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# Product and Process Innovation, Keynesian Unemployment, and Economic Growth\*

Hiroaki Sasaki<sup>†</sup>

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## Abstract

In economic growth theory, product innovation and process innovation are important factors behind technological progress. This study builds an economic growth model that considers product and process innovation and theoretically investigates how these two types of innovation affect the economic growth rate and unemployment rate. Our model, based on that developed by Zagler (2004), allows us to make the following three main findings. We find that (1) an increase in the efficiency of product innovation increases both the economic growth rate and the unemployment rate; (2) an increase in the efficiency of process innovation increases the economic growth rate and does not affect the unemployment rate; and (3) in the R&D sector, a decrease in the labor allocation to product innovation and an increase in the labor allocation to process innovation increase the economic growth rate and decrease the unemployment rate depending on the conditions. These findings suggest that to both raise employment and increase economic growth, we need not only a policy for fostering product innovation but also another policy to improve employment.

*JEL Classification:* E12; O31; O41

*Keywords:* effective demand; notional demand; unemployment; economic growth; product and process innovation; monopolistic competition

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# 1 Introduction

In economic growth theory, product innovation and process innovation are important factors behind technological progress. The endogenous growth models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) have been used to investigate the effect of product or process innovation on economic growth. However, full labor employment is assumed in these models, and thus they cannot be employed to investigate how innovation affects unemployment. Aghion and Howitt (1994) and Mortensen (2005) examine how innovation affects unemployment. However, unemployment in these models is that caused by friction in labor markets as opposed to involuntary unemployment due to the shortage of effective demand advocated by Keynes (1936).<sup>1)</sup> Based on these gaps in the literature, the present study theoretically investigates the effect of product and process innovation on economic growth and unemployment.

These two types of innovation differ in their natures and affect economic growth and unemployment differently. Product innovation produces a new good that does not exist in the economy and accordingly may increase employment. On the contrary, process innovation increases productivity and accordingly may decrease employment because it saves labor inputs if demand is constant. However, unemployment and economic growth are determined through the interactions between demand and supply.<sup>2)</sup> Consequently, to investigate the effect of innovation on economic growth and unemployment, we must build a model that captures such interactions.

The Kaleckian growth model, a type of post-Keynesian growth model, models economic growth based on the principle of effective demand and considers involuntary unemployment due to the shortage of effective demand. Lima (2004) uses a specification such that the growth rate of labor productivity is a function of the wage share, whereas Dutt (2006), Flaschel and Skott (2006), and Sasaki (2010, 2013) employ spec-

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1) Parello (2011) build a no-shirking model of innovation-based growth and investigate equilibrium employment and growth. Aricó (2003) provides a comprehensive survey of the relationship between economic growth and unemployment. Hashimoto and Ono (2020) develop an aggregate demand analysis with dynamic optimization. Most demand-led models lack dynamic optimization, but their model incorporates dynamic optimization behaviors of households and firms, which can provide a rigorous welfare analysis.

2) For the effect of innovation on employment, see Harrison *et al.* (2014) and Zhu *et al.* (2020). The former study empirically investigates the effects of process and product innovation on employment growth with data from Germany, Spain and the United Kingdom for 1998–2000. Process innovation reduces employment, holding output fixed, but output expansion of the old products overcomes this and raises employment. The latter study examines the different impact of product and process innovation on employment in China. Their results show that process innovation exerts a positive effect on employment while product innovation has a negative effect.

ifications such that the growth rate of labor productivity is an increasing function of the employment rate to investigate how technical change affects economic growth and unemployment. Hence, technical change in these models is process innovation that improves productivity, whereas process innovation is not considered.

The models of Aoki and Yoshikawa (2002, 2007), Kurose (2009), and Murakami (2017) are growth models in which product innovation relaxes the demand constraint. Since demand for existing goods saturates over time, economic growth will stagnate due to the demand constraint. However, if a new good is born in the economy due to product innovation, it alleviates the demand constraint, leading to economic growth. In each model, demand for each good expands according to the logistic curve, and a new good is born stochastically. Murakami (2017) considers the R&D costs related to the invention of new goods and endogenously derives the economic growth rate. Kurose (2009) considers process and product innovation in an extended model of Pasinetti (1993). He specifies process innovation by assuming that the labor productivity of each sector is an increasing function of time and investigates the relationship between the growth rate of the macroeconomic employment rate and the macroeconomic growth rate.<sup>3)</sup> When demand for each good rapidly increases, the growth rate of employment is high, whereas the growth rate of labor productivity is low. By contrast, when demand for each good slowly increases, the growth rate of employment is low, whereas the growth rate of labor productivity is high. These results suggest that the unemployment rate is high when the economic growth rate is high. Moreover, an increase in the probability of the birth of a new product increases the employment rate, thereby decreasing the unemployment rate, but lowers the economic growth rate.<sup>4)</sup> However, the abovementioned models do not consider factor inputs in innovation activities, and hence insufficiently examine the interactions among innovation, economic growth, and unemployment.

Our model, based on that developed by Zagler (2004), bridges this gap in the literature. He models the situation in which the number of goods produced increases due to product innovation and investigates how product innovation affects economic growth and unemployment. The mechanism by which unemployment occurs in Zagler (2004) is as follows. As the economy is growing, income increases, creating demand for new products. This endogenously increases the amount of potential investment projects, thus avoiding the emergence of secular stagnation. However, when firms fail to provide consumers with ever-new varieties of consumption products, effective

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3) Kurose (2013), an extended version of Kurose (2009), incorporates capital inputs and investigates the income distribution between wages and profits.

4) This result is not provided in his paper but is explained by the author personally.

demand will decline, changing the pattern of economic growth and, consequently, the unemployment rate. However, Zagler (2004) does not consider process innovation such that the production technology of each sector improves, thus lowering the price of each good. Therefore, we extend Zagler (2004) by considering process innovation as well as product innovation.

The key factor behind the analysis of Zagler (2004) is the specification of the price index and this specification produces some interesting results. As the number of existing goods increases, the expenditure share of each good used to calculate the price index decreases. For this reason, even if the price of each good is constant, the price index decreases as variety increases, which raises real income. Such an increase in real income increases consumption demand, which raises economic growth. He terms this mechanism the “aggregate demand externality.” His idea is considered to be based on the love of variety. Therefore, we incorporate Dixit and Stiglitz’s (1977) utility function that captures the love of variety, specify the price index consistent with utility maximization, and investigate unemployment and economic growth.<sup>5)</sup>

We find that (1) an increase in the efficiency of product innovation increases both the economic growth rate and the unemployment rate; (2) an increase in the efficiency of process innovation increases the economic growth rate and does not affect the unemployment rate; and (3) in the R&D sector, a decrease in the labor allocation to product innovation and an increase in the labor allocation to process innovation increase the economic growth rate and decrease the unemployment rate depending on the conditions.

The contributions of this study are summarized as follows. First, we introduce the Dixit–Stiglitz utility function to conduct a detailed analysis. We also consider the profit maximization of monopolistic competitive firms. As a result, the mark-up rate is determined by the elasticity of substitution between goods of the utility function. Second, we incorporate fixed costs into production of differentiated goods and consider a decrease in fixed costs due to knowledge spillovers according as the number of the differentiated goods increase. Third, we incorporate process innovation as well as product innovation and analyze how these two types of innovation affect unemployment and economic growth.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 derives the steady-state equilibrium value. Section 4 presents the results of a comparative static analysis and investigates the effect of an increase in a parameter

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5) Matsuyama (1994, 1997) are excellent surveys of monopolistic competition models that investigate economic growth and the business cycle.

on the unemployment rate and economic growth rate. Section 5 concludes.

## 2 Model

In this section, we explain the utility maximization of consumers, profit maximization of firms, specifications of the two types of innovation, specification of aggregate demand, and derivations of the unemployment and employment rates.

### 2.1 Utility maximization of consumers

Suppose an economy in which there exist  $n(t)$  types of differentiated goods, where  $t$  denotes time. Suppose also that households have a love of variety. This preference can be captured by the Dixit–Stiglitz utility function. Labor is the sole factor of production. Households obtain wage income from working and allocate all the wage income to the purchase of consumer goods. Households obtain wage income  $w(t)$  if employed, but nothing if unemployed. Suppose that labor supply is fixed to unity and that the unemployment rate is given by  $u(t)$ . Then, we consider the following utility maximization problem of households.<sup>6)</sup> In the following analysis, for simplicity, we assume that the level of utility is equal to the level of aggregate consumption  $C$ . The same results are obtained as long as the utility function is increasing in aggregate consumption:

$$\max C = \left[ \int_0^{n(t)} x(i, t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \quad (1)$$

$$\text{s.t.} \quad \int_0^{n(t)} p(i, t)x(i, t) di = [1 - u(t)]w(t), \quad (2)$$

where  $p(i, t)$  denotes the price of each good and  $x(i, t)$  is the quantity of demand for each good. The parameter  $\theta$  denotes the elasticity of substitution between goods. The assumption  $\theta > 1$  means that each good is not absolutely essential for consumption. Under this assumption, aggregate consumption  $C$  is feasible even if a good is not consumed at all. This assumption is necessary when we consider the situation in which the range of variety can change.

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6) This specification leads to the conclusion that an increase in  $n(t)$  increases aggregate consumption. However, according to the empirical studies of Ardelean (2006) and Broda *et al.* (2017), an increase in variety affects economic growth only marginally.

We define the price index as follows:

$$P(t) = \left[ \int_0^{n(t)} p(i, t)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \quad (3)$$

From equations (1) and (2), solving the utility maximization problem, we obtain the demand function for each good as follows:

$$x(i, t) = \frac{p(i, t)^{-\theta}}{P(t)^{1-\theta}} [1 - u(t)] w(t). \quad (4)$$

Using equation (4), the price elasticity of demand is given by  $\theta$ . Moreover, from equation (4), the expenditure share of each good is given by

$$\frac{p(i, t)x(i, t)}{[1 - u(t)]w(t)} = \left[ \frac{p(i, t)}{P(t)} \right]^{1-\theta}. \quad (5)$$

When  $\theta > 1$ , an increase in  $p(i, t)/P(t)$  decreases the expenditure share of each good.

Consider the symmetrical equilibrium in which the prices of all goods are equal. Then, when  $p(j, t) = p(t)$ , from equation (3), the price index is given by

$$P(t) = \left[ \int_0^{n(t)} p(t)^{1-\theta} di \right]^{\frac{1}{1-\theta}} = n(t)^{-\frac{1}{\theta-1}} p(t). \quad (6)$$

When  $\theta > 1$ , an increase the variety  $n(t)$  decreases price index  $P(t)$  even if the price of each good  $p(t)$  is constant. Here,  $1/(\theta - 1)$  corresponds to the aggregate demand externality of Zagler (2004). In the following analysis, we confine ourselves to the symmetrical equilibrium.

When  $p(i, t) = p(t)$ , substituting equation (6) into equation (5), the expenditure share of each good is rewritten as follows:

$$\frac{p(i, t)x(i, t)}{[1 - u(t)]w(t)} = \frac{1}{n(t)}. \quad (7)$$

Zagler (2004) assumes that the prices of all goods are equal and that households equally allocate their wage income to each good,  $1/n(t)$ . Accordingly, Zagler's (2004) assumption that the expenditure share of each good is constant and equal is consistent with the results of the Dixit–Stiglitz-type utility function. Moreover, in our specifications,

an increase in variety decreases the expenditure share of each good.<sup>7)</sup>

## 2.2 Profit maximization of firms

Suppose that the goods market is monopolistically competitive. The production function of each good is assumed to be given by

$$h(t) = \frac{x(t)}{a(t)} + \frac{F}{n(t)^\lambda}, \quad \lambda > 0, F \in [0, 1), \quad (8)$$

where  $h(t)$  denotes the employment of goods production sectors,  $x(t)$  is output,  $a(t)$  is labor productivity with regard to variable costs, and  $F$  is fixed costs. Here,  $F$  is measured in terms of labor units. As stated above, we consider the symmetrical equilibrium, and hence all firms face the same production technology. The restriction  $F \in [0, 1)$  is necessary for the existence of the steady state, which is explained later. By contrast, Zagler (2004) assumes that  $F = 0$ . The specification  $F/n(t)^\lambda$  shows a learning effect and captures the situation in which fixed costs decline as the number of firms increases because of knowledge spillover effects. According to Matsuyama (1994), the labor required to start a business declines as the number of firms increases. We can thus assume that the information and knowledge needed to start new businesses accumulate as the number of firms increases because new entrepreneurs can learn from the experiences of others. In the following, to simplify the analysis, we assume that  $\lambda = 1$ .

We consider firms' profit maximization. The profit of a firm is given by

$$\pi(t) = p(t)x(t) - w(t) \left[ \frac{x(t)}{a(t)} + \frac{F}{n(t)} \right]. \quad (9)$$

Nominal wage  $w(t)$  is common to all firms by assuming that workers are free to move between firms. From equation (9), the first-order condition for profit maximization is given by

$$\frac{\partial \pi(t)}{\partial p(t)} = x(t) + \left[ p(t) - \frac{w(t)}{a(t)} \right] \frac{\partial x(t)}{\partial p(t)} = 0 \quad (10)$$

$$\implies p(t) = \frac{w(t)}{a(t)} - \frac{x(t)}{\frac{\partial x(t)}{\partial p(t)}}. \quad (11)$$

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7) Zagler (2005) uses the Dixit–Stiglitz-type utility function and investigates the relationship between economic growth and unemployment. However, unemployment in Zagler (2005) is that due to negotiations between labor unions and firms rather than to a demand shortage.



With  $p(i, t) = p(t)$  in demand function (4), we obtain  $x(t)/[\partial x(t)/\partial p(t)] = -p(t)/\theta$ , which is substituted into equation (11). Then, the price of each good is given by the mark-up pricing over the marginal cost:

$$p(t) = \frac{\theta}{\theta - 1} \cdot \frac{w(t)}{a(t)} = \gamma \cdot \frac{w(t)}{a(t)}. \quad (12)$$

In the following analysis, to ease readability, we use the expression  $\gamma \equiv \theta/(\theta - 1) > 1$  if necessary.

The mark-up rate is more than unity, and thus firms obtain positive extra-profits. Since our model does not include physical capital, firms invest all extra-profits into innovation, as specified in the next section.

### 2.3 Product and process innovation

In this economy, product and process innovation are carried out in the R&D sector. Let  $s(t)$  denote employment in the R&D sector. Suppose that the constant fractions  $\delta \in [0, 1]$  and  $1 - \delta$  of  $s(t)$  are devoted to product and process innovation, respectively.<sup>8)</sup> Then, we can specify these two types of innovation as follows:

$$\dot{n}(t) = \phi n(t) \delta s(t), \quad \phi \in (0, 1), \quad (13)$$

$$\dot{a}(t) = \beta a(t) (1 - \delta) s(t), \quad \beta \in (0, 1), \quad (14)$$

where  $\phi$  denotes the efficiency of product innovation and  $\beta$  is the efficiency of process innovation. Equation (13) shows that the higher the number of existing goods and the higher the number of researchers, the more likely a new good is produced. Equation (14) shows that the higher labor productivity and the higher the number of researchers, the more likely labor productivity increases. These specifications show that when  $s(t)$  is constant at the steady state, the growth rates of  $n$  and  $a$  are constant. In R&D endogenous growth models, scale effects are a disputable problem: the higher the number of researchers, the higher is the per capita growth rate of output. In our model, the employment share rather than the employment level is a variable in the innovation functions; hence, scale effects do not arise.<sup>9)</sup>

Suppose that product innovation fails at a constant rate  $\varphi$ . Then, the number of

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8) The endogenization of  $\delta$  is a subject for future research.

9) For scale effects in endogenous growth theory, see Jones (1999).

goods actually sold in goods markets can be specified as

$$m(t) = n(t) - \varphi \dot{n}(t), \quad \varphi \in (0, 1). \quad (15)$$

From this, the equation of motion of  $m(t)$  is given by

$$\dot{m}(t) = \dot{n}(t) - \varphi \ddot{n}(t). \quad (16)$$

Consumers plan to purchase  $n(t)$  types of goods beforehand. However, if they judge that a newly invented good is not useful based on their acquired knowledge, they do not purchase it.<sup>10)</sup> In addition, even if a good is invented, consumers do not purchase it unless they have the knowledge to use it well.

When firms innovate, they face uncertainty, such as whether their products will succeed when they appear in goods markets. Hence, they must sign labor contracts with workers. If newly invented products fail in goods markets, firms have to fire workers, and hence these workers cannot find other jobs in that period. In this case, they become unemployed and do not obtain a wage income.

## 2.4 Notional demand and effective demand

Benassy (1982) defines the idea of “notional demand.” Here, we assume that notional demand is the level of aggregate consumption at which all consumers realize their desired planned consumption. In our model, notional demand  $C_N(t)$  can be specified as follows:

$$\begin{aligned} C_N(t) &= \frac{1}{P(t)} \int_0^{n(t)} p(t)x(t) di = \frac{1}{P(t)} \int_0^{n(t)} \frac{[1 - u(t)]w(t)}{n(t)} di \\ &= \frac{1}{P(t)} [1 - u(t)]w(t). \end{aligned} \quad (17)$$

Here, the price index is given by  $P(t) = m(t)^{1/(1-\theta)}p(t)$  since  $m(t)$  is the number of the goods in the economy.

Effective demand can be specified as follows:

$$\begin{aligned} C(t) &= \frac{1}{P(t)} \int_0^{m(t)} p(t)x(t) di = \frac{1}{P(t)} \int_0^{m(t)} \frac{[1 - u(t)]w(t)}{n(t)} di \\ &= \frac{1}{P(t)} \frac{[1 - u(t)]w(t)}{n(t)} m(t), \end{aligned} \quad (18)$$

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10) In Zagler (2004), telephones, personal computers, and medicines are provided as examples.

$$\begin{aligned}
&= m(t)^{\frac{1}{\theta-1}} p(t)^{-1} \frac{m(t)}{n(t)} [1 - u(t)] w(t) \\
&= \frac{\theta - 1}{\theta} m(t)^{\frac{\theta}{\theta-1}} n(t)^{-1} a(t) [1 - u(t)].
\end{aligned} \tag{19}$$

In this specification, the price index is given by  $P(t) = m(t)^{1/(1-\theta)} p(t)$  since  $m(t)$  is the number of the goods in the economy. Moreover, from the definition of notional demand, the expenditure share of each good is given by  $1/n(t)$ . From equations (17) and (18), the ratio of effective demand to notional demand is given by  $C(t)/C_N(t) = m(t)/n(t)$ .

Taking the logarithms of equation (19) and differentiating the resultant expression with respect to time, we obtain

$$\hat{C}(t) = \frac{\theta}{\theta - 1} \hat{m}(t) - \hat{n}(t) + \hat{a}(t) - \frac{u(t)}{1 - u(t)} \hat{u}(t), \tag{20}$$

where  $\hat{z}(t) = \dot{z}(t)/z(t)$  denotes the rate of change in variable  $z(t)$ . In the following analysis, we call  $\hat{C}(t)$  the economic growth rate.

## 2.5 Unemployment and sectoral employment rates

The unemployment rate, employment rate of the goods sector, and employment rate of the R&D sector are respectively given by

$$u(t) = \int_{m(t)}^{n(t)} h(t) di = \int_{m(t)}^{n(t)} \frac{1 - u(t) + \gamma F}{\gamma n(t)} di \tag{21}$$

$$= \left( \frac{1 - u(t) + \gamma F}{\gamma} \right) \varphi \hat{n}(t) \tag{22}$$

$$= \left( \frac{1 - u(t) + \gamma F}{\gamma} \right) \varphi \phi \delta s(t), \tag{23}$$

$$e(t) = \int_0^{m(t)} h(t) di = \frac{1 - u(t) + \gamma F}{\gamma} - u(t), \tag{24}$$

$$s(t) = 1 - \frac{1 - u(t) + \gamma F}{\gamma}, \tag{25}$$

where we use  $u(t) + e(t) + s(t) = 1$ . From equation (24),  $e$  and  $u$  change in the opposite direction. From equation (25),  $s$  and  $u$  change in the same direction.

Summarizing equations (23), (24), and (25), we obtain the following quadratic equation of  $u(t)$ :

$$u(t)^2 + \frac{1}{\varphi \phi \delta} [\gamma^2 - \varphi \phi \delta \{2 - \gamma(1 - 2F)\}] u(t) + [1 - \gamma + 2\gamma F - \gamma^2 F(1 - F)] = 0. \tag{26}$$

Solving equation (26), we obtain the steady-state equilibrium value of the unemployment rate. Since  $u^*$  becomes a function of the constant parameters, it does not depend on time. Here, “\*” denotes the steady-state equilibrium value of a variable. From  $u^*$ , the values of  $e^*$  and  $s^*$  are determined. Since the parameter  $\beta$  does not appear in  $u^*$ ,  $e^*$ , or  $s^*$ , the unemployment rate, employment rate of the goods sector, and employment rate of the R&D sector do not depend on  $\beta$  that captures the efficiency of process innovation.

The unemployment rate must lie within the interval of  $u \in [0, 1)$ . Let the left-hand side of equation (26) be  $f(u)$ . The graph of  $f(u)$  is a parabola that is convex downward. The end points  $f(0)$  and  $f(1)$  are given by

$$f(0) = 1 - \gamma(1 - 2F) - \gamma^2 F(1 - F), \quad (27)$$

$$f(1) = \gamma^2 \left[ \frac{1}{\varphi\phi\delta} - F(1 - F) \right]. \quad (28)$$

From the restriction on the parameters, we have  $1/(\varphi\phi\delta) > 1$  and from  $F \in [0, 1)$ , we have  $0 \leq F(1 - F) \leq 1/4$ . Accordingly, we necessarily obtain  $f(1) > 0$ . From this, the necessary and sufficient condition for  $f(u) = 0$  to have one root within the interval of  $u \in [0, 1)$  is given by  $f(0) < 0$ .

Here, we regard  $f(0)$  as a function of  $F$  and set  $g(F) = \gamma^2 F^2 + \gamma(2 - \gamma)F - (\gamma - 1)$ . The domain of  $F$  is given by  $F \in [0, 1)$ . We obtain  $g(0) = -(\gamma - 1) < 0$  and  $g(1) = \gamma + 1 > 2$ . In addition, we obtain  $g(1/\theta) = 0$ . From this, we obtain  $g(F) < 0$  within the interval of  $F \in [0, 1/\theta)$ . This implies that  $F \in [0, 1/\theta)$  and  $f(0) < 0$  are equivalent. Therefore, we assume the following inequality.

**Assumption 1.** *The restriction  $F < 1/\theta$  holds.*

### 3 Steady state

In our model, the equilibrium value of  $s^*$  is determined before the economic growth rate. We consider the dynamics of  $x(t) = m(t)/n(t)$ , that is, the ratio of the number of goods actually purchased  $m(t)$  to the number of goods invented by product innovation  $n(t)$ :

$$\hat{x}(t) = \hat{m}(t) - \hat{n}(t). \quad (29)$$

As stated above, note that  $x(t) = C(t)/C_N(t)$  also holds. Here, from  $m(t) = n(t) - \varphi\dot{n}(t)$  and  $\dot{n}(t) = \phi\delta n(t)s^*$ , we obtain the following equation:

$$\dot{m}(t) = \phi\delta n(t)s^* - \varphi\phi^2\delta^2(s^*)^2n(t). \quad (30)$$

From this, the differential equation of  $x$  is given by

$$\dot{x}(t) = -\phi\delta s^*x(t) + \phi\delta s^*(1 - \varphi\phi\delta s^*). \quad (31)$$

The steady state is a state in which  $\dot{x}(t) = 0$  holds. Since  $x^* = 1 - \varphi\phi\delta s^*$  and  $d\dot{x}(t)/dx(t) = -\phi\delta s^* < 0$ , the steady state is stable and the steady-state value of  $x$  is given by

$$x^* = \left(\frac{m}{n}\right)^* = \left(\frac{C}{C_N}\right)^* = 1 - \varphi\phi\delta s^*. \quad (32)$$

Under Assumption 1, we obtain  $s^* \in (0, 1)$  and hence,  $x^* \in (0, 1)$  from the restrictions on the parameters. Note that  $s^*$  itself depends on  $\phi$ ,  $\varphi$ ,  $F$ ,  $\delta$ , and  $\theta$ .

The result that  $x(t)$  converges to the content value means that the growth rates of  $m(t)$  and  $n(t)$  are equalized at the steady state. Substituting  $\hat{m} = \hat{n}$  into equation (20), we obtain the economic growth rate at the steady state as follows:

$$\hat{C}^* = \frac{1}{\theta - 1} \hat{n}^* + \hat{a}^* \quad (33)$$

$$= \left[ \frac{\phi\delta}{\theta - 1} + \beta(1 - \delta) \right] s^*. \quad (34)$$

Equation (33) shows that the economic growth rate is the sum of the growth rate of the new invention multiplied by the aggregate demand externality and the growth rate of labor productivity.

## 4 Comparative static analysis

As Section 2 shows, solving equation (26) for  $u$ , we obtain the steady-state equilibrium value of the unemployment rate from which we obtain the steady-state value of the economic growth rate. However, analyzing the solution of quadratic equation (26) is rather complicated. Therefore, following Zagler (2004), we use two graphs that relate  $u$  to  $\hat{C}$  to investigate changes in the parameters on  $u^*$  and  $\hat{C}^*$ .

First, supposing that the economy is in the steady state, substituting equation (25)

into equations (13) and (14) and substituting the resultant expression into equation (20), we obtain the EC curve that represents the equilibrium condition. Next, from equation (20), we obtain  $\hat{n} = (\theta - 1)(\hat{C} - \hat{a})$ . Substituting this equation into  $\hat{n}$  of equation (23) and substituting equations (14) and (25) into the resultant expression, we obtain the ED curve that represents the effective demand constraint. Summarizing the above results, we obtain the following two curves:

$$\text{EC : } \hat{C} = \frac{1}{\theta} [\phi\delta + \beta(1 - \delta)(\theta - 1)]u + \left( \frac{1 - \theta F}{\theta} \right) \left[ \beta(1 - \delta) + \frac{\phi\delta}{\theta - 1} \right], \quad (35)$$

$$\text{ED : } \hat{C} = \frac{\theta u}{\varphi(\theta - 1)[(\theta - 1)(1 - u) + \theta F]} + \frac{\beta(1 - \delta)(\theta - 1)}{\theta} u + \left( \frac{1 - \theta F}{\theta} \right) \beta(1 - \delta). \quad (36)$$

The EC curve is a straight line with a positive slope and a positive intercept.

We investigate the slope of the ED curve:

$$\left. \frac{d\hat{C}}{du} \right|_{\text{ED}} = \frac{\theta}{\varphi(\theta - 1)^2} \cdot \frac{1 + \gamma F}{(1 - u + \gamma F)^2} + \frac{\beta(1 - \delta)(\theta - 1)}{\theta} > 0, \quad (37)$$

$$\left. \frac{d^2\hat{C}}{du^2} \right|_{\text{ED}} = \frac{2\theta(1 + \gamma F)}{\varphi(\theta - 1)^2(1 - u + \gamma F)^3} > 0. \quad (38)$$

From these, the slope of the ED curve is positive and increasing with respect to  $u$ .

The intercepts of both curves are determined by the sign of  $1 - \theta F$ . From Assumption 1, we obtain  $1 - \theta F > 0$ , which means that both intercepts are positive. Moreover, in this case, we know that the intercept of the EC curve is larger than that of the ED curve. Therefore, we obtain Figure 1.

Since both curves are upward sloping, the relationship between the unemployment rate and economic growth rate is a trade-off. In the following analysis, we investigate how a change in a parameter affects the EC and ED curves to examine the changes in the equilibrium values of the unemployment rate and economic growth rate. We also use numerical simulations to supplement the analysis since the graphical analysis does not produce definite results in some cases.

We note the equilibrium values of the employment rate of the goods sector and that of the R&D sector. From equation (24), except for  $F$  and  $\theta$ ,  $e^*$  and  $u^*$  change in the opposite directions when a parameter changes. From equation (25), except for  $F$  and  $\theta$ ,  $s^*$  and  $u^*$  change in the same direction when a parameter changes.

From equation (32), the effect of a change in the parameter  $\phi$ , for example, on the

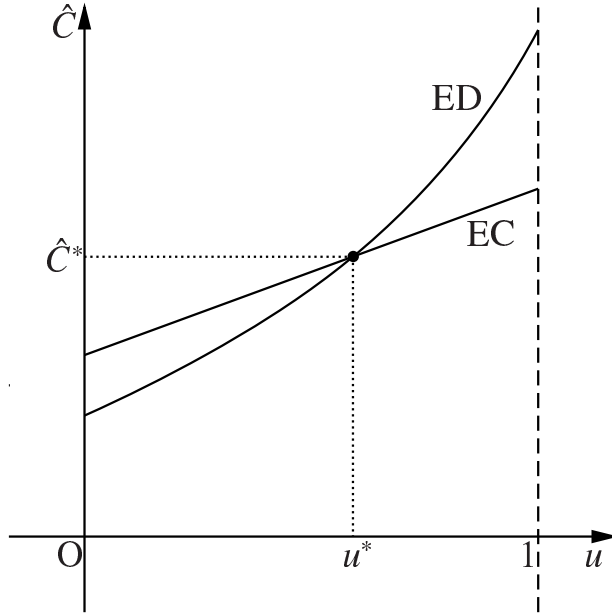


Figure 1: Determination of the unemployment rate and economic growth rate in the equilibrium

demand ratio  $C/C_N$ , is given by

$$\frac{d(C/C_N)^*}{d\phi} = -\varphi\delta s^* - \phi\varphi\delta \frac{ds^*}{d\phi}. \quad (39)$$

The first term on the right-hand side of equation (39) shows the direct effect and the second term on the right-hand side of equation (39) shows the indirect effect. For the other parameters, we can use similar procedures.

#### 4.1 Increase in the efficiency of product innovation

An increase in  $\phi$  does not move the ED curve but shifts the EC curve upward because both the slope and the intercept of the EC curve become large. Therefore, both the economic growth rate and the unemployment rate increase.

An increase in  $\phi$ , which is a supply-side factor, shifts the EC curve upward, which increases the economic growth rate for a given unemployment rate. The increase in  $\phi$ , other things being equal, increases the growth rate of  $n$ , which decreases effective demand  $m/n$  because it changes  $\dot{n}/n = \dot{m}/m$  to  $\dot{n}/n > \dot{m}/m$ . The decrease in effective demand increases unemployment  $u$  from which the employment rate of the R&D sector  $s$  increases. The increase in  $s$  raises the growth rates of both types of innovation, which increases the economic growth rate through the aggregate demand externality. There-

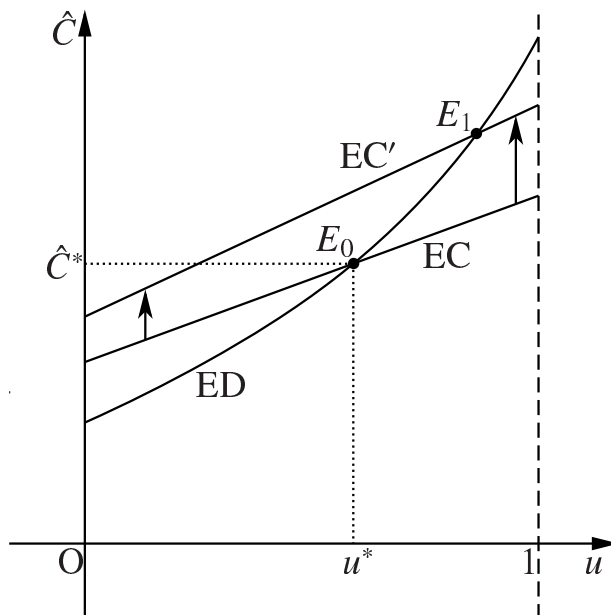


Figure 2: Effect of an increase in  $\phi$  on  $u$  and  $\hat{C}$

fore, an increase in the efficiency of product innovation increases both the economic growth rate and the unemployment rate.

## 4.2 Increase in the efficiency of process innovation

An increase in  $\beta$  shifts the EC and ED curves upward. From equation (26) that determines  $u^*$ , we see that  $\beta$  does not affect  $u^*$ . Therefore, an increase in  $\beta$  increases the economic growth rate but does not affect the unemployment rate.

An increase in  $\beta$ , which is a supply-side factor, shifts the ED curve as well as the EC curve. In this respect, it is different than an increase in  $\phi$  (another supply-side factor). An increase in  $\beta$  increases labor productivity, which decreases the prices of the goods, leading to an increase in the real wage. The increase in the real wage increases effective demand, and hence decreases the unemployment rate. In our model, the increase in the unemployment rate arising from an upward shift of the EC curve exactly offsets the decrease in the unemployment rate arising from an upward shift of the ED curve. Therefore, the equilibrium value of the unemployment rate does not change.

## 4.3 Increase in the failure rate of product innovation

An increase in  $\varphi$  does not move the EC curve but rotates the ED curve clockwise. Therefore, both the economic growth rate and the unemployment rate increase.



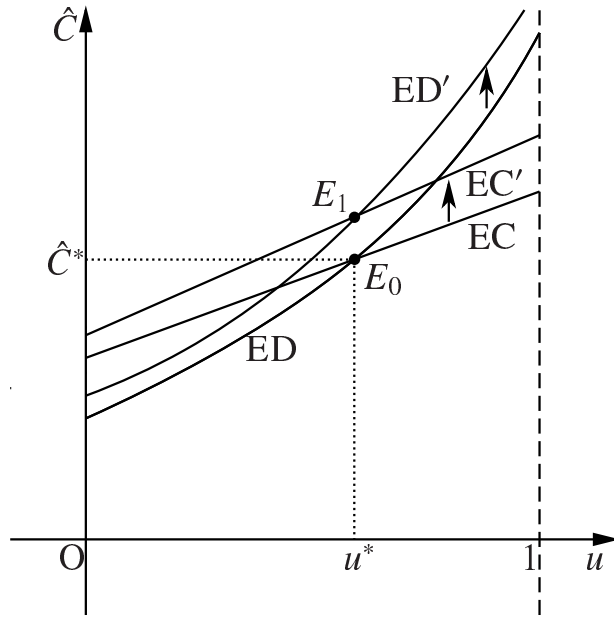


Figure 3: Effect of an increase in  $\beta$  on  $u$  and  $\hat{C}$

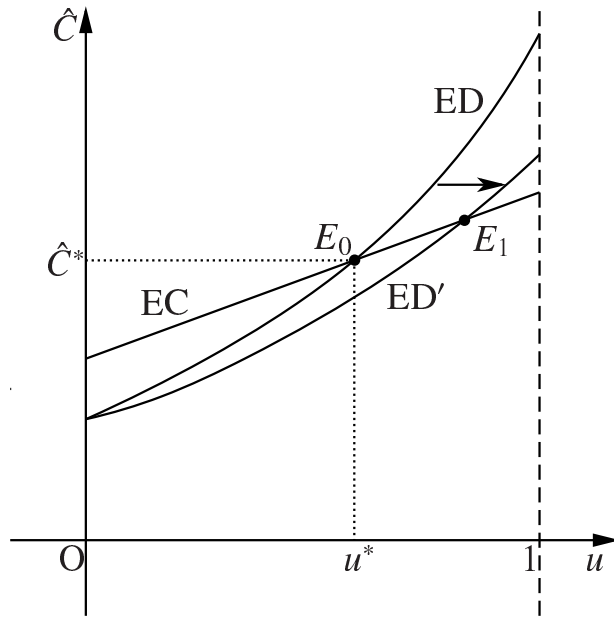


Figure 4: Effect of an increase in  $\varphi$  on  $u$  and  $\hat{C}$

An increase in  $\varphi$ , which is a demand-side factor, decreases effective demand. This shifts the ED curve to the right, which increases the unemployment rate for a given economic growth rate. The increase in  $\varphi$  decreases the growth rate of  $m$ , which decreases effective demand  $m/n$  because it changes  $\dot{n}/n = \dot{m}/m$  to  $\dot{n}/n > \dot{m}/m$ . From this, the growth rate of innovation increases, which increases the economic growth rate through the aggregate demand externality. Therefore, at the new steady-state equilibrium, both the unemployment rate and the economic growth rate increase.

#### 4.4 Increase in the fixed costs of goods production

With an increase in  $F$ , the slope of the EC curve does not change and its intercept becomes smaller, which shifts the EC curve downward. The slope of the ED curve becomes flatter and its intercept becomes smaller, which shifts the ED curve downward. Combining these results, the economic growth rate is likely to decrease. For the unemployment rate, the effect is ambiguous. Accordingly, we resort to numerical simulations.

We set the benchmark parameters as follows:

$$\theta = 3.62, \phi = 0.6, \delta = 0.5, \beta = 0.5, F = 0.05, \varphi = 0.5. \quad (40)$$

For the elasticity of substitution, Ardelean (2006) estimates 3.79 for the average of countries and 5.33 for the United States, whereas Broda *et al.* (2017) estimates 3.45. Hence, we use  $\theta = 3.62$ , which is the average of 3.79 and 3.45. We set the efficiency of product innovation higher than the value of process innovation. For the labor allocation parameter  $\delta$  in the R&D sector, we use 0.5; that is, the labor allocation between the two types of innovation is equal. For fixed costs, we use a value that satisfies Assumption 1. Following the findings of NISTEP (2016), we suppose that the failure rate of product innovation is about 50% and use the value  $\varphi = 0.5$ . Under these settings, we obtain  $u^* = 0.0278532$  and  $\hat{C}^* = 0.0898145$ .

If we increase  $F = 0.05$  to  $F = 0.06$ , we obtain  $u^* = 0.0270303$  and  $\hat{C}^* = 0.0859523$ . Therefore, both the economic growth rate and the unemployment rate decrease.

An increase in  $F$ , which is a supply-side factor, increases the employment rate of the goods sector and decreases the employment rate of the R&D sector as a direct effect. The decrease in the employment rate of the R&D sector decreases the growth rates of the two types of innovation, which decreases the economic growth rate. Moreover, the decrease in the employment rate of the R&D sector increases effective demand  $m/n$ , which decreases the unemployment rate. Therefore, both the economic growth rate

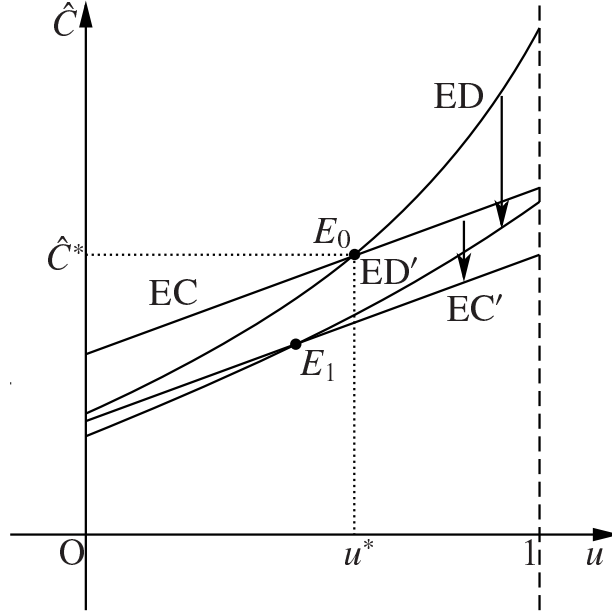


Figure 5: Effect of an increase in  $F$  on  $u$  and  $\hat{C}$

and the unemployment rate decrease.

#### 4.5 Increase in the employment allocation rate to the product innovation sector

With an increase in  $\delta$ , the slope of the ED curve becomes flatter and its intercept becomes smaller, which shifts the ED curve downward. An increase in  $\delta$  affects the slope and intercept of the EC curve as follows:

$$\frac{d(\text{EC's slope})}{d\delta} = \frac{1}{\theta} [\phi - \beta(\theta - 1)], \quad (41)$$

$$\frac{d(\text{EC's intercept})}{d\delta} = \frac{1 - \theta F}{\theta} \left[ \frac{\beta + \theta(1 - \beta)}{\theta - 1} \right] > 0. \quad (42)$$

Moreover, an increase in  $\delta$  affects  $\hat{C}$  when  $u = 1$  as follows:

$$\left. \frac{d\hat{C}}{d\delta} \right|_{u=1} = \frac{1 - F}{\theta - 1} [\phi - \beta(\theta - 1)]. \quad (43)$$

Hence, when  $\phi > \beta(\theta - 1)$ , both the slope and the intercept of the EC curve become larger. Then, combining this result with the shift of the ED curve, both the economic growth rate and the unemployment rate increase.

On the contrary, when  $\phi < \beta(\theta - 1)$ , the slope of the EC curve becomes smaller, the intercept at  $u = 0$  becomes larger, and the intercept at  $u = 1$  becomes smaller. Accordingly, an increase in  $\delta$  rotates the EC curve clockwise. Then, the effects on the economic growth rate and unemployment rate are ambiguous.

In the numerical examples, we obtain  $\phi < \beta(\theta - 1)$ . If we increase  $\delta = 0.5$  to  $\delta = 0.6$ , we obtain  $u^* = 0.338144$  and  $\hat{C}^* = 0.0845929$ ; that is, the economic growth rate decreases and the unemployment rate increases. This implies that a decrease in  $\delta$  increases the economic growth rate and decreases the unemployment rate in some cases. In other words, an increase in the labor allocation toward process innovation produces the favorable result that the economic growth rate increases and the unemployment rate decreases.

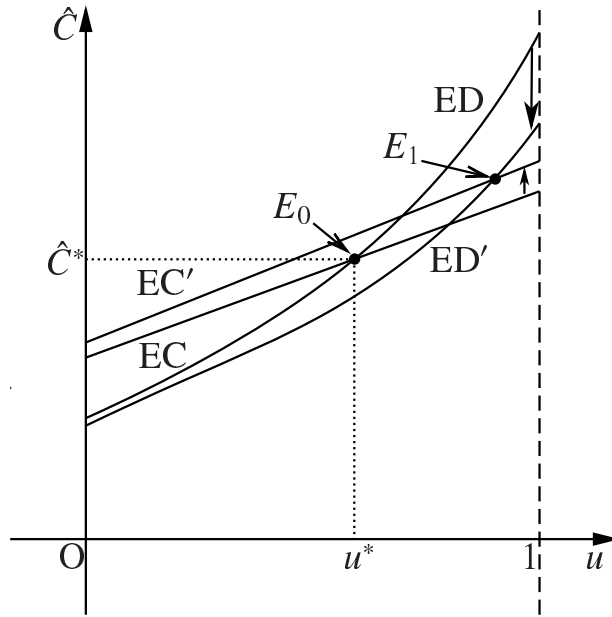


Figure 6: Effect of an increase in  $\delta$  on  $u$  and  $\hat{C}$  when  $\phi > \beta(\theta - 1)$

If we change the values of the parameters, an increase in  $\delta$  increases both the economic growth rate and the unemployment rate.<sup>11)</sup> Therefore, an increase in the labor allocation for process innovation is likely to increase the unemployment rate and has an ambiguous effect on the economic growth rate.

11) For example, if we use  $\theta = 6$ ,  $\phi = 0.8$ ,  $\delta = 0.8$ ,  $\beta = 0.2$ ,  $F = 0.05$ , and  $\varphi = 0.6$ , we obtain  $u^* = 0.0515009$  and  $\hat{C}^* = 0.0268101$ . If we set  $\delta = 0.9$ , we obtain  $u^* = 0.06$  and  $\hat{C}^* = 0.0273333$ . In this case, the relation  $\phi < \beta(\theta - 1)$  holds.

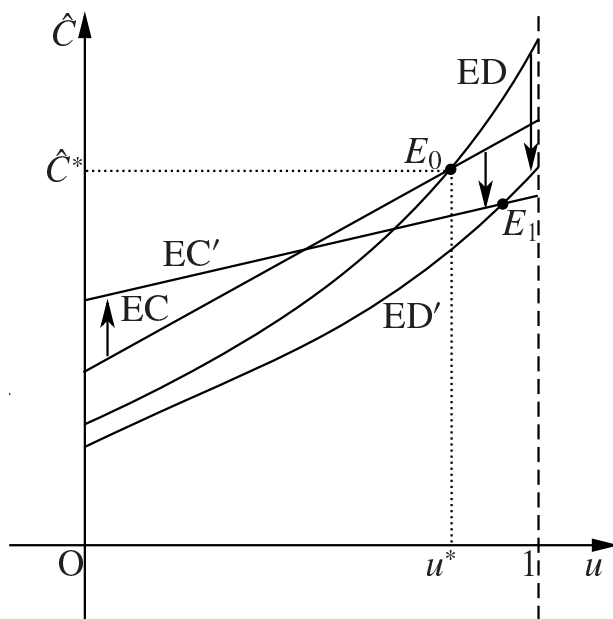


Figure 7: Effect of an increase in  $\delta$  on  $u$  and  $\hat{C}$  when  $\phi < \beta(\theta - 1)$  and  $\hat{C}$  eventually decreases

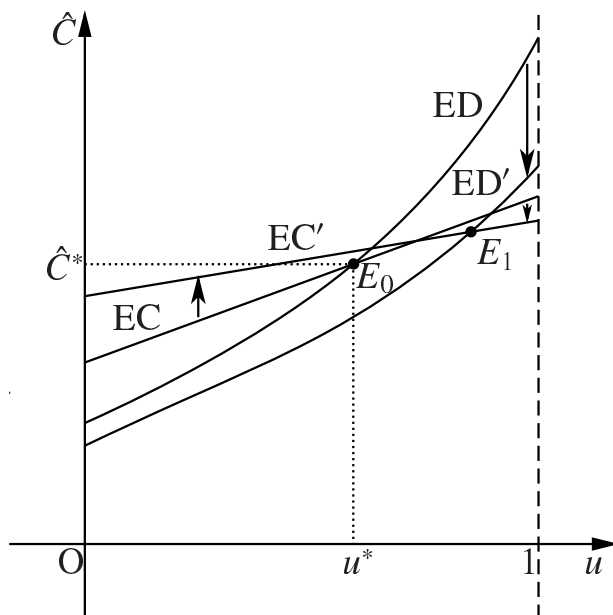


Figure 8: Effect of an increase in  $\delta$  on  $u$  and  $\hat{C}$  when  $\phi < \beta(\theta - 1)$  and  $\hat{C}$  eventually increases

The above results can be understood by rewriting equation (34) as follows:

$$\hat{C}^* = \left[ \underbrace{\left( \frac{\phi}{\theta - 1} - \beta \right)}_{\star} \delta + \beta \right] s^*(\delta). \quad (44)$$

An increase in  $\delta$  affects the part  $\star$  (direct effect) and  $s^*(\delta)$  (indirect effect). If  $\phi > \beta(\theta - 1)$ , the direct effect is positive. By contrast, if  $\phi < \beta(\theta - 1)$ , the direct effect is negative. Since the indirect effect  $ds^*/d\delta$  is positive, an increase in  $\delta$  increases the economic growth rate when the direct effect is positive. When the direct effect is negative, an increase in  $\delta$  either increases or decreases the economic growth rate. The part of  $\phi/(\theta - 1)$  of the direct effect is the efficiency of product innovation multiplied by the aggregate demand externality, and  $\beta$  is the efficiency of process innovation. Therefore, an increase in the labor allocation to product innovation increases the economic growth when the effect of product innovation on the economic growth rate is larger than the effect of process innovation on the economic growth rate. On the contrary, it decreases the economic growth rate when the effect of product innovation on the economic growth rate is smaller than the effect of process innovation on the economic growth rate.

An increase in  $\delta$ , which is a supply-side factor, has two combined effects: an increase in  $\phi$  and a decrease in  $\beta$ . As stated above, an increase in  $\phi$  increases both the economic growth rate and the unemployment rate. A decrease in  $\beta$  decreases the economic growth rate. An increase in  $\beta$  does not change  $u^*$ ,  $e^*$ , or  $s^*$ . Summarizing the above results, an increase in  $\delta$  increases the unemployment rate. Moreover, an increase in  $\delta$  either increases or decreases the economic growth rate according to which effect is dominant: the growth-enhancing effect of product innovation or the growth-stagnating effect of process innovation.

## 4.6 Increase in the elasticity of substitution

An increase in  $\theta$  affects the intercept of the ED curve as follows:

$$\frac{d(\text{EC's intercept})}{d\theta} = -\phi\delta \frac{\theta(\theta - 1)F + (1 - \theta F)(2\theta - 1)}{\theta(\theta - 1)} - \frac{\beta(1 - \delta)}{\theta^2} < 0. \quad (45)$$

Equation (45) is negative for the following reason. Regarding the numerator of the first term on the right-hand side as a function of  $F$ , we set  $h(F) = \theta(\theta - 1)F + (1 - \theta F)(2\theta - 1)$ . The graph of this function is a straight line with a negative slope and a positive intercept. From Assumption 1, we have  $F < 1/\theta$  and  $h(1/\theta) = \theta - 1 > 0$ ,

and accordingly  $h(F) > 0$  for  $F \in (0, 1/\theta]$ . This means that an increase in  $\theta$  makes the intercept of the EC curve smaller.

An increase in  $\theta$  affects the slope of the EC curve as follows:

$$\frac{d(\text{EC's slope})}{d\theta} = \frac{\beta(1 - \delta) - \phi\delta}{\theta^2}. \quad (46)$$

From this, we obtain the following relationship:

$$\beta(1 - \delta) \gtrless \phi\delta \implies \frac{d(\text{EC's slope})}{d\theta} \gtrless 0 \quad (\text{double-sign corresponds}). \quad (47)$$

With the employment rate of the R&D sector  $s^*$  as given,  $\beta(1 - \delta)$  represents the speed of process innovation and  $\phi\delta$  represents the speed of product innovation. When the speed of process innovation exceeds that of product innovation, an increase in  $\theta$  makes the slope of the EC curve steeper. On the contrary, when the speed of product innovation exceeds that of process innovation, an increase in  $\theta$  makes the slope of the EC curve flatter. In our numerical example,  $\beta(1 - \delta) < \phi\delta$  holds, and hence the speed of product innovation exceeds that of process innovation.

An increase in  $\theta$  affects the intercept of the ED curve as follows:

$$\frac{d(\text{ED's intercept})}{d\theta} = -\frac{\beta(1 - \delta)}{\theta^2} < 0. \quad (48)$$

This means that an increase in  $\theta$  decreases the intercept of the ED curve.

However, it is difficult to obtain the effect of an increase in  $\theta$  on the slope of the ED curve.<sup>12)</sup>

The numerical simulations show that if we increase  $\theta = 3.62$  to  $\theta = 3.63$ , we obtain  $u^* = 0.0277935$  and  $\hat{C}^* = 0.0894221$ . Therefore, both the economic growth rate and the unemployment rate decrease.

Even if we set  $\phi = 0.5$  and  $\beta = 0.6$  to obtain  $\beta(1 - \delta) > \phi\delta$ , both the economic growth rate and the unemployment rate decrease if we increase  $\theta = 3.62$  to  $\theta = 3.63$ .

The elasticity of substitution between differentiated goods is a parameter that is a demand-side factor, and it corresponds to the price elasticity of demand. An increase in  $\theta$  increases effective demand  $m/n$  because demand for each differentiated good largely increases when the price of each good decreases due to process innovation. This decreases the unemployment rate, increases the employment rate of the goods sector, and decreases the employment rate of the R&D sector. This, in turn, leads to

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12) In the quadratic equation  $f(u) = 0$ , an increase in  $\theta$  increases the value of  $f(0)$  and decreases the value of  $f(1)$ . However, it is uncertain whether  $u^*$  such that  $f(u^*) = 0$  increases or decreases.

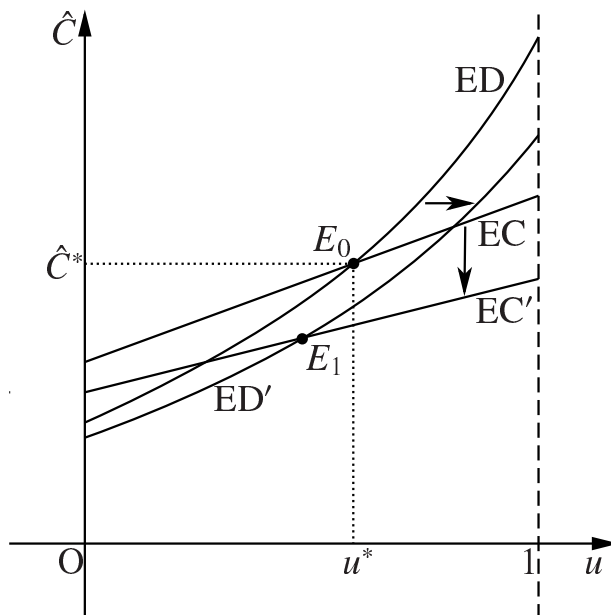


Figure 9: Effect of an increase in  $\theta$  on  $u$  and  $\hat{C}$

the stagnation of innovation activities, which decreases the economic growth rate. For these reasons, an increase in the elasticity of substitution decreases both the economic growth rate and the unemployment rate.

Table 1 summarizes the results obtained from Sections 4.1 to 4.6. For the three parameters,  $F$ ,  $\delta$ , and  $\theta$ , we combine analytical and numerical methods.

Table 1: Results of the comparative static analysis

	$\phi$	$\beta$	$\varphi$	$F$	$\delta$	$\theta$
$\hat{C}$	+	+	+	-	$\pm$	-
$u^*$	+	0	+	-	+	-
$e^*$	-	0	-	+	-	+
$s^*$	+	0	+	-	+	-
$(C/C_N)^*$	-	0	-	+	-	+

## 4.7 Special cases

When  $\delta = 1$  in the R&D sector, process innovation is not carried out; only product innovation is conducted. This case is similar to that of Zagler (2004). In our model, with  $\delta = 1$ , irrespective of the existence of fixed costs, an increase in  $\phi$  or  $\varphi$  increases both the economic growth rate and the unemployment rate. This result is the same



as that of Zagler (2004).<sup>13)</sup>

When  $\delta = 0$  in the R&D sector, product innovation is not carried out; only process innovation is conducted. In this case, the two curves are rewritten as follows:

$$\text{EC} : \hat{C} = \frac{\beta(\theta - 1)}{\theta} u + \frac{\beta(1 - \theta F)}{\theta}, \quad (49)$$

$$\text{ED} : \hat{C} = \frac{\theta u}{\varphi(\theta - 1)[(\theta - 1)(1 - u) + \theta F]} + \frac{\beta(\theta - 1)}{\theta} u + \frac{\beta(1 - \theta F)}{\theta}. \quad (50)$$

The intersection of these two curves is given by  $u = 0$ . Therefore, the steady-state equilibrium values of the economic growth rate and unemployment rate are given by

$$u^* = 0, \quad (51)$$

$$\hat{C}^* = \frac{\beta(1 - \theta F)}{\theta} > 0. \quad (52)$$

Accordingly, the unemployment rate is zero. This means that our model is reduced to a supply-constrained growth model that is not restricted by the demand constraint. A positive supply-side factor such as an increase in  $\beta$  or a decrease in  $F$  increases the economic growth rate.

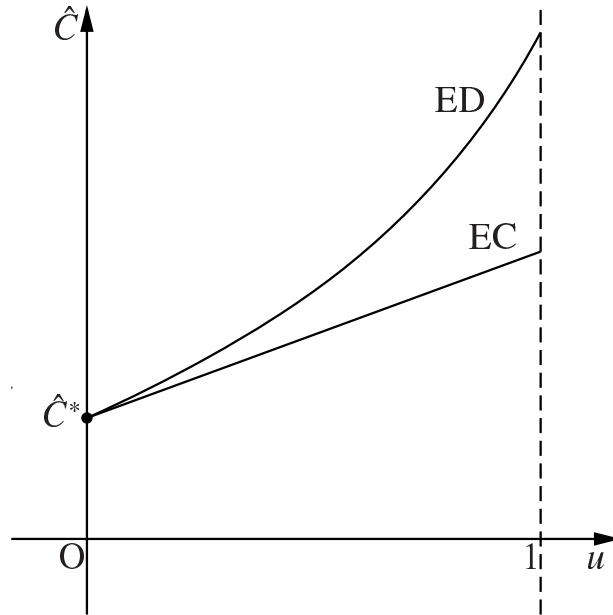


Figure 10: Equilibrium when only process innovation is conducted

When the demand constraint is effective (i.e.,  $\delta > 0$ ), the parameters related to

13) For the detailed analysis of the case of  $\delta = 1$ , see the Appendix.

product innovation such as  $\phi$ ,  $\varphi$ , and  $\delta$  affect the economic growth rate and unemployment rate. As the results of the comparative statics show, for most parameters, the relationship between the economic growth rate and unemployment rate is a trade-off. Accordingly, if one wants to increase the economic growth rate, the unemployment rate worsens. An increase in the efficiency of process innovation increases the economic growth rate and does not change the unemployment rate. An increase in the labor allocation to process innovation in the R&D sector increases the economic growth rate and decreases the unemployment rate depending on the conditions.

## 5 Concluding remarks

This study presented a growth model that considers both product and process innovation and investigated the relationship between the economic growth rate and unemployment rate. To build our model, we supposed that a new good is born with product innovation, but consumers cannot immediately judge its quality, usefulness, or convenience. Accordingly, not all newly invented products are purchased by consumers; some are not purchased and effective demand is constrained.

We find that an increase in the efficiency of product innovation increases both the economic growth rate and the unemployment rate. An increase in the efficiency of process innovation increases the economic growth rate but does not change the unemployment rate. An increase in the labor allocation to process innovation and a decrease in the labor allocation to product innovation increases the economic growth rate and decreases the unemployment rate, or decreases both the economic growth rate and the unemployment rate. An increase in the success probability of product innovation decreases both the economic growth rate and the unemployment rate.

In our model, there are three ways to foster product innovation: an increase in the efficiency of product innovation, an increase in the labor allocation to product innovation, and an increase in the success probability of product innovation. As stated above, these policies affect the economic growth rate and unemployment rate differently. In general, the birth of a new product due to product innovation creates new demand and raises employment and economic growth. However, our analysis that considers the demand and supply interactions shows that fostering product innovation does not lower the unemployment rate and increases the economic growth rate, but at most either lowers the unemployment rate or increases the economic growth rate. This finding suggests that to both raise employment and increase economic growth, we need not only a policy for fostering product innovation but also another policy to improve

employment.

As mentioned in the text, in our model, the employment allocation parameter of the R&D sector  $\delta$  is fixed for the ease of comparative static analysis. However, in reality, firms determine the employment allocation of the R&D sector to maximize their profits. This suggests that employment allocation of the R&D sector should be endogenized, from which we can investigate the effects of product and process innovation in more detail. This will be left for future research.

## Appendix: Only product innovation is conducted

When  $\delta = 1$  in the R&D sector, process innovation is not carried out, and only product innovation is conducted. This is the case Zagler (2004) considers. The two curves are rewritten as

$$\text{EC} : \hat{C} = \frac{\phi}{\theta} u + \frac{\phi(1 - \theta F)}{\theta(\theta - 1)}, \quad (53)$$

$$\text{ED} : \hat{C} = \frac{\theta u}{\varphi(\theta - 1)[(\theta - 1)(1 - u) + \theta F]}. \quad (54)$$

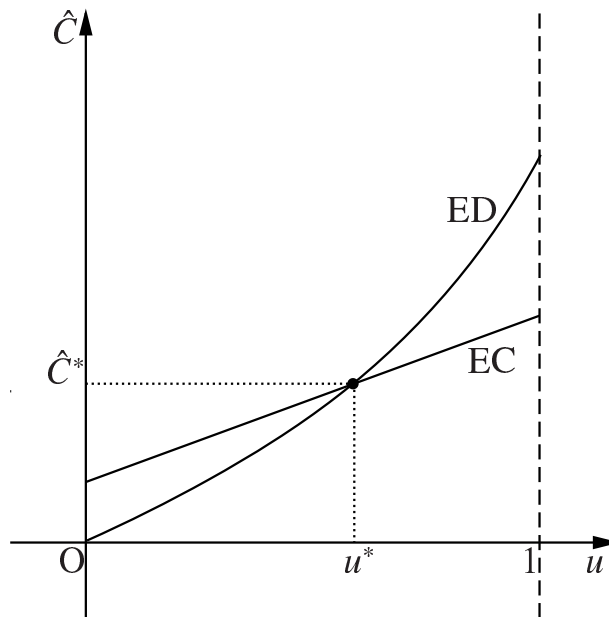


Figure 11: Equilibrium when only product innovation is conducted

The economic growth rate of Zagler (2004) is given by

$$\hat{C} = \chi \hat{n}, \quad \chi > 0, \quad (55)$$

where  $\chi$  represents the aggregate demand externality. In our model,  $1/(\theta - 1) > 0$  corresponds to the aggregate demand externality, and a decrease in  $\theta$  corresponds to an increase in  $\chi$ . In Zagler's model, an increase in  $\chi$  does not affect the unemployment rate and increases the economic growth rate. In our model, a decrease in  $\theta$  increases both the economic growth rate and the unemployment rate even if  $F = 0$ .

Zagler (2004) investigates the effect of an increase in the mark-up rate  $\gamma$  on the economic growth rate and unemployment rate independently of the aggregate demand externality  $\chi$ . However, in our analysis, the aggregate demand externality is related to the mark-up rate through the elasticity of substitution. From  $\chi = 1/(\theta - 1)$  and  $\gamma = \theta/(\theta - 1)$ , we have  $\gamma = 1 + \chi$ , and therefore we cannot analyze  $\chi$  and  $\gamma$  independently.

## References

- Aghion, P. and Howitt, P. (1992) "A model of growth through creative destruction," *Econometrica* 60, pp. 323–351.
- Aghion, P. and Howitt, P. (1994) "Growth and unemployment," *The Review of Economic Studies* 61 (3), pp. 477–494.
- Aoki, M. and Yoshikawa, H. (2002) "Demand saturation-creation and economic growth," *Journal of Economic Behavior & Organization* 48, pp. 127–154.
- Aoki, M. and Yoshikawa, H. (2007) *Reconstructing Macroeconomics: A Perspective from Statistical Physics and Combinational Stochastic Processes*, Cambridge, UK: Cambridge University Press.
- Ardelean, A. (2006) "How strong is the love of variety?" Purdue CIBER Working Papers, Purdue University.
- Aricó, F. (2003) "Growth and unemployment: Towards a theoretical integration," *Journal of Economic Surveys* 17 (3), pp. 419–455.
- Benassy, J. P. (1982) *The Economics of Market Disequilibrium*, New York: Academic Press.
- Broda, C., Greenfield, J., and Weinstein, D. E. (2017) "From groundnuts to globalization: A structural estimate of trade and growth," *Research in Economics* 71, pp. 759–783.

- Dixit, A. K. and Stiglitz, J. E. (1977) “Monopolistic competition and optimum product diversity,” *American Economic Review* 67 (3), pp. 297–308.
- Dutt, A. K. (2006) “Aggregate demand, aggregate supply and economic growth,” *International Review of Applied Economics* 20, pp. 319–336.
- Flaschel, P. and Skott, P. (2006) “Steindlian models of growth and stagnation,” *Metroeconomica* 57 (3), pp. 303–338.
- Grossman, G. M. and Helpman, E. (1991) *Innovation and Growth in the Global Economy*, Cambridge, MA: MIT Press.
- Harrison, R., Jaumandreu, J., Mairesse, J., and Peters, B. (2014) “Does Innovation Stimulate Employment? A Firm-level Analysis Using Comparable Micro-data from Four European Countries,” *International Journal of Industrial Organization* 35, pp. 29–43.
- Hashimoto, K. and Ono, Y. (2020) “A Simple Aggregate Demand Analysis with Dynamic Optimization in a Small Open Economy,” *Economic Modelling*, <https://doi.org/10.1016/j.econmod.2020.105111>.
- Jones, C. I. (1999) “Growth: With or without scale effects?” *American Economic Review* 89, pp. 139–144.
- Keynes, J. M. (1936) *The General Theory of Employment, Interest and Money*, London: Macmillan.
- Kurose, K. (2009) “The relation between the speed of demand saturation and the dynamism of the labour market,” *Structural Change and Economic Dynamics* 20, pp. 151–159.
- Kurose, K. (2013) “The dynamics of the labour market and income distribution in relation to the speed of demand saturation,” *Structural Change and Economic Dynamics* 24, pp. 101–111.
- Lima, G. T. (2004) “Endogenous technological innovation, capital accumulation and distributional dynamics,” *Metroeconomica* 55, pp. 386–408.
- Matsuyama, K. (1994) “General equilibrium models of monopolistic competition” in K. Iwai and Motoshige, I. (eds,) *Modern Economic Theory*, Tokyo: University of Tokyo Press. (in Japanese)
- Matsuyama, K. (1997) “Complementarity, instability and multiplicity,” *Japanese Economic Review* 48 (3), pp. 240–266.
- Mortensen, D. T. (2005) “Growth, unemployment, and labor market efficiency,” *Journal of the European Economic Association* 3 (2–3), pp. 236–258.

- Murakami, H. (2017) “Economic growth with demand saturation and ‘endogenous’ demand creation,” *Metroeconomica* 68 (4), pp. 966–985.
- NISTEP (2016) “Report on the Fourth Round of the Japanese National Innovation Survey (J-NIS 2015),” *NISTEP REPORT*, No. 170, National Institute of Science and Technology Policy Tokyo.
- Parelo, C. P. (2011) “Labor Market Rigidity and Productivity Growth in a Model of Innovation-Driven Growth,” *Economic Modelling* 28, pp. 1058–1067.
- Pasinetti, L. L. (1993) *Structural Economic Dynamics: A Theory of the Economic Consequences of Human Learning*, Cambridge, UK: Cambridge University Press.
- Romer, P. M. (1990) “Endogenous technological change,” *Journal of Political Economy* 98 (5), pp. 71–102.
- Sasaki, H. (2010) “Endogenous technological change, income distribution, and unemployment with inter-class conflict,” *Structural Change and Economic Dynamics* 21 (2), pp. 123–134.
- Sasaki, H. (2013) “Cyclical growth in a Goodwin-Kalecki-Marx model,” *Journal of Economics* 108 (2), pp. 145–171.
- Zagler, M. (2004) “A new look at old issues: Keynesian unemployment revisited,” *International Review of Applied Economics* 18 (2), pp. 209–224.
- Zagler, M. (2005) “Wage pacts and economic growth,” *Journal of Economic Studies* 32 (5), pp. 420–434.
- Zhu, C., Qiu, Z., and Liu, F. (2020) “Does Innovation Stimulate Employment? Evidence from China,” *Economic Modelling*, <https://doi.org/10.1016/j.econmod.2020.02.041>