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Abstract

This study explores how agricultural technology affects the endogenous takeoff of an economy in the Schumpeterian growth model. Due to the subsistence requirement for agricultural consumption, an improvement in agricultural technology reallocates labor from agriculture to the industrial sector. Therefore, agricultural improvement expands the firm size in the industrial sector, which determines innovation and triggers an endogenous transition from stagnation to growth. Calibrating the model to data, we find that without the reallocation of labor from agriculture to the industrial sector in the early 19th century, the takeoff of the US economy would have been delayed by about four decades.

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> The spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it. [...] The introduction of the turnip [...] made possible a change in crop rotation which [...] brought about a tremendous rise in agricultural productivity. As a result, more food could be grown with much less manpower. Manpower was released for capital construction. The growth of industry would not have been possible without the turnip and other improvements in agriculture. Nurkse (1953, p. 52-53)

1 Introduction

According to Nurkse (1953), among many others, improvements in agricultural technology that released labor from agriculture were crucial for the industrial revolution. The industrial revolution in turn sparked the great divergence (Pomeranz 2001) and centuries of sustained economic growth. History thus suggests that improvements in agricultural technology propagate pervasively throughout the economy and have momentous consequences that far exceed what one can see by looking at the sector in isolation.

Modern growth economics has investigated extensively the forces driving the process, typically building on the theory of endogenous technological change (Romer 1990). Since at its core the theory has dynamic increasing returns, it identifies the size of the market in which firms operate as a, if not the, crucial factor determining incentives to innovate. A spectacular application of these ideas is the Unified Growth Theory of Galor and Weil (2000); see also Galor (2005, 2011). Models in this tradition produce an endogenous takeoff and a transition from stagnation to growth. Following these two influential branches of growth economics, and to place industry solidly at the forefront of the analysis, Peretto (2015) has developed an IO-based Schumpeterian growth model with endogenous takeoff in which firm size determines the incentives to innovate; see, e.g., Cohen and Klepper (1996a, b) and Laincz and Peretto (2006) for evidence on this channel. We use this model to formalize Nurkse's idea and then investigate the role that agriculture plays in shaping the growth path of the economy. This strikes us as a first-order question in light of studies like, among others, Lagakos and Waugh (2013) that document large and persistent productivity differences in agriculture across countries.

In the baseline Schumpeterian model firm size is increasing in population size and decreasing in the number of firms. All else equal, a larger population causes an earlier transition from stagnation to growth. However, countries with large population, such as China and India, did not experience an early industrial takeoff, arguably because the vast majority of their population was in agriculture and thus not contributing to firm size in industry. To capture this idea we introduce an agricultural sector and investigate how it affects the takeoff and the subsequent growth pattern. We preserve the analytical tractability of the original model and derive a closed-form solution for the equilibrium growth rate throughout the entire transition from stagnation to balanced growth. We find that higher agricultural productivity causes an earlier takeoff with faster post-takeoff growth and final convergence to scale-invariant growth.

At the heart of the mechanism driving this result is a subsistence requirement for agricultural consumption, which yields that when agricultural productivity improves labor moves from agriculture to industry. This reallocation alone can be sufficient to ignite industrialization. More generally, we have that: (i) for given agricultural technology, the model predicts a finite takeoff date with an associated wait time that is co-determined by initial firm size and decreasing in agricultural productivity; (ii) for given firm size, the model identifies the minimum size of the improvement in agricultural technology—an Agricultural Revolution—that triggers an immediate Industrial Revolution. The combination of (i) and (ii) says that low agricultural productivity *delays* industrialization and creates a *temporary drag* on post-industrialization growth. The drag is only temporary and not permanent because our Schumpeterian growth model with endogenous market structure sterilizes the scale effect.

These properties provide a new lens for interpreting the empirical evidence. As mentioned, economies with large populations (e.g., China, India) failed to industrialize for decades after smaller ones did (e.g., UK, USA). Growth theories based on increasing returns have problems explaining this fact. The typical argument is that they had bad institutions (e.g., Acemoglu and Robinson, 2012). Our analysis develops the complementary hypothesis that their large, relatively unproductive agricultural sectors played an important role in determining their industrialization lags. Moreover, the scale-invariance of steady-state growth implies that while agricultural productivity does not affect income growth asymptotically, it has permanent and large effects on the overall time-profile of income. This property sheds new light on the debate about the role that agriculture (more generally, the primary sector) plays in shaping the dynamics of cross-country income differences.

We calibrate the model to US data to perform an illustrative quantitative analysis. The agricultural share of the US workforce was about 80% in the early 19th century (see Baten 2016) and decreased to about 70% in 1830 and 60% in 1840 (see Lebergott 1966 and Weiss 1986). We find that this reallocation of labor from agriculture to industry was a powerful push toward the takeoff of the US economy. In line with our analytical result, absent this reallocation the takeoff of the US economy would have occurred four decades later. Finally, we derive a formula that shows that a one-fifth increase in industrial employment reduces the wait time to takeoff by about a decade.

This study relates to the literature on endogenous technological change. Romer (1990) develops the first R&D-based growth model driven by the invention of new products (horizontal innovation). Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) develop the creative-destruction Schumpeterian growth model driven by the improvement of the quality of products (vertical innovation). Peretto (1994, 1998, 1999), Smulders (1994), and Smulders and van de Klundert (1995) combine the two dimensions of innovation to develop the creative-accumulation Schumpeterian growth model with endogenous market structure.[1] We contribute to this literature by incorporating an agricultural sector in the creative-accumulation model. We find that the scale-invariance property arising from the two dimensions of innovation is important in allowing the allocation of resources to affect the endogenous takeoff but not economic growth in the long run.

This study also relates to the literature on endogenous takeoff. The seminal study in this literature is Galor and Weil (2000) that develops unified growth theory, which shows that

[1] Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008, 2010) and Ang and Madsen (2011) provide early evidence for this class of models. Garcia-Macia *et al.* (2019) provide the latest evidence that growth is driven by the in-house innovation activity of existing firms. Howitt (1999) combines the two dimensions of technology to develop a creative-destruction version of the theory.

the quality-quantity trade-off in childrearing and the accumulation of human capital enable an economy to escape the Malthusian trap and experience an endogenous transition from stagnation to growth.[2] We focus, instead, on the role of Schumpeterian technological progress as a complementary channel for the endogenous takeoff of the economy. More generally, and in line with the overall thrust of this literature, we formalize the idea of Nurkse (1953), and the related big push idea of Murphy *et al.* (1989), in a very tractable dynamic general equilibrium model that allows us to obtain analytical results and then quantify the effects of agricultural technology on the industrialization path of the economy—a path consisting of an endogenous takeoff followed by post-takeoff accelerating growth, with final convergence from below to scale-invariant innovation-led steady-state growth.

2 A Schumpeterian model of endogenous takeoff

The model features both improvement of existing intermediate goods (vertical innovation) and creation of new intermediate goods (horizontal innovation). Incentives to undertake these activities depend on firm size. Consequently, whether the economy experiences the endogenous takeoff depends on the size of the market for intermediate goods. In the original version (Peretto, 2015) the size of this market is proportional to the size of the labor force. By incorporating an agricultural sector with subsistence consumption, we disentangle the size of the market for intermediate goods from the size of the labor force and obtain a structure where the size of the intermediate sector, and therefore the size of intermediate firms, depends on the reallocation of labor from agriculture.

2.1 Household

There is a representative household with $L_t = L_0 e^{\lambda t}$ identical members, where $L_0 = 1$ and $\lambda > 0$ is population growth rate. The household has Stone-Geary preferences

$$U_0 = \int_0^\infty e^{-(\rho-\lambda)t} \left[\ln c_t + \beta \ln(q_t - \eta)\right] dt, \tag{1}$$

where c_t and q_t denote, respectively, consumption per capita of an industrial and of an agricultural good. The parameter $\beta > 0$ determines the importance of industrial consumption relative to agricultural consumption. The latter features a subsistence requirement $\eta > 0$.[3] The parameter $\rho > \lambda$ is the subjective discount rate.

The household maximizes utility subject to the asset-accumulation equation

$$\dot{a}_t = (r_t - \lambda)a_t + w_t - c_t - p_t q_t, \tag{2}$$

[2] See also Galor and Moav (2002), Galor and Mountford (2008), Galor *et al.* (2009) and Ashraf and Galor (2011). Galor (2011) provides a comprehensive review of unified growth theory.

[3] This is a common feature of structural change models (see, e.g., Matsuyama (1992), Laitner (2000) and Kongsamut *et al.* (2001)), which study the implications of structural change for long-run (i.e., asymptotic) growth but not for endogenous takeoff. See Herrendorf *et al.* (2014) for an excellent survey of this literature and Herrendorf *et al.* (2020) for a recent contribution.

where a_t is wealth per capita and r_t is the real interest rate. Each member of the household supplies inelastically one unit of labor to earn the wage w_t. Let the industrial good be our numeraire and p_t be the price of the agricultural good. The household sets:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho; \tag{3}$$

$$q_t = \eta + \frac{\beta c_t}{p_t}. \tag{4}$$

The first equation summarizes the intertemporal consumption-saving decision as the growth path of industrial consumption c_t. The second summarizes the intratemporal allocation of expenditure across the two goods as the demand for agricultural consumption q_t.

2.2 Agriculture

We follow Lagakos and Waugh (2013) and model agriculture as a competitive sector operating a linear technology

$$Q_t = AL_{q,t}, \tag{5}$$

where the parameter $A > \eta$ is labor productivity and $L_{q,t}$ is employment in agriculture. Profit maximization yields

$$w_t = p_t A, \tag{6}$$

which says that the wage in agriculture is equal to the marginal product of labor.

We omit land for simplicity. Including land produces the same qualitative results about endogenous takeoffs but the analysis is much more algebra-intensive. Vollrath (2011), among many others, studies the effects of land intensity and labor intensity in agriculture on industrialization. Our results are in line with the general insights produced by that work.

2.3 Industrial production

A representative competitive firm operates the assembly technology

$$Y_t = \int_0^{N_t} X_t^{\theta}(i) \left[Z_t^{\alpha}(i) \, Z_t^{1-\alpha} L_{y,t} / N_t^{1-\sigma} \right]^{1-\theta} di, \tag{7}$$

where $\{\theta, \alpha, \sigma\} \in (0,1)$. The key features are: (i) there is a continuum of non-durable differentiated intermediate goods $i \in [0, N_t]$; (ii) $X_t(i)$ is the quantity of intermediate good i; (iii) the productivity of good i depends on its own quality $Z_t(i)$ and on average quality $Z_t \equiv \int_0^{N_t} Z_t(j) \, dj / N_t$; (iv) overall productivity in assembly depends on product variety N_t. Two parameters regulate technological spillovers: α captures the private return to quality and hence $1 - \alpha$ determines vertical technological spillovers; $1 - \sigma$ captures a congestion effect of product variety so that the social return to variety is σ.

Let $P_t(i)$ be the price of $X_t(i)$. Profit maximization yields the conditional demands:

$$L_{y,t} = (1 - \theta) \frac{Y_t}{w_t}; \tag{8}$$

$$X_t(i) = \left(\frac{\theta}{P_t(i)}\right)^{1/(1-\theta)} \frac{Z_t^{\alpha}(i)\, Z_t^{1-\alpha} L_{y,t}}{N_t^{1-\sigma}}. \tag{9}$$

These expressions yield that the competitive industrial firm pays $(1-\theta) Y_t = w_t L_{y,t}$ for industrial labor and $\theta Y_t = \int_0^{N_t} P_t(i)\, X_t(i)\, di$ for intermediate goods.

2.4 Intermediate goods and in-house R&D

A monopolistic firm produces differentiated intermediate good i with a linear technology that requires $X_t(i)$ units of the industrial good to produce $X_t(i)$ units of intermediate good i at quality $Z_t(i)$, that is, the marginal cost of production is one. The firm also pays $\phi Z_t^{\alpha}(i)\, Z_t^{1-\alpha}$ units of the industrial good as a fixed operating cost. To improve the quality of its product, the firm devotes $I_t(i)$ units of the industrial good to in-house R&D. The innovation technology is

$$\dot{Z}_t(i) = I_t(i). \tag{10}$$

The firm's gross profit (i.e., profit before-R&D) is

$$\Pi_t(i) = [P_t(i) - 1]\, X_t(i) - \phi Z_t^{\alpha}(i)\, Z_t^{1-\alpha}. \tag{11}$$

The value of the monopolistic firm is

$$V_t(i) = \int_t^{\infty} \exp\left(-\int_t^s r_u du\right) [\Pi_s(i) - I_s(i)]\, ds. \tag{12}$$

The monopolistic firm maximizes (12) subject to (9) and (10).

We solve this dynamic optimization problem in Appendix A and find that the unconstrained profit-maximizing markup ratio is $1/\theta$. However, we assume that competitive fringe firms can produce $X_t(i)$ at quality $Z_t(i)$ but at the higher marginal cost $\mu \in (1, 1/\theta)$.[4] The monopolistic firm then sets

$$P_t(i) = \min\{\mu, 1/\theta\} = \mu \tag{13}$$

and prices fringe firms out of the market. The optimization problem also delivers the firm's rate of return to innovation,

$$r_t^q(i) = \alpha \frac{\Pi_t(i)}{Z_t(i)} = \alpha \left[(\mu - 1) \frac{X_t(i)}{Z_t(i)} - \phi Z_t^{\alpha-1}(i)\, Z_t^{1-\alpha}\right],$$

which is linear in quality-adjusted firm size $X_t(i)/Z_t(i)$. This property is at the heart of the mechanism that we study: incentives to innovate depend on quality-adjusted firm size, which in turn depends on the size of the market. We now turn to this component of the logical chain.

In models of this class the equilibrium of the market for intermediate goods is symmetric, that is, intermediate firms start with the same initial quality $Z_0(i) = Z_0$ for $i \in [0, N_t]$ and, facing a symmetric environment, make identical decisions. Consequently, they grow at the

[4] Specifically, we allow for diffusion of knowledge from monopolistic firms to fringe firms that enables the latter to constrain the pricing behavior of the former. This structure disentangles markups from the technological parameter θ.

same rate and symmetry holds at any point in time. Using the limit price (13), quality-adjusted firm size is

$$\frac{X_t\,(i)}{Z_t\,(i)} = \frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_{y,t}}{N_t^{1-\sigma}} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}} \frac{L_{y,t}}{L_t}.$$

We define the industrial employment share $l_{y,t} \equiv L_{y,t}/L_t$ and the composite variable

$$x_t \equiv \theta^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}. \tag{14}$$

This variable compresses the two state variables L_t (population) and N_t (mass of firms) to the ratio $L_t/N_t^{1-\sigma}$ and, therefore, makes the analysis of the model's dynamics simple.

With this notation, quality-adjusted firm size becomes

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{x_t}{\theta^{1/(1-\theta)}} \frac{L_{y,t}}{L_t} = \frac{x_t l_{y,t}}{\mu^{1/(1-\theta)}}.$$

Accordingly, the rate of return to innovation is

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t l_{y,t} - \phi \right]. \tag{15}$$

To summarize, this structure captures two sides of the idea explored in this paper. First, agricultural employment implies $l_{y,t} < 1$ and thus reduces firm size in the intermediate sector and thereby depresses incentives to innovate. Second, the reallocation of labor from agriculture to industrial production is an essential component of the dynamics of takeoff and subsequent sustained growth: as $l_{y,t}$ rises, the return to innovation rises faster than in the absence of structural change.

2.5 Entrants

Upon payment of a sunk cost of δX_t, $\delta > 0$, units of the industrial good, a new firm enters the market and offers a new differentiated good of average quality. This structure preserves the symmetry of the intermediate goods market equilibrium at all times. The asset-pricing equation governing the value of firms (old and new) is

$$r_t = \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{16}$$

Entry is positive when the free-entry condition holds, i.e., when

$$V_t = \delta X_t. \tag{17}$$

Substituting (9) and (13) in (11) and then using the resulting expression, (10), (16) and (17) yield the return to entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\delta} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t l_{y,t}} \right) + z_t + \frac{\dot{x}_t}{x_t} + \frac{\dot{l}_{y,t}}{l_{y,t}}, \tag{18}$$

where $z_t \equiv \dot{Z}_t/Z_t$ is the growth rate of average quality.

2.6 Aggregation

We define the general equilibrium in Appendix A. (9) and (13) yield the reduced-form representation of industrial production

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{\sigma} Z_t L_{y,t}. \tag{19}$$

The associated growth rate of industrial output per capita, $y_t = Y_t/L_t$, is

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \sigma n_t + z_t + \frac{\dot{l}_{y,t}}{l_{y,t}}. \tag{20}$$

This growth rate has three components: (i) the growth rate of the variety of intermediate goods, $n_t \equiv \dot{N}_t/N_t$; (ii) the growth rate of the average quality of intermediate goods, z_t; (iii) the growth rate of the industrial labor share $l_{y,t}$.

2.7 Labor allocation

The combination of labor demand from agriculture (6) and industry (8) yields

$$p_t = \frac{(1-\theta)\, Y_t}{AL_{y,t}}. \tag{21}$$

Substituting the agricultural technology (5) and the relative price (21) in the demand function for q_t in (4) yields the industrial labor share $l_{y,t}$ as

$$l_{y,t} = \left(1 + \frac{\beta}{1-\theta}\frac{c_t}{y_t}\right)^{-1}\left(1 - \frac{\eta}{A}\right). \tag{22}$$

This equation says that for given consumption-output ratio c_t/y_t, the industrial labor share $l_{y,t}$ is *increasing* in A if and only if $\eta > 0$. This property produces sectoral reallocation whereby an improvement in the agricultural technology releases labor from agriculture to the industrial sector.

3 Agriculture, takeoff and long-run growth

We now develop the main insight of the paper. We first show that the economy begins in a pre-industrial era in which the growth rate of industrial output per capita is zero. It then enters the industrial era, which consists of two phases. In the first, only the development of new products marketed by new firms drives the growth rate of industrial output per capita. In the second, product-quality improvement by existing firms adds its contribution and produces an acceleration of the growth rate.[5] The economy finally converges to a balanced growth path that features constant growth of income per capita fueled by both vertical and horizontal innovation.

[5] We consider the realistic case in which product creation happens before quality improvement. See Peretto (2015) for details on this property of the baseline growth model.

Next, we show that agriculture shapes this process of phase transitions and convergence: agricultural productivity determines the timing of the first phase transition, the endogenous takeoff of the economy, and of the second phase transition, the activation of vertical innovation. This *timing effect* has momentous consequences: although agricultural productivity does not affect steady-state growth due to the model's sterilization of the scale effect, it has permanent and large effects on the economy's time-profile of income. This property sheds new light on the debate about the role that agriculture plays in shaping the dynamics of cross-country income differences.

3.1 The model's global dynamics

The equilibrium law of motion of the state variable x_t is

$$\dot{x}_t = [\lambda - (1-\sigma)n_t]\, x_t, \tag{23}$$

where the entry rate n_t is either zero or an increasing function of x_t (see Appendix A). The process is thus initially explosive and then becomes implosive. It converges to the balanced growth path if the following condition holds

$$\delta\phi > \frac{1}{\alpha}\left[\mu - 1 - \delta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right] > \mu - 1. \tag{24}$$

Specifically, the state variable x_t converges to

$$x^* = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\,[\rho + \sigma\lambda/(1-\sigma)]} \frac{1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)}{\left(1 - \frac{\eta}{A}\right)}$$

as the growth rate of product variety converges to $n^* = \lambda/(1-\sigma)$. Steady-state firm size and income per capita growth are:

$$x^* l_y^* = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\,[\rho + \sigma\lambda/(1-\sigma)]}; \tag{25}$$

$$g^* = \alpha\left[(\mu-1)\frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta\,[\rho + \sigma\lambda/(1-\sigma)]} - \phi\right] - \rho > 0. \tag{26}$$

This structure has two properties worth stressing.

First, the existence condition (24) consists of two inequalities that ensure that the steady state x^* exists. To establish whether it is the attractor of the model's dynamics, we need to investigate the conditions for the occurrence of the two phase transitions discussed above. We do so in the remainder of this section, placing the role of agriculture at the center of the investigation. The exercise shows that the two inequalities also provide the condition for the occurrence of the second phase transition. The two conditions in (24) are then jointly sufficient for the full transition to the steady state x^*.

Second, (26) says that steady-state growth is independent of the sectoral allocation of labor due to the scale-invariance of the Schumpeterian growth model with endogenous market structure. This property is central to the paper's insight. As we investigate the role of agriculture in driving the phase transitions, we find that because steady-state growth is invariant to A, cross-country differences in agricultural technology produce a pattern of *divergence-convergence*, namely: (i) differences in A generate differences in growth that are solely due to differences in the timing of takeoff; (ii) such differences are only temporary and eventually vanish so that all else equal there is long-run growth equalization. It is worth stressing that differences in growth rates vanish, not differences in income levels. That is, differences in agricultural productivity imprint themselves on income levels and are amplified by the initial divergence in income dynamics caused by the different takeoff times. The amplification can be large since it leverages differences in growth rates that last several decades due to the model's slow convergence to the steady state.

3.2 The pre-industrial era

In the pre-industrial era firm size $x_t l_{y,t}$ is small and there are two possible configurations of the intermediate-good sector. First, initially demand for each intermediate good is so small that a would-be monopolist operating the increasing-returns technology would earn negative profit (see Appendix A for details). Since the increasing-returns technology is not viable, the existing N_0 intermediate goods are produced by competitive firms that do not innovate and make zero profit at the equilibrium price $P_t(i) = \mu$. Anticipating this, entrepreneurs are not willing to pay the sunk entry cost and thus there is no variety innovation either. Initially, therefore, all technologies in this economy exhibit constant returns to scale and firm size grows only because of exogenous population growth.

The second possible configuration occurs when the size of the market for intermediate goods grows sufficiently large that a would-be monopolist operating the increasing-returns technology could earn positive profit. We assume, however, that although the increasing-returns technology is now viable agents do not deploy it yet because doing so requires payment of the sunk entry cost.[6] The idea is that only innovation, in this case a process innovation, allows a new firm to monopolize an existing market. Hence, the pre-industrial era ends only when the present value of monoplistic firms is sufficiently large that the free-entry condition (17) holds.

As a result of the pre-industrial market structure outlined above, in the pre-industrial era the household's industrial consumption is $c_t = w_t l_{y,t} = (1-\theta) y_t$, which yields

$$\frac{c_t}{y_t} = 1 - \theta. \tag{27}$$

Substituting this result in (22) yields

$$l_y = \frac{1}{1+\beta}\left(1 - \frac{\eta}{A}\right). \tag{28}$$

This says that the industrial labor share in the pre-industrial era is stationary and increasing in agricultural productivity A. The associated growth rate of industrial output per capita is

$$g_t = \sigma n_t + z_t + \frac{\dot{l}_{y,t}}{l_{y,t}} = 0 \tag{29}$$

[6] In Appendix B, we consider an extension of the model that does not rely on this assumption and show that the dynamics are less realistic.

because $n_t = z_t = \dot{l}_{y,t}/l_{y,t} = 0$.

3.3 The industrial era: phase 1

Horizontal innovation (but not yet vertical innovation) activates when firm size $x_t l_{y,t}$ grows sufficiently large. To see this, note that when the free-entry condition holds the consumption-output ratio c_t/y_t and the industrial labor share $l_{y,t}$ jump to the steady-state values (derivation in Appendix A):

$$\left(\frac{c}{y}\right)^* = \frac{(\rho-\lambda)\delta\theta}{\mu} + 1 - \theta; \tag{30}$$

$$l_y^* = \frac{1}{1+\beta\left(1+\frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)}\left(1-\frac{\eta}{A}\right). \tag{31}$$

The growth rate of product variety is (derivation in Appendix A)

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta}\left(\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t l_y^*}\right) + \lambda - \rho > 0, \tag{32}$$

which is positive if

$$x_t > \frac{\left[1+\beta\left(1+\frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\mu^{1/(1-\theta)}\phi}{\mu-1-\delta(\rho-\lambda)}\left(1-\frac{\eta}{A}\right)^{-1} \equiv x_N. \tag{33}$$

Note that n_t is increasing in the agricultural technology A via the industrial labor share l_y^*, which is increasing in A, and increasing in the state variable x_t so that (23) describes a stable process. The growth rate of industrial output per capita is $g_t = \sigma n_t$.

The interpretation of this property in terms of the baseline growth model is that there exists a threshold of x_t below which the economy operates under pre-industrial conditions and firm size grows only because of exogenous population growth. Eventually, the economy crosses the threshold x_N but it takes

$$T_N = \frac{1}{\lambda}\log\left(\frac{x_N}{x_0}\right) \tag{34}$$

years to achieve such takeoff (derivation in Appendix A). Since x_N is decreasing in A, the combination of (32) and (34) says that economies with higher agricultural productivity A take off earlier and exhibit faster post-takeoff growth than economies with lower A.

An alternative interpretation is as follows. We write (33) as

$$A > \frac{\eta}{1 - \frac{1}{\mu-1-\delta(\rho-\lambda)}\left[1+\beta\left(1+\frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\mu^{1/(1-\theta)}\phi/x_t}. \tag{35}$$

This now says that, given x_t, when the agricultural technology A is below this critical threshold the economy remains in the pre-industrial equilibrium. However, if A rises above the threshold, the economy takes off immediately. In this sense, we have a condition determining when and how an Agricultural Revolution can trigger the Industrial Revolution. The two interpretations are complementary. The first holds A constant and uses the model's dynamics to compute the wait time to industrialization, i.e., how long it takes for x_t to go from its initial value x_0 to the threshold value x_N. As shown, the wait time is lower the larger is A. The second interpretation fixes x_t and asks how large an improvement in A is needed to trigger immediately the activation of Schumpeterian innovation. (35) says that economies with larger firms require smaller agricultural improvements to take off.

The important component of this mechanism is that when the agricultural technology improves the economy reallocates labor from the agricultural sector to the industrial sector and that *this reallocation alone can be sufficient to ignite industrialization.* Figure 1 presents the time path of the growth rate g_t when A increases at time t and causes the economy to escape the pre-industrial era and enter the first phase of the industrial era. The figure highlights the two complementary interpretations discussed above: (i) for given A, the model predicts a finite takeoff date with an associated wait time determined by the initial condition x_0 (equivalently, initial firm size $x_0 l_y$); (ii) for given firm size $x_t l_y^*$, the model identifies the minimum size of the improvement in A—an Agricultural Revolution—that triggers an immediate Industrial Revolution. The combination of (i) and (ii) says that low agricultural productivity *delays* industrialization and creates a *temporary drag* on post-industrialization growth. The drag is only temporary because our Schumpeterian growth model with endogenous market structure sterilizes the scale effect.

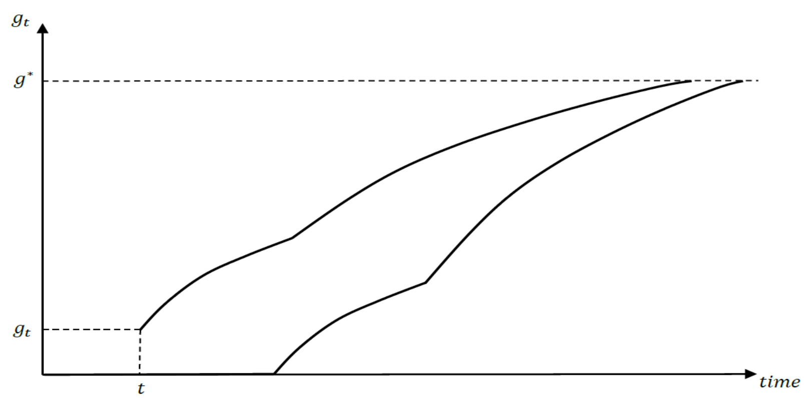

Figure 1: Agricultural revolution and industrialization

3.4 The industrial era: phase 2

When firm size $x_t l_y^*$ is sufficiently large, horizontal and vertical innovation occur simultaneously. This is the second phase of the industrial era. Given active horizontal innovation, the consumption-output ratio and the industrial labor share remain at the steady-state values (30)-(31). The growth rate (derivation in Appendix A),

$$g_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t l_y^* - \phi \right] - \rho > 0, \tag{36}$$

is increasing in the agricultural technology A via the industrial labor share l_y^* and increasing in firm size $x_t l_y^*$. The entry process (derivation in Appendix A) driving the dynamics of x_t is

$$n_t = \frac{\mu^{1/(1-\theta)}}{\delta}\left(\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t l_y^*}\right) + \lambda - \rho > 0, \tag{37}$$

where

$$z_t = \left(1 - \frac{\mu^{1/(1-\theta)}\sigma}{\delta x_t l_y^*}\right)^{-1}\left\{\left[\frac{\mu-1}{\mu^{1/(1-\theta)}} x_t l_y^* - \phi\right]\left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta x_t l_y^*}\right] - \rho + \sigma(\rho - \lambda)\right\}.$$

Given (24), this phase transition occurs when

$$x_t > \left[1 + \beta\left(1 + \frac{\rho - \lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\Omega\left(1 - \frac{\eta}{A}\right)^{-1} \equiv x_Z > x_N, \tag{38}$$

where

$$\Omega \equiv \underset{\omega}{\arg\operatorname{solve}}\left\{\left[\frac{\mu-1}{\mu^{1/(1-\theta)}}\omega - \phi\right]\left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta\omega}\right] = \rho - \sigma(\rho - \lambda)\right\}.$$

As in the previous case, the standard interpretation of this condition is that for given A there exists a threshold of firm size above which firms invest in-house and growth accelerates due to quality innovation.

The complementary interpretation of the threshold follows from rewriting (38) as

$$A > \frac{\eta}{1 - \left[1 + \beta\left(1 + \frac{\rho-\lambda}{\mu}\frac{\delta\theta}{1-\theta}\right)\right]\Omega / x_t}. \tag{39}$$

This says that for given x_t a sufficiently large level-change in the agricultural technology A can cause the immediate activation of quality innovation if it causes the threshold x_Z to fall below x_t.

3.5 Summary

We can summarize our main global dynamics result as follows.

Proposition 1 *Given (24) and $x_0 < x_N < x_Z$, the economy begins in the pre-industrial era with no innovation of any kind. It then experiences the endogenous takeoff and enters the first phase of the industrial era where horizontal innovation alone fuels industrial growth. Finally, the economy enters the second phase of the industrial era with both vertical and horizontal innovation and converges to the balanced growth path. Agricultural productivity A determines the timing of the two-phase transitions but does not affect the steady-state growth rate of the economy. Specifically, economies with higher agricultural productivity take off earlier and exhibit temporarily faster post-takeoff growth than economies with lower A, eventually converging to the scale-invariant growth rate g^*.*

Proof. See Appendix A. ■

These properties are important when looking at the data. As mentioned, economies with large populations (e.g., China, India) failed to industrialize for decades after smaller ones did (e.g., UK, USA). Growth theories based on increasing returns have obvious problems explaining this fact. The typical argument is that they had bad institutions (e.g., Acemoglu and Robinson, 2012). Our analysis says that their reliance on a large, relatively unproductive agricultural sector played an important role in determining their industrialization lags both in term of the timing of the takeoff and of the steepness of the post-takeoff income profile. The scale-invariance of steady-state growth implies that while agricultural productivity does not affect income growth asymptotically, it has permanent and large effects on the overall time-profile of income.

4 Quantitative analysis

In the early 19th century, the agricultural share of the US workforce decreased from about 80% to 60%.[7] We perform a counterfactual analysis to assess how large an effect this reallocation of labor from agriculture to industry had on the takeoff of the US economy.

Recall that firm size, which determines the timing of the takeoff, is

$$x_t l_{y,t} = x_t(1 - l_{q,t}),$$

where $l_{q,t} \equiv L_{q,t}/L_t$ is the agricultural labor share. The takeoff occurs when x_t reaches the threshold x_N. In terms of firm size we have

$$x_t l_{y,t} > x_N l_y^*.$$

A decrease in the agricultural labor share $l_{q,t}$ from 80% to 60% yields an increase in the industrial labor share $l_{y,t}$ from 20% to 40%.[8] This expands firm size $x_t l_{y,t}$ by a factor of 2 for given x_t. In the pre-industrial era the state variable x_t grows at rate λ. In the US, the long-run population growth rate is 1.8%.[9] Therefore, without the increase in the industrial labor share, x_t would take

$$t = \frac{\ln 2}{\lambda} = \frac{0.7}{1.8\%} = 39 \text{ years}$$

to increase by a factor of 2. In other words, without the reallocation of labor from agriculture to industry in the early 19th century, the takeoff of the US economy would have been delayed

[7] See Baten (2016), Lebergott (1966) and Weiss (1986).

[8] Here we are putting manufacturing and services together as the industrial sector that requires innovation; see e.g., United Nations (2011) for a review on the importance of innovation in the services sector. Kongsamut *et al.* (2001) show that manufacturing and services require the same technology growth rate in order for a balanced growth path to exist in their model.

[9] Data source: Maddison Project Database. The waiting time to takeoff is lower if the population growth rate is higher.

by about four decades. Furthermore, we can define $\chi \equiv dl_{y,t}/l_{y,t}$, i.e., the percent change in $l_{y,t}$, and for χ small obtain the approximation

$$t = \frac{\ln(1+\chi)}{\lambda} \approx \frac{\chi}{\lambda} \text{ years.}$$

This says that, given a population growth rate λ of 1.8%, a one-fifth increase in industrial employment reduces the wait time to takeoff by about a decade.

We now calibrate the rest of the model to data in the US economy in order to perform a quantitative analysis. In addition to the population growth rate λ, the model also features the following parameters: $\{\rho, \alpha, \sigma, \beta, \theta, \delta, \phi, \mu\}$.[10] We set the discount rate ρ to a conventional value of 0.05. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers $1-\alpha$ to 0.833 and the social return of variety σ to 0.25. Then, we calibrate β using the current agricultural share of GDP in the US, which is about 1%.[11] Furthermore, we calibrate $\{\theta, \delta, \phi\}$ by matching the following moments of the US economy: 60% for the labor income share of GDP, 62% for the consumption share of GDP, and 1% for the long-run growth rate. Finally, we calibrate the markup ratio μ by matching the average growth rates of the simulated path from our model and the historical path in the US. The calibrated parameter values are $\{\beta, \theta, \delta, \phi, \mu\} = \{0.016, 0.404, 2.547, 1.212, 1.630\}$.

To explore how well our model matches the historical path of the growth rate in the US, we first use historical data to calibrate a time path for the subsistence ratio η/A. Specifically, we calibrate the initial value of η/A using an agricultural labor share of 80% at the beginning of the 19th century; see Baten (2016). Then, we use an agricultural labor share of 60% in 1840 and 53% in 1860 in Lebergott (1966) and Weiss (1986) and also an agricultural share of GDP of 30% in 1900, 20% in 1920-1930, 10% in 1950 and 2% in 1980 in Kongsamut *et al.* (2001) to compute a piecewise linear path of η/A. We model these changes in A as MIT shocks (i.e., a sequence of unanticipated, permanent changes). Based on this imputed path of η/A, Figure 2 simulates the path of the agricultural share of GDP, which decreases from about 70% in the early 19th century to 1% at the end of the 20th century as in the US data.

Figure 3 presents the simulated path of the growth rate of industrial output per worker and the HP-filter trend of the US growth rate[12] along with a simulated path of the growth rate without agricultural improvement (i.e., η/A remains at its initial value). Here we pick an initial value x_0 such that the takeoff of the economy occurs before the mid-19th century. Following the occurrence of horizontal innovation, vertical innovation also starts to happen half a decade later. After that the economy keeps growing and reaches a growth rate as high as 3% due to the expansion of the industrial sector, which helps to accelerate the rate of innovation. Around the time of the Great Depression in the 20th century, there is a pause in the reallocation of labor from agriculture to the industrial sector, which translates into a temporary slow down in technological progress before a recovery. Before the end of the 20th century, the growth rate of the economy gradually falls towards the long-run growth rate due to the deceleration of sectoral reallocation. This simulated pattern replicates the data reasonably well with the average growth rate increasing from 1.08% in the 19th century to

[10] There is also the subsistence ratio η/A, which we will calibrate using historical data.

[11] Here we assume that the subsistence requirement is no longer binding in modern days; i.e., $\eta/A \to 0$.

[12] Unfortunately, we don't have historical data on labor productivity growth in the US, so we use data on the growth rate of output per capita as a proxy.

2.24% in the 20th century before decreasing to 1.04% in the 21st century, whereas the corresponding data are 1.20%, 2.12% and 1.13% in the 19th, 20th and 21st centuries respectively. In contrast, the simulated path of the growth rate without agricultural improvement cannot capture this inverted-U pattern in the data.

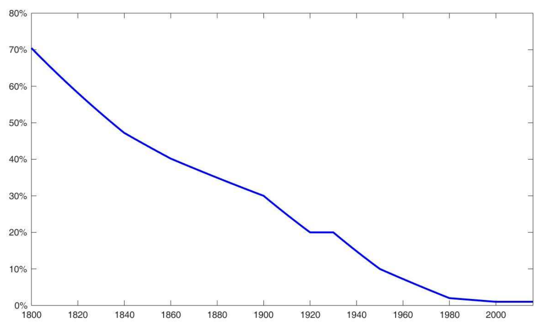

Figure 2: Agricultural share of GDP

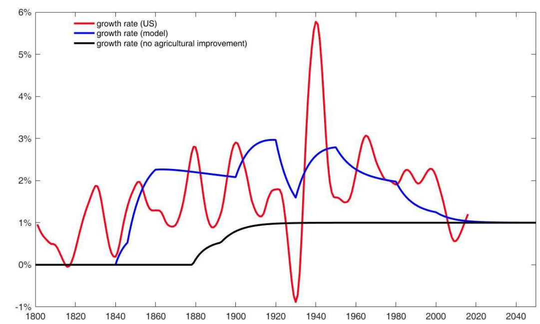

Figure 3: Economic growth

5 Conclusion

In this study, we have developed a Schumpeterian growth model with an agricultural sector in which the size of firms in the industrial sector determines the endogenous takeoff of the economy. The primary goal of the exercise is to shed new light on the important role of agriculture in a dynamic process that historians describe narratively as follows (e.g., Nurkse 1953): at the heart of industrialization, large improvements in agricultural productivity liberate labor from food production and reallocate it to industrial production. The secondary goal is to shed new light on the role of agriculture in explaining why countries with large populations, such as China and India, did not experience an early industrial takeoff. Our explanation is that the vast majority of their population being in agriculture did not contribute to firm size in the industrial sector. The model delivers analytical insights on the mechanism through which how an agricultural revolution determines the timing of the endogenous takeoff. A sectoral reallocation that expands firm size in the industrial sector produces an earlier transition from stagnation to growth. Our quantitative analysis indicates that the decline in the agricultural share of the US workforce in the early 19th century contributed to the takeoff of the US economy. Without the reallocation of labor from agriculture to industry, the takeoff of the US economy would have been delayed by four decades.

References

[1] Acemoglu, D., and Robinson, J., 2012. *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*. New York: Crown Business.

[2] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica*, 60, 323-351.

[3] Ang, J., and Madsen, J., 2011. Can second-generation endogenous growth models explain the productivity trends and knowledge production in the Asian miracle economies?. *Review of Economics and Statistics*, 93, 1360-1373.

[4] Ashraf, Q., and Galor, O., 2011. Dynamics and stagnation in the Malthusian epoch. *American Economic Review*, 101, 2003-2041.

[5] Baten, J., 2016. *A History of the Global Economy: 1500 to the Present*. Cambridge University Press.

[6] Cohen, W., and Klepper, S., 1996a. A reprise of size and R&D. *Economic Journal*, 106, 925-951.

[7] Cohen, W., and Klepper, S., 1996b. Firm size and the nature of innovation within industries: The case of process and product R&D. *Review of Economics and Statistics*, 78, 232-243.

[8] Galor, O., 2005. From stagnation to growth: Unified growth theory. *Handbook of Economic Growth*, 1, 171-293.

[9] Galor, O., 2011. *Unified Growth Theory*. Princeton University Press.

[10] Galor, O., and Moav, O., 2002. Natural selection and the origin of economic growth. *Quarterly Journal of Economics*, 117, 1133-1192.

[11] Galor, O., Moav, O., and Vollrath, D., 2009. Inequality in landownership, the emergence of human-capital promoting institutions, and the great divergence. *Review of Economic Studies*, 76, 143-179.

[12] Galor, O., and Mountford, A., 2008. Trading population for productivity: Theory and evidence. *Review of Economic Studies*, 75, 1143-1179.

[13] Galor, O., and Weil, D., 2000. Population, technology and growth: From the Malthusian regime to the demographic transition. *American Economic Review*, 110, 806-828.

[14] Garcia-Macia, D., Hsieh, C., and Klenow, P., 2019. How destructive is innovation?. *Econometrica*, 87, 1507-1541.

[15] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies*, 58, 43-61.

[16] Ha, J., and Howitt, P., 2007. Accounting for trends in productivity and R&D: A Schumpeterian critique of semi-endogenous growth theory. *Journal of Money, Credit, and Banking*, 33, 733-774.

[17] Herrendorf, B., Rogerson, R., and Valentinyi, A., 2014. Growth and structural transformation. *Handbook of Economic Growth*, 2, 855-941.

[18] Herrendorf, B., Rogerson, R., and Valentinyi, A., 2020. Structural change in investment and consumption: A unified analysis. *Review of Economic Studies*, forthcoming.

[19] Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy*, 107, 715-730.

[20] Iacopetta, M., Minetti, R., and Peretto, P., 2019. Financial markets, industry dynamics and growth. *Economic Journal*, 129, 2192-2215.

[21] Kongsamut, P., Rebelo, S., and Xie, D., 2001. Beyond balanced growth. *Review of Economic Studies*, 68, 869-882.

[22] Lagakos, D., and Waugh, M., 2013. Selection, agriculture, and cross-country productivity differences. *American Economic Review*, 103, 948-80.

[23] Laincz, C., and Peretto, P., 2006. Scale effects in endogenous growth theory: An error of aggregation not specification. *Journal of Economic Growth*, 11, 263-288.

[24] Laitner, J., 2000. Structural change and economic growth. *Review of Economic Studies*, 67, 545-561.

[25] Lebergott, S., 1966. Labor force and employment, 1800-1960. *NBER Book Series Studies in Income and Wealth*.

[26] Madsen, J., 2008. Semi-endogenous versus Schumpeterian growth models: Testing the knowledge production function using international data. *Journal of Economic Growth*, 13, 1-26.

[27] Madsen, J., 2010. The anatomy of growth in the OECD since 1870. *Journal of Monetary Economics*, 57, 753-767.

[28] Matsuyama, K., 1992. Agricultural productivity, comparative advantage, and economic growth. *Journal of Economic Theory*, 58, 317-334.

[29] Murphy, K., Shleifer, A., and Vishny, R., 1989. Income distribution, market size and industrialization. *Quarterly Journal of Economics*, 104, 537-564.

[30] Nurkse, R., 1953. *Problems of Capital Formation in Underdeveloped Countries*. New York: Oxford University Press.

[31] Peretto, P., 1994. *Essays on Market Structure and Economic Growth*. Ph.D. dissertation, Yale University.

[32] Peretto, P., 1998. Technological change and population growth. *Journal of Economic Growth*, 3, 283-311.

[33] Peretto, P., 1999. Cost reduction, entry, and the interdependence of market structure and economic growth. *Journal of Monetary Economics*, 43, 173-195.

[34] Peretto, P., 2015. From Smith to Schumpeter: A theory of take-off and convergence to sustained growth. *European Economic Review*, 78, 1-26.

[35] Pomeranz, K., 2001. *The Great Divergence: China, Europe, and the Making of the Modern World Economy*. Princeton, NJ: Princeton University Press.

[36] Romer, P., 1990. Endogenous technological change. *Journal of Political Economy*, 98, S71-S102.

[37] Segerstrom, P., Anant, T., and Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review*, 80, 1077-91.

[38] Smulders, S., 1994. Growth, Market Structure and the Environment: Essays on the Theory of Endogenous Economic Growth. Ph.D. dissertation, Tilburg University.

[39] Smulders, S. and van de Klundert, T., 1995. Imperfect competition, concentration and growth with firm-specific R&D. *European Economic Review*, 39, 139-160.

[40] United Nations, 2011. *Promoting Innovation in the Services Sector: Review of Experiences and Policies*. United Nations Economic Commission for Europe.

[41] Vollrath, D., 2011. The agricultural basis of comparative development. *Journal of Economic Growth*, 16, 343-370.

[42] Weiss, T., 1986. Revised estimates of the United States workforce, 1800-1860. *NBER Book Series Studies in Income and Wealth*.

Appendix A

Equilibrium. The equilibrium is a time path of allocations $\{a_t, q_t, c_t, Y_t, X_t, I_t, L_{y,t}, L_{q,t}\}$ and prices $\{r_t, w_t, p_t, P_t, V_t\}$ such that:

- the household consumes $\{q_t, c_t\}$ to maximize utility taking $\{r_t, w_t, p_t\}$ as given;
- competitive firms produce Q_t to maximize profits taking $\{w_t, p_t\}$ as given;
- competitive firms produce Y_t to maximize profits taking $\{w_t, P_t\}$ as given;
- monopolistic intermediate-good firms choose $\{P_t, I_t\}$ to maximize V_t taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the aggregate value of monopolistic firms equals the household's wealth, $a_t L_t = N_t V_t$;
- the labor market clears, $L_{q,t} + L_{y,t} = L_t$;
- the market for the agricultural good clears, $q_t L_t = A L_{q,t}$;
- the market for the industrial good clears, $Y_t = c_t L_t + N_t \left(X_t + \phi Z_t + I_t\right) + \dot{N}_t \delta X_t$.

Dynamic optimization of monopolistic firms. The current-value Hamiltonian for monopolistic firm i is

$$H_t(i) = \Pi_t(i) - I_t(i) + \zeta_t(i)\,\dot{Z}_t(i) + \xi_t(i)\left[\mu - P_t(i)\right], \tag{A1}$$

where $\xi_t(i)$ is the multiplier on $P_t(i) \leq \mu$. We substitute (9)-(11) into (A1) and derive

$$\frac{\partial H_t(i)}{\partial P_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial P_t(i)} = \xi_t(i), \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial I_t(i)} = 0 \Rightarrow \zeta_t(i) = 1, \tag{A3}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha \left\{ \left[P_t(i) - 1\right] \left[\frac{\theta}{P_t(i)}\right]^{1/(1-\theta)} \frac{L_{y,t}}{N_t^{1-\sigma}} - \phi \right\} Z_t^{\alpha-1}(i)\, Z_t^{1-\alpha} = r_t \zeta_t(i) - \dot{\zeta}_t(i). \tag{A4}$$

If $P_t(i) < \mu$, then $\xi_t(i) = 0$. In this case, $\partial \Pi_t(i) / \partial P_t(i) = 0$ yields $P_t(i) = 1/\theta$. If the constraint on $P_t(i)$ is binding, then $\xi_t(i) > 0$. In this case, we have $P_t(i) = \mu$. Therefore, we have proven (13). Then, the assumption $\mu < 1/\theta$ implies $P_t(i) = \mu$. Substituting (A3), (14) and $P_t(i) = \mu$ into (A4) and imposing symmetry yield (15), where $l_{y,t} \equiv L_{y,t}/L_t$. ■

Monopolistic profit in the pre-industrial era. In the pre-industrial era, the firm size $x_t l_{y,t}$ is so small that monopolistic firms with increasing returns technology cannot earn a positive profit; i.e.,

$$x_t l_{y,t} < \phi \mu^{1/(1-\theta)}/(\mu - 1) \Leftrightarrow \Pi_t < 0,$$

where l_y is given in (28). In this case, the existing intermediate goods N_0 are produced by competitive firms that make zero profit. When $x_t l_y$ reaches $\phi\mu^{1/(1-\theta)}/(\mu-1)$, we assume that the increasing returns technology is not yet deployed until x_t reaches x_N; see Appendix B for the case without this assumption. ■

Dynamics of the consumption-output ratio in the industrial era. The value of assets owned by each member of the household is

$$a_t = V_t N_t / L_t. \tag{A5}$$

If $n_t > 0$, then $V_t = \delta X_t$ in (17) holds. Substituting (17) and $\mu X_t N_t = \theta Y_t$ into (A5) yields

$$a_t = \delta X_t N_t / L_t = (\theta/\mu)\,\delta Y_t / L_t = (\theta/\mu)\,\delta y_t, \tag{A6}$$

which implies that a_t/y_t is constant. Substituting (A6), (3) and (8) into (2) yields

$$\begin{aligned}\frac{\dot{y}_t}{y_t} = \frac{\dot{a}_t}{a_t} &= r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - c_t - p_t q_t}{a_t} \\ &= \frac{\dot{c}_t}{c_t} + \rho - \lambda + \frac{(1-\theta)\,\mu}{\delta\theta} - \frac{\mu}{\delta\theta}\frac{c_t}{y_t},\end{aligned} \tag{A7}$$

where we have also used $w_t L_{q,t} = p_t Q_t$. Equation (A7) can be rearranged as

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = \frac{\mu}{\delta\theta}\frac{c_t}{y_t} - \frac{(1-\theta)\,\mu}{\delta\theta} - (\rho - \lambda)\,, \tag{A8}$$

which shows that the dynamics of c_t/y_t is characterized by saddle-point stability such that c_t/y_t jumps to its steady-state value in (30) whenever $n_t > 0$. Then, substituting (30) into (22) yields l_y^* in (31). ■

Proof of Proposition 1. In the pre-industrial era, the firm size $x_t l_y$ is not sufficiently large for horizontal and vertical innovation to be viable such that the variety growth rate and the quality growth rate are both zero (i.e., $n_t = z_t = 0$). In this case, the industrial labor share l_y is given by (28) and the state variable $x_t = \theta^{1/(1-\theta)} L_t / N_0^{1-\sigma}$ increases at the population growth rate λ. Therefore, in the pre-industrial era, the dynamics of x_t is simply

$$\dot{x}_t = \lambda x_t > 0. \tag{A9}$$

In the first phase of the industrial era, the firm size $x_t l_y^*$ becomes sufficiently large for horizontal innovation (but not vertical innovation) to be viable such that $n_t > 0$ and $z_t = 0$. In this case, the variety growth rate n_t is given by (32), which is positive if and only if

$$x_t > \frac{\mu^{1/(1-\theta)}\phi/l_y^*}{\mu - 1 - \delta(\rho - \lambda)} \equiv x_N > x_0, \tag{A10}$$

where l_y^* is given by (31) and increasing in A. Given x_0, the state variable x_t increases at the rate λ until it reaches x_N; therefore, the time this process takes is

$$T_N = \frac{1}{\lambda}\log\left(\frac{x_N}{x_0}\right).$$

After reaching x_N, the dynamics of x_t in (23) becomes

$$\dot{x}_t = [\lambda - (1-\sigma)n_t]\, x_t = \frac{1-\sigma}{\delta}\left\{\frac{\phi\mu^{1/(1-\theta)}}{l_y^*} - \left[\mu - 1 - \delta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right] x_t\right\} > 0, \quad \text{(A11)}$$

which uses (32) for n_t.

In the second phase of the industrial era, the firm size $x_t l_y^*$ becomes sufficiently large for both horizontal and vertical innovation to be viable such that $n_t > 0$ and $z_t > 0$. In this case, the quality growth rate z_t is positive if and only if

$$x_t > \frac{\Omega}{l_y^*} \equiv x_Z > x_N, \quad \text{(A12)}$$

where l_y^* is given by (31) and the composite parameter Ω is defined as before:

$$\Omega \equiv \underset{\omega}{\arg\text{solve}}\left\{\left[\frac{\mu-1}{\mu^{1/(1-\theta)}}\omega - \phi\right]\left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\delta\omega}\right] = \rho - \sigma(\rho - \lambda)\right\}.$$

In this regime, the equilibrium growth rate in (36) is derived from $g_t = r_t^q - \rho$, where r_t^q is given in (15). Then, we use (36), (37) and $z_t = g_t - \sigma n_t$ to derive n_t and the linearized dynamics of x_t as

$$\dot{x}_t = \frac{1-\sigma}{\delta}\left\{\left[(1-\alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right]\frac{\mu^{1/(1-\theta)}}{l_y^*} - \left[(1-\alpha)(\mu-1) - \delta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right] x_t\right\} \geq 0, \quad \text{(A13)}$$

where we have used $\sigma\mu^{1/(1-\theta)}/\left(x_t l_y^*\right) \cong 0$. Then, we can use n_t to derive $z_t = g_t - \sigma n_t$.

Given (24), the autonomous dynamics of x_t is stable and captured by (A9), (A11) and (A13). Given an initial value x_0, the state variable x_t increases according to (A9) until x_t reaches the first threshold x_N, which is decreasing in A via l_y^*. Then, x_t increases according to (A11) until x_t reaches the second threshold x_Z, which is also decreasing in A via l_y^*. Finally, x_t increases according to (A13) until x_t converges to its steady state

$$x^* = \frac{\mu^{1/(1-\theta)}}{l_y^*}\frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(\mu-1) - \delta[\rho + \sigma\lambda/(1-\sigma)]}, \quad \text{(A14)}$$

where l_y^* is given in (31). ■

Appendix B

In this appendix, we extend the baseline model to allow for the possibility that in the pre-industrial era (i.e., $n_t = z_t = 0$), monopolistic profits become positive (i.e., $\Pi_t > 0$) before the takeoff occurs. When $n_t = 0$, the entry condition in (17) does not hold. However, the asset-pricing equation in (16) still holds and becomes

$$r_t = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t}, \tag{B1}$$

where $I_t = z_t = 0$. We use (A5) and $n_t = 0$ to derive $\dot{a}_t/a_t = \dot{V}_t/V_t - \lambda$ and then substitute this equation into (2) to obtain

$$\frac{\dot{V}_t}{V_t} - \lambda = \frac{\dot{a}_t}{a_t} = r_t - \lambda + \frac{w_t l_{y,t} + w_t l_{q,t} - p_t q_t - c_t}{a_t}. \tag{B2}$$

Substituting (B1) into (B2) yields

$$c_t = \frac{\Pi_t}{V_t} a_t + w_t l_{y,t} = \frac{N_t}{L_t} \Pi_t + (1-\theta)\, y_t, \tag{B3}$$

where we have used (A5), $w_t l_{q,t} = p_t q_t$ and $w_t l_{y,t} = (1-\theta)\, y_t$. Then, substituting (11) and $P_t = \mu$ into (B3) yields

$$c_t = \frac{N_t X_t \left(\mu - 1 - \phi Z_t/X_t\right)}{L_t} + (1-\theta)\, y_t = \theta \mu^{\theta/(1-\theta)} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t l_{y,t}}\right) y_t + (1-\theta)\, y_t, \tag{B4}$$

where the second equality uses $\theta Y_t = \mu N_t X_t$ and (14). The consumption-output ratio is

$$\frac{c_t}{y_t} = \theta \mu^{\theta/(1-\theta)} \left(\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t l_{y,t}}\right) + 1 - \theta, \tag{B5}$$

which would increase from (27) to (30) if the firm size $x_t l_{y,t}$ increases from $\phi\mu^{1/(1-\theta)}/(\mu - 1)$ to $\phi\mu^{1/(1-\theta)}/[\mu - 1 - \delta(\rho - \lambda)]$. Finally, we substitute (B5) into (22) and manipulate the equation to obtain the equilibrium firm size:

$$x_t l_{y,t} = \frac{\frac{\beta\theta\phi}{1-\theta}\mu^{\theta/(1-\theta)} + \left(1 - \frac{\eta}{A}\right) x_t}{1 + \beta\left(1 + \frac{\theta}{1-\theta}\frac{\mu - 1}{\mu}\right)}, \tag{B6}$$

which continues to be increasing in the level of agricultural technology A.

Given that the dynamics of x_t is still given by (A9) in the pre-industrial era, the firm size $x_t l_{y,t}$ gradually increases towards the threshold in (A10) to trigger the takeoff as before. The only difference is that as x_t increases overtime, $l_{y,t}$ in (B6) is gradually decreasing from l_y in (28) to l_y^* in (31) (instead of jumping from l_y to l_y^* at the time of the takeoff). This additional dynamics in $l_{y,t}$ gives rise to negative growth in the industrial output per capita before the takeoff, which is less realistic than the dynamics in the baseline model.