Optimal Capital Taxation in an Economy with Innovation-Driven Growth

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Abstract

This paper investigates optimal capital taxation in an innovation-driven growth model. We examine how the optimal capital tax rate varies with externalities associated with R&D and innovation. Our results show that the optimal capital tax rate is higher when (i) the "stepping on toes effect" is smaller, (ii) the "standing on shoulders effect" is stronger, or (iii) the extent of creative destruction is greater. Moreover, the optimal capital tax rate and the monopolistic markup exhibit an inverted-U relationship. By calibrating our model to the US economy, we find that the optimal capital tax rate is positive, at a rate of around 11.9 percent. We also find that a positive optimal capital tax rate is more likely to be the case when there is underinvestment in R&D.

Keywords: Optimal capital taxation, R&D externalities, innovation

JEL classification: E62, H21, O31

1 Introduction

Capital income is taxed worldwide. The estimated effective average tax rates on capital income are around 40% in the United States and 30% in EU countries. In some countries, such as the United Kingdom and Japan, the capital income tax rates are even up to nearly 60%. From the perspective of welfare maximization, whether these capital tax rates are too high or too low is an important policy question.

Despite the fact that capital taxes are commonly levied in the real world, a striking theory put forth by Judd (1985) and Chamley (1986) suggests that the government should only tax labor income and leave capital income untaxed in the long run. A number of subsequent studies, including Chari et al. (1994), Jones et al. (1997), Atkeson et al. (1999), and Chari and Kehoe (1999), relax key assumptions in Judd (1985) and Chamley (1986), and find their result to be quite robust. The idea of a zero optimal capital tax has then been dubbed the Chamley-Judd result, which turns out to be one of the most well-established and important results in the optimal taxation literature.[1]

In this paper, we revisit the Chamley-Judd result in an innovation-driven growth model. There are several reasons as to why we choose this environment to study optimal taxation. First, as stressed by Aghion et al. (2013), it appears that the consideration of growth does not play much of a role in the debate on the Chamley-Judd result. However, given that the recent empirical evidence suggests that the tax structure has a significant impact on economic growth (e.g., Arnold et al., 2011), it is more plausible to bring the role of growth into the picture. Second, along the line of the optimal taxation literature, production technology is treated as exogenously given. The role of endogenous technological change driven by R&D has thus been neglected in previous models. In view of the fact that innovation is a crucial factor in economic development as well as in the improvement of human well-being, overlooking this element could lead to a suboptimal design of tax policies. Our study thus aims to fill this gap. Third, as pointed out by Domeij (2005), a key premise in early contributions supporting the Chamley-Judd result is that there exist no inherent distortions and externalities in the economy. If market failures are present, the optimal capital income tax might be different from zero. Thus, we introduce an innovation market that featues various R&D externalities put forth by Jones and Williams (2000). Within this framework, we can study how the optimal capital taxation and R&D externalities interact in ways not

[1] More recently, Chari et al. (2020) further support the result that capital should not be taxed by extending the model to include richer tax instruments that the government can access.

so far understood.

By calibrating the model to the US economy, our numerical analysis shows that the optimal capital income tax rate is around 12 percent. The reason for a positive optimal capital income tax in our R&D-based growth model can be briefly explained as follows. In essence, the Chamley-Judd result involves a tax shift between capital income tax and labor income tax. The basic rationale behind a zero optimal capital tax is that taxing capital generates more distortion than taxing labor, because taxing capital creates a dynamic inefficiency for capital accumulation. In our R&D-based growth model, by contrast, innovation requires R&D labor, as typically specified in standard R&D-based growth models (e.g., Romer, 1990; Jones, 1995; Acemoglu, 1998).[2] Under such a framework, taxing labor has a detrimental effect on the incentives for innovation and growth. This introduces a justification for taxing capital income instead of labor income. On these grounds, it might be optimal to have a non-zero capital income tax rate.

Although the result of a positive capital income tax rate is not new in the literature, our study provides insights by examining with what features of the innovation process would the optimal capital tax rate be positive. By varying the parameters capturing important R&D externalities to see how the optimal capital income tax responds, our analysis reveals the following findings. First, under the benchmark parameters, the optimal capital tax rate is positive, but this result can be sensitive to the parameter that determines the monopolistic markup. Second, when knowledge spillovers are large or R&D duplication externalities are small (thereby increasing the chances of underinvestment in R&D), it is more likely that a positive optimal capital income tax rate will result. Third, when creative destruction is more important in the R&D process, the optimal capital income tax rate should be higher (lower) if the monopolistic markup is constrained (unconstrained) by the degree of creative destruction. Fourth, a higher government spending ratio pushes toward a positive optimal capital income tax.

Another contribution of this paper is that we identify the role of the monopolistic markup played in determining optimal capital taxation. Our numerical analysis shows that the optimal capital income tax and the markup display an inverted-U shaped relationship. In

[2] There are two specifications regarding the innovation process in typical R&D-based growth models: the knowledge-driven specification (i.e., R&D using labor/scientists as inputs) and the lab-equipment specification (i.e., R&D using final goods as inputs). Our analysis adopts the former approach by following the viewpoint of Romer (1990) and Jones (1995) and also the empirical viewpoint of Einiö (2014) who points out that R&D is a labor-intensive activity. If we instead adopt the lab-equipment specification, the numerical values of the optimal capital tax rate would be different. However, the nature of the relationships between R&D externalities and the optimal capital tax, which is our central goal in this paper, will not change.

existing studies, a well-known result is that when the intermediate firms are imperfectly competitive, capital investment is too low compared to the socially optimal level (e.g., Aiyagari, 1995; Judd, 1997, 2002; Coto-Martínez et al., 2007). Accordingly, the government should subsidize capital income to induce a higher level of capital investment, implying that the optimal capital income tax tends to decrease when the monopolistic markup increases. In addition to capturing this traditional effect, our present R&D-based growth model also discloses another effect. In our model, the markup is inversely determined by the elasticity of substitution between intermediate goods. A reduction in the substitution elasticity that raises the markup amplifies the productivity of differentiated varieties in the production of final goods and hence increases the social value of R&D. As a result of this, the government is inclined to subsidize labor by taxing capital given that the R&D sector uses labor. In consideration of this R&D effect, an increase in the monopolistic markup is not necessarily accompanied by a lower optimal capital income tax.

There is a vast literature that attempts to overturn the Chamley-Judd result.[3] Two papers studying the optimal factor tax within the framework of an innovation-based endogenous growth model are closely related to our present paper. Aghion et al. (2013) is the first attempt that introduces R&D-based growth into the debate of the Chamley-Judd result. They find that a positive optimal capital income tax can be the case when the government-spending-to-output ratio exceeds 38%, which is much larger than the empirical value. In our analysis, by contrast, the optimal capital income tax is positive even if the government spending ratio is quite small (around 14%). Long and Pelloni (2017) also find a sizable positive optimal capital tax by using a standard expanding-variety R&D model a la Romer (1990). In contrast to previous literature on the Chamley-Judd result, in Long and Pelloni (2017) the role of physical capital is dismissed and the capital tax is imposed on the return of financial assets related to R&D investment.

In sum, our paper contributes to the above studies in the following ways. First, by following the specification of Jones and Williams (2000), our model is free of the "scale effect" which is often not observed in reality (Jones, 1995).[4] Second, our model features R&D

[3]The majority of this literature obtains a positive optimal capital income tax; see, e.g., Chamley (2001), Erosa and Gervais (2002), Cozzi (2004), Domeij (2005), Golosov et al. (2006), Conesa et al. (2009), Aghion et al. (2013), Chen and Lu (2013), Piketty and Saez (2013), Long and Pelloni (2017), and Straub and Werning (2020). Alternatively, a few studies overturn the Chamley-Judd result by proposing a negative optimal capital tax; see, e.g., Judd (1997, 2002), Coto-Martínez et al. (2007), and Petrucci (2015).

[4]The earlier R&D-based growth models (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) have a feature that changes in the size of an economy's population affect the long-run growth rate. Jones (1995) argues that such a "scale effect" is not supported by empirical evidence.

externalities in various dimensions, which fits our goal to examine the relation between the optimal capital income tax and the innovation process. In particular, our analysis examines how the optimal capital tax rate changes with the monopolistic markup. These are important issues that are not fully investigated in previous studies.

Our paper is also related to a group of studies that examine the effects of factor taxes in R&D-based growth models. Zeng and Zhang (2002) examine the long-run growth effects of various taxes including the capital, labor, and consumption tax. Scrimgeour (2015) examines the effects of reforming taxes on government revenues and welfare. Iwaisako (2016) explores the effects of patent protection on optimal corporate income and consumption taxes. These papers, however, do not focus on the normative analysis of optimal capital taxation.

The rest of the paper proceeds as follows. In Section 2 we describe the R&D-based growth model featuring creative destruction and various types of R&D externalities elucidated by Jones and Williams (2000). In Section 3 we analyze how capital tax changes affect the economy in the long run. In Section 4 we quantify the optimal capital income tax rate and examine how its value depends on various R&D externalities. Section 5 concludes.

2 The model

Our framework builds on the scale-invariant R&D-based growth model in Jones and Williams (2000). The main novelty of the Jones-Williams model is that it introduces a variety of R&D externalities into the original variety-expanding R&D-based growth model in Romer (1990). In this paper, we extend their model by incorporating (i) an elastic labor supply and (ii) factor income taxes, namely, capital and labor income taxes. To conserve space, the familiar components of the Romer variety-expanding model will be briefly described, while new features will be described in more detail.

2.1 Households

We consider a continuous-time economy that is inhabited by a representative household. At time t, the population size of the household is N_t, which grows at an exogenous rate n. Each member of the household is endowed with one unit of time that can be used to supply labor to a competitive market or enjoy leisure. The lifetime utility function of the representative household is given as:[5]

$$U=\int_0^\infty e^{-\beta t}\left[\ln c_t+\chi\ln(1-l_t)\right]\,dt,\quad \beta>0,\quad \chi\geq 0, \tag{1}$$

where c_t is per capita consumption and l_t is the supply of labor per capita. The parameters β and χ denote, respectively, the subjective rate of time preference and leisure preference. The representative household maximizes (1) subject to the following budget constraint:

$$\dot{k}_t+\dot{e}_t=[(1-\tau_K)r_{K,t}-n-\delta]k_t+(r_{e,t}-n)\,e_t+(1-\tau_{L,t})w_tl_t-c_t, \tag{2}$$

where a dot hereafter denotes the derivative with respect to time, k_t is physical capital per capita, δ is the physical capital depreciation rate, e_t is the value of equity shares of R&D owned by each member, $r_{K,t}$ is the capital rental rate, $r_{e,t}$ is the rate of dividend, and w_t is the wage rate. The policy parameters τ_K and $\tau_{L,t}$ are respectively the capital and labor income tax rate.[6]

Solving the dynamic optimization problem yields the following first-order conditions:

$$\frac{1}{c_t}=q_t, \tag{3}$$

$$(1-\tau_{L,t})w_t(1-l_t)=\chi c_t, \tag{4}$$

$$r_{e,t}=(1-\tau_K)r_{K,t}-\delta. \tag{5}$$

where q_t is the Hamiltonian co-state variable on eq. (2). Equations (3) and (4) are respectively the optimality conditions for consumption and labor supply, and eq. (5) is a no-arbitrage condition which states that the net returns on physical capital and equity shares must be equalized. We denote the common net return on both assets as r_t (i.e., $r_t=r_{e,t}=(1-\tau_K)r_{K,t}-\delta$). The typical Keynes-Ramsey rule is:

$$\frac{\dot{c}_t}{c_t}=r_t-n-\beta. \tag{6}$$

[5] Here we assume that household welfare depends on per capita utility. See, e.g., Chu and Cozzi (2014) for a similar specification.

[6] We drop the subscript t for τ_K because it is treated as an exogenous policy parameter throughout the paper.

2.2 The final-goods sector

A perfectly-competitive final-good sector produces a single final output Y_t (treated as the numéraire) by using labor and a continuum of intermediate capital goods, according to the CES technology:

$$Y_t = L_{Y,t}^{1-\alpha}\left(\sum_{i=1}^{A_t} x_t^{\alpha\rho}(i)\right)^{\frac{1}{\rho}}, 1 > \alpha > 0,\ 1/\alpha > \rho > 0, \tag{7}$$

where $L_{Y,t}$ is the labor input employed in final goods production, $x_t(i)$ is the i-th intermediate capital good, and A_t is the number of varieties of the intermediate goods.

Profit maximization yields the following conditional demand functions for the labor input and intermediate goods:

$$w_t = (1-\alpha)\frac{Y_t}{L_{Y,t}}, \tag{8}$$

$$p_t(i) = \alpha L_{Y,t}^{1-\alpha}\left(\sum_{i=1}^{A_t} x_t^{\alpha\rho}(i)\right)^{\frac{1}{\rho}-1} x_t^{\alpha\rho-1}(i), \tag{9}$$

where $p_t(i)$ is the price of the i-th intermediate good.

2.3 The intermediate-goods sector

Each intermediate good is produced by a monopolistic producer that owns a perpetually protected patent for that good. The producer uses one unit of physical capital to produce one unit of intermediate goods; that is, the production function is $x_t(i) = v_t(i)$, where $v_t(i)$ denotes the capital input employed by monopolistic intermediate firm i. Accordingly, the profit of intermediate goods firm i is:

$$\pi_{x,t}(i) = p_t(i)x_t(i) - r_{K,t}v_t(i). \tag{10}$$

Let $\eta_t(i)$ denote the gross markup that the i-th intermediate firm can charge over its marginal cost; that is:

$$p_t(i) = \eta_t(i)r_{K,t}. \tag{11}$$

Then, the profit of the i-th intermediate firm can be obtained as:

$$\pi_{x,t}(i) = \frac{\eta_t(i) - 1}{\eta_t(i)} \alpha \frac{Y_t}{A_t}. \tag{12}$$

In subsection 2.5, we will elucidate how $\eta_t(i)$ is determined.

2.4 The R&D sector

R&D creates new varieties of intermediate goods for final-good production. The production technology we adopt incorporates the knowledge-driven specification of Romer (1990) and Jones (1995), i.e., innovation using the labor input (scientists and engineers), with the Jones and Williams (2000)'s specification which features fruitful R&D externalities:

$$(1 + \psi)\dot{A}_t = \tilde{\varsigma}_t L_{A,t}, \quad \psi \geq 0, \tag{13}$$

where $L_{A,t}$ is the labor input used in the R&D sector, and $\tilde{\varsigma}_t$ is the productivity of R&D which the innovators take as given. The parameter ψ represents the size of the innovation clusters.[7]

We follow Jones (1995) to specify that the productivity of R&D takes the following functional form:

$$\tilde{\varsigma}_t = \varsigma L_{A,t}^{\lambda-1} A_t^{\phi}, \quad \varsigma > 0, \quad 1 \geq \lambda > 0, \quad 1 > \phi > 0, \tag{14}$$

where ς is a constant productivity parameter. In addition to ς, eqs. (13) and (14) contain three parameters λ, ϕ and ψ. These parameters capture salient features of the R&D process, as proposed by Jones and Williams (1998).

First, the parameter $1 \geq \lambda > 0$ reflects a (negative) duplication externality or a congestion effect of R&D. It implies that the social marginal product of research labor can be less than the private marginal product. This may happen because of, for example, a patent race, or if two researchers accidentally work out a similar idea. Jones and Williams (1998) refer to this negative duplication externality as the *stepping on toes effect*. Notice that this effect is stronger with a smaller λ, and it vanishes when $\lambda = 1$.

Second, the parameter $1 > \phi > 0$ reflects a (positive) knowledge spillover effect due to the fact that richer existing ideas are helpful to the development of new ideas. A higher ϕ means that the spillover effect is greater. In his pioneering article, Romer (1990) specifies

[7] In the later analysis, we will provide a more detailed explanation for this parameter.

$\phi = 1$; however, Jones (1995) argues that $\phi = 1$ exhibits a scale effect which is inconsistent with the empirical evidence. We follow Jones (1995) and assume that $\phi < 1$ in order to remove this scale effect. The knowledge spillover effect is dubbed by Jones and Williams (1998) as the *standing on shoulders effect*.

Finally, the parameter $\psi \geq 0$ denotes the size of the innovation clusters, which captures the concept of creative destruction formalized in the Schumpeterian growth model developed by Aghion and Howitt (1992). The basic idea is that innovations must come together in clusters, some of which are new, while others simply build on old fashions. More specifically, suppose that an innovation cluster, which contains $(1+\psi)$ varieties, has been invented. Out of these $(1+\psi)$ varieties, only one unit of variety is entirely new and thus increases the mass of the variety of intermediate goods. The remaining portion, of size ψ, simply replaces the old versions. This portion captures the spirit of creative destruction since new versions are created with the elimination of old versions. However this part does not contribute to an increase in existing varieties. In other words, for $(1 + \psi)$ intermediate goods invented, the actual augmented variety is 1, while there are ψ repackaged varieties.

Given $\tilde{\varsigma}_t$, the R&D sector hires $L_{A,t}$ to create $(1+\psi)$ varieties. Thus, the profit function is $\pi_{A,t} = P_{A,t}(1 + \psi)\dot{A}_t - w_t L_{A,t}$. By assuming free entry in the R&D sector, we can obtain:

$$P_{A,t} = \frac{s_t}{1 - s_t} \frac{(1-\alpha)Y_t}{(1+\psi)\dot{A}_t}, \tag{15}$$

where $s_t \equiv L_{A,t}/L_t$ is the ratio of research labor to total labor supply L_t. Moreover, the no-arbitrage condition for the value of a variety is:

$$r_t P_{A,t} = \pi_{x,t} + \dot{P}_{A,t} - \psi \frac{\dot{A}_t}{A_t} P_{A,t}. \tag{16}$$

In the absence of creative destruction ($\psi = 0$), the familiar no-arbitrage condition reports that, for each variety, the return on the equity shares $r_t P_{A,t}$ will be equal to the sum of the flow of the monopolistic profit $\pi_{x,t}$ plus the capital gain or loss $\dot{P}_{A,t}$. When creative destruction is present, existing goods are replaced. Accompanied by $\dot{A}_t$ new varieties being invented, the amount of $\psi\dot{A}_t$ existing varieties will be replaced. Therefore, for each variety, the expected probability of being replaced is $\psi\dot{A}_t/A_t$, which gives rise to the expected capital loss expressed by the last term in eq. (16).

2.5 The monopolistic markup

This subsection explains how the monopolistic markup $\eta_t(i)$ is determined. As identified by Jones and Williams (2000), there are two scenarios in which the markup is decided. The first is the "unconstrained" case. In this case, the monopolistic intermediate firm freely sets the price by maximizing eq. (10) subject to the production function $x_t(i) = v_t(i)$ and eq. (9), which yields the pricing rule $p_t(i) = \frac{1}{\rho\alpha} r_{K,t}$. We refer to $\frac{1}{\rho\alpha}$ as the "unconstrained" markup. The second case is the "constrained" case, which may occur if the new designs are linked together in the *innovation cluster*. Specifically, a larger size of innovation clusters ψ serves as a constraint that controls the magnitude of the monopolistic markup. The intuition underlying this idea requires a more detailed explanation. Consider that the current number of varieties is A_t. Now an innovation cluster with size $(1+\psi)$ is developed. This increases the mass of varieties to A_t+1; at the same time it also replaces old-version varieties by ψ units. Subsequently, the final-good firm faces two choices. It can either adopt the new innovation cluster and then use A_t+1 intermediate goods priced at a markup, or part with the new innovation cluster and still use A_t intermediate goods in the production process. If the final-good firm chooses the latter, since ψ varieties have now been displaced, the final-good firm only needs to purchase $A_t - \psi$ units of intermediate goods at a markup price, while the other ψ units of displaced intermediate goods can be purchased at a lower (competitive) price. When the size of an innovation cluster is high (a large value of ψ), the final-good firm will not tend to adopt the new innovation cluster because sticking to old clusters is cheaper. As a result, the intermediate-good firms have to set a lower price so as to attract the final-good firm to adopt the new innovation cluster. This adoption constraint explains why an increase in the size of the innovation clusters reduces the markup.

In an appendix, Jones and Williams (2000) demonstrate that the constrained markup is negatively related to both the size of the innovation clusters and the elasticity of substitution between capital goods. Specifically, they demonstrate that, in order to attract the final-good firm to adopt the new innovation cluster, the intermediate-good firms cannot set a markup that is higher than $[(1+\psi)/\psi]^{1/\rho\alpha-1}$. A profit-maximizing firm thus always tends to set the highest price $p_t(i) = [(1+\psi)/\psi]^{1/\rho\alpha-1} r_{K,t}$. We refer to $[(1+\psi)/\psi]^{1/\rho\alpha-1}$ as the "unconstrained" markup. By combining the constrained markup pricing with the unconstrained markup pricing rule mentioned earlier (i.e., $p_t(i) = \frac{1}{\rho\alpha} r_{K,t}$), we can conclude that the equilibrium markup is:

$$\eta_t(i) = \min\left\{ \frac{1}{\rho\alpha} \, , \, \left(\frac{1+\psi}{\psi}\right)^{\frac{1}{\rho\alpha}-1} \right\}, \tag{17}$$

which is independent of i and t. Combining eqs. (10) and (17) implies that all intermediate-good firms are symmetric. Hence, the notation i can be dropped from now on.

2.6 The government and aggregation

The government collects capital income taxes and labor income taxes to finance its public spending. The balanced budget constraint faced by the government is:

$$N_t(\tau_K r_{K,t} k_t + \tau_{L,t} w_t l_t) = G_t, \tag{18}$$

where G_t is the total government spending. We assume that government spending is a fixed proportion of final output, i.e., $G_t = \zeta Y_t$, where $\zeta \in (0,1)$ is the ratio of government spending to output. As in Conesa et al. (2009), Aghion et al. (2013) and Long and Pelloni (2017), equation (18) puts aside the role of government debt when examining the optimal factor taxes. That is, we mainly focus on the trade-off between the capital and labor income tax.[8]

Now let us define the aggregate capital stock as $K_t = N_t k_t$, aggregate consumption $C_t = N_t c_t$, and total labor supply $L_t = N_t l_t$. After some derivations, we can obtain the following resource constraint in the economy: $\dot{K}_t = Y_t - C_t - G_t - \delta K_t$.

2.7 The decentralized equilibrium

The decentralized equilibrium in this economy is an infinite sequence of allocations $\{C_t,\, K_t,\, A_t,\, Y_t,\, L_t,\, L_{Y,t},\, L_{A,t},\, x_t,\, v_t\}_{t=0}^{\infty}$, prices $\{w_t,\, r_{K,t},\, r_t,\, p_t,\, P_{A,t}\}_{t=0}^{\infty}$, and policies $\{\tau_K,\, \tau_{L,t}\}$, such that at each instant of time:

a. households choose $\{c_t,\, k_t,\, e_t,\, l_t\}$ to maximize lifetime utility, eq. (1), taking prices and policies as given;

b. competitive final-good firms choose $\{x_t,\, L_{Y,t}\}$ to maximize profit taking prices as given;

[8] We do not consider time-variant tax rates as done by Chamley (1985). Introducing time-variant tax rates would be interesting, but also brings substantial technical complexity. Moreover, Aghion et al. (2013) point out that time-dependent policies are not empirically relevant.

c. monopolistic intermediate firms $i \in [0, A_t]$ choose $\{v_t,\, p_t\}$ to maximize profit taking $r_{K,t}$ as given;

d. the R&D sector chooses $L_{A,t}$ to maximize profit taking $\{P_{A,t},\, w_t\}$ and the productivity $\tilde{\varsigma}_t$ as given;

e. the labor market clears, i.e., $N_t l_t = L_{A,t} + L_{Y,t}$;

f. the capital market clears, i.e., $N_t k_t = A_t v_t$;

g. the stock market for variety clears, i.e., $N_t e_t = P_{A,t} A_t$

h. the resource constraint is satisfied, i.e., $\dot{K}_t = Y_t - C_t - G_t - \delta K_t$;

i. the government budget constraint is balanced, i.e., $N_t(\tau_K r_{K,t} k_t + \tau_{L,t} w_t l_t) = G_t$.

3 Balanced growth path

In this section, we explore the balanced growth path along which each variable grows at a constant rate, which can be zero. We denote the growth rate of any generic variable Z by g_Z, and drop the time subscript to denote any variable in a steady state. The steady-state growth rates of varieties and output are given by (see Appendix A):

$$g_A = \frac{\lambda}{1-\phi} n, \quad g_Y = \frac{1}{1-\alpha}\left(\frac{1}{\rho} - \alpha\right) g_A + n. \tag{19a}$$

Moreover, in order to obtain stationary endogenous variables, it is necessary to define the following transformed variables:

$$\hat{k}_t \equiv \frac{K_t}{N_t^{\sigma}}, \quad \hat{c}_t \equiv \frac{C_t}{N_t^{\sigma}}, \quad \hat{y}_t \equiv \frac{Y_t}{N_t^{\sigma}}, \quad \hat{a}_t \equiv \frac{A_t}{N_t^{\lambda/(1-\phi)}}, \tag{19b}$$

where $\sigma \equiv 1 + \frac{(1/\rho - \alpha)\lambda}{(1-\alpha)(1-\phi)} > 0$ is a composite parameter. For ease of exposition, in line with Eicher and Turnovsky (2001), $\hat{k}$, $\hat{c}$, $\hat{y}$, and $\hat{a}$ are dubbed the scale-adjusted capital, consumption, output, and R&D varieties, respectively. Based on the transformed variables and the equilibrium defined in subsection 2.5, the economy in the steady state can be described by the following set of equations:

$$
\begin{aligned}
r &= (1-\tau_K)r_K - \delta = \beta + g_Y, && \text{(20a)}\\
s &= \frac{\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}(1+\psi)g_A}{r - g_Y + \left(1+\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}\right)(1+\psi)g_A}, && \text{(20b)}\\
\frac{\hat{k}}{\hat{y}} &= \frac{\alpha}{\eta r_K}, && \text{(20c)}\\
(1-\zeta)\frac{\hat{y}}{\hat{k}} &= \frac{\hat{c}}{\hat{k}} + g_Y + \delta, && \text{(20d)}\\
\hat{y} &= \hat{a}^{1/\rho-\alpha}\hat{k}^{\alpha}\left((1-s)l\right)^{1-\alpha}, && \text{(20e)}\\
g_A &= \frac{1}{1+\psi}\frac{\varsigma\,(sl)^{\lambda}}{\hat{a}^{1-\phi}}, && \text{(20f)}\\
\frac{\chi l}{(1-l)} &= \frac{(1-\tau_L)(1-\alpha)}{(1-s)}\frac{\hat{y}}{\hat{c}}, && \text{(20g)}\\
\tau_L &= \frac{1-s}{1-\alpha}\left(\zeta - \tau_K\frac{\alpha}{\eta}\right), && \text{(20h)}
\end{aligned}
$$

in which eight endogenous variables r, s, $\hat{c}$, $\hat{k}$, $\hat{a}$, $\hat{y}$, l, τ_L are determined.

Of particular note, our main focus is on the examination of the capital tax. By holding the proportion of the government spending constant, an increase in the capital income tax will be coupled with a reduction in the labor income tax. Therefore, the literature on the Chamley-Judd result generally assumes that the labor income tax endogenously adjusts to balance the government budget. This approach has been dubbed as "tax shifting" or "tax swap" in the literature. Our analysis follows this standard approach in the literature.

3.1 Comparative static analysis

In this section, we analyze the effects of capital taxation on the R&D share s of labor, the endogenous labor income tax rate, labor supply, and other scale-adjusted variables: $\hat{a}, \hat{k}, \hat{c}$, and $\hat{y}$.[9]

The long-run R&D labor share, s, is given by

[9] We solve the dynamic system in Appendix B, and a detailed derivation of the comparative static analysis is presented in Appendix C.

$$s = \frac{\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}(1+\psi)g_A}{r - g_Y + \left(1 + \frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}\right)(1+\psi)g_A}. \tag{21a}$$

It follows from the above equation that, in the steady state, a change in the capital income tax rate (21a) does not affect the R&D labor share (i.e., $\partial s/\partial\tau_K = 0$). The intuition underlying $\partial s/\partial\tau_K = 0$ can be grasped as follows. The non-arbitrage condition between physical capital and R&D equity reported in (20a) requires that the return on physical capital be equal to the return on R&D equity. Given that the return on R&D equity, $r = \beta + \frac{1}{1-\alpha}\left(\frac{1}{\rho} - \alpha\right) g_A + n$, is independent of the capital tax rate, the capital income tax rate does not affect the return on R&D equity and the R&D labor share. Therefore, our analysis does not rely on capital taxation having a direct effect on the allocation of R&D and production labor. Instead, our analysis is based on the trade-off between labor supply and capital investment as in the standard Chamley-Judd setting.

From (20h), we have:

$$\tau_L = \frac{1-s}{1-\alpha}\left(\zeta - \tau_K\frac{\alpha}{\eta}\right), \tag{21b}$$

Based on (21a), we have:

$$\frac{\partial\tau_L}{\partial\tau_K} = -\frac{1-s}{\eta}\frac{\alpha}{1-\alpha} < 0. \tag{21c}$$

The above equation shows that an increase in the capital income tax rate is coupled with a reduction in the labor income tax rate.

Given a constant capital income tax rate τ_K, labor supply in the steady state is given by:

$$l = 1 - \frac{\chi}{\chi + \frac{1}{[(1-\zeta)-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]}\frac{(1-\tau_L)(1-\alpha)}{(1-s)}}. \tag{22a}$$

It is straightforward from eq.(22a) to infer the following result:

$$\frac{\partial l}{\partial\tau_K} = \frac{\alpha\beta(\frac{1-s}{1-\alpha})[1-\zeta+\frac{\eta-1}{\eta}\frac{\alpha(\delta+g_Y)}{\beta+(1+\psi)g_A}](1-l)l}{\eta(\beta+\delta+g_Y)(1-\tau_L)[1-\zeta-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]} > 0. \tag{22b}$$

Equation (22b) indicates that, when taxes shift from a labor income tax to a capital income tax, a rise in the capital income tax rate leads to an increase in labor supply. The intuition underlying this result can be explained as follows. In response to a rise in the capital income tax rate, the following effect emerges. Raising the capital tax rate reduces the labor income tax rate (see eq. (21b)) and raises the after-tax wage income, thereby exerting a positive effect on labor supply. Therefore, a rise in the capital income tax rate is accompanied by an increase in labor supply.

Moreover, the scale-adjusted R&D varieties $\hat{a}$ is given by:

$$\hat{a} = \left[\frac{\varsigma}{(1+\psi)g_A} \right]^{1/(1-\phi)} (sl)^{\lambda/(1-\phi)}, \tag{23a}$$

where s and l are reported in eqs. (21a) and (22a). With $\partial s/\partial \tau_K = 0$, it is quite easy from eq. (23a) to derive that:

$$\frac{\partial \hat{a}}{\partial \tau_K} = \frac{\lambda \hat{a}}{(1-\phi)l} \frac{\partial l}{\partial \tau_K} > 0. \tag{23b}$$

Equation (23b) indicates that a rise in the capital income tax rate boosts scale-adjusted R&D varieties. The intuition is clear. Following a rise in the capital income tax rate that is coupled with a decline in the labor income tax rate, the household is motivated to increase its labor supply. This in turn increases labor input allocated to the R&D sector ($L_A = Nsl$). Then, as reported in eq. (23a), given that scale-adjusted R&D varieties $\hat{a}$ is increasing in the R&D labor input sNl, $\hat{a}$ will increase in response to a rise in τ_K.

From eqs. (20a), (20c), (20d), (23a), and (20e), we can infer that:

$$\hat{y} = \left[\frac{\varsigma}{(1+\psi)g_A} \right]^{\frac{1/\rho-\alpha}{(1-\alpha)(1-\phi)}} (sl)^{\frac{1/\rho-\alpha}{1-\alpha}\frac{\lambda}{1-\phi}} \left[\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)} \right]^{\frac{\alpha}{1-\alpha}} (1-s)l, \tag{24a}$$

where

$$\frac{\partial \hat{y}}{\partial \tau_K} = \left[-\frac{\alpha}{(1-\alpha)(1-\tau_K)} + \frac{\sigma}{l}\frac{\partial l}{\partial \tau_K} \right] \hat{y} \gtrless 0. \tag{24b}$$

Equation (24b) indicates that a rise in the capital income tax rate has ambiguous effects on the scale-adjusted output $\hat{y}$. As shown in eq. (24b), two conflicting effects emerge following a rise in the capital income tax rate. First, a rise in the capital income tax rate shrinks capital investment, which in turn generates a negative impact on output. Second, a rise in the capital income tax rate is accompanied by a fall in the labor income tax rate, which motivates the household to provide more labor supply. This increase in labor supply implies that more labor input is available for the R&D sector and in turn boosts R&D varieties, thereby contributing to a positive effect on output. If labor supply is exogenous ($\chi = 0$), the second positive effect is absent ($\partial l/\partial \tau_K = 0$), and a higher capital income tax rate lowers output. However, if labor supply is endogenous ($\chi > 0$), the two opposing effects are present, and the output effect of capital income taxation depends upon the relative strength between these two effects.

From eqs. (20a), (20c), and (20d), we have:

$$\hat{k} = \frac{(1-\tau_K)\Phi}{(\delta + g_Y)}\hat{y}, \tag{25a}$$

$$\hat{c} = [(1-\zeta) - (1-\tau_K)\Phi]\hat{y}, \tag{25b}$$

where $\Phi \equiv \frac{\alpha(\delta+g_Y)}{\eta(\beta+\delta+g_Y)}$ is a composite parameter. Based on eqs. (25a) and (25b), the effects of τ_K on $\hat{k}$ and $\hat{c}$ can be expressed as:

$$\begin{aligned}\frac{\partial \hat{k}}{\partial \tau_K} &= -\frac{\Phi}{(\delta+g_Y)}\hat{y} + \frac{(1-\tau_K)\Phi}{(\delta+g_Y)}\frac{\hat{y}}{\partial \tau_K} \qquad (26a)\\ &= [\sigma\frac{\partial l}{l\partial \tau_K} - \frac{1}{(1-\alpha)(1-\tau_K)}]\frac{(1-\tau_K)\Phi}{(\delta+g_Y)}\hat{y} \gtreqless 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial \hat{c}}{\partial \tau_K} &= \Phi\hat{y} + [(1-\zeta) - (1-\tau_K)\Phi]\frac{\partial \hat{y}}{\partial \tau_K} \qquad (26b)\\ &= \{\Phi + [(1-\zeta) - (1-\tau_K)\Phi][\sigma\frac{\partial l}{l\partial \tau_K} - \frac{\alpha}{(1-\alpha)(1-\tau_K)}]\}\hat{y} \gtreqless 0.\end{aligned}$$

The intuition behind eqs. (26a) and (26b) can be explained as follows. It is clear in eq. (25a) that capital income taxation affects scale-adjusted capital $\hat{k}$ through two channels. The first channel is the capital-output ratio $\hat{k}/\hat{y} = \frac{(1-\tau_K)\Phi}{(\delta+g_Y)}$, and the second channel is the level of scale-adjusted output $\hat{y}$. The first term after the first equality in eq. (26a) indicates that the first channel definitely lowers the level of $\hat{k}$. Moreover, as shown in eq. (24b), the second channel may either raise or lower the level of $\hat{k}$ since capital taxation leads to an ambiguous effect on $\hat{y}$. As a consequence, the net effect of capital taxation on the scale-adjusted capital stock $\hat{k}$ is still uncertain. Similarly, as indicated in eq. (25b), capital income taxation also affects $\hat{c}$ through two channels. The first channel is the consumption-output ratio $\hat{c}/\hat{y} = [(1-\zeta) - (1-\tau_K)\Phi]$, and the second channel is the level of scale-adjusted output $\hat{y}$. The first channel definitely boosts the level of $\hat{c}$, while the second channel may either raise or lower the level of $\hat{c}$ since capital taxation leads to an ambiguous effect on $\hat{y}$. As a consequence, the net effect of capital taxation on scale-adjusted consumption $\hat{c}$ remains ambiguous.

4 Quantitative results

In this section, we simulate the transitional dynamic effects of capital taxation and compute the optimal capital tax rate by performing a quantitative analysis.[10] We calibrate the parameters of our theoretical model based on US data to quantify the optimal capital tax. Then we explore how the optimal capital tax responds to important parameters that feature R&D externalities and the government size.[11]

By dropping the exogenous terms, the life-time utility of the representative household reported in eq. (1) can be expressed as:

$$U = \int_0^\infty e^{-\beta t} \left[\ln \hat{c}_t + \chi \ln(1 - l_t) \right] dt, \tag{27}$$

in which $\hat{c}_t$ and l_t are functions of τ_K. The government chooses the capital income tax rate τ_K to maximize eq. (27) that includes the transitional dynamics, while balancing the budget, eq. (18), using the labor tax.[12]

4.1 Calibration

To carry out a numerical analysis, we first choose a baseline parameterization, as reported in Table 1. Our model has eleven parameter values to be assigned. These parameters are either set to a commonly used value in the existing literature or calibrated to match some empirical moments in the US economy. We now describe each of them in detail. In line with Andolfatto et al. (2008) and Chu and Cozzi (2018), the labor income share $1 - \alpha$ and the discount rate β are set to standard values 0.4 and 0.05, respectively. The population growth rate n is set to 0.011 as used by Conesa et al. (2009). The physical capital depreciation rate is set to 0.0318 so that the initial capital-output ratio is 2.5 as in Lucas (1990). The initial capital tax rate τ_K is set to 0.3 based on the average US effective tax rate estimated by Carey and Tchilingurian (2000). A similar value of the capital income tax rate has been adopted in

[10] We describe the dynamic system of the model in Appendix B.

[11] We start from the same initial steady state when we vary the value of each parameter.

[12] The numerical approach is basically consistent with Aghion et al. (2013), except that their initial capital income tax rate is set to 0%, while we consider a value of 30% to fit the US data.

Domeij (2005) and Chen and Lu (2013). As for the government size (the ratio of government spending to output), data for the US indicate that this is around 20 percent (Gali, 1994), and has slightly increased in recent years. We therefore set ζ to be 0.22, which is the average level during 2001-2013, to reflect its increasing trend. The parameter for leisure preference χ is chosen as 1.5901 to make hours worked around one third of total hours.

Table 1. Benchmark Parameterization

Definition	Parameter	Value	Source/Target
Labor income share	$1-\alpha$	0.6	Andolfatto et al. (2008)
Discount rate	β	0.05	Chu and Cozzi (2018)
Population growth rate	n	0.011	Conesa et al. (2009)
Initial capital tax rate	τ_K	0.3	Carey and Tchilingurian (2000)
Government size	ζ	0.22	Data
Leisure preference	χ	1.5901	Total hours worked = 1/3
R&D productivity	ς	1	Normalized[13]
Standing on toes effect	λ	0.5	Benchmark
Substitution parameter	ρ	2.2727	Monopolistic markup = 1.1
Standing on shoulders effect	ϕ	0.9593	Output growth rate = 2%
Size of innovation cluster	ψ	0.25	Comin (2004)
Physical capital depreciation rate	δ	0.0318	Capital-output ratio = 2.5

Our parameterization regarding the R&D process basically follows the approach in Jones and Williams (2000). First, we normalize the R&D productivity ς to unity. The value of the parameter for the standing on toes effect λ is somewhat difficult to calibrate because, as argued by Stokey (1995), the empirical literature does not provide much guidance on such a parameter. In our analysis, we thus choose an intermediate value $\lambda = 0.5$ as a benchmark, but we will allow it to vary over the whole interval from 0 to 0.564.[14] The substitution parameter ρ is closely related to the markup of the intermediate firms. We set ρ to be 2.2727 such that, given $1-\alpha$, the (unconstrained) markup in our economy is 1.1, which lies within the reasonable range estimated for US industries (e.g., Laitner and Stolyarov, 2004; Yang, 2018). Next, we use the output growth rate to calibrate the extent of the standing on

[13] Our results are independent of the value of ς.

[14] If the value of λ is over 0.564, the second-order condition of the government's maximization with respect to τ_K will not be satisfied.

shoulders effect ϕ. In our model we have:

$$g_Y = \frac{1}{1-\alpha}\left(\frac{1}{\rho} - \alpha\right) g_A + n. \tag{28}$$

Given that $g_A = \lambda n/(1-\phi)$ and that we have already assigned values to $1-\alpha$, ρ, n and λ, we can then choose ϕ to target the empirical level of the output growth rate in the US, which is around 2%. This results in $\phi = 0.9593$ as our baseline value. Finally, as a benchmark we choose the size of the innovation cluster $\psi = 0.25$ by following Comin (2004). In this case the markup is not bound by the adoption constraint. If the value of ψ is large, the markup will then be constrained and determined by this parameter. In subsection 4.3 we will run ψ from 0 to 0.515 as a robustness check.

4.2 The optimal capital tax with transitional dynamics

Under our benchmark parameterization, Figure 1 plots the relationship between the level of welfare and the rate of capital income tax, which exhibits an inverted-U shaped relationship. Of particular note, the optimal capital tax is positive under our benchmark parameters, and its value is around 11.9%. The Chamley-Judd result of zero capital tax does not hold in our R&D-based growth model.

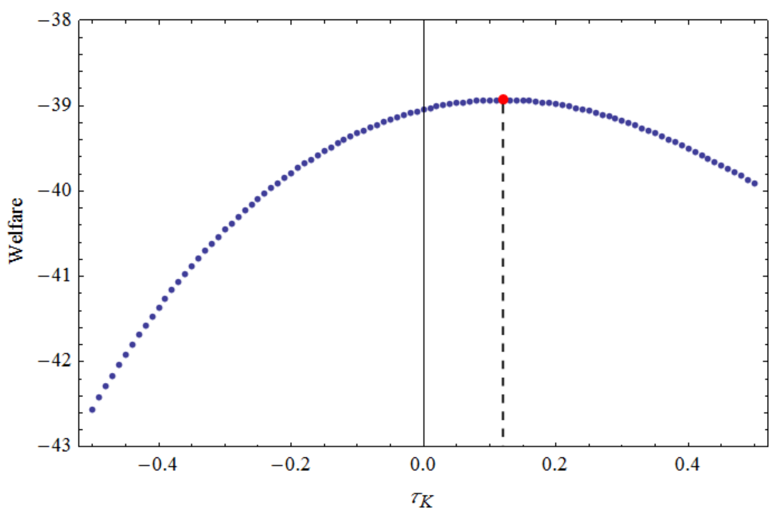

Figure 1: The level of welfare and the rate of capital income tax

The intuition underlying this result is as follows. Given that the government is limited to capital and labor taxation to finance a fix amount of the government expenditure, not taxing capital income implies that the labor income must be taxed at a higher rate. Although a zero capital tax efficiently leaves the capital market undistorted, a high labor tax distorts the labor market severely by decreasing the after-tax wage income and in turn reduces total labor supply. As a consequence, there is less labor devoted to the production in the R&D sector, which then results in fewer equilibrium varieties for the final-good production, and ultimately depresses the level of consumption and welfare.

Although Figure 1 suggests a positive optimal capital tax, we should note that this result is obtained under our benchmark parameters, and it may change when the innovation process exhibits different degrees of R&D externalities. Thus, our goal is not to conclude that it is always right to tax capital, but to highlight that in achieving the social optimum, it is necessary to balance both distortions in the capital and labor markets. In view of this, an extreme case of the zero capital tax is often suboptimal. More importantly, we make an attempt to give guidance on which R&D mechanisms are at play in influencing the optimal capital tax, which we will show in the next subsection.

4.3 Policy implications of R&D externalities

In this subsection, we investigate how the optimal capital tax responds to relevant parameters, in particular those related to the innovation process. More importantly, we shed some light on the roles of R&D externalities in the design of optimal tax policies. To this end, we provide a robustness check for whether the positive optimal capital tax still survives under various scenarios. In what follows, we propose some relevant parameters that need to be considered by the policy-makers. The results are depicted in Figures 2 to 6. Our robustness analysis generates several implications.

First, Figures 2 and 3 show that the optimal capital tax rate is increasing in λ (the stepping on toes effect) and ϕ (the standing on shoulders effect). With sufficiently small values of λ and ϕ, the optimal capital income tax is negative. Notice that a higher λ implies that the negative duplication externality is small, and a higher ϕ means that the positive spillover effect of R&D is relatively strong. Both cases imply a similar circumstance in which the innovation process is more productive, and in which underinvestment in R&D is more likely. Under such a situation, the welfare cost of depressing innovation by raising the labor income tax is larger. Therefore, the government is inclined to increase the capital tax while reducing the labor tax.

Second, Figure 4 shows that the optimal capital income tax and the substitution para-

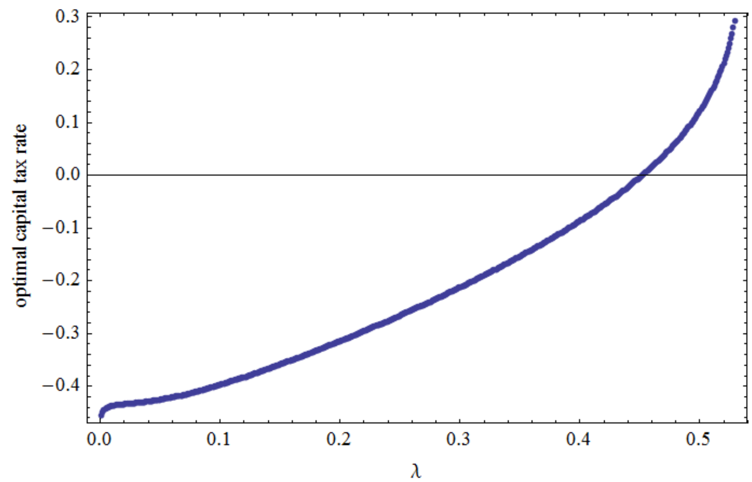

Figure 2: The optimal capital tax rate and the stepping on toes effect

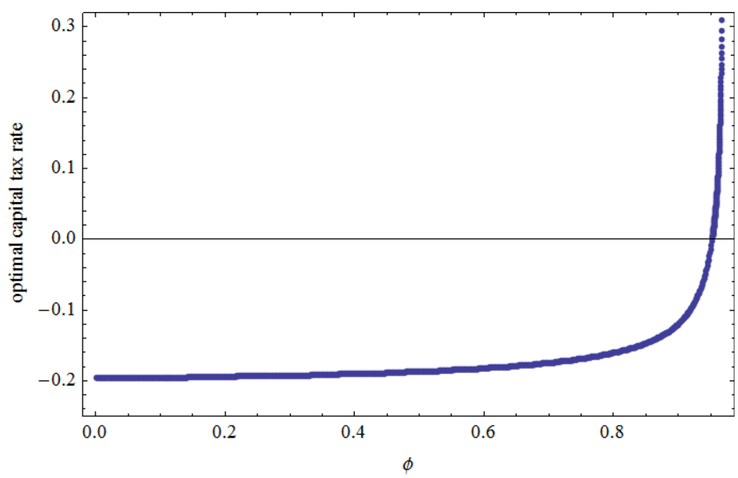

Figure 3: The optimal capital tax rate and the standing on shoulders effect

meter ρ exhibit an inverted-U shaped relationship. A lower ρ is associated with a higher monopolistic markup η, regardless of whether the adoption constraint is binding or not. The substitution parameter mainly affects the optimal capital tax in three different ways. First, when η is large (when ρ is small), the degree of the intermediate firms' monopoly power is strong. To correct this distortion, the government tends to subsidize capital to offset the gaps between price and the marginal cost; see Judd (1997, 2002). Second, when η is large (when ρ is small), the private value of inventions increases. As a result, equilibrium R&D increases, which in turn makes R&D overinvestment more likely. Therefore, the government tends to raise the tax on labor because R&D uses labor in our model. These two effects indicate that the optimal capital tax should be decreasing in the markup as in previous studies. Third, a small ρ amplifies the productivity of varieties in final-good production and thus amplifies the effect of g_A on g_Y (see eq. (28)). In this case, the government is inclined to subsidize labor by taxing capital since the R&D sector uses labor. This last effect indicates that the optimal capital tax rate is decreasing in the elasticity of substitution between intermediate goods (or increasing in the markup). Figure 4 shows that the first two effects dominate when ρ is small and the third effect dominants when ρ becomes sufficiently large. Thus the optimal capital tax reverses as ρ exceeds a threshold value.

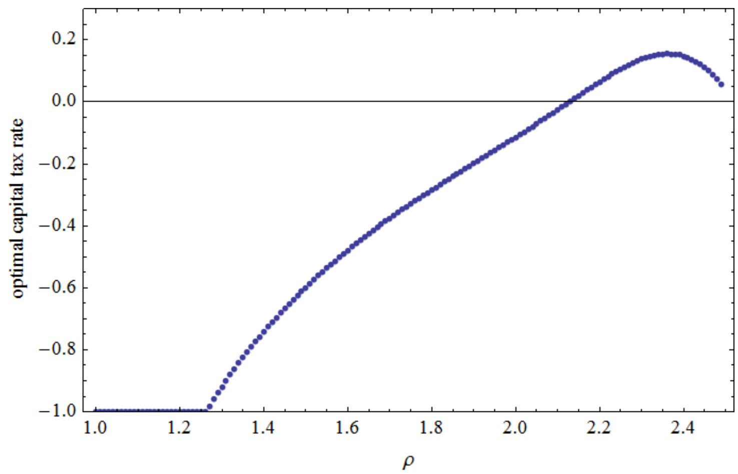

Figure 4: The optimal capital tax rate and the substitution parameter

Third, Figure 5 shows that the optimal capital tax increases in response to a rise in the size of the innovation cluster (creative destruction). To explain the intuition, we first distinguish three effects that creative destruction may have on the incentive to engage in R&D. The first positive effect comes from the R&D firm being able to earn profits even for those of its products that do not really increase the variety of intermediate goods (note that $\pi_{A,t} = P_{A,t}(1+\psi)\dot{A}_t - w_t L_{A,t}$).[15] This is referred to as the "carrot" by Jones and Williams (2000). The second negative effect arises, as exhibited in eq. (15), from a higher ψ that decreases the equilibrium price of the products in the presence of free entry, even though it increases the products sold by the R&D firm. The third negative effect is associated with the no-arbitrage condition for the value of a variety, which is displayed in eq. (16). Due to creative destruction, existing goods have a probability of being replaced by new goods, and this probability increases with the degree of creative destruction. Therefore, creative destruction increases the expected capital loss in terms of the return on the equity shares,

[15]The R&D firm can earn profits from its whole products $(1+\psi)\dot{A}$, in which $\psi\dot{A}$ does not contribute to the increase of varieties.

and in turn reduces the incentive to engage in R&D. Jones and Williams (2000) dub this effect as the "stick". In the model, the first and second effects approximately offset each other, leaving the stick effect as the main influence of creative destruction on R&D. As a result, a higher ψ discourages R&D, and hence the government should increase the capital tax and reduce the labor tax in order to boost labor supply and R&D labor.

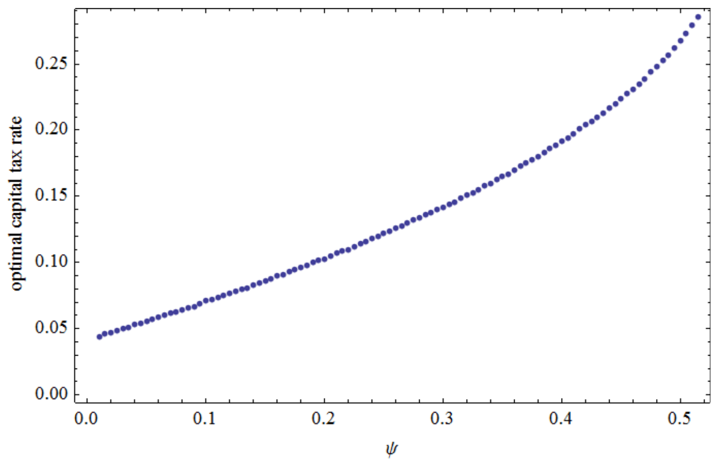

Figure 5: The optimal capital tax rate and creative destruction

Finally, the optimal capital tax is increasing in the government spending ratio ζ (see Figure 6). This result is consistent with Aghion et al. (2013) and Lu and Chen (2015).[16] When the need for public expenditure is sufficiently small, the government can collect labor tax revenues to finance the government spending and also to subsidize capital. Note that in this case the monopoly effect dominates the R&D effect so that the optimal capital tax rate becomes negative. As the size of government expenditure increases, it is not promising to rely solely on raising the labor tax, because the distortion to the R&D sector would be too strong. In this case, it becomes optimal to shift some of the tax burden to capital.

As we have noted earlier, our result of a positive optimal capital income tax is obtained under the benchmark parameters. Before ending this section, it is worthwhile to briefly discuss how plausible the above parameters fall into the range that implies a negative optimal capital tax rate. First, the optimal capital tax rate becomes negative if $\lambda < 0.454$, i.e., the stepping on toes effect is larger. This seems not very likely, however, as Jones and Williams

[16] Lu and Chen (2015) show that in an exogenous growth model with a given share of government expenditure in output, the optimal capital income tax is positive and increasing with the the share of government expenditure. The intuition is that capital accumulation reduces the discounted net marginal product of next period's capital by way of increasing government expenditure. Thus, the government should tax capital to correct this distortion.

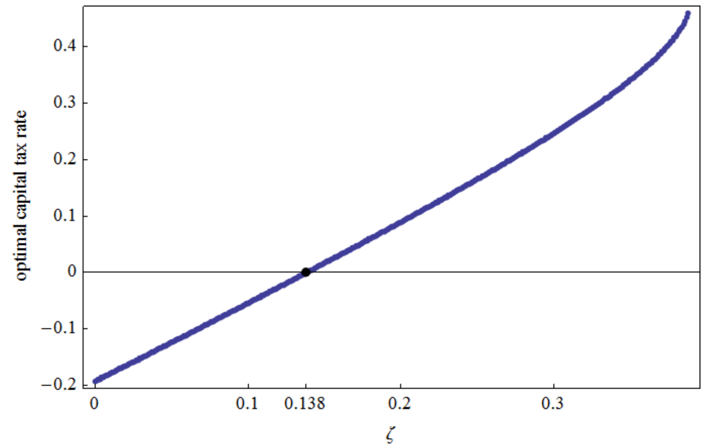

Figure 6: The optimal capital tax and the government size

(2000) point out that the lower bound of λ is about 0.5. Second, the optimal capital tax rate becomes negative if the standing on shoulders effect is smaller, i.e., $\phi < 0.948$. For the first-generation R&D-based growth models a la Romer (1990), $\phi = 1$, so that our result of a positive optimal capital income tax always holds. However, given that ϕ can take a wide range of values in the literature, it calls for further consideration on the positive optimal capital tax before identifying the value of this parameter. Third, for the substitution parameter ρ, the threshold value that will result in a negative optimal capital tax is $\rho < 2.14$. This implies a monopolistic markup higher than 1.17. Fourth, the optimal capital tax rate is positive for the whole possible values of the size of the innovation cluster ψ. Finally, the optimal capital tax rate is negative if the government spending ratio is less than 13.8%, i.e., $\zeta < 0.138$. This threshold value is much smaller than that in Aghion et al. (2013), in which the government spending ratio required for a positive optimal capital tax rate is around 40%.

5 Conclusion

In this paper, we have examined whether the Chamley-Judd result of zero optimal capital taxation is valid in a non-scale innovation-based growth model. By calibrating our model to the US economy, our result shows that the optimal capital income tax is positive, at a rate of around 11.9 percent. We examine how the optimal capital tax rate responds to various R&D externalities. The optimal capital tax rate is higher when (i) the "stepping on toes effect" is smaller, (ii) the "standing on shoulders effect" is stronger, or (iii) the extent of creative destruction is greater. We also find that the optimal capital tax is sensitive to the parameter that determines the monopolistic markup. An inverted-U relationship is found between these two variables.

Some extensions for future research are worth noting. First, since R&D investment usually has liquidity problems (Lach, 2002), it would be relevant to introduce a credit constraint on R&D investment into our model. Second, it would be interesting to examine the optimal capital tax in an endogenous growth model where both innovation and capital accumulation are the driving forces of economic growth (see, e.g., Iwaisako and Futagami, 2013; Chu et al., 2019). These directions will no doubt generate new insights into the debate on the Chamley-Judd result.

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Appendix A. Deriving the steady-state growth rate

To solve for the steady-state growth rate of the economy, from eqs. (13) and (14) we have:

$$\frac{\dot{A}_t}{A_t} = \frac{\varsigma}{1+\psi}\frac{L_{A,t}^{\lambda}}{A_t^{1-\phi}}. \tag{A1}$$

where $g_{A,t} = \dot{A}_t/A_t$. Let g_Z denote $g_{Z,t} = \frac{\dot{Z}}{Z}$ the growth rate of any generic variable Z, and drop the time subscript when referring to any variables in the steady state. The steady-state growth rate of varieties is given by:

$$g_A = \frac{\varsigma}{1+\psi}\frac{L_A^{\lambda}}{A^{1-\phi}}. \tag{A2}$$

Moreover, the R&D labor share is $s_t = L_{A,t}/(N_t l_t)$. In so doing, eq. (A2) can alternatively be expressed as:

$$g_A = \frac{\varsigma}{1+\psi}\frac{(sNl)^{\lambda}}{A^{1-\phi}}. \tag{A3}$$

By taking logarithms of eq. (A3) and differentiating the resulting equation with respect to time, we have the following steady-state expression:

$$g_A = \frac{\lambda}{1-\phi}n. \tag{A4}$$

Equipped with the symmetric feature $x(i) = x$, the equilibrium condition for the capital market $K = Av$, and the production in the intermediate-good sector $x = v$, the aggregate production function can be rewritten as:

$$Y_t = A_t^{\frac{1}{\rho}-\alpha} L_t^{\alpha} K_t^{1-\alpha}. \tag{A5}$$

Taking logarithms of eq. (A5) and differentiating the resulting equation with respect to time, we can infer the following result:

$$g_Y = \frac{(\frac{1}{\rho}-\alpha)}{1-\alpha}g_A + n. \tag{A6}$$

Inserting eq. (A4) into eq. (A6) yields:

$$g_Y = \sigma n, \tag{A7}$$

where $\sigma \equiv 1 + \frac{(\frac{1}{\rho}-\alpha)}{1-\alpha}\frac{\lambda}{1-\phi}$ is a composite parameter.

We now turn to solve the steady-state R&D labor share. In the long run, substituting $\dot{A}_t = g_A A_t$ and differentiating the resulting equation with respect to time gives rise to:

$$\dot{P}_A/P_A = g_Y - g_A \tag{A8}$$

From eqs. (12), (15), (17), in the steady state we have:

$$\pi_x = \frac{\eta-1}{\eta}\alpha\frac{Y}{A} \tag{A9}$$

$$P_A = \frac{s}{1-s}\frac{(1-\alpha)Y/A}{(1+\psi)g_A} \tag{A10}$$

$$r = \frac{\pi_x}{P_A} + \frac{\dot{P}_A}{P_A} - \psi g_A \tag{A11}$$

Substituting eqs. (A8), (A9), and (A10) into eq. (A11) yields the result:

$$r = \frac{\frac{\eta-1}{\eta}\alpha Y/A}{\frac{s}{1-s}\frac{(1-\alpha)Y/A}{(1+\psi)g_A}} + g_Y - (1+\psi)g_A \tag{A12}$$

Based on eq. (A12), we have the stationary R&D labor share s as follows:

$$s = \frac{\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}(1+\psi)g_A}{r - g_Y + (1+\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha})(1+\psi)g_A} \tag{A13}$$

Appendix B. Transition dynamics

This appendix solves the dynamic system of the model under *tax shifting from labor income taxes to capital income taxes*. The set of equations under the model is expressed by:

$$\frac{1}{c_t} = q_t, \tag{B1}$$

$$\chi = q_t(1-\tau_{L,t})w_t(1-l_t), \tag{B2}$$

$$r_t = (1-\tau_K)r_{K,t} - \delta, \tag{B3}$$

$$\frac{\dot{c}_t}{c_t} = r_t - n - \beta, \tag{B4}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_{Y,t}}, \tag{B5}$$

$$\eta r_{K,t} = \alpha A_t^{\frac{1}{\rho}-1} L_{Y,t}^{1-\alpha} x_t^{\alpha-1}, \tag{B6}$$

$$r_{K,t}K_t = \frac{\alpha}{\eta}Y_t, \tag{B7}$$

$$\pi_{x,t} = \frac{\eta-1}{\eta}\alpha\frac{Y_t}{A_t}, \tag{B8}$$

$$r_t P_{A,t} = \pi_{x,t} + \dot{P}_{A,t} - \psi\frac{\dot{A}_t}{A_t}P_{A,t}, \tag{B9}$$

$$G_t = \zeta Y_t, \tag{B10}$$

$$G_t = N_t(\tau_K r_{K,t} k_t + \tau_{L,t} w_t l_t), \tag{B11}$$

$$Y_t = A_t^{1/\rho-\alpha} L_{Y,t}^{1-\alpha} K_t^{\alpha}, \tag{B12}$$

$$\dot{K}_t = Y_t - C_t - G_t - \delta K_t, \tag{B13}$$

$$\frac{\dot{A}_t}{A_t} = \frac{\varsigma}{1+\psi}\frac{L_{A,t}^{\lambda}}{A_t^{1-\phi}}, \tag{B14}$$

$$P_{A,t} = \frac{s_t}{1-s_t}\frac{(1-\alpha)Y_t}{(1+\psi)\dot{A}_t}, \tag{B15}$$

$$N_t l_t = L_{Y,t} + L_{A,t}. \tag{B16}$$

The above 16 equations determine 16 unknowns $\{c_t,\, l_t,\, A_t,\, K_t,\, L_{Y,t},\, x_t,\, r_{K,t},\, \pi_{x,t}\; r_t,\, G_t,\, \tau_{L,t},\, Y_t,\, q_t,\, L_{A,t},\, P_{A,t},\, w_t\}$, where q_t is the Hamiltonian multiplier, $C_t = N_t c_t$, $K_t \equiv N_t k_t = A_t x_t$, and $s_t = L_{A,t}/N_t l_t$. Based on $K_t = N_t k_t = A_t x_t$, and eqs. (B1), (B2), (B5), and (B12), we can obtain:

$$\chi = \frac{1}{c_t}(1-\tau_{L,t})(1-\alpha)\frac{Y_t}{L_{Y,t}}(1-l_t). \tag{B17a}$$

From eqs. (B5), (B7), and (B11), we have:

$$\tau_{L,t} = (1-s_t)\frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}. \tag{B17b}$$

Moreover, to solve the balanced growth rate, we define the following transformed variables:

$$\hat{k}_t \equiv \frac{K_t}{N_t^{\sigma}}, \quad \hat{c}_t \equiv \frac{C_t}{N_t^{\sigma}}, \quad \hat{y}_t \equiv \frac{Y_t}{N_t^{\sigma}}, \quad \hat{a}_t \equiv \frac{A_t}{N_t^{\lambda/(1-\phi)}}, \quad s_t \equiv L_{A,t}/N_t l_t. \tag{B18}$$

Based on eqs. (B16), (B15), (B17a), and the above definitions, we can obtain:

$$\frac{\chi}{(1-l_t)} = \frac{1}{\hat{c}_t}[1-(1-s_t)\frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}](1-\alpha)\hat{a}_t^{1/\rho-\alpha}(\hat{k}_t)^{\alpha}[1-s_t)l_t]^{-\alpha}. \tag{B19a}$$

From eq. (B19a), we can infer the following expression:

$$l_t = l_t(\hat{k}_t, \hat{a}_t, \hat{c}_t, s_t; \tau_K), \tag{B19b}$$

where

$$\frac{\partial l_t}{\partial \hat{k}_t} = \frac{\alpha}{\hat{k}_t(\frac{l_t}{1-l_t}+\alpha)} l_t, \tag{B20a}$$

$$\frac{\partial l_t}{\partial \hat{a}_t} = \frac{(1/\rho-\alpha)}{\hat{a}_t(\frac{l_t}{1-l_t}+\alpha)} l_t, \tag{B20b}$$

$$\frac{\partial l_t}{\partial \hat{c}_t} = -\frac{l_t}{\hat{c}_t(\frac{l_t}{1-l_t}+\alpha)}, \tag{B20c}$$

$$\frac{\partial l_t}{\partial s_t} = \frac{\frac{\tau_{L,t}}{(1-\tau_{L,t})}+\alpha}{(1-s_t)(\frac{l_t}{1-l_t}+\alpha)} l_t, \tag{B20d}$$

$$\frac{\partial l_t}{\partial \tau_K} = \frac{(1-s_t)\frac{\alpha}{\eta(1-\alpha)}}{(1-\tau_{L,t})(\frac{l_t}{1-l_t}+\alpha)} l_t. \tag{B20e}$$

Based on (B3), (B4), (B7), (B12), (B18), and $C_t = N_t c_t$, we have:

$$g_{\hat{c},t} \equiv \frac{d\hat{c}_t/dt}{\hat{c}_t} = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a}_t)^{1/\rho-\alpha}[\frac{(1-s_t)l_t(\hat{k}_t, \hat{a}_t, \hat{c}_t, s_t; \tau_K)}{\hat{k}_t}]^{1-\alpha} - \delta - \beta - g_Y. \tag{B21}$$

From eqs. (B10), (B12), (B13), and (B18), we can directly infer:

$$g_{\hat{k},t} \equiv \frac{d\hat{k}_t/dt}{\hat{k}_t} = (1-\zeta)(\hat{a}_t)^{1/\rho-\alpha}[\frac{(1-s_t)l_t(\hat{k}_t,\hat{a}_t,\hat{c}_t,s_t;\tau_K)}{\hat{k}_t}]^{1-\alpha} - \frac{\hat{c}_t}{\hat{k}_t} - \delta - g_Y. \tag{B22}$$

According to eqs. (B14) and (B18), we can further obtain:

$$g_{\hat{a},t} \equiv \frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{\varsigma}{1+\psi}\frac{[s_t l_t(\hat{k}_t,\hat{a}_t,\hat{c}_t,s_t;\tau_K)]^{\lambda}}{\hat{a}_t^{1-\phi}} - g_A. \tag{B23}$$

In what follows, to simplify the notation we suppress those arguments of the labor supply function. From eq. (B18), taking logarithms of eqs. (B19a) and (B12) and differentiating the resulting equations with respect to time, we have:

$$g_{\hat{y},t} = (1/\rho-\alpha)g_{\hat{a},t} + \alpha g_{\hat{k},t} + (1-\alpha)(\dot{l}_t/l_t - \frac{\dot{s}_t}{1-s_t}), \tag{B24}$$

$$\dot{l}_t/l_t = \{(1/\rho-\alpha)g_{\hat{a},t} + \alpha g_{\hat{k},t} - g_{\hat{c},t} - [\alpha + \tau_{L,t}/(1-\tau_{L,t})]\}/[\alpha + l_t/(1-l_t)]. \tag{B25}$$

Taking logarithms of eq. (B15) differentiating the resulting equation with respect to time, we obtain:

$$\frac{\dot{P}_{A,t}}{P_{A,t}} = (1/\rho-\alpha-\phi)g_{\hat{a},t} + \alpha g_{\hat{k},t} + (1-\lambda+\alpha\frac{s_t}{1-s_t})\frac{\dot{s}_t}{s_t} + (1-\lambda-\alpha)\frac{\dot{l}_t}{l_t} + g_Y - g_A. \tag{B26}$$

Combining eqs. (B9), (B15), (B18), (B21), (B24), (B25), and (B26) together, we obtain:

$$\begin{aligned}\frac{ds_t/dt}{s_t} &= \{\beta - [\frac{(\eta-1)\alpha(1+\psi)(1-s_t)}{(1-\alpha)\eta s_t} - \psi](g_A + g_{\hat{a},t}) + \phi g_{\hat{a},t} + g_A - [1 + \frac{1-\lambda-\alpha}{\alpha + l_t/(1-l_t)}] \\ &\quad \times[(1/\rho-\alpha)g_{\hat{a},t} + \alpha g_{\hat{k},t} - g_{\hat{c},t}]\}/\{1-\lambda+\alpha\frac{s_t}{1-s_t} + \frac{1-\lambda-\alpha}{\alpha+l_t/(1-l_t)}(\alpha + \frac{\tau_{L,t}}{1-\tau_{L,t}})\frac{s_t}{1-s_t}\}.\end{aligned} \tag{B27}$$

Note that $r_t - g_Y - g_{\hat{c},t} = \beta$. As a result, in the steady state we have $r - g_Y = \beta$.
Inserting eq. (B18) into eq. (B17b) yields:

$$\tau_{L,t} = (1-s_t)\frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}. \tag{B28}$$

Based on eqs. (B21), (B22), (B23),(B27), and (B28), the dynamic system can be expressed as:

$$\frac{d\hat{k}_t/dt}{\hat{k}_t} = (1-\zeta)(\hat{a}_t)^{1/\rho-\alpha}[\frac{(1-s_t)l_t}{\hat{k}_t}]^{1-\alpha} - \frac{\hat{c}_t}{\hat{k}_t} - \delta - g_Y, \tag{B29a}$$

$$\frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{\varsigma}{1+\psi}\frac{(s_t l_t)^\lambda}{\hat{a}_t^{1-\phi}} - g_A, \tag{B29b}$$

$$\frac{d\hat{c}_t/dt}{\hat{c}_t} = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a}_t)^{1/\rho-\alpha}[\frac{(1-s_t)l_t}{\hat{k}_t}]^{1-\alpha} - \delta - \beta - g_Y, \tag{B29c}$$

$$\begin{aligned}\frac{ds_t/dt}{s_t} = {} & \{\beta - [\frac{(\eta-1)\alpha(1+\psi)(1-s_t)}{(1-\alpha)\eta s_t} - \psi](g_A + g_{\hat{a},t}) + \phi g_{\hat{a},t} + g_A - [1 + \frac{1-\lambda-\alpha}{\alpha + l_t/(1-l_t)}] \\ & \times[(1/\rho-\alpha)g_{\hat{a},t} + \alpha g_{\hat{k},t} - g_{\hat{c},t}]\}/\{1-\lambda+\alpha\frac{s_t}{1-s_t} + \frac{1-\lambda-\alpha}{\alpha+l_t/(1-l_t)}(\alpha + \frac{\tau_{L,t}}{1-\tau_{L,t}})\frac{s_t}{1-s_t}\}.\end{aligned} \tag{B29d}$$

Linearizing eqs. (B29a), (B29b), (B29c), and (B29d) around the steady-state equilibrium yields:

$$\begin{pmatrix} d\hat{k}_t/dt \\ d\hat{a}_t/dt \\ d\hat{c}_t/dt \\ ds_t/dt \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \begin{pmatrix} \hat{k}_t - \hat{k} \\ \hat{a}_t - \hat{a} \\ \hat{c}_t - \hat{c} \\ s_t - s \end{pmatrix} + \begin{pmatrix} b_{15} \\ b_{25} \\ b_{35} \\ b_{45} \end{pmatrix} d\tau_K, \tag{B30}$$

where
$b_{11} = \frac{\partial(d\hat{k}_t/dt)}{\partial\hat{k}_t}$, $b_{12} = \frac{\partial(d\hat{k}_t/dt)}{\partial\hat{a}_t}$, $b_{13} = \frac{\partial(d\hat{k}_t/dt)}{\partial\hat{c}_t}$, $b_{14} = \frac{\partial(d\hat{k}_t/dt)}{\partial s_t}$, $b_{15} = \frac{\partial(d\hat{k}_t/dt)}{\partial\tau_K}$,
$b_{21} = \frac{\partial(d\hat{a}_t/dt)}{\partial\hat{k}_t}$, $b_{22} = \frac{\partial(d\hat{a}_t/dt)}{\partial\hat{a}_t}$, $b_{23} = \frac{\partial(d\hat{a}_t/dt)}{\partial\hat{c}_t}$, $b_{24} = \frac{\partial(d\hat{a}_t/dt)}{\partial s_t}$, $b_{25} = \frac{\partial(d\hat{a}_t/dt)}{\partial\tau_K}$,
$b_{31} = \frac{\partial(d\hat{c}_t/dt)}{\partial\hat{k}_t}$, $b_{32} = \frac{\partial(d\hat{c}_t/dt)}{\partial\hat{a}_t}$, $b_{33} = \frac{\partial(d\hat{c}_t/dt)}{\partial\hat{c}_t}$, $b_{34} = \frac{\partial(d\hat{c}_t/dt)}{\partial s_t}$, $b_{35} = \frac{\partial(d\hat{c}_t/dt)}{\partial\tau_K}$,
$b_{41} = \frac{\partial(ds_t/dt)}{\partial\hat{k}_t}$, $b_{42} = \frac{\partial(ds_t/dt)}{\partial\hat{a}_t}$, $b_{43} = \frac{\partial(ds_t/dt)}{\partial\hat{c}_t}$, $b_{44} = \frac{\partial(ds_t/dt)}{\partial s_t}$, $b_{45} = \frac{\partial(ds_t/dt)}{\partial\tau_K}$.

Due to the complicated calculations, we do not list the analytical results for b_{ij}, where $i \in \{1, 2, 3, 4, 5\}$ and $j \in \{1, 2, 3, 4, 5\}$.

Let ℓ_1, ℓ_2, ℓ_3, and ℓ_4 be the four characteristic roots of the dynamic system. Due to the complexity involved in calculating the four characteristic roots, we do not try to prove the saddle-point stability analytically. Instead, via a numerical simulation, we show that the dynamic system has two positive and two negative characteristic roots. For expository convenience, in what follows let ℓ_1 and ℓ_2 be the negative root, and ℓ_3 and ℓ_4 be the positive roots. The general solution is given by:

$$\begin{pmatrix} \hat{k}_t \\ \hat{a}_t \\ \hat{c}_t \\ s_t \end{pmatrix} = \begin{pmatrix} \hat{k}(\tau_K) \\ \hat{a}(\tau_K) \\ \hat{c}(\tau_K) \\ s(\tau_K) \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 & 1 \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix} \begin{pmatrix} D_1 e^{\ell_1 t} \\ D_2 e^{\ell_2 t} \\ D_3 e^{\ell_3 t} \\ D_4 e^{\ell_4 t} \end{pmatrix}. \tag{B31a}$$

where D_1, D_2, D_3, and D_4 are undetermined coefficients and

$$\triangle_j = \begin{vmatrix} b_{12} & b_{13} & b_{14} \\ b_{22}-\ell_j & b_{23} & b_{24} \\ b_{32} & b_{33}-\ell_j & b_{34} \end{vmatrix} \; ; j \in \{1,\ 2,\ 3,\ 4\}, \tag{B31b}$$

$$h_{2j} = \begin{vmatrix} \ell_j - b_{11} & b_{13} & b_{14} \\ -b_{21} & b_{23} & b_{24} \\ -b_{31} & b_{33}-\ell_j & b_{34} \end{vmatrix} / \triangle_j \; ; j \in \{1,\ 2,\ 3,\ 4\}, \tag{B31c}$$

$$h_{3j} = \begin{vmatrix} b_{12} & -b_{11} & b_{14} \\ b_{22}-\ell_j & -b_{21} & b_{24} \\ b_{32} & -b_{31} & b_{34} \end{vmatrix} / \triangle_j \; ; j \in \{1,\ 2,\ 3,\ 4\}, \tag{B31d}$$

$$h_{4j} = \begin{vmatrix} b_{12} & b_{13} & \ell_j - b_{11} \\ b_{22}-\ell_j & b_{23} & -b_{21} \\ b_{32} & b_{33}-\ell_j & -b_{31} \end{vmatrix} / \triangle_j \; ; j \in \{1,\ 2,\ 3,\ 4\}. \tag{B31e}$$

The government changes the capital tax rate τ_K from τ_{K0} to τ_{K1} at t=0. Based on eqs. (B31a)-(B31e), we employ the following equations to describe the dynamic adjustment of $\hat{k}_t$, $\hat{a}_t$, $\hat{c}_t$ and s_t:

$$\hat{k}_t = \begin{cases} \hat{k}(\tau_{K0}); & t = 0^- \\ \hat{k}(\tau_{K1}) + D_1 e^{\ell_1 t} + D_2 e^{\ell_2 t} + D_3 e^{\ell_3 t} + D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32a}$$

$$\hat{a}_t = \begin{cases} \hat{a}(\tau_{K0}); & t = 0^- \\ \hat{a}(\tau_{K1}) + h_{21} D_1 e^{\ell_1 t} + h_{22} D_2 e^{\ell_2 t} + h_{23} D_3 e^{\ell_3 t} + h_{24} D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32b}$$

$$\hat{c}_t = \begin{cases} \hat{c}(\tau_{K0}); & t = 0^- \\ \hat{c}(\tau_{K1}) + h_{31} D_1 e^{\ell_1 t} + h_{32} D_2 e^{\ell_2 t} + h_{33} D_3 e^{\ell_3 t} + h_{34} D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32c}$$

$$s_t = \begin{cases} s(\tau_{K0}); & t = 0^- \\ s(\tau_{K1}) + h_{41} D_1 e^{\ell_1 t} + h_{42} D_2 e^{\ell_2 t} + h_{43} D_3 e^{\ell_3 t} + h_{44} D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32d}$$

where 0^- and 0^+ denote the instant before and instant after the policy implementation, respectively. The values for D_1, D_2, D_3 and D_4 are determined by:

$$\hat{k}_{0^-} = \hat{k}_{0^+}, \tag{B33a}$$

$$\hat{a}_{0^-} = \hat{a}_{0^+}, \tag{B33b}$$

$$D_3 = D_4 = 0. \tag{B33c}$$

Equations (B33a) and (B33b) indicate that both $\hat{k}_t$ $(= \frac{K_t}{N_t^\sigma})$ and $\hat{a}_t$ $(= \frac{A_t}{N_t^{\lambda/(1-\phi)}})$ remain intact at the instant of policy implementation since K_t, A_t, and N_t are predetermined variables. Equation (B33c) is the stability condition which ensures that all $\hat{k}_t$, $\hat{a}_t$, $\hat{c}_t$ and s_t converge to their new steady-state equilibrium. By using eqs. (B33a) and (B33b), we can obtain:

$$D_1 = \frac{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]}{h_{22} - h_{21}}, \tag{B34a}$$

$$D_2 = \frac{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}}{h_{22} - h_{21}}. \tag{B34b}$$

Inserting eqs. (B33c), (B34a), and (B34b) into eqs. (B32a)-(B32d) yields:

$$\hat{k}_t = \begin{cases} \hat{k}(\tau_{K0}); & t = 0^- \\ \hat{k}(\tau_{K1}) + \frac{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]}{h_{22} - h_{21}} e^{\ell_1 t} & t \geq 0^+ \\ \qquad + \frac{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}}{h_{22} - h_{21}} e^{\ell_2 t}; & \end{cases}$$

$$\hat{a}_t = \begin{cases} \hat{a}(\tau_{K0}); & t = 0^- \\ \hat{a}(\tau_{K1}) + \frac{\{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]\} h_{21} e^{\ell_1 t}}{h_{22} - h_{21}} & t \geq 0^+ \\ \qquad + \frac{\{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}\} h_{22} e^{\ell_2 t}}{h_{22} - h_{21}}; & \end{cases}$$

$$\hat{c}_t = \begin{cases} \hat{c}(\tau_{K0}); & t = 0^- \\ \hat{c}(\tau_{K1}) + \frac{\{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]\} h_{31} e^{\ell_1 t}}{h_{22} - h_{21}} & t \geq 0^+ \\ \qquad + \frac{\{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}\} h_{32} e^{\ell_2 t}}{h_{22} - h_{21}}; & \end{cases}$$

$$s_t = \begin{cases} s(\tau_{K0}); & t = 0^- \\ s(\tau_{K1}) + \frac{\{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]\} h_{41} e^{\ell_1 t}}{h_{22} - h_{21}} & t \geq 0^+ \\ \qquad + \frac{\{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}\} h_{42} e^{\ell_2 t}}{h_{22} - h_{21}}; & \end{cases}$$

Appendix C. Proof of comparative statics

From eqs. (B29a)-(B29d), we have:

$$\frac{d\hat{k}_t/dt}{\hat{k}_t} = (1-\zeta)(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \frac{\hat{c}_t}{\hat{k}_t} - \delta - g_Y, \tag{C1a}$$

$$\frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{\varsigma}{1+\psi}\frac{[l_t(\hat{k}_t, \hat{a}_t, \hat{c}_t, l_{Y,t}; \tau_K) - l_{Y,t}]^\lambda}{\hat{a}_t^{1-\phi}} - g_A, \tag{C1b}$$

$$\frac{d\hat{c}_t/dt}{\hat{c}_t} = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \delta - \beta - g_Y, \tag{C1c}$$

$$\begin{aligned}\frac{ds_t/dt}{s_t} = {} & \{\beta - [\frac{(\eta-1)\alpha(1+\psi)(1-s_t)}{(1-\alpha)\eta s_t} - \psi](g_A + g_{\hat{a},t}) + \phi g_{\hat{a},t} + g_A - [1 + \frac{1-\lambda-\alpha}{\alpha + l_t/(1-l_t)}] \\ & \times[(1/\rho-\alpha)g_{\hat{a},t} + \alpha g_{\hat{k},t} - g_{\hat{c},t}]\}/\{1-\lambda+\alpha\frac{s_t}{1-s_t} + \frac{1-\lambda-\alpha}{\alpha+l_t/(1-l_t)}(\alpha + \frac{\tau_{L,t}}{1-\tau_{L,t}})\frac{s_t}{1-s_t}\}.\end{aligned} \tag{C1d}$$

In the steady state $\frac{d\hat{k}_t/dt}{\hat{k}_t} = \frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{d\hat{c}_t/dt}{\hat{c}_t} = \frac{ds_t/dt}{s_t} = 0$, we then have the following steady-state results:

$$\frac{\hat{c}}{\hat{k}} = (1-\zeta)(\hat{a})^{1/\rho-\alpha}[\frac{(1-s)l}{\hat{k}}]^{1-\alpha} - \delta - g_Y, \tag{C1e}$$

$$g_A = \frac{\varsigma}{1+\psi}\frac{(sl)^\lambda}{\hat{a}^{1-\phi}}, \tag{C1f}$$

$$\beta = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a})^{1/\rho-\alpha}[\frac{(1-s)l}{\hat{k}}]^{1-\alpha} - \delta - g_Y, \tag{C1g}$$

$$0 = \beta - [\frac{(\eta-1)\alpha(1+\psi)(1-s)}{(1-\alpha)\eta s} - \psi]g_A + g_A. \tag{C1h}$$

Based on eq. (C1h), we have:

$$s = \frac{\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}(1+\psi)g_A}{\beta + \left(1 + \frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}\right)(1+\psi)g_A}. \tag{C2}$$

From eqs. (B3) and (C1g), we can obtain

$$r - g_Y = \beta > 0. \tag{C3}$$

Equation eq. (C1g) can be rearranged as:

$$\hat{y}/\hat{k} = (\hat{a})^{1/\rho-\alpha}[\frac{(1-s)l}{\hat{k}}]^{1-\alpha} = \frac{\eta(\beta+\delta+g_Y)}{\alpha(1-\tau_K)}. \tag{C4a}$$

Substituting eq. (C4a) into eq. (C1e) gives rise to:

$$\frac{\hat{c}}{\hat{y}} = \{(1-\zeta)\frac{\eta(\beta+\delta+g_Y)}{\alpha(1-\tau_K)} - \delta - g_Y\}\frac{\hat{k}}{\hat{y}} = (1-\zeta) - (\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}. \tag{C5a}$$

To ensure that the steady-state consumption-output ratio $\hat{c}/\hat{y}$ is positive, we impose the restriction $(1-\zeta)-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)} > 0$ for all values of the time preference rate β. As a consequence, $\lim_{\beta\to 0}\hat{c}/\hat{y}>0$ implies:

$$(1-\zeta) - \frac{\alpha(1-\tau_K)}{\eta} > 0. \tag{C5b}$$

From eq (C1f), we can derive:

$$\hat{a} = [\frac{\varsigma}{(1+\psi)g_A}]^{1/(1-\phi)}(sl)^{\lambda/(1-\phi)}. \tag{C6}$$

Based on eq. (B28), we can infer the following expression:

$$\tau_L = (1-s)\frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}, \tag{C7a}$$

where

$$\frac{\partial\tau_L}{\partial\tau_K} = -(1-s)\frac{\frac{\alpha}{\eta}}{1-\alpha} < 0. \tag{C7b}$$

Equipped with eqs. (B1), (B2), (B5), and $L_Y = N(1-s)l$, we can obtain:

$$\frac{l}{1-l}\chi = \frac{\hat{y}}{\hat{c}}\frac{(1-\tau_L)(1-\alpha)}{(1-s)}. \tag{C8}$$

Inserting eqs. (C5a) and (C7a) into eq. (C8) yields:

$$l = \begin{cases} 1 - \frac{\chi}{\chi + \frac{1}{[(1-\zeta)-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]}\frac{(1-\tau_L)(1-\alpha)}{(1-s)}} & ; \ \chi > 0 \\ 1 & ; \ \chi = 0 \end{cases}, \tag{C9a}$$

where

$$\frac{\partial l}{\partial \tau_K}=\begin{cases}\frac{\alpha\beta(\frac{1-s}{1-\alpha})[1-\zeta+\frac{\eta-1}{\eta}\frac{\alpha(\delta+g_Y)}{\beta+(1+\psi)g_A}](1-l)l}{\eta(\beta+\delta+g_Y)(1-\tau_L)[1-\zeta-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]}>0 & ;\quad \chi>0\\ 0 & ;\quad \chi=0\end{cases}. \tag{C9b}$$

Combining eqs. (C2), (C6), and (C9b) together, we can derive

$$\hat{a}=[\frac{\varsigma}{(1+\psi)g_A}]^{1/(1-\phi)}(sl)^{\lambda/(1-\phi)}, \tag{C10a}$$

where

$$\frac{\partial \hat{a}}{\partial \tau_K}=\frac{\lambda}{(1-\phi)}\hat{a}\frac{\partial l}{l\partial \tau_K}>0. \tag{C10b}$$

Based on eqs. (C4a), (C9b), (B12), and (B18), we have:

$$\hat{y}=\hat{a}^{\frac{1/\rho-\alpha}{1-\alpha}}[\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]^{\frac{\alpha}{1-\alpha}}(1-s)l, \tag{C11a}$$

where

$$\frac{\partial \hat{y}}{\partial \tau_K}=[\sigma\frac{\partial l}{l\partial \tau_K}-\frac{\alpha}{(1-\alpha)(1-\tau_K)}]\hat{y}\gtrless 0,\quad \sigma\equiv 1+\frac{1/\rho-\alpha}{1-\alpha}\frac{\lambda}{1-\phi}. \tag{C11b}$$

According to eqs. (C4a), (C5a), and (C11b), we obtain:

$$\hat{k} = \frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}\hat{y}, \tag{C12a}$$

$$\hat{c} = [(1-\zeta)-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]\hat{y}, \tag{C12b}$$

Inserting eq. (C11a) into (C12a) and (C12b), we can derive the following comparative statics:

$$\frac{\partial \hat{k}}{\partial \tau_K} = \frac{\alpha(1-\tau_K)\hat{y}}{\eta(\beta+\delta+g_Y)}\{\sigma\frac{\partial l}{l\partial \tau_K}-\frac{1}{(1-\alpha)(1-\tau_K)}\}\gtrless 0, \tag{C12c}$$

$$\begin{aligned}\frac{\partial \hat{c}}{\partial \tau_K} = {} & \{\frac{\alpha(\delta+g_Y)}{\eta(\beta+\delta+g_Y)}+[(1-\zeta)\\ & -\frac{\alpha(1-\tau_K)(\delta+g_Y)}{\eta(\beta+\delta+g_Y)}][\sigma\frac{\partial l}{l\partial \tau_K}-\frac{\alpha}{(1-\alpha)(1-\tau_K)}]\}\hat{y}\gtrless 0.\end{aligned} \tag{C12d}$$