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Abstract

We develop an endogenous fertility model of social stratification with two hereditary classes: warriors and peasants. Our model shows that the extra cost warriors must incur to raise their children and to equip them for war is the key determinant of (1) the relative sizes of both classes, and (2) the warriors' economic privileges in terms of income and consumption. The higher the cost of warrior children, the greater the economic privileges of warriors will be, and the smaller the ratio of warriors to peasants will be. Historical evidence confirms this prediction.

Keywords: Social stratification; income inequality; warfare; military participation ratio; Malthus; economic history; population economics.

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1 Introduction

Social stratification is a central concern of both Sociology and Anthropology. Although these disciplines have gone a long way describing and explaining social stratification, their theories eschew generalization and are, in consequence, largely devoid of testable predictions.[1] Believing there are general principles underlying all social phenomena, we take the opposite road and develop a microeconomic model that answers two questions: what determines the class composition in a stratified society, and what determines the degree of economic privilege of its upper classes. This model produces a set of predictions that can be tested against historical evidence. Also, the formal approach of economics allows us to identify some of the mechanisms behind the stratification of human societies.

Most anthropological and sociological theories of early social stratification share three recurrent themes. First, early stratified societies divide labor between warriors that fight wars and a peasants that work the land. Second, a society must be able to produce a sizable food surplus (i.e., more food than is needed to feed the peasants and their families) in order to support the non-food-producing warrior class. Third, social positions in early stratified societies are, for the most part, hereditary. Our model integrates these three themes into an endogenous-fertility framework, which takes into account the demographic forces that influence social stratification.[2]

The main features of our model are as follows. There are two states, always at war with each other. Populations in both states are unisex. Each state is divided into two hereditary social classes: warriors, who own the land, protect their own state, and plunder its neighbor; and peasants, who work for the warriors producing the food necessary to support both classes. People maximize utility by choosing how many children to have and how much food to consume, constrained by their food income and the cost of children in units of food. Children and food consumption are substitutes and are both normal goods (their demand is an increasing function of income when relative prices are fixed). Warrior children are more expensive than peasant children because they must be equipped and trained for war. There are diminishing returns to labor in food production. In the long-run, population must adapt itself to the amount of food that is available from production and from plunder.

Our model produces three main predictions:

1. *The average warrior enjoys higher income and higher consumption than the average peasant.* In the long-run, the average warrior and the average peasant must have exactly one child. Otherwise, the absolute and relative sizes of the two classes will not be stable. These averages include people who have no children because they die prematurely and survivors with more

[1] See subsections 1.1 and 1.2 for a brief review of these theories, and Summers (2005) for a more detailed account.

[2] Endogenous fertility models were pioneered by Razin and Ben-Zion (1975). Among these models, ours is closely related to those in which diminishing returns to labor operate as a Malthusian population check; for example, Boldrin and Jones (2002), Eckstein et al. (1988), and Nerlove et al. (1986).

than one child. Because warrior children are more expensive than peasant children, warriors must receive extra income in order to persuade them to have the same number of children as peasants. Since children and food are normal goods, warriors spend some of their extra income on food and hence on average enjoy a higher level of consumption than peasants.

2. *As the relative cost of warrior children increases, the size of the warrior class relative to the peasantry falls and the warriors' economic privileges increase.* In the long run, the average warrior or peasant must support exactly one child. Since children are normal goods, an increase in their price will induce warriors to substitute food for children, and that will bring average warrior fertility below one. In order to restore equilibrium, the per capita income of warriors must rise up to a point where they are again willing to have one child. This is accomplished by reducing the relative size of the warrior class: the reduction in warrior fertility induces a decline in warrior population, and as land is divided among fewer warriors than before, each of them will earn a bigger rent. Because consumption is a normal good and a substitute for children, part of this bigger rent will be used to finance the more expensive children and part will be destined to finance more consumption.

3. *Taking technology and prices as given, total population will be lower in a stratified state than in an unstratified state.* There are two reasons for this. First, stratified states employ less people on the land and therefore produce less food than unstratified states, and less food means fewer people can be fed. Second, the per capita income of warriors is higher than the per capita income of peasants, so even the same amount of food would support fewer people in a stratified state.

The rest of this introduction reviews the sociological and anthropological theories of social stratification, and presents the main empirical regularities. It also covers the basics of Malthusian population theories, necessary to understand the demographic aspects of our model. We invite readers already familiar with this literatures to go straight to Section 2, where we outline our model, discuss its predictions, and test them against historical evidence.

1.1 Agriculture and social stratification developed together

Evidence of early social stratification, mostly in the form of artifacts interred with the dead, is first found in the same archeological stratum as the oldest vestiges of agriculture (Angle, 1986). Ethnographic studies reveal that most contemporary bands of hunter-gatherers, such as the !Kung in the Kalahari or the Yolngu in Arnhem Land, are egalitarian and devoid of leadership (Boehm, 1999; Knauft, 1994; Winterhalder, 2001). We do not know for sure how prehistoric hunter-gatherers organized themselves, but the archeological evidence and the observation of their contemporary remnants indicate prehistoric hunter-gatherers lacked social stratification. All early agrarian states, on the contrary, were socially stratified (e.g., Sumer, Ancient Egypt, and Mycenaean Greece). The parallel emergence of agriculture and social stratification suggests that both developments were in some way related.

The ability to produce a surplus of food lies at the core of many theories of social stratification. The argument runs as follows. Surplus food is required to support a non-food-producing upper class. Agriculture has the potential to yield a surplus, and that explains why agrarian societies can be stratified. Hunter-gatherers, on the other hand, are always living on the edge of subsistence: chronically undernourished and constantly threatened by famine [evidence of this is surveyed by Kaplan (2000)]. They are thus unable to afford a class of non-food-producers. Gordon Childe (1942, p. 18; 1954) is the foremost surplus theorist of social stratification, and his ideas are (not surprisingly) very popular among Marxists thinkers (e.g., Beaucage, 1976, pp. 409–410; Mandel, 1962, pp. 26, 43).

The surplus theories of social stratification have several critics. Pearson (1957), for instance, argues that all societies have the potential to produce food in excess of biological necessity. It is the social organization what generates a surplus, and not the other way round. Sahlins (1972/1998) maintains that the key precondition for social stratification is not the ability to produce surplus food, but the feasibility of food storage [see also Cashdan (1980) and Hayden (1995)]. Without storage, Sahlins argues, there is no accumulation of wealth, and without wealth, social inequalities cannot exist. Most hunter-gatherers are nomads: as they quickly deplete local resources, they have no other choice but to keep moving. Nomadism makes storing food, and thus stratification, impossible. It comes to Sahlins as no surprise that the few reported cases of stratified hunter-gatherer communities are all located in exceptionally favorable ecological niches, where the abundance of food allows for permanent settlement and thus for food storage [Testart (1982) surveys the evidence]. The Pomo people of Central California are a classic example of a stratified gathering society. Acorns, the staple of the Pomo diet before modernization, were only available during one month in autumn. During that month the Pomo gathered the acorn and stored it for the rest of the year. The acorn stores were controlled by the chiefs (Kniffen, 1939).

Whatever the preconditions to stratification may be (surplus, storage, or both), it remains to be explained how an upper class of non-food-producers can emerge. Two opposing explanations have been proposed: a conflict-based explanation, advanced by Fried (1967; see also Hayden, 1995), and a functionalist explanation, attributed to Service (1962; also found in Davis, 1949, p. 367). Conflict theorists hold that 'aggrandizers' seized control of the means of production, and then used the surplus to obtain a superior standard of living. The functionalists, on the other hand, believe the upper classes provide goods that benefit society as whole: they lead war parties and organize defenses, build and maintain irrigation systems, store food as famine relief, and manage intergroup trade. As a reward for their services, the lower classes allow the upper classes a greater share of society's wealth.

Between these extremes, intermediate positions have emerged. For instance, Johnson and Earle (2000) maintain that the intensification of agriculture and consequent population growth pose a number of problems that can only be solved through hierarchy and the centralization of power: resource competition leading to raidings and warfare, the risk of failure in food production, inefficient use of resources that call for major technology investments, and resource deficiencies that can only be made up by foreign trade (pp. 29–32). Once power is acquired by an upper class, that group uses its power to establish privileges for itself (pp. 266–277, 301–303). At the same time, the lower classes face a trade-off between the benefits they derive from the public goods provided by the upper classes, and the burden of inequality net of the cost of revolting (Boone, 1992).

1.2 The rise of an hereditary warrior class

The intensification of agriculture required people to abandon nomadism and become sedentary. Sedentism, in turn, created competition for the most productive soils, and the opportunity to ransack the food stores of neighbouring communities. As a result, warfare escalated among early agrarian societies (Johnson and Earle, 2000, p. 252). The mounting demands of war triggered dramatic enhancements in military technology (Ferguson, 2003), creating the need for professional warriors that could handle it (Carneiro, 1970; Webster, 1975). The increased effectiveness of weaponry widened the fighting advantage of warriors over peasants. It is a common view among anthropologist and sociologists that the monopoly of weapons allowed warriors to prevent upward social mobility and become an hereditary social class (Summers, 2005).

According to Andreski (1968, pp. 31–32), a class of warriors can emerge in two ways: either by gradual differentiation of warriors from the rest of the population, or by conquest and subjugation of another group. Gradual differentiation occurs when a group manages to monopolize arms-bearing in order to secure a privileged position in society, or if the professionalization of warriors is necessary for society to augment its military power. Andreski maintains that conquest was the most common mechanism of social stratification, and provides a long list of historical cases to back up his claim: the subjugation of one city by another in Sumer (p. 42), the Dorian invasions in Greece (p. 43–44), and the Norse conquest of Russian Slavic tribes (p. 62), to mention just a few. Perhaps the chemically purest examples of stratification by conquest can be found in East Africa and Sudan. In that regions, ample kingdoms where founded through the conquest of negroid agriculturalists by hamitic pastoralists (p. 32). In Ankole, for instance, the pastoralist Hima conquered the agricultural Iru sometime before the British colonization. The Hima forced the Iru to pay tribute and allowed them no political rights. Only Iru men were allowed to bear arms and participate in war.

1.3 Malthusian principles of social stratification

The size and quality of the professional army that a society can support depends on the size of the surplus that the society can generate. In turn, the size of the surplus depends on three factors: first, the number of workers; second, the productivity of the average worker; and third, the amount consumed by the average worker and his dependants. From Malthus onwards, there has been a lively debate on the interplay between these factors (Coleman and Scholfield, 1986). The classical tradition, exemplified by Malthus and Ricardo, assumed a perfectly elastic supply of population at a constant subsistence wage rate together with diminishing returns to labor. If wages rise above subsistence, population will expand, leading to more employment. That will force downwards the marginal product of labor and thus wages. This process will only come to a halt when the marginal product of labor equals the subsistence wage, at which point the laboring population will stop growing. This is also the point at which the surplus product, in the form of rent, will be maximized.

To the extent that the adoption of agriculture involves the development of a more costly military technology and the emergence of a class of specialist warriors, not all of the extra output produced by agriculture can be translated into support for more producers and their families. A fraction of total production must be used to maintain and equip the warrior class. This point was made by Sauvy (1999), who argued that achieving a "power optimum" requires maximizing the surplus that is available to support the military and the government apparatus more generally.

How is population to be regulated so as to generate a surplus, subject to the limits set by technology and the environment? The growth of population within a given territory is determined by a combination of fertility, mortality, and migration, the relative importance of which has varied widely across time and space. The role of migration is obvious and is uncontroversial, so we shall focus on the other factors.

The original Malthusian theory assumed that population is automatically regulated through some kind of homeostatic mechanism. If population gets too large relative to available resources, there will be malnutrition, famine, disease, and warfare causing premature deaths. This formulation is based on the biological analogy that an animal species will blindly multiply up to the limits set by the carrying capacity of its habitat. Other formulations rely on conscious choice or social convention to limit population. Malthus himself in his later writings suggested prudential restraint involving late marriage or celibacy (Malthus 1820, pp. 248–252). Abortion, infanticide, and prolonged breast-feeding may also serve to space out births or get rid of unwanted children, and were all common practices among pre-modern societies (Douglas, 1966; Cashdan, 1985; Macfarlane, 1997). Some of these practices were deliberately designed to limit population, whereas others were social practices that were followed without any such objective in mind. However, even non deliberate social practices may have a homeostatic effect. Different societies compete with each other and those with practices that most effectively regulate population may outcompete their rivals. Thus, group selection may lead to the emergence of population practices that are well-adapted to the prevailing environment (Wrigley, 1978).

A controversial notion that needs to be clarified at this point is that of 'subsistence consumption'. Some versions of Malthusian theory interpret this notion in biological terms, assimilating it to the minimum food intake that allows a human being to survive and to produce an average of one offspring (e.g., Wolf, 1966, p. 6). The later Malthus regarded such an idea as simplistic, and he stressed the influence of socially-conditioned preferences on reproductive behavior (Malthus, pp. 248–252; Costabile and Rowthorn, 1985). This was a common view among classical economists, such as Ricardo, who expressed himself as follows:

> "It is not to be understood that the natural price of labour, estimated even in food and necessaries, is absolutely fixed and constant... It essentially depends on the habits and customs of the people. An English labourer would consider his wages under their natural rate, and too scanty to support a family, if they enabled him to purchase no other food than potatoes... Many of the conveniences now enjoyed in an English cottage, would have been thought luxurious at an earlier period of our history." (Ricardo, 1821, p. 91).

2 The Model

2.1 Setup

An agrarian state called Home is composed of $N \geq 0$ adults.[3] The adults are divided into $N_w \geq 0$ warriors and $N_p \geq 0$ peasants, where $N = N_w + N_p$. Warriors wage war against Home's neighboring state, Foreign. The purpose of war is to capture food, which peasants produce by working the land. Warriors own the land in equal shares, hire labor from the peasants, and protect them from pillaging by Foreign warriors. Social positions are hereditary: the children of warriors become warriors, and the children of peasants become peasants. Adults are averse to death, and derive utility from eating food and having children.

2.1.1 War

Let N_w^{F} be the the number of Foreign warriors and N_p^{F} the number of Foreign peasants. Every year Home and Foreign go to war. During war, Home captures a fraction Φ of Foreign's food surplus and keeps a fraction $1 - \Phi^{\mathrm{F}}$ of its own. These fractions depend on the relative fighting strengths of the two sides, as follows:

$$\Phi = f\left(\frac{N_w}{N_w + N_w^{\mathrm{F}}}\right), \tag{1}$$

$$\Phi^{\mathrm{F}} = f\left(\frac{N_w^{\mathrm{F}}}{N_w + N_w^{\mathrm{F}}}\right). \tag{2}$$

where $f' > 0$. Note that $\Phi = \Phi^F$ if $N_w = N_w^F$.

Nothing is directly destroyed by war. Yet war reduces food production because it removes people from farming to place them in the battlefield.

2.1.2 Food production and allocation

Home food production is given by $Y = AL^{\alpha} N_p^{1-\alpha}$, where $A > 0$ is the total factor productivity, and $\alpha \in (0, 1)$ measures the intensiveness of land in production. Following Andreski (1968, pp. 75–76), we assume that the amount of land controlled by Home is positively related to the relative effectiveness of attack over defense, represented by the parameter $\phi > 0$. Letting $L = L_0 \phi^{\beta}$, where $\beta > 0$, and normalising L_0 to 1 we get

$$Y = AL^{\alpha} N_p^{1-\alpha} = A\phi^{\alpha\beta} N_p^{1-\alpha}. \tag{3}$$

[3] We use Keeley's (1996) definition of states: "class-stratified political units that maintain a 'monopoly of deadly force' —a monopoly institutionalized as permanent police and military forces." (p. 27)

If Home is unstratified (i.e., if $N_w = 0$), peasants will earn their average product. If, on the contrary, Home is stratified (i.e., if $N_w > 0$), total food production must be divided between Home warriors and Home peasants. Assume peasants earn a fraction $1 - \alpha$ of total food production and warriors take the rest. This allocation can be justified in two ways.

First, it could result from a crop-sharing rule established by social convention. Crop-sharing rules are frequent among early agriculturalists (Raper and Reid, 1941, pp. 35–36). Of all possible crop-sharing rules, dividing production between warriors and peasants in fractions α and $1 - \alpha$ will maximize the military power of the state (see proof in Appendix A.1), which is equivalent in our model to N_w. In a warlike environment it is to be expected that group selection will favour those social conventions that maximize military power.

Second, the same allocation will result if labor markets are competitive. Competitiveness implies peasants will earn their marginal product, which is equal to $(1 - \alpha) Y$. The assumption of a competitive labor market may seem unrealistic at a first glance, since historically peasants were often bound to the land. In practice, escape was often easy, and peasants deserted their lord and sought a new one when they felt mistreated (North and Paul, 1973, pp. 30, 79, 200). In China, for instance, massive desertions of peasants were not only possible but indeed frequent, turning the tide of war against the deserted lord and in favour of the new one (Andreski, 1868, p. 48).

Let $\sigma_p \in (0, 1]$ be the probability of a peasant reaching reproductive age. Assuming the incomes of peasants who die before reproducing are inherited by their relatives, the income of the typical surviving peasant is given by

$$y_p = \begin{cases} \dfrac{(1-\alpha) Y}{\sigma_p N_p} & \text{if } N_w > 0, \\ \dfrac{Y}{\sigma_p N_p} & \text{if } N_w = 0. \end{cases} \tag{4}$$

Surviving warriors earn a rent from land and also get a share of the net spoils of war:

$$y_w = \underbrace{\frac{\alpha Y}{\sigma_w N_w}}_{\text{Rent of land}} + \underbrace{\frac{\Phi \alpha Y^{\text{F}} - \Phi^{\text{F}} \alpha Y}{\sigma_w N_w}}_{\text{Net spoils of war}}, \tag{5}$$

where $\sigma_w \in (0, 1]$ is a warriors' probability of reaching reproductive age and $Y^{\text{F}} = A\phi^{\alpha\beta}(N_p^{\text{F}})^{1-\alpha}$ is Foreign's food production.

2.1.3 Consumption, reproduction, and utility

People who die prematurely do not eat and do not bear children. The utility of a person is given by

$$u = \frac{\sigma^{1+\delta} c^{\theta} n^{1-\theta}}{\theta^{\theta} (1-\theta)^{1-\theta}},$$

where where σ is the probability of reaching reproductive age, $c \geq 0$ is his consumption, and $n \geq 0$ is the number of his children. The parameter $\theta \in (0, 1)$ represents the weight of consumption in utility, and $\delta > 0$ is a measure of death aversion. Consumption may not consist exclusively of food. Both warriors and peasants may use part of their incomes to purchase manufactured goods that are obtained at a fixed relative price by trading with the outside world. We do not explore this issue and assume that imported manufactures are represented in the utility function in terms of their food equivalent.

Historical evidence suggests that fertility among pre-modern peoples depended on the income available to them. The methods they used to limit the number of their children included abstinence, celibacy, prolonged breast-feeding, abortion, and infanticide (Douglas, 1966; Cashdan, 1985; Macfarlane, 1997). Recourse to such methods was more frequent when times were hard than in good times. To capture the link between fertility and income we shall assume that each adult solves

$$\begin{aligned} \max_{\{c,n\}} \quad & \sigma^{1+\delta} c^{\theta} n^{1-\theta} \\ \text{s.t.} \quad & c + \kappa n = y, \\ & c, n > 0, \end{aligned}$$

where $\kappa > 0$ is the price of a child in units of food and y is the adult's income.

The solution to an adult's problem is given by

$$c = \theta y, \tag{6}$$

$$n = \frac{(1-\theta) y}{\kappa}, \tag{7}$$

$$u = \frac{\sigma^{1+\delta} y}{\kappa^{1-\theta}}. \tag{8}$$

This is the standard result of the consumer problem with a Cobb-Douglas utility function. Expenditures on consumption and children are constant fractions of income, and indirect utility is increasing in food income and decreasing in the relative price of children.

Children are cheaper for peasants than for warriors: $\kappa_p < \kappa_w$. This difference can be read as the extra cost that warriors face in order to equip and train their children for war. As an example, consider the case of Spartans, who were taken away from their mothers to start their military training as young as seven years old (O'Connell, 2002, p. 42). Another example is given by Prestwich (1996), who reports that a complete armour in Middle Ages would cost the equivalent of a 1939 light tank. According to Andreski (1968, p. 58), the total cost of equipping one knight amounted to the annual income of a whole village, making knighthood a heavy financial burden.

There is some evidence that before Modern Times warriors had a lesser chance than peasants of reaching reproductive age. According to Wrigley (1997, p. 206), for example, the expectation of life at birth among the English aristocracy lagged behind that of the population as whole until the eighteenth century; among other reasons, because the children of aristocrats were weaned earlier that the children of commoners. Hollingsworth (1957), reports that during the 14th and 15th centuries, 46% of the sons of English dukes died violent deaths. The local peasants, on the other hand, were free from the hazards of continual combat (although they were occasionally prey to marauding lords). In line with this evidence, we shall assume that $\sigma_w < \sigma_p$.

2.1.4 Population dynamics

There is no migration and no mobility between social classes. Population dynamics is therefore governed by the following laws of motion:

$$\begin{aligned} N_w^{\text{next}} &= \underbrace{n_w \sigma_w N_w}_{\substack{\text{Children of} \\ \text{surviving warriors}}}, \\ N_p^{\text{next}} &= \underbrace{n_p \sigma_p N_p}_{\substack{\text{Children of} \\ \text{surviving peasants}}}, \end{aligned}$$

where N_w^{next} and N_p^{next} are the sizes of the warrior class and of the peasantry during the next generation. Survival probabilities σ_p and σ_w are exogenously determined parameters. These assumptions imply that any endogenous changes in population growth must come about through variations in the birth rates n_w and n_p.

2.2 Equilibria

Assume Home and Foreign are identical: $N_w = N_w^{\text{F}}$, $N_p = N_p^{\text{F}}$, and $Y = Y^{\text{F}}$. The existence of a rough equilibrium among competing societies can be traced back to the early stages of the adoption of agriculture. O'Connell (2002, p. 32) gives an example from the Sumerian world, which a man of that times characterized as follows: "You go and carry off the enemy's land; the enemy comes and carries off your land."

Our model has three symmetric equilibria, an unstratified equilibrium where $N_w = 0$ and $N_p > 0$, a stratified equilibrium where $N_w > 0$ and $N_p > 0$, and a trivial equilibrium where $N_p = N_w = 0$. We will discuss in detail both non-trivial equilibria. The unstratified equilibrium will serve as a benchmark to which the implications of the stratified equilibrium can be compared.

2.2.1 Unstratified equilibrium

In equilibrium, warrior and peasant populations remain constant through time:

$$\begin{aligned} N_w^* &= \sigma_w n_w^* N_w^*, \\ N_p^* &= \sigma_p n_p^* N_p^*, \end{aligned}$$

where the asterisks indicate the equilibrium value of a variable. The first of these conditions is immediately satisfied in an unstratified society, where $N_w^* = 0$. The second condition implies that

$$\sigma_p n_p^* = 1,$$

or equivalently,

$$n_p^* = \frac{1}{\sigma_p}.$$

The average peasant must have, in expected terms, exactly one child. Since a fraction $1 - \sigma_p$ of peasants does not survive to reproduce, the remaining peasants must compensate by having more than one child: $n_p^* > 1$. Plugging this equilibrium value of n_p into the solutions of the adult problem—given in equations (6), (7), and (8)—we get the equilibrium levels of income, consumption, and utility:

$$\begin{aligned} y_p^* &= \frac{\kappa_p}{(1-\theta)\,\sigma_p}, \qquad (9) \\ c_p^* &= \frac{\theta\kappa_p}{(1-\theta)\,\sigma_p}, \\ u_p^* &= \frac{\sigma_p^\delta \kappa_p^\theta}{1-\theta}. \end{aligned}$$

The equilibrium value of y_p is our model's equivalent to subsistence income. No matter how much the production technology improves (i.e., how much A increases), the income of peasants will always return to y_p^*. Equilibrium income y_p^* fits nicely into Malthus and Ricardo's view of subsistence income: instead of being a biological minimum, equilibrium income is determined in our model by preferences and prices.

In the long-run, population adjusts to keep income at a subsistence level. Combining equations (3), (4), and (9) we get the equilibrium level of population:

$$N^* = N_p^* = \left(\frac{(1-\theta) A L^{\alpha}}{\kappa_p} \right)^{\frac{1}{\alpha}}. \tag{10}$$

Observe that population is the only variable affected in the long-run by total factor productivity, represented by parameter A. Technological improvements translate into nothing but larger populations:

$$\frac{\partial \ln N^*}{\partial \ln A} = \frac{1}{\alpha} > 0.$$

Initially, a higher value of A allows more food to be produced by a given amount of labor. Peasants consume part of that extra food and use the rest to have more children. As a result, population begins to grow. But diminishing returns to labor imply population growth will eventually offset the productivity enhancement. In the end, more peasants will be employed on the land and the average amount of food produced by each of them will return to its original level. Consumption will also fall back to where it started, as the price of children remains unaltered through the whole process. This is the typical Malthusian result when there are diminishing returns to labor in production.

The effects of an increase in the price of children, represented by κ_p, are more interesting. Both long-run consumption and utility increase when children become more expensive:

$$\frac{\partial \ln c_p^*}{\partial \ln \kappa_p} = 1 > 0,$$
$$\frac{\partial \ln u_p^*}{\partial \ln \kappa_p} = \theta > 0.$$

This results, which may seem paradoxical at first, can be explained as follows. An increase in κ_p leads initially to decreased fertility and in the long-run to a reduction in population, so that fewer adults are employed on the land and hence labor productivity (y_p^*) is higher:

$$\frac{\partial \ln N_p^*}{\partial \ln \kappa_p} = \frac{1}{\alpha} > 0,$$
$$\frac{\partial \ln y_p^*}{\partial \ln \kappa_p} = 1 > 0.$$

When κ_p first increases, adults are no longer able to afford their current mix of children and consumption, and hence they experience a reduction in their utility. But when the system reaches the new equilibrium, adults end up with more income but the same number of children as they had before the shock. This implies that food consumption, and therefore utility, must be higher. The existence of diminishing returns entails that the short-run and long-run impacts of an increase in the price of children must run in opposite directions. Figure 1 illustrates the process. Under the plausible assumption that children are normal goods (Tzannatos and Symons, 1989), this comparative statics does not depend on the particular production and utility functions we use throughout this paper. We provide a general proof in Appendix A.2.

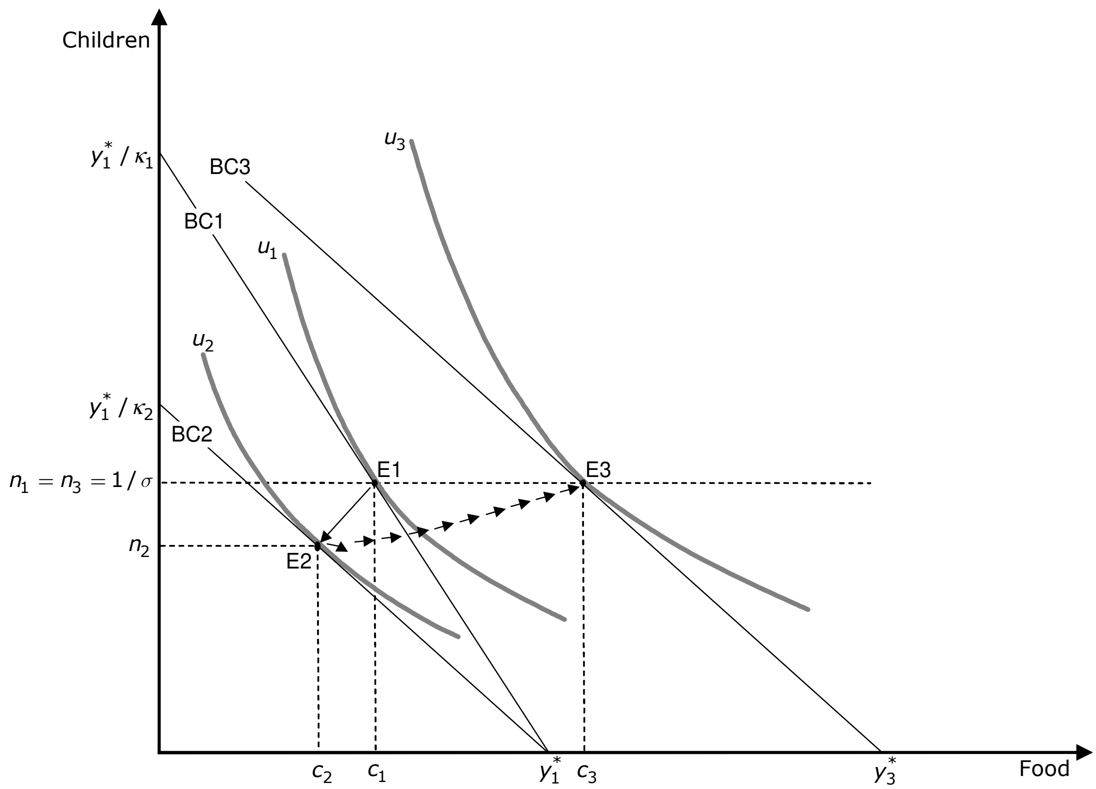

Figure 1: *An increase in the cost of children reduces welfare in the short-run but increases it in the long-run.* Point E1 represents the initial equilibrium. The income of a surviving adult, given by y_1^*, is just enough to induce him to have $1/\sigma$ children at their current price κ_1. That level of fertility ensures population will remain constant. All of a sudden the price of children raises from κ_1 to κ_2. Population and thus incomes are fixed in the short-run, so the adult's budget constrain rotates inwards from BC1 to BC2. As a result, the adult moves to point E2, where his consumption, his fertility, and his utility are all lower than before. The reduction in fertility causes population to decline through the generations. Eventually, the decline in population will increase the returns to labour up to y_3^*, pushing outwards the budget constrain from BC2 to BC3. Equilibrium is reestablished at point E3, where income is just enough to induce the descendents of the original adult to have $1/\sigma$ children given their new price κ_2. In the new equilibrium, fertility is the same but consumption is higher than it was in the beginning. Consequently, utility is also higher.

2.2.2 Stratified equilibrium

Both N_w and N_p are positive in a stratified state. In the short run, the values of N_w and N_p are fixed. Using the symmetry between Home and Foreign, together with equations (1) to (8), we obtain the short-run equilibrium:

$$\begin{aligned} y_w &= \frac{\alpha A \phi^{\alpha\beta} N_p^{1-\alpha}}{\sigma_w N_w}, & y_p &= \frac{(1-\alpha)\,\phi^{\alpha\beta} N_p^{1-\alpha}}{\sigma_p N_p}, \\ c_w &= \frac{\theta\alpha A \phi^{\alpha\beta} N_p^{1-\alpha}}{\sigma_w N_w}, & c_p &= \frac{\theta\,(1-\alpha)\,\phi^{\alpha\beta} N_p^{1-\alpha}}{\sigma_p N_p}, \\ n_w &= \frac{(1-\theta)\,\alpha A \phi^{\alpha\beta} N_p^{1-\alpha}}{\sigma_w N_w \kappa_w}, & n_p &= \frac{(1-\theta)\,(1-\alpha)\,\phi^{\alpha\beta} N_p^{1-\alpha}}{\sigma_p N_p \kappa_p}, \\ u_w &= \frac{\sigma_w^{\delta}\alpha A \phi^{\alpha\beta} N_p^{1-\alpha}}{\kappa_w^{1-\theta} N_w}, & u_p &= \frac{\sigma_p^{\delta}\,(1-\alpha)\,A \phi^{\alpha\beta} N_p^{1-\alpha}}{\kappa_p^{1-\theta} N_p}. \end{aligned}$$

(Short-run equilibrium)

In the long-run, surviving warriors and surviving peasants must bear just enough children to keep their numbers constant:

$$\begin{aligned} N_w^* &= n_w^* \sigma_w N_w^*, \\ N_p^* &= n_p^* \sigma_p N_p^*. \end{aligned}$$

The above conditions imply that the typical warrior and the typical peasant must have, in expected terms, the same number of children:

$$n_w^* \sigma_w = n_p^* \sigma_p. \tag{11}$$

If peasants were to breed faster than warriors, the ratio of warriors to peasants would contract until warriors became extinct and all the population was composed by peasants. Conversely, if warriors were to breed faster than peasants, the ratio of warriors to peasants would expand until peasants were no longer able to support the warrior class. Eventually, the warrior class would collapse under its own weight. Hence, condition (11) is necessary for a stratified equilibrium to obtain.

Substituting the short-run equilibrium values of n_w and n_p into condition (11) we obtain the long-run equilibrium level of population and the class composition in Home:

$$N^* = \phi^\beta \left(\frac{(1-\theta)(1-\alpha)A}{\kappa_p} \right)^{\frac{1}{\alpha}} \left[1 + \frac{\alpha}{1-\alpha} \left(\frac{\kappa_w}{\kappa_p} \right)^{-1} \right], \tag{12}$$

$$\frac{N_w^*}{N_p^*} = \frac{\alpha}{1-\alpha} \left(\frac{\kappa_w}{\kappa_p} \right)^{-1}. \tag{13}$$

Following Andreski (1968, p. 33), we will refer to N_w/N_p as Home's *military participation ratio*, or MPR.

Finally, plugging N^* and N_w^*/N_p^* into the short-run equilibrium values of income, consumption, fertility, and utility we get the long-run equilibrium values of these variables:

$$\begin{aligned}
y_w^* &= \frac{\kappa_w}{(1-\theta)\sigma_w}, & y_p^* &= \frac{\kappa_p}{(1-\theta)\sigma_p}, \\
c_w^* &= \frac{\theta\kappa_w}{(1-\theta)\sigma_w}, & c_p^* &= \frac{\theta\kappa_p}{(1-\theta)\sigma_p}, \\
n_w^* &= \frac{1}{\sigma_w}, & n_p^* &= \frac{1}{\sigma_p}, \\
u_w^* &= \frac{\sigma_w^\delta \kappa_w^\theta}{1-\theta}, & u_p^* &= \frac{\sigma_p^\delta \kappa_p^\theta}{1-\theta}.
\end{aligned}$$

(Long-run equilibrium)

The effect of social stratification on economic privilege

The average warrior or peasant must support exactly one child. These averages include people who have no children because they die prematurely and survivors with more than one child. Because warrior children are more expensive than peasant children, equilibrium requires that the average warrior has a higher income than the average peasant. Let $\overline{y}_w$ be the per capita income of warriors and $\overline{y}_p$ be the per capita income of peasants. In equilibrium we have

$$\overline{y}_w^* = \sigma_w y_w^* = \frac{\kappa_w}{1-\theta} > \frac{\kappa_p}{1-\theta} = \sigma_p y_p^* = \overline{y}_p^*,$$

since $\kappa_w > \kappa_p$. Note that income inequality is entirely determined by the additional costs warriors must incur in training and equipping their children.

As a result of $\overline{y}_w^* > \overline{y}_p^*$, warriors will have higher per capita consumption than peasants:

$$\overline{c}_w^* \quad = \quad \sigma_w c_w^* \quad = \quad \frac{\theta \kappa_w}{1-\theta} \quad > \quad \frac{\theta \kappa_p}{1-\theta} \quad = \quad \sigma_p c_p^* \quad = \quad \overline{c}_p^*,$$

The economic inequality between surviving warriors and surviving peasants is more pronounced than the economic inequality between pre-war warriors and pre-war peasants:

$$\begin{aligned}
\frac{y_w^*}{y_p^*} \quad &= \quad \frac{\kappa_w/\sigma_w}{\kappa_p/\sigma_p} \quad > \quad \frac{\kappa_w}{\kappa_p} \quad = \quad \frac{\overline{y}_w^*}{\overline{y}_p^*} \\
\frac{c_w^*}{c_p^*} \quad &= \quad \frac{\kappa_w/\sigma_w}{\kappa_p/\sigma_p} \quad > \quad \frac{\kappa_w}{\kappa_p} \quad = \quad \frac{\overline{c}_w^*}{\overline{c}_p^*}.
\end{aligned}$$

This is due to the fact that a surviving warrior must finance σ_w^{-1} children whereas a surviving peasant must finance σ_p^{-1} $(< \sigma_w^{-1})$ children.

The warriors' degree of privilege in terms of utility is given by:

$$\frac{u_w^*}{u_p^*} \quad = \quad \underbrace{\left(\frac{\sigma_w}{\sigma_p}\right)^{\delta}}_{<1} \cdot \underbrace{\left(\frac{\kappa_w}{\kappa_p}\right)^{\theta}}_{>1} \quad \gtrless \quad 1.$$

As warriors face a larger risk of death than peasants ($\sigma_w < \sigma_p$), the warriors degree of privilege could be lower than 1, meaning that the peasantry would be the privileged class. This is a possible though unlikely state of affairs, as warrior children would probably shun the military career. Some form of compulsion or indoctrination would thus be required to keep the warrior class from disbanding.

If, on the contrary, warriors enjoy more utility than peasants, coercion may be needed to enforce property rights or, *in extremis*, a peasant revolution. This is probably why the vast majority of warrior nobilities kept their peasants disarmed. Before the westernization of Japan, for example, the bearing of arms was a strict prerogative of the nobles (with the very brief exception of the Taikwa reforms period, during the 7th century). It was no coincidence that no peasant rebellion ever succeeded in Japan (Andreski, 1958, p. 50). Some authors suggest other mechanisms that would allow a privileged upper class to subsist without coercing the lower classes. For example, if groups are segregated and investments in human capital generate within-groups positive externalities, then individual choices may lead to self-perpetuating economic differences between groups (Lundberg and Startz, 1998); or upper-class propaganda could deceive the lower classes into believing economic inequalities are in their best interest (Cronk, 1994; DeMarrais et al., 1996); or if people tend be influenced by members of their own social classes, lower-class people could just learn to play, without further questioning, their disadvantaged role in society (Henrich and Boyd, 2007).

Stratification reduces total population and maximizes military power

Maximal peasant population cannot be an equilibrium for a stratified state. When peasant population is at its maximum, the peasantry does not produce the surplus of food that is needed to support a warrior class. To produce a surplus, a stratified state must employ fewer peasants on the land, and therefore produce less food than would be possible with the existing technology. This can be observed in Figure 2, by comparing points E and H.

Everything else being equal, total population will be lower in a stratified state than in an unstratified state, for two reasons. First, stratified states produce less food than unstratified states, and less food means fewer people can be fed. Second, the per capita income of warriors is higher than the per capita income of peasants, so even the same amount of food would support fewer people in a stratified state. Figure 2 illustrates this point. In algebraic terms: from equations (10) and (12) it follows that population will be lower with than without stratification if and only if

$$\frac{\left[(1-\alpha)\,\kappa_w + \alpha\kappa_p\right](1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\kappa_w} \quad < \quad 1,$$

which is always true since $\alpha \in (0,1)$ and $\kappa_p < \kappa_w$.

Would peasants be better off without social stratification? The answer is: only temporarily, as can be inferred from Figure 2. Imagine that peasants organized a revolution and proceeded to execute all warriors. The per capita income of peasants would immediately rise from $\overline{y}_p^*$ to $Y(N_p^*)/N_p^*$, with the consequent increase of welfare. But the higher per capita income will sooner or later translate into higher fertility rates, and population will begin to grow. Consequently, the per capita income will fall until it reaches the subsistence level, at which point population growth will come to a halt (point H in the figure). After a period of increased welfare, peasants will end up as poor as before. In the new, unstratified equilibrium the state produces no surplus, and population is maximal. Were population any larger, the average product of labor would be forced below subsistence, and the population would decline until equilibrium was restored. This is confirmed by observing that the equilibrium value of u_p is equal to $(1-\theta)^{-1}\,\sigma_p^{\delta}\kappa_p^{\theta}$ both with and without stratification.

Sauvy (1952–54/1969, pp. 51–59) argues that maximizing military power requires maximizing the surplus that is available to support an army. This is clearly the case in our model, as the assumption that peasants get their marginal product entails that surplus, and hence warrior population, will be maximized in equilibrium (see proof in Appendix A.1).

Finally, note that all results above hold in our model irrespective of the existence of war, because war does not affect Home's equilibrium income:

$$\text{Home's income} \quad = \quad Y + \underbrace{\Phi\alpha Y^{\mathrm{F}} - \Phi^{\mathrm{F}}\alpha Y}_{\text{Net spoils of war}} \quad = \quad Y_{\text{Home}},$$

since $Y = Y^{\mathrm{F}}$ and $\Phi = \Phi^{\mathrm{F}}$.

Technological progress in food production will only increase population

Just as in the unstratified equilibrium, a technological improvement in food production has no effect but to increase total population:

$$\frac{\partial \ln N^*}{\partial \ln A} = \frac{1}{\alpha} > 0.$$

This result explains why the shift form hunting-and-gathering to agriculture (8,000 to 3,000 BC) was accompanied by the first demographic explosion ever to be recorded (Bocquet-Appel, 2002). The yield of the land steadily increased as a result of a series of technological improvements that can be read as increases in parameter A: fertilizers, fallowing, irrigation systems, the plow, and crop rotation (Vasey, 1992, pp. 44–57, 111). Suddenly, food was available to our ancestors in quantities they never dreamt before (Price and Gebauer, 1995). Just a few centuries after adopting agriculture, typical communities saw its numbers grow from about 30 people to 300 or more. Population densities also increased in those places where agriculture was adopted: from less than one hunter-gatherer per square mile, to 20 or more farmers on the same area (Johnson and Earle, 2000, pp. 43, 125, 246).

The present model also explains why the adoption of agriculture was not accompanied by improvements in our ancestors' nutrition, as studies of the human fossil record have revealed (Armelagos et al., 1991; Cohen, 1989):

$$\frac{\partial \ln c^*}{\partial \ln A} = 0.$$

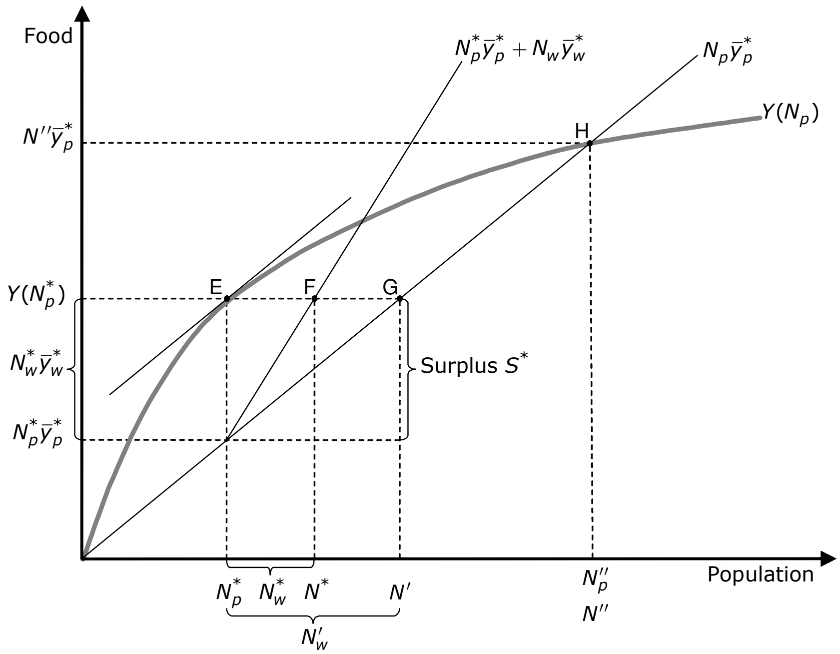

Figure 2: *Stratification reduces total population and maximizes military power.* Point H corresponds to the unstratified equilibrium (i.e., $N''_w = 0$, so $N'' = N''_p$). At point H, the average product of peasants equals their per capita income [i.e., $Y(N''_p)/N''_p = \overline{y}_p^*$], and per capita income is fixed at the subsistence level $\overline{y}_p^*$ by the long-run equilibrium conditions. The state produces no surplus. Points E and F correspond to the stratified equilibrium. At point E, the marginal product of peasants is equal to their per capita income, so the surplus $S^* = Y(N_p^*) - N_p^* \overline{y}_p^*$ is maximal. Also, total food production is lower with stratification than without it [i.e., $Y(N_p^*) < Y(N''_p)$]. Point G indicates what total population would be if warriors had the same per capita income as peasants (i.e., $\overline{y}_p^*$). If this were the case, the number of warriors would be equal to $N'_w = S^*/\overline{y}_p^*$ and total population would be $N' = N'_w + N_p^*$. In the long-run equilibrium, however, the per capita income of warriors is equal to $\overline{y}_w^*$. Hence the equilibrium number of warriors is $N_w^* = S^*/\overline{y}_w^*$ and total population is $N^* = N_w^* + N_p^*$. Since $\overline{y}_w^* > \overline{y}_p^*$, it follows that $N_w^* < N'_w$ and hence $N^* < N'$. Thus, stratification reduces total population for two reasons: total food production is lower, and the per capita income of warriors is higher than the per capita income of peasants.

More expensive weapons imply a lower MPR and higher income inequality

An increase in the costs of weapons and military training is represented in our model by an increase in the ratio κ_w/κ_p. Our model predicts that an increase in κ_w/κ_p will reduce the MPR, reduce total population, and sharpen income inequality in favor of warriors:

$$\begin{aligned}
\frac{\partial \ln\left(N_w^*/N_p^*\right)}{\partial \ln\left(\kappa_w/\kappa_p\right)} &= -1 & < 0. \\
\frac{\partial \ln\left(N^*\right)}{\partial \ln\left(\kappa_w\right)} &= \frac{-\alpha\kappa_p}{\left(1-\alpha\right)\kappa_w+\alpha\kappa_p} & < 0. \\
\frac{\partial \ln(\overline{y}_w^*/\overline{y}_p^*)}{\partial \ln\left(\kappa_w/\kappa_p\right)} &= 1 & > 0.
\end{aligned}$$

Andreski (1962, pp. 40–41) observes that and increase in the cost of weapons tends to reduce the MPR and to increase the economic advantage of warriors, exactly as our model predicts. He provides historical examples from a wide array of civilizations (pp. 39–72). We reproduce here three of Andreski's examples, to give the reader a taste of the historical evidence.

Persia— In times of the Achaemenid empire the Persian army consisted of nobles and freemen. The MPR was high since the freemen were very numerous, maybe even more than the peasants. Most of the army battled on foot, supported by a minimal cavalry. The main weapons in use were the bow and the long spear. Protective armors were scanty and uncommon. When the Sassanid dynasty rose to power in the third century A.D., it introduced a series of effective but very expensive military innovations; most importantly the stirrup and heavy protective armors. As a result, the freemen disappeared, the warrior nobility shrunk while its privileges expanded, and the peasants were reduced to harsh servitude (pp. 46–47).

Poland— The original Polish kingdom was despotic. Freemen and the king's personal guard, the Druzhina, conformed the army. Both groups were armed with primitive weapons and did not wear body armors. Gradually, the army incorporated more advanced equipment. Heavily armed horsemen were the mainstay of the Polish forces that repelled the Teutonic Knights in Grünewald (1410 A.D.). The modernization of the army was accompanied by a reduction of the MPR and an increase in social inequalities: peasants were reduced to the status of serfs, and military service was restricted to the nobility (pp. 59–60).

England— The Norman conquest of England, which introduced heavy cavalry to the country, sharpened social inequalities relative to the preceding Anglo-Saxon period. This process began to be reversed during the wars against the Welshmen, when English warriors learned how to use the long bow. An inexpensive yet formidable weapon, the long bow was far superior to any other type of bow. In combination with cavalry, it was able to inflict enormous damage on enemy forces. The adoption of the long bow forced profound changes in military tactics and organization. As a consequence of these changes, serfdom virtually disappeared from England, yeomen thrived, the MPR increased, and social inequality became much less pronounced (pp. 64–65).

As offensive weapons become more effective, states become more populous

A relative improvement of offensive weapons has only one effect: increasing population.

$$\frac{\partial \ln N^*}{\partial \ln \phi} \quad = \quad \beta \quad > \quad 0.$$

Historical evidence, as presented by Andreski (1968, p. 76), is consistent with this implication of our model. Andreski observes that when the art of fortifications surpasses the existing siege-craft, the number of independent states within a given area tends to increase, and the population of these states to decrease. Andreski provides examples from several cultures. Here we mention just a few. The Assyrians, who were the first to develop effective siege-engines (e.g., battering rams) and created the first cavalry, built an empire much bigger than any other seen hitherto. The creation of the Chinese empire was also related to the appearance of siege-engines. Before that, the strongholds that protected the dominions of Chinese feudal lords were almost impregnable. Philip of Macedon introduced catapults and balistas into the Balkan Peninsula, and with their aid he wielded the thus far independent greek cities into one empire (pp. 76–78).

The MPR and the warriors' degree of economic privilege are negatively correlated

The main stylized fact detected by Andreski (1968) is a negative correlation between the MPR and the warriors' degree of economic privilege (pp. 40, 73). This fact is consistent with the predictions of our model, as shown in Table 1. It is never the case in our model that an increase in the MPR accompanies an increase in the privileges of warriors. Moreover, when the relative price of warrior children increases, MPR goes down while privileges go up.

Table 1
Response of the military participation ratio (N_w^*/N_p^*), income inequality ($\overline{y}_w^*/\overline{y}_p^*$), and population ($N^*$) to changes in the parameters of the model

Parameter	N_w^*/N_p^*	$\overline{y}_w^*/\overline{y}_p^*$	N^*
Price of warrior children relative to peasant children (κ_w/κ_p)	$-$	$+$	0
Effectiveness of offensive relative to defensive weapons (ϕ)	0	0	$+$
Total factor productivity in agriculture (A)	0	0	$+$

3 Concluding remarks

Population dynamics plays a central role in our model. In its present form, the model assumes that the numbers of warriors and peasants are regulated exclusively through endogenous variations in fertility. Mortality rates are exogenous, and there is no migration across the frontiers of the state or mobility between classes. The model could be modified to allow for endogenous mortality by making death rates a function of per capita income. Provided the endogenous fertility mechanism continued to operate, the inclusion of endogenous mortality would not affect the basic results. Allowing for migration or social mobility would be more problematic. The effect of such movements would depend on how they were modelled and on the scale of the human flows. If migration and mobility were on a small scale, their inclusion in the model would not significantly affect the results. However, very large scale migration or inter-class mobility might radically alter the conclusions. Both the effects of migration and of inter-class mobility deserve further exploration.

Greater realism could also be achieved by dropping the unisex assumption and distinguishing between men and women. Polygyny and the division of labor between the sexes could then be incorporated into the model. The class structure could be made more realistic as well, by including servants and craftsmen that provide goods and services to the rest of the population. Administrators, technicians, scientists, and priests could also be included in the model. Most of these modifications would not affect the basic results. The one area where there might be a significant effect is agricultural technology and the variables that depend on it. In our model, stratification results in less food production and a smaller total population. This might not be true if the model were to include administrators, technicians, and scientists. Although not directly employed in agriculture, such people might contribute indirectly to food production by organizing public works and promoting the use of more productive technologies, thereby raising total food production and allowing the economy to support a larger population. Our conclusion that stratification reduces population assumes that the indirect contribution of the upper classes to agricultural production is of limited importance.

Another area worth exploring is that of peasant incomes. Our model assumes that in the stratified society peasants receive their marginal product. Two alternative justifications were given for this assumption: (1) there is competition in the labor market; (2) there is a crop-sharing rule that in the long-run maximizes the surplus available to finance the army. The second justification presumes that competition between states will favor the evolution of social practices that maximize military power. This group selection argument deserves examination in its own right.

A Appendix

A.1 A crop-sharing rule that maximizes military power

Suppose that warriors take a share γ of total food production Y. This leaves $(1-\gamma)Y$ to be divided among $\sigma_p N_p$ surviving peasants. The income of typical peasant is therefore

$$y_p = \frac{(1-\gamma)Y}{\sigma_p N_p}.$$

From the solution to the adult's problem we know the fertility of each peasant is given by

$$n_p = \frac{(1-\theta)y_p}{\kappa_p}.$$

Also recall that total production is $Y = A\phi^{\alpha\beta}N_p^{1-\alpha}$. Hence,

$$\sigma_p n_p = \frac{(1-\theta)(1-\gamma)A\phi^{\alpha\beta}N_p^{-\alpha}}{\kappa_p}.$$

Long-run equilibrium requires $n_p^* \sigma_p N_p^* = N_p^*$. From this condition and the above equations it follows that

$$N_p^* = \left(\frac{(1-\theta)(1-\gamma)A\phi^{\alpha\beta}}{\kappa_p}\right)^{\frac{1}{\alpha}}.$$

Rearranging terms we get the amount of food taken by the warrior class:

$$\text{Surplus} = \gamma Y = \gamma(1-\gamma)^{\frac{1-\alpha}{\alpha}}\phi^{\beta}A^{\frac{1}{\alpha}}\left(\frac{1-\theta}{\kappa_p}\right)^{\frac{1-\alpha}{\alpha}}.$$

The right-hand side of the above equation is maximized by taking $\gamma = \alpha$. Since N_w^* is proportional to the available surplus, N_w^* will also be maximal.

A.2 Malthusian equilibrium with a general utility function

Assume the utility of a typical adult is given by function $u(c, n)$, were $c \geq 0$ is the adults consumption and $n \geq 0$ the number of his children. Function u is strictly increasing in both its arguments, and also quasiconcave. The adult maximizes his utility choosing c and n, subject to the budget constrain $c + \kappa n \leq y$. Parameter $\kappa \geq 0$ represents the price of children, and $y \geq 0$ is the adult's income. The maximization yields the following first order condition:

$$\frac{u_n}{u_c} = \kappa, \tag{14}$$

where subscripts denote partial differentiation.

The model in the text assumes that fertility must in the long-run converge to a certain level $\overline{n}$ ($= 1/\sigma$) that keeps population constant. This demographic equilibrium is achieved through the interplay of two forces: demographic pressures and diminishing returns to labor. These two forces will carry y to a value that induces adults to choose $n = \overline{n}$, given price κ. That means the long-run value of c is fully determined by the following expression:

$$\frac{u_n(c, \overline{n})}{u_c(c, \overline{n})} = \kappa,$$

What happens if the price of children increases? Differentiating the above expression with respect to c and rearranging yields

$$\frac{\partial c}{\partial \kappa} = \left[\frac{\partial}{\partial c}\left(\frac{u_n(c, \overline{n})}{u_c(c, \overline{n})}\right)\right]^{-1}.$$

Hence, long-run consumption will be an increasing function of κ if and only if

$$\frac{\partial}{\partial c}\left(\frac{u_n(c, \overline{n})}{u_c(c, \overline{n})}\right) > 0. \tag{15}$$

Condition (15) can be interpreted in terms of indifference curves, as shown in figure 1. The condition states that the slope of the indifference curve must be flatter at E3 than at E1. Observe that if condition (15) holds, long-run utility will also rise when κ increases, simply because $u_c > 0$.

A bit of algebra proves that condition (15) will hold if children are normal goods. First we use first order condition (14) to reformulate condition (15) as follows:

$$\frac{\partial}{\partial c}\left(\frac{u_n}{u_c}\right) = \frac{u_{cn}}{u_c} - \frac{u_n u_{cc}}{u_c^2} = \frac{u_{nc} - \kappa u_{cc}}{u_c} > 0.$$

Since $u_c > 0$, the above inequality boils down to

$$u_{nc} - \kappa u_{cc} \quad > \quad 0.$$

On the other hand, the effect of income on the adult's demand for children is given by

$$\frac{dn}{dy} \quad = \quad \frac{u_{cn} - \kappa u_{cc}}{-\kappa^2 u_{cc} + 2\kappa u_{cn} - u_{nn}}. \tag{16}$$

Since u is quasi-concave and $\kappa = u_n/u_c$, the denominator of the right hand side of equation (16) must be positive. Therefore, children will be normal if $u_{cn} - \kappa u_{cc} > 0$, a condition that subsumes condition (15).

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