Pairwise forward trading and bilateral oligopoly

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Abstract

This paper investigates pairwise efficient forward trading followed by spot market competition. The model finds that forward trading rules out a "bilateral oligopoly" spot market where at least one net seller under-supplies and least one net buyer under-procures. If not, both firms, by exercising market power, would hurt each other, a negative externality problem which can be mitigated by pairwise forward trading. Next, a configuration is analyzed where firms' marginal costs increase linearly with slopes inversely related to their capacities. It is shown that assuming market shares equal capacity shares overstates the Hirschman-Herfindahl Index, a result useful for merger evaluation.

Keywords: forward contracts, bilateral oligopoly, mergers

JEL classification: D43, G13, L13

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1. Introduction

In this paper, a model of pairwise forward trading is developed that delivers clear predictions about the strategic interaction in the spot market. The model does not impose restrictions regarding who can sign forward contracts with whom. The analysis demonstrates that pairwise forward trading serves the desirable purpose of ruling out that the spot market is a bilateral oligopoly market (with both seller and buyer power). The analysis of a parametric configuration delineates how the degree of market concentration depends on firms' capacity shares and yields results useful for merger investigation.

Forward contracts are known to decisively affect firms' incentives to exercise market power in commodity spot markets. The theoretical mechanism through which this happens is well-understood and has been extensively documented in influential empirical work.[2] The natural follow-up question—how do firms sign forward contracts in equilibrium?—has accordingly received much attention and is also the topic of this paper.

We currently have two main theories to explain forward trade. First, in the field of finance, spot market prices are modelled to be volatile. In this type of environment forward sales can serve as an instrument to hedge against risk, as laid out in seminal work by Holthausen (1979) and Bessembinder and Lemmon (2002). Second, the literature following Allaz and Vila (1993) is based on the idea that forward obligations, whenever perfectly observable, can act as a commitment device.[3] The commitment value, however, is known to break down when firms can also sign contracts secretly, for example in an over-the-counter market. Bagwell (1995), for

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[2] The theoretical mechanism is presented in the pioneering study by Allaz and Vila (1993). Green (1999), Wolak (2000), Bushnell et al. (2008), and Ito and Reguant (2016) are examples of influential studies highlighting the importance of contractual arrangements in electricity markets.
[3] Mahenc and Salanié (2004) show that the competitive effect of strategic commitment crucially depends on the presence of strategic substitutes or strategic complements in the spot market. Ferreira (2003), Liski and Montero (2006), and Wölfing (2019) are examples of studies investigating the relationship between observable forward contracts and collusion.

example, shows that small amounts of noise can eliminate a firm's first-mover advantage, and Hughes and Kao (1997) demonstrate that unobservability has critical implications in the analysis of forward contracting as commitment device.

This paper considers a two-period model where a forward trading stage (first stage) is followed by a spot market à la Cournot (second stage). The main novelty lies in the first stage and is to investigate the solution concept pairwise efficiency, which works as follows. Imagine a mesh topology where each strategic firm can trade pairwise with all other strategic firms. In this setting, pairwise efficiency requires that pairs of strategic firms should not be able to gain from additionally exchanging forward obligations bilaterally, considering how doing so would affect their own behavior in the spot market and taking as given the other firms' forward obligations and behavior in the spot market.[4] The concept does not suppose that all forward trade occurs bilaterally—in practice over-the-counter markets and centralized auctions oftentimes coexist. Instead, the concept imposes stability conditions with respect to the outcome of the forward trading process. These stability conditions are relevant whenever there is an over-the-counter market where firms can trade secretly, which is the case for many commodity markets in the world. The analysis thus has the potential to deliver basic predictions that are robust across markets.

A key result of the analysis is that strategic firms are either all net sellers or all net buyers in the spot market. The intuition is as follows. If the finding would not hold, the market would be a *bilateral oligopoly*, where net sellers exercise seller power by under-supplying and net buyers exercise buyer power by under-procuring (see Hendricks and McAfee (2010) on a theory of bilateral oligopoly).[5] Now consider a pairwise forward contract between a prospective net seller and a prospective net buyer. The net seller, by selling an additional unit through the forward contract, reduces its volume subject to the spot market price. Consequently, it has fewer incentives to exercise seller power. Similarly, the net buyer, by purchasing an

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[4] In section 3 a formal definition is presented.
[5] See also e.g. the studies by Wolak (2000), Mansur (2007), Bushnell et al. (2008), Hortacsu and Puller (2008), and Hortacsu et al. (2019).

additional unit from the net seller through the forward contract, reduces its volume procured in the spot market and has fewer incentives to exercise buyer power.

Such a pairwise contract is always profitable for the following reason. Bilateral oligopoly markets have the unique feature that firms, when exercising market power, impose negative externalities on the firms that have net positions of the opposite sign. A net buyer is hurt by a net seller's exercise of seller power and a net seller is hurt by a net buyer's exercise of buyer power. By transacting through a forward contract rather than in the spot market, firms rule out these negative competitive externalities. The analysis thus establishes that a prospective net seller and a prospective net buyer have incentives to transact pairwise through a forward contract rather than to wait for the spot market to balance their positions. This result offers a new explanation why forward contracting is prevalent in markets where firms behave strategically.

Section 4 presents an analysis of a configuration where firms' marginal costs are increasing linearly. The slope of a firm's marginal cost function is an inverse measure of its capacity, such that larger (smaller) firms have a marginal cost function characterized by a flatter (steeper) slope.[6] The analysis starts by considering forward trade between two representative firms with heterogenous capacities. It is found that larger firms, as they can modify output more for a given change in marginal cost, respond more strongly to a change in forward contract position than smaller firms. Consequently, a forward contract between firms with asymmetric capacities involves a total output effect. When signing a forward contract, asymmetrically sized firms trade off this total output effect against production efficiency considerations.

Next, the analysis is scaled up to cover all pairs in the industry. A key finding in this respect is that the property pairwise efficiency is transitive: when two firms each trade pairwise-efficiently with a common outsider who has non-zero capacity,

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[6] One can analogously think of firms as having marginal valuation functions which are linearly decreasing, such that larger (smaller) firms have a marginal valuation function with a flatter (steeper) slope. This interpretation is detailed in the Appendix.

the pair consisting of those two firms satisfies pairwise efficiency as well. With this finding as a stepping stone, section 4 is able to deliver precise predictions regarding firms' market shares.

What is the relationship between firms' capacities and their market shares? It is found that firms who have more capacity, i.e., whose marginal cost function is characterized by a flatter slope, have a larger market share. Since firms with larger market shares have more incentives to exercise market power, marginal costs are not equalized across firms. The analysis thus establishes that, with pairwise efficient forward trade, firms do not fully achieve production efficiency.

It is also demonstrated that firms' market shares are invariant with respect to the intercepts of their marginal cost functions. This finding can be understood as a Coasean result. To gain intuition, one can think of a pairwise forward contract as a transfer of output between two firms. Additional output shifts a firm's marginal cost function horizontally to the right, such that a higher amount of output corresponds to a lower marginal cost function intercept. In the model, outputs are reallocated across firms pairwise-efficiently, and it is found that the outcome of this process (i.e., firms' market shares) is independent of firms' initial marginal cost function intercepts.

This insight has implications for understanding competition in electricity markets with renewables. Production from renewable energy sources could be regarded as shifting firms' marginal cost functions horizontally to the right, thereby affecting the marginal cost intercepts but not the slopes. The model thus suggests that, with pairwise efficient forward trade, the presence of renewable energy generation would not affect firms' market shares in equilibrium.

Next, the analysis investigates the relationship between pairwise efficient forward trading and market concentration. Does pairwise forward trading exacerbate market power, such that dominant firms with large capacity shares have an even larger market share? Or does it rather balance firms' market shares to improve production efficiency? To address this question, the predictions about firms' market shares are used to construct the Hirschman-Herfindahl Index (HHI), defined as the sum of the squared market shares. The HHI is a widely used indicator for market concentration and, in Cournot oligopoly, it is intimately related to firms' markups.[7] The analysis demonstrates that, with pairwise efficient forward trading, the HHI is smaller than the one which would be calculated by naïvely assuming that spot market shares equal capacity shares. As such, the analysis has obtained a reassuring result: in the parametric configuration with linear marginal costs, pairwise forward trading does not lead to the monopolization of the spot market. Put simply, firms' net sales in the spot market are less concentrated than firms' capacities.

This finding can be useful in the context of merger evaluation. In the spirit of Perry and Porter (1985), mergers can be modelled such that merging partners combine their pre-merger capacities without affecting the sum of their capacities.[8] The analysis of the theoretical model suggests that it would be meaningful to naïvely calculate the post-merger HHI by assuming post-merger market shares equal post-merger capacity shares. Disregarding synergies, that naïve calculation would overstate actual concentration post-merger. So, capacity shares can serve to screen mergers in industries with forward contracting and where marginal costs are reasonably characterized as linear. If the naïve calculation indicates that a merger poses a competition concern, a deeper investigation may be warranted.

This approach has two features which are attractive in the context of merger evaluation. First, the approach does not require data regarding firms' net market shares. Such data is in practice not always straightforwardly available, as a full view on firms' contract positions is hard to obtain.[9] Instead, firms' market shares are predicted from the model with pairwise efficient forward trade. Second, when analyzing a proposed merger, there is a need to account for how firms' forward contracting behavior differs pre-merger versus post-merger. The solution concept

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[7] The average Lerner index equals the HHI divided by the price elasticity of demand.

[8] Stated with the terminology of Farrell and Shapiro (1990), such mergers make it possible to reallocate production across facilities but do not generate synergies.

[9] van Eijkel et al. (2016) study the Dutch wholesale market for natural gas using a dataset that consists of "a substantial fraction of all forward and spot contracts". Hortacsu and Puller (2008) infer firms' forward positions from detailed firm-level bidding data.

pairwise efficiency can serve as a theoretical basis for such an analysis and proposes an upper bound for the post-merger HHI which can be calculated with information about firms' capacity shares.[10]

The framework developed in this paper is distinct from Allaz and Vila (1993) in two important respects. First, by studying the solution concept pairwise efficiency, the analysis focusses on firms' incentives to sign secret rather than publicly observable contracts. Second, Allaz and Vila (1993) consider forward contracts between producers and speculators[11], whereas this paper concerns pairwise forward trade among strategic firms more generally. To facilitate comparison, the framework is set up such that it also incorporates speculators. On this front the following result is obtained: a pair consisting of a regular firm (with non-zero capacity) and a speculator signs a forward contract that closes the speculator's position. Doing so is optimal as it makes sure that the regular firm is the residual claimant on all variable profits in the spot market. With this result, the parametric configuration predicts that the HHI is invariant to the presence of speculators.

The setup is also related in spirit to Anderson and Hu (2008) and Ruddell et al. (2018). These papers study forward trade between producers and large retailers who are sellers and buyers in the spot market, respectively. In their analyses, however, retailers are assumed not to behave strategically in the spot market, an assumption which rules out that the spot market is a bilateral oligopoly. In this paper I study the effects of forward contracting when both parties engaging in the transaction behave strategically.

The solution concept pairwise efficiency can be relevant not only for modelling forward trade in commodity markets but also for modelling forward trade in

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[10] Previous work on mergers with endogenous forward contracting by Miller and Podwol (2019) relies on the strategic commitment motive for forward contracting with publicly observable contracts. Their analysis predicts that mergers reduce the use of forward contracting which can amplify the adverse effect on consumers.

[11] The distinguishing characteristic of a speculator is that it does not have the flexibility to adjust its production, rather than that it would not have any production at all. Section 2, which sets up the model, therefore categorizes firms using the label flexible or the label inflexible.

financial securities markets. In this respect, the study relates to Coutinho (2013) who investigates a treasury auction preceded by a when-issued forward market which is modelled as a uniform-price auction. In Coutinho (2013), firms' marginal valuation functions have symmetric slopes and asymmetric intercepts. Section 4, which develops the parametric configuration, allows for asymmetries on both dimensions and shows that their distinction is qualitatively important. For example, allowing trading partners to have marginal valuation functions with asymmetric slopes is essential to capture the effect of a forward contract on total output.

The framework also bears similarities with Spiegel (1993) and Van Moer (2019) who analyze how two producers optimally sign an ex ante horizontal subcontract (upstream) before competing against each other in the product market (downstream).[12] The condition which characterizes the optimal contract between both producers obtained in Spiegel (1993) also appears in section 3 in the analysis of a representative pair of firms with non-zero capacities. The contribution compared to his paper is to incorporate the possibility of seller *and* buyer power, which is essential for the results in section 3 on the topic of bilateral oligopoly. Also, by studying more than two firms, section 4 is able to characterize the extent to which the property pairwise efficiency is transitive and is able deliver predictions regarding firms' market shares and the HHI.

Gans (2007) and Hendricks and McAfee (2010) are two papers which study concentration-based indicators of competitiveness in models that account for vertical structure. Gans (2007) presents a model of bilateral bargaining which bears similarity with the notion pairwise efficiency used in this paper. His model differs, however, by having the feature that, when two firms bargain about which quantity of inputs to supply each other, they hold fixed their internal supplies. That assumption would not be well-suited to investigate forward markets, where it is critical to incorporate the effect of a firm's forward contract position on its optimal behavior in the spot market. By incorporating the latter effect, I demonstrate that

[12] Van Moer (2019) shows that two capacity-constrained producers can sign a supply contract to escape head-to-head competition in the downstream market.

pairwise efficient forward trade rules out bilateral oligopoly. In addition, section 4 demonstrates that when firms are asymmetrically sized, firms fail to achieve production efficiency.

The remainder of this paper is structured as follows. Section 2 analyzes the spot market and demonstrates the potential for bilateral oligopoly. Section 3 shows that pairwise efficient forward trade rules out that the spot market is a bilateral oligopoly. Section 4 analyzes the parametric configuration and presents the predictions regarding firms' market shares and the HHI. Section 5 concludes.

2. The spot market

There are N strategic firms, indexed by $i = 1,...,N$, which interact in a two-stage game. The first stage is the forward trading stage and the second stage is the spot market which is modelled à la Cournot. This section models the spot market for an exogenous outcome of the forward trading stage.

The volume offered by firm i in the spot market equals

$$Q_i = x_i + q_i,$$

where x_i denotes firm i's net forward purchases and q_i denotes firm i's net production such that

$$\begin{aligned} x_i &= \text{forward purchases}_i - \text{forward sales}_i \\ q_i &= \text{production}_i - \text{consumption}_i. \end{aligned}$$

To understand the components of Q_i, start with the special case where firm i is a producer and has decided not to engage in forward trade. Firm i's volume in the spot market then simply equals its production. The above formulation generalizes that special case in two respects. First, when firm i has positive consumption, the volume it trades in the spot market is more generally determined by the difference between production and consumption. Second, the formulation accounts for the possibility of forward trading: forward purchases act as a substitute for production, and firms must cover their forward sales just as they must cover their consumption. Firm i is a *net seller* in the spot market when $Q_i > 0$ and a *net buyer* in the spot market when $Q_i < 0$. Finally, let t_i denote firm i's net transfer received in the forward market such that $t_i = \text{received transfers}_i - \text{paid transfers}_i$.

There are two types of strategic firms: *flexible firms* whose net production is adjustable and *inflexible firms* whose net production is not adjustable. The set of flexible firms is denoted by L, the set of inflexible firms is denoted by M and the set of all strategic firms is N such that $L + M = N$. Flexible firms optimize q_i in the spot market whereas for inflexible firms q_i is exogenously determined.

Assumption 1: When firm i is flexible ($i \in L$), its cost of attaining net production q_i is denoted by $C_i(q_i)$, assumed twice continuously differentiable such that $C_i''(q_i) > 0$.

The Appendix presents the microfoundation for Assumption 1 and details how net production follows from adjustable production and adjustable consumption.[13] It is shown that $C_i''(q_i) > 0$ follows from marginal production costs being strictly increasing and marginal valuation functions being strictly decreasing.

Assumption 2: The inverse demand function equals $P = a - b \times \sum_{i \in N} Q_i$ where P is the uniform price and $b > 0$.

The demand function can represent non-strategic final consumers or a competitive fringe. The demand curve is specified as linear to guarantee a unique equilibrium in the spot market.[14]

[13] Since this study is on the topic of bilateral oligopoly, it is valuable to highlight both interpretations.
[14] Assuming that the second derivative satisfies $P'' \leq 0$ is insufficient in this regard because firms' volumes in the spot market can be positive or negative.

We are now ready to analyze the equilibrium in the spot market. We can write a flexible firm's profits as

$$\pi_i = t_i + P \times Q_i - C_i\left(q_i\right) \text{ for } i \in L,$$

Assumptions 1 and 2 guarantee that the first-order conditions for maximization with respect to q_i are necessary and sufficient. They equal

$$P - b \times \left(x_i + q_i^*\right) = C_i^{'}\left(q_i^*\right) \text{ for } i \in L. \tag{1}$$

This shows that flexible firms who are net sellers ($Q_i > 0$) have a marginal net production cost *below* the market-clearing price. In contrast, flexible firms who are net buyers ($Q_i < 0$) have a marginal net production cost *above* the market-clearing price.

The spot market is said to be a *bilateral oligopoly* when at least one flexible firm is a net buyer and at least one flexible firm is a net seller. We are now ready to state Result 1.

Result 1: *For exogenous forward contract positions, the spot market can be a bilateral oligopoly.*

The proof of this possibility result is by example. Since the example is instructive, it is presented here. The example considers an industry consisting of two flexible firms, 1 and 2 with $1,2 \in L$. The inflexible firms, for simplicity, sell a zero volume in the spot market ($Q_i = 0$ for all $i \in M$). Firms 1 and 2 differ regarding their marginal cost to attain a net production equal to $q_i = -x_i$ such that $C_1^{'}\left(-x_1\right) < a < C_2^{'}\left(-x_2\right)$. In words, attaining a volume of zero in the spot market, i.e., balancing internally, requires little effort for firm 1 but great effort for firm 2. Such a situation might arise, for example, when firm 1 can produce cheaply or has purchased many units through forward contracts, and the situation for firm 2 is less favorable.

The proof, which is presented formally in the Appendix, shows that in this example firm 1 acts as net seller in the spot market and firm 2 acts as net buyer in the spot market. Intuitively, for the market to clear, the price can neither be too low ($P \leq C_1^{'}\left(-x_1\right)$) nor too high ($P \geq C_2^{'}\left(-x_2\right)$). Indeed, in the former case, no market participant would be selling in the spot market, a situation which would be in violation with the market-clearing condition. In the latter case, no market participant would be buying, thereby also violating the market-clearing condition. The proof shows that in the example firm 1 acts as net seller and firm 2 acts as net buyer.

Importantly, from a welfare point of view, the combined presence of seller power and buyer power does not cancel each other out. In the example, firm 1 is a net seller in the spot market and accordingly under-supplies the market ($C_1^{'}\left(q_1^*\right) < P$). Firm 2 is a net buyer in the spot market and so under-procures ($C_2^{'}\left(q_2^*\right) > P$). The wedge between firms' marginal costs indicates an inefficiency: there is potential to improve welfare if firm 1 increases its net production and firm 2 decreases its net production.

Remark also that there are negative competitive externalities in the following sense. Firm 1, being net seller, would benefit if firm 2 under-procures to a lesser extent. Firm 2, being net buyer, would benefit if firm 1 under-supplies to a lesser extent. This idea will be formalized in the next section, which endogenizes firms' forward trading behavior.

In summary, when forward contract positions are exogenous, the spot market can be composed of flexible firms who are net sellers and other flexible firms who are net buyers. Such a market structure is a bilateral oligopoly which involves inefficiencies.

3. Pairwise efficient forward trade

This section investigates pairwise efficient forward trade among strategic firms, which is modelled according to the following definition.

Definition 1 (Pairwise efficiency). A vector of net forward purchases $\left(x_1,...,x_N\right)$ is pairwise efficient if, for any $i, j \in \{1,2,...,N\}$ such that $i \neq j$, firms i and j cannot increase their joint profit by bilaterally exchanging additional units through a forward contract, holding fixed the net forward purchases of the other firms in the industry (x_k is fixed for all $k \neq i, j$) and the beliefs of all firms about the forward contracts they are not involved in, so that net production q_k for all $k \neq i, j$ is unaffected by the contract.

The first part of definition 1 captures the idea that the network of bilateral links has a mesh topology: all firms can trade pairwise with all other firms. Pairwise efficiency then requires firms' forward contract positions to be such that each pair of strategic firms that can be formed in a mesh topology cannot gain by engaging in additional bilateral trade. In the parametric configuration presented in section 4, it is demonstrated that the analysis is equivalent for a star topology with one central flexible firm.

Pairwise efficiency relates closely to Jeon and Lefouili (2018)'s concept bilateral efficiency in their analysis of cross-licensing agreements between horizontal competitors. Similar concepts have also been used to study vertical relations.[15] One difference in comparison with the vertical relations literature is that in this framework one cannot satisfactorily classify firms as either upstream or downstream.[16] Therefore, it seems most natural that the solution concept in the

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[15] The concept pairwise efficiency bears similarities with e.g. Crémer and Riordan (1987) on contract equilibrium, Hart and Tirole (1990), McAfee and Schwartz (1994) on passive beliefs and pairwise-proof equilibrium, and Collard-Wexler et al. (2019) on Nash-in-Nash bilateral bargaining.
[16] One approach would be to classify firms based on whether they are active in the forward market or in the spot market. Firms, however, are typically active in both markets, and so could not be classified as upstream or downstream with this approach. Another approach could be to classify firms according to whether they are net sellers or net buyers. Such an approach, however, also does not seem very appealing in this context, as a firm's role as net seller or net buyer is the result of optimality conditions rather than that it is imposed by the model.

forward market places no restrictions regarding who can trade with whom, which is the approach taken here.

There exist three types of pairs: pairs consisting of two flexible firms, pairs consisting of a flexible and an inflexible firm, and pairs consisting of two inflexible firms.

The following notation is used to distinguish the different types of pairs. The indices e and f are used to indicate flexible firms, such that $e \in L$ and $f \in L$ with $e \neq f$. Next, the indices g and h indicate inflexible firms such that $g \in M$ and $h \in M$ with $g \neq h$. Firm e and firm f then form a representative pair of flexible firms, the type of pairs analyzed in subsection 3.1. Firm e and firm g form a representative pair consisting of a flexible and an inflexible firm, the pairs which are the topic of subsection 3.2. Finally, firm g and firm h form a representative pair of inflexible firms and are analyzed in subsection 3.3.

3.1. Pairs consisting of two flexible firms

This subsection investigates the representative pair of flexible firms (e, f). The goal is to obtain a necessary condition for pairwise efficiency and to show that it rules out that the spot market is a bilateral oligopoly. The result is shown by contradiction. Suppose the spot market would be a bilateral oligopoly. Then, there must exist a pair of flexible firms that consists of a net seller in the spot market and a net buyer in the spot market. Without loss of generality, denote the net seller in the spot market by firm e and denote the net buyer in the spot market by firm f.

The remainder of this subsection is devoted to showing that firms e and f could have gained from transacting pairwise through a forward contract, a finding which demonstrates the contradiction. We denote firms' *candidate* net forward purchases by superscript ^{c}: x_e^c for firm e and x_f^c for firm f. Next, we investigate whether firms have incentives to sign an additional forward contract where firm e sells γ

units to firm f.[17] Consequently, $x_e = x_e^c - \gamma$ and $x_f = x_f^c + \gamma$. Similarly, firms' candidate net transfers are denoted by t_e^c and t_f^c, and with the forward contract firms can exchange τ monetary units such that $t_e = t_e^c + \tau$ and $t_f = t_f^c - \tau$. Since the transfer paid by one firm is received by the other firm, the sum $t_e + t_f$ is unaffected.

Two remarks are useful. First, as can be seen from the first-order conditions (1), neither t_e nor t_f affects q_e^* or q_f^*. Financial transfers occur in the forward trading stage and are therefore regarded as sunk when firms compete in the spot market. Second, according to Definition 1, the pairwise contract does not affect the behavior of outsiders to the contract. Consequently, we have that $q_i^*(\gamma,\tau) = q_i^*$ for $i \in L, i \neq e, f$.

The pair of firms solves the following problem:

$$\max_{\gamma,\tau} \pi_e + \pi_f ,$$

where firms' variable profits are

$$\pi_e = t_e^c + \tau + PQ_e - C_e\left(q_e^*(\gamma)\right) \tag{2}$$

$$\pi_f = t_f^c - \tau + PQ_f - C_f\left(q_f^*(\gamma)\right), \tag{3}$$

and

$$\begin{gathered} P = a - b\left[Q_e + Q_f + \sum_{\substack{i\in L \\ i\neq e,f}}\left(x_i + q_i^*\right) + \sum_{i\in M}\left(x_i + q_i\right)\right] \\ Q_e = x_e^c - \gamma + q_e^*(\gamma) \\ Q_f = x_f^c + \gamma + q_f^*(\gamma). \end{gathered} \tag{4}$$

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[17] Denoting firm e as the seller and firm f as the buyer of the forward contract is without loss of generality because γ can be positive or negative.

Bilateral profits (2) + (3) can be seen to be independent of τ. Consequently, the pair of firms maximizes bilateral profits with respect to λ. When the gains from bilateral trade are positive, there exists a transfer τ that shares these gains from trade such that both firms are willing to engage in the forward contract. Maximizing with respect to γ, we obtain the necessary first-order condition

$$\frac{d\left(\pi_e(\gamma)+\pi_f(\gamma)\right)}{d\gamma}=0,$$

which can be written as

$$\underbrace{\frac{\partial \pi_e}{\partial \gamma}+\frac{\partial \pi_f}{\partial \gamma}}_{=0}+\underbrace{\frac{\partial \pi_e}{\partial q_e^*}}_{=0}\frac{dq_e^*(\gamma)}{d\gamma}+\frac{\partial \pi_e}{\partial q_f^*}\frac{dq_f^*(\gamma)}{d\gamma}+\frac{\partial \pi_f}{\partial q_e^*}\frac{dq_e^*(\gamma)}{d\gamma}+\underbrace{\frac{\partial \pi_f}{\partial q_f^*}}_{=0}\frac{dq_f^*(\gamma)}{d\gamma}=0,$$

where the notation $\frac{\partial \pi_i}{\partial q_j^*}$ is used as a shorthand notation for $\left.\frac{\partial \pi_i}{\partial q_j}\right|_{q_j=q_j^*}$. The first two terms represent the direct effects, which equal $\frac{\partial \pi_e}{\partial \gamma}+\frac{\partial \pi_f}{\partial \gamma}=-P+P=0$. Moreover, we know that firm e and firm f 's quantity choices in the spot market satisfy their first-order conditions, so that $\frac{\partial \pi_e}{\partial q_e^*}=\frac{\partial \pi_f}{\partial q_f^*}=0$ (the envelope theorem). Therefore, we obtain that pairwise efficiency requires

$$\frac{\partial \pi_e}{\partial q_f^*}\frac{dq_f^*(\gamma)}{d\gamma}+\frac{\partial \pi_f}{\partial q_e^*}\frac{dq_e^*(\gamma)}{d\gamma}=0. \tag{5}$$

We next explore each of the four terms in (5). Firms' first-order conditions in the spot market equal

$$P-b\left(x_e^c-\gamma+q_e^*(\gamma)\right)-C_e'\left(q_e^*(\gamma)\right)=0$$
$$P-b\left(x_f^c+\gamma+q_f^*(\gamma)\right)-C_f'\left(q_f^*(\gamma)\right)=0,$$

where P is characterized by equation (4). Totally differentiating with respect to γ gives

$$b+\frac{dq_e^*(\gamma)}{d\gamma}\left(-2b-C_e^{"}\left(q_e^*(\gamma)\right)\right)+\frac{dq_f^*(\gamma)}{d\gamma}(-b)=0$$
$$-b+\frac{dq_e^*(\gamma)}{d\gamma}(-b)+\frac{dq_f^*(\gamma)}{d\gamma}\left(-2b-C_f^{"}\left(q_f^*(\gamma)\right)\right)=0.$$

This is a system of equations which can be solved to obtain

$$\begin{cases}\dfrac{dq_e^*(\gamma)}{d\gamma}=\left(3b+C_f^{"}\left(q_f^*(\gamma)\right)\right)\Big/C \;>\; 0\\[2ex] \dfrac{dq_f^*(\gamma)}{d\gamma}=-\left(3b+C_e^{"}\left(q_e^*(\gamma)\right)\right)\Big/C \;<\; 0\end{cases} \tag{6}$$

where $C\equiv 3b+2\left(C_e^{"}\left(q_e^*(\gamma)\right)+C_f^{"}\left(q_f^*(\gamma)\right)\right)+C_e^{"}\left(q_e^*(\gamma)\right)C_f^{"}\left(q_f^*(\gamma)\right)\Big/b$.

Relation (6) characterizes how firms alter their net production in response to the forward contract. First consider firm e, who sells the forward contract. To gain intuition, imagine what would happen when firms' levels of net production would be unchanged. Any forward sales by firm e to firm f would then cause a one-to-one reduction in firm e's spot market volume (Q_e). The forward contract would thus make firm e a smaller seller. Holding smaller stakes in the spot market, firm e has less incentives to exercise seller power. Therefore, firm e raises net production. According to (6), we have that $0<dq_e^*(\gamma)\big/d\gamma<1$, meaning that the increase in net production mitigates - but does not fully offset - the decrease in spot market volume. Second, in an analogous way, relation (6) finds that firm f who buys the forward contract reduces its net production according to $-1<dq_f^*(\gamma)\big/d\gamma<0$.

Next, we can use (2), (3) and (4) to write

$$\begin{aligned}\frac{\partial \pi_e}{\partial q_f^*} &= -b\left(x_e^c - \gamma + q_e^*(\gamma)\right)\\ \frac{\partial \pi_f}{\partial q_e^*} &= -b\left(x_f^c + \gamma + q_f^*(\gamma)\right),\end{aligned} \tag{7}$$

so that

$$\begin{aligned}&\frac{\partial \pi_e}{\partial q_f^*} < 0 \text{ when } x_e^c - \gamma + q_e^*(\gamma) > 0\\ &\frac{\partial \pi_f}{\partial q_e^*} > 0 \text{ when } x_f^c + \gamma + q_f^*(\gamma) < 0.\end{aligned} \tag{8}$$

Relation (7) characterizes how a firm's profits depend on the quantity choice of the trading partner. It states that a net seller (buyer) in the spot market suffers (benefits) when the trading partner increases its level of net production.

The combination of (6) and (8) demonstrates that, whenever firm e is net seller and firm f is net buyer in the spot market, the necessary condition for pairwise efficiency (5) fails to hold. Consequently, we have arrived the following result.

Result 2: *With pairwise efficient forward trade, the spot market is never a bilateral oligopoly. Formally,* $Q_i \geq 0$ *for all* $i \in L$ *or* $Q_i \leq 0$ *for all* $i \in L$.

When firm e is a prospective net seller in the spot market and firm f is a prospective net buyer in the spot market, firms e and f can always gain from letting firm e sell to firm f in the forward market, which establishes the contradiction. Such a forward contract reduces firm f's volume procured in the spot market, which reduces its incentives to exercise buyer power (firm f's net production decreases according to (6)). This effect benefits firm e, who is a net seller in the spot market, and is formally represented by the first term of equation (5). Also, with the forward contract, firm e's volume in the spot market shrinks, so that it has fewer incentives to exercise seller power in the spot market (firm e's net production increases according to (6)). This effect benefits firm f, who is a net buyer in the spot market, and is formally represented by the second term of equation

(5). So, the forward contract reduces the competitive externalities that firms inflict upon each other, which is mutually beneficial. For this reason, firms always have incentives to sign forward contracts to rule out a bilateral oligopoly spot market.

The model setup in this subsection compares closely to Spiegel (1993) who investigates how two duopoly producers optimally sign an ex ante horizontal subcontract before competing against each other in the product market.[18] In his paper both firms are sellers in the product market. An ex ante subcontract between two producers is then not always profitable as the seller of the ex ante subcontract responds to the contract by raising production, which hurts the other producer. The analysis here differs by studying commodity markets where trade occurs between a prospective net seller and a prospective net buyer in the spot market. Both firms then respond to the contract in a way that benefits the counterparty. With this finding it is demonstrated that pairwise efficient forward contracting rules out bilateral oligopoly.

In summary, a prospective net seller and a prospective net buyer have incentives to sign a forward contract rather than to wait to balance their positions in the spot market. This result offers a new explanation why forward contracting is prevalent in commodity markets with strategic firms.

3.2. Pairs consisting of a flexible and an inflexible firm

This subsection considers the representative pair of a flexible and an inflexible firm $\left(e, g\right)$. It is shown that a necessary condition for pairwise efficiency is that the inflexible firm sells a volume equal to zero in the spot market ($Q_g = 0$).

We apply the same techniques: we denote firms' *candidate* net forward purchases by superscript ^{c}, and we suppose that firm e sells γ units to firm g, so that

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[18] The condition for a pair of flexible firms to satisfy pairwise efficiency (5) is similar to the condition characterizing the optimal ex ante subcontract in Spiegel (1993, p. 581). To see the comparison, use that $dQ_e/d\gamma = -1 + dq_e^*/d\gamma$, $dQ_f/d\gamma = 1 + dq_f^*/d\gamma$, $P' = -b$, and $C_e'\left(q_e^*\right) - C_f'\left(q_f^*\right) = -b\left(Q_e - Q_f\right)$ which follows from (1).

$x_e = x_e^c - \gamma$ and $x_g = x_g^c + \gamma$. Firms' net financial transfers equal $t_e = t_e^c + \tau$ and $t_g = t_g^c - \tau$. As before, according to Definition 1, the spot market behavior of outsiders to the contract is independent of γ. We can thus write that $q_i^*(\gamma,\tau) = q_i^*$ for $i \in L, i \neq e$.

The pair of firms solves the following problem: $\max_{\gamma,\tau} \pi_e + \pi_g$, where firms' variable profits equal

$$\pi_e = t_e^c + \tau + PQ_e - C_e\left(q_e^*(\gamma)\right) \tag{9}$$

$$\pi_g = t_g^c - \tau + PQ_g, \tag{10}$$

where

$$\begin{aligned} P &= a - b\left(Q_e + Q_g + \sum_{\substack{i\in L\\ i\neq e}}\left(x_i + q_i^*\right) + \sum_{\substack{i\in M\\ i\neq g}}\left(x_i + q_i\right)\right) \\ Q_e &= x_e^c - \gamma + q_e^*(\gamma) \\ Q_g &= x_g^c + \gamma + q_g. \end{aligned} \tag{11}$$

As in subsection 3.1., the pair of firms maximizes the gains from bilateral trade with respect to λ and the transfer τ shares the gains from trade but leaves bilateral profits unaffected. We obtain the necessary first-order condition

$$\underbrace{\frac{\partial \pi_e}{\partial \gamma} + \frac{\partial \pi_g}{\partial \gamma}}_{=0} + \underbrace{\frac{\partial \pi_e}{\partial q_e^*}}_{=0}\frac{dq_e^*(\gamma)}{d\gamma} + \frac{\partial \pi_g}{\partial q_e^*}\frac{dq_e^*(\gamma)}{d\gamma} = 0.$$

The first two terms represent the direct effects, which sum up to zero. The third term is zero because of the envelope theorem. Consequently, additional forward trade affects bilateral profits only through firm e's quantity choice (the fourth term). We work out and obtain

$$-b\underbrace{\left(x_g^c + \gamma + q_g\right)}_{Q_g}\frac{dq_e^*(\gamma)}{d\gamma} = 0. \tag{12}$$

We are now ready to state Result 3.

Result 3: *Pairwise efficient forward trade requires that the inflexible firms sell a volume equal to zero in the spot market:*

$$Q_i = 0 \text{ for all } i \in M \ . \tag{13}$$

The proof is in the Appendix. If Result 3 would not hold, there would exist an inflexible firm with a non-zero volume in the spot market. Without loss of generality, denote that firm by g and consider the pair of firms e and g. Firm e would pose an externality on firm g because its quantity decision affects the price that firm g receives or pays in the spot market. Under pairwise efficiency, firms sign the forward contract that eliminates this externality by closing firm g's position.

Comparing with Allaz and Vila (1993), the analysis here differs by studying the possibility for firms to sign secret contracts, which do not affect the behavior of outsiders (according to Definition 1). Result 3 finds that inflexible firms do not serve as a vehicle for strategic commitment: they sign forward contracts to close any open position they might otherwise have. With pairwise efficient forward trade they hold no stakes in the spot market.

3.3. Pairs consisting of two inflexible firms

Finally, consider pairwise trade between two inflexible firms g and h. As in the previous subsections, we investigate firms' incentives to trade γ additional units in the forward market in exchange for transfer τ, such that $x_g = x_g^c - \gamma$, $x_h = x_h^c + \gamma$, $t_g = t_g^c + \tau$, and $t_h = t_h^c - \tau$.

Firms' bilateral profits equal $\pi_g + \pi_h = t_g^c + \tau + PQ_g + t_h^c - \tau + PQ_h$, where

$$P = a - b\left[Q_g + Q_h + \sum_{i \in L}\left(x_i + q_i^* \right) + \sum_{\substack{i \in M \\ i \neq g,h}} \left(x_i + q_i \right) \right]$$

$$Q_g = x_g^c - \gamma + q_g \tag{14}$$

$$Q_h = x_h^c + \gamma + q_h,$$

and are unaffected by γ or τ. Consequently, pairs consisting of two inflexible firms always satisfy pairwise efficiency.

In summary, from Result 2, we know that flexible firms have incentives to transact with each in the forward market rather than in the spot market, in order to avoid inefficiencies from bilateral oligopoly. Result 3 states that, with pairwise efficiency, inflexible firms close their position entirely in the forward market. Consequently, the counterparties of strategic firms in the spot market cannot be strategic firms themselves. At least one "side" of the spot market—all net sellers or all net buyers—must belong to the non-strategic fringe.

4. A parametric configuration

This section analyzes a parametric configuration and obtains predictions regarding firms' market shares in the spot market. Suppose that flexible firms are characterized by quadratic net production cost functions

$$C_i\left(q_i\right) = c_i q_i + \frac{0.5}{k_i} q_i^2 \text{ for } i \in L, \tag{15}$$

where c_i is firm i's marginal cost intercept and $k_i > 0$ represents a measure of firm i's flexible capacity. The marginal net production cost can be written as $C_i^{'}\left(q_i\right) = c_i + q_i / k_i$. Consequently, with a doubling of flexible capacity k_i, firm i's marginal cost function increases at half the rate, or

$$k_i = \frac{1}{C_i^{"}(q_i)} \tag{16}$$

The Appendix presents the microfoundation showing that the amount of flexible capacity can be interpreted as the sum of adjustable production capacity and adjustable consumption capacity. The sum of all flexible firms' capacities is denoted by

$$K \equiv \sum_{i \in L} k_i \ . \tag{17}$$

The setup allows firms to be heterogenous in two respects. First, firms can be subject to firm-specific output shifters, as captured by c_i which measures parallel shifts of firm i's marginal cost function. Second, firms can be asymmetrically sized in terms of flexible capacity as captured by k_i.

The analysis of the parametric configuration starts by investigating the relationship between a flexible firm's size (k_i) and the extent to which it alters net production in response to a change in forward contract position. In this respect, consider the representative pair of flexible firms (e, f) as analyzed in subsection 3.1, which engages in a forward contract where firm e sells γ units to firm f. Using (6), both firms adjust net production according to the following relation

$$\begin{cases} \dfrac{dq_e^*(\gamma)}{d\gamma} = \left(3b + \dfrac{1}{k_f}\right) \Big/ C \\ \dfrac{dq_f^*(\gamma)}{d\gamma} = -\left(3b + \dfrac{1}{k_e}\right) \Big/ C \end{cases} \tag{18}$$

$$\text{where } C = 3b + 2\left(\frac{1}{k_e} + \frac{1}{k_f}\right) + \frac{1}{bk_e k_f}.$$

When $k_e = k_f$, joint net production is invariant to the forward contract ($dq_e^*(\gamma)/d\gamma + dq_f^*(\gamma)/d\gamma = 0$). When $k_e > k_f$, firm e who is larger adjusts net production more strongly than firm f who is smaller, so that

$dq_e^*(\gamma)/d\gamma > -dq_f^*(\gamma)/d\gamma$. Intuitively, when k_e is large, firm e can increase (decrease) its net production more strongly for a given increase (decrease) in marginal cost. When $k_e < k_f$, the opposite relation holds true ($dq_e^*(\gamma)/d\gamma < -dq_f^*(\gamma)/d\gamma$).

We are now ready to state Result 4, proven in the Appendix, which presents necessary and sufficient conditions for pairwise efficiency.

Result 4: *In the parametric configuration, the following conditions are necessary and sufficient for the representative pairs (e, f) and (e, g) to satisfy pairwise efficiency:*

$$Q_e\left(3b + \frac{1}{k_e}\right) = Q_f\left(3b + \frac{1}{k_f}\right) \tag{19}$$

$$Q_g = 0\,. \tag{20}$$

According to relation (19), each flexible firm's volume in the spot market is of the same sign, which is consistent with the prediction of the more general model (Result 2). Also, firms who have less flexible capacity have a smaller market share, and in the limit a firm with an infinitesimally small capacity has an almost zero market share in the spot market. This insight is qualitatively in line with Result 3, which has shown that with pairwise efficiency inflexible firms – who can be thought of as having zero capacity - must sell a zero volume in the spot market.

Why do larger firms have a larger market share? In other words, why don't forward contracts equalize firms' market shares and thereby equalize firms' marginal net production costs? The reason is that, when firms are asymmetrically sized, industry output is not invariant to how forward obligations are allocated. More specifically, firms who have more capacity increase (decrease) their net production more strongly in response to an increase (decrease) in their forward obligations, as was demonstrated in (18). For asymmetrically sized firms this generates a tradeoff between production efficiency and reducing competition.

For example, if firms are net sellers, the model predicts that larger firms have larger net sales. Consequently, due to seller power, larger firms "under-supply" to a higher extent. A large firm could have considered selling additional units to a smaller firm using a pairwise forward contract. As a result, both firms' marginal net production costs would have converged, thereby improving production efficiency. However, according to (18), the larger firm would have increased its supply by more than the amount by which the smaller firm would have decreased its supply. Consequently, the market-clearing price would have decreased, hurting both firms. The relative market volumes according to (19) are the result of asymmetrically sized firms balancing both considerations.

Result 4 also reports on the conditions for pairwise efficient trade between a flexible and an inflexible firm. From Result 3, we already know that (20) is necessary for pairwise efficiency. Result 4 shows in the parametric configuration that (20) is also sufficient for pairwise efficiency.

We are now ready to report Result 5 which is an essential ingredient of the analysis. The solution concept as defined in Definition 1 requires all possible pairs in a mesh topology to satisfy pairwise efficiency. The next result, proven in the Appendix, states that for such pairwise efficiency to obtain, it is not necessary that all pairs of firms actively engage in bilateral trade. When two firms each trade pairwise-efficiently with a flexible firm as common counterparty, the bilateral link composed of those two firms satisfies pairwise efficiency by transitivity.

Result 5: (transitivity). *In the parametric configuration, consider any flexible firm $j \in L$. When firms i and j trade pairwise-efficiently and when firms j and k trade pairwise-efficiently, it follows that the bilateral link between i and k is also pairwise efficient.*

The transitivity result implies that the predictions of pairwise efficiency are not sensitive to the assumption that each firm should actively trade bilaterally with each other firm. For example, the predictions of pairwise efficiency are unaltered when forward trading occurs through a star topology where any flexible firm is the central firm.

Result 5 is also useful from a technical perspective. It states that, to analyze the implications of pairwise efficiency, one can restrict attention to requiring that firms trade pairwise-efficiently with a central flexible firm. This insight drastically reduces the number of bilateral links that need to be considered in the analysis.

Denote firms' market shares in the spot market by $s_i = Q_i \Big/ \sum_{j=1}^{N} Q_j$. The following result, proven in the Appendix, characterizes firms' market shares with pairwise efficiency.

Result 6: *In the parametric configuration, pairwise efficient forward trade holds if and only if firms' market shares in the spot market equal*

$$s_i = \begin{cases} \beta_i \Big/ \sum_{j \in L} \beta_j \text{, where } \beta_i \equiv \left(3b + \dfrac{1}{k_i}\right)^{-1} \text{, for } i \in L \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{for } i \in M. \end{cases} \tag{21}$$

Result 6 presents the predictions from pairwise efficiency regarding firms' market shares in the spot market. Those market shares are found to be invariant to firms' marginal cost function intercepts (c_i). To interpret this finding, imagine that firm i, next to having access to adjustable net production, is also endowed with an amount of non-adjustable net production. The endowment would shift firm i's marginal cost function horizontally to the right, such that a higher endowment corresponds to a lower c_i. Next, think of a pairwise forward contract as an exchange of endowments between two firms. In the model, endowments are reallocated across firms pairwise-efficiently. The invariance finding can now be understood as a Coasean result: with pairwise efficient forward trading, firms' market shares are independent of their initial endowments of non-adjustable net production.

Remark that the framework has thus far not introduced an assumption regarding the sum of all strategic firms' net forward purchases ($\sum_{i \in N} x_i$). Since any forward purchase held by one firm must imply a forward liability by another firm, it must be true that the competitive fringe holds net forward purchases equal to $-\sum_{i \in N} x_i$. The results derived in this section are thus robust with respect to the contract position held by the competitive fringe.[19]

The market shares according to (21) allow to construct the traditional indicators of competitiveness such as the C4 ratio or the HHI. The following result concerns the HHI and can be useful in the context of merger analysis. The Appendix presents the proof.

Result 7. *Denote the number of flexible firms by n and consider the parametric configuration. Assuming that each flexible firm has a market share equal to $1/n$ understates the HHI and assuming that market shares equal capacity shares overstates the HHI. Formally,*

$$\sum_{i \in L} (1/n)^2 < \sum_{i \in N} s_i^2 < \sum_{i \in L} (k_i/K)^2 . \tag{22}$$

The first inequality in (22) follows from Result 4 which has demonstrated that, with pairwise efficiency, inflexible firms sell a zero volume in the spot market. Consequently, the HHI is independent of inflexible firms. Assuming that all flexible firms have a symmetric market share equal to $1/n$ thus understates the HHI.

The second inequality in (22) relates to the conditions for pairwise efficient forward trading between flexible firms. Such forward trading involves a tradeoff between production efficiency and reducing competition. On the one hand, firms benefit

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[19] According to Result 5, pairwise efficiency holds if a central flexible firm trades pairwise-efficiently with each of the $N-1$ other firms. So, a consequence of not making an assumption about the level of $-\sum_{i \in N} x_i$ is that the vector of net forward purchases $(x_1, ..., x_N)$ is underdetermined with degree of freedom one.

from symmetric market shares, as more symmetric market shares imply that firms have similar marginal net production costs, which is cost-efficient. On the other hand, it also implies a higher intensity of competition, and therefore means that fewer rents are extracted from the fringe. In the parametric configuration, the former effect is sufficiently strong so that the spot market volumes are more symmetric than capacity shares. Calculating the HHI based on the naïve assumption that market shares equal capacity shares overstates the actual HHI.

Relationship (22) has implications for the evaluation of mergers in industries where marginal costs are reasonably characterized as linear. It demonstrates that, disregarding synergies, calculating a post-merger HHI with the assumption that market shares equal capacity shares is conservative by overstating post-merger concentration. Capacity shares, whenever such data are available, may thus be suitable to screen which mergers require more careful investigation.

In summary, the parametric configuration has obtained predictions regarding firms' market shares under pairwise efficient forward trade. With these predictions a lower and upper bound for the HHI were constructed, bounds which can be calculated with information regarding the number of strategic flexible firms and their capacity shares.

5. Conclusion and discussion

Many commodity products are traded through forward contracts. This paper has studied the implications of pairwise efficient forward trading, a solution concept which requires each strategic firm to trade pairwise-efficiently with all other strategic firms, on the strategic interaction in the spot market. The model has demonstrated that, with pairwise efficient forward trading, the spot market is never a bilateral oligopoly. If the finding would not hold, there would exist a pair of strategic firms consisting of a prospective net seller and a prospective net buyer. The net seller, by exercising seller power, would hurt the net buyer, and the net buyer, by exercising buyer power, would hurt the net seller. Pairwise forward trading mitigates this negative externality problem and is always in both firms' joint interest. It also involves an efficiency gain as it reduces incentives for the net seller to withhold supply and reduces incentives for the net buyer to withhold demand. This theory offers a new explanation why forward contracting is prevalent in markets with strategic firms and, in contrast with the strategic commitment mechanism proposed by Allaz and Vila (1993), it does not require contracts to be publicly observable.

The parametric configuration has considered marginal cost functions which are increasing linearly with slopes inversely related to firms' capacities. It is shown that the property pairwise efficiency is transitive: when two firms each trade pairwise-efficiently with a common counterparty who has non-zero capacity, their direct bilateral link also satisfies pairwise efficiency. With this result the analysis has yielded clear predictions regarding firms' market shares in the spot market. These predictions can be used to investigate concentration-based indicators of competitiveness. The Hirschman-Herfindahl Index (HHI) is shown to be smaller than the HHI following from a naïve calculation which would assume that market shares equal capacity shares. This result can be useful for merger evaluation in commodity markets where marginal costs can reasonably be characterized as linear. It proposes that whenever capacity shares data are available, they can serve to screen which mergers require closer attention. Another insight from the analysis is that the HHI is invariant to firms' levels of non-adjustable production and non-adjustable consumption. In the electricity industry, generation sourcing from e.g. nuclear, wind, or solar is oftentimes characterized as must-run (i.e., non-adjustable). Accordingly, the model suggests that the HHI is invariant to the amount of generation from these sources.

Whereas this paper has investigated the implications of pairwise efficient forward trading, it has not addressed the question which market microstructures lead to pairwise efficient forward trading. Insights on this front, though being beyond the scope of this paper, seem worthwhile pursuing. Also, it would be useful to have a more comprehensive model of forward trade that includes not only strategic aspects but also the risk-hedging motive for forward trading. Perhaps such a model could explore the interactions between both trading motives and generate empirical predictions that allow to measure the relative importance of the different theories.

6. References

Allaz, B., & Vila, J. L. (1993). Cournot competition, forward markets and efficiency. *Journal of Economic theory*, *59*(1), 1-16.

Anderson, E. J., & Hu, X. (2008). Forward contracts and market power in an electricity market. *International Journal of Industrial Organization*, *26*(3), 679-694.

Bagwell, K. (1995). Commitment and observability in games. *Games and Economic Behavior*, *8*(2), 271-280.

Bessembinder, H., & Lemmon, M. L. (2002). Equilibrium pricing and optimal hedging in electricity forward markets. *Journal of Finance*, *57*(3), 1347-1382.

Bushnell, J. B., Mansur, E. T., & Saravia, C. (2008). Vertical arrangements, market structure, and competition: An analysis of restructured US electricity markets. *American Economic Review*, *98*(1), 237-66.

Collard-Wexler, A., Gowrisankaran, G., & Lee, R. S. (2019). “Nash-in-Nash” bargaining: a microfoundation for applied work. *Journal of Political Economy*, *127*(1), 163-195.

Coutinho, P. B. (2013). When-Issued Markets and Treasury Auctions. *PhD dissertation, UCLA*.

Crémer, J., & Riordan, M. H. (1987). On governing multilateral transactions with bilateral contracts. *RAND Journal of Economics, 18*(3), 436-451.

van Eijkel, R., Kuper, G. H., & Moraga-González, J. L. (2016). Do firms sell forward for strategic reasons? An application to the wholesale market for natural gas. *International Journal of Industrial Organization*, *49*, 1-35.

Farrell, J., & Shapiro, C. (1990). Horizontal mergers: an equilibrium analysis. *American Economic Review, 80*(1), 107-126.

Ferreira, J. L. (2003). Strategic interaction between futures and spot markets. *Journal of Economic Theory*, *108*(1), 141-151.

Gans, J. S. (2007). Concentration-based merger tests and vertical market structure. *The Journal of Law and Economics*, *50*(4), 661-681.

Green, R. (1999). The electricity contract market in England and Wales. *The Journal of Industrial Economics*, *47*(1), 107-124.

Hart, O., & Tirole, J. (1990). Vertical integration and market foreclosure. *Brookings papers on economic activity. Microeconomics*, 205-286.

Hendricks, K., & McAfee, R. P. (2010). A theory of bilateral oligopoly. *Economic Inquiry*, *48*(2), 391-414.

Holthausen, D. M. (1979). Hedging and the competitive firm under price uncertainty. *American Economic Review*, *69*(5), 989-995.

Hortacsu, A., & Puller, S. L. (2008). Understanding strategic bidding in multi-unit auctions: a case study of the Texas electricity spot market. *RAND Journal of Economics*, *39*(1), 86-114.

Hortaçsu, A., Luco, F., Puller, S. L., & Zhu, D. (2019). Does strategic ability affect efficiency? Evidence from electricity markets. *American Economic Review*, *109*(12), 4302-42.

Hughes, J. S., & Kao, J. L. (1997). Strategic forward contracting and observability. *International Journal of Industrial Organization*, *16*(1), 121-133.

Ito, K., & Reguant, M. (2016). Sequential markets, market power, and arbitrage. *American Economic Review*, *106*(7), 1921-57.

Jeon, D. S., & Lefouili, Y. (2018). Cross-licensing and competition. *RAND Journal of Economics*, *49*(3), 656-671.

Liski, M., & Montero, J. P. (2006). Forward trading and collusion in oligopoly. *Journal of Economic Theory*, *131*(1), 212-230.

Mahenc, P., & Salanié, F. (2004). Softening competition through forward trading. *Journal of Economic Theory*, *116*(2), 282-293.

Mansur, E. T. (2007). Upstream competition and vertical integration in electricity markets. *The Journal of Law and Economics*, *50*(1), 125-156.

McAfee, R. P., & Schwartz, M. (1994). Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. *American Economic Review, 84*(1), 210-230.

Miller, N., & Podwol, J. (2019). Forward contracts, market structure, and the welfare effects of mergers. *Journal of Industrial Economics, Forthcoming*

Perry, M. K., & Porter, R. H. (1985). Oligopoly and the incentive for horizontal merger. *American Economic Review*, *75*(1), 219-227.

Ruddell, K., Downward, A., & Philpott, A. (2018). Market power and forward prices. *Economics Letters*, *166*, 6-9.

Spiegel, Y. (1993). Horizontal subcontracting. *RAND Journal of Economics*, *24*(4), 570-590.

Van Moer, G. (2018). The industrial organization of horizontal subcontracting with applications to electricity markets. *PhD dissertation, University of Antwerp.*

Van Moer, G. (2019). *Can the input supplier credibly compete downstream?* Retrieved from http://bit.ly/2PeXW8q

Wolak, F. A. (2000). An empirical analysis of the impact of hedge contracts on bidding behavior in a competitive electricity market. *International Economic Journal*, *14*(2), 1-39.

Wölfing, N. M. (2019). Forward trading and collusion in supply functions. *ZEW-Centre for European Economic Research Discussion Paper*, (19-003).

7. Appendix

Microfoundation for Assumption 1

Consider a flexible firm $i \in L$. Decompose firm i's net production as $q_i = r_i - s_i$ where r_i equals firm i's production and s_i equals firm i's consumption. It is shown that $C_i^{''}(q_i) > 0$ follows from marginal production costs being strictly increasing and marginal valuation functions being strictly decreasing.

Two special cases are straightforward to analyze. First, when firm i only has adjustable production, its consumption s_i is fixed. Denote production costs by $\rho_i(r_i)$ and marginal production costs by $\rho_i^{'}(r_i)$, where $r_i = q_i + s_i$. Since consumption is fixed, raising net production occurs exclusively by raising production. We then have that $C_i^{''}(q_i) = \rho_i^{''}(q_i + s_i) > 0$ because of strictly increasing marginal production costs. Second, when firm i only has adjustable consumption, production r_i is fixed. Denote the valuation function by $\sigma_i(s_i)$ and the marginal valuation function by $\sigma_i^{'}(s_i)$, where $s_i = r_i - q_i$. Since production is fixed, raising net production occurs exclusively by reducing consumption. We then have that $C_i^{''}(q_i) = -\sigma_i^{''}(r_i - q_i) > 0$ because the marginal valuation function is strictly decreasing.

Next consider the case where both production and consumption are adjustable. To obtain a net production equal to q_i, firm i solves the following cost minimization problem:

$$\min_{r_i, s_i} C_i(q_i) = \rho_i(r_i) - \sigma_i(s_i), \tag{A1}$$

subject to $s_i = r_i - q_i$. Plugging in, we get $\min_{r_i}\left[\rho_i\left(r_i\right)-\sigma_i\left(r_i-q_i\right)\right]$, which yields the first-order condition

$$\rho_i'\left(r_i^*\right)-\sigma_i'\left(r_i^*-q_i\right)=0. \tag{A2}$$

Firm i thus optimally chooses a production and a consumption level such that its marginal production cost equals its marginal consumption value (see also Hendricks and McAfee (2010)). Differentiating (A2) with respect to q_i gives

$$\rho_i''\left(r_i^*\left(q_i\right)\right)r_i^{*}{}'\left(q_i\right)-\sigma_i''\left(r_i^*\left(q_i\right)-q_i\right)\left[r_i^{*}{}'\left(q_i\right)-1\right]=0. \tag{A3}$$

We can rewrite (A3) as

$$\frac{\rho_i''\left(r_i^*\right)}{\sigma_i''\left(r_i^*-q_i\right)}=\frac{r_i^{*}{}'\left(q_i\right)-1}{r_i^{*}{}'\left(q_i\right)}. \tag{A4}$$

Since $\rho_i''\left(r_i^*\right)>0$ and $\sigma_i''\left(r_i^*-q_i\right)<0$ the left-hand side of (A4) is negative. Consequently, it must be true that the right-hand side is also negative, or $0<r_i^{*}{}'\left(q_i\right)<1$. This means that attaining a higher net production occurs partly by increasing production (at rate $r_i^{*}{}'\left(q_i\right)$) and partly by reducing consumption (at rate $1-r_i^{*}{}'\left(q_i\right)$).

Finally, we look for the sign of $C_i''\left(q_i\right)$. Differentiating (A1), we can write $C_i'\left(q_i\right)=\rho_i'\left(r_i^*\right)r_i^{*}{}'\left(q_i\right)-\sigma_i'\left(r_i^*-q_i\right)\left(r_i^{*}{}'\left(q_i\right)-1\right)$, which by using (A2) simplifies to $C_i'\left(q_i\right)=\rho_i'\left(r_i^*\right)$. Differentiating again yields

$$C_i''\left(q_i\right)=\rho_i''\left(r_i^*\right)r_i^{*}{}'\left(q_i\right). \tag{A5}$$

The feature that $C_i''\left(q_i\right)>0$ now follows from using that $\rho_i''\left(r_i\right)>0$ and that $0<r_i^{*}{}'\left(q_i\right)<1$.

Proof of Result 1.

The proof considers the example laid out in the main text, which has the feature that

$$C_1'(-x_1) < a < C_2'(-x_2)\,. \tag{A6}$$

Step 1 establishes four ingredients which are needed. Step 2 demonstrates that the spot market is a bilateral oligopoly.

Step 1.

1a. We first show that, whenever $C_i'(-x_i) < P$, we must have that $Q_i > 0$. Indeed, suppose not, then $Q_i \leq 0$. Consequently, $C_i'(q_i^*) \leq C_i'(-x_i)$, and by plugging in this inequality into the first-order condition (1) we get $P - b \times (x_i + q_i^*) \leq C_i'(-x_i)$. Since $C_i'(-x_i) < P$, we must thus have that $P - b \times (x_i + q_i^*) < P$, equivalently written as $-b \times (x_i + q_i^*) < 0$, which violates that $Q_i \leq 0$ and hence proves the contradiction.

1b. Second, in an analogous way, we show that whenever $P < C_i'(-x_i)$, we must have that $Q_i < 0$. Indeed, suppose not, then $Q_i \geq 0$. Consequently, $C_i'(q_i^*) \geq C_i'(-x_i)$, and by the first-order condition (1) we get $P - b \times (x_i + q_i^*) \geq C_i'(-x_i)$. Since $P < C_i'(-x_i)$, we must have that $P - b \times (x_i + q_i^*) > P$, equivalently written as $-b \times (x_i + q_i^*) > 0$, which violates that $Q_i \geq 0$ and hence proves the contradiction.

1c. Third, we establish that whenever $P = C_i'(-x_i)$ we must have $Q_i = 0$. Indeed, when $Q_i = 0$, we have $C_i'(q_i^*) = C_i'(-x_i)$ and the first-order condition (1) which is necessary and sufficient holds.

1d. Fourth, from Assumption 2, $P < a$ implies that $\sum_{i=1,2} Q_i > 0$ and $P > a$ implies that $\sum_{i=1,2} Q_i < 0$.

Step 2.

We now establish that the equilibrium price must lie within the following interval $C_1^{'}\left(-x_1\right) < P < C_2^{'}\left(-x_2\right)$ by showing that either of the extremes violates that the market clears. With this insight, it follows from 1a and 1b that firm 1 must be a net seller and firm 2 must be a net buyer.

First, consider the possibility that $P \le C_1^{'}\left(-x_1\right)$. By (A6), 1a, 1b and 1c, we must have $Q_1 \le 0$ and $Q_2 < 0$, so that $\sum_{i=1,2} Q_i < 0$. However, from (A6), $P \le C_1^{'}\left(-x_1\right)$ also implies that $P < a$, so that from 1d we would have that $\sum_{i=1,2} Q_i > 0$. This demonstrates a contraction. Therefore, $P > C_1^{'}\left(-x_1\right)$ and, by 1a, firm 1 must be a net seller.

Second, consider the possibility that $P \ge C_2^{'}\left(-x_2\right)$. By (A6), 1a, 1b and 1c, we must have $Q_1 > 0$ and $Q_2 \ge 0$, so that $\sum_{i=1,2} Q_i > 0$. However, using (A6), $P \ge C_2^{'}\left(-x_2\right)$ must also imply that $P > a$. From 1d, therefore, $\sum_{i=1,2} Q_i < 0$, a contradiction. Consequently, $P < C_2^{'}\left(-x_2\right)$ and, by 1b, firm 2 must be a net buyer. This establishes that the market is a bilateral oligopoly.

Proof of Result 3.

Step one establishes that $\dfrac{dq_e^*\left(\gamma\right)}{d\gamma} > 0$. Step two establishes that $Q_g = x_g^c + \gamma + q_g = 0$.

Step 1.

We start from firm e's first-order condition in the spot market, which equals

$$P - b\left(x_e^c - \gamma + q_e^*\left(\gamma\right)\right) - C_e^{'}\left(q_e^*\left(\gamma\right)\right) = 0,$$

where P is characterized by equation (11). Totally differentiating with respect to γ gives

$$b + \frac{dq_e^*(\gamma)}{d\gamma}\left(-2b - C_e''\left(q_e^*(\gamma)\right)\right) = 0,$$

rewritten as

$$\frac{dq_e^*(\gamma)}{d\gamma} = \frac{b}{2b + C_e''\left(q_e^*\right)} > 0. \tag{A7}$$

Step 2.

The necessary condition for pairwise efficiency is (12). Since $\frac{dq_e^*(\gamma)}{d\gamma} > 0$, as demonstrated in step one, (12) is satisfied if and only if

$$-b\left(x_g^c + \gamma + q_g\right) = 0,$$

equivalently written as

$$Q_g = x_g^c + \gamma + q_g = 0.$$

Microfoundation for k_i

We build on the microfoundation for Assumption 1 presented earlier in this Appendix. Suppose that the marginal production cost function is linear so that its slope $\rho_i'' > 0$ is constant and suppose that the marginal valuation function is also linear with constant slope $\sigma_i'' < 0$. These slopes can be interpreted as (inverse) measures of installed adjustable production and consumption capacities, defined as

$$k_i^p \equiv \frac{1}{\rho_i''} \text{ and } k_i^c \equiv \frac{1}{-\sigma_i''}. \tag{A8}$$

We are now ready to show that k_i as characterized according to (16) can be decomposed as $k_i = k_i^p + k_i^c$. Starting from (A5) and plugging in (16) and (A8) we obtain

$$\frac{1}{k_i} = \frac{r_i^*{}'(q_i)}{k_i^p}. \tag{A9}$$

Next, we can plug (16) into (A4) and obtain $\frac{-k_i^c}{k_i^p} = \frac{r_i^*{}'(q_i)-1}{r_i^*{}'(q_i)}$, which can be rewritten as

$$\frac{r_i^*{}'(q_i)}{k_i^p} = \frac{1-r_i^*{}'(q_i)}{k_i^c}. \tag{A10}$$

Combining (A9) and (A10) gives

$$\frac{1}{k_i} = \frac{r_i^*{}'(q_i)}{k_i^p} = \frac{1-r_i^*{}'(q_i)}{k_i^c}, \tag{A11}$$

rewritten as

$$k_i = \frac{k_i^p}{r_i^*{}'(q_i)} = \frac{k_i^c}{1-r_i^*{}'(q_i)}. \tag{A12}$$

It follows that (i) $k_i^p = r_i^*{}'(q_i)k_i$ and that (ii) $k_i^c = \left(1-r_i^*{}'(q_i)\right)k_i$. Adding up (i) and (ii), we can conclude that $k_i^p + k_i^c = k_i$.

Proof of Result 4.

The proof first considers the condition for pairwise efficiency between two flexible firms according to (5) in the context of the parametric configuration. We can plug in (18) and (7) to rewrite equation (5) as

$$-b\left(x_e^c-\gamma+q_e^*(\gamma)\right)(-1)\left(3b+\frac{1}{k_e}\right)\Big/C$$
$$-b\left(x_f^c+\gamma+q_f^*(\gamma)\right)\left(3b+\frac{1}{k_f}\right)\Big/C=0. \tag{A13}$$

The second derivative with respect to γ equals

$$\left(-1+\frac{dq_e^*(\gamma)}{d\gamma}\right)b\left(3b+\frac{1}{k_e}\right)\Big/C-\left(1+\frac{dq_f^*(\gamma)}{d\gamma}\right)b\left(3b+\frac{1}{k_f}\right)\Big/C,$$

which is negative because $\frac{dq_e^*(\gamma)}{d\gamma}<1$ and $\frac{dq_f^*(\gamma)}{d\gamma}>-1$ (from (18)). Consequently, the second-order condition holds, and (A13) is necessary as well as sufficient for pairwise efficiency. We can rewrite (A13) according to (19).

Second, the proof considers the condition for pairwise efficiency between a flexible and an inflexible firm according to (12) in the context of the parametric configuration. The second derivative with respect to γ equals $-b\frac{dq_e^*(\gamma)}{d\gamma}-b\left(x_g^c+\gamma+q_g\right)\frac{d^2q_e^*(\gamma)}{d\gamma^2}$, which is negative because $\frac{dq_e^*(\gamma)}{d\gamma}>0$ and $\frac{d^2q_e^*(\gamma)}{d\gamma^2}=0$ (from (A7)). Consequently, the volumes reported according to Result 3 represent a necessary and sufficient condition.

Proof of Result 5.

There are three scenarios to consider.

Scenario 1: $i,j,k\in L$.

When firms i and j trade pairwise-efficiently and when j and k trade pairwise-efficiently, it follows from using (19) that

$$\begin{cases} \left(x_i + q_i^*\right)\left(3b + \dfrac{1}{k_i}\right) = \left(x_j + q_j^*\right)\left(3b + \dfrac{1}{k_j}\right) \\ \left(x_j + q_j^*\right)\left(3b + \dfrac{1}{k_j}\right) = \left(x_k + q_k^*\right)\left(3b + \dfrac{1}{k_k}\right). \end{cases}$$

Consequently, the bilateral link between i and k is also pairwise efficient, or

$$\left(x_i + q_i^*\right)\left(3b + \frac{1}{k_i}\right) = \left(x_k + q_k^*\right)\left(3b + \frac{1}{k_k}\right).$$

Scenario 2: $i, j \in L$ and $k \in M$.

Using (20), we know that whenever j and k trade pairwise-efficiently, it follows that $Q_k = 0$, so that the bilateral link between i and k is also pairwise efficient.

Scenario 3: $j \in L$ and $i, k \in M$.

Subsection 3.3 has shown that there are no gains from trade between two inflexible firms. Consequently, the bilateral link between i and k is always pairwise efficient.

Proof of Result 6.

From Result 5 we know that it is sufficient to require that firms trade pairwise-efficiently with a central flexible firm. Without loss of generality, denote the central flexible firm as firm e. The necessary and sufficient conditions for pairwise efficiency are obtained from Result 4. By defining $\beta_i \equiv \left(3b + \frac{1}{k_i}\right)^{-1}$, these can be written as

$$Q_i = \beta_i Q_e \beta_e^{-1} \text{ for all } i \in L, i \neq e .$$

$$Q_i = 0 \text{ for all } i \in M .$$

Firms' market shares become

$$
s_i = \begin{cases} \dfrac{\beta_i Q_e \beta_e^{-1}}{\sum_{j\in L} \beta_j Q_e \beta_e^{-1} + \sum_{j\in M} Q_j} & \text{for all } i \in L, i \neq e \\ 0 & \text{for all } i \in M. \end{cases}
$$

Since $\sum_{j\in M} Q_j = 0$, we can divide the numerator and the denominator by $Q_e \beta_e^{-1}$ and obtain $s_i = \dfrac{\beta_i}{\sum_{j\in L}\beta_j}$ for all $i \in L$ with $i \neq e$. This establishes Result 6 for all $i \neq e$.

Finally, the market share of firm e is obtained by using that $s_e = 1 - \sum_{\substack{i\in N \\ i\neq e}} s_i$.

Proof of Result 7.

The proof consists of three steps. Step 1 shows that $d\sum_{i\in N} s_i^2 \Big/ db < 0$. With this result, the bounds for the HHI which are reported in Result 7 can be established by investigating the limit as b approaches infinity (Step 2) and the limit as b approaches zero (Step 3).

Step 1.

From (21), we know that $d\sum_{i\in N} s_i^2 \Big/ db = d\sum_{i\in L} s_i^2 \Big/ db$. By using the chain rule, we obtain

$$
d\sum_{i\in N} s_i^2 \Big/ db = \sum_{i\in L} 2s_i \times \left(ds_i / db \right), \tag{A14}
$$

where

$$
ds_i / db = \frac{d\beta_i / db}{\sum_{j\in L} \beta_j} + \beta_i (-1) \left(\sum_{j\in L} \beta_j \right)^{-2} \sum_{j\in L} \left(d\beta_j / db \right) \tag{A15}
$$

and

$$
d\beta_i / db = -3\beta_i^2. \tag{A16}
$$

We can plug in (A15) and (A16) into (A14) and obtain

$$d\sum_{i\in N} s_i^2 \Big/ db = \sum_{i\in L}\left[2s_i \times \left(\frac{-3\beta_i^2}{\sum_{j\in L}\beta_j} + \beta_i(-1)\left(\sum_{j\in L}\beta_j\right)^{-2}\sum_{j\in L}\left(-3\beta_j^2\right)\right)\right]. \text{ (A17)}$$

To simplify notation, define the constant $D \equiv \sum_{j\in L}\beta_j$, where we know that $D > 0$.

Using (21), we thus have that $\beta_i = s_i D$ and that $\beta_i^2 = s_i^2 D^2$. With this notation we can rewrite (A17) as

$$d\sum_{i\in N} s_i^2 \Big/ db = \sum_{i\in L}\left[2s_i \times \left(\frac{-3s_i^2 D^2}{D} + s_i D(-1) D^{-2}\sum_{j\in L}\left(-3s_j^2 D^2\right)\right)\right].$$

In the above expression, the positive constant $6D$ can be isolated. To show that $d\sum_{i\in N} s_i^2 \Big/ db < 0$, it therefore suffices to show that

$$\sum_{i\in L}\left[s_i \times \left(-s_i^2 - s_i \sum_{j\in L}\left(-s_j^2\right)\right)\right] < 0$$

$$\Leftrightarrow \sum_{i\in L}\left[-s_i^3 + s_i^4 + \left(\sum_{\substack{j\in L\\ j\neq i}} s_j^2\right) s_i^2\right] < 0.$$

Since the sum of the flexible firms' market shares equals one ($s_i + \sum_{\substack{j\in L\\ j\neq i}} s_j = 1$) we can equivalently write

$$\sum_{i\in L}\left[-s_i^3\left(s_i+\sum_{\substack{j\in L\\ j\neq i}}s_j\right)+s_i^4+\left(\sum_{\substack{j\in L\\ j\neq i}}s_j^2\right)s_i^2\right]<0$$

$$\Leftrightarrow \sum_{i\in L}\left[-s_i^3\sum_{\substack{j\in L\\ j\neq i}}s_j+\left(\sum_{\substack{j\in L\\ j\neq i}}s_j^2\right)s_i^2\right]<0$$

$$\Leftrightarrow \sum_{i\in L}\left[\sum_{\substack{j\in L\\ j\neq i}}\left[s_j s_i\left(-s_i^2+s_j s_i\right)\right]\right]<0.$$

By splitting up the sum into two parts, we obtain

$$\sum_{i\in L}\left[\sum_{\substack{j\in L\\ j<i}}\left[s_j s_i\left(-s_i^2+s_j s_i\right)\right]\right]+\sum_{i\in L}\left[\sum_{\substack{j\in L\\ j>i}}\left[s_j s_i\left(-s_i^2+s_j s_i\right)\right]\right]<0,$$

which is equivalent to

$$\sum_{i\in L}\left[\sum_{\substack{j\in L\\ j<i}}\left[s_j s_i\left(-s_i^2+s_j s_i\right)\right]\right]+\sum_{j\in L}\left[\sum_{\substack{i\in L\\ i>j}}\left[s_i s_j\left(-s_j^2+s_i s_j\right)\right]\right]<0. \quad \text{(A18)}$$

Let P denote the set of all pairs of flexible firms (i,j) with $i,j\in L, i>j$. Both terms in (A18) are summations over all pairs in P, so (A18) can be written as

$$\sum_{(i,j)\in P}\left[s_j s_i\left(-s_i^2+s_j s_i\right)\right]+\sum_{(i,j)\in P}\left[s_i s_j\left(-s_j^2+s_i s_j\right)\right]<0$$

$$\Leftrightarrow \sum_{(i,j)\in P}\left[s_j s_i\left(-s_i^2+s_j s_i\right)+s_i s_j\left(-s_j^2+s_i s_j\right)\right]<0 \qquad .$$

$$\Leftrightarrow \sum_{(i,j)\in P}\left[s_j s_i\left(-1\right)\left(s_i-s_j\right)^2\right]<0,$$

which always holds.

Step 2.

From (21), we can obtain that a flexible firm i's market share ($i\in L$) equals

$$s_i = \frac{1/(3b + 1/k_i)}{\sum_{j\in L}\left(1/(3b+1/k_j)\right)} = \frac{1/(3 + 1/bk_i)}{\sum_{j\in L}\left(1/(3+1/bk_j)\right)}. \qquad \text{(A19)}$$

The limit of the HHI as b approaches infinity is calculated, by using (A19), as

$$\lim_{b\to\infty}\sum_{i\in N} s_i^2 = \lim_{b\to\infty}\sum_{i\in L} s_i^2 = \sum_{i\in L}\lim_{b\to\infty}\left(\frac{1/(3+1/bk_i)}{\sum_{j\in L}\left(1/(3+1/bk_j)\right)}\right)^2 = \sum_{i\in L}\left(\frac{1/3}{\sum_{j\in L}(1/3)}\right)^2 = \sum_{i\in L}\left(\frac{1}{n}\right)^2,$$

where n equals the number of flexible firms.

Step 3.

The HHI as b approaches zero can be calculated by using (21) and plugging in $b = 0$. We obtain

$$\lim_{b\to 0}\sum_{i\in N} s_i^2 = \lim_{b\to 0}\sum_{i\in L} s_i^2 = \lim_{b\to 0}\sum_{i\in L}\left[\beta_i \Big/ \sum_{j\in L}\beta_j\right]^2 = \sum_{i\in L}\left[k_i/K\right]^2,$$

where K is according to (17).