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Abstract

This paper sets up a New Keynesian model in which the monetary authority implements a zero lower bound interest rate policy, and uses it to explore whether the supportive fiscal instruments (including expansionary government spending, a payroll tax cut, and a financial assets tax cut) are effective in overcoming a deep recession. The salient feature of this study is that it provides a new dynamic viewpoint of regime switching by evaluating each of several supportive fiscal policies in terms of their performance in alleviating a deep recession. Two main findings emerge from the analysis. First, when the monetary authority implements the zero lower bound interest rate policy to dampen the negative natural rate shock, the economy will sink into a deep recession with deflation. Second, to overcome the deep recession, of the three supportive fiscal tools (i.e., expansionary government spending, a payroll tax cut, and a financial assets tax cut), only expansionary government spending is effective in alleviating the deep recession. More specifically, the implementation of fiscal policy in the form of either the payroll tax cut or the financial assets tax cut will only further deepen the recession.

Keywords: Zero lower bound, New Keynesian model, fiscal stimulus, regime switching

JEL Classification: E62, E63, H20

1. Introduction

During the financial crisis in 2008-2009, the monetary and fiscal authority in the U.S. implemented various monetary and fiscal measures in an attempt to dampen the adverse macroeconomic effects of the crisis. The monetary policy took the form of lowering the interest rate. However, in spite of the Fed funds rate being lowered to a sufficiently low level (between 0-0.25%), this measure was found to be insufficient to pull the economy out of the deep recession in the face of such a catastrophic crisis. Thus, various fiscal policies involving expansionary government spending and tax cuts were quickly and repeatedly implemented to support the monetary policy during the crisis period.

When the Fed funds rate had been lowered to close to zero during the financial crisis period, various fiscal policies (including expansionary government spending, a payroll tax cut, and a financial assets tax cut) were proposed by economists to dampen the adverse effects. A question naturally arose: When the economy experiences a serious adverse shock, are these supportive fiscal instruments able to effectively alleviate the negative effect on the economy when the monetary authority implements a zero lower bound (henceforth ZLB) interest rate policy? To answer this question, this paper builds up a New Keynesian framework to evaluate the performance of each of the proposed supportive fiscal policies from the viewpoint of transitional dynamics. To be more precise, this paper sets up a perfect foresight dynamic general equilibrium model that is able to describe the dynamic adjustment of relevant macroeconomic variables during the financial crisis period when the monetary authority implemented the ZLB interest rate policy. It is worth mentioning that, to make our analysis of the transitional dynamics more clear, this paper will provide not only a complete analytical solution but also a simple graphical exposition when we examine whether the supportive fiscal policies are effective in overcoming a deep recession.

In essence, the time interval of the financial crisis embodies a *temporary* characteristic since it will only last for a specific period. As a consequence, even though the monetary authority implements an interest rate peg policy at the ZLB during the time interval of the financial crisis, the forward-looking public fully recognizes that the financial crisis will come to an end at a specific date in the future. At that time, the monetary authority will once again implement the original interest rate adjustment rule (the Taylor rule). With this understanding, our analysis involves regime switching between the interest rate peg regime and the interest rate adjustment regime.[1] Compared to the existing literature on the financial crisis, the dynamic analysis of this paper has the following three distinctive traits. First, this paper provides a complete analytical solution of the dynamic analysis to explain whether each of the supportive fiscal instruments is helpful in alleviating the negative impact of the financial crisis. Second, this paper develops a simple graphical exposition, and uses it to provide an intuitive explanation for the analytical solution. Third, this paper proposes a new dynamic viewpoint of regime switching to evaluate the stabilizing effect of fiscal policies.

This paper is related to three strands of the existing literature on monetary policy with the binding of the ZLB. Firstly, in their recent articles, Carlstrom et al. (2015) and Cochrane (2017) also set up a New Keynesian model in which the economy will sink into a deep recession with deflation when the monetary authority implements the ZLB interest rate policy. However, their analysis focuses on whether forward guidance regarding the central bank's action is helpful in dampening the recession. This paper instead discusses which kinds of fiscal policies (rather than the forward guidance announcement implemented by the central bank) would be able to alleviate the economy's deep recession when the nominal interest rate is constrained at the ZLB.

Secondly, by building up a Markov switching model, some studies, such as Eggertsson (2011) and Woodford (2011), pay special attention to the fiscal multiplier when the monetary authority implements the ZLB interest rate policy. However, these studies do not explore the transitional dynamics of policy implementation, and only focus on whether a fiscal stimulus would generate a large multiplier at the ZLB constraint.[2] This paper instead provides the transitional analysis with a graphical illustration and highlights that, if the fiscal authority does not adopt any supportive policies and the monetary authority is forced to implement the ZLB interest rate policy, the economy will tend to fall into a deep recession throughout the entire period in which a temporary negative shock is present. Moreover, this paper comprehensively

[1] For the traditional analysis on regime switching (or regime change), see, e.g., Sargent and Wallace (1981), Krugman (1979), Drazen (1985), Obstfeld and Stockman (1985), Agénor and Flood (1992), and Lai and Chang (1994).

[2] In a celebrated article by Eggertsson (2011), the short run is defined as the period in which the economy is subject to temporary disturbance, and the long run is defined as the period in which the shock reverts to the steady-state value with the probability $1-\mu$ in each period. Based on the feature of the Markov process, Eggertsson (2011) cannot discuss the transition dynamics of policy implementation, and instead focuses attention on the kind of fiscal policy that would generate a larger multiplier in association with *two points in time* (i.e., the short run and the long run). This paper instead highlights the economy's dynamic adjustment *during the whole time period* in association with the implementation of different kinds of supportive fiscal policies.

depicts the transitional dynamics and shows that certain kinds of supportive fiscal policies can serve as an effective tool in helping to pull the economy out of a deep recession.

Thirdly, some recent studies including Eusepi (2010), Davig and Leeper (2011), Werning (2012), Schmidt (2016), and Shen and Yang (2018) set up New Keynesian models, and discuss how the coordination of monetary and fiscal policies will govern the transitional adjustment of relevant macroeconomic variables by resorting to *numerical* analysis.[3] In departing from these studies, this paper provides a detailed analytical solution, coupled with a simple diagrammatic exposition, to explain whether the fiscal instruments are helpful in alleviating the negative impact of the financial crisis when the monetary authority implements the ZLB interest rate policy.

The remainder of this paper is organized as follows. Section 2 builds up a standard continuous-time New Keynesian model. Section 3 examines the dynamic properties under two distinct regimes, and then shows that, faced with the negative natural rate shock, the economy would sink into a deep recession if the government were to implement the ZLB interest rate policy without any supportive fiscal policies. Section 4 discusses whether there exist feasible fiscal policies that will enable the economy to escape from the deep recession at the ZLB interest rate. Finally, the main findings of our analysis are presented in Section 5.

2. The New Keynesian model

In this section, we first develop is a continuous-time version of a standard New Keynesian model, which can be treated as an integration of Eggertsson (2011) and Farhi and Werning (2016).[4] Similar to Farhi and Werning (2016), the New Keynesian model, summarized by the New Keynesian Phillips Curve (NKPC) and the IS curve, can be represented by the following linearized differential equations:[5]

$$\dot{\pi}_t = \rho\pi_t - \kappa c_t - \delta_g g_t - \delta_w \tilde{\tau}_t^w, \tag{1}$$

$$\dot{c}_t = \hat{\sigma}^{-1}\left((1-\tau_0^a)i_t - i_0\tilde{\tau}_t^a - r_t^n - \pi_t\right). \tag{2}$$

To make the notation more compact, the variable with the subscript "0" refers to its initial

[3] It should be noted that Werning (2012) focuses on the normative analysis from the viewpoint of social loss minimization (our analysis instead engages in a positive analysis). More specifically, Werning (2012) shows that, under the liquidity trap scenario, a monetary policy without commitment would lead the economy into a depression coupled with deflation, while a monetary policy with commitment (i.e., the monetary authority commits to implementing the ZLB policy over a period longer than the liquidity trap) could lead the economy out of the depression accompanied by deflation. However, due to the difficulty in determining the optimal ZLB lagged period so as to minimize the social loss, Werning (2012) depicts the dynamic path in association with the optimal ZLB lagged period by resorting to numerical analysis (see Werning (2012, Fig. 2) for a more detailed discussion. Moreover, among the available supportive fiscal policies, Werning (2012) only deals with expansionary government spending.

[4] To be more precise, the model we develop can be treated as an integration of the Farhi and Werning (2016) perfect-foresight model and a variety of fiscal policies proposed by Eggertsson (2011).

[5] See Appendix A for a detailed mathematical derivation.

steady-state value. In Eqs. (1) and (2), π_t is the inflation rate, $c_t = (C_t - C_0)/Y_0$ is the ratio between the deviation of consumption C_t from its steady-state C_0 and the steady-state output Y_0, $g_t = (G_t - G_0)/Y_0$ is the ratio between the deviation of government spending G_t from its steady-state G_0 and the steady-state output Y_0, i_t is the nominal interest rate set by the monetary authority, and r_t^n denotes the natural (interest) rate, which is treated as an exogenous variable. In addition, τ_t^a denotes financial assets taxes and τ_t^w denotes payroll taxes, $\tilde{\tau}_t^a$ and $\tilde{\tau}_t^w$ are respectively defined as $\tilde{\tau}_t^a \equiv \tau_t^a - \tau_0^a$ and $\tilde{\tau}_t^w = \tau_t^w - \tau_0^w$, and i_0 is the steady-state nominal interest rate. The coefficients σ, φ, ρ and θ are the inverse of the intertemporal consumption substitution elasticity, the inverse of the labor supply elasticity, the subjective discount rate, and the probability of resetting prices. In addition, $\hat{\sigma} = \sigma/(1-\alpha_g)$, $\kappa = \theta(\rho+\theta)(\hat{\sigma}+\varphi)$, $\delta_g = \theta(\rho+\theta)\varphi$ and $\delta_w = \theta(\rho+\theta)/(1-\tau_0^w)$, where α_g is the ratio between the steady-state government spending and the steady-state GDP, i.e., $\alpha_g = G_0/Y_0$.

In addition, the economy's resource constraint is given by:[6]

$$y_t = c_t + g_t. \tag{3}$$

3. Transitional dynamics and deep recession

Based on the New Keynesian model reported in Eqs. (1) and (2), we now turn to examine how the economy will react in response to a negative natural rate shock when the monetary authority implements the ZLB interest rate policy.[7] Similar to Carlstrom et al. (2015), the experiment we conduct can be briefly described as follows. We assume that, prior to the presence of the negative natural rate shock, the monetary authority implements the interest rate rule. The implementation of the interest rate rule refers to the monetary authority's adjustment of the nominal interest rate in response to a change in the inflation rate (i.e., $i_t = i_0 + \phi_\pi \pi_t$, where ϕ_π is the responsiveness of i_t to π_t, which is a policy parameter determined by the monetary authority). Then, when the economy faces an anticipated shock so that the natural rate experiences a temporary reduction, the monetary authority will be forced to take action to implement the ZLB interest rate policy (i.e., $i_t = i^*$, where i^* is a sufficiently constant low and non-negative level of the nominal interest rate). Finally, when the negative natural rate shock vanishes, the monetary authority will once again implement the interest rate rule.

As described above, the regime switch that we consider involves the interest rate peg regime and the interest rate rule regime. In what follows in this subsection, we will first discuss the dynamic system under these two regimes. Then, we will show that, in the face of a negative natural rate shock, the economy will fall into a deep recession when the monetary authority implements the zero lower bound (ZLB) interest rate policy.

[6] See Appendix A for a detailed derivation of the economy's resource constraint.

[7] Eggertsson (2011) indicates that a negative natural rate shock is associated with an exogenous increase in the borrower's default probability, which was the main feature of the subprime crisis.

3.1. Transitional dynamics under the interest rate rule regime

We first analyze the dynamic behavior of the economy under an interest rate rule. By using Eqs. (1) and (2) together with the interest rule $i_t = i_0 + \phi_\pi \pi_t$, we have the following dynamic system:

$$\begin{pmatrix} \dot{\pi}_t \\ \dot{c}_t \end{pmatrix} = \begin{pmatrix} \rho & -\kappa \\ \hat{\sigma}^{-1}(1-\tau_0^a)(\phi_\pi - 1) & 0 \end{pmatrix} \begin{pmatrix} \pi_t \\ c_t \end{pmatrix} - \begin{pmatrix} \delta_g g_t + \delta_w \tilde{\tau}_t^w \\ \hat{\sigma}^{-1}(i_0 \tilde{\tau}_t^a + r_t^n - (1-\tau_0^a) i_0) \end{pmatrix}. \tag{4}$$

Based on Eq. (4), we can easily infer the two eigenvalues λ_1 and λ_2 of the dynamic system as follows:

$$\lambda_1 = \left(\rho - \sqrt{\rho^2 - 4\hat{\sigma}^{-1}\kappa(1-\tau_0^a)(\phi_\pi - 1)} \right) / 2, \tag{5}$$

$$\lambda_2 = \left(\rho + \sqrt{\rho^2 - 4\hat{\sigma}^{-1}\kappa(1-\tau_0^a)(\phi_\pi - 1)} \right) / 2. \tag{6}$$

In line with Galí (2015), to ensure that the dynamic system achieves a unique-stable equilibrium, we impose the restriction $(1-\tau_0^a)\phi_\pi > 1$. Accordingly, given $\lambda_1 > 0$ and $\lambda_2 > 0$, the dynamic system under the interest rate rule is associated with the feature of global instability. Based on Eqs. (4), (5), and (6), the general solution for π_t and c_t is given by:

$$\pi_t = \bar{\pi} + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}, \tag{7}$$

$$c_t = \bar{c} + \left(\frac{\rho - \lambda_1}{\kappa} \right) A_1 e^{\lambda_1 t} + \left(\frac{\rho - \lambda_2}{\kappa} \right) A_2 e^{\lambda_2 t}, \tag{8}$$

where $\bar{\pi}$ and $\bar{c}$ are the steady-state values of π_t and c_t under the interest rate rule regime, respectively, and A_1 and A_2 are undetermined coefficients. We now turn to solve $\bar{\pi}$ and $\bar{c}$ under the interest rate rule regime. At the steady-state equilibrium, the economy is characterized by $\dot{\pi}_t = \dot{c}_t = 0$. Given that $g_t = g$, $\tilde{\tau}_t^w = \tilde{\tau}^w$ and $\tilde{\tau}_t^a = \tilde{\tau}^a$ at the steady-state equilibrium, it is quite easy from Eq. (4) to derive the following steady-state results:

$$\bar{\pi} = (i_0 \tilde{\tau}^a + r^n - \rho) / \left((1-\tau_0^a)\phi_\pi - 1 \right), \tag{9}$$

$$\bar{c} = \left[\rho (i_0 \tilde{\tau}^a + r^n - \rho) / \left((1-\tau_0^a)\phi_\pi - 1 \right) - \delta_g g - \delta_w \tilde{\tau}^w \right] / \kappa, \tag{10}$$

To simplify the mathematical notation, we assume that the economy is initially (at time $t = 0$) in its steady state where the natural rate shock is absent and fiscal policies remain unchanged; that is $r_0^n = \rho$ and $g_0 = \tilde{\tau}_0^a = \tilde{\tau}_0^w = 0$. By substituting these initial conditions into Eqs. (9) and (10) we can infer that the initial inflation and consumption are respectively given by:

$$\pi_0 = \bar{\pi} = 0,$$

$$c_0 = \bar{c} = 0.$$

Then, based on $c_0 = \bar{c} = 0$, $g_0 = 0$, and the economy's resource constraint $c_t + g_t = y_t$,[8] the initial output is given by $y_0 = 0$.

The dynamic behavior of π_t and c_t under the interest rate rule can be described in terms of a phase diagram as shown in Figure 1. It is clear from Eq. (4) that the slopes of the loci $\dot{\pi}_t = 0$ and $\dot{c}_t = 0$ displayed in π_t and c_t space are given by:

$$\left.\frac{\partial c_t}{\partial \pi_t}\right|_{\dot{\pi}_t=0} = \frac{\rho}{\kappa} > 0,$$

$$\left.\frac{\partial c_t}{\partial \pi_t}\right|_{\dot{c}_t=0} = \frac{\hat{\sigma}^{-1}(1-\tau_0^a)(\phi_\pi - 1)}{0} \to \infty .$$

Equipped with information regarding the direction of the arrows, we can sketch all possible trajectories in Figure 1. In the phase space plane, the unstable branches UU^* and UU are associated with $A_1 = 0$ and $A_2 = 0$ in Eqs. (7) and (8), respectively. All other unstable trajectories in the figure correspond to the values with $A_1 \neq 0$ and $A_2 \neq 0$ in (7) and (8). The common feature of these divergent trajectories is that they start from the unstable node E_0 with the slope UU, and their slope asymptotically approaches that of the UU^* schedule.

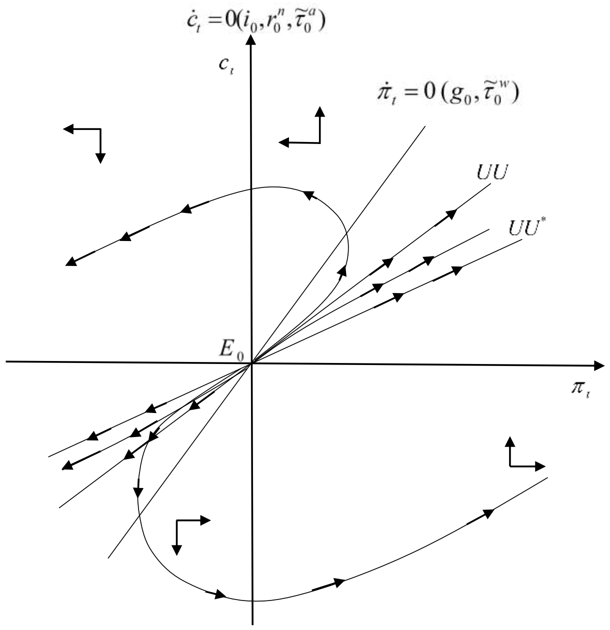

Figure 1. Phase diagram under the interest rate rule regime

3.2 Transitional dynamics under the interest rate peg regime

This subsection turns to analyze the dynamic behavior of the economy under an interest rate peg regime. Using Eqs. (1) and (2) and the interest rate peg policy $i_t = i^*$, we have:

$$\begin{pmatrix} \dot{\pi}_t \\ \dot{c}_t \end{pmatrix} = \begin{pmatrix} \rho & -\kappa \\ -\hat{\sigma}^{-1} & 0 \end{pmatrix} \begin{pmatrix} \pi_t \\ c_t \end{pmatrix} + \begin{pmatrix} -(\delta_g g_t + \delta_w \tilde{\tau}_t^w) \\ \hat{\sigma}^{-1}((1-\tau_0^a)i^* - i_0\tilde{\tau}_t^a - r_t^n) \end{pmatrix}. \tag{11}$$

[8] See Appendix A for a derivation of the economy's resource constraint.

From Eq. (11) we can easily infer the two eigenvalues λ_1^* and λ_2^* of the dynamic system as follows:

$$\lambda_1^* = \left(\rho - \sqrt{\rho^2 + 4\hat{\sigma}^{-1}\kappa}\right)/2 < 0, \tag{12}$$

$$\lambda_2^* = \left(\rho + \sqrt{\rho^2 + 4\hat{\sigma}^{-1}\kappa}\right)/2 > 0. \tag{13}$$

Eqs. (12) and (13) indicate that, just as in Farhi and Werning (2016) and Wieland (2019), the dynamic system under the interest rate peg regime has one positive and one negative root, i.e., $\lambda_1^* < 0 < \lambda_2^*$. Equipped with Eqs. (11), (12), and (13), the general solutions for π_t and c_t are:

$$\pi_t = \hat{\pi} + A_1^* e^{\lambda_1^* t} + A_2^* e^{\lambda_2^* t}, \tag{14}$$

$$c_t = \hat{c} + \left(\frac{\rho - \lambda_1^*}{\kappa}\right) A_1^* e^{\lambda_1^* t} + \left(\frac{\rho - \lambda_2^*}{\kappa}\right) A_2^* e^{\lambda_2^* t}, \tag{15}$$

where $\hat{\pi}$ and $\hat{c}$ denote the stationary values of inflation and consumption under the interest rate peg regime, and A_1^* and A_2^* are undetermined coefficients.

We now proceed to examine the determination of $\hat{\pi}$ and $\hat{c}$ under the interest rate peg regime. Given that $g_t = g$, $\tilde{\tau}_t^w = \tilde{\tau}^w$ and $\tilde{\tau}_t^a = \tilde{\tau}^a$ at the steady-state equilibrium, it follows from Eq. (11) with $\dot{\pi}_t = \dot{c}_t = 0$ that the following steady-state expressions can be derived:

$$\hat{\pi} = (1 - \tau_0^a) i^* - i_0 \tilde{\tau}^a - r^n, \tag{16}$$

$$\hat{c} = \left[\rho\left((1 - \tau_0^a) i^* - i_0 \tilde{\tau}^a - r^n\right) - \delta_g g - \delta_w \tilde{\tau}^w\right] / \kappa. \tag{17}$$

Eqs. (16) and (17) indicate how government spending, payroll taxes, and financial assets taxes affect $\hat{\pi}$ and $\hat{c}$. To simplify the notation, the above relationships can then be expressed as the following implicit functional forms:

$$\hat{\pi} = \hat{\pi}(\tilde{\tau}^a); \partial\hat{\pi} / \partial\tilde{\tau}^a = -i_0 < 0, \tag{16a}$$

$$\hat{c} = \hat{c}(g, \tilde{\tau}^w, \tilde{\tau}^a); \quad \partial\hat{c} / \partial g = -\delta_g / \kappa < 0, \partial\hat{c} / \partial\tilde{\tau}^w = -\delta_w / \kappa < 0,$$

$$\partial\hat{c} / \partial\tilde{\tau}^a = -\rho i_0 / \kappa < 0. \tag{17a}$$

By inserting Eqs. (16a) and (17a) into Eqs. (14) and (15), the general solutions for π_t and c_t can then be rewritten as:

$$\pi_t = \hat{\pi}(\tilde{\tau}^a) + A_1^* e^{\lambda_1^* t} + A_2^* e^{\lambda_2^* t}, \tag{14a}$$

$$c_t = \hat{c}(g, \tilde{\tau}^w, \tilde{\tau}^a) + \left(\frac{\rho - \lambda_1^*}{\kappa}\right) A_1^* e^{\lambda_1^* t} + \left(\frac{\rho - \lambda_2^*}{\kappa}\right) A_2^* e^{\lambda_2^* t}. \tag{15a}$$

The evolution of π_t and c_t under the interest rate peg can be illustrated by a phase diagram as shown in Figure 2. It is clear from Eq. (11) that the slopes of the loci $\dot{\pi}_t = 0$ and $\dot{c}_t = 0$ are given by:

$$\left.\frac{\partial c_t}{\partial \pi_t}\right|_{\dot{\pi}_t=0} = \frac{\rho}{\kappa} > 0,$$

$$\left.\frac{\partial c_t}{\partial \pi_t}\right|_{\dot{c}_t=0} = \frac{\hat{\sigma}^{-1}}{0} \to \infty.$$

As indicated by the direction of the arrows in Figure 2, the lines UU and SS represent the unstable and stable branches, respectively. The UU curve and the SS curve respectively trace the locus of π_t and c_t that satisfies $A_1^* = 0$ and $A_2^* = 0$ in Eqs. (14a) and (15a). Evidently, the convergent saddle path SS is upward sloping, while the divergent branch UU is downward sloping. All other unstable trajectories in the figure correspond to the values with $A_1^* \neq 0$ and $A_2^* \neq 0$ in Eqs. (14a) and (15a). A common feature of these divergent trajectories is that they start with the slope of the SS line and end with the slope of the UU schedule.

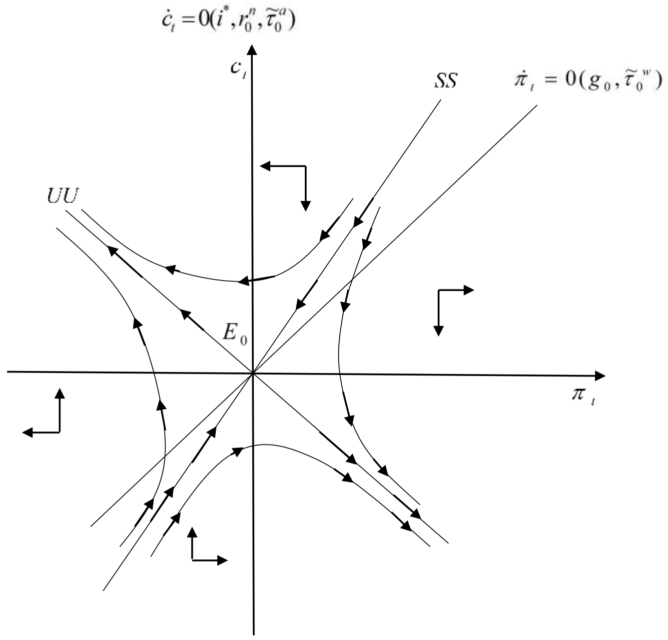

Figure 2. Phase diagram under the interest rate peg regime

3.3 Deep recession

The previous subsection illustrates the dynamic behavior of the economy under two distinct monetary regimes (i.e., the interest rate rule regime and the interest rate peg regime). In this subsection, we turn to discuss how the economy will react following a *temporary* negative natural rate shock when the monetary authority implements the temporary interest rate peg policy. Our analysis shows that the economy will experience a deep recession during the period in which a negative natural rate shock is present. Two points deserve mention here. First, according to Real Gross Domestic Product data provided by the database of the Federal Reserve Bank of St. Louis, the time interval from 2008Q1 to 2009Q2-Q3 is marked as the financial crisis period. This practical data reveals that the financial crisis period lasts around 5 or 6 quarters, and the duration is longer than the normal lasting time for the recession (two consecutive quarters of decline in real gross domestic output).[9] With this observation, to make our analysis more clear, in this paper "the deep recession" is defined as a substantial fall in output at the beginning of the negative natural rate shock and the duration of the shock is moderately prolonged. Second, in this subsection we will show that the economy falls into a deep recession if the monetary authority implements the zero lower bound (ZLB) interest rate policy and the fiscal authority does not implement any fiscal policies. To shed light on whether the coordination between the ZLB interest rate policy and fiscal policies could lessen the extent of the deep recession, the situation discussed in this subsection (i.e., the ZLB interest rate policy without any supportive fiscal policies) is treated as the benchmark case.

To make our analysis clearer, Figure 3 coupled with Eqs. (18) and (19) reported below are depicted to describe the process of regime switching. For expository convenience, throughout the remainder of the paper 0^- and 0^+ denote the instant before and instant after the beginning of an unanticipated shock, respectively; T^- and T^+ denote the instant before and that after the ending of the shock, respectively. As indicated in Figure 3 as well as Eqs. (18) and (19), the process of regime switching is depicted by means of the three stages. In the first stage, at time 0^- the economy is initially in its steady-state equilibrium and the monetary authority implements the interest rate rule regime (i.e., $r_t^n = r_0^n$ and $i_t = i_0 + \phi_\pi \pi_t$). In the second stage, the economy experiences an anticipated shock where the natural rate temporarily falls from r_0^n to $r_1^n (< 0 < r_0^n)$ during the period from 0^+ to T^-.[10] Faced with such an unfavorable situation, the monetary authority takes action to implement the ZLB interest rate policy under the interest rate peg regime (i.e., $i_t = i^*$). Finally, in the third stage during the period from T^+ onwards, the negative natural rate shock vanishes (i.e., the natural rate reverts back to r_0^n), and the monetary authority once again implements the interest rate rule regime (i.e., $i_t = i_0 + \phi_\pi \pi_t$).

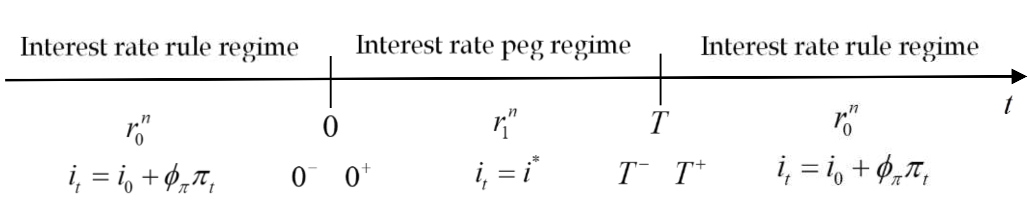

Figure 3. The timing of regime switching

Based on the above description, the evolution of the natural rate r_t^n can be expressed as:

[9] In their calibration analysis, Cochrane (2017) and Boneva et al. (2018) also set the crisis period to be around 5 to 6 quarters.

[10] Eggertsson (2011) deals with the uncertainty surrounding the ending date of the shock, i.e., the shock may revert back to its initial level with a specific probability. To highlight the transitional dynamics during the entire period in which the shock is present, just as in Wieland (2019), we assume that the ending date of the shock is constant for simplicity.

$$r_t^n = \begin{cases} r_0^n\,; & t = 0^- \\ r_1^n\,; & 0 \le t \le T^- \\ r_0^n\,; & T^+ \le t \end{cases}, \tag{18}$$

Moreover, the evolution regarding the switch in the interest rate policy can be written as:

$$i_t = \begin{cases} i_0 + \phi_\pi \pi_t\,; & t = 0^- \\ i^*\,; & 0 \le t \le T^- \\ i_0 + \phi_\pi \pi_t\,; & T^+ \le t \end{cases}. \tag{19}$$

It should be noticed once again that ϕ_π is the responsiveness of i_t to π_t, and is a policy parameter determined by the monetary authority. Eq. (19) indicates that during the period in which an adverse natural rate shock is present (the time horizon between 0^+ and T^-), the monetary authority implements a transient interest rate peg policy to make the interest rate remain intact at the sufficiently low and non-negative level i^*.

By using Eqs. (7), (8), (14a), and (15a) and recalling that the economy is initially in its steady-state equilibrium at time $t = 0^-$, the evolution of π_t and c_t in association with distinct time intervals can be described as follows:

$$\pi_t = \begin{cases} \pi_0\,; & t = 0^- \\ \hat{\pi}(\tilde{\tau}_0^a = 0) + A_1^* e^{\lambda_1^* t} + A_2^* e^{\lambda_2^* t}\,; & 0^+ \le t \le T^- \\ \pi_0 + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}\,; & T^+ \le t \end{cases}, \tag{20}$$

$$c_t = \begin{cases} c_0\,; & t = 0^- \\ \hat{c}(g_0 = \tilde{\tau}_0^w = \tilde{\tau}_0^a = 0) + A_1^* \left((\rho - \lambda_1^*)/\kappa\right) e^{\lambda_1^* t} + A_2^* \left((\rho - \lambda_2^*)/\kappa\right) e^{\lambda_2^* t}\,; & 0^+ \le t \le T^- \\ c_0 + A_1 \left((\rho - \lambda_1)/\kappa\right) e^{\lambda_1 t} + A_2 \left((\rho - \lambda_2)/\kappa\right) e^{\lambda_2 t}\,; & T^+ \le t \end{cases}, \tag{21}$$

where $\pi_0 = 0$, $c_0 = 0$, $\hat{\pi}(\tilde{\tau}_0^a = 0) = (1 - \tau_0^a) i^* - r^n$, and $\hat{c}(g_0 = \tilde{\tau}_0^w = \tilde{\tau}_0^a = 0) = \rho\left((1 - \tau_0^a) i^* - r^n\right)/\kappa$. One point should be mentioned here. To highlight that, in this benchmark case, the implementation of the ZLB interest rate policy is not matched with any supportive fiscal policy tools, we thus set $g_t = \tilde{\tau}_t^w = \tilde{\tau}_t^a = 0$ for all t.

As indicated in Eqs. (20) and (21), to trace the time paths of π_t and c_t from 0^+ to T^- and from T^+ onwards, we must solve for the appropriate values of A_1, A_2, A_1^*, and A_2^*. These values are determined by two continuity conditions, $\pi_{T^-} = \pi_{T^+}$ and $c_{T^-} = c_{T^+}$, and two stability conditions, $A_1 = 0$ and $A_2 = 0$. The two continuity conditions indicate that the forward-looking variables π_t and c_t cannot exhibit an anticipated discontinuity at the instant of realizing regime switching T^+. The two stability conditions state that the economy should move to its steady-state equilibrium at the instant when regime switching is realized, namely,

T^+, because the dynamic system from T^+ onwards is characterized by global instability.[11,12]

Inserting $\hat{\pi}(\tilde{\tau}_0^a = 0) = (1-\tau_0^a)i^* - r_1^n$, $\hat{c}(g_0 = \tilde{\tau}_0^w = \tilde{\tau}_0^a = 0) = \rho\big((1-\tau_0^a)i^* - r_1^n\big)/\kappa$, $\pi_0 = 0$, $c_0 = 0$, $\pi_{T^-} = \pi_{T^+}$, $c_{T^-} = c_{T^+}$, $A_1 = 0$, and $A_2 = 0$ into Eqs. (20) and (21), we have:

$$\pi_t = \begin{cases} 0; & t = 0^- \\ \dfrac{\lambda_2^*(1-e^{-\lambda_1^*(T-t)}) - \lambda_1^*(1-e^{-\lambda_2^*(T-t)})}{\lambda_2^* - \lambda_1^*}\big((1-\tau_0^a)i^* - r_1^n\big); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}, \tag{20a}$$

$$c_t = \begin{cases} 0; & t = 0^- \\ \dfrac{\lambda_2^*(\rho-\lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho-\lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\big((1-\tau_0^a)i^* - r_1^n\big); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}. \tag{21a}$$

Based on the economy's resource constraint ($y_t = c_t + g_t$) together with $g_t = 0$ in this benchmark case, the evolution of y_t can then be expressed as:

$$y_t = \begin{cases} 0; & t = 0^- \\ \dfrac{\lambda_2^*(\rho-\lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho-\lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\big((1-\tau_0^a)i^* - r_1^n\big); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}. \tag{22}$$

The evolution of both π_t and c_t reported in (20a) and (21a) can be clearly illustrated by means of a graphical apparatus. Before proceeding to study the dynamic adjustment of the economy, three points should be addressed. First, as indicated in Figure 4, at time 0^- the economy is assumed to be at its steady-state equilibrium E_{0^-}, where the $\dot{\pi}_t = 0\,(g_0, \tilde{\tau}_0^w)$ line intersects the $\dot{c}_t = 0\,(i_0, r_0^n, \tilde{\tau}_0^a)$ line, and the initial inflation rate and consumption level are $\pi_{0^-} = 0$ and $c_{0^-} = 0$, respectively.[13]

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[11] See, e.g., Turnovsky (2000, Ch. 7) for a detailed discussion on the continuity condition and the stability condition.

[12] The stability condition is referred to as "the boundary conditions" by Wieland (2019).

[13] To avoid cluttering the diagram, we leave out labels π_{0^-} and c_{0^-}.

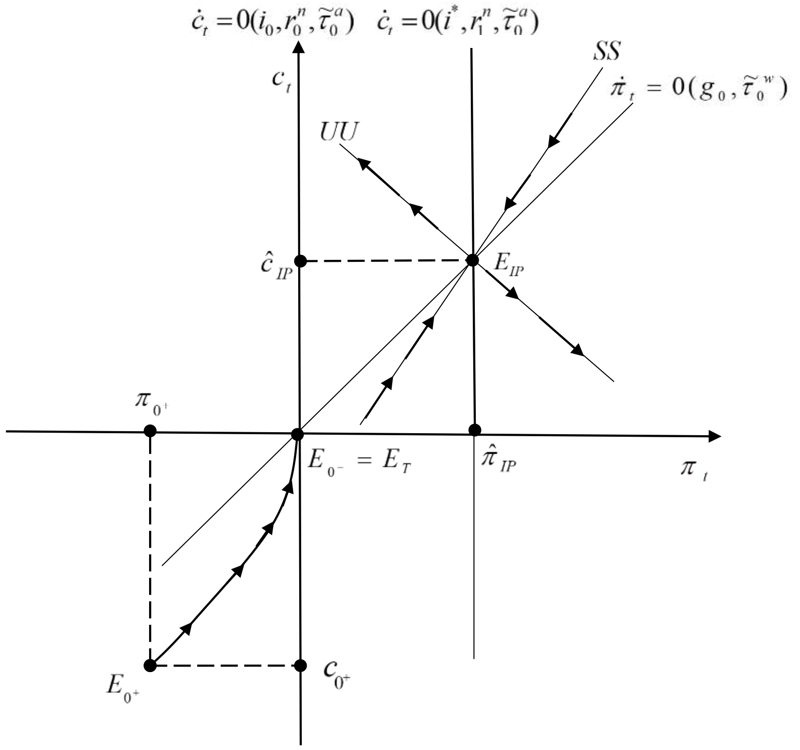

Figure 4. The ZLB interest rate policy

Second, during the time periods from 0^+ to T^-, in response to the negative natural rate shock (i.e., a reduction in the natural rate from r_0^n to r_1^n), to stabilize the economy the monetary authority implements its interest rate peg policy (i.e., the nominal interest rate is pegged to a sufficiently low level $i^*(<i_0)$) in response. Accordingly, following a decline of the natural rate from r_0^n to r_1^n coupled with the nominal interest rate remaining fixed at the level $i^*(<i_0)$, the $\dot{c}_t = 0(i_0, r_0^n, \tilde{\tau}_0^a)$ line shifts rightwards to $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_0^a)$, while the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line remains intact. The $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line and the $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_0^a)$ line intersect at point E_{IP}, where the inflation rate and the consumption level are $\hat{\pi}_{IP}$ and $\hat{c}_{IP}$, respectively.[14] Given that during the dates between 0^+ to T^- the natural rate remains at the r_1^n level and the nominal interest rate remains at the $i^*(<i_0)$ level, point E_{IP} should be treated as the reference point to govern the dynamic adjustment of π_t and c_t.

Third, since the agents universally know that, at the moment T^+, the negative natural rate shock will vanish and the monetary authority will recover to implement the interest rate rule, the economy should exactly move to its initial steady-state equilibrium point E_{0^-} at that instant of time because the dynamic system from T^+ onwards is characterized by global instability.

Based on the above three considerations, in Figure 4 at time 0^+ the economy will jump from point E_{0^-} to E_{0^+} on impact, and both the inflation rate and the consumption level will

[14] The subscript "IP" indicates that the monetary authority implements an interest rate peg policy.

instantly decrease from π_{0^-} to π_{0^+} and from c_{0^-} to c_{0^+}, respectively. Subsequently, from 0^+ to T^-, as the arrow indicates, the economy will gradually move from E_{0^+} to E_T (it coincides with point E_{0^-}) in which both π_t and c_t keep on rising. At time T^+, the negative natural rate shock vanishes and the monetary authority once again implements the interest rate rule such that the economy reaches point E_T, which exactly coincides with the economy's initial steady-state equilibrium point E_{0^-}. Thereafter, from T^+ onwards, the economy remains intact at point E_T, and both π_t and c_t remain intact at their initial values $\pi_{0^-}=0$ and $c_{0^-}=0$, respectively.

It is quite clear from Figure 4 that, during the time interval between 0^+ and T^-, even though the monetary authority implements the interest rate peg policy to lessen the unfavorable impact arising from the negative natural rate shock, the consumption level (which is equal to the output level) during this time interval is lower than its initial level. In particular, the consumption (output) level of the earlier periods is sufficiently smaller than its initial level. This reveals that the economy experiences a deep recession coupled with deflation during the period in which the natural rate shock is present (i.e., the time interval between 0^+ to T^-). Appendix B provides a detailed analytical result to confirm the presence of a deep recession during the time period between 0^+ and T^-, which is totally matched by the graphical exposition in Figure 4.

For the economic intuition behind the outcome of the deep recession we can refer to the New Keynesian Phillips curve and the IS curve reported in Eqs. (1) and (2). Equipped with these two equations, we can infer that both c_t and π_t are characterized by the following forward-looking traits:

$$c_t=-\hat{\sigma}^{-1}\int_t^T\left((1-\tau_0^a)i^*-\pi_s-r_1^n\right)ds\ ;\ \ 0^+\leq t\leq T^-, \tag{23a}$$

$$\pi_t=\kappa\int_t^T e^{-\rho(s-t)}c_s ds\ ;\ \ 0^+\leq t\leq T^-. \tag{23b}$$

Eq. (23a) indicates that, upon the news of a temporary decline in r^n from r_0^n to r_1^n during the time periods between 0^+ and T^-, the forward-looking household knows that the real interest rate will rise during this specific time frame. By means of the intertemporal substitution effect in consumption, the household is motivated to increase its savings and lower its current consumption discretely on impact. Moreover, we can infer from Eq. (23a) that c_t will keep on rising during the periods between 0^+ and T^- since r_1^n remains intact during this time interval.

Eq. (23b) reveals the following forward-looking feature. Based on the result that a reduction in the consumption level takes place (compared with its initial value) between 0^+ and T^-, the forward-looking firm recognizes that the goods market exhibits a significant shortage in demand, thereby leading the firm to lower the pricing (and hence the inflation rate) with a discrete adjustment at time 0^+. In addition, given that c_t displays a rising tendency over time from 0^{+} to T^{-}, we can infer from Eq. (23b) that π_t also exhibits an increasing adjustment pattern during this period.

Based on the discussions in this subsection, we can establish the following Proposition:

Proposition 1. *When the monetary authority implements a zero interest rate policy to dampen the negative natural rate shock, the economy will sink into a deep recession with deflation.*

In order to clearly show the time paths of output and the inflation rate during the period of the deep recession for this benchmark case, we offer a quantitative assessment by resorting to numerical analysis. The parameters we set are adopted from commonly-used values in the existing New Keynesian literature. In line with Kaszab (2016), the inverse of the intertemporal consumption substitution elasticity σ and the inverse of the intertemporal labor supply substitution elasticity φ are both set to 1. As in Wieland (2019) the discount rate ρ is set to 0.02. According to Galí (2015), the probability of resetting prices θ is set to 1/4. In line with Corsetti et al. (2010) and Collard et al. (2017), the government spending share $\alpha_g (= G_0 / Y_0)$ is set to 0.2, which is consistent with the U.S. data. By following Boneva et al. (2018), to match a fall in output of around 7.5% in the U.S. during the financial crisis period, the temporary natural rate shock is calibrated to decrease from 2% to -0.83%. According to Real Gross Domestic Product data provided by the database of the Federal Reserve Bank of St. Louis, the time interval from 2008Q1 to 2009Q2-Q3 is marked as the financial crisis period, and hence the time period for the presence of a temporary natural rate shock T is set to 6 (in terms of the number of quarters). As in Eggertsson (2011), the initial payroll tax rate τ_0^w is set to 0.2. Finally, by following Bullard and Russell (1999), the initial financial assets tax rate τ_0^a is set to 0.2.

In Figure 5, in association with the benchmark case where the government implements the sole ZLB interest rate policy, the gray curve in the upper and lower panels depicts the time paths of output and the inflation rate, respectively. As exhibited by the gray curve in both panels of Figure 5, at the instant where the economy experiences an anticipated temporary natural rate shock, output will immediately fall by about 7.5% and the inflation rate will immediately decline by about 2.5%. During the time periods in which the temporary natural rate shock is present, both output and the inflation rate will keep on rising and will gradually return to their initial levels. Obviously, as displayed in the upper panel, the output level along the transition path (in particular, the first few periods) will fall far below its initial level, and the economy will thereby suffer from a deep recession even though the monetary authority implements the ZLB interest rate policy to dampen the negative effect of the natural rate shock.

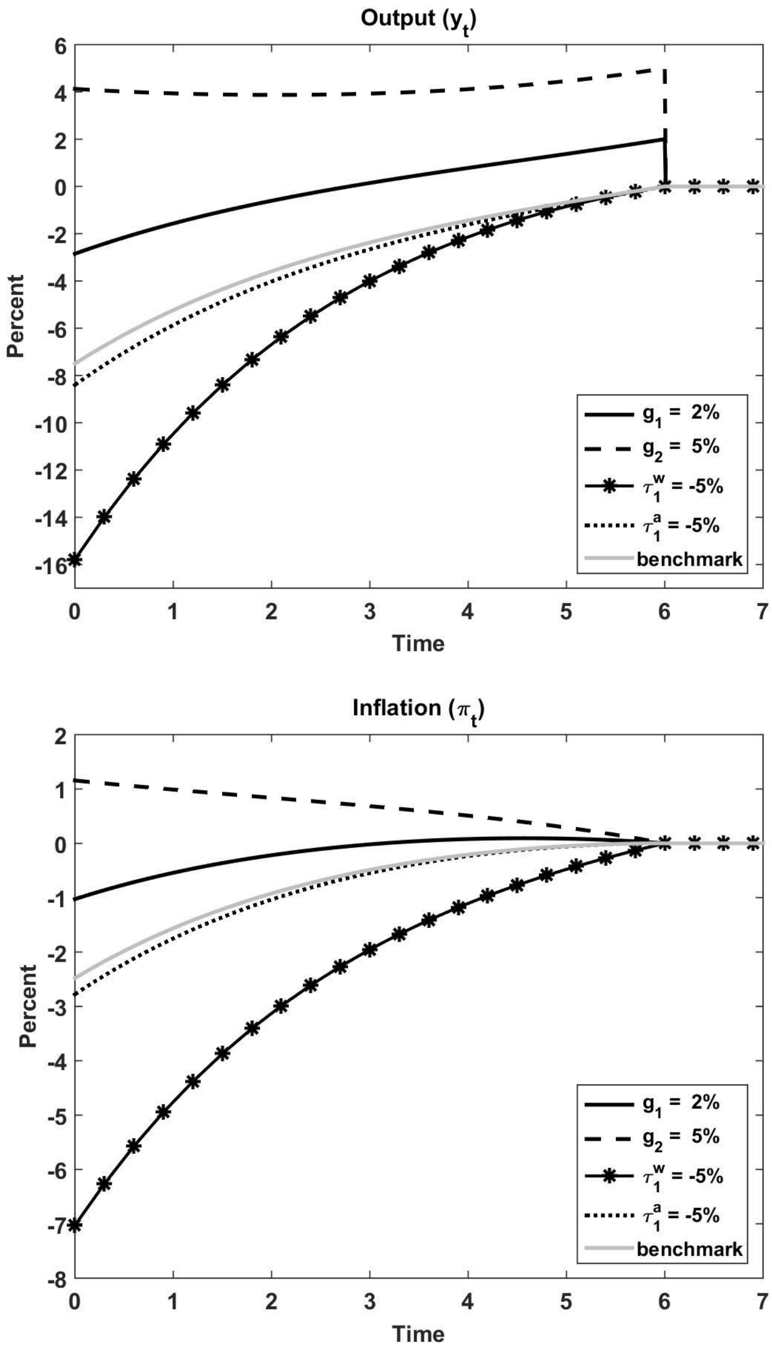

Figure 5: The time paths of output and the inflation rate.

4. Feasible fiscal policies

As shown in the previous section, the sole ZLB interest rate policy is incapable of fully dampening the impact of the natural rate shock, thereby causing the economy to sink into a deep recession. Based on the inability of the monetary policy to overcome the deep recession, in this section we turn to discuss whether there exist feasible fiscal policies that are able to lead the economy to escape from the deep recession at the ZLB interest rate.

In this section, the available fiscal policies that we consider include expansionary government spending, a payroll tax cut, and a financial assets tax cut. It needs to be emphasized here that, as pointed out in Eq. (A15) reported in Appendix A, the government will balance its budget by means of adjusting the lump-sum tax when it implements each of the available fiscal policies. Given that in the previous section we have provided a detailed analysis regarding why in the benchmark case (i.e., the ZLB interest rate in the absence of fiscal policies) the economy would suffer from a deep recession during the period in which a negative natural rate shock is present, to save space this section will briefly discuss which kinds of fiscal policies would be able to attenuate the economy's deep recession following a temporary natural rate shock.

4.1 Dynamic adjustment of expansionary government spending

The first fiscal policy we consider is expansionary government spending. Under such a situation, both the payroll tax and financial assets tax remain intact at their initial levels (i.e., $\tilde{\tau}_t^w = \tilde{\tau}_t^a = 0$) and government spending rises from its initial level g_0 to g_1. Given that the mathematical derivations for the evolutional dynamics of π_t, c_t, and y_t are similar to those in association with the sole ZLB interest rate policy, for convenience of presentation, the detailed proof underlying this case is relegated to Appendix C.

We now present a graphical analysis to discuss how the economy will react following a temporary negative natural rate shock when the government implements an expansionary fiscal policy coupled with the ZLB interest rate policy. Our analytical result shows that the economy will generate different types of dynamic paths according to the increased size of government spending. Accordingly, in what follows, we will deal with two distinctive scenarios in light of the relatively increased size in government spending (this reflects the different extents of the downward shift in the $\dot{\pi}_t$=0 locus).

The upper panel in Figure 6 depicts the scenario where an expansion in government spending is relatively low (hence a fiscal expansion leads to a relatively small shift in the $\dot{\pi}_t$=0 locus). In the upper panel of Figure 6, similar to the graphical illustration in Figure 4, at time 0^- the economy is initially established at its steady-state equilibrium E_{0^-}, where the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line intersects the $\dot{c}_t = 0(i_0, r_0^n, \tilde{\tau}_0^a)$ line, and the initial inflation rate and the consumption level are $\pi_{0^-} = 0$ and $c_{0^-} = 0$, respectively.

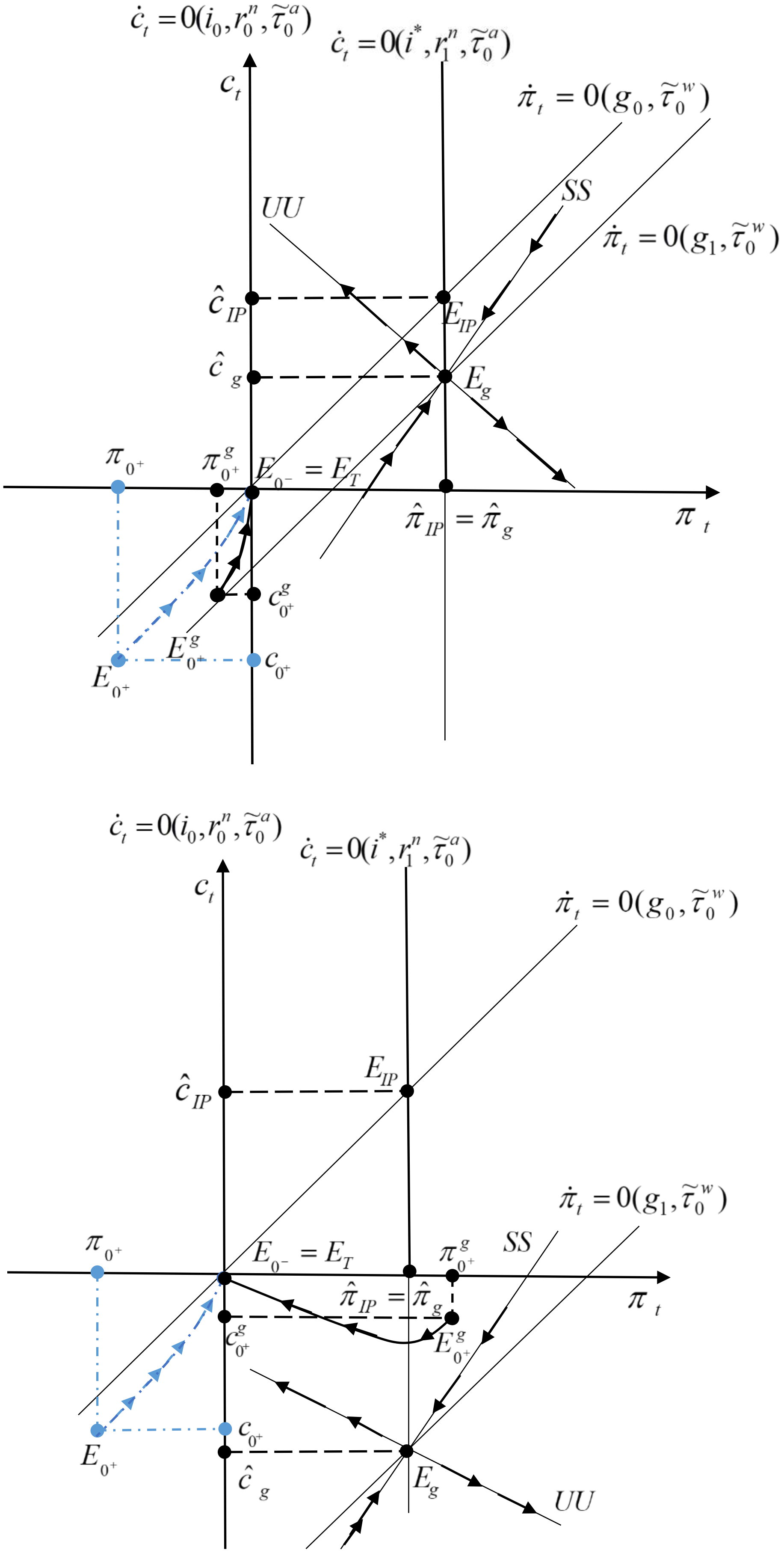

Figure 6. Expansionary government spending coupled with the ZLB interest rate policy

During the time periods from 0^+ to T^-, when the negative natural rate shock is enacted (i.e., a reduction in the natural rate from r_0^n to r_1^n), to stabilize the economy the monetary authority implements the ZLB interest rate policy (i.e., the nominal interest rate is pegged at a sufficiently low level $i^*(<i_0)$) and the fiscal authority implements an expansionary fiscal policy to increase its spending from g_0 to g_1. Accordingly, following a decline in the natural rate from r_0^n to r_1^n, a reduction in the nominal interest rate from i_0 to i^*, and an expansion in government spending from g_0 to g_1, the $\dot{c}_t = 0(i_0, r_0^n, \tilde{\tau}_0^a)$ line and $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line shift rightward to $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_0^a)$ and $\dot{\pi}_t = 0(g_1, \tilde{\tau}_0^w)$, respectively. The $\dot{\pi}_t = 0\,(g_1, \tilde{\tau}_0^w)$ line and the $\dot{c}_t = 0\,(i^*, r_1^n, \tilde{\tau}_0^a)$ line intersect at point E_g, where the inflation rate and the consumption level are $\hat{\pi}_g$ and $\hat{c}_g$, respectively. As the upper panel in Figure 6 describes the scenario where an expansion in government spending is relatively low, a fiscal expansion from g_0 to g_1 then leads to a relatively small shift in the $\dot{\pi}_t = 0$ curve from $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ to $\dot{\pi}_t = 0\,(g_1, \tilde{\tau}_0^w)$.

Given that during the dates between 0^+ to T^-, the natural rate, the nominal interest rate, and government spending remain intact at the levels r_1^n, i^*, and g_1, respectively, point E_g should be treated as the reference point to govern the dynamic adjustment of π_t and c_t. Since the public becomes aware that, at the moment T^+, the natural rate will revert back to its initial level r_0^n, the monetary authority will once again implement the interest rate rule (i.e., $i_t = i_0 + \phi_\pi \pi_t$), and government spending will be restored to its initial level g_0, the economy should exactly move to its initial steady-state equilibrium point E_{0^-} at that instant in time because the dynamic system from T^+ onwards is characterized by global instability. As clearly indicated in the upper panel of Figure 6, the dynamic path $E_{0^+}^g E_T$ (point E_T coincides with point E_{0^-}) could lead the economy back to its initial steady-state equilibrium E_{0^-} when E_g is treated as the reference point for the dynamic adjustment of π_t and c_t. Accordingly, upon the arrival of the news of a temporary negative natural rate shock at the instant 0^+, the economy will instantly jump from point E_{0^-} to $E_{0^+}^g$ on impact, and both the inflation rate and the consumption level will decline instantly from $\pi_{0^-} = 0$ to $\pi_{0^+}^g$ and $c_{0^-} = 0$ to $c_{0^+}^g$, respectively. Subsequently, from 0^+ to T^-, as the arrow indicates, the economy will move from point $E_{0^+}^g$ to E_T (it coincides with point E_{0^-}) along which π_t first increases and then decreases over time, while c_t keeps on rising. At time T^+, the negative natural rate shock vanishes,[15] and the economy arrives at point E_T, which exactly coincides with the economy's initial steady-state equilibrium point E_{0^-}. Thereafter, from T^+ onwards, the economy remains intact at point E_T, and both π_t and c_t remain intact at their initial values $\pi_{0^-} = 0$ and $c_{0^-} = 0$, respectively.

[15] It should be noted that, when the negative natural rate shock vanishes at the instant T^+, the monetary authority once again implements the interest rate rule and the fiscal authority restores the government spending to its initial level.

The lower panel in Figure 6 portrays the scenario where an expansion in government spending is relatively high. When compared with the upper panel in Figure 6, it is clearly observed that the vertical shift in the $\dot{\pi}_t = 0$ curve from $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ to $\dot{\pi}_t = 0(g_1, \tilde{\tau}_0^w)$ in the lower panel is significantly greater than that in the upper panel. Following a similar illustration to that for the upper panel, during the dates between 0^+ to T^-, E_g is treated as the reference point to govern the dynamic adjustment of π_t and c_t, and only one dynamic path $E_{0^+}^g E_T$ (point E_T coincides with point E_{0^-}) could lead the economy back to its initial steady-state equilibrium E_{0^-}. As a result, at the instant 0^+, the economy will instantly jump from point E_{0^-} to $E_{0^+}^g$ on impact, the inflation rate will instantly rise from $\pi_{0^-} = 0$ to $\pi_{0^+}^g$ and the consumption level will fall from $c_{0^-} = 0$ to $c_{0^+}^g$. Subsequently, from 0^+ to T^-, as the arrow indicates, the economy will move from point $E_{0^+}^g$ to E_T (it coincides with point E_{0^-}) along which c_t first decreases and then increases over time, while π_t keeps on falling. At time T^-, the economy reaches its initial steady-state equilibrium point E_{0^-}. Thereafter, from T^+ onwards, the economy remains intact at point E_{T^+}, and both π_t and c_t remain intact at their initial values $\pi_{0^-} = 0$ and $c_{0^-} = 0$, respectively.

By comparing both panels in Figure 6, it is observed that the dynamic paths are quite different. In particular, during the dates between 0^+ to T^-, the economy is characterized by deflation inertia if the rise in government spending is relatively low, while it is characterized by inflation inertia if the rise in government spending is relatively high.

To highlight whether the additional fiscal policy in the form of expansionary government spending is helpful in leading the economy to escape from a deep recession at the ZLB interest rate, in both panels of Figure 6 we re-portray the $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_0^a)$ line, the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line, and the dynamic path $E_{0^+} E_T$ in Figure 4 under the sole ZLB interest rate policy. By comparing the dynamic path of $E_{0^+}^g E_T$ with that of $E_{0^+} E_T$, it is clear that at any point in time $t \in [0^+, T^-]$ the consumption level in association with the dynamic path of $E_{0^+}^g E_T$ is higher than that in association with the dynamic path of $E_{0^+} E_T$. Coupled with the fact that the government spending level in association with the dynamic path of $E_{0^+}^g E_T$ (i.e., g_1) exceeds that in association with the dynamic path of $E_{0^+} E_T$ (i.e., g_0), we can conclude that at any point in time $t \in [0^+, T^-]$ the output level in association with the dynamic path of $E_{0^+}^g E_T$ is greater than that in association with the dynamic path of $E_{0^+} E_T$. As a result, the expansionary government spending policy could mitigate the extent of the deep recession, or may possibly lead to a boom in output if the rise in government spending is relatively high. Appendix C provides a detailed analytical result to confirm the graphical exposition in Figure 6.

For the economic intuition as to why the additional fiscal policy in the form of expansionary government spending can alleviate the deep recession, we can refer to the New Keynesian Phillips curve and the IS curve reported in Eqs. (1) and (2). Based on these two equations, we can infer that both c_t and π_t can be represented by the following forward-looking expressions:

$$c_t = -\hat{\sigma}^{-1} \int_t^T \left((1-\tau_0^a) i^* - \pi_s - r_1^n \right) ds \,; \quad 0^+ \le t \le T^- \,, \tag{24a}$$

$$\pi_t = \int_t^T e^{-\rho(s-t)} (\kappa c_s + \delta_g g_1) ds \,; \quad 0^+ \le t \le T^- \,. \tag{24b}$$

Eq. (24b) indicates that, in the face of a temporary decline in r^n from r_0^n to r_1^n during the period between 0^+ and T^-, the forward-looking household knows that, with an additional rise in government spending from g_0 to g_1, there are two possibilities. First, π_t will fall if the rise in government spending is relatively low, and the magnitude of the reduction in inflation π_t will decrease compared to the sole ZLB interest rate policy. Second, π_t will rise if the rise in government spending is relatively high. As indicated in Eq. (24a), both possibilities will lead to a reduction in the real interest rate, and hence result in a decline in the return on savings. As a consequence, with an additional fiscal policy in the form of expansionary government spending, the household is motivated to lower its savings by raising its consumption level.

Summing up the above discussion, we have the following Proposition:

Proposition 2. *When the monetary authority implements the ZLB interest rate policy to dampen the negative natural rate shock, the additional fiscal policy in the form of expansionary government spending could lessen the extent of the deep recession.*

The result exhibited in Figure 6 whereby the expansionary government spending policy could mitigate the extent of a deep recession can be confirmed with the help of numerical analysis. By using the same parameters as those in the benchmark case, we can portray the time path of output and the inflation rate in association with an additional policy in the form of a rise in government spending. As our analytical and graphical results in Figure 6 reveal that the economy will generate different types of dynamic paths depending upon whether an expansion in government spending is relatively low or high, we thus present two different values for a rise in government spending. In both panels of Figure 5, the black line depicts the responses to a 2% rise in government spending, while the dashed line portrays the responses to a 5% rise in government spending. As indicated by the black curve in both panels of Figure 5, at the moment of the enactment of an anticipated temporary natural rate shock, output will instantly decrease by about 2.8% and the inflation rate will immediately fall by about 1.0%. Thereafter, during the time periods in which the temporary natural rate shock is present, both output and the inflation rate will continue to rise and will gradually return to their initial levels. At the instant in which the negative natural rate shock vanishes, output experiences a discrete reduction since government spending reverts back to its initial level.

In addition, as indicated by the dashed curve in both panels of Figure 5, at the instant in which an anticipated temporary natural rate shock is enacted, output will instantly rise by about

4.1% and the inflation rate will immediately rise by about 1.2%. Thereafter, during the time periods in which the temporary natural rate shock is present, output will first rise then fall, while the inflation rate will keep on falling. As exhibited by the dashed curves in both panels of Figure 5, following a 5% rise in government spending (this reflects the fact that the expansion in government spending is relatively high), a boom in output coupled with inflation (rather than deflation) is present during the time periods in which the temporary natural rate shock is enacted. At the moment in which the negative natural rate shock vanishes, output exhibits a discrete reduction since government spending reverts back to its initial level.

By comparing the gray curve, the black curve, and the dashed line in the upper panel of Figure 5, we can find that both the dashed curve and the black curve lie entirely above the gray curve, implying that, during the period in which the anticipated temporary natural rate shock is present, the output level in association with an additional expansionary government spending policy is greater than that in association with the benchmark case. As a consequence, the additional fiscal policy in the form of expansionary government spending could serve as a powerful policy instrument to lessen the extent of the deep recession.[16]

4.2 Dynamic adjustment of the payroll tax cut

The second fiscal policy that we consider is the payroll tax cut. Under such a situation, both government spending and the financial assets tax remain unchanged at their initial levels (i.e., $g_t = \tilde{\tau}_t^a = 0$) and the payroll tax rate declines from its initial level of $\tilde{\tau}_0^w$ to $\tilde{\tau}_1^w$. Similar to subsection 4.1, we only provide graphical analysis for the evolutional dynamics of π_t and c_t, and the relevant mathematical derivations underlying this situation are relegated to Appendix D.

[16] One point should be noted regarding the time path of y_t in the upper panel of Figure 5. As the negative natural rate shock disappears at the instant T^+, the fiscal authority tends to lower its government spending from g_1 to the initial level g_0. Therefore, as displayed by the black curve in the upper panel of Figure 5, the level y_t reveals a discrete reduction at the instant T^+.

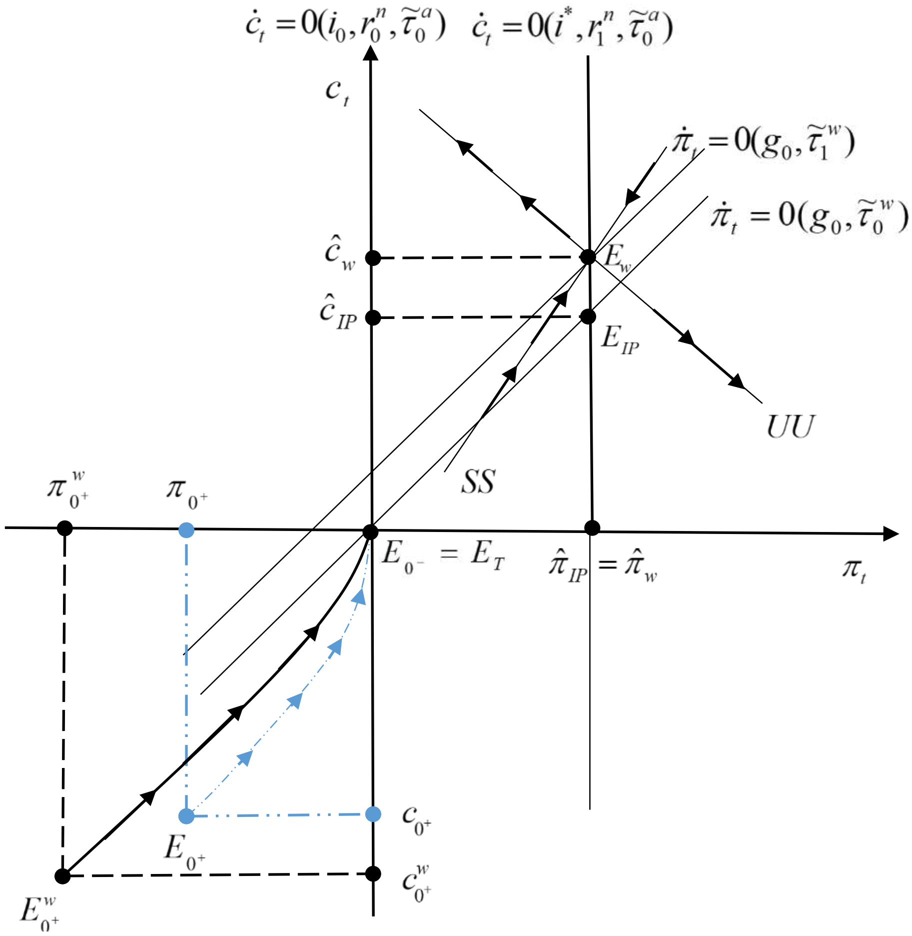

Figure 7. The payroll tax cut coupled with the interest rate peg policy

Figure 7 presents a graphical illustration to describe the adjustment process of the economy in response to a temporary negative natural rate shock when the government implements the payroll tax cut policy coupled with the ZLB interest rate policy. In Figure 7, similar to the graphical illustration in Figure 4, at time 0^- the economy is initially established at its steady-state equilibrium E_{0^-}, where the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line intersects the $\dot{c}_t = 0(i_0, r_0^n, \tilde{\tau}_0^a)$ line, and the initial inflation rate and the consumption level are $\pi_{0^-} = 0$ and $c_{0^-} = 0$, respectively.

During the dates between 0^+ to T^-, in response to the negative natural rate shock (i.e., a reduction in the natural rate from r_0^n to r_1^n), to stabilize the economy the monetary authority implements the ZLB interest rate policy (i.e., the nominal interest rate is pegged at a sufficiently low level $i^*(<i_0)$) and the fiscal authority implements the payroll tax cut policy (i.e., the payroll tax rate falls from $\tilde{\tau}_0^w$ to $\tilde{\tau}_1^w$). Accordingly, given that the natural rate decreases from r_0^n to r_1^n, the nominal interest rate remains constant at the level $i^*(<i_0)$, and the payroll tax rate falls from $\tilde{\tau}_0^w$ to $\tilde{\tau}_1^w$, the $\dot{c}_t = 0(i_0, r_0^n, \tilde{\tau}_0^a)$ line shifts rightward to $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_0^a)$ and the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line shifts leftward to $\dot{\pi}_t = 0(g_0, \tilde{\tau}_1^w)$, respectively. The $\dot{\pi}_t = 0(g_0, \tilde{\tau}_1^w)$ line and the $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_0^a)$ line intersect at point E_w, where the inflation rate and the consumption level are $\hat{\pi}_w$ and $\hat{c}_w$, respectively. Given that $r^n = r_1^n$, $i = i^*$, and $\tilde{\tau}^w = \tilde{\tau}_1^w$ during the dates

between 0^+ to T^-, point E_w should be treated as the reference point to govern the dynamic adjustment of π_t and c_t.

Based on a similar illustration in Figure 6, upon the arrival of the news of a temporary negative natural rate shock at the instant 0^+, the economy jumps from point E_{0^-} to $E_{0^+}^w$ on impact, and both the inflation rate and the consumption level immediately decline from $\pi_{0^-}=0$ to $\pi_{0^+}^w$ and $c_{0^-}=0$ to $c_{0^+}^w$, respectively. Subsequently, from 0^+ to T^-, as the arrows indicate, the economy moves from $E_{0^+}^w$ to E_T (it coincides with point E_{0^-}) along which both π_t and c_t continue to rise. At time T^+, the negative natural rate shock vanishes, the monetary authority once again implements the interest rate rule and the fiscal authority restores the payroll tax rate to its initial level. At that moment, the economy reaches point E_T, which exactly coincides with the economy's initial steady-state equilibrium point E_{0^-}. Thereafter, from T^+ onwards, the economy stays intact at point E_T, and both π_t and c_t remain intact at their initial values $\pi_{0^-}=0$ and $c_{0^-}=0$, respectively.

To shed light on whether the additional fiscal policy in the form of the payroll tax cut is beneficial to mitigate the extent of the deep recession at the ZLB, in Figure 7 we re-portray the $\dot{c}_t=0(i^*,r_1^n,\tilde{\tau}_0^a)$ line, the $\dot{\pi}_t=0(g_0,\tilde{\tau}_0^w)$ line, and the dynamic path $E_{0^+}E_T$ exhibited in Figure 4 under the sole ZLB interest rate policy. By comparing the dynamic path of $E_{0^+}^w E_T$ with that of $E_{0^+}E_T$, it is clear that at any point in time $t\in[0^+,T^-]$ the consumption level in association with the dynamic path of $E_{0^+}^w E_T$ is lower than that in association with the dynamic path of $E_{0^+}E_T$. Given that the government spending level in association with $E_{0^+}^w E_T$ is equal to that in association with $E_{0^+}E_T$ (i.e., $g_0=0$), we can assert that at any point in time $t\in[0^+,T^-]$ the output level in association with the dynamic path of $E_{0^+}^w E_T$ is smaller than that in association with the dynamic path of $E_{0^+}E_T$. As a result, the additional fiscal policy in the form of the payroll tax cut tends to further deepen (rather than help alleviate) the recession. Appendix D provides a detailed analytical result to confirm the graphical exposition in Figure 7.

For the intuition underlying the reinforcement of the deep recession in association with the payroll tax cut policy, we can also refer to the New Keynesian Phillips curve and the IS curve expressed in Eqs. (1) and (2). Based on these two equations, both c_t and π_t can be expressed in the following forward-looking manner:

$$c_t=-\hat{\sigma}^{-1}\int_t^T\left((1-\tau_0^a)i^*-\pi_s-r_1^n\right)ds\ ;\ \ 0^+\le t\le T^-, \tag{25a}$$

$$\pi_t=\int_t^T e^{-\rho(s-t)}(\kappa c_s+\delta_w\tilde{\tau}_1^w)ds\ ;\ \ 0^+\le t\le T^-. \tag{25b}$$

Eq. (25b) clearly indicates that, in the face of a temporary decline in r^n from r_0^n to r_1^n during the period between 0^+ and T^-, the forward-looking household knows that, with an additional reduction in the payroll tax from $\tilde{\tau}_0^w$ to $\tilde{\tau}_1^w$, the magnitude of the reduction in inflation π_t will be greater compared to that for the sole ZLB interest rate policy. By referring to Eq. (25a), this will lead to a higher real interest rate, and hence the household will be motivated to increase its savings via a contraction in its consumption level.

The above discussion leads us to establish the following Proposition:

Proposition 3. *When the monetary authority implements the ZLB interest rate policy to dampen the negative natural rate shock, a fiscal policy in the form of a payroll tax cut will further exacerbate the deep recession.*

The result displayed in Figure 7 can also be confirmed by resorting to numerical simulations. By using the same parameters as those in the benchmark case, we can depict the starred curve in the upper and lower panels in Figure 5, which respectively illustrate the time path of output and the inflation rate in association with an additional fiscal policy in the form of a 5% cut in the payroll tax. As indicated by the starred curve in both panels of Figure 5, at the moment that an anticipated temporary natural rate shock is enacted, output will instantly decrease by about 15.8% and the inflation rate will immediately fall by about 7.0%. Thereafter, during the time periods in which the temporary natural rate shock is present, both output and the inflation rate will continue to rise and will gradually revert back to their initial levels. By comparing the gray curve and the starred curve in the upper panel of Figure 5, it is found that the starred curve lies entirely below the gray curve, and this result clearly indicates that a fiscal policy in the form of a payroll tax cut will further exacerbate the extent of the deep recession.

4.3 Dynamic adjustment of the financial assets tax cut

The third fiscal policy that we deal with is the financial assets tax cut. Under such a situation, both government spending and the payroll tax remain fixed at their initial levels (i.e., $g_t = \tilde{\tau}_t^w = 0$) and the financial assets tax falls from its initial level $\tilde{\tau}_0^a$ to $\tilde{\tau}_1^a$. Similar to the previous two subsections, we only provide a graphical analysis for the evolutional dynamics of π_t and c_t, and the detailed mathematical derivations underlying this situation are presented in Appendix E.

Figure 8 provides a graphical treatment regarding the transitional dynamics of the economy, in which the government implements the financial tax cut policy coupled with the ZLB interest rate policy during the period in which a temporary negative natural rate shock is present. In Figure 8, similar to the graphical illustration in Figure 4, at time 0^- the economy is initially established at its steady-state equilibrium E_{0^-}, where the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line intersects the $\dot{c}_t = 0(i_0, r_0^n, \tilde{\tau}_0^a)$ line, and the initial inflation rate and the consumption level are $\pi_{0^-} = 0$ and $c_{0^-} = 0$, respectively.

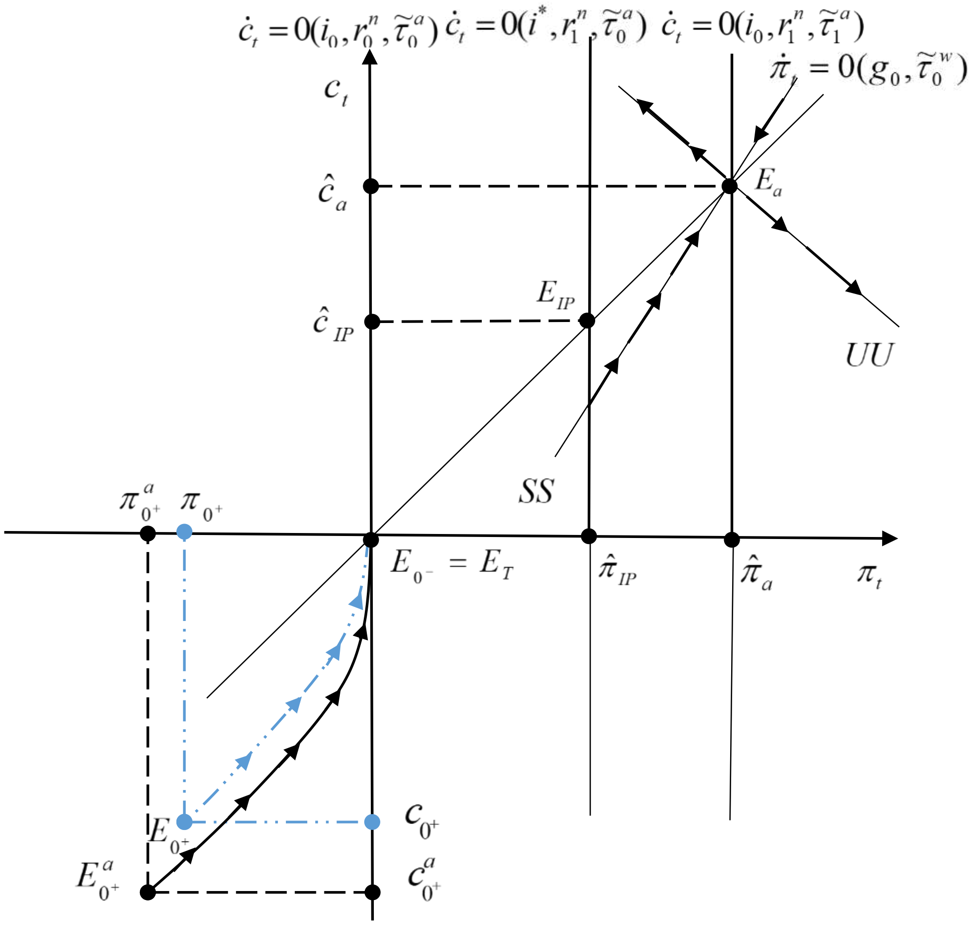

Figure 8. The financial assets tax cut coupled with the ZLB interest rate policy

During the dates between 0^+ to T^-, in response to the negative natural rate shock (i.e., a reduction of the natural rate from r_0^n to r_1^n), to stabilize the economy the monetary authority will implement the ZLB interest rate policy (i.e., the nominal interest rate is pegged to a sufficiently low level $i^*(<i_0)$) and the fiscal authority will implement the financial tax cut policy (i.e., the payroll tax rate declines from $\tilde{\tau}_0^a$ to $\tilde{\tau}_1^a$). Accordingly, following a decline in the natural rate from r_0^n to r_1^n, a reduction in the nominal interest rate from i_0 to i^*, and a cut in the financial tax rate from $\tilde{\tau}_0^a$ to $\tilde{\tau}_1^a$, the $\dot{c}_t = 0(i_0, r_0^n, \tilde{\tau}_0^a)$ line shifts rightwards to $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_1^a)$, while the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line remains intact. The $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line and the $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_1^a)$ line intersect at point E_a with the inflation rate and the consumption level being $\hat{\pi}_a(>\hat{\pi}_{IP})$ and $\hat{c}_a(>\hat{c}_{IP})$, respectively. Given that $r^n = r_1^n$, $i = i^*$, and $\tilde{\tau}^a = \tilde{\tau}_1^a$ during the dates between 0^+ to T^-, point E_a should be treated as the reference point to govern the dynamic adjustment of π_t and c_t.

In Figure 8, similar to the graphical illustration in Figure 4, at the instant 0^+ the economy jumps from point E_{0^-} to $E_{0^+}^a$ on impact, and both the inflation rate and the consumption level immediately decline from $\pi_{0^-} = 0$ to $\pi_{0^+}^a$ and $c_{0^-} = 0$ to $c_{0^+}^a$, respectively. Thereafter, from 0^+ to T^-, as the arrows indicate, the economy gradually moves from $E_{0^+}^a$ to E_T (it coincides with point E_{0^-}), along which both π_t and c_t keep on rising. At time T^+, the negative natural rate shock vanishes, the monetary authority once again implements the interest rate rule, and the fiscal authority restores the financial tax rate to its initial level. At that moment, the economy reaches point E_T, which exactly coincides with the economy's initial steady-state equilibrium point E_{0^-}. Thereafter, from T^+ onwards, the economy remains intact at point E_T, and both π_t and c_t remain intact at their initial values $\pi_{0^-} = 0$ and $c_{0^-} = 0$, respectively.

To examine whether an additional fiscal policy in the form of a financial asset tax cut is able to lessen the extent of the deep recession at the ZLB, in Figure 8 we re-portray the $\dot{c}_t = 0(i^*, r_1^n, \tilde{\tau}_0^a)$ line, the $\dot{\pi}_t = 0(g_0, \tilde{\tau}_0^w)$ line, and the dynamic path $E_{0^+}E_T$ exhibited in Figure 4 under the sole ZLB interest rate policy. By comparing the dynamic path of $E_{0^+}^a E_T$ with that of $E_{0^+}E_T$, it is clear that at any point in time $t \in [0^+, T^-]$ the consumption level in association with the dynamic path of $E_{0^+}^a E_T$ is lower than that in association with the dynamic path of $E_{0^+}E_T$. Given that the government spending level in association with the dynamic path of $E_{0^+}^a E_T$ is equal to that in association with the dynamic path of $E_{0^+}E_T$ (i.e., $g_0 = 0$), we can then infer that at any point in time $t \in [0^+, T^-]$ the output level in association with the dynamic path of $E_{0^+}^a E_T$ is smaller than that in association with the dynamic path of $E_{0^+}E_T$. Accordingly, the additional fiscal policy in the form of the financial assets tax cut will further deepen the already deep recession. Appendix E provides a detailed analytical result to confirm the graphical exposition in Figure 8.

We can briefly describe the economic intuition underlying the reinforcement of the deep recession by referring to the IS curve reported in Eq. (2). It is quite easy to infer from Eq. (2) that c_t is characterized by the following forward-looking feature:

$$c_t = -\hat{\sigma}^{-1} \int_t^T \left((1-\tau_0^a) i^* - i_0 \tilde{\tau}_1^a - \pi_s - r_1^n \right) ds \, ; \quad 0^+ \leq t \leq T^- . \tag{26}$$

Eq. (26) indicates that, in the face of a temporary decline in r^n from r_0^n to r_1^n during the period between 0^+ and T^-, the forward-looking household knows that, with an additional reduction in the financial assets tax from $\tilde{\tau}_0^a$ to $\tilde{\tau}_1^a$, the return on holding assets (i.e., the return on savings) will rise in response. This will encourage the forward-looking household to increase its savings matched by a reduction in its consumption.

The above discussion leads us to establish the following Proposition:

Proposition 4. *When the monetary authority implements the ZLB interest rate policy to dampen the negative natural rate shock, a fiscal policy in the form of a financial assets tax cut will further deepen the deep recession.*

By resorting to the numerical analysis, we can verify the result displayed in Figure 8. The dotted lines in the upper and lower panels in Figure 5 respectively depict the time path of output and the inflation rate in association with an additional fiscal policy in the form of a 5% cut in the financial assets tax. As indicated by the dotted curve in both panels of Figure 5, at the moment where an anticipated temporary natural rate shock is enacted, output will instantly decrease by about 8.4% and the inflation rate will immediately fall by about 2.8%. Thereafter, during the time interval in which the temporary natural rate shock is present, both output and the inflation rate will continue to rise and gradually revert back to their initial levels. By comparing the gray curve and the dotted curve in the upper panel of Figure 5, it is found that the dotted curve is fully located below the gray curve, and this result clearly reveals that a fiscal policy in the form of a financial tax cut will exacerbate the deep recession.

5. Conclusion

To lessen the severity of a deep recession during a financial crisis period, many countries have been forced to implement a ZLB interest rate policy. However, they have usually found that the sole ZLB interest rate policy has been insufficient to dampen such a recession in the face of a catastrophic crisis. Faced with this problem, various fiscal policies involving expansionary government spending and tax cuts have been proposed and implemented to support the ZLB interest rate policy during the crisis period.

Even though the monetary authority has implemented a zero lower bound (ZLB) interest rate policy during the time interval of the financial crisis, the public fully recognizes that the financial crisis will come to an end at a future date. From then on, the monetary authority will once again implement the original interest rate adjustment rule (the Taylor rule). As a result, the implementation of the ZLB interest rate policy during the financial crisis period is characterized as being *temporary* since it only lasts for a specific period. Accordingly, it would be plausible for the implementation of the ZLB interest rate policy to involve regime switching between the interest rate peg regime and the interest rate adjustment regime.

Based on the above observations, this paper constructs a New Keynesian model, and uses it to explore whether supportive fiscal instruments (including expansionary government spending, a payroll tax cut, and a financial assets tax cut) are effective in alleviating the adverse impact of a deep recession from the regime-switching viewpoint. Compared to the existing literature on the financial crisis, the dynamic analysis of this paper has the following two distinctive traits. First, this paper provides a complete analytical solution of the dynamic analysis in explaining whether each of the supportive fiscal instruments can help alleviate the negative impact of the financial crisis. Second, this paper develops a simple graphical exposition, and uses it to provide an intuitive explanation for the analytical solution. Third, this paper proposes a new dynamic viewpoint of regime switching to evaluate the performance of each supportive fiscal policy.

Two main findings emerge from the analysis. First, when the monetary authority implements the ZLB interest rate policy to dampen the negative natural rate shock, the economy will sink into a deep recession with deflation. Second, to overcome the deep recession, of the three supportive fiscal tools (i.e., expansionary government spending, a payroll tax cut, and a financial assets tax cut), only expansionary government spending is effective in terms of helping pull the economy out of the deep recession. More specifically, the implementation of a fiscal policy in the form of either a payroll tax cut or a financial assets tax cut will only serve to further deepen the recession.

Appendix A

This Appendix provides a detailed derivation of the New Keynesian Phillips curve, the IS curve, and the economy's resource constraint reported in Eqs. (1), (2), and (3).

A.1 Households

The representative household derives utility from consumption C_t and incurs disutility from working L_t. The lifetime utility of the representative household is given by:

$$\int_0^\infty e^{-\rho t}\xi_t \left\{\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{L_t^{1+\varphi}}{1+\varphi}\right\}dt\ ;\quad \rho,\sigma,\varphi>0\,. \tag{A.1}$$

The parameter ρ is the constant rate of the time preference, σ and φ are the inverse of the intertemporal consumption substitution elasticity and the inverse of the intertemporal labor supply substitution elasticity, respectively, and ξ_t is an exogenous preference shock.

The household's budget constraint can be expressed as:

$$\dot{B}_t=(1-\tau_t^a)i_tB_t+(1-\tau_t^w)W_tL_t+D_t-P_tC_t-T_t\,, \tag{A.2}$$

where P_t is the aggregate price level, W_t is the nominal wage, i_t is the nominal interest rate of the bond which is controlled by the monetary authority, B_t is the holdings of the risk-free government bond, D_t is the distributed nominal profits from the firms, τ_t^a is the financial assets tax rate, τ_t^w is the payroll tax rate, and T_t is the lump-sum tax.

Let η_t denote the shadow value of the bonds. The optimum conditions for the representative household with respect to the indicated variables are:

$$C_t:\ \xi_tC_t^{-\sigma}=\eta_tP_t\,, \tag{A.3}$$

$$L_t:\ \xi_tL_t^{\varphi}=(1-\tau_t^w)W_t\eta_t\,, \tag{A.4}$$

$$B_t:\ (1-\tau_t^a)\,i_t\eta_t=-\dot{\eta}_t+\rho\eta_t\,, \tag{A.5}$$

$$\eta_t:\ \dot{B}_t=(1-\tau_t^a)i_tB_t+(1-\tau_t^w)W_tL_t+D_t-P_tC_t-T_t\,. \tag{A.6}$$

From Eqs. (A.3) and (A.4), the optimality condition for labor supply is given by:

$$L_t^{\varphi}=\left((1-\tau_t^w)W_t/P_t\right)C_t^{-\sigma}\,. \tag{A.7}$$

Equipped with Eqs. (A.3) and (A.5), the usual Keynes-Ramsey rule can be written as:

$$\dot{C}_t/C_t=\sigma^{-1}\left((1-\tau_t^a)i_t-r_t^n-\pi_t\right) \tag{A.8}$$

where $r_t^n=\rho-(\dot{\xi}_t/\xi_t)$ and π_t is the inflation rate. It is quite clear that r_t^n is a combination of the *exogenous* discount rate ρ and the change rate in the *exogenous* preference shock $\dot{\xi}_t/\xi_t$, and hence r_t^n is treated as an *exogenous* natural rate shock in our analysis.

A.2 Firms

We are now in a position to deal with the production side of the economy. The production side consists of two sectors (final goods and intermediate goods) and is characterized as follows: (a) the final goods market is perfectively competitive, and each of the final goods firms utilizes a continuum of intermediate goods to produce its goods; and (b) the intermediate goods market is monopolistically competitive, and each of the intermediate goods firms utilizes the labor input to produce its goods. We then deal with the optimal behavior of these two kinds of firms.

The competitive final goods producer uses a continuum of intermediate goods $Y_t(j)$, $j \in [0,1]$, to produce its goods Y_t. The technology for producing the final good can be described by a standard CES production function with a constant elasticity of substitution $\varepsilon > 0$:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}; \quad \varepsilon > 0. \tag{A.9}$$

Let $P_t(j)$ denote the price of $Y_t(j)$. The maximization problem of the final goods firm can be expressed as:

$$\underset{Y_t(j)}{Max} \, [P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj]. \tag{A.10}$$

The first-order condition of Eq. (A.10) yields the standard demand function for $Y_t(j)$ as:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t. \tag{A.11}$$

Given the fact that the final goods market is perfectly competitive, the zero-profit condition for the final good sector implies that the price index is given by:

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{A.12}$$

We then deal with the optimal behavior of the intermediate goods firms. The intermediate goods market is monopolistically competitive. In line with Wieland (2019), we suppose that each of the intermediate goods firms uses only labor to produce its output, and one unit of labor produces one unit of the intermediate good. The production function can then be expressed as $Y_t(j) = L_t(j)$, where $L_t(j)$ is the labor input used by the jth firm.

Following Calvo (1983) and Wieland (2019), we assume that the jth intermediate goods firm resets its price with probability θ in each period, for $\theta \in [0,1]$. The intermediate goods producer j determines its optimal price subject to its demand functions reported in Eq. (A.11). Thus, the optimal price setting problem of the jth firm can be expressed as:

$$\underset{P_t^*(j)}{Max} \int_t^\infty e^{-(\rho+\theta)(s-t)} C_s^{-\sigma} \left(\frac{P_t^*(j)}{P_s} Y_s(j) - \frac{W_s}{P_s} L_s(j) \right) ds, \tag{A.13}$$

where $P_t^*(j)$ is the optimal price set by the jth intermediate goods firm. As in Wieland

(2019), $C_t^{-\sigma}$ captures the contingent valuation of the firm's profit. The optimality condition for the j th producer is to reset its real price as follows:

$$\frac{P_t^*(j)}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{\int_0^\infty e^{-(\rho+\theta)(s-t)}C_s^{-\sigma}(W_s / P_s)(P_t / P_s)^{-\varepsilon}Y_s ds}{\int_0^\infty e^{-(\rho+\theta)(s-t)}C_s^{-\sigma}(P_t / P_s)^{1-\varepsilon}Y_s ds}. \quad \text{(A.14)}$$

Eq. (A.14) indicates that the firm's optimal pricing decision reflects the forward-looking trait. To be more specific, as indicated in Eq. (A.14), the producer makes its price-setting decision at time t in light of the discounted value of the stream of nominal marginal cost in all future periods.

A.3 Fiscal authority

The government finances its expenditures on fiscal spending and interest payments by levying taxes (including the payroll tax, financial assets tax and lump-sum tax) and by issuing risk-free bonds. Therefore, the government's budget constraint can be written as:

$$\dot{B}_t = P_t G_t + (1-\tau_t^a) i_t B_t - \tau_t^w W_t L_t - T_t . \quad \text{(A.15)}$$

This study assumes that the government implements fiscal policies (in the form of an expansion in G_t, or a decline in τ_t^w, or a decline in τ_t^a) by means of adjusting the lump-sum tax rather than issuing risk-free bonds.

A.4 Monetary authority

In line with Carlstrom et al. (2015), we specify that the monetary authority implements distinct interest rate rules in normal times and during the crisis period. To be specific, in normal times (in the absence of a negative natural rate shock), the monetary authority follows a simple interest rate rule that may be expressed as:

$$\frac{1+i_t}{1+i_0} = (1+\pi_t)^{\phi_\pi} , \quad \text{(A.16)}$$

where ϕ_π represents the policy parameters of the inflation targeting which is determined by the monetary authority, i_0 is the steady-state interest rate and $i_0 = \rho / (1-\tau_0^a)$.

During the crisis period (in the presence of the negative natural rate shock), the monetary authority implements the interest rate peg rule at the ZLB level i^*, that is:

$$i_t = i^* . \quad \text{(A.17)}$$

A.5 Aggregation

Based on Eqs. (A.6), (A.10), (A.15), the zero profit condition of final goods producers $P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$, and the nominal profits of all intermediate goods firms distributed to the household $D_t = \int_0^1 P_t(j) Y_t(j) dj - \int_0^1 W_t L_t(j) dj$, the economy's resource constraint (i.e., the market clearing condition for the final goods) is given by:

$$Y_t = C_t + G_t . \quad \text{(A.18)}$$

A.6 Derivation of the IS curve and interest rate rule

Let the variable with the subscript "0" refer to its steady-state value. In line with Farhi and Werning (2016), we state that $c_t \equiv (C_t - C_0)/Y_0$, $g_t = (G_t - G_0)/Y_0$, and $y_t = (Y_t - Y_0)/Y_0$. Then, from the Keynes-Ramsey rule reported in Eq. (A.8) we can infer the following IS curve with the linearized deviation form:

$$\dot{c}_t = \hat{\sigma}^{-1}\left((1-\tau_0^a)i_t - i_0\tilde{\tau}_t^a - r_t^n - \pi_t\right), \tag{A.19}$$

where $\hat{\sigma} = \sigma/(1-\alpha_g)$, $\alpha_g = G_0/Y_0$, and $\tilde{\tau}_t^a = \tau_t^a - \tau_0^a$.

In a similar way, from Eq. (A.16) we can derive the following interest rate rule:

$$i_t = i_0 + \phi_\pi \pi_t. \tag{A.20}$$

A.7 Derivation of the New Keynesian Phillips curve

We then follow Wieland (2019) to derive the New Keynesian Phillips curve in terms of the linearized deviation form. Based on Eq. (A.14), we have:[17]

$$\dot{\pi}_t = \rho\pi_t - \theta(\rho+\theta)\omega_t, \tag{A.21}$$

where ω_t represents the real wage rate, $\omega_t = \ln(W_t/P_t) - \ln(W_0/P_0)$. By taking the deviation of the variables from their steady-state values in Eq. (A.7) and substituting the result, $\omega_t = \hat{\sigma}c_t + \varphi l_t + \tilde{\tau}_t^w/(1-\tau_0^w)$ as well as $y_t = l_t$ into Eq. (A.21), we obtain the NKPC with the linearized deviation form:

$$\dot{\pi}_t = \rho\pi_t - \kappa c_t - \delta_g g_t - \delta_w \tilde{\tau}_t^w, \tag{A.22}$$

where $\kappa = \theta(\rho+\theta)(\hat{\sigma}+\varphi)$, $\tilde{\tau}_t^w = \tau_t^w - \tau_0^w$, $\delta_g = \theta(\rho+\theta)\varphi$ and $\delta_w = \theta(\rho+\theta)/(1-\tau_0^w)$.

A.7 The linearized deviation form of the economy's resource constraint

Then, based on the definitions: $c_t \equiv (C_t - C_0)/Y_0$, $g_t = (G_t - G_0)/Y_0$, and $y_t = (Y_t - Y_0)/Y_0$, the economy's resource constraint in Eq. (A.18) can be expressed as the deviation form:

$$y_t = c_t + g_t. \tag{A.23}$$

Eq. (A.23) is identical to Eq. (3) in the main text.

Appendix B

Based on Eqs. (12) and (13), we know that $\lambda_1^* < 0 < \lambda_2^*$. Then, by using Eqs. (20a), (21a)

[17] See Wieland (2019) for a detailed derivation which is reported in the online Appendix E. The reader should be reminded that in deriving the New Keynesian Phillips curve in terms of the linearized deviation form we ignore the effect of the productivity shock term.

and (22), we can derive the following results for the time interval $t \in [0^+, T^-]$:

$$\pi_{t,IP} = \frac{\lambda_2^* \lambda_1^* (\int_t^T e^{-\lambda_1^*(T-s)} ds - \int_t^T e^{-\lambda_2^*(T-s)} ds)}{\lambda_2^* - \lambda_1^*} \left((1-\tau_0^a) i^* - r_1^n \right) < 0 , \tag{B1}$$

$$c_{t,IP} = \frac{\rho \lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds) + \lambda_1^* \lambda_2^* (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a) i^* - r_1^n \right) < 0 , \tag{B2}$$

$$y_{t,IP} = \frac{\rho \lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds) + \lambda_1^* \lambda_2^* (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a) i^* - r_1^n \right) < 0 , \tag{B3}$$

where $\pi_{t,IP}$, $c_{t,IP}$, $y_{t,IP}$ denote the inflation rate, consumption and output under the sole ZLB policy, respectively. Eqs. (B1) and (B3) show that, under the sole ZLB policy, the economy will sink into a deep recession with deflation.

Appendix C

This Appendix provides a detailed derivation of the time paths of π_t and c_t during the whole transitional periods when the government implements the fiscal expansion policy coupled with the ZLB interest rate policy. Similar to the mathematical derivations in association with the sole ZLB interest rate policy in Section 3, by referring to Eqs. (20) and (21), the evolution of π_t and c_t in association with distinct periods can be expressed as follows:

$$\pi_t = \begin{cases} \pi_0 ; & t = 0^- \\ \hat{\pi}(\tilde{\tau}_0^a = 0) + A_1^* e^{\lambda_1^* t} + A_2^* e^{\lambda_2^* t} ; & 0^+ \le t \le T^- \\ \pi_0 + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} ; & T^+ \le t \end{cases} , \tag{C1}$$

$$c_t = \begin{cases} c_0 ; & t = 0^- \\ \hat{c}(g_1, \tilde{\tau}_0^w = \tilde{\tau}_0^a = 0) + A_1^* \left((\rho - \lambda_1^*)/\kappa \right) e^{\lambda_1^* t} + A_2^* \left((\rho - \lambda_2^*)/\kappa \right) e^{\lambda_2^* t} & ; 0^+ \le t \le T^- \\ c_0 + A_1 \left((\rho - \lambda_1)/\kappa \right) e^{\lambda_1 t} + A_2 \left((\rho - \lambda_2)/\kappa \right) e^{\lambda_2 t} ; & T^+ \le t \end{cases} . \tag{C2}$$

We can infer from Eq. (17) that $\hat{c}(g_1, \tilde{\tau}_0^w = \tilde{\tau}_0^a = 0)$ in Eq. (C2) can be expressed as $\hat{c}(g_1, \tilde{\tau}_0^w = \tilde{\tau}_0^a = 0) = \rho \left[\left((1-\tau_0^a) i^* - r_1^n \right) - \delta_g g_1 \right] / \kappa$.

Following the same reasoning as in Section 3, four undetermined parameters (A_1, A_2, A_1^*, and A_2^*) in Eqs. (C1) and (C2) can be solved by two continuity conditions, $\pi_{T^-} = \pi_{T^+}$ and $c_{T^-} = c_{T^+}$, and two stability conditions, $A_1 = 0$ and $A_2 = 0$.

Inserting $\hat{\pi}(\tilde{\tau}_0^a = 0) = (1-\tau_0^a) i^* - r_1^n$, $\hat{c}(g_1, \tilde{\tau}_0^w = \tilde{\tau}_0^a = 0) = \rho \left[\left((1-\tau_0^a) i^* - r_1^n \right) - \delta_g g_1 \right] / \kappa$, $\pi_0 = 0$, $c_0 = 0$, $\pi_{T^-} = \pi_{T^+}$, $c_{T^-} = c_{T^+}$, $A_1 = 0$, and $A_2 = 0$ into Eqs. (C1) and (C2), we have:

$$\pi_t = \begin{cases} 0; & t = 0^- \\ \left(\begin{aligned} &\frac{\lambda_2^*(1-e^{-\lambda_1^*(T-t)}) - \lambda_1^*(1-e^{-\lambda_2^*(T-t)})}{\lambda_2^* - \lambda_1^*}\left((1-\tau_0^a)i^* - r_1^n\right) \\ &+\frac{e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)}}{\lambda_2^* - \lambda_1^*}\delta_g g_1 \end{aligned} \right); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}, \tag{C3}$$

$$c_t = \begin{cases} 0; & t = 0^- \\ \left(\begin{aligned} &\frac{\lambda_2^*(\rho - \lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho - \lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\left((1-\tau_0^a)i^* - r_1^n\right) \\ &+\frac{\lambda_1^*(1-e^{-\lambda_2^*(T-t)}) - \lambda_2^*(1-e^{-\lambda_1^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\delta_g g_1 \end{aligned} \right); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}, \tag{C4}$$

According to the resource constraint $y_t = c_t + g_t$ reported in Eq. (3), the evolution of y_t can then be expressed as:

$$y_t = \begin{cases} 0; & t = 0^- \\ \left(\begin{aligned} &\frac{\lambda_2^*(\rho - \lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho - \lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\left((1-\tau_0^a)i^* - r_1^n\right) \\ &+\left(1+\frac{\lambda_1^*(1-e^{-\lambda_2^*(T-t)}) - \lambda_2^*(1-e^{-\lambda_1^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\delta_g\right) g_1 \end{aligned} \right); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}. \tag{C5}$$

Equipped with Eqs. (C3), (C4) and (C5), we can infer the following results for the time interval $t \in [0^+, T^-]$:

$$\pi_{t,g} = \frac{\lambda_2^*\lambda_1^*(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds)}{\lambda_2^* - \lambda_1^*}\left((1-\tau_0^a)i^* - r_1^n\right) + \frac{e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)}}{\lambda_2^* - \lambda_1^*}\delta_g g_1 \gtrless 0, \tag{C6}$$

$$\begin{aligned} c_{t,g} = &\frac{\rho\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds) + \lambda_1^*\lambda_2^*(e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\left((1-\tau_0^a)i^* - r_1^n\right) \\ &+\frac{\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_2^*(T-s)} - e^{-\lambda_1^*(T-s)} ds)}{\kappa(\lambda_2^* - \lambda_1^*)}\delta_g g_1 \gtrless 0, \end{aligned} \tag{C7}$$

$$y_{t,g}=\frac{\rho\lambda_1^*\lambda_2^*\left(\int_t^T e^{-\lambda_1^*(T-s)}-e^{-\lambda_2^*(T-s)}ds\right)+\lambda_1^*\lambda_2^*(e^{-\lambda_1^*(T-t)}-e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^*-\lambda_1^*)}\left((1-\tau_0^a)i^*-r_1^n\right)$$
$$+\left(1+\frac{\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_2^*(T-s)}-e^{-\lambda_1^*(T-s)}ds)}{\kappa(\lambda_2^*-\lambda_1^*)}\right)\delta_g g_1 \gtrless 0, \tag{C8}$$

where $\pi_{t,g}$, $c_{t,g}$, $y_{t,g}$ denote the inflation rate, consumption and output under the expansionary government spending coupled with the ZLB interest rate policy. By comparing (B1) with (C6), (B2) with (C7) and (B3) with (C8), we can obtain:

$$\pi_{t,g}-\pi_{t,IP}=\frac{e^{-\lambda_1^*(T-t)}-e^{-\lambda_2^*(T-t)}}{\lambda_2^*-\lambda_1^*}\delta_g g_1>0\,, \tag{C9}$$

$$c_{t,g}-c_{t,IP}=\frac{\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_2^*(T-s)}-e^{-\lambda_1^*(T-s)}ds)}{\kappa(\lambda_2^*-\lambda_1^*)}\delta_g g_1>0\,, \tag{C10}$$

$$y_{t,g}-y_{t,IP}=\left(1+\frac{\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_2^*(T-s)}-e^{-\lambda_1^*(T-s)}ds)}{\kappa(\lambda_2^*-\lambda_1^*)}\right)\delta_g g_1>0\,. \tag{C11}$$

It is clear from Eq. (C11) that the additional fiscal policy in the form of expansionary government spending could mitigate the extent of the deep recession.

Appendix D

This Appendix provides a detailed derivation of the time paths of π_t and c_t during the overall transitional periods when the government implements the payroll tax cut policy accompanied by the ZLB interest rate policy. Equipped with Eqs. (20) and (21), the evolution of π_t and c_t in association with distinct periods can be described as follows:

$$\pi_t=\begin{cases} 0\,; & t=0^- \\ \left(\begin{array}{l}\dfrac{\lambda_2^*(1-e^{-\lambda_1^*(T-t)})-\lambda_1^*(1-e^{-\lambda_2^*(T-t)})}{\lambda_2^*-\lambda_1^*}\left((1-\tau_0^a)i^*-r_1^n\right)\\ +\dfrac{e^{-\lambda_1^*(T-t)}-e^{-\lambda_2^*(T-t)}}{\lambda_2^*-\lambda_1^*}\delta_w\tilde{\tau}_1^w\end{array}\right); & 0^+\le t\le T^- \\ 0\,; & T^+\le t\end{cases}\,, \tag{D1}$$

$$c_t = \begin{cases} 0; & t = 0^- \\ \left(\begin{array}{l} \dfrac{\lambda_2^*(\rho - \lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho - \lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a) i^* - r_1^n \right) \\ + \dfrac{\lambda_1^*(1 - e^{-\lambda_2^*(T-t)}) - \lambda_2^*(1 - e^{-\lambda_1^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \delta_w \tilde{\tau}_1^w \end{array} \right) & ; 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases} . \tag{D2}$$

Based on the resource constraint $y_t = c_t + g_t$ together with $g_t = 0$, the evolution of y_t can then be expressed as:

$$y_t = \begin{cases} 0; & t = 0^- \\ \left(\begin{array}{l} \dfrac{\lambda_2^*(\rho - \lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho - \lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a) i^* - r_1^n \right) \\ + \dfrac{\lambda_1^*(1 - e^{-\lambda_2^*(T-t)}) - \lambda_2^*(1 - e^{-\lambda_1^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \delta_w \tilde{\tau}_1^w \end{array} \right) & ;\ 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases} . \tag{D3}$$

By using Eqs. (D1), (D2) and (D3), we can derive the following expressions for the time interval $t \in [0^+, T^-]$:

$$\pi_{t,w} = \frac{\lambda_2^* \lambda_1^* (\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds)}{\lambda_2^* - \lambda_1^*} \left((1-\tau_0^a) i^* - r_1^n \right) + \frac{e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)}}{\lambda_2^* - \lambda_1^*} \delta_w \tilde{\tau}_1^w < 0, \tag{D4}$$

$$\begin{aligned} c_{t,w} = & \frac{\rho \lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds) + \lambda_1^* \lambda_2^* (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a) i^* - r_1^n \right) \\ & + \frac{\lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_2^*(T-s)} - e^{-\lambda_1^*(T-s)} ds)}{\kappa(\lambda_2^* - \lambda_1^*)} \delta_w \tilde{\tau}_1^w < 0, \end{aligned} \tag{D5}$$

$$\begin{aligned} y_{t,w} = & \frac{\rho \lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds) + \lambda_1^* \lambda_2^* (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a) i^* - r_1^n \right) \\ & + \frac{\lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_2^*(T-s)} - e^{-\lambda_1^*(T-s)} ds)}{\kappa(\lambda_2^* - \lambda_1^*)} \delta_w \tilde{\tau}_1^w < 0, \end{aligned} \tag{D6}$$

where $\pi_{t,w}$, $c_{t,w}$, $y_{t,w}$ denote the inflation rate, consumption and output under the payroll tax cut coupled with the ZLB interest rate policy. Comparing (B1) with (D4), (B2) with (D5) and (B3) with (D6) yields:

$$\pi_{t,w} - \pi_{t,IP} = \frac{e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)}}{\lambda_2^* - \lambda_1^*} \delta_w \tilde{\tau}_1^w < 0, \tag{D7}$$

$$c_{t,w} - c_{t,IP} = \frac{\lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_2^*(T-s)} - e^{-\lambda_1^*(T-s)} ds)}{\kappa(\lambda_2^* - \lambda_1^*)} \delta_w \tilde{\tau}_1^w < 0, \tag{D8}$$

$$y_{t,w} - y_{t,IP} = \frac{\lambda_1^* \lambda_2^* (\int_t^T e^{-\lambda_2^*(T-s)} - e^{-\lambda_1^*(T-s)} ds)}{\kappa(\lambda_2^* - \lambda_1^*)} \delta_w \tilde{\tau}_1^w < 0. \tag{D9}$$

Eq. (D9) clearly indicates that a payroll tax cut will further exacerbate the deep recession.

Appendix E

This appendix provides a detailed derivation of the time paths of π_t and c_t under the situation where the government implements the financial assets tax cut policy accompanied by the ZLB interest rate policy. By referring to Eqs. (20) and (21), the evolution of π_t and c_t in association with distinct periods can be described as follows:

$$\pi_t = \begin{cases} 0; & t = 0^- \\ \dfrac{\lambda_2^*(1 - e^{-\lambda_1^*(T-t)}) - \lambda_1^*(1 - e^{-\lambda_2^*(T-t)})}{\lambda_2^* - \lambda_1^*} \left((1-\tau_0^a)i^* - r_1^n - i_0 \tilde{\tau}_1^a\right); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}, \tag{E1}$$

$$c_t = \begin{cases} 0; & t = 0^- \\ \dfrac{\lambda_2^*(\rho - \lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho - \lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a)i^* - r_1^n - i_0 \tilde{\tau}_1^a\right); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}. \tag{E2}$$

By using the resource constraint $y_t = c_t + g_t$ together with $g_t = 0$, the evolution of y_t can then be expressed as:

$$y_t = \begin{cases} 0; & t = 0^- \\ \dfrac{\lambda_2^*(\rho - \lambda_2^* e^{-\lambda_1^*(T-t)}) - \lambda_1^*(\rho - \lambda_1^* e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)} \left((1-\tau_0^a)i^* - r_1^n - i_0 \tilde{\tau}_1^a\right); & 0^+ \le t \le T^- \\ 0; & T^+ \le t \end{cases}. \tag{E3}$$

Equipped with Eqs. (E1), (E2) and (E3), we can infer the following results for the time interval $t \in [0^+, T^-]$:

$$\pi_{t,a} = \frac{\lambda_2^* \lambda_1^* (\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)} ds)}{\lambda_2^* - \lambda_1^*} \left((1-\tau_0^a)i^* - r_1^n - i_0 \tilde{\tau}_1^a\right) < 0, \tag{E4}$$

$$c_{t,a} = \frac{\rho\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)}ds) + \lambda_1^*\lambda_2^*(e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\left((1-\tau_0^a)i^* - r_1^n\right) - \frac{\lambda_1^*\lambda_2^*\left(\rho(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)}ds) + (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})\right)}{\kappa(\lambda_2^* - \lambda_1^*)} i_0\tilde{\tau}_1^a < 0, \tag{E5}$$

$$y_{t,a} = \frac{\rho\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)}ds) + \lambda_1^*\lambda_2^*(e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})}{\kappa(\lambda_2^* - \lambda_1^*)}\left((1-\tau_0^a)i^* - r_1^n\right) - \frac{\lambda_1^*\lambda_2^*\left(\rho(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)}ds) + (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})\right)}{\kappa(\lambda_2^* - \lambda_1^*)} i_0\tilde{\tau}_1^a < 0, \tag{E6}$$

where $\pi_{t,a}$, $c_{t,a}$, $y_{t,a}$ denote inflation, consumption and output under the payroll tax cut coupled with the ZLB interest rate policy. By comparing (B1) with (E4), (B2) with (E5) and (B3) with (E6), we have:

$$\pi_{t,a} - \pi_{t,IP} = -\frac{\lambda_1^*\lambda_2^*(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)}ds)}{\lambda_2^* - \lambda_1^*} i_0\tilde{\tau}_1^a < 0, \tag{E7}$$

$$c_{t,a} - c_{t,IP} = -\frac{\lambda_1^*\lambda_2^*\left(\rho(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)}ds) + (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})\right)}{\kappa(\lambda_2^* - \lambda_1^*)} i_0\tilde{\tau}_1^a < 0, \tag{E8}$$

$$y_{t,a} - y_{t,IP} = -\frac{\lambda_1^*\lambda_2^*\left(\rho(\int_t^T e^{-\lambda_1^*(T-s)} - e^{-\lambda_2^*(T-s)}ds) + (e^{-\lambda_1^*(T-t)} - e^{-\lambda_2^*(T-t)})\right)}{\kappa(\lambda_2^* - \lambda_1^*)} i_0\tilde{\tau}_1^a < 0. \tag{E9}$$

It is quite obvious from Eq. (E9) that a financial assets tax cut tends to further exacerbate the deep recession.

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