of the effective cost distributions allows me to approximate the inverse bid functions. The first step in doing so is determining the equilibrium high bid,  $\bar{b}$ , which I find by solving equation (7) for  $b_0$  and setting  $\bar{b}$  to the minimum of  $b_0$ , the upper bound of the reserve price distribution, and the least efficient prime contractor's highest effective cost. When the states differ between prime contractors, the auction is asymmetric, so I then use projection methods in conjunction with the mathematical programming with equilibrium constraints (or MPEC) optimization strategy suggested by Hubbard and Paarsch (2009) in the case of asymmetric auctions. In particular, I assume that the inverse bid functions can be represented by an eighth-degree Chebyshev polynomial of the first kind:

$$\hat{\xi}_i(b_l, \omega) = \sum_{m=0}^M \alpha_{i,m} \mathbb{T}_m[x(b_l)], \quad i = 1, 2; l = 1, \dots, L,$$

where  $\alpha_{i,m}$  is the  $m^{\text{th}}$  Chebyshev coefficient for prime contractor  $i$ ,  $x(\cdot)$  is in the interval  $[-1, 1]$ , and  $\mathbb{T}_m$  is the  $m^{\text{th}}$ -degree Chebyshev polynomial. Here, the function  $x(\cdot)$  maps the interval of equilibrium bids,  $[\underline{b}, \bar{b}]$ ,

into  $[-1, 1]$  and is given by

$$x(b_l) = \frac{2(b_l - \underline{b})}{\bar{b} - \underline{b}} - 1.$$

Following Judd (1998), I evaluate the inverse bid functions on the Chebyshev grid, which contains the zeros of the Chebyshev polynomials.

To then get approximations of the inverse bid functions, my objective is to choose the Chebyshev coefficients and an equilibrium low bid such that the first-order conditions in equation (5) are as close to zero as possible, subject to the boundary conditions in equation (6) as well as monotonicity (bid functions are increasing) and rationality (bidders do not bid below their effective costs) constraints. Let  $\boldsymbol{\alpha}$  be a vector that collects all Chebyshev coefficients,  $k = 1, \dots, K$  be a grid to evaluate the constraint points, and  $R_i(b | \boldsymbol{\alpha})$  be the residual given by equation (5)– except using the Chebyshev approximations for the effective cost distributions and inverse bid functions. My objective function is then

$$\begin{aligned} \min_{\underline{b}, \boldsymbol{\alpha}} \quad & \sum_{l=1}^L R_1(b_l | \boldsymbol{\alpha})^2 + R_2(b_l | \boldsymbol{\alpha})^2 \\ \text{s.t.} \quad & \hat{\xi}_i(\underline{b}, \boldsymbol{\omega}) = \underline{\phi}_i, \quad i = 1, 2 \\ & \hat{\xi}_i(\bar{b}, \boldsymbol{\omega}) = \min\{\bar{\phi}_i, \bar{b}, \bar{r}\}, \quad i = 1, 2 \\ & \underline{b} < \bar{b} \\ & \hat{\xi}'_i(b_k, \boldsymbol{\omega}) > 0, \quad i = 1, 2; \quad k = 1, \dots, K \quad (\text{Monotonicity}) \\ & \hat{\xi}_i(b_k, \boldsymbol{\omega}) < b_k, \quad i = 1, 2; \quad k = 1, \dots, K \quad (\text{Rationality}). \end{aligned}$$

In practice, I use a grid of 10 uniformly spaced points on  $[\underline{b}, \bar{b}]$  to evaluate the monotonicity and rationality constraints.

## B.2 Value Function

Working backwards to the value function, I use the parametric value function procedure advocated by Judd (1998) to approximate the value function. In particular, I assume that a prime contractor's value function can be represented by a tensor product between two fourth-degree Chebyshev polynomials:

$$V(\omega_1, \omega_2) \approx \hat{V}(\omega_1, \omega_2 | \boldsymbol{\zeta}) = \sum_{m_1}^{M_1} \sum_{m_2}^{M_2} \zeta_{m_1, m_2} \mathbb{T}_{m_1}(x_1(\omega_{1, l_1})) \mathbb{T}_{m_2}(x_2(\omega_{2, l_2})),$$

where

$$x_i(\omega_{i,l_i}) = \frac{2(\omega_{i,l_i} - \underline{\omega})}{\bar{\omega} - \underline{\omega}} - 1.$$

Let the current iteration's value function be denoted by  $V_i^{(n)}(\omega)$  and the next iteration's value function be denoted by  $V_i^{(n+1)}(\omega)$ . To approximate the next iteration's value function I solve

$$\begin{aligned} V_i^{(n+1)}(\omega) = & \int_{c_i} \max_{b,s} \left\{ W_i(b) \left[ (1 - F_R(b)) (b - (1-s)c_i - P(s) \right. \right. \\ & \left. \left. + \delta \mathbb{E}_\epsilon V_i^{(n)}(\sigma_i(s), \sigma_{-i}(0)) + F_R(b) \delta \mathbb{E}_\epsilon V_i^{(n)}(\sigma_i(0), \sigma_{-i}(0)) \right] \right. \\ & \left. + (1 - W_i(b)) \delta \int_{\underline{c}_{-i}}^{\xi_{-i,c}(b,\omega)} (1 - F_R(B_{-i}(c_{-i}))) \mathbb{E}_\epsilon V_i^{(n)}(\sigma_i(0), \sigma_{-i}(S_{-i}(c_{-i}))) \right. \\ & \left. + F_R(B_{-i}(c_{-i})) \mathbb{E}_\epsilon V_i^{(n)}(\sigma_i(0), \sigma_{-i}(0)) dF_c(c_{-i}) \right\} dF_c(c_i) \end{aligned}$$

given my inverse bid and subcontracting approximations on a two-dimensional grid of state values, where  $\xi_{-i,c}(\cdot)$  is the inverse bid function of the competing prime contractor in terms of unregulated costs. In practice, I use a Chebyshev grid with the boundary points in each state. To deal with the potential of multiple equilibria, I use the robust equilibrium selection rule described in Chen et al. (2009), which involves backwards inducting on the value function with a starting value of 0 to find the limit of a finite game. I continue to solve the value function and re-approximate the bids and subcontracting until  $\|V^{(n)}(\omega) - V^{(n+1)}(\omega)\|_\infty < \epsilon$ , where  $\|x\|_\infty = \sup |x|$ .

### B.3 Simulation

Given that the state space is continuous, I need a method to interpolate the bids and subcontracting for all states in  $\Omega$  for the simulations. I do so by linearly interpolating the approximated bids and subcontracting functions across the missing states. Specifically, I fix a grid of 100 unregulated cost draws and the two-dimensional Chebyshev grid of states from the value function, yielding a three-dimensional grid. Observe that I have estimates of the inverse bid functions in terms of effective costs and the fine grid of equilibrium subcontracting shares for each state combination in the state grid at this point. Hence, I use the fine subcontracting grid to interpolate the subcontracting shares corresponding to the 100 unregulated costs in each state combination to get the shares in each state. I then linearly interpolate the states outside of

the state grid. For bids, I find the effective costs corresponding to each unregulated cost for a given state combination and then use the inverse bid function to ascertain the bids. Similar to the subcontracting shares, I linearly interpolate bids outside of the grid.

To simulate an industry, I draw 1,100 unregulated costs for each prime contractor and 1,100 reserve price values. Given the current state, I use the three-dimensional grid to interpolate bids and subcontracting shares and then update the state using the state transition equation. I repeat these simulations 100 times and average them to get the time paths and industry statistics. For the stationary distribution, I assume that the industry reaches the limiting distribution after 1,000 periods, which is generous given the time paths, and I assume that the remaining draws are from the stationary distribution.

## C Inverse Bid Function Error

This section evaluates the error associated with approximating the inverse bid functions. In an exact solution, the first-order conditions would be zero at every bid. Therefore, to evaluate the approximation error, I measure how close the first-order conditions are to zero given the approximated inverse bids. Figure 8 plots the residual function,  $R_i(b | \hat{\alpha})$ , for each prime contractor given the inverse bid function approximations used to generate Figure 3. In general, I find that the first-order conditions are close to zero except at the left boundary. Even the errors for the least accurate approximation, which occurs when  $\bar{s} = 0.15$  and  $\omega = (1.25, 0.5)$ , are comparable to the ones from the third example in Hubbard and Paarsch (2014), which suggests that my approximations are reasonable.

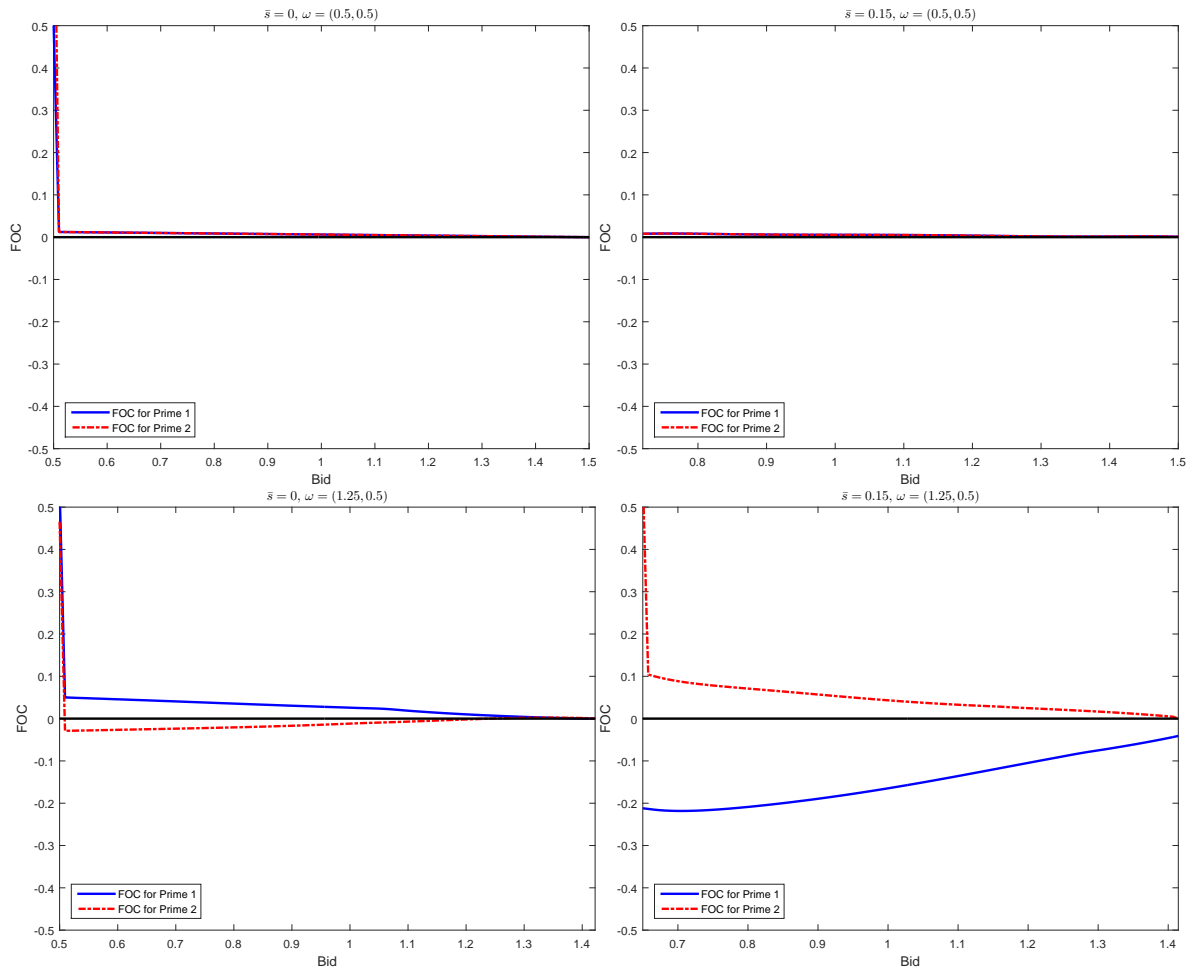


Figure 8: Bid Function Error