Compromising to share the revenues from broadcasting sports leagues

Bergantiños, Gustavo and Moreno-Ternero, Juan D.

Universidade de Vigo, Universidad Pablo de Olavide

4 June 2019

Compromising to share the revenues from broadcasting sports leagues

Gustavo Bergantiños[1].

ECOSOT. Universidade de Vigo.

Facultade de Económicas. Universidade de Vigo. 36310. Vigo. Spain.

E-mail: gbergant@uvigo.es

Juan D. Moreno-Ternero

Department of Economics, Universidad Pablo de Olavide.

Carretera de Utrera, Km. 1 41013 Seville, Spain.

E-mail: jdmoreno@upo.es

---

[1]Corresponding author.

Abstract

We study the problem of sharing the revenues raised from the collective sale of broadcasting rights for sports leagues. We characterize the sharing rules satisfying three basic and intuitive axioms: *symmetry*, *additivity* and *maximum aspirations*. They convey a natural compromise between two focal rules, arising from polar estimations of teams' loyal viewers. We also show that these *compromise rules* have further interesting properties, such as allowing for the existence of a majority voting equilibrium. We bring some of the testable implications from our axiomatic analysis to the real case of European football leagues.

Keywords: *resource allocation, broadcasting, sports leagues, compromise rules, testable implications.*

JEL numbers: *D63, D72, H80, Z20.*

1   Introduction

Sources report that half of the world's population watched the final game of the 2010 FIFA World Cup (e.g., Palacios-Huerta, 2014). The increasing popularity during the last two decades of televised sports events has had significant effects on the broadcasting sectors and sports leagues in North America and Europe (e.g., Cave and Crandall, 2001). According to Kantar Media estimates, sports programming in 2014-15 generated $8.47 billion in sales for ABC, CBS, NBC and Fox, a 35% increase from five years before, accounting for more than one-third of the Big Four's overall ad revenue for the period.[2] As for sports organizations, the sale of broadcasting and media rights is currently their biggest source of revenue. According to Statista, more than 50 percent of the revenue that the (US) National Football League as a whole generated in 2015 is attributable to television rights deals.

The sale of broadcasting rights for sports leagues is often carried out through some sort of collective bargaining involving all participating organizations (teams) in a given competition on the one hand, and broadcasting companies on the other hand.[3] Thus, an ensuing key problem arises in which the revenues collected from the sale have to be shared among the teams. This is, by no means, a straightforward problem, mostly because the individual contribution to the revenues is not known. Furthermore, the revenue is sizable, which renders the solution of the problem crucial for the management of most sports organizations.

The revenue sharing rules used by the leagues are actually strikingly different between North America and Europe. In North America, contracts essentially involve equal sharing, whereas in Europe, performance-based reward schemes are widespread. The latter is rationalized by Palomino and Szakovics (2004) due to the competitive environment in which European leagues operate.[4] On the other hand, as explained by Fort and Quirk (1995), in a one-team-one-vote environment, such as the one in North America, equal sharing is more or less guaranteed because the national contract can be approved only if there is a virtual consensus among league teams.[5]

---

[2]In the era of streaming, sports has become the cornerstone to television programming, playing the role of a *defensive wall* against online disruption (e.g., Lee, 2019).

[3]Falconieri et al., (2004) provide a welfare analysis of collective vs. individual sale of TV rights.

[4]In Europe, as opposed to North America, teams but also leagues have incentives to compete for talent.

[5]Weak-drawing teams can block unequal sharing by refusing to permit televising of games involving them and strong-drawing teams. This happened to be a profitable bargaining strategy for Betis in the pre-collective sale era of TV rights for La Liga when they were paired with Real Madrid for the first game of the 2003/2004 season, which was the highly anticipated debut of David Beckham in such a tournament.

In this paper, we align with the European case, assuming that broadcast-revenue sharing goes beyond equal sharing. More precisely, as in our recent papers (Bergantiños and Moreno-Ternero, 2020a, 2020b), we consider a simple formal model in which the sharing process is based on the (broadcasting) audiences that games throughout the season generate. We then take the axiomatic approach for such a model to derive *appropriate* sharing rules.

To wit, we consider three basic and intuitive axioms for sharing rules: *symmetry*, *additivity* and *maximum aspirations*. The first one says that if two teams have the same overall audiences, then they should receive equal amounts.[6] The second one says that revenues should be additive on audiences.[7] The third axiom says that no team can receive more than its *claim*, i.e., the total revenue obtained from all the games in which the team was involved. These three axioms, which seem to be innocuous independently, have a strong bite when combined. We actually show (in the main result of this paper) that they characterize a family of rules that offer a compromise between two focal and somewhat polar rules (that is why we call them *compromise rules*). On the one hand, the so-called *equal-split* rule which splits the audience of each game equally among the two teams. On the other hand, the so-called *concede-and-divide*, which concedes each team the audience coming from its fan base (the loyal viewers watching all games played by that team) and divides equally the residual. The two rules have distinguishing merits, but they treat fans in two opposite and somewhat extreme ways. The *equal-split* rule essentially ignores the existence of fan bases as it considers, de facto, that both teams participating in a game contributed equally to the revenues collected from broadcasting that game. On the other hand, *concede-and-divide* essentially ignores the existence of casual viewers as it considers, de facto, that viewers watching a game are either fans of one participating team, or compulsive viewers, who watch all games in the season. The scenarios underlying the *equal-split* rule and *concede-and-divide* can be thought of as meaningful lower and upper bounds, depending on whether the team has a weak or strong fan base. Reality seems to be somewhat in between those two scenarios. Thus, compromising between both rules seems to be a natural move. That is precisely what we do in this paper.

---

[6] As we shall argue later, revenues can be reduced to audiences provided one assumes a constant pay-per-view fee for each game.

[7] An interpretation is that the aggregation of the revenue sharing in two seasons (involving the same teams) is equivalent to the revenue sharing in the hypothetical combined season aggregating the audiences of the corresponding games in both seasons.

Each rule in the family we characterize is simply defined by a certain convex combination of the *equal-split* rule and *concede-and-divide*. More precisely, for a given parameter $\lambda \in [0, 1]$, the rule R^λ selects, for each problem, the convex combination of the solutions suggested by the *equal-split* rule and *concede-and-divide* for that problem, with weights λ and $1 - \lambda$, respectively. Note that, when the set of options is equipped with a convex structure (as in this case), averaging between different positions that people may take concerning the best way of approaching problems is an appealing way of finding some common ground between them.[8] What is remarkable in our setting is that this position is normatively supported by three simple and intuitive principles, as our characterization shows.

We then explore the family so derived and discover further interesting features of it.

First, we show that, if we allow teams to vote for any rule within the family, then a majority voting equilibrium exists, i.e., a rule that cannot be overturned by any other rule within the family through majority rule. This feature avoids the existence of disturbing majority cycles and it is a consequence of the fact that the rules within the family satisfy the so-called *single-crossing property*, which allows one to separate those teams that benefit from the application of one rule or the other rule, depending on the rank of their claims.[9]

Second, we show that the rules within the family yield outcomes that are fully ranked according to the Lorenz dominance criterion, the most fundamental principle for the evaluation of inequality (e.g., Dasgupta et al., 1973; Atkinson and Bourguignon, 1987). More precisely, for each problem, and each pair of rules within the family, the outcome suggested by the rule associated with a higher parameter dominates (in the sense of Lorenz) the outcome suggested by the other rule, which is equivalent to saying that the former will be more egalitarian than the latter.

Due to the previous feature, the parameter describing the family can be considered as an index of egalitarianism within the family. As a matter of fact, the parameter can also be considered as an estimation of the percentage of viewers who watch a game without being a fan of one of the teams playing the game.[10] In other words, a low value of the parameter is associated with a large fan base for participating teams, whereas a high value of the parameter

---

[8] Averaging to compromise is a recurrent theme in game theory and resource allocation (Thomson, 2019a).

[9] It is well known that a sufficient condition for the existence of a majority voting equilibrium is that voters exhibit *intermediate* preferences over the set of alternatives (e.g., Gans and Smart, 1996).

[10] This is reminiscent of the concept of *neutral* (as opposed to *hard-core*) fans introduced by Szymanski (2001).

is associated with a small fan base for participating teams.

A third interesting feature of the family is that it can be applied to real-life situations. More precisely, we consider the case of the Spanish Football League and apply the family to explore several allocation schemes therein. An important aspect of the rules in our family is that they have a minimal informational basis, which allows us to use them even when there is limited available data (as, unfortunately, happens to be the case with European football leagues).

One-parameter families, such as the one we derive in this paper, have been frequently highlighted in the literature. Atkinson (1970) famously introduced a family of inequality measures, characterized by a weighting parameter measuring aversion to inequality. Somewhat related, Donaldson and Weymark (1980) generalized the social-evaluation function corresponding to the focal Gini inequality index to derive the (one-parameter) family of generalized Gini inequality indices.[11] In a context more similar to ours, Moulin (1987) characterized a family compromising between the equal and proportional surplus sharing methods. As a matter of fact, his family is the convex combination of those two methods and one of the axioms used for its characterization is precisely *additivity*. Thus, the parallelism with our result is strong. Something similar happens in minimum cost spanning tree problems, where Trudeau (2014) characterizes the convex combination of the folk rule (e.g., Bergantiños and Vidal-Puga, 2007) and the so-called cycle-complete rule (e.g., Trudeau, 2002), also making use of *additivity*. Compromises between the proportional and *constrained* equal-award rules (thus, satisfying the standard non-negativity condition for claims problems) have also been considered by Thomson (2015a,b). Moreno-Ternero and Villar (2006) introduced a one-parameter family of rules for claims problems generalizing the so-called Talmud rule (e.g., Aumann and Maschler, 1985) and encompassing (as extreme cases) the polar constrained equal awards and losses rules. The rules within such a family also happen to satisfy the single-crossing property and be fully ranked according to the Lorenz dominance criterion (e.g., Moreno-Ternero, 2011).

The rest of the paper is organized as follows. We introduce the model in Section 2. We present the axiomatic characterization leading to the family in Section 3. Section 4 is devoted to explore additional properties of the rules within the family. In Section 5, we bring testable implications from our analysis to the case of the Spanish Football League. We conclude in Section 6. Some technical aspects have been deferred to an Appendix.

---

[11]See also Weymark (1981) and Bossert (1990).

6

2   The model

We consider the model introduced by Bergantiños and Moreno-Ternero (2020a). Let N describe a finite set of teams. Its cardinality is denoted by n. We assume $n \geq 3$. For each pair of teams $i, j \in N$, we denote by a_{ij} the broadcasting audience (number of viewers) for the game played by i and j at i's stadium. We use the notational convention that $a_{ii} = 0$, for each $i \in N$. Let $A \in \mathcal{A}_{n \times n}$ denote the resulting matrix of broadcasting audiences generated in the whole tournament involving the teams within N.[12] Each matrix $A \in \mathcal{A}_{n \times n}$ with zero entries in the diagonal will thus represent a *problem* and we shall refer to the set of problems as $\mathcal{P}$.[13]

Let $\alpha_i(A)$ denote the total audience achieved by team i, i.e.,

$$\alpha_i(A) = \sum_{j \in N}(a_{ij} + a_{ji}).$$

Without loss of generality, we normalize the revenue generated from each viewer to 1 (to be interpreted as the "pay per view" fee). Thus, we sometimes refer to $\alpha_i(A)$ by the *claim* of team i. When no confusion arises, we write α_i instead of $\alpha_i(A)$. We define $\overline{\alpha}$ as the average audience of all teams. Namely,

$$\overline{\alpha} = \frac{\sum\limits_{i \in N} \alpha_i}{n}.$$

For each $A \in \mathcal{A}_{n \times n}$, let $||A||$ denote the total audience of the tournament. Namely,

$$||A|| = \sum_{i,j \in N} a_{ij} = \frac{1}{2} \sum_{i \in N} \alpha_i = \frac{n\overline{\alpha}}{2}.$$

A (sharing) **rule** is a mapping that associates with each problem the list of the amounts the teams get from the total revenue. Thus, formally, $R : \mathcal{P} \to \mathbb{R}^n$ is such that, for each $A \in \mathcal{P}$,

$$\sum_{i \in N} R_i(A) = ||A||.$$

Two rules stand out as focal for this problem (e.g., Bergantiños and Moreno-Ternero, 2020a, 2020b). First, the so-called *equal-split rule*, which splits equally the audience of each game

---

[12] We are therefore assuming a round-robin tournament in which each team plays in turn against each other team twice: once home, another away, which is the format of most of the national football leagues. Our model could be extended though to account for tournaments in which some teams play other teams a different number of times. In such a case, a_{ij} would denote the broadcasting audience in all games played by i and j at i's stadium.

[13] As the set N will be fixed throughout our analysis, we shall not explicitly consider it in the description of each problem.

(among the two teams), thus ignoring the existence of fans for each team. Second, the so-called *concede-and-divide*, which concedes each team its number of fans and divides equally the rest. They are, nevertheless, defined in a similar way. First, each team i tentatively receives its claim (α_i). Second, they each subtract from it an amount associated to the remaining $n-1$ teams. In the case of the *equal-split* rule, an equal share of half of the team's total audience ($\beta_i = \frac{\alpha_i/2}{n-1}$); in the case of *concede-and-divide*, the average audience per game that the remaining teams played ($\gamma_i = \frac{\sum\limits_{j,k \in N \setminus \{i\}} \left(a_{jk}+a_{kj}\right)}{(n-2)(n-1)}$).[14] Formally,

Equal-split rule, ES: for each $A \in \mathcal{P}$, and each $i \in N$,

$$ES_i(A) = \alpha_i - (n-1)\beta_i = \frac{\alpha_i}{2}.$$

Concede-and-divide, CD: for each $A \in \mathcal{P}$, and each $i \in N$,

$$CD_i(A) = \alpha_i - (n-1)\gamma_i = \frac{(n-1)\alpha_i - ||A||}{n-2}.$$

We now consider a family of rules that offer a compromise between the *equal-split rule* and *concede-and-divide*. They are defined as convex combinations of the two rules. Formally,

Compromise rules, $\left\{C^\lambda\right\}_{\lambda \in [0,1]}$: for each $\lambda \in [0,1]$, each $A \in \mathcal{P}$, and each $i \in N$,

$$C_i^\lambda(A) = \lambda ES_i(A) + (1-\lambda)CD_i(A).$$

At the risk of stressing the obvious, note that when $\lambda = 0$ then C^λ coincides with *concede-and-divide*, whereas when $\lambda = 1$ then C^λ coincides with the *equal-split* rule. That is, $C^0 \equiv CD$ and $C^1 \equiv ES$.

Note also that, with straightforward algebraic computations, we can obtain that, for each $A \in \mathcal{P}$, each $i \in N$, and each $\lambda \in [0,1]$,

$$C_i^\lambda(A) = \frac{\alpha_i}{2} + \frac{n(1-\lambda)}{2(n-2)}(\alpha_i - \bar{\alpha}). \tag{1}$$

One can easily infer from the previous expression that, if $\alpha_i < \bar{\alpha}$, then $C_i^\lambda(A)$ is an increasing function of λ, thus maximized at $\lambda = 1$. If, instead, $\alpha_i > \bar{\alpha}$, then $C_i^\lambda(A)$ is a decreasing function of λ, thus maximized at $\lambda = 0$. Finally, if $\alpha_i = \bar{\alpha}$, then $C_i^\lambda(A) = \frac{\alpha_i}{2}$ for each $\lambda \in [0,1]$.

---

[14]The term *concede-and-divide*, which was coined by Thomson (2003) in a different setting, is justified here by an intuitive procedure, based on a form of statistical estimation aiming to capture the loyal viewers of each team, which leads to this rule (see Bergantiños and Moreno-Ternero (2020a) for further details).

Generalizing what we said above for its two extreme rules, each rule within the family can be obtained with a two-step process: first, each team i tentatively receives its claim (α_i), and then we subtract from it an amount associated to the remaining $n-1$ teams (in this case, $\lambda_i = \lambda\beta_i + (1-\lambda)\gamma_i$). Formally,

$$C_i^\lambda(A) = \alpha_i - (n-1)\lambda_i = \alpha_i - (n-1)(\lambda\beta_i + (1-\lambda)\gamma_i).$$

3 The characterization

We now introduce three natural axioms for rules.

The first axiom is a minimal requirement of *impartiality*, a basic requirement of justice (e.g., Moreno-Ternero and Roemer, 2006). It says that if two teams have equal total audience, then they should receive equal amounts.

Symmetry: For each $A \in \mathcal{P}$, and each pair $i, j \in N$, such that $\alpha_i = \alpha_j$,

$$R_i(A) = R_j(A).$$

The second axiom is a *robustness* axiom indicating that when two equally valid perspectives can be taken in evaluating a situation, it seems natural to require that these two perspectives result in the same outcome (e.g., Thomson, 2019b). More precisely, the axiom states that revenues should be additive on A.[15] Formally,

Additivity: For each pair A and $A' \in \mathcal{P}$,

$$R(A + A') = R(A) + R(A').$$

The third axiom says that each team should receive, at most, the total audience of the games played by the team. It therefore formalizes a natural upper bound, akin to the standard requirement of claims boundedness for the problem of adjudicating conflicting claims (e.g., O'Neill, 1982; Thomson, 2019a).

Maximum aspirations: For each $A \in \mathcal{P}$ and each $i \in N$,

$$R_i(A) \le \alpha_i.$$

---

[15] One might argue that subadditivity, i.e., for each pair A and $A' \in \mathcal{P}$, $R(A + A') \le R(A) + R(A')$, is a more reasonable axiom. It turns out that both axioms are equivalent in our setting due to the definition of rules.

The next result states that just the combination of the three previous axioms characterizes the family of *compromise* rules. This is remarkable as the three axioms are intuitive and basic and none of them seem to convey strong implications individually.

Theorem 1 *A rule satisfies symmetry, additivity, and maximum aspirations if and only if it is a compromise rule.*

Proof. It is straightforward to show that each *compromise* rule satisfies the three axioms.

Conversely, let R be a rule satisfying the three axioms. Let $A \in \mathcal{P}$. For each pair $i, j \in N$, with $i \neq j$, let 1^{ij} denote the matrix with the following entries:

$$1^{ij}_{kl} = \begin{cases} 1 & \text{if } (k, l) = (i, j) \\ 0 & \text{otherwise.} \end{cases}$$

Notice that 1^{ij}_{ji} is the *zero matrix*, i.e., the matrix with only zero entries.

Let $k \in N$. By *additivity*,

$$R_k(A) = \sum_{i,j \in N: i \neq j} a_{ij} R_k \left(1^{ij}\right). \tag{2}$$

By *symmetry*, for each pair $k, l \in N \setminus \{i, j\}$ we have that $R_i \left(1^{ij}\right) = R_j \left(1^{ij}\right) = x^{ij}$, and $R_k \left(1^{ij}\right) = R_l \left(1^{ij}\right) = z^{ij}$. As $\sum_{k \in N} R_j \left(1^{ij}\right) = ||1^{ij}|| = 1$, we deduce that

$$z^{ij} = \frac{1 - 2x^{ij}}{n - 2}.$$

Let $k \in N \setminus \{i, j\}$. By *additivity*, $R_j \left(1^{ij} + 1^{ik}\right) = x^{ij} + z^{ik}$, and $R_k \left(1^{ij} + 1^{ik}\right) = z^{ij} + x^{ik}$. By *symmetry*, $R_j \left(1^{ij} + 1^{ik}\right) = R_k \left(1^{ij} + 1^{ik}\right)$. Thus,

$$
\begin{aligned}
x^{ij} + \frac{1 - 2x^{ik}}{n - 2} &= x^{ik} + \frac{1 - 2x^{ij}}{n - 2} \Leftrightarrow \\
(n - 2) x^{ij} + 1 - 2x^{ik} &= (n - 2) x^{ik} + 1 - 2x^{ij} \Leftrightarrow \\
x^{ij} &= x^{ik}
\end{aligned}
$$

Therefore, there exists $x \in \mathbb{R}$ such that for each $\{i, j\} \subset N$,

$$
\begin{aligned}
R_i \left(1^{ij}\right) &= R_j \left(1^{ij}\right) = x, \text{ and} \\
R_l \left(1^{ij}\right) &= \frac{1 - 2x}{n - 2} \text{ for each } l \in N \setminus \{i, j\}.
\end{aligned}
$$

Let $k \in N$. By (2),

$$
\begin{aligned}
R_k(A) &= \alpha_k x + (||A|| - \alpha_k) \frac{1 - 2x}{n - 2} \\
&= \alpha_k x + (2x - 1) \left[\frac{(n - 1)\alpha_k - ||A||}{n - 2} - \alpha_k \right] \\
&= \alpha_k x + (2x - 1) CD_k(A) - (2x - 1)\alpha_k \\
&= \frac{\alpha_k}{2} 2(x - 2x + 1) + (2x - 1) CD_k(A) \\
&= (2 - 2x) ES(A) + (2x - 1) CD_k(A).
\end{aligned}
$$

Let $\{i, j, l\} \subset N$ be a set of three different teams. By *maximum aspirations*,

$$
\begin{aligned}
x &= R_i\left(1^{ij}\right) \leq \alpha_i\left(1^{ij}\right) = 1 \text{ and} \\
\frac{1 - 2x}{n - 2} &= R_l\left(1^{ij}\right) \leq \alpha_l\left(1^{ij}\right) = 0.
\end{aligned}
$$

Thus, $\frac{1}{2} \leq x \leq 1$. Let $\lambda = 2 - 2x$. Then, $1 - \lambda = 2x - 1$. As x ranges from $1/2$ to 1, it then follows that λ ranges from 0 to 1. Consequently,

$$
R_k(A) = \lambda ES_k(A) + (1 - \lambda) CD_k(A) = C_k^{\lambda}(A),
$$

as desired. ∎

We prove in the Appendix that the three axioms are independent.

Theorem 1 shows that the family of *compromise* rules is characterized only by three basic and intuitive axioms, which, when combined, have strong implications to single out a one-parameter family ranging from the *equal-split* rule to *concede-and-divide*.

In Bergantiños and Moreno-Ternero (2020a), we characterized the *equal-split* rule and *concede-and-divide*. Two properties were common in both characterizations. One (*equal treatment of equals*) was a weakening of the *symmetry* axiom considered here.[16] The other was the same *additivity* axiom we consider here. The third property in each characterization came from a pair of polar properties modeling the effect of *null* or *essential* teams.[17] What we show with

---

[16] Formally, we say that a rule R satisfies *equal treatment of equals* if, for each $A \in \mathcal{P}$, and each pair $i, j \in N$ such that $a_{ik} = a_{jk}$, and $a_{ki} = a_{kj}$, for each $k \in N \setminus \{i, j\}$, $R_i(A) = R_j(A)$.

[17] The *null team* property states that if each game played by a team has no audience, then such a team (called null) receives nothing. The *essential team* property states that if only the games played by one team have positive audience, then such a team (called essential) receives all its audience. Formally, a rule R satisfies *null team* if, for each $(N, A) \in \mathcal{P}$, and each $i \in N$, such that $a_{ij} = 0 = a_{ji}$, for each $j \in N$, $R_i(N, A) = 0$. It satisfies *essential team* if, for each $(N, A) \in \mathcal{P}$, and each $i \in N$ such that $a_{jk} = 0$ for each pair $\{j, k\} \in N \setminus \{i\}$, $R_i(N, A) = \alpha_i$.

Theorem 1 is that, considering the axiom of *maximum aspirations* instead of any of the last two polar properties, allows us to move from characterizing the *equal-split* rule and *concede-and-divide* to characterize the family of all rules generated by their convex combinations.

The compromise rules just characterized are not very demanding from an informational viewpoint. Although they are defined over matrices, they do not require all the entries from the matrices to be defined. Just the claims are enough to define the rules. This will be a valuable asset for the empirical analysis in Section 5, as, in general, full information about audiences is rare to exist. On the other hand, information on aggregate audiences for each team is usually available.

The compromise rules obviously also satisfy other properties beyond those in the statement of Theorem 1. We highlight two of them here because they provide interesting testable implications for our empirical analysis in Section 5.

The first one is a strengthening of *symmetry* and it states that rules yield amounts that preserve the ranking of claims.

Order Preservation: For each $A \in \mathcal{P}$ and each pair $i, j \in N$, such that $\alpha_i \leq \alpha_j$,

$$R_i(A) \leq R_j(A).$$

The second one states that rules provide amounts that, for each team, lie between the extreme amounts provided by the *equal-split* rule and *concede-and-divide* for each of them.[18]

Lower and Upper Bounds: For each $A \in \mathcal{P}$ and each $i \in N$,

$$\min\{ES_i(A), CD_i(A)\} \leq R_i(A) \leq \max\{ES_i(A), CD_i(A)\}.$$

4 Further insights

4.1 Majority preferences

We have provided in the previous section normative foundations for a family of rules to share revenues raised from broadcasting. The axiomatic analysis is, nevertheless, silent regarding the specific rule to choose within the family. We explore such a problem in this section, taking

---

[18]Note that, for some teams, the amount that the equal-split rule yields will be smaller than the amount concede-and-divide yields, whereas for others it will be the opposite.

a decentralized approach. More precisely, we study whether the choice of a rule within the family could be made by means of simple majority voting, letting each team vote for a rule within the family.[19] Due to the overwhelming existence of majority cycles, one should normally not expect a positive answer to this question. Surprisingly, we do get a positive answer in our setting, thanks to the following feature that compromise rules exhibit.

In what follows, we assume, without loss of generality, that, for each $A \in \mathcal{P}$, $N = \{1, \ldots, n\}$ and $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n$, with at least one strict inequality.[20] We show next that the *compromise* rules satisfy the so-called *single-crossing* property. Formally,

Proposition 1 *Let $0 \leq \lambda_1 \leq \lambda_2 \leq 1$, and $A \in \mathcal{P}$. Then, there exists $i^* \in N$ such that:*
(i) $C_i^{\lambda_1}(A) \leq C_i^{\lambda_2}(A)$ for each $i = 1, ..., i^*$ and
(ii) $C_i^{\lambda_1}(A) \geq C_i^{\lambda_2}(A)$ for each $i = i^* + 1, ..., n$.

Given a problem $A \in \mathcal{P}$, we say that $C^\lambda(A)$ is a *majority winner* (within the *compromise* rules) for A if there is no other compromise rule $C^{\lambda'}$ such that $C_i^{\lambda'}(A) > C_i^\lambda(A)$ for a majority of teams. We say that the family of *compromise* rules has a *majority voting equilibrium* if there is at least one majority winner (within the *compromise* rules) for each problem $A \in \mathcal{P}$. It is well known that the single-crossing property of preferences is a sufficient condition for the existence of a majority voting equilibrium (e.g., Gans and Smart, 1996). Thus, we have the following corollary from Proposition 1.

Corollary 1 *There is a majority voting equilibrium for the family of compromise rules.*

We now study which specific *compromise* rule could be a majority winner for each problem. We obtain three different scenarios, depending on the characteristics of the problem at stake; more precisely, the partition of agents with respect to their claims. For some problems (those in which there is a high concentration of small claims), only the *equal-split* rule is a majority winner. For other problems (those in which there is a low concentration of small claims), only *concede-and-divide* is a majority winner. For the remainder of the problems, each *compromise* rule is a majority winner.

---

[19]This way of proceeding is somewhat realistic in one-team-one-vote environments, such as the one in North America (e.g., Fort and Quirk, 1995).

[20]Otherwise, all rules within our family would yield the same allocation.

For each $A \in \mathcal{P}$, we consider the following partition of N, with respect to the average claim $(\bar{\alpha})$: $N_l(A) = \{i \in N : \alpha_i < \bar{\alpha}\}$, $N_u(A) = \{i \in N : \alpha_i > \bar{\alpha}\}$, and $N_e(A) = \{i \in N : \alpha_i = \bar{\alpha}\}$. That is, taking the average claim (within the tournament) as the benchmark threshold, we consider three groups referring to individuals with claims below, above, or exactly at, the threshold. When no confusion arises, we simply write N_l, N_u, and N_e. Note that $n = |N_l| + |N_u| + |N_e|$.

Proposition 2 *Let $A \in \mathcal{P}$. The following statements hold:*

(i) If $2|N_l| > n$, then $ES(A)$ is the unique majority winner.

(ii) If $2|N_u| > n$, then $CD(A)$ is the unique majority winner.

(iii) Otherwise, each $C^\lambda(A)$ is a majority winner.

Proposition 2 implies that if the distribution of claims is skewed to the left (i.e., the median claim is below the mean claim), then the *equal-split* allocation (the most equal allocation within the family) is the majority winner, whereas if it is skewed to the right (i.e., the median claim is above the mean claim), then the *concede-and-divide* allocation (the most unequal allocation within the family, as proved below) is the majority winner. If it is not skewed, then any *compromise* allocation can be a majority winner.[21]

The single-crossing property also guarantees that the social preference relationship obtained under majority voting is transitive, and corresponds to the median voter's. In our setting, the median voter corresponds to the team with the median overall audience (claim). Thus, depending on whether this median overall audience is below or above the average audience, the median voter's preferred rule (and, thus, the majority winner) will either be the *equal-split* rule or *concede-and-divide*. In other words, a tournament with a small number of very strong teams (i.e., with very high claims in relative terms) will proclaim the *equal-split* allocation (the one favoring weaker teams more within the family) as the majority winner, whereas a tournament with a small number of very weak teams (i.e., with very small claims in relative terms) will proclaim the *concede-and-divide* allocation (the one favoring stronger teams more within the family). The reader is referred to the Appendix for the details.

---

[21]This is somewhat consistent with empirical evidence recently found in voting experiments on redistribution (e.g., Jiménez-Jiménez et al., 2019).

4.2 Distributive power

We now turn to the distributional effects of the rules within the family. More precisely, we show that the rules within the family are completely ranked according to the so-called Lorenz dominance criterion, the most fundamental criterion of income inequality.

Formally, given $x, y \in \mathbb{R}^n$ satisfying $x_1 \leq x_2 \leq ... \leq x_n$, $y_1 \leq y_2 \leq ... \leq y_n$, and $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$, we say that x *is greater than* y *in the Lorenz ordering* if $\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i$, for each $k = 1, ..., n - 1$, with at least one strict inequality. This criterion induces a partial ordering on allocations which reflects their relative spread. When x is greater than y in the Lorenz ordering, x is unambiguously "more egalitarian" than y (e.g., Dasgupta et al., 1973; Atkinson and Bourguignon, 1987).

In our setting, we say that a rule R is **more egalitarian** than another R' if for each $A \in \mathcal{P}$, $R(A)$ is greater than $R'(A)$ in the Lorenz ordering.

As mentioned above, the Lorenz ordering is only a partial ordering. Thus, one should not expect many rules to be ranked according to the egalitarian criterion just described. Nevertheless, as the next result shows, the *compromise* rules are fully ranked according to the parameter that defines the family. This parameter can therefore be interpreted as an index of the distributive power of the rule.

Proposition 3 *If* $0 \leq \lambda_1 \leq \lambda_2 \leq 1$*, then* C^{λ_2} *is more egalitarian than* C^{λ_1}.

In particular, one obtains from Proposition 3 that the *equal-split* rule is the most egalitarian rule within the family, as expected, whereas *concede-and-divide* is the least egalitarian rule within the family. Nevertheless, this does not imply that the *equal-split* rule is always the fairest (or *concede-and-divide* the least fair) rule within the family. Egalitarianism (here formalized as Lorenz dominance of allocations) does not need to be considered as a synonym of fairness (e.g., Sen, 1980; Roemer, 1998). In our environment, fairness of an allocation might depend on several factors (such as the partition of the audience among loyal fans and casual viewers). Thus, a proper analysis on fairness in this setting would require an axiomatic approach addressing some of these factors and thus extending the one we offered in Section 3.

4.3 Non-negativity

Another interesting aspect of the *compromise* rules is that they can provide negative amounts for some teams. Now, given a problem $A \in \mathcal{P}$, and $i \in N$, one might be interested in identifying the set of rules within the family that yield a positive amount to team i. Here is a clear-cut answer to that question:

Proposition 4 *For each $A \in \mathcal{P}$ and each $i \in N$, we have the following statements:*

(a) *If $\alpha_i \geq \overline{\alpha}$, then $C_i^\lambda(A) \geq 0$, for each $\lambda \in [0, 1]$.*

(b) *If $\alpha_i < \overline{\alpha}$, then $C_i^\lambda(A) \geq 0$ if and only if*

$$\lambda \geq 1 - \frac{(n-2)\alpha_i}{n(\overline{\alpha} - \alpha_i)}.$$

We have the following definition:

Non negativity. For each $A \in \mathcal{P}$ and each $i \in N$,

$$R_i(A) \geq 0.$$

Proposition 4 says that, for each rule within the family, teams with an audience above average will get a non-negative amount. Teams with an audience below average will get a non-negative amount depending on the relationship between α_i and $\overline{\alpha}$. When α_i is relatively small with respect to $\overline{\alpha}$, we need a large λ for *non-negativity*. The only case always guaranteeing a non-negative allocation to each team is the case in which $\lambda = 1$, i.e., the *equal-split* rule. As a consequence of Theorem 1, we can actually give a characterization of the *equal-split rule* based on this property.

Proposition 5 *A rule satisfies additivity, symmetry, maximum aspirations and non negativity if and only if it is the equal-split rule.*

As mentioned in the previous section, we would need additional axioms (reflecting principles with normative appeal too) to extend the analysis in Section 3 and be able to discriminate within the family (eventually, singling out a member of it as "superior" to the others in terms of fairness). If one would consider non-negativity as an axiom of that kind (it is, after all, formalizing another meaningful bound somewhat complementary to that of *maximum aspirations*), then we would indeed conclude that the equal-split rule (which happens to be the more egalitarian rule within the family, as shown in the previous section) is the *fairest* among the rules within the family as it is the only one passing the additional test that axiom imposes.

4.4 On the interpretation of λ

As mentioned above, compromise rules are defined by means of a parameter $\lambda \in [0, 1]$ indicating the relative weight of the *equal-split* rule in the convex combination with *concede-and-divide*. We showed in Section 4.2 that this parameter λ can also be interpreted as an index of the distributive power of the rule, as all rules within the family are fully ranked (in terms of the Lorenz dominance criterion) according to λ. We also discuss now that the parameter can be interpreted as a measure of teams' fan bases.

In general, individuals watching a game can be classified as fans of one of the teams involved in the game, or as *neutral* viewers. In practice, the above information is not available and we only know the total audience of the game. We can conjecture several plausible scenarios. For instance, In the extreme scenario in which the first group is empty (i.e., no team has fans), it seems natural to divide viewers of each game equally, which is what the *equal-split rule* proposes. In the polar extreme case, in which the second group is empty, it seems natural to concede each team the amount generated by its fans, which is the allocation proposed by *concede-and-divide*.[22] In other words, $\lambda = 1$ is associated to 100% of *neutral* (non-fan) viewers, whereas $\lambda = 0$ is associated to 0% of *neutral* (non-fan) viewers.

In practice, we know the total number of viewers of each game, but not the partition as fans and no fans. Now, it is feasible to estimate the average number of fans and no fans watching the games. For instance, we can take a sample of viewers and ask them to report the games they have watched, and if they are fans of some team. Let f denote the number of people who have watched a game being a fan of some of the teams. Let f^n denote the number of people who have watched a game without being a fan of any of the teams. Then, $\bar{\lambda} = \frac{f^n}{f+f^n}$ is the percentage of *neutral* (non-fan) viewers of a game. Similarly, $1 - \bar{\lambda} = \frac{f}{f+f^n}$ is the percentage of fans watching a game.

In general, λ can be considered as an estimation of the percentage of *neutral* viewers (those who watch a game without being a fan of one of the teams playing the game). Similarly, $1 - \lambda$ can be considered as an estimation of the percentage of viewers who watch a game because they are fans of one of the teams playing the game. In other words, a low value of λ is associated with a large fan base for participating teams, whereas a high value of λ is associated with a small fan base for participating teams.

---

[22] See Bergantiños and Moreno-Ternero (2020a) for further details.

5 An empirical application

In this section, we present an empirical application of our model resorting to La Liga, the Spanish Football League.

La Liga is a standard round robin tournament involving 20 teams. Thus, each team plays 38 games, facing each time one of the other 19 teams (once home, another away). The 20 teams, and the overall audience (in millions) of each team during the season 2017-2018, are listed in the first two columns of Table 1.[23]

<center>Insert Table 1 about here</center>

Note that the total audience of the entire season is 197.05 millions, and the total revenue was 1325.6 millions of euros. Thus, in order to accommodate the premises of our model and identify total audience with total revenue, we have to assume that each viewer paid a pay-per-view fee of 6.73 euros (instead of only one) per game. This normalizing assumption appears in Column 3. The resulting scaling will be implicit in the next tables describing the allocations.

Columns 4 and 5 yield the allocation put in practice for the season 2017-18 (in millions of euros and in percentage terms).[24] As we can see, two teams (Barcelona and Real Madrid) dominated the sharing, collecting (when combined) almost 23% of the pie.

An important conclusion one can derive from Table 1 is that the testable implication we formalized by the axiom of *Maximum Aspirations* is verified, as all teams obtain amounts below their claims (i.e., the amount in Column 4 is always below the corresponding amount in Column 3). On the other hand, the testable implication we formalized by the axiom of *Symmetry* is not verified, as two teams (Real Sociedad and Girona) have equal claims but obtain different amounts. Obviously, this infers that the testable implication we formalized by the axiom of *Order Preservation* is not verified either. As a matter of fact, we have several violations of it. For instance, Real Madrid has a higher claim than Barcelona but receives a smaller amount. Betis actually receives a smaller amount than 7 other teams with a lower claim (Atlético de Madrid, Valencia, Sevilla, Málaga, Athletic de Bilbao, Real Sociedad, and Villarreal).

Table 2 lists again the allocation put in practice for the season 2017-18, but now together with the ones proposed by the *equal-split* rule and *concede-and-divide* (the two extreme compro-

---

<center>18</center>

mise rules). In the last column of this table we explore whether the amount obtained by each team in the allocation used in practice corresponds to some *compromise* rule. For instance, Barcelona receives the amount that the rule $C^{0.98}$ would yield for this setting. In contrast, Real Madrid receives less than the amount proposed by any rule within the family because $148 < \min\{158.43, 260.81\}$. On the other hand, Atlético de Madrid receives more than the amount proposed by any rule within the family because $110.60 > \max\{85.77, 107.43\}$.

Insert Table 2 about here

Several conclusions can be derived from Table 2. Maybe the most obvious one is that the testable implication of *Lower and Upper Bounds* is far from being verified: less than half of teams obtain amounts within the interval determined by those bounds. More precisely, nine teams are favored by the actual allocation, in the sense that the amount each gets is above the amounts suggested by both bounds. Apart from Real Madrid, only one team (Betis) obtains amounts below those two bounds.[25] The remaining nine teams obtain amounts that can therefore be rationalized by some *compromise* rule. However, the rule would be different for each team. For instance, for Celta, it would be the rule corresponding to $\lambda = 0.02$ (which means that it receives something quite similar to the *concede-and-divide* outcome), whereas, for Barcelona, it would be the rule corresponding to $\lambda = 0.98$ (which means that it receives something quite similar to the *equal-split* outcome).[26] Note that if the parameter λ is interpreted as the percentage of *neutral* viewers (as argued in Section 4.4), the number for Barcelona is quite counterintuitive because the audiences of Barcelona games are much larger than the audiences of all other games (excluding those involving Real Madrid).

Another conclusion is that, contrary to what some might argue, the actual revenue sharing seems to be biased against the two powerhouses. Barcelona receives approximately the minimum it could receive, whereas Real Madrid receives even less than the minimum. With *concede-and-divide* (one of the extreme rules within the family), Barcelona and Real Madrid

---

[25]It is actually a remarkable case, as the allocation yields 3.99%, whereas the two rules would recommend 7.1% and 9.44%, respectively.

[26]Somewhat surprisingly, the *compromise* rule yielding a closer allocation (according to the Euclidean distance) to the real allocation is the rule corresponding to $\lambda = 1$, i.e., the *equal-split* rule. If we compare both allocations, one team (Betis) obtains much less (41 millions). Other nine teams (including Real Madrid) also obtain less (between 1 and 10 millions). The remaining ten teams (including Barcelona) obtain more (between 0 and 25 millions).

together would receive 38.28% of the pie (instead of the 22.78% they actually receive). We believe that fairness considerations regarding this issue can only be made with a proper normative analysis of how to split revenues. We have actually provided normative foundations for a whole family of rules on sharing revenues. Although the family encompasses a wide array of views, all of them agree suggesting that the two teams combined should receive something more than what they currently receive now. Thus, we can indeed argue that, based on our analysis, one cannot state that Barcelona and Real Madrid are unfairly favored.

6   Discussion

We have studied the problem of sharing the revenues from the collective sale of broadcasting rights for sports leagues, as recently considered by Bergantiños and Moreno-Ternero (2020a, 2020b). We have considered three basic and intuitive axioms for such a problem. Together, the three axioms characterize a family of rules that offer a compromise between two focal and somewhat polar rules: the *equal-split* rule and *concede-and-divide.* As such, the family is flexible enough to accommodate a wide variety of views regarding the existence of fans associated to each participating team. It ranges from the extreme view that, de facto, dismisses the existence of those fan bases (as exemplified by the *equal-split* rule) to the polar (and, thus, extreme too) view that minimizes the number of *casual viewers*, who simply watch a game because they are interested into the specific pair of teams involved in it (as exemplified by *concede-and-divide*).

We have also shown that the family has other merits. For instance, it constitutes a domain of rules for which majority voting equilibrium exists. This is important as the ultimate decision to approve a sharing rule might come to a vote among participating teams. Thus, guaranteeing the existence of a majority voting equilibrium, avoiding disturbing majority cycles, seems crucial. Especially so, when the voting decision is made among a wide variety of options, as in this case.

Our family of rules is reminiscent of some other families that have been considered in the literature on related topics (such as income inequality measurement, surplus sharing, cost allocation, or claims problems). Some of these families also offer compromises between focal and somewhat polar rules. Others share with ours the structure regarding the order of their members (according to the spread of the outcomes they yield), or the majority preferences (with respect to the members of the family).

We have also applied the rules within our family to a real-life situation. More precisely, we have explored the allocation of the (joint) revenues collected from selling broadcasting rights in the case of La Liga, the Spanish Football League. We have observed that some of the testable implications of our analysis are not verified for this case, which casts doubts on the allocation implemented by the Spanish Football League Association. It would be interesting to extend our empirical analysis to other major leagues such as the English Premier League.

The schemes we obtained for La Liga reflected a considerable level of inequality of outcomes. Much has been written about what constitutes *distributive justice* (see, for instance, Konow (2001) and the references therein). Depending on the specific view that one might hold on distributive justice, the above schemes might be considered fair or not. For instance, it might be considered fair that teams with a stronger bargaining power (either due to outside options, better performance, or higher ratings) receive much more.[27] On the other hand, inequality aversion is a widespread phenomenon that might make some consider the above schemes as unfair.[28] Our view is that a proper analysis on distributive justice for this setting requires an axiomatic approach in which further aspects, such as performance, are considered. New axioms in such a more general structure, with normative appeal, should help derive new rules. For instance, *hybrid* rules in which a portion of the pie is divided according to performance and another portion according to audiences. Alternatively, generalizations of the equal-split rule, in which the revenues from each game are split among the two teams playing the game, according to some weight reflecting performance. As this analysis is beyond the scope of this paper, we leave it for further research.

We conclude acknowledging that our paper is silent about the potential effects of the sharing process (of revenues raised from selling broadcasting rights) into several aspects of sports leagues. Much has been studied, for instance, about its effects on competitive balance.[29] It is also left for further research to study whether the rules in our family perform in a structured way with respect to their effect on competitive balance. To begin with, this would require to consider an appropriate measure of competitive balance, as the literature on sports economics is flooded with different measures and no consensus has been reached yet (e.g., Moreno-Ternero

---

[27]Rodríguez-Lara (2016) mentions precisely this case in his study on equity and bargaining power.

[28]As Garner (1986) puts it, "individuals become distressed when they participate in unfair relationships, and the greater the inequity, the more distress they feel".

[29]See, for instance, Késenne (2000), Szymanski (2001), Szymanski and Késenne (2004), or Peeters (2011).

and Weber, 2019; Pawlowski and Nalbantis, 2019). A more general model than ours, in which teams would be modeled as profit-maximizing clubs, determining talent investments resulting in win probabilities (with playing talent sold on a competitive market), would also be required.[30]

Finally, it would also be interesting to explore the interdependence of broadcasting revenues and transfer fees (the other major source of revenues in professional sports).[31] It is conventionally argued that the increase in the amounts raised from broadcasting has boosted transfer fees. Interesting patterns emerge too as a consequence of international differences. For instance, weak teams competing in the English Premier League (the most powerful domestic football competition worldwide to raise revenues from broadcasting, with a very egalitarian sharing system) are able to pay high transfer fees to reasonably strong teams in other strong domestic competitions (such as La Liga, or Serie A) with a less egalitarian sharing system of revenues from broadcasting rights. Partly because of this, it might not have been a surprise to observe that, for the first time in history, four teams from the same domestic competition (in this case, the English Premier League) dominated the two international competitions in a given season (2018-2019).

7 Acknowledgements

---

[30]Peeters (2012), for instance, is a model along these lines.

[31]Tervio (2006) rationalizes transfer fees as an efficient way to allocate scarce playing opportunities among players of different levels of known and potential ability.

References

[1] Atkinson, A., 1970. On the Measurement of Inequality, Journal of Economic Theory 2, 244-63.

[2] Atkinson, A., Bourguignon, F., 1987. Income Distribution and Differences in Needs. In: Feiwel G.R. (eds) Arrow and the Foundations of the Theory of Economic Policy. Palgrave Macmillan, London.

[3] Aumann R.J., Maschler M., 1985. Game theoretic analysis of a bankruptcy problem from the Talmud. Journal of Economic Theory 36, 195-213.

[4] Bergantiños, G., Moreno-Ternero, J.D., 2020a. Sharing the revenues from broadcasting sport events. Management Science 66 (6), 2417-2431.

[5] Bergantiños, G., Moreno-Ternero, J.D., 2020b. Allocating extra revenues from broadcasting sports leagues. Journal of Mathematical Economics. Forthcoming.

[6] Bergantiños, G., Vidal-Puga, J., 2007. A fair rule in minimum cost spanning tree problems. Journal of Economic Theory 137, 26–352.

[7] Bossert, W., 1990. An Axiomatization of the Single-Series Ginis, Journal of Economic Theory 50, 83-92.

[8] Cave, M., Crandall, R., 2001. Sports rights and the broadcast industry. The Economic Journal 111, F4-F26.

[9] Dasgupta, P.S., Sen, A.K., Starret, D., 1973. Notes on the Measurement of Inequality. Journal of Economic Theory 6, 180-187.

[10] Donaldson D., Weymark, J., 1980. A single-parameter generalization of the Gini indices of inequality. Journal of Economic Theory 22, 67- 86.

[11] Falconieri, S., Palomino, F., Sákovics, J., 2004. Collective versus individual sale of television rights in league sports. Journal of the European Economic Association 2, 833-862.

[12] Fort, R., Quirk, J., 1995. Cross subsidization, incentives and outcome in professional team sports leagues. Journal of Economic Literature 33, 1265-1299.

[13] Gans, J. S., Smart, M., 1996. Majority voting with single-crossing preferences, Journal of Public Economics 59, 219-237.

[14] Garner, A., 1986. Equity in economic relationships. Towards a Positive Theory. Journal of Economic Behavior and Organization 7, 253-264.

[15] Jiménez-Jiménez, N., Molis, E., Solano-García, A., 2019. The effect of initial inequality on meritocracy: A voting experiment on tax redistribution. Journal of Economic Behavior and Organization. Forthcoming.

[16] Késenne, S., 2000. Revenue sharing and competitive balance in professional team sports. Journal of Sports Economics 1, 56-65.

[17] Konow, J., 2001. Fair and square: the four sides of distributive justice. Journal of Economic Behavior and Organization 46, 137-164.

[18] Lee, E., 2019. Selling TV's Fall Lineup. New York Times

[19] Moreno-Ternero J.D., 2011. Voting over piece-wise linear tax methods. Journal of Mathematical Economics 47, 29-36.

[20] Moreno-Ternero, J., Roemer, J., 2006. Impartiality, priority and solidarity in the theory of justice. Econometrica 74, 1419-1427.

[21] Moreno-Ternero J.D., Villar A., 2006. The TAL-family of rules for bankruptcy problems. Social Choice and Welfare 27, 231-249.

[22] Moreno-Ternero, J., Weber, S., 2019. The interdependence of domestic and international success: the case of the UEFA Champions League, CPER DP 13927.

[23] Moulin, H., 1987. Equal or proportional division of a surplus, and other methods, International Journal of Game Theory 16, 161-186.

[24] O'Neill, B., 1982. A problem of rights arbitration from the Talmud. Mathematical Social Sciences 2, 345-371.

[25] Palacios-Huerta, I., 2014. Beautiful game theory. How soccer can help economics. Princeton University Press. Princeton. NJ.

[26] Palomino, F., Sákovics, J., 2004. Inter-league competition for talent vs. competitive balance. International Journal of Industrial Organization 22, 783-797.

[27] Pawlowski, T., Nalbantis, G., 2019. Competitive balance: measurement and relevance. In Downward, P, Frick, B., Humphreys, B. R., Pawlowski, T., Ruseski, J., Soebbing, B. P. (eds), The SAGE Handbook of Sports Economics (p. 154-162). Thousand Oaks (California): SAGE Publications Ltd.

[28] Peeters, T., 2011. Broadcast rights and competitive balance in European soccer. International Journal of Sport Finance 6, 23-39.

[29] Peeters, T., 2012. Media revenue sharing as a coordination device in sports leagues. International Journal of Industrial Organization 30, 153-163.

[30] Rodríguez-Lara, I., (2016) Equity and bargaining power in ultimatum games. Journal of Economic Behavior and Organization 130, 144-165.

[31] Roemer, J., 1998. Equality of Opportunity. Harvard University Press. Cambridge, Ma.

[32] Sen, A., 1980. Equality of what? In Tanner Lectures on Human Values, Vol. I. Cambridge University Press.

[33] Szymanski, S., Késenne, S., 2004. Competitive balance and revenue sharing in team sports. The Journal of Industrial Economics 52, 165-177.

[34] Szymanski, S., 2001. Income inequality, competitive balance and the attractiveness of team sports: some evidence and a natural experiment from English soccer. The Economic Journal 111, F69-F84.

[35] Tervio, M., 2006. Transfer Fee Regulations and Player Development. Journal of the European Economic Association 4, 957-987.

[36] Thomson, W., 2015a. For claims problems, a compromise between the constrained equal awards and proportional rules. Economic Theory 60, 495-520.

[37] Thomson, W., 2015b. For claims problems, another compromise between the constrained equal awards and proportional rules. Journal of Dynamics and Games 2, 366-382.

[38] Thomson W., 2019a. How to divide when there isn't enough: from Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation, Econometric Society Monograph. Cambridge University Press.

[39] Thomson, W., 2019b. On the Axiomatics of Resource Allocation: Classifying Axioms and Mapping Out Promising Directions. RCER WP 596.

[40] Trudeau, C., 2012. A new stable and more responsive cost sharing solution for mcst problems. Games and Economic Behavior 75, 402–412.

[41] Trudeau, C., 2014. Characterizations of the cycle-complete and folk solutions for minimum cost spanning tree problems. Social Choice and Welfare 42, 941-957.

[42] Weymark, J., 1981. Generalized Gini Inequality Indices, Mathematical Social Sciences 1, 409-430.

To save space, we have included in this appendix, which is not for publication, some technical aspects of our analysis, as well as secondary proofs.

8   Appendix

Remark 1 *The axioms of Theorem 1 are independent.*

Let R^1 be the rule that arises as a convex combination between the equal split rule and concede-and-divide, but with the (endogenous) weight obtained by the ratio between the maximum audience and the overall audience. Formally, for each problem $A \in \mathcal{P}$, let $\bar{A} = max_{i,j \in N} a_{ij}$. Then, for each $i \in N$,

$$R_i^1(A) = \frac{\bar{A}}{||A||} ES_i(A) + \left(1 - \frac{\bar{A}}{||A||}\right) CD_i(A).$$

R^1 satisfies symmetry and maximum aspirations, but not additivity.

Let R^2 be the rule in which, for each game $(i,j) \in N \times N$, the revenue a_{ij} goes to the team with the lowest number of the two. Namely, for each problem $A \in \mathcal{P}$, and each $i \in N$,

$$R_i^2(A) = \sum_{j \in N : j > i} (a_{ij} + a_{ji}).$$

R^2 satisfies maximum aspirations and additivity, but not symmetry.

The uniform rule, which divides the total audience equally among the teams, satisfies additivity and symmetry, but not maximum aspirations.

Proof of Proposition 1

Let $0 \leq \lambda_1 \leq \lambda_2 \leq 1$, and $A \in \mathcal{P}$.

We consider two cases:

1. **Case $\alpha_i \leq \bar{\alpha}$.** In this case,

$$\begin{aligned} C_i^{\lambda_1}(A) &= \frac{\alpha_i}{2} + \frac{n(1 - \lambda_1)}{2(n-2)} (\alpha_i - \bar{\alpha}) \\ &= \frac{\alpha_i}{2} + \frac{n(\lambda_1 - 1)}{2(n-2)} (\bar{\alpha} - \alpha_i) \\ &\leq \frac{\alpha_i}{2} + \frac{n(\lambda_2 - 1)}{2(n-2)} (\bar{\alpha} - \alpha_i) \\ &= C_i^{\lambda_2}(A). \end{aligned}$$

2. **Case** $\alpha_i > \overline{\alpha}$. In this case,

$$
\begin{aligned}
C_i^{\lambda_1}(A) &= \frac{\alpha_i}{2} + \frac{n(1-\lambda_1)}{2(n-2)}\left(\alpha_i - \overline{\alpha}\right) \\
&\geq \frac{\alpha_i}{2} + \frac{n(1-\lambda_2)}{2(n-2)}\left(\alpha_i - \overline{\alpha}\right) \\
&= C_i^{\lambda_2}(A).
\end{aligned}
$$

It turns out that i^* is precisely the team whose overall audience is closest (from below) to the average overall audience. $\qquad\square$

Proof of Proposition 2

Let $0 \leq \lambda \leq 1$, and $A \in \mathcal{P}$. By (1), for each $i \in N$,

$$
C_i^{\lambda}(A) = \frac{\alpha_i}{2} + \frac{n(1-\lambda)}{2(n-2)}\left(\alpha_i - \overline{\alpha}\right).
$$

If $\alpha_i < \overline{\alpha}$, then $C_i^{\lambda}(A)$ is an increasing function of λ, thus maximized at $\lambda = 1$. This implies that, for each $i \in N_l$, $ES_i(A)$ is the most preferred outcome (among those provided by the family).

If $\alpha_i > \overline{\alpha}$, then $C_i^{\lambda}(A)$ is a decreasing function of λ, thus maximized at $\lambda = 0$. This implies that, for each $i \in N_u$, $CD(A)$ is the most preferred outcome (among those provided by the family).

If $\alpha_i = \overline{\alpha}$, then $C_i^{\lambda}(A) = \frac{\alpha_i}{2}$ for each $\lambda \in [0,1]$. This implies that, for each $i \in N_e$, all rules in the family yield the same outcome.

From the above, statements (i) and (ii) follow trivially. Assume, by contradiction, that statement (iii) does not hold. Then, there exists $A \in \mathcal{P}$ and $\lambda \in [0,1]$ such that C^{λ} is not a majority winner for A. Thus, we can find $\lambda' \in [0,1]$ such that $C_i^{\lambda'}(A) > C_i^{\lambda}(A)$ holds for the majority of the teams. We then consider two cases:

Case $\lambda' > \lambda$.

In this case, $C_i^{\lambda'}(A) > C_i^{\lambda}(A)$ if and only if $i \in N_l$. Now,

$$
\begin{aligned}
|N_l| &= \left|\left\{i \in N : C_i^{\lambda'}(A) > C_i^{\lambda}(A)\right\}\right| \\
&> \left|\left\{i \in N : C_i^{\lambda'}(A) \leq C_i^{\lambda}(A)\right\}\right| \\
&= |N_u| + |N_e|
\end{aligned}
$$

which is a contradiction.

Case $\lambda' < \lambda$.

In this case, $C_i^{\lambda'}(A) > C_i^{\lambda}(A)$ if and only if $i \in N_u$. Now,

$$
\begin{aligned}
|N_u| &= \left| \left\{ i \in N : C_i^{\lambda'}(A) > C_i^{\lambda}(A) \right\} \right| \\
&> \left| \left\{ i \in N : C_i^{\lambda'}(A) \leq C_i^{\lambda}(A) \right\} \right| \\
&= |N_l| + |N_e|
\end{aligned}
$$

which is a contradiction. $\square$

We now reformulate Proposition 2 in terms of the median voter. Depending on whether the number of teams is odd or even, the median can be uniquely determined or not. To avoid ambiguity, we consider in each case the median to be the mean of the two middle values. Formally, the *median overall audience* is defined by

$$
\alpha_m = \begin{cases} \alpha_{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ \frac{1}{2}\left(\alpha_{\frac{n}{2}} + \alpha_{\frac{n+2}{2}} \right) & \text{otherwise.} \end{cases}
$$

Depending on whether this median overall audience is below or above the average audience, the median voter's preferred rule (and, thus, the majority winner) will either be the *equal-split* rule or *concede-and-divide*. More precisely,

Corollary 2 *Let $A \in \mathcal{P}$ be such that n is odd. The following statements hold:*

(i) *If $\alpha_m < \bar{\alpha}$, then $ES(A)$ is the unique majority winner.*

(ii) *If $\alpha_m > \bar{\alpha}$, then $CD(A)$ is the unique majority winner.*

(iii) *If $\alpha_m = \bar{\alpha}$, then any $C^{\lambda}(A)$ is a majority winner.*

Proof. If $\alpha_m < \bar{\alpha}$, then $|N_l| \geq m$. Hence $|N_l| > |N_u| + |N_e|$. By Proposition 2, statement (i) holds.

If $\alpha_m > \bar{\alpha}$, then $|N_u| \geq m$. Hence $|N_u| > |N_l| + |N_e|$. By Proposition 2, statement (ii) holds.

If $\alpha_m = \bar{\alpha}$, then $|N_l| < m$, $|N_u| < m$, and $|N_e| > 0$. Hence, we are in case (iii) of the statement of Proposition 2, which concludes the proof. $\blacksquare$

Corollary 3 *Let $A \in \mathcal{P}$ be such that n is even. The following statements hold:*

(i) *If $\alpha_{\frac{n+2}{2}} < \bar{\alpha}$, then $ES(A)$ is the unique majority winner.*

(ii) *If $\alpha_{\frac{n}{2}} > \bar{\alpha}$, then $CD(A)$ is the unique majority winner.*

(iii) *If $\alpha_{\frac{n}{2}} \leq \bar{\alpha} \leq \alpha_{\frac{n+2}{2}}$, then any $C^{\lambda}(A)$ is a majority winner.*

Proof. If $\alpha_{\frac{n+2}{2}} < \bar{\alpha}$, then $|N_l| \geq m$. Hence $|N_l| > |N_u| + |N_e|$. By Proposition 2, statement (i) holds.

If $\alpha_{\frac{n}{2}} > \bar{\alpha}$, then $|N_u| \geq m$. Hence $|N_u| > |N_l| + |N_e|$. By Proposition 2, statement (ii) holds.

Suppose now that $\alpha_{\frac{n}{2}} \leq \bar{\alpha} \leq \alpha_{\frac{n+2}{2}}$. Then, it is enough to prove that we are in case (iii) of the statement of Proposition 2. That is, we have to prove that neither $|N_l| > |N_u| + |N_e|$ nor $|N_u| > |N_l| + |N_e|$ hold. We consider several subcases:

1. If $\bar{\alpha} = \alpha_{\frac{n}{2}}$, then $|N_l| < \frac{n}{2}$, $|N_u| \leq \frac{n}{2}$ and $|N_e| > 0$.

2. If $\alpha_{\frac{n}{2}} < \bar{\alpha} < \alpha_{\frac{n+2}{2}}$, then $|N_l| = \frac{n}{2}$, $|N_u| = \frac{n}{2}$ and $|N_e| = 0$.

3. If $\bar{\alpha} = \alpha_{\frac{n+2}{2}}$, then $|N_l| \leq \frac{n}{2}$, $|N_u| < \frac{n}{2}$ and $|N_e| > 0$.

In either case, the desired conclusion holds. ∎

Proof of Proposition 3

Let $A \in \mathcal{P}$.

We first prove that $ES(A)$ is greater than $CD(A)$ in the Lorenz ordering.

Let $i \in N$. By equation (1),

$$CD_i(A) = \frac{\alpha_i}{2} + \frac{n}{2(n-2)}(\alpha_i - \bar{\alpha}).$$

Thus,

$$ES_1(A) \leq ES_2(A) \leq ... \leq ES_n(A) \text{ and}$$
$$CD_1(A) \leq CD_2(A) \leq ... \leq CD_n(A). \tag{3}$$

It then suffices to show that, for each $k = 1, ..., n-1$,

$$\sum_{i=1}^{k} \frac{\alpha_i}{2} \geq \sum_{i=1}^{k}\left(\frac{\alpha_i}{2} + \frac{n}{2(n-2)}(\alpha_i - \bar{\alpha})\right).$$

But this is simply a consequence of the fact that

$$\sum_{i=1}^{k} \alpha_i \leq k\bar{\alpha},$$

for each $k = 1, ..., n-1$.

We now prove that $C^{\lambda_2}(A)$ is greater than $C^{\lambda_1}(A)$ for each $0 \leq \lambda_1 \leq \lambda_2 \leq 1$. By (3), we have that

$$
\begin{aligned}
C_1^{\lambda_1}(A) &\leq C_2^{\lambda_1}(A) \leq ... \leq C_n^{\lambda_1}(A) \text{ and} \\
C_1^{\lambda_2}(A) &\leq C_2^{\lambda_2}(A) \leq ... \leq C_n^{\lambda_2}(A).
\end{aligned}
$$

Then, it suffices to show that, for each $k = 1, ..., n - 1$,

$$
\sum_{i=1}^{k} C_i^{\lambda_2}(A) \geq \sum_{i=1}^{k} C_i^{\lambda_1}(A).
$$

Now,

$$
\begin{aligned}
\sum_{i=1}^{k} \left[\frac{\alpha_i}{2} + \frac{n(1 - \lambda_2)}{2(n-2)} (\alpha_i - \overline{\alpha}) \right] &\geq \sum_{i=1}^{k} \left[\frac{\alpha_i}{2} + \frac{n(1 - \lambda_1)}{2(n-2)} (\alpha_i - \overline{\alpha}) \right] \Leftrightarrow \\
\sum_{i=1}^{k} \frac{n(1 - \lambda_2)}{2(n-2)} (\alpha_i - \overline{\alpha}) &\geq \sum_{i=1}^{k} \frac{n(1 - \lambda_1)}{2(n-2)} (\alpha_i - \overline{\alpha}) \Leftrightarrow \\
(1 - \lambda_2) \sum_{i=1}^{k} (\alpha_i - \overline{\alpha}) &\geq (1 - \lambda_1) \sum_{i=1}^{k} (\alpha_i - \overline{\alpha}).
\end{aligned}
$$

As $\sum_{i=1}^{k} (\alpha_i - \overline{\alpha}) \leq 0$ and $\lambda_1 \leq \lambda_2$, the above follows. $\qquad\square$

Proof of Proposition 4

Let $A \in \mathcal{P}$, $i \in N$, and $\lambda \in [0, 1]$. By equation (1), $C_i^{\lambda}(A) \geq 0$ if and only if

$$
\lambda \frac{\alpha_i}{2} + (1 - \lambda) \frac{(n-1)\alpha_i - ||A||}{n-2} \geq 0.
$$

Or, equivalently,

$$
(n - 2) \lambda \alpha_i + 2(1 - \lambda) [(n-1)\alpha_i - ||A||] \geq 0.
$$

As

$$
||A|| = \frac{\sum\limits_{i \in N} \alpha_i}{2}, \text{ and } \overline{\alpha} = \frac{\sum\limits_{i \in N} \alpha_i}{n},
$$

we deduce that

$$
||A|| = \frac{n\overline{\alpha}}{2}.
$$

Then, $C_i^{\lambda}(A) \geq 0$ if and only if

$$
(n - 2) \lambda \alpha_i + 2(1 - \lambda) \left[(n-1)\alpha_i - \frac{n\overline{\alpha}}{2} \right] \geq 0.
$$

31

Equivalently,

$$\lambda n\alpha_i - 2\lambda\alpha_i + 2n\alpha_i - 2\alpha_i - 2\lambda n\alpha_i + 2\lambda\alpha_i - n\overline{\alpha} + \lambda n\overline{\alpha} \geq 0,$$

or

$$\lambda n \left(\overline{\alpha} - \alpha_i\right) \geq n\overline{\alpha} - 2n\alpha_i + 2\alpha_i. \tag{4}$$

We now consider three cases:

Case $\alpha_i > \overline{\alpha}$.

In this case, (4) is equivalent to

$$\lambda \leq \frac{n\overline{\alpha} - 2n\alpha_i + 2\alpha_i}{n\left(\overline{\alpha} - \alpha_i\right)} = 1 - \frac{(n-2)\alpha_i}{n\left(\overline{\alpha} - \alpha_i\right)}.$$

As $\overline{\alpha} - \alpha_i < 0$ we deduce that

$$1 - \frac{(n-2)\alpha_i}{n\left(\overline{\alpha} - \alpha_i\right)} \geq 1,$$

and hence (4) holds for any $\lambda \in [0,1]$.

Case $\alpha_i = \overline{\alpha}$.

In this case, (4) is equivalent to $0 \geq (2-n)\alpha_i$, which always holds.

Case $\alpha_i < \overline{\alpha}$.

In this case, (4) is equivalent to

$$\lambda \geq \frac{n\overline{\alpha} - 2n\alpha_i + 2\alpha_i}{n\left(\overline{\alpha} - \alpha_i\right)} = 1 - \frac{(n-2)\alpha_i}{n\left(\overline{\alpha} - \alpha_i\right)},$$

as stated in the proposition. $\qquad\square$

Proof of Proposition 5

By Theorem 1, we know that the *equal-split* rule satisfies *symmetry*, *additivity* and *maximum aspirations*. It is obvious that it also satisfies *non negativity*.

Conversely, let R be a rule satisfying the four properties. By Theorem 1, R is a *compromise rule*. Thus, there exists $\lambda \in [0,1]$ such that, for each $A \in \mathcal{P}$,

$$R(A) = \lambda ES(A) + (1-\lambda) CD(A).$$

Suppose, by contradiction, that $\lambda < 1$. Then,

$$R_3 \left(\{1,2,3\}, 1^{12}\right) = (1-\lambda)(-1) < 0,$$

which contradicts *non negativity*. Thus, $\lambda = 1$ and, hence, $R \equiv ES$. $\qquad\square$

Table 1: Audiences and revenues for the Spanish Football League. Season 2017-18

Teams	Alpha (millions)	Alpha normalized	Allocation 17-18 (millions euros)	Allocation 17-18 (%)
Real Madrid	47,10	316,85	148,00	11,16
Barcelona	45,10	303,40	154,00	11,62
Betis	28,00	188,36	52,90	3,99
Atlético Madrid	25,50	171,54	110,60	8,34
Valencia	19,50	131,18	65,70	4,96
Sevilla	18,50	124,45	74,00	5,58
Celta	17,80	119,74	52,90	3,99
Málaga	17,60	118,40	53,50	4,04
Athletic Bilbao	17,20	115,71	73,20	5,52
Español	16,70	112,34	52,40	3,95
Las Palmas	15,90	106,96	46,80	3,53
Levante	15,10	101,58	45,10	3,40
Real Sociedad	14,90	100,24	61,50	4,64
Girona	14,90	100,24	43,30	3,27
Dep. Coruña	14,30	96,20	46,00	3,47
Villareal	13,80	92,84	65,50	4,94
Alavés	13,70	92,16	46,10	3,48
Getafe	13,50	90,82	44,50	3,36
Eibar	13,10	88,13	46,30	3,49
Leganés	11,90	80,05	43,30	3,27
Total	197,05	1325,60	1325,60	100.00

Table 2: The allocation rule and the EC family.

Team	Alloc. 17-18	ES	CD	lambda
Real Madrid	148,00	158,43	260,81	Below
Barcelona	154,00	151,70	246,61	0,98
Betis	52,90	94,18	125,18	Below
Atlético Madrid	110,60	85,77	107,43	Above
Valencia	65,70	65,59	64,82	Above
Sevilla	74,00	62,23	57,72	Above
Celta	52,90	59,87	52,75	0,02
Málaga	53,50	59,20	51,33	0,28
Athletic Bilbao	73,20	57,85	48,49	Above
Español	52,40	56,17	44,94	0,66
Las Palmas	46,80	53,48	39,26	0,53
Levante	45,10	50,79	33,58	0,67
Girona	61,50	50,12	32,16	Above
Real Sociedad	43,30	50,12	32,16	0,62
Deportivo Coruña	46,00	48,10	27,90	0,90
Villareal	65,50	46,42	24,35	Above
Alavés	46,10	46,08	23,64	Above
Getafe	44,50	45,41	22,22	0,96
Eibar	46,30	44,06	19,38	Above
Leganés	43,30	40,03	10,86	Above