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Abstract

In this study, we present a baseline monetary growth model for disequilibrium macroeconomics. Our model is similar to the existing Keynes-Wicksell models, but we highlight a characteristic of disequilibrium (non-Walrasian) macroeconomics, that is, the regime dividing in the static model. In addition, since we synthesize demand-side factors (Keynesian) and supply-side factors (neo-classical), we find a new effect on dynamical feedback loops, that is, the dual-decision effect. This new effect stabilizes (resp. destabilizes) an unstable (resp. a stable) feedback loop when the regime switches from the demand-side to the supply-side. Moreover, this dual-decision effect partly works on the real wage adjustment process and it enhances the instability if the economy is in Keynesian regime. We implement numerical experiments to confirm these results, and find that Walrasian equilibrium itself is not always stable.

Keywords: Disequilibrium macroeconomics, Non-Walrasian analysis, Keynes-Wicksell model, Economic growth

1 Introduction

Today, monetary economics is one of the most important areas for macroecomics. Many researchers utilize the issues of monetary economics as the "Keynesian" features for macrodynamics, and they have developed so called New-Keynesian economics. The model of New-Keynesian economics is usulally called Dynamic Stochastic General Equilibrium (DSGE) model, and it has "a core structure that corresponds to a Real Business Cycle (RBC) model" (Galí, 2015). New-Keynesian researchers introduce the nominal rigidities into RBC framework and manage to prove the non-neutrality of monetary policy at least in the short term.[1]

While equilibrium monetary macroeconomics prospers as mentioned above, the research on disequilibrium (or non-Walrasian) economics is not promoted these days.[2] In

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[1]See Christiano et al. (2005). For the brief summary of DSGE analyses today, see Christiano et al. (2018).

[2]Strictly speaking, the disequilibrium macrodynamics has been often analyzed in Keynesian economics; see Chiarella and Flaschel (2000), Chiarella et al. (2000), Chiarella et al. (2005), Asada et al.

the early days of the monetary growth, however, the difference between equilibrium school and disequilibrium school is not so distinct. Since Tobin (1965) specified the portfolio mechanism on the neo-classical growth framework, the relationship between the capital intensity in the steady state and the existence of the money has been one important issue.[3] By contrast with the neo-classical monetary growth, in which the planned saving and the planned investment always match, Stein (1969) constructs a "Keynes-Wicksell" monetary growth model. In his model, the speed of the price adjustment is finite and the gap between saving and investment determines the price dynamics.[4] This short-run disequilibrium adjustment often derives a growth-cycle dynamics, while the neo-classical (equilibrium) model usually has a unique path which converges into a steady state. However, the coexistence of the two different approaches does not mean the separation between equilibrium and disequilibrium dynamics: Villanueva (1971) explores the IS disequilibrium dynamics on the neo-classical monetary growth, for instance. As Bénassy (1986, Chapter. 1) argues, we should note that the disequilibrium dynamics is rather a kind of expansion of the equilibrium model than the counterpart against the equilibrium models.

Therefore, it is necessary to explore the disequilibrium macrodynamics on the monetary economics to evaluate today's prosper of equilibrium-type monetary growth models. As I mentioned above, however, the non-Walrasian disequilibrium research on monetary macroeconomics is insufficient. So this paper constructs a baseline disequilibrium monetary growth model and explore how the disequilibrium macrodynamics is affected by the money, comparing to the existing monetary growth models.

Non-Walrasian economics, which is influenced by Clower (1965), treats the quantity constrained transactions of goods under rigid prices. After Barro and Grossman (1971) built a basic general disequilibrium model, many researchers worked on it.[5] Since the goods transaction occurs under the prevailing prices, the quantity of demand and supply must be adjusted in each market. This adjustment induces the demand-supply gaps (and then a quantity constraint for the long side individual) in the markets. For the quantity constraint, the individuals reconsider the demands or supplies in the other markets. This dual decision hypothesis is a core of the disequilibrium models. The mathematical expression is as follows:

$$\tilde{x}_i = \min\{x_i^s(P, \tilde{x}_{-i}), x_i^d(P, \tilde{x}_{-i})\}, \quad \forall i,$$

where $\tilde{x}_i$ is the realized transaction of good i, $-i$ is a set of goods index except i, and P is the prevailing price vector. Subscription s means supply, and d is demand. This

(2006) and Asada et al. (2011). However, so called non-Walrasian economics, which synthesizes Keynesian and neo-classical regimes, is scarcely studied today: the exceptions are Chiarella et al. (2012, Chapter. 8, 9), Böhm (2017) and Ogawa (2019a). This seems to be because Flaschel (1999) as well as Malinvaud (1980) showed the dominance of Keynesian regime in the disequilibrium dynamics.

[3] Tobin's earlier work (Tobin (1955)) had reffered it. For the works of neo-classical monetary growth, see Sidrauski (1967a,b), Levhari and Patinkin (1968), Hadjimichalakis (1970), Benhabib and Miyao (1981) and Hayakawa (1984).

[4] For Keynes-Wicksell monetary growth, see Stein (1966), Rose (1967, 1969), Fischer (1972), Franke (1992) and Flaschel and Sethi (1996). Burmeister and Dobell (1970, Chapter. 6), Orphanides and Solow (1990) and Chiarella and Flaschel (2000) summerize neo-classical type and Keynes-Wicksell type analyses.

[5] For early studies in macroeconomic model, see Korliras (1975), Malinvaud (1977, 1980), Hildenbrand and Hildenbrand (1978) and Muellbauer and Portes (1978). In perticular, Böhm (1978), Ito (1980), Honkapohja and Ito (1980, 1982), Blad and Zeeman (1982) and Picard (1983) study dynamics of disequilibrium macroeconomics. For microeconomic features such as exchange and money, see Younès (1974), Bénassy (1975) and Grandmont and Laroque (1976). For the history of disequilibrium analyses, see Backhouse and Boianovsky (2012).

expression explicitly shows the strong spillover effect among the realized transactions. Non-Walrasian economists highlight this characteristics to distinguish the effective demand derived from the dual decisions from the notional demand derived from normal optimization problems without quantity constraints.

For disequilibrium monetary growth, Azam (1980) uses IS-LM framework and portfolio equilibrium suggested by Tobin (1969) and found that the slow price adjustment stabillizes the convergence into the steady state. However, his dynamic analysis is limited since he only shows some example paths which converge to the steady state. Sgro (1984) compares the disequilibrium growth without money to the other with money, and shows the non-neutrality of money on the steady state and that the steady state becomes a saddle-point, which are often refered in normal (equilibrium) monetary growth models. Although the dynamic analysis is conducted in detail in that paper, the gooods market is supposed to be always in equilibrium: disequilibrium dynamics is not completely analyszed. Therefore, we should construct a model which allows the disequilibria in the both goods and labor markets and conduct the complete dynamic analysis.

Our framework is based on the classical monetary mondel in Sargent (1987, Chapter. 1). We extend his model by adding the possibility of demand-supply gap in each market. As we adopt the dual decision machanism to the static model and formulate the static transactions in (dis)equilibrium , the model in this paper is the further generalization of the (dis)equilibrium dynamics of Tobinian or Keynes-Wicksell monetary growth. Notice that our model is also very similar with the sophisticated monetary growth model in Chiarella et al. (2000, Chapter. 5), which extends the non-Walrasian monetray growth model in Picard (1983). However, we omit the inventory dynamics and the flexible workforce (they allow the possibility of the over-time work), which work as the buffers and weaken the potential disequilibria. Our simplifications specifies the characteristics of non-Walrasian economics such as regime switching so that we find new characteristics of the dynamic feedback loops. Therefore, this paper should be regarded as one suggestion of a baseline model which treats non-Walrasian monetary growth, rather than the development of the existing (old) Keynesian dynamic models.

The rest of this paper is organized as follows. In Section 2, we construct the static model which defines the regimes of the temporary equilibria. The temporary equilibrium uniquely exists under the given fixed price vectors and the stock variables. In section 3, we formulate the dynamics of the stock variables and the adjustment process of wage, price and expectation. We show the feedback loops of these variables and how the dynamics is (de)stabilized. We discuss the new effect on the feedback loops: the dual-decision effect. As the dynamicall system is five dimensional in our model, we implement numerical experiment in Section 4. In Section 5, we summerize the results of our disequilibrium monetary growth model.

2 The Model

In this section, we construct a static model. Before the analysis, we set the following rules regarding the mathematical conditions and notations; unless specifically mentioned, all the functions in this paper are at least twice continuously differentiable; let $\dot{x}$ denote the time derivative of x, or $\dot{x} = dx/dt$; let f_i denote the partial derivative of function f with respect to the ith variable, that is, $f_1 = \partial f(x_1, x_2, x_3)/\partial x_1$; the double partial derivative is described as $f_{ij} = \partial^2 f/\partial x_j \partial x_i$.

The model consists of identical households, the representative firm, and the government. These economic agents trade labor, goods, and assets (money, bond and equity) under the fixed price and the fixed wage. The nominal interest rate responces to the disequilibrium immediately so that the asset market equilibrium is always ensured unlike the real markets. As the capital stock K is fixed in the static model, we describe the static equilibrium (temporary equilibrium) in the intensive form by dividing the quantity variables by K.

2.1 The firm

The representative firm produces goods Y using the employed labor E and their own capital stock K. The firm's production technology is expressed as the followng neoclassical type production function F:

$$Y = F(K, E), \text{where } \begin{aligned}[t] & F(0,0) = 0, \\ & F_1, F_2 > 0, \\ & F_{11}, F_{22} < 0, \\ & F(\lambda K, \lambda E) = \lambda F(K, E), \ \forall \lambda > 0. \end{aligned} \tag{2.1}$$

Each time the firm gains the net real revenue $F(K, E) - \delta K$ and pays the real wage w to the employees and devidend to the shareholders. We suppose that the firm does not reserve the money so that the net real devidend flow ρK is

$$\rho K = F(K, E) - wE - \delta K, \tag{2.2}$$

where $\delta > 0$ is the cosntant positive depreciation rate. The firm intends to maximize the real devidend flow ρK in each time, and therefore the firm solves the following profit maximization problem with quantity constraint:

$$\max_E F(K, E) - wE \ \text{ subject to } F(K, E) \leq Y^d \text{ and } w, K \text{ are given.} \tag{2.3}$$

Notice that when the demand quantity constraint $F(K, E) \leq Y^d$ is bounded, the solution is different from the usual maximum which follows the first order condition. The solution of E is the labor demand fucntion L^d, which is composed of the two different labor demand:

$$L^d = \min\{L^{d*}, \tilde{L}^d\}, \text{ where } L^{d*} = (F')^{-1}(w; K) \text{ and } \tilde{L}^d = F^{-1}(Y^d; K) \tag{2.4}$$

The first one L^{d*}, which is an interior solution and derived from the first order condition without the demand constraint, is the *notional* labor demand. Since the production function is supposed to be linear homogeneous, L^{d*} could be rearrenged $v(w)K$, where $v' < 0$. The second one (corner solution) is the *effective* labor demand since it depends on the quantity of the goods demand.

For the goods supply, we use the variable Y^s as follows:

$$Y^s = \min\{F(K, L^{d*}), \ F(K, L^s)\}. \tag{2.5}$$

The firm purchases the produced goods for investment issuing equities:[6]

$$P\dot{K} = \dot{V} - V\pi, \tag{2.6}$$

[6] For the case in which the firm uses the debt financing, see Picard (1983) and Chiarella et al. (2000, Chapter. 5).

where V is the total amount of nominal equity value of the firm, P is the price of the goods, and π is the expected inflation rate. The real equity is equal to the exisiting capital of the firm's own which is valued in the market:

$$V/P = qK, \tag{2.7}$$

where q is the market-valued price of the exisiting capital.[7] The firm invests following the investment function below:

$$I = \dot{K} + \delta K = \psi(q-1)K + (n+\delta)K, \quad \psi(0) = 0, \quad \psi > -(n+\delta), \quad \psi' > 0. \tag{2.8}$$

This investment function implies that the capital and the population grow at the same rate when $q = 1$ and is used in Chiarella and Flaschel (2000). q is what is called "Tobin's (average) q," and it depends on the (expected) net cash flow stream on divident payment in future. If the quantity constraint on goods demand could be expected, q at the moment t would be written as follows:

$$q(t)K(t) = \int_t^\infty \mathbb{E}_t \left[\rho(\tau)K(\tau)e^{-(r(t)-\pi(t))\tau}\right] d\tau,$$

where $\mathbb{E}_t$ is expectation operator at t and r is the nominal interest rate.

However, the calculatable forward-looking expectation on the goods demand does not seem suitable in "Keynesian" disequilibrium models (Murakami, 2016). As Neary and Stiglitz (1983) shows, the pessimistic expectation on the goods demand in future might shrink the actual goods demand in both present and future. This is a kind of self-fulfilling prophecy or sun-spot equilibrium[8]. We should consider that the goods demand today affects the expectation about the future and that the ample goods demand would make the expectation optimistic.

In this paper, we use the following ad-hoc function:[9]

$$q = q(\rho, Y^d/Y^s, r-\pi), \; q_1 > 0, \; q_2 > 0, \; q_3 < 0. \tag{2.9}$$

The term $q_2 > 0$ argues that the today's excess demand ratio Y^d/Y^s is a cryterion for the expected goods demand in future. As is in equation (2.8), the investment is directly affected by q. Our formulation implies that the investment depends both on the return rate terms $r-\pi$ and ρ and on the goods demand expectation term Y^d/Y^s. This is a kind of reconciliation between Wicksellian- and Keynesian investments.

As our stady is an extention of equilibrium models, we suppose that the q fucntion in equation (2.9) be equal to the normal q function in equilibrium theories such as Yoshikawa (1980) and Hayashi (1982), as long as the situation is "Walrasian."

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[7]We could use the notations for the price of equilities and the issued equities, such as P_eE in Chiarella and Flaschel (2000) and Asada et al. (2011) instead. Although this notation is more correct and intuitive, we use the notation in equation (2.7) for the simplicity of calculations.

[8]For the discussion of this phenomenon in equilibrium theory, see Azariadis (1981), Woodford (1986) and Farmer (1999). Howitt and McAfee (1985) refers to the sun-spot equilibrium as the business cycle driven by *Animal Spirits*. For the simplicity, they often utilize the Markov process, which means the state variable today is the most important factor to determine how optimistic (pessimistic) the expectation is. Our formulation is suitable for an extention of the investment function of Bénassy (1984), which utilizes the adoptive expectation.

[9]For the simple example of formulation of q in this paper, see AppendixA.

Assumption 1. The q function in equation (2.9) satisfies the following condition:

$$q(\rho, 1, r - \pi) = 1 \Leftrightarrow \rho = r - \pi + \xi, \tag{2.10}$$

where $\xi > 0$ is a constant risk premium.

This assumption says that when the goods market is in equilibrium (which implies the expectation on excess goods demand would be stationary), the condition for $q = 1$ is equivalent to the normal condition in equilibrium theory such as Sargent (1987, Chapter. 1).

2.2 Households

The homogeneous households supply labor and buy goods for consumption. The labor is inelastically supplied so that the labor supply L^s is equal to the population which grows constantly:[10]

$$\dot{L}^s/L^s = n > 0, \quad n = \text{const.} \tag{2.11}$$

They hold assets, which consists of money, bond, and equity. The real asset holding A is defined as follows:

$$A = (M + B + V)/P, \tag{2.12}$$

where M is the holding money and B is the government issued bond.

The households plan the consumptions and the savings and express them under the budget constraint of the perceived real disposable income concept. The perceived real disporsable income Y_{di} is equal to the real wage payment on the reallized employment wE *plus* divident payments ρK *minus* total real tax collection T *plus* the real return on the bond rB/P *minus* the anticipated capital loss on the real value of government debt $(M + B)P^{-1}\pi$ *plus* the rate at which the real value of equities $\dot{V}/P$ *minus* the rate at which the firm is issuing equities to finance investment $\dot{K}$.

$$\begin{aligned} C^d + \dot{A}^d = Y_{di} &\equiv wE + \rho K - T + rB/P - (M + B)\pi/P + \dot{V}/P - \dot{K} \\ &= Y - \delta K - T + rB/P - A\pi, \end{aligned} \tag{2.13}$$

where C^d is expressed consumption demand and $\dot{A}^d$ is equal to the *ex ante* saving.

In this paper, we omit the utility-maximization problem and suppose the consumption demand function follows Azam (1980) and Sargent (1987, Chapter. 1):

$$C^d = C^d(Y_{di}, A, r - \pi) > 0, \quad 0 < C_1^d < 1, \ C_2^d > 0, \ C_3^d < 0, \tag{2.14}$$

and the aggregate consumption function C^d is linear homogeneous to the aggregate variables Y_{di} and A. The first term on partial derivative C_1^d shows that the consumption is increasing in the perceived real disporsal income and the marginal propensity to consume out of Y_{di} is positive but less than unity.[11] The fact that the planned consumption depends on the realized income implies that C^d is an effective demand function. The term

[10] If the labor supply is affected by the quantity constraint on the goods purchases (if the labor supply become the effective supply), the multiplier effect emerges on the goods supply as well as the goods demand (Barro and Grossman, 1971).

[11] As is in Sargent (1987, Chapter. 1), it gets along with Clower (1965), which shows that the consumption demand is the function of the realized income. This is an interpretation for "dual decision hypothesis."

on C^d_2 shows what is called real balance effect (*Pigou* effect) on consumption. The third term on C^d_3 is about the substitution effect of future consumption.

To clarify the implications induced in the latter sections, we formulate the consumption demand fucntion as follows:

$$C^d = f^c(A, r - \pi, Y_{di})Y_{di},\ 0 < f^c < 1,\ f^c_1 > 0,\ f^c_2 < 0,\ -f^c/Y_{di} < f^c_3 \leq 0, \tag{2.15}$$

and the propensity-to-consume function f^c is homogeneous with degree zero to A and Y_{di}. The negativity of f^c_3 is about the substitution effect, but this effect is not stronger than the income effect.

For the simplicity, we suppose that the consumption demand function is increasing in A and π.

Assumption 2. When $\pi > 0$ holds, the following condition holds:

$$\epsilon_j > (1 + \epsilon_{Ydi})\pi A/Y_{di},\ j = A, \pi, \tag{2.16}$$

where $\epsilon_j = (\partial f^c/j) \cdot (j/f^c)$, or j elasticity of consumption propensity.

This assumption implies that the effect of capital loss $A\pi$ in the perceived disposable income is not strong for consumption demand.

2.3 The government

The government purchases the goods G and pays net real interest rB/P, by collecting real tax T and issuing bonds and money.

$$G + rB/P = T + \dot{B}/P + \dot{M}/P.\ \ \dot{M}/M = \mu > 0, \quad \mu = \text{const.} \tag{2.17}$$

In this paper, we suppose the money supply grows at the cosntant rate for the simpicity. Following Sargent (1987) and Asada et al. (2011), we suppose the government purchase is proportional to the existing capital:

$$G = gK, \quad g = \text{const} > 0. \tag{2.18}$$

The tax payments are imposed on the household's net real income cash flow for the positive constant rate plus the same amount of the net real interest:

$$T = \tau_w wE + \tau_\rho \rho K + rB/P.$$

We suppose that the tax rate is common $\tau_w = \tau_\rho = \tau > 0$ so that the taxation is proportional to the realized income.

$$T = \tau(Y - \delta K) + rB/P. \tag{2.19}$$

For the static analysis, the following condition is satisfied:

$$dM = -dB,$$

which says that the government or the central bank implements open market operations. Finally, we define the effective goods demand as follows:

$$Y^d = C^d + I + G = C^d + \dot{K} + \delta K + G. \tag{2.20}$$

Notice that the investment which are financed by issuing equities and the goverment purchase is not quantity rationed: if $Y < Y^d$, then the consumption is rationed.[12]

[12] This assumption is also utilized by Böhm (1978). Ogawa (2019b) analyzes the case in which the investmemt is quantity constrained, using a two-sector framework.

2.4 Asset market

The equation (2.12) shows that the aggregative asset is consist of the money M, the bond B, and the equity V. Following Sargent (1987, Chapter. 1), the households desire a division of their asset between M and $B+V$ (the latter two assets are supposed perfect substitutable) and this division is described as the following functions:

$$M^d/P = f^m(r, Y, A) \tag{2.21}$$

$$(B^d + V^d)/P = f^b(r, Y, A) \tag{2.22}$$

$$(M^d + B^d + V^d)/P = A \tag{2.23}$$

Then, the portfolio equiibrium condition is characterized by the following money balance condition:

$$M/P = M^d(r, Y)/P, \quad M_1^d < 0, \quad M_2^d > 0, \quad M^d(r, Y/K) = M^d(r, Y)/K \tag{2.24}$$

where M^d/P is the real money demand fucntion.[13] The partial derivative conditions are supposed to express the speculative motive and the transaction motive for monay holding. For the boundedness and positiveness of r, we suppose that

$$\forall X > 0, \quad \exists r > 0, \quad \lim_{Y \to 0} M^d(r, Y) = X \tag{2.25}$$

2.5 Temporary equilibrium

From the formulations above, we define a temporary equilibrium of goods, labor, and money. As the capital K is given in the short term, we utilize the intensive form description by dividing variables by K. Note $y = Y/K$, $c^d = C^d/K$, $i = I/K$, $l^j{=}L^j/K$, $e = E/K$, $f(e) = F(1, e)$, $m = M/(PK)$, and $b = B/(PK)$.

Definition 1. A temporary equilibrium is the solutuon $(y, e, m) \in (0, f(l^s)] \times (0, l^s] \times \mathbb{R}_{++}$ for the following system:[14]

$$y = \min\{y^d, f(l^{d*}), f(l^s)\}, \tag{2.26}$$

$$e = \min\{l^{d*}, \tilde{l}^d, l^s\}, \tag{2.27}$$

$$m = m^d(r, y), \tag{2.28}$$

where $(l^s, m, b, g, w, \pi) \in \mathbb{R}^5_{++} \times \mathbb{R}$ is given and $y^d = c^d + i + g$.

Proposition 1. when the consumption propensity is not too large and the consumption and the investment is not too sensitive to q, the temporary equilibrium $(y, e, m) \in (0, f(l^s)] \times (0, l^s] \times \mathbb{R}_{++}$ is uniquely determined for any given $(l^s, m, b, g, w, \pi) \in \mathbb{R}^5_{++} \times \mathbb{R}$.

[13] The last equation says the money balance condition could be rearranged with the real balaces ratio $M/(PK)$. For the brief discussions of the money demand fucntion M^d, see Burmeister and Dobell (1970, Chapter. 6)

[14] The dynamics of each real asset is also determined. However, the accumulation demand such as $\dot{m}^d$ does not directly appears in the system: the actual dynamics of money and bond holdings are determined by the supply side. Notice that, however, the accumulation demand indirectly works in the static model since the asset accumulation demand inversely works on the expressed consumption demand c^d.

Proof. Obviously, the employment e is uniquely determined when the production y is determined since $y = f(e)$ always holds and f is monotonically increasing. When the exogeneous variables are omitted, the realized transaction-of-goods function is reduced into $y = y(y, q, r)$ since $y_{di} = Y_{di}/K = (1-\tau)(y-\delta) - \pi(m+b+q)$. As q is the function of y, y^d, $y^s = \min\{f(l^{d*}), f(l^s)\}$ and r, the endogenous variables could be reduced into the two variables, (r, y).

To prove the unique determination of (r, y), we use the IS-LM framework following Azam (1980) and Sargent (1987). First, we can check that y^s is exogenously determined in the short term so that we should prove that the production which satisfies $y = y^d(y; r)$ uniquely exists for all $r > 0$. From equations (2.8) and (2.14), $y^d(0; r) > 0$. The solution for $y = y^d$ exists when the two curves $y = y$ and $y = y^d(y; r)$ uniquely crosses, and the sufficient condition for it is

$$1 > (1-\tau)f^c - (q_1 + q_2)(c_q^d + i_q), \tag{2.29}$$

where $c_q^d = (\partial c^d/\partial q)$ and $i_q = (\partial i/\partial q)$.[15] This condition is satisfied when the consumption propensity is not too large and the consumption and the investment are not too sensitive to q. We suppose this stability condition holds hereinafter. From the equation (2.14), the "IS" curve $r_{IS}(y)$ which satisfies $y = \min\{y^d(r), y^s\}$ is downward sloping when $y = y^d$ and vertical when $y = y^s$ on the y - r plane (see Figure 1).

The LM curve $r_{LM}(y)$ satisifies equation (2.24) and is upward sloping. From the inequality $y^d(0; r) > 0$, there exists $\underline{y} > 0$ which satisfies $r_{IS} \to \infty$ for $y \to \underline{y}$. Therefore IS curve and LM curve uniquely crosses and the solution for the temporary equilibrium (r, y) is the crossing point.

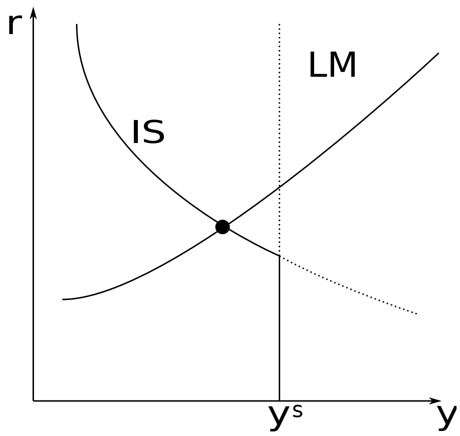

Figure 1: IS-LM interpretation for the temporary equilibrium

□

The figure implies the mechanism to determine the regime of the economy, as well as the exisitence of a tenporary equilibrium.[16] The segment of IS curve on which the slope is downward, is the region in which $y = y^d$ and $e = \tilde{l}^d \leq l^s$. When LM curve across IS curve on this segment, the output and the employment is determined by the goods

[15]Strictly speaking, the slope of $c^d(y) + i(y, y^d(y)) + g$ under $y = y^d$ is $(1-\tau)(f_3^c y_{di} + f^c) + (q_1(1 - w/f') + q_2)(f_1^c y_{di} - \pi f^c + \psi')$.

[16]IS-LM description for disequilibrium macroeconomics is shown in Bénassy (1983) and Sneessens (1984).

demand. On the other hand, the employment is determined following the goods supply constraint when LM curve across the vertical segment of IS curve. How the two curves cross is affected by the exogenous variables.

From the budget constraints of the three economic individuals, we derive the extended "Walrasian law." Aggregating equations (2.2), (2.13) and (2.17),

$$Y^d - Y = C^d - C = \dot{A} - \dot{A}^d, \tag{2.30}$$

which means the excess goods demand is equal to the difference between the realized (or *ex post*) saving and the expressed (or *ex ante*) saving.[17]

2.6 The regimes and comparative statics

The realized production and the realized employment are determined by the maginitude correlation among y^d, $f(l^{d*})$, and $f(l^s)$.

We define the regimes of the economy following Malinvaud (1977):

- **Keynesian unemployment (KU)**

  In this regime, the goods production is constrained by the effective demand y^d and the effective labor demand is less than the supply. The involuntary unemployment happens due to the insufficient goods demand, and the conditions are

$$y^d \leq y^s = \min\{f(l^{d*}), f(l^s)\}. \tag{2.31}$$

- **classical unemployment (CU)**

  Although this regime also have involuntary unemployment, the unemployment mechanism is different from KU. The firm restricts the employmemt due to the high wage w:

$$y = f(l^{d*}) \leq y^d, f(l^s). \tag{2.32}$$

- **repressed infration (RI)**

  In this regime, the production is constrained since the labor supply is insufficient. The both markets have excess demands and

$$y = f(l^s) < y^d, f(l^{d*}). \tag{2.33}$$

- **equilibrium(EQ)**

  When the both markets are in equilibrium, the economy belongs to equilibrium regime.

$$y = y^d = f(l^s). \tag{2.34}$$

  In particular, *Walrasian equilibrium*(WE) is the regime in which the economy is at EQ regime and $f(l^s) = f(l^{d*})$. In WE, the notional demand and the effective demand match.

[17] This equation is the extention of the rearranged Walrasian laws in Azam (1980, equation (11)) and Ogawa (2019b, equation (2.20)).

The economy always belongs to some regime on the above, and what regime realizes is determined by the set of the five exogeneous variables (l^s, m, b, w, π) in the static model. To simplify the comparative statics, we use some natural assumptions.

Assumption 3. The slope of LM curve is not too steep and the goods demand is not too sensitive for the interest rate: $|m_2^d/m_1^d|$, f_2^c, and q_2 is sufficiently small so that $(\partial y^d)/(\partial y) > 0$ always holds.

Assumption 4. The effective goods demand is strongly affected by the realized income so that $(\partial y^d)/(\partial y) + (\partial y^d)/(\partial y^s) > 0$ holds when $y = y^s < y^d$.

The large goods supply declines the effective goods demand as the invest demand weakens. However, the large goods suppy also means the large realized income when $y = y^s < y^d$. Assumption 4 says that the latter impact is the stronger for the effective goods demand.

Now we move to comparative statics. We utiize IS-LM framework again, and see Appendix B for the detailed calculations.

In the case $y = y^d$, IS curve is downward sloping and shifts by the change of the exogeneous variables including policy parameters g and τ. Using the totally difference techniques, we calculate the equatin for IS-LM temporary equilibrium as follows:[18]

$$r_{IS}(y; \underbrace{m, b, g, \pi}_{\oplus}, \underbrace{w, y^s, \tau}_{\ominus}) = r_{LM}(y; \underbrace{m}_{\ominus}) \tag{2.35}$$

The signatures below the exogenous variables show the signs of partial differentiations. This equation is depicted in Figure 2.

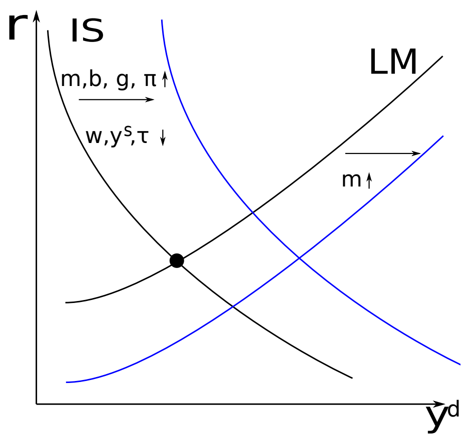

Figure 2: Comparative statics under $y = y^d$

In the case $y = y^s$, meanwhile, IS curve become a vertical line. the size of production is determined by w or l^s and any other changes of the exogeneous variables does not affect the production.

[18]Notice that $dm = -db$ holds when we concider about the effects of monetary policy. When the monetary authority implements an open market operation, IS curve would not shift.

Therefore, we can formulate the realized production as the below functions:

$$y^d = y^d(\underbrace{m, b, g, \pi}_{\oplus}, \underbrace{w, y^s, \tau}_{\ominus}) \tag{2.36}$$

$$f(l^{d*}) = v(\underbrace{w}_{\ominus}) \tag{2.37}$$

$$f(l^s) = f(\underbrace{l^s}_{\oplus}) \tag{2.38}$$

Using these equations, we can illustrate the regime dividings on a plane. As the number of the exogeneous variables is too much, Figure 3 shows the simple version of the regime dividing.

Notice that high real wage always induces the both types of unemployment, while other disequilibrium models such as Böhm (1978), Weddepohl and Yildirim (1993) and Ogawa (2019b) show the lower real wage induces Keynesian unemployment. In our model, the consumption demand is not affected by the income distribution and investment fucntion is decreasing in w since the high wage rate lowers profitability. These formulations make y^d decreasing in w.[19]

Intuitively, the expansion of g and the shrink of τ remedy Keynesian unemeployment regime since y^d increases. Notice that, however, these fiscal policies increase the production only when the economy is trapped into Keynesian regime. The expansion of real assets m and b stimulats the real economy throuth the *Pigou* effect channel (y^d increases), and the money market is also stimulated when m is expanded. The equilibrium

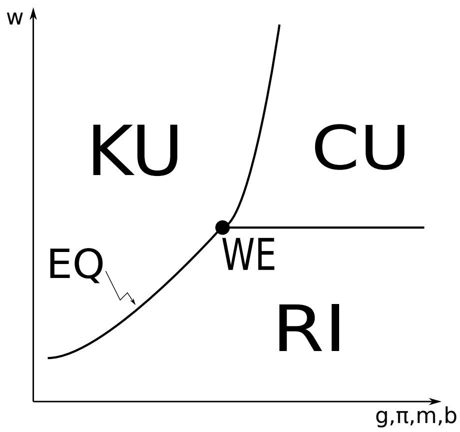

Figure 3: Regime dividing on g, π, m, b - w plane

regime locates between KU regime and RI regime as the border, and Walrasian equilibrium is at the center among the regimes. These regional characteristics are common with disequilibrium models; see Bénassy (1986) for instance.

3 Dynamic analysis

We have seen how the statically given exogeneous variables (l^s, m, b, w, π) determine the regime of the temporary equilibrium. In this section, we analyze the dynamics of these

[19]Several Keynesian monetary growth models like Chiarella and Flaschel (2000) and Asada et al. (2011) adopt income distribution - effective demand system by setting the two classes; workers and asset holders.

variables in order to check the stability of balanced growth path, to see how the regime changes in the growth path, and to ascertain which regime is dominant in our economy.

To complete the dynamical system, we should formulate the dynamics of the wage, the price, and the expectation for the price change. In this paper, we use the Phillips curve methodology, which is often utilized in Keynesian monetary growth theories.[20] We also refer to Fischer (1972), which presents the wage - price dynamics for Keynes-Wicksell models. The change of price P and the nominal wage W are formulated as following Walrasian adjustments:

$$\dot{P}/P = \pi + \nu_P(y^d - y^s), \ \ \nu_P = \text{const} > 0, \tag{3.1}$$

$$\dot{W}/W = \pi + \nu_W(l^{d*} - l^s), \ \ \nu_W = \text{const} > 0. \tag{3.2}$$

As Orphanides and Solow (1990) pointed out, these Fischer relations enable the market-clearing steady state with price inflation. Notice that the nominal wage adjustment is intended to match the labor supply and the notional labor demand. This formulation implies that the wage dynamics would adjust the employment to the "potential" level in Chiarella et al. (2000, Chapter. 5).[21]

The development of inflation expectation, or the dynamics of π, is described as the combination of the adaptive- and the forward-looking development:

$$\dot{\pi} = \beta[\alpha(\dot{P}/P - \pi) + (1-\alpha)(\pi_0 - \pi)], \quad \beta > 0, \quad 0 \le \alpha \le 1, \tag{3.3}$$

where π_0 is the steady state value of π. If $\alpha = 0$, then the economy is characterized with myophic perfect foresight. If α becomes unity, on the other hand, the expectation adjustment is completely adaptive.

3.1 Dynamical system

From Equations (2.8), (2.11), (2.17) - (2.19) and (3.1) - (3.3), the dynamics of (l^s, m, b, w, π) is described as follows:

$$\dot{l}^s = l^s(n - \psi - n) = -l^s\psi \tag{3.4}$$

$$\dot{m} = m\{\mu - \pi - \nu_P(y^d - y^s) - \psi - n\} \tag{3.5}$$

$$\dot{b} = \{g - \tau(y - \delta) - \mu m\} - b\{\pi + \nu_P(y^d - y^s) + \psi + n\} \tag{3.6}$$

$$\dot{w} = w\{\nu_W(l^{d*} - l^s) - \nu_P(y^d - y^s)\} \tag{3.7}$$

$$\dot{\pi} = \beta[\alpha\nu_P(y^d - y^s) + (1-\alpha)(\mu - n - \pi)] \tag{3.8}$$

Notice that the dynamcal system is consist of "differential equations with discontinuous righthand sides" (Filippov, 1988) since the goods supply y^s and the realized production y (and then y^d) is determined through the minimum function. In dynamics, the economic regime swithes several times and the right hand sides of the above dynamic equations also switch. Therefore, the normal analytical tools will be sometimes invalid for our system.

[20] See Chiarella and Flaschel (2000), Asada et al. (2006), and Asada et al. (2011). For the details of wage-price modules, see Chiarella et al. (2005, Chapter. 5)

[21] Chiarella et al. (2000, Chapter. 5) use the term "potential" as the level at which the production is conducted under the marginal profit principle. Ogawa (2019a, Appendix B) uses the bargaining framework to justify the nominal wage adjustment in which the employment would go into the notional demand level.

The (usual) steady state condition of this dynamical system is

$$y^d = y^s \tag{3.9}$$
$$l^{d*} = l^s \tag{3.10}$$
$$q = 1 \tag{3.11}$$
$$\pi = \mu - n \tag{3.12}$$
$$0 = g - \tau(y - \delta) - \mu(m + b) \tag{3.13}$$

The first two equations say that the steady state is on Walrasian equilibrium regime.[22] Furthermore, the real interest rate plus the risk premium $r - \pi + \xi$ equals to the real return rate of holding capital ρ^* from equation (2.10). Therefore, the "Wicksellian" equilibrium condition $r - \pi + \xi = \rho$ as well as the real equilibrium conditions is satisfied at the steady state.

The last equation in the steady state conditions says that $\dot{B}/B = \dot{M}/M = \mu$, or the nominal bond is issued at the same rate of the money printing. Then the net government deficit $G - \tau(Y - \delta K)$ grows at the same rate as n at the steady state.

Definition 2. The steady state value of dynamic variables is the set $(l^s_0, m_0, b_0, w_0, \pi_0) \in \mathbb{R}^4_{++} \times \mathbb{R}$ which satisfies equations (3.9) - (3.13).

Proposition 2. The steady state value $(l^s_0, m_0, b_0, w_0, \pi_0)$ uniquely exists.

Proof. Obviously, $\pi_0 = \mu - n$ is uniquely determined. Let x_0 denote tha value of x when the exogeneous variables are $(l^s_0, m_0, b_0, w_0, \pi_0)$. The real equilibrium equations (3.9) and (3.10) imply that $y^d_0 = v(w_0) = f(l^s_0)$, which generates two independent equations. From equation (2.10), $\rho_0 = y_0 - l^s_0 f'(l^s_0) - \delta = r_0 - \pi_0 + \xi$ holds. So we have the four independent equations for the four variables (l^s_0, m_0, b_0, w_0). □

Notice that our dynamical system is discontinuous and therefore the dynamics could stop elsewhere; what is called pseudo-equilibrium in Filippov (1988). Our system also has the possibility of this pseudo-steady state.

3.2 Stability and basic feedback loops

When we check a local stability condition for the dynamical system, we should explore the Jacobian matrix $\mathbf{J}$, which is the coeffcent matrix of the linerized dynamical system at the steady state. Notice that, however, the values of the factors of $\mathbf{J}$ change in the different regimes since the steady state is located on the intersection of the regime boundaries, on which the vector field becomes discontinuous. Due to this discontinuity, the local stability analysis with the Jacobian matrix is difficult for the high dimensional system: we could not use the graphical analysis to detect the shapes of the (un)stable manifolds. [23]

[22] Notice that the regime in the steady state often depends on the formulations of dynamical system in the disequilibrium school. If we adopt the Walrasian adjustment process both in wage and price dynamics, as is in equation (3.7), the persistent existences of the demand-supply gaps in the both markets are easily enabled. The regime in the steady state is determined by the other dynamic equations and the goods market clearing is ensured in our system. This is because our model is based on neo-classical monetary growth in Sargent (1987): if we would adopt frictions such as searching process, another result was easily gained.

[23] Exceptionally, the following theorem is worthwhile for our system: when the two-dimensional dynamical system is locally stable for all three regimes, then the whole system is also so in our model

Instead, we concentrate on how the stabilizing - destabilizing feedback loops of each economic variabls works grobally. Each feedback channel has been summerized in Chiarella et al. (2000), Chiarella et al. (2005) and Asada et al. (2006). We should notice that these channels might work differently in each disequilibrium regime; one feedback loop stabilizes in *Keynesian* regime but destabilizes in other regime, for instance.

1. *The Keynes (and Pigou) effect.* When the price level goes up, the nominal (and then the real) interest rate in LM market become high. The high real interest rate decreases q and today's consumption demand. Furthermore, the low q weakens the investment and the consumption demand again; this is the Keynes effect. The high price level also depreciates the real asset holding A, which weakens the consumption demand. This is the result of the Pigou effect. The decline of the effective demand induces the low price inflation, and therefore the Keynes effect and the Pigou effect stabilize the price dynamics.

2. *The Mundell effect.* As our model formulation adopts IS-LM framework, this effect works as usual. When the economy expects the higher inflation, the incentive to holding money lowers and the capital accumulation is aroused since q increases. The increase in q expands the goods demand (notice that the actual production expands only if $y = y^d$), and then the actual price inflation is stimulated. This price inflation pulls up the inflation expectation as long as $\alpha \neq 0$, or the expectation has an adoptive characteristics. Thus, the Mundell effect destabilizes the expectation dynamcis.

3. *The real wage effect.* As shown in the static model analysis, the effective goods demand as well as supply is decreasing in the real wage w. This negativity induces an ambiguity with the price inflatiom against the real wage dynamics. Notice that if the goods demand is more sensitive to the real wage than the suppy, the high real wage lowers the price inflation pressure. This unstable feedback is called *the Rose effect*, as is in Flaschel and Sethi (1996). Furthermore, the instability become stronger in Keynesian regime ($y = y^d$); see Appendix B. As we suppose the Walrasian adjustment process in the labor market, meanwhile, the nominal wage moves into an opposite direction against the real wage. Summing up, the direction of the real wage adjustment is ambiguous when the price dynamics is not too slow. Therefore, the real wage feedback loop in the two Walrasian adjustments in equations (3.1) and (3.2) could be both stabilizing and distabilizing.

The feedback loops above are same as the ones of the ordinal Keynesian dynamic models. The next one is characteristic with the non-Walrasian regime switching phenomenon.

4. *The dual-decision effect.* To specify this effect, we should see the two feedback loops of y^s; the dynamics of w and l^s. The first case $y^s = v(w)$ is included in the loop in *the real wage effect.* Suppose that $y = y^d$. As the large y^s decreases y^d and the price goes down, the real wage increases. Therefore the feedback loop of $v(w)$ is stable, as long as we see the real economy and $y = y^d$ holds. The second case $y^s = f(l^s)$ is in contrast. When $y = y^d$, the high y^s directly lowers y^d/y^s and decreases the investment (and then increases l^s). As the excess demand term works

(Eckalbar, 1980). However, this sufficient condition could be violated by the unstable expectation dynamics and our model is five-dimensional and therefore we do not adopt it. For the Jacobian matrix, see Appendix B.

in the investment function, the feedback loop of l^s is destabilized by y^s. This is a kind of Harrodian instability.
These (in)stabilities of y^s feedback loops are damped when $y = y^s$, comparing with the case $y = y^d$. In the first case $y^s = v(w)$, the large y^s obviously increases y, and the increased y enlarges y^d through the effective demand principle $y^d = y^d(y)$. This new path works as the price inflation pressure, which is in the opposite way against the stable feedback in the case $y = y^d$. This *dual-decision effect* works similarly in the case $y^s = f(l^s)$. The large y^s (and then large y) increases the investment and has a negative pressure of l^s.

As the descriptions in the item are mathematical, we should discuss how the dual-decision effect works in detail. As we adopt the dual decision hypothesis, the goods demand is *effective* in the sense it depends on the realized income y. In ordinal Keynesian models, the production and income is always determined by the effective goods demand. The supply side is usually regarded as the criterion of the potential production and the gap between the potential and the realized production (e.g., capital utilization rate) is an important issue for macrodynamics; not the goods supply itself. In contrast, the production is always determined by the supply side in the usual neo-classical models since full capacity is realized.

Non-Walrasian models synthesize the two perspectives. What is different from an ordinal Keynesian model? The answer: we could treat the supply side directly as well as the demand side. In our model, the effective demand principle always works; $y^d = y^d(y)$. However, the realized production could be determined by the supply side and $y^d = y^d(y^s)$ holds in that case. Certainly the (relatively) large y^s intends the large gap between the potential and the realized production and it usually decreases the effective demand since the investment demand declins. When $y = y^s$, however, it also means the large realized income; the *effective* demand is aroused. This composite effect complicates the pure feedbacks in the Keynesian case $y = y^d$. The dual-decision effect works as both a stabilizer and a destabilizer for the feedback loops.

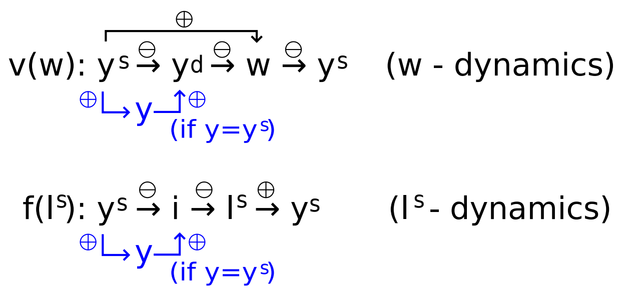

Figure 4: The y^s feedback loops with the dual-decision effects

Although the stability analysis of our high dimensional dynamical system is difficult, we can derive the sufficient condition for unstability which is so limited but similar with that of Chiarella et al. (2000, Chapter. 5).

Proposition 3. The steady state will be unstable when the speed of the expectation adjustment β is large, the expenctation adjustment is near to adaptive, and the real wage effect is moderately unstable.

Proof. Take the 2 by 2 principal minor which is consists of the forth and the fifth rows (and columns) $\mathbf{J}_{45}$ of the Jacobian matrix. If $\nu_W v'(w_0)(\nu_P w_0)^{-1} < (d/dw)(y^d - y^s) < 0$

and $\alpha\nu_P(\partial y^d/\partial\pi) > 1-\alpha$ hold, then $\det\mathbf{J}_{45} = \beta\ominus$, where $\ominus < 0$; see Appendix B. When β is sufficiently large, the sum of all the 2 by 2 principal minors would be negative. Then Routh-Hurwitz stability condition is violated in every regime. □

From this propostion, we confirm our dynamics overlaps the usual (old) Keynesian models. The sensitive adoptive adjustment of the inflation expectation usually destabilizes the monetary dynamics(Burmeister and Dobell, 1970; Chiarella and Flaschel, 2000). However, the instablility condition hardly holds in our model, as shown in the next section.

4 Numerical experiments

In this section, we implement numerical experiments of the canonical disequilibrium monetary growth model and simulate the dynamical system presented in the previous section. As we can not use the graphical and analytical deductions for five dimensinal dynamical system, the discontinuity should be accurately detected and dealed properly in the simulation. Therefore, we use `DISODE45` algorithm of MATLAB, produced by Calvo et al. (2016).

In the beginning, we should specify the parameter values and the functional forms. In this paper, we utilize the empirical studies in Flaschel et al. (2001), who construct a disequilibrium monetary growth model and specifies the parameter values following postwar US data.

First, we formulate the fucntions in our system. We suppose that the production function is Cobb-Douglas type: $F(K,E) = K^aE^{1-a},\ a > 0$. Following the ordinal neo-classical studies, we set $a = 0.34$ which says the profit share rate is near one third around the steady state. We consider E as the efficient labor which includes the labor productivity and therefore n is the sum of the population growth rate and the labor productivity growth rate. In the next, we formulate the q function as follows: $q = (y^d/y^s)^\gamma(\rho/(r-\pi+\xi)),\ \gamma > 0$. This formulation is compatible with Appendix A. The consumption demand function c^d is estimated from US postwar data as is in Appendix C and is $c^d = 0.6483\exp(0.9044(r-\pi))((m+b+q)/y_{di})^{0.1866}y_{di}$. In this estimation, we arbitrarily set $\tau = 0.15$ and $\nu_P = 0.010$, and therefore we use this value in the following simulation.

The rest of the functions are set same as the linearized functions in Flaschel et al. (2001): $m^d = h_1y + h_2(r_0 - r)$; $\psi = i_1(\rho - r + \pi - \xi) + i_2((y^d/y^s) - 1)$. The parameters also follows that empirical work: $h_1 = 0.1769$, $h_2 = 2.1400$, $i_1 = 0.1363$, $i_2 = 0.0340$ and $\nu_W = 0.0958$.

Second, we set the residual parameters to make the steady state values of y and r compatible with the empirical study. Flaschel et al. (2001) shows that $n = 0.0081$, $\mu = 0.0154$, $\delta = 0.0468$ and $\xi = 0.1500$ and we utilize them for calculation.

The steady state value of y_0 and r_0 (and of course $(l_0^s, m_0, b_0, w_0, \pi_0)$) now depend on the undecided parameter g. Since Flaschel et al. (2001) shows the value of them as $y_0 = 0.5091$ and $r_0 = 0.0221$, we set g as 0.1250 which indicates

$$y_0 = 0.6276,\ r_0 = 0.0239,\ l_0^s = 0.4937,\ m_0 = 0.1110,\ b_0 = 2.3491,\ w_0 = 0.8390,\ \pi_0 = 0.0073.$$

Using these results, finally, we estimate the value of γ. Taylor expansion around the steady state implies

$$\psi \simeq \psi'\rho_0^{-1}(\rho - r + \pi - \xi) + \psi'\gamma((y^d/y^s) - 1).$$

Then we gain $\gamma = i_1/(i_2\rho_0) = 1.4976$.

4.1 Two examples: persistent Keynesian unemployment and cyclical growth

As we have not set the parameters α and β, we should explore how they affect the stability of the steady state. Before that, we introduce two characteristic examples here.

First, we set the initial value as $(l^s, m, b, w, \pi) = (0.5159, 0.1157, 2.2440, 0.8590, 0.0071)$ and the adjustment parameters as $\alpha = 0.400$ and $\beta = 0.280$. The simulated path are illustrated in figures 5 and 6. The dashed line in figure 5 shows the steady state value, and the dots in figure 5 and the vertical lines in figure 6 correspond with the discontinuous points of the dynamical system. These figures show that the economy is initially in classical unemployment regime and moves into Keynesian unemployment regime. During $y = y^d < y^s = v(w)$, or the time from $t = 5$ to $t = 80$, w recede from the steady state value. This is obviously the dual decision effect on $y^s (= v(w))$ feedback loop (destabilizing effect). After the effective supply switches to $f(l^s)$, y^s is gradually adjusted to y^d (stabilizing effect). When the regime switches from classical to Keynesian, the shortage of goods demand makes the expectation on goods sales pessimistic and therefore the real value of capital q quickly falls even though the nominal interest rate r keeps low. The small investment demand as well as the small consumption demand causes further shortage of effective demand with the multiplier effect. This instability induces the consistent Keynesian unemployment. The sticky low values of interest rate and real wage rate reflect "secular stagnation." This example implies that Keynesian unemployment is more persistent than classical unemployment, similar with Malinvaud (1980).

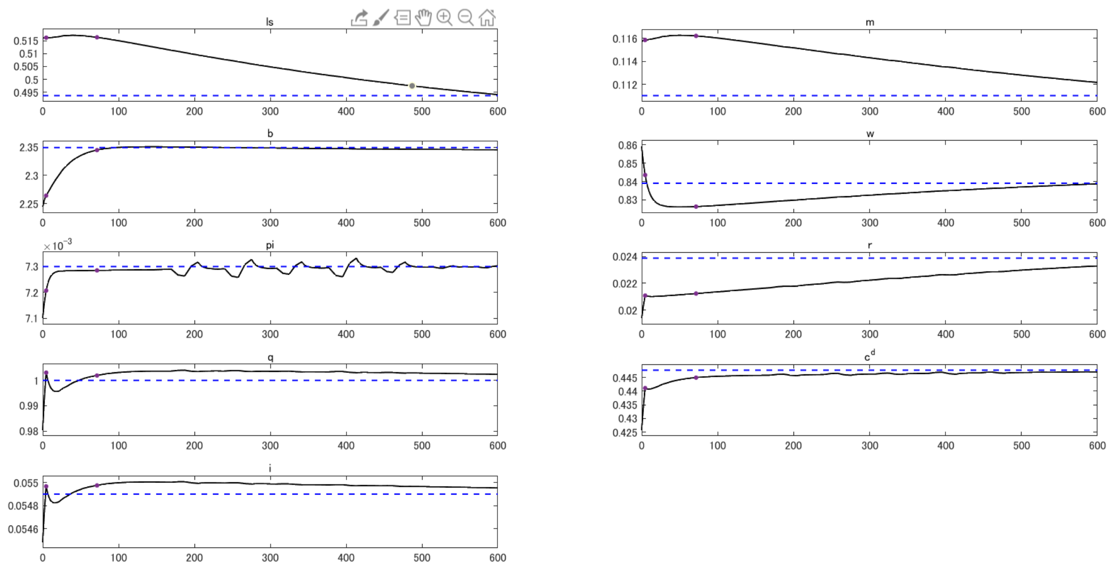

Figure 5: The dynamics of variables in example 1

The second example is a cyclical dynamics. we suppose the mostly adoptive expectation adjustment $\alpha = 0.9500$ with the adjustment speed $\beta = 0.2000$, which is near to the one in example 1. We set the initial value as $(l^s, m, b, w, \pi) = (0.4755, 0.1159, 2.4562, 0.8453, 0.0070)$. As figures 7 and 8 show, the (long-run) cyclical dynamics happen and the scale is enhanced

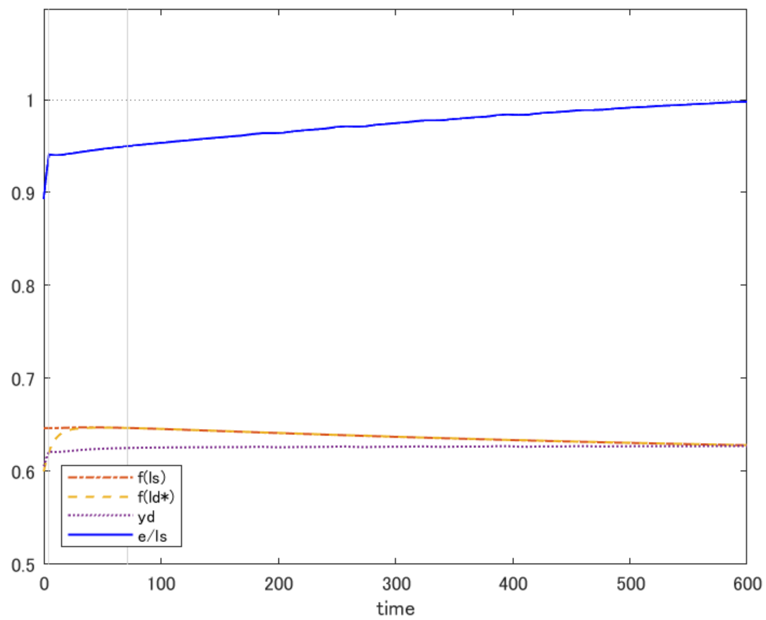

Figure 6: The dynamics of employment and production in example 1

as time goes. In the case $y^s < y^d$, the dynamics of y^s and y^d make similar patterns (the effect is stabilizing). Classical unemployment occurs in the latter half of the term $y^s < y^d$ but the employment rate does not reduce so much. In the case $y^d < y^s$, on the other hand, y^s excessively moves upward and the Keynesian unemployment become more serious than classical one. The cyclical regime switchings WE $\to$ RI $\to$ CU $\to$ KU $\to$ WE $\to \cdots$ continue and the cycle gains momentum as time goes.

4.2 The strong local stability of the steady state

We should check how much the expectation adjustment parameters α and β affect the local stability of the steady state. Figure 9 implies the strong local stability. In figure 9, the dot is pointed when all the Jacobian matrix of the system which correspond with the all six possible cases (e.g., $y^d < f(l^s) < v(w)$), have only negative eigenvalues under the combination (α, β) which is rationed.

The figure shows the steady state of our dynamical system is locally (maybe asymptiocally) stable unless the way of expectation adjustment is completely adaptive. This result implies the strong local stability of the steady state. However, the global stability is not ensured as shown in the second example above.

Besides, the second example shows that Warlasian equilibrium (full equilibria in all markets) is not always stable. The steady state in our system lies in the set of all possible Warlasian equilibria, but the economy might move into a disequilibvrium regime unless it happens to reach the steady state. This result implies that assumption that the normal state of the economy is (Warlasian) equilibrium is doubtful. We should reconsider how to justify (excessive) equilibrium models as the starting point of the macroeconomics.

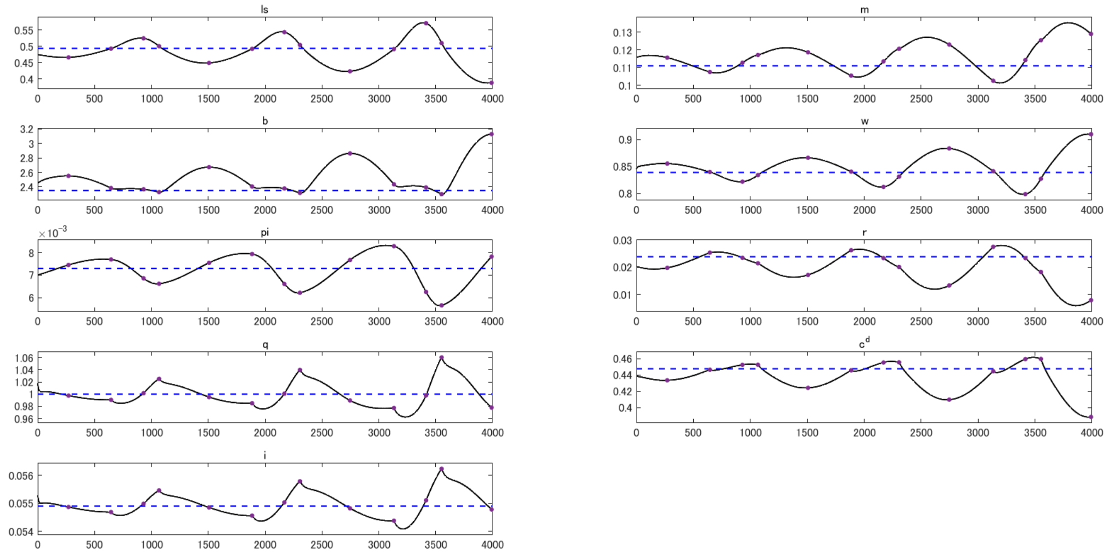

Figure 7: The dynamics of variables in example 2

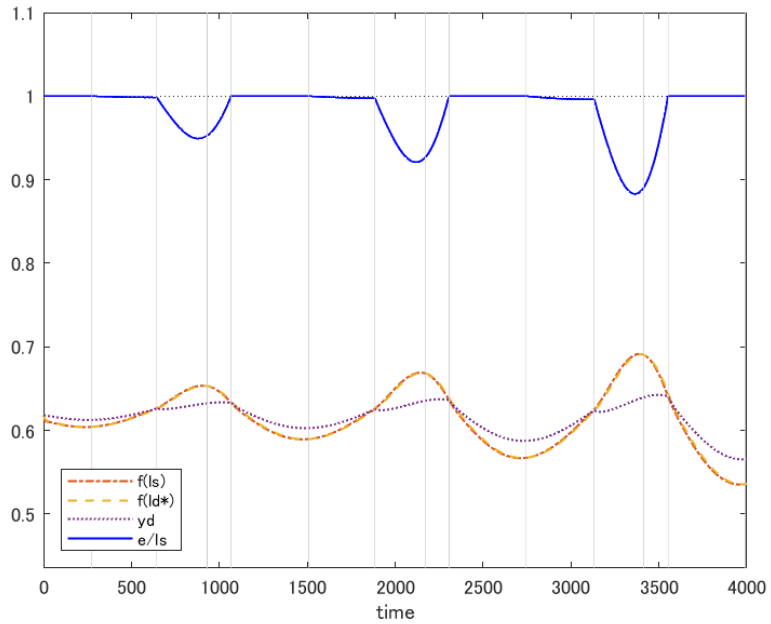

Figure 8: The dynamics of employment and production in example 2

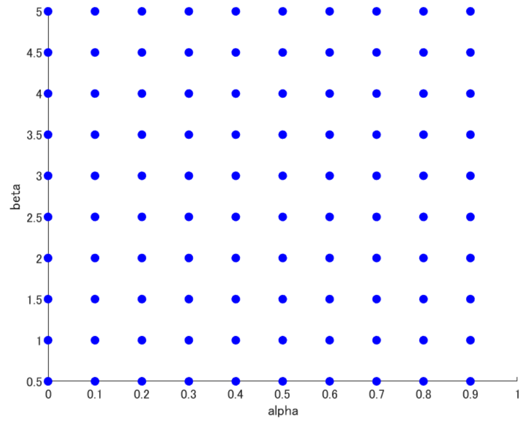

Figure 9: The point on which the steady state is locally stable

5 Concluding remarks

In this paper, we have analytically explored the non-Walrasian monetary growth model and found two important conclusions for dynamical perspective:

- The real wage dynamics which is based on the Walrasian adjustment intends to be unstable in Keynesian regime than that in classical regime, around the steady state.
- For the feedback loops of the goods supply, the dual-decision effect works and it could become both a stabilizer and a destabilizer.
- Although the locally stable steady state is in Warlasian regime, the economy at another point of Warlasian regime might move into a disequilibrium regime.

In fact, the first one is related to the dual decision effect. This conclusion is a direct resulut of the fact that y^d is more (negatively) sensitive to the real wage in Keynesian regime around the steady state. To see the mechanism for this fact, we should notice that there are two paths through which the real wage works negatively on the effective goods demand. The first one is the profitability factor ρ for Tobin's q, which uniformly works in both Keynesian and classical regimes. The second one is the effective demand principle. When $y = y^d$, first, the effective demand is expressed as a kind of recursive expression $y^d = y^d(y^d)$, which generates the multiplier effect on the goods market. The negative effect of the real wage on q is multiplied when $y = y^d$. When $y = y^s$, however, this multiplier effect disappears. Even though the goods supply (and then the realized production) does not positively depends on the real wage, the negative effect of the real wage on the effective demand is not multiplied in the path $y^d = y^d(y^s)$. Obviously, this difference comes from the dual decision hypothesis in which the effective demand depends on the realized income.

We have constructed a baseline model, which means that our model is too crude and that it is difficulut to regard the model as an approximation for the real world economy. To make more sophisticated non-Walrasian macrodynamic model, we should integrate the following issues. The first one is friction in markets. Almost all non-Walrasian models ignore it to simplify the model analyses. However, the frictions such as searching process are important issues for unemployment, which is the main problem for macroeconomics.

We should unify the frictions and the dual decision hypothesis. The second one is the inventory dynamics. Ordinal Keynesian models such as Chiarella et al. (2000) often adopt the inventory dynamics stimulted by Metzler (1941). As Green and Laffont (1981) and Honkapohja and Ito (1980) formulate the issue of the inventory in disequilibrium macroeconomics, we should extend the model referring to their works. This paper is just a first step. We hope the further development of disequilibrium dynamics, and our model would help them.

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A A simple example of q

We present a simple example of the formulation of q which suggests the function in equation 2.9 here. We follow the caluculation in Sargent (1987).

The nominal value of the firm V at time t is determined by the stream of the net cash flows:

$$V(t) = \int_t^\infty \mathbb{E}_t \left[\{P(\tau)F(K(\tau), E(\tau)) - W(\tau)E(\tau) - \delta P(\tau)K(\tau)\} e^{-(\int_t^\tau (r(s)+\xi)\, ds)} \right] d\tau. \tag{A.1}$$

The value is assessed with the constant positive risk premium ξ.

We suppose that all the individuals expect that the nominal interest rate is constant and that the price inflates at constant rate $\pi(t)$ in long run. Then

$$V(t) = P(t) \int_t^\infty \mathbb{E}_t \left[F(K(\tau), E(\tau)) - w(\tau)E(\tau) - \delta K(\tau)\right] e^{-(r(t)-\pi(t)+\xi)(\tau-t)}\, d\tau. \tag{A.2}$$

Furthermore, we introduce the following arbitrary assumption:

$$\mathbb{E}_t \left[F(K(\tau), E(\tau)) - w(\tau)E(\tau) - \delta K(\tau)\right] = \Theta(Y^d(t)/Y^s(t)) \left[F(K(t), E(t)) - W(t)E(t) - \delta K(t)\right], \tag{A.3}$$

where $\Theta' > 0$ and $\Theta(1) = 1$. This assumption implies that the future expected value of ρ is composed of the today's value of ρ and the measure of optimism Θ. If the goods market has the excess supply, the future profitability is underestimated relative to the today's profitability because the individuals become pessimistic for future sales. If we utilize this assumption, the value of V becomes

$$V(t) = P(t)K(t)\Theta(Y^d(t)/Y^s(t))[\rho(t)/(r(t) - \pi(t) + \xi)]. \tag{A.4}$$

Therefore

$$q(t) \equiv V(t)/(P(t)K(t)) = \Theta(Y^d(t)/Y^s(t))\rho(t)/(r(t) - \pi(t) + \xi)$$

. This equation is compatible with assumption 1.

B Comparative statics of the model and Jacobian matrix

The temporary equilibrium is characterized with the following seven simultaneous equations:

$$y = \min\{y^d, y^s\} \tag{B.1}$$
$$e = f^{-1}(y) \tag{B.2}$$
$$m = m^d(r, y) \tag{B.3}$$
$$y^d = f^c(m + b + q, r - \pi, y_{di})y_{di} + \psi(q - 1) + n + \delta + g \tag{B.4}$$
$$y^s = \min\{v(w), f(l^s)\} \tag{B.5}$$
$$y_{di} = (1 - \tau)(y - \delta) - \pi(m + b + q) \tag{B.6}$$
$$q = q(y - we - \delta, y^d/y^s, r - \pi) \tag{B.7}$$

As is in the proof for Proposition 1, this system has a unique solution as long as the variables (l^s, m, b, w, π) are exogenous and the parameters (g, τ, δ, n) are given.

For the dynamic analysis, we should know how the scale of the effective goods demand term y^d is affected by the exogenous and the other endogenous varibles. We use the total difference approach for equation (B.4) as follows:

$$\begin{aligned}(1 - q_2\phi/y^s)dy^d &= (\phi - \psi' + (f_2^c y_{di} + q_3\phi)/m_1^d)dm + (\phi - \psi')db + dg - (f_3^c y^d i + f^c)(y - \delta)d\tau \\ &\quad - [(f_3^c y_{di} + f^c)(m + b + q) + (f_2^c y_{di} + q_3\phi)]d\pi - eq_1\phi dw - (y^d/y^s)q_2\phi/y^s dy^s \\ &\quad + [1 - G_y - q_2\phi/y^s - (f_2^c y_{di} + q_3\phi)m_2^d/m_1^d]dy, \\ \text{where } \phi &= f_1^c y_{di} + \psi' - \pi(f_3^c y^d i + f^c) = (\partial c^d/\partial q) + (\partial i/\partial q) > 0, \\ G_y &= 1 - (1 - \tau)(f_3^c y^d i + f^c) - [q_1(1 - w/f') + q_2/y^s]\phi > 0.\end{aligned} \tag{B.8}$$

Notice that this equation is not valid when $y = y^d = y^s$ (the equation is not totally differentiable and therefore we should use the limitation calculation) and still has indeterminant terms dy^s, dy. When $y = y^d < y^s = f(l^s) < v(w)$, for instance, dy in equation (B.8) changes into dy^d and dy^s becomes $f'dl^s$.

This discontinuity makes the correlations between the variables complicated and therefore we should use "differential equation with discontinuous-righthand-side" techniqus for the dynamic analysis. Now we move to Jacobian matrix analyses.

To see the signs of the factors in the first row of $\mathbf{J}$, we check how Tobin's q is affected by the other variables:

$$dq = [q_1(1 - w/f') + q_3 m_2^d/m_1^d]dy + (q_2/y^s)dy^d - (y^d/y^s)(q_2/y^s)dy^s - eq_1 dw + (q_3/m_1^d)dm - q_3 d\pi \tag{B.9}$$

As $(dq)/(dy^d) > 0$, the signs of partial derivatives of q with respect to (l^s, m, b, π) are same as those of y^d. (dq/dw) is summerized as follows:

$$\begin{aligned}(dq/dw)|_{y=y^d} &= [-eq_1 - (y^d/y^s)(q_2/y^s)(\partial y^s/\partial w)]\oplus, \\ (dq/dw)|_{y=y^s} &= [-eq_1 - (y^d/y^s)(q_2/y^s)(\partial y^s/\partial w) + \ominus]\oplus,\end{aligned} \tag{B.10}$$

where $\oplus > 0$ and $\ominus < 0$. The maginitude relation between the two cases is ambiguous due to the nonlinear term y^d/y^s but $(dq/dw)|_{y=y^d} < 0$ always holds from Assumption 4. In the steady state $y^d = y^s$, $(dq/dw) < 0$ is ensured in each regime.

In the next, we implement the comparative statics for the Walrasian price adjustment term $\nu_P(y^d - y^s)$. As y^s is unaffeted by m, b, and π, we can adopt the results of y^d directly for them. For l^s, y^d is not directly affeted and so that

$$dy^d/dl^s = (\partial y^d/\partial y^s)(\partial y^s/\partial l^s) \tag{B.11}$$

holds. Therefore the partial derivative of $\nu_P(y^d - y^s)$ with respect to l^s is 0 if $y^s = v(w)$ and negative value if $y^s = f(l^s)$. By contrast, the effect of the real wage w on the Walrasianprice adjustment is complicated. Using equation (B.8), $(d/dw)(y^d - y^s)$ is summerized as follows:

$$\begin{aligned}
(d/dw)(y^d - y^s)|_{y=y^d} &< [-eq_1\phi - (\partial y^s/\partial w)]/[G_y + (f_2^c y_{di} + q_3\phi)m_2^d/m_1^d] \\
(d/dw)(y^d - y^s)|_{y=y^s} &= [-eq_1\phi - (\partial y^s/\partial w)(1+\ominus)]/(1 - q_2\phi/y^s) \\
&\geq [-eq_1\phi - (\partial y^s/\partial w)]/(1 - q_2\phi/y^s)
\end{aligned} \tag{B.12}$$

and the denominators in the above equations are postive since $G_y > 0$. Notice that the sign of the numerator $-eq_1\phi - (\partial y^s/\partial w)$ is ambiguous when $y^s = v(w)$.[24] We only conclude that the real wage effect would be more unstable in Keynesian regime than which in classical regime.

As we have seen the signs of the important factors, we would summerize the Jacobian matrix $\mathbf{J}$:

$$\mathbf{J} = \begin{bmatrix}
\oplus(\partial y^s/\partial l^s) & \ominus & \ominus & \ominus & \ominus \\
\oplus(\partial y^s/\partial l^s) & \ominus & \ominus & -\nu_P\rho_w + \oplus & \ominus \\
-\tau(\partial y/\partial l^s) + \oplus(\partial y^s/\partial l^s) & \ominus & \ominus & \ominus - \nu_P\rho_w & \ominus \\
-w_0\nu_W + \nu_P \oplus (\partial y^s/\partial l^s) & \ominus & \ominus & \nu_W v' - w_0\nu_P\rho_w & \ominus \\
\ominus(\partial y^s/\partial l^s) & \oplus & \oplus & \beta\alpha\nu_P\rho_w & \beta[\alpha\nu_P(\partial y^d/\partial\pi) - (1-\alpha)]
\end{bmatrix} \tag{B.13}$$

where $\rho_w = (d/dw)(y^d - y^s)$. Notice that this expression of the Jacobian matrix is common among the regimes.

C Estimation of consumption demand function

In this appendix, we carry out a rough estimation of consumotion demand function $c^d = f^c y_{di}$ from postwar (1982Q1 – 2017Q4) US data. The data on private consumption expenditure is taken from NIPA Table, (realized) inflation rate from OECD data, (10 year-) expected inflation rate and expected real interest rate from Federeal Reserve Bank of Cleveland, and the rest data are taken from FRED Economy Data, published by Federal Reserve Bank.

We use Net national product as $Y - \delta K$, Net worth (Households and nonprofit organizations) as A, Private consumption expenditure as C, and Capital stock as K. We use geometrical mean of the monthly data of π and $r - \pi$ and directly use the annual capital stock data for every quertly of each year.

[24] As $1 - q_2\phi/y^s > 0$ holds, $-eq_1 - \phi^{-1}(\partial y^s/\partial w) > -eq_1\phi - (q_2/y^s)(\partial y^s/\partial w)$ holds. Assumption 3 only ensures the negativity of the right hand side of the inequality.

Notice that, first, the observed consumption C is not always same as the consumption demand C^d in our model.[25] We estimate the value of the consumption demand using equation (3.1):

$$f^c = c^d/y_{di} = c/y_{di} + y_{di}^{-1}\max\{(\dot{P}/P - \pi)/\nu_P, 0\}. \quad \text{(C.1)}$$

We use the average propensity to consume f^c calculated from above equation as the explained variable. From the assumptions, we set the following equation for OLS estimation:

$$\ln f_t^c = c_0 + c_1(r_t - \pi_t) + c_2 \ln(A_t/Y_{di\ t}) + \varepsilon_t, \quad \text{(C.2)}$$

where $\varepsilon_t \sim N(0, \sigma)$. In this equation, the change rate of the average propensity to cosume is determined by the change of the real return rate on the safe asset and the change rate of the asset - disposable income ratio. From usual OLS etimation, we gain the following table 1. therefore we set the consumption demand function as follows:

$$f^c = 0.6483 \exp(0.9044(r - \pi))((m + b + q)/y_{di})^{(0.1866)} \quad \text{(C.3)}$$

variables	coefficient (standard error)
intercept	$-0.4334(0.0389)$
$r - \pi$	0.9044(0.0938)
$\ln(A/Y_{di})$	0.1865(0.0201)
$\bar{R}^2$	0.4676

Table 1: Estimation result

[25] For estimation methods, see Quandt (1988).