# Construction and Evaluation of Performance Measures for Bayesian Chain Sampling Plan (BChSP-1)

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# CONSTRUCTION AND EVALUTION OF PERFORMANCE MEASURES FOR BAYESIAN CHAIN SAMPLING PLAN (BChSP-1)

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## Abstract:

Bayesian Acceptance Sampling Approach is associated with utilization of prior process history for the selection of Distributions (viz., Gamma Poisson, Beta Binomial) to describe the random fluctuations involved in Acceptance Sampling. Calvin (1984) provides procedures and tables for implementing Bayesian Sampling Plan. Dodge (1955) has proposed Chain Sampling Plan in which Chain Sampling Plan allows significant reduction in sample size and the condition for a continuing succession of lots from a stable and trusted supplier. Usha (1991) has proposed procedure for Bayesian Chain Sampling Plan. Latha (2002) has further studied Bayesian Chain Sampling Plan – 1 involving designing of Bayesian Chain Sampling Plan indexed through AQL, LQL, OAOQL, and MAAPD.

The main thrust of this paper is to account for the possibility of dependence among the items of a sample. This paper mainly relates with the procedure for designing Bayesian Chain Sampling Plan indexed with acceptable and limiting quality levels. Tables and Procedures are also provided for the selection of the parameters for the plan with specified $h_1$, $h_0$ and $h_2$. Numerical Illustration are also provided for the shop floor applications of these procedures.

## Keywords:

BAYESIAN ACCEPTANCE SAMPLING, CHAIN SAMPLING PLAN, BETA-BINOMIAL DISTRIBUTION, GAMMA-POISSON DISTRIBUTION,

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# Introduction

## Acceptance Sampling by Attributes

Acceptance Sampling is necessarily a defensive measure, instituted as protective device against the threat of deterioration in quality. Bayesian methodology offers a rational theory of personalistic beliefs in contexts of uncertainty. American National Standards Institutes/American Society for Quality Control Standard A2 (1987) defines Acceptance Sampling "as the methodology that deals with procedure through which decisions to accept or reject are based on the results of the inspection of samples". According to Dodge (1969), the major areas of acceptance samplings are

1. Lot-by-lot sampling by the methods of attributes, in which each unit in a sample is inspected on a go-no-go basis for one or more characteristics;
2. Lot-by-lot sampling by the methods of variables, in which each unit in a sample is measured for a single characteristic, such as weight or strength;
3. Continuous sampling of a flow of units by the methods of attributes; and
4. Special purpose plans including Chain Sampling, Skip-lot sampling and small-sample plans etc.,

## Bayesian Statistics

Bayesian Statistics is directed towards the use of sample information. Thomas Bayes (1702-1761) was the first to use prior information in inductive inference and the approach to statistics, which formally seeks to utilize prior information, is called Bayesian analysis. Suppose a product in a series is supplying a product, due to random fluctuations these lots will differ in quality even though the process is stable and in control. These fluctuations can be separated in to within lot (sampling) variations of individual units and between lot (sampling and process) variations.

## Bayesian Acceptance Sampling

Bayesian Acceptance Sampling Approach is associated with utilization of prior process history for the selection of distributions (viz., Gamma Poisson, Beta Binomial) to describe the random fluctuations involved in Acceptance Sampling. Bayesian sampling plans requires the user to specify explicitly the distribution of defectives from lot to lot.

The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The distribution is called prior because it is formulated prior to the taking of samples. The combination of prior knowledge, represented with the prior distribution, and the empirical knowledge based on the sample leads o the decision on the lot.

## Gamma-Poisson Distribution

Let x be the number of defects with λ as expected number of defects per unit, and let the corresponding Poisson probability be denoted by

$$g\,(x,\, n\lambda) = e^{-n\lambda}\,(n\lambda)^{x} / x!$$

Assuming that λ has a Gamma -Poisson distribution is given as

$$g_w\,(x,\, n\lambda) = \int_0^{8} g\,(x,\, n\lambda)\;\; w(\lambda)\; d\lambda$$

## Beta - Binomial Distribution

Let x be the outcome of n Bernoulli trials with a fixed probability p, and let the corresponding binomial probability be denoted by

$$b\,(x,\, n,\, p) = {}^{n}C_{x}\; p^{x}\, q^{n-x}$$

Assuming that p has a prior distribution with density w(p), the marginal distribution of x, the mixed binomial distribution is given as

$$b_w\,(x,\, n) = \int_0^1 b(x, n, p)\;\; w(p)\; dp$$

Calvin (1984) provides procedures and tables for implementing Bayesian Sampling Plans. A set of tables presented by Oliver and Springer (1972) are based on assumption of Beta prior distribution with specific posterior risk to achieve minimum sample size, which avoids the problem of estimating cost parameters. It is generally true that Bayesian Plan requires a smaller sample size than a conventional sampling plan with the same producer and consumer risks. Schafer (1967) discusses single sampling by attributes using three prior distributions of lot quality. Hald (1960) gives an extensive account of sampling plans based on discrete prior distributions of product quality. Case and Keats (1982) have provided a table for the classification of attributes sampling plan design methodologies.

Deepa (2002) has studied the formulation of a Bayesian Sampling Plan using acceptance probability with Gamma prior distribution for product quality using producer and consumer quality levels with selection procedure for Bayesian Special type of Double Sampling Plan with MAAOQ and also the sum of weighted risks and designing sampling plans of the one plan suspension system and Quick switching system.

Latha (2002) has studied average probability of acceptance function for single sampling plan with Gamma Prior distribution. Formula of inflection point and tangent at the inflection point are also derived. A selection procedure for Bayesian Single sampling attributes plan (with Gamma prior distribution) based on AQL and LQL, point of control and relative slope at that point, MAAPD and K, measure of sharpness are also explained, and lot acceptance procedures are developed for Bayesian Single sampling attributes plans when the acceptance number is fixed and when the sample size is fixed.

Tables of average probability of acceptance for BChSP-1 are constructed and the selection of BChSP-1 plan, which are based on AQL, LQL, IQL, Inflection Point and Overall Average Outgoing Quality Limit (OAOQL), are designed. Selection Procedures for BChSP-1 plan by minimizing the sum of weighted risks, the average cost and regrets functions for BChSP-1 have been derived and the minimum regret function is also obtained.

## Chain Sampling Plan (ChSP– 1)

Sampling inspection in which the criteria for acceptance and non acceptance of the lot depend in part on the results of the inspection of immediately preceding lots is adopted in Chain Sampling Plan. Chain Sampling Plan (ChSP-1) proposed by Dodge (1955) making use of cumulative results of several samples help to overcome the short comings of the Single Sampling Plan.

## Conditions for application of ChSP -1:

The cost of destructiveness of testing is such that a relatively small sample sizes I necessary, although other factors make a large sample desirable.

1) The product to be inspected comprises a series of successive lots produced by a continuing process.
2) Normally lots are expected t be of essentially the same quality.
3) The consumer has faith in the integrity of the producer.

## Operating Procedure

The plan is implemented in the following way:

1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2) Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d > 1.
3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceding i samples of size n.

Dodge (1955) has given the operating characteristic function of ChSP-1 as

$$P_a(p) = P_0 + P_1\,(P_0)^i,$$

Where $P_j$ = probability of finding j nonconforming units in a sample of n units for j = 0.1.

The Chain sampling Plan is characterized by the parameters n and i.

When i= , the OC function of a ChSP -1 plan reduces to the OC function of the Single Sampling Plan with acceptance number zero and when i = 0, the OC function of ChSP-1 plan reduces to the OC function of the Single Sampling Plan with acceptance number 1.

## Bayesian Chain Sampling Plan (BChSP-1)

According to Dodge (1955) the operating characteristic function of ChSP-1 is

$$P_a(p) = P_0 + P_1\,(P_0)^i,$$

The Chain Sampling Plan (ChSP-1) is characterized by two parameters n and i, where n is the sample size and i is the number of preceding samples with zero defective, using the OC curve, Dodge (1955) has studied the properties of the Chain Sampling Plan. Clark (1960) has presented additional OC curves, which cover most of the situations. Soundararajan (1978 a, b) has described procedures and tables for construction and selection of Chain sampling plans (ChSP-1) indexed by specified parameters.

The probability of acceptance of ChSP-1 based on Poisson Model is provided as

$$P(n,i/p) = e^{-np} + e^{-np(1+i)} np$$

Using the Past history of inspection, it is observed that p follows Gamma distribution with density function,

$$w(p) = e^{-pt} p^{s-i} t^{s} / \Gamma(s), \quad s,t > 0, \quad p > 0$$

The average probability of acceptance is given as

$$\overline{P} = \int_{0}^{\infty} P(n,i/p) w(p) dp$$

$$= s^{S} / (s+n\mu)^{S} + ns^{S+1}\mu / (s+n\mu+ni\mu)^{S+1}$$

Where $\mu = s/t$, is the mean value of the product quality p.

## Designing of Bayesian Chain Sampling Plan (BChSP-1) indexed with Relative slopes of Acceptable and Limiting Quality Levels:

### Selection at the Acceptable Quality Level

Table 1 is used to select the parameters for Bayesian Chain Sampling Plan indexed by $\mu_1$ and $h_1$. For example, for given $\mu_1$ = 0.01 and $h_1$ = 0.07, from Table 1 under the column headed $h_1$, locate the value equal to or just greater than specified $h_1$. Corresponding to this $h_1$, the values of associated are $n\mu_1$ = 0.0808, s = 1 and i = 9. From this one can obtain the sample size as n= $n\mu_1/\mu_1$ 8. Thus the parameters are n= 8, s = 1 and i = 9.

**Selection at the Limiting Quality Level**

Table 1 is used to select the parameters for Bayesian Chain Sampling Plan indexed by $\mu_2$ and $h_2$. For example, for given $\mu_2 = 0.02$ and $h_2 = 0.9$, from Table 1 under the column headed $h_2$, locate the value equal to or just greater than specified $h_2$. Corresponding to this $h_2$, the values associated are $n\mu_2 = 9.1086$, s = 1 and i = 9. From this one can obtain the sample size as n= $n\mu_2/ \mu_2$   10. Thus the parameters are n = 10, s = 1 and i= 9.

**Selection through Inflection Point**

Table 1 is used to select the parameters for Bayesian Chain Sampling Plan indexed by $\mu_0$ and $h_0$. For example, for given $\mu_0 = 0.01$ and $h_0 = 0.5$, from Table 1 under the column headed $h_0$, locate the value equal to or just greater than specified $h_0$. Corresponding to this $h_0$, the values associated are $n\mu_0 = 1.0326$, s = 1 and i = 9. From this one can obtain the sample size as n= $n\mu_0/ \mu_0$   10. Thus the parameters are n =10, s = 1 and i= 9.

**Selection through ratio of relative slopes**

Table 1 is used to select the Bayesian Chain Sampling Plan parameters for specified AQL (or LQL) with $h_2$ and $h_1$. For example, for given $\mu_1 = 0.01$; $h_1 = 0.07$; $h_2 = 0.9$, $h_0 = 0.5$, one can find that $h_2 / h_1 = 12.86$. Using Table 1 under the column headed $h_2 / h_1$, locate the value which is equal to or just greater than desired ratio. Corresponding to this located ratio, the values associated are $n\mu_1 = 0.0808$, s = 1 and i = 9. From this one can obtain the sample size as n= $n\mu_1 / \mu_1$   8. Thus the selected parameters are n = 8, s = 1 and i= 9. For the ratio $h_0 / h_1 = 7.14$, using Table 1 under the column headed $h_0 / h_1$, locate the value which is equal to or just greater than desired ratio. Corresponding to this located ratio, the values associated are $n\mu_1 = 0.0917$, s = 1 and i =6. From this one can obtain the sample size as n= $n\mu_1 / \mu_1$   9. Thus the selected parameters are n = 9, s = 1 and i= 6.

## Conversion of Parameters

It is necessary to convert a given set of parameters to another familiar set providing information on other related parameters. Conversions are arrived using Table 1. For example when $\mu_1 = 0.01$; $h_1 = 0.07$ are specified the other set of parameters are found using these tables. Corresponding to $h_1 = 0.07$ one finds using table 1, s = 1, i = 9, $n\mu_1 = 0.0808$, $n\mu_0 = 1.0326$, $n\mu_2 = 9.1086$, $h_1 = 0.0701$, $h_2 = 0.9019$, $h_0 = 0.5131$. Dividing $n\mu_1$ by $\mu_1$ one gets n = 8 for the Bayesian Multiple Deferred State Sampling Plan with $\mu_2 = 1.1386$, $\mu_0 = 0.1291$. When $\mu_1 = 0.01$ and $h_1 = 0.07$ the other set of parameters are

$\mu_1 = 0.01$; $h_1 = 0.07$

$\mu_0 = 0.1291$; $h_0 = 0.5131$

$\mu_2 = 1.1386$; $h_2 = 0.9019$

$\mu_* = 0.0117$; $h_* = 0.0821$

## Construction of Tables.

The expression for APA function for Bayesian Chain Sampling Plan $\overline{P}$ is given in equation

$$\overline{P} = \int_0^\infty P(n, i / p) w(p) dp$$

$$\overline{P} = \frac{s^s}{(s + n\mu)^s} + \frac{n\mu s^{s+1}}{(s + n\mu + in\mu)^{s+1}}$$

Where $\mu = s/t$, is the mean value of the product quality p.

Differentiating the APA function with respect to μ gives

$$\frac{d\overline{P}}{d\mu} = \frac{-ns^{s+1}}{(s + n\mu)^{s+1}} + \frac{ns^{s+2}(1 - n\mu - n\mu i)}{(s + n\mu + n\mu i)^{s+2}}$$

The relative slope h at μ is,

$$h = \frac{-\mu}{\overline{P}(\mu)} \frac{d\overline{P}(\mu)}{d\mu}$$

Differentiating the APA function with respect to μ and evaluating at μ we get various values of (i, s) and their corresponding $n\mu_1$, $n\mu_0$, $n\mu_2$ and $n\mu_*$ values are substituted in the equation and the relative slopes at $\mu = \mu_o, \mu_1, \mu_2, \mu_*$, the values $h_0, h_1, h_2$ and $h_*$ are obtained and tabulated in Table 1.

## Comparison with Conventional Plan

The values obtained from Bayesian MDS are compared with Conventional MDS. On comparison it is observed that n $\mu_1$ value is much less than n $p_1$ for smaller values of s and as s increases n $\mu_1$ tends to n $p_1$. But n $\mu_2$ is much greater than n $p_2$ for smaller values of s and as increases n $\mu_2$ tends to n $p_2$. So, when it is known from the past history that the product is very good, more lots will be accepted under the current process of production. OAOQL for Bayesian plan is lesser than AOQL values for conventional plan for small values of s and as s increases OAOQL reaches AOQL of conventional plan.

## Conclusion

Bayesian Acceptance sampling is the technique, which deals with procedures in which decision to accept or reject lots or process is based on the examination of past history or knowledge of samples. The present work mainly relates to the construction and selection of tables for Bayesian Chain Sampling plan indexed through relative slopes of Acceptable and Limiting Quality Levels. Conversions of parameters are also given for convenience. Tables are provided here which tailor-made, handy and ready-made use to the industrial shop-floor condition.

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**Table 1. Relative slopes for Acceptable and Limiting Quality Levels**

| s | i | $n\mu_1$ | $n\mu_0$ | $n\mu_2$ | $h_0$ | $h_1$ | $h_2$ | $h_2/h_1$ | $h_2/h_0$ | $h_0/h_1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.2880 | 2.4142 | 18.4868 | 0.5858 | 0.0817 | 0.9237 | 11.3029 | 1.57686 | 7.167955 |
| 1 | 1 | 0.1686 | 1.4675 | 11.4957 | 0.5752 | 0.0808 | 0.9193 | 11.3831 | 1.598206 | 7.122411 |
| 1 | 2 | 0.1340 | 1.2491 | 10.1442 | 0.5580 | 0.0791 | 0.9129 | 11.5449 | 1.636213 | 7.055847 |
| 1 | 3 | 0.1163 | 1.1574 | 9.6554 | 0.5444 | 0.0774 | 0.9088 | 11.7376 | 1.669439 | 7.030851 |
| 1 | 4 | 0.1052 | 1.109 | 9.4242 | 0.5346 | 0.0759 | 0.9063 | 11.9422 | 1.695349 | 7.044114 |
| 1 | 5 | 0.0975 | 1.0809 | 9.2969 | 0.5277 | 0.0745 | 0.9047 | 12.1412 | 1.714447 | 7.08172 |
| 1 | 6 | 0.0917 | 1.0614 | 9.2194 | 0.5223 | 0.0732 | 0.9036 | 12.3474 | 1.730038 | 7.137067 |
| 1 | 7 | 0.0873 | 1.0466 | 9.1687 | 0.5179 | 0.0721 | 0.9028 | 12.5264 | 1.743158 | 7.186033 |
| 1 | 8 | 0.0837 | 1.0394 | 9.1337 | 0.5154 | 0.0710 | 0.9023 | 12.7105 | 1.750615 | 7.260586 |
| 1 | 9 | 0.0808 | 1.0326 | 9.1086 | 0.5131 | 0.0701 | 0.9019 | 12.8750 | 1.757766 | 7.324617 |
| 3 | 0 | 0.3245 | 1.8838 | 6.3615 | 0.8277 | 0.0884 | 1.8236 | 20.6213 | 2.20335 | 9.359061 |
| 3 | 1 | 0.1892 | 1.138 | 4.0192 | 0.8014 | 0.0876 | 1.7606 | 20.1015 | 2.196971 | 9.149642 |
| 3 | 2 | 0.1493 | 0.9585 | 3.6417 | 0.7565 | 0.0861 | 1.6836 | 19.5451 | 2.225514 | 8.782285 |
| 3 | 3 | 0.1286 | 0.8841 | 3.5363 | 0.7206 | 0.0847 | 1.6455 | 19.4382 | 2.283468 | 8.512559 |
| 3 | 4 | 0.1155 | 0.8452 | 3.4983 | 0.6936 | 0.0832 | 1.6279 | 19.5587 | 2.347107 | 8.333102 |
| 3 | 5 | 0.1063 | 0.823 | 3.4821 | 0.6741 | 0.0819 | 1.6192 | 19.7732 | 2.402007 | 8.231961 |
| 3 | 6 | 0.0994 | 0.8095 | 3.4742 | 0.6602 | 0.0806 | 1.6147 | 20.0271 | 2.445666 | 8.188816 |
| 3 | 7 | 0.0940 | 0.8009 | 3.4701 | 0.6502 | 0.0794 | 1.6121 | 20.2940 | 2.479298 | 8.185371 |
| 3 | 8 | 0.0897 | 0.7951 | 3.4677 | 0.6429 | 0.0784 | 1.6106 | 20.5437 | 2.50511 | 8.200734 |
| 3 | 9 | 0.0861 | 0.7912 | 3.4663 | 0.6376 | 0.0774 | 1.6097 | 20.8061 | 2.524568 | 8.241444 |
| 5 | 0 | 0.3353 | 1.7975 | 5.2107 | 0.9034 | 0.0902 | 2.1998 | 24.3994 | 2.435087 | 10.01994 |
| 5 | 1 | 0.1953 | 1.0835 | 3.3079 | 0.8714 | 0.0894 | 2.0806 | 23.2835 | 2.387492 | 9.752266 |
| 5 | 2 | 0.1538 | 0.9102 | 3.0245 | 0.8166 | 0.0880 | 1.9444 | 22.1004 | 2.381138 | 9.281463 |
| 5 | 3 | 0.1322 | 0.8367 | 2.9571 | 0.7700 | 0.0865 | 1.8858 | 21.7888 | 2.448976 | 8.897114 |
| 5 | 4 | 0.1185 | 0.7992 | 2.9330 | 0.7351 | 0.0852 | 1.8615 | 21.8552 | 2.532404 | 8.630201 |
| 5 | 5 | 0.1089 | 0.7783 | 2.9300 | 0.7099 | 0.0839 | 1.8537 | 22.0906 | 2.611288 | 8.459665 |
| 5 | 6 | 0.1017 | 0.766 | 2.9271 | 0.6921 | 0.0827 | 1.8496 | 22.3580 | 2.672395 | 8.366283 |
| 5 | 7 | 0.0960 | 0.7584 | 2.9258 | 0.6795 | 0.0815 | 1.8476 | 22.6611 | 2.718816 | 8.334916 |
| 5 | 8 | 0.0915 | 0.7537 | 2.9252 | 0.6708 | 0.0805 | 1.8466 | 22.9270 | 2.752781 | 8.328669 |
| 5 | 9 | 0.0877 | 0.7506 | 2.9249 | 0.6646 | 0.0795 | 1.8460 | 23.2183 | 2.777763 | 8.358636 |
| 7 | 0 | 0.2880 | 1.7625 | 4.7894 | 0.9409 | 0.0685 | 2.4044 | 35.1010 | 2.555517 | 13.73539 |
| 7 | 1 | 0.1686 | 1.0608 | 3.0473 | 0.9058 | 0.0688 | 2.2451 | 32.6351 | 2.478641 | 13.16652 |
| 7 | 2 | 0.1340 | 0.8895 | 2.8000 | 0.8455 | 0.0693 | 2.0696 | 29.8488 | 2.447909 | 12.19359 |
| 7 | 3 | 0.1163 | 0.8167 | 2.7667 | 0.7935 | 0.0698 | 2.0103 | 28.7866 | 2.533539 | 11.36221 |
| 7 | 4 | 0.1052 | 0.7798 | 2.7331 | 0.7540 | 0.0702 | 1.9766 | 28.1521 | 2.621442 | 10.73915 |
| 7 | 5 | 0.0975 | 0.7596 | 2.7289 | 0.7257 | 0.0705 | 1.9681 | 27.9056 | 2.712155 | 10.28907 |
| 7 | 6 | 0.0917 | 0.7478 | 2.7275 | 0.7056 | 0.0707 | 1.9649 | 27.8051 | 2.784527 | 9.985559 |
| 7 | 7 | 0.0873 | 0.7408 | 2.7269 | 0.6918 | 0.0709 | 1.9634 | 27.7048 | 2.838189 | 9.76143 |
| 7 | 8 | 0.0837 | 0.7365 | 2.7267 | 0.6823 | 0.0709 | 1.9628 | 27.6772 | 2.876979 | 9.620232 |
| 7 | 9 | 0.0808 | 0.7339 | 2.7266 | 0.6758 | 0.0710 | 1.9625 | 27.6438 | 2.904191 | 9.518589 |