A Residual-based Test For Multicointegration In Models With Structural Breaks And Threshold Adjustment To Steady State

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Abstract

In this paper I derive a test of Multicointegration of I (2) series that takes into account both structural breaks and threshold adjustment to steady state. I extend the I(2) – multicointegration test proposed by Berenguer-Rico and Carrion-i-Silvestre (2005), by relaxing the assumption of symmetric adjustment. In a way, I adapt the Engsted et al. (1997) approach to the concept of multicointegration and following Enders and Siklos (2001) I model the multicointegration relation while allowing for asymmetric adjustment to long run equilibrium. Further, use is made of the multivariate invariance principle, the weak convergence to stochastic integrals for dependent heterogeneous processes, and the continuous mapping theorem in order to derive an augmented Dickey-Fuller type of multicointegration test for I (2) series. I find that the limiting distributions of the estimators and test statistics associated with multicointegration depend on the cut-off point of the asymmetric response and the break point. I illustrate the test by applying it to understanding interest rate pass-through in Malawi. The derived multicointegration test confirms the presence of multicointegration among lending rates, policy rate and Treasury bill rates in Malawi in which lending rates adjust asymmetrically to steady state following a positive or negative policy rate adjustment.

Keywords: Multicointegration; Threshold Adjustment; I (2) series; ADF test

1. Introduction

Most economic time series are I (1) containing deterministic trends. Recent research, however, has shown that some economic time series(for instance nominal variables like money balances, prices, wages, and stock variables) may better be characterized as integrated of order two(see Haldrup,1994;Johansen,1995;Kitamura,1995; Paruolo,1996; Engsted et al,1997).Time series that are integrated of order two are essential in analysing multicointegration, a very important time series property that is rarely tested empirically. As explained by Engsted et al (1997) the concept of multicointegration follows from the work of Granger and Lee (1980, 1990). Basically, Granger and Lee (1980; 1990) define multicointegration of time series that are integrated of order one as follows: assume that two I (1) flow series X_t and Y_t co-integrate such that $Z_t = Y_t - \alpha X_t$ is I (0). It follows that $\sum_{i=1}^{T} Z_i$ is I (1) which might co-integrate with X_t and Y_t such that $U_t = \sum_{i=1}^{t} Z_i - Y_t - \alpha X_t$ is I (0). Thus, there are essentially two levels of cointegration between just two I (1) time series. Granger and Lee denote this sort of cointegration 'multicointegration'. Although Granger and Lee (1990) defined multicointegration for I (1) series, Engsted et al (1997) extended the definition analogously to I (2) variables.

The existence of multicointegration presents serious statistical implication, but surprisingly, many researchers do not check for its presence. According to Lee (1990) and Engsted and Johansen (1997) the presence of multicointegration undermines usual procedures for estimation and testing in cointegrated systems. This is because, by ignoring the second level cointegration, the standard error correction model is mis specified. Engsted et al (1997) also points out that multi-cointegration has serious consequences on power of forecasting and hypothesis testing. Engsted et al(1997) emphasises the fact that 1(2) cointegration is relevant for the analysis of multi-cointegrated time series since implicitly it involves the cumulation of I(1) series which by definition are 1(2).

Basically, there are two approaches to testing the presence of multi-cointegration in the literature; first, the two-step approach, developed by Granger and Lee (1990) and second, the one-step approach proposed by Engsted et al (1997).While Granger and Lee (1990) assume I (1) series in their proposition of testing multicointegration, Engsted et al (1997) seek to test multicointegration using one-step procedure for I(2) series and this presents the first difference between these two approaches . Secondly, Granger and Lee(1990) only consider the case where the cointegrating vector at the first level is known in advance, i.e. where $\alpha = 1$, and, hence, does not need to be estimated. This implies that the statistical analysis only consists of investigating whether the directly observable I(1) series X , Y, and U, are cointegrated. Considering the statistical implications that comes when one has to estimate α first in the two-stage approach, Engsted et al (1997) extends, by proposing one-step approach, the analysis by assuming that the first level cointegrating vector is not known and therefore has to be estimated.

Technically, the two-step procedure proposed by Granger and Lee(1990) is conducted as follows; First, an almost sure consistent estimate of α is obtained in a regression of Y_t on X_t. The residuals from this regression, $\hat{Z}_t$, then provide the estimated Z-series. In the second step, the cumulated sum of $\hat{Z}_t$, is generated and thus giving the U-series, which subsequently is regressed onto X_t, and/or Y_t, resulting in an almost sure consistent estimate of α, provided there is multicointegration. The problem with this procedure is that the limiting theory to test for multicointegration is complicated by the fact that the auxiliary regression is based on cumulated regression residuals from another regression (see Engsted et al,1997 for proof) . This means that standard methods to test for cointegration therefore become invalidated for this particular type of models, since the asymptotics will be expressed in terms of functionals of a Brownian bridge process rather than a Brownian motion process as is normally the case (Engsted et al,1997). Hence, if the first step cointegration parameter is not known in advance, the size of standard residual based cointegration tests will be incorrect.

One notices that there are two possible solutions to this problem: Either, the actual distributions and new critical values can be tabulated to account for the particular distributions or one may develop a one-step procedure which estimates both levels of cointegration almost surely. Engsted et al(1997) developed such a one-step estimation procedure with favourable statistical properties compared to the two-step Granger and Lee (1990) procedure.

The study by Engsted et al (1997), being an extension of the Granger and Lee (1990) proposition, is itself not without proposed extensions in the literature. Most importantly, Berenguer-Rico and Carrion-i-Silvestre (2005) propose a more general type of multicointegration that allows for the existence of a structural break in the cointegrating vector. Following Gregory and Hansen (1996), who link the concept of cointegration to the idea of structural change, and using the Engsted et al (1997) approach to the concept of multicointegration, Berenguer-Rico and Carrion-i-Silvestre (2005) model the multicointegration relationship while allowing the possibility of regime shifts. They (Berenguer-Rico and Carrion-i-Silvestre, 2005) make two observations in substantiating their contribution to literature; First, they observe that multicointegration is a concept that appears in the long-run. And second, the previous definition of multicointegration had assumed invariant parameters in time. Considering that the longer the time period that is analysed the higher the probability of finding a structural change a more general type of multicointegration that allows for the existence of a changing relation in time, according to them, was imperative.

2. Contribution to literature

While building upon the work of Granger and Lee (1980; 1990) and Engsted et al (1997), I observe that previous studies on multicointegration assume symmetric adjustment to long run equilibrium. In other words, they assume that positive and negative disequilibria values in the short run adjust similarly towards long run steady state. I observe however that a large number of studies recently have shown that key macroeconomic time series such as real gross domestic product, unemployment, and industrial production display asymmetric adjustment over the course of the business cycle (see Neftci, 1984; DeLong & Summers,1986; Sichel,1993;Ramsey & Rothman, 1996;and Bradley & Jensen,1997). Pippenger and Goering (1993), Balke and Fomby (1997), and Enders and Granger (1998) showed that cointegration tests have low power in the presence of asymmetric adjustment. According to Enders and Siklos (2001) cointegration tests and indeed their eventual error correction models are mis specified if they do not consider asymmetry when there happens to be asymmetric adjustment in the series. Therefore, the extension of Engsted et al(1997) one step approach to include asymmetric adjustment to steady state is imperative. In this paper, I extend Enders and Siklos (2001) approach combined with Berenguer-Rico and Carrion-i-Silvestre (2005) approach to extend the Engsted et al (1997) one-step approach to multi-cointegration and develop a more general approach to test for multi-cointegration for I(2) series that takes into account both structural breaks and asymmetric adjustment to long run equilibrium.

3. The Model

In this section, I develop single-equation regression models that allow for multicointegration with structural change and threshold adjustment. Following Berenguer-Rico and Carrion-i-Silvestre (2005), consider a one-dimensional time series $\{y_t\}_0^{+\infty} \sim I(2)$ and k-dimensional time series $\{x_t\}_0^{+\infty} \sim I(2)$. I present the model of multicointegration with the structural break (SB) "known" to have occurred at some time period t_b ;

$$Y_t = \mu' C_{m_t} + \beta' X_t + \gamma' x_t + \delta SB_t + \eta_t$$

$$SB_t = \begin{cases} 1, t \geq t_b \\ \\ 0, t < t_b \end{cases}$$

When $s_t = \Delta^{-1}\hat{\eta}_t = \sum_{j=1}^{t} \hat{\eta}_j$ cointegrate with $\{x_t\}_0^{+\infty}$ and $\{y_t\}_0^{+\infty}$ I $s_t = \alpha' m_t + \gamma' x_t + \theta y_t + \delta SB_t + \xi_t$, from-which $\hat{\xi}_t \sim I(0)$.

Taking into consideration the asymmetric adjustment from short run disequilibrium to long run equilibrium, I model the error correction model as;

$$\Delta Y_t = \mu' C_{m_t} + \sum_{n=1}^{N} \theta_n \Delta Y_{t-n} + \sum_{j=1}^{J} \beta_j \Delta X_{t-j} + \sum_{h=1}^{H} \vartheta_h \Delta x_{t-h} + \delta SB_t + D_t \rho_{1t} \hat{\eta}_{t-1} + (1 - D_t) \rho_2 \hat{\eta}_{t-1} + \varepsilon_t$$

$$D_t = \begin{cases} 1, \hat{\eta}_{t-1} \geq 0 \\ 0, \hat{\eta}_{t-1} < 0 \end{cases}$$

To test for asymmetric adjustment one uses the Wald test statistic. The null hypothesis posits that the coefficients of the error-correction terms in the short run equation are equal, that is $H_0 : \rho_1 = \rho_2$

4. Underlying assumptions

In this section, I present the underlying assumptions that I make to analyse the limiting properties of the processes outlined in the previous sections. I heavily follow Berenguer-Rico and Carrion-i-Silvestre (2005) and Allan w. Gregory and Bruce E. Hansen (1992) in formulating these assumptions. Following Berenguer-Rico and Carrion-i-Silvestre (2005), I make significant use of the following three results in our theoretical developments: the multivariate invariance principle based on Herrndorf (1984), Phillips and Durlauf (1986), Haldrup (1994) and Gregory and Hansen (1996); the weak convergence to stochastic integrals for dependent heterogeneous processes studied in Hansen (1992) and applied in Gregory and Hansen (1996); and the continuous mapping theorem (CMT) from Billingsley (1968,Thm. 5.1). As such, I make the following assumptions;

Assumption 1: For some $p > \beta > 2.5$ $\{\eta_t\}$ is mean-zero and strong mixing with mixing coefficients of size $-{}^{p\beta}\!/_{p-\beta}$

Assumption 2: $Sup_{t\geq 1}\|\eta_t\|_p = C < \infty$

Assumption 3: $\lim_{n\to\infty} n^{-1}E\left(\left(\sum_{j=1}^{t}\eta_j\right)\left(\sum_{j=1}^{t}\eta_j\right)'\right) = K < \infty$

Assumption 4: $\rho_1 < 0$ *and* $\rho_2 < 0$ and $(\rho_1 + 1)(\rho_2 + 1) < 1$

I make this assumption since Petrucelli and Woolford (1984) showed that $\rho_1 < 0$ *and* $\rho_2 < 0$ and $(\rho_1 + 1)(\rho_2 + 1) < 1$ *are* the necessary and sufficient conditions for stationarity of innovations with asymmetric adjustment.

Assumption 5: Let $[.]$define an integer part of its argument , let $\gamma = (SB', D')'$ and let T be the sample size, then $(p = 1 + m_1 + m_2)$-dimensional stochastic process $B_t(r,\gamma) = T^{-1/2}\sum_{t=1}^{[Tr]}\eta_t$ defined on $[0,1]$will converge weakly in distribution to a vector Brownian motion process with a long run covariance matrix K i.e. $B_T(r,\gamma) \Rightarrow B(r,\gamma) \equiv BM(K,\gamma)$ as $T \to \infty$

5. Lemmas

In this section, I make propositions that based on the underlying assumptions lead to the theoretical developments of this paper.

Lemma 1: $\lim_{T\to\infty} T^{-1}D_T^{-1}\sum_{t=1}^{T} z_t z_t' D_T^{-1} = T^{-1}\sum_{t=1}^{T} \underline{z_t}\,\underline{z_t'} \Rightarrow \int_0^1 B_*(r,\gamma)B_*'(r,\gamma)$

$$\lim_{T\to\infty} T^{-1/2}D_T^{-1}\sum_{t=1}^{T} z_t\eta_t = T^{-1}\sum_{t=1}^{T} z_t\eta_t \Rightarrow \int_0^1 B_*(r,\gamma)dB_0(r,\gamma) + (0',(1-\gamma)\Delta_{10},0')$$

$$\lim_{T\to\infty} T^{-3/2}D_T^{-1}\sum_{t=1}^{T} z_t\Delta^{-1}\eta_t = T^{-1}\sum_{t=1}^{T} \underline{z_t\Delta^{-1}\eta_t} \Rightarrow \int_0^1 B_*(r,\gamma)B_0(r,\gamma)$$

$$\lim_{T\to\infty} T^{-5/2}D_T^{-1}\sum_{t=1}^{T} z_t\Delta^{-2}\eta_t = n^{-1}\sum_{t=1}^{T} \underline{z_t\Delta^{-2}\eta_t} \Rightarrow \int_0^1 B_*(r,\gamma)\overline{B_0(r,\gamma)}$$

Proof: The proof of this lemma is analogous to that in Berenguer-Rico and Carrion-i-Silvestre (2005), which is itself a component of the proof of Lemma 1 in Haldrup (1994).

6. Results

In this section I provide some theoretical results derived from the aforementioned lemma and underlying assumptions.

6.1 Estimation of the cointegrating equation

Following Berenguer-Rico and Carrion-i-Silvestre (2005), the cointegrating equation $Y_t = \mu' C_{m_t} + \beta' X_t + \gamma' x_t + \delta SB_t + \eta_t$ can be written as $Y_t = \phi' X_t^* + \eta_t$. This implies that one can apply OLS $Y_t = \phi' X_t^* + \eta_t$ and get;

$$\hat{\phi}_{OLS} = \left(\sum_{t=1}^{T} X_t^* X_t^{*\prime}\right)^{-1}\left(\sum_{t=1}^{T} X_t^* Y_t\right)$$

Theorem 1: $\lim_{T\to\infty} T^{-1}D_T^{-1}\sum_{t=1}^{T} z_t z_t' D_T^{-1} = T^{-1}\sum_{t=1}^{T} \underline{z_t}\,\underline{z_t'} \Rightarrow \int_0^1 B_*(r,\gamma)B_*'(r,\gamma)$

Remark: This theorem shows that the asymptotics of OLS estimators are expressed as Brownian motion process. This ensures the validity of the standard residual based cointegration tests.

Proof: As noted by Berenguer-Rico and Carrion-i-Silvestre (2005), the proof is entirely analogous to Haldrup (1994).

6.2 Testing for multicointegration with threshold adjustment

In this section I provide the asymptotic properties of residual-based Dickey-Fuller class of tests for multicointegration taking into account the presence of regime shifts and asymmetric adjustment. To achieve this objective, just like Berenguer-Rico and Carrion-i-Silvestre (2005), I follow again the Engsted, Gonzalo and Haldrup (1997) approach to the concept of multicointegration. For the purposes of this paper, I follow Perron (1990) and Enders and Siklos (2001) and suppose that the break point is known a prior and the cut-off point for asymmetric adjustment is also known. Our goal is to analyse how the limiting distribution of the residual-based Dickey-Fuller test for asymmetric response multicointegration or I(2) cointegration, studied in Haldrup (1994), is modified when the long-run equilibrium relationship has changed at a one known point in time.

It is a known fact now that if the series in a system are multicointegrated the innovations must be integrated of order zero. However, as also noted by Berenguer-Rico and Carrion-i-Silvestre (2005), practically, it is likely that cointegration to at least I(1) level will occur. For this reason I follow Berenguer-Rico and Carrion-i-Silvestre (2005) and assume that the null hypothesis is that there is cointegration at the first level;

$$H_0 : \hat{\eta} \sim I(1)$$
$$H_1 : \hat{\eta} \sim I(0)$$

In order to test this hypothesis, I conduct the augmented Dickey-Fuller regression;

$$\Delta\hat{\eta}_t = D_t \rho_{1t} \hat{\eta}_{t-1} + (1 - D_t)\rho_2 \hat{\eta}_{t-1} + \sum_{j=1}^{P-1} \gamma \Delta\hat{\eta}_{t-j} + \mu_t$$

Where D_t is a Heaviside indicator function such that;

$$D_t = \begin{cases} 1, \hat{\eta}_{t-1} \geq 0 \\ \\ 0, \hat{\eta}_{t-1} < 0 \end{cases}$$

In this case I assume that the cut-off point for asymmetric adjustment is at 0. In other words, positive disequilibria values adjust differently compared to negative disequilibria values towards steady state.

Theorem 2: $\quad ADF \Rightarrow \left(\int_0^1 W^2(r,\gamma) \right)^{-\frac{1}{2}} \left(\int_0^1 W(r,\gamma) dW(r,\gamma) \right)$

Where $W(r,\gamma) = W_0(r,\gamma) - \left(\int_0^1 W_0(r,\gamma) W_*^{'}(r,\gamma) \right) \left(\int_0^1 W_*(r,\gamma) W_*^{'}(r,\gamma) \right)^{-1} W_*(r,\gamma)$

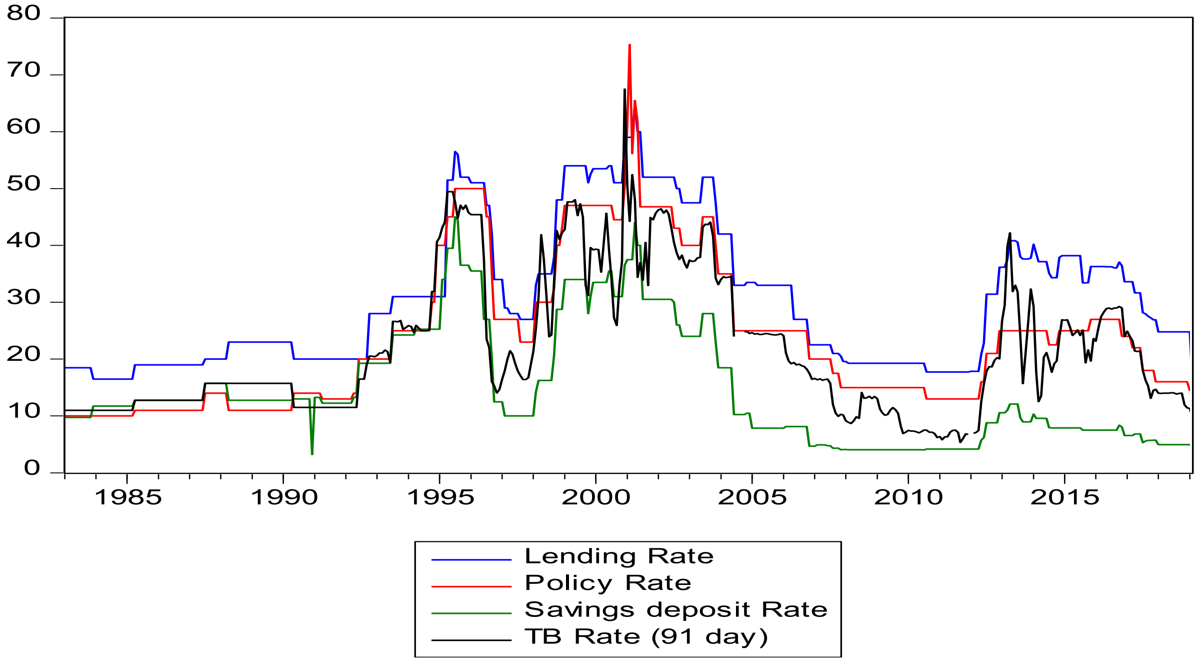
Remark: The limiting distribution of this test to Multicointegration depends on γ, the break fraction parameter, i.e. the cut-off point of the asymmetric response and the break point

Proof: The proof is entirely analogous to that of Phillips and Ouliaris (1990).

7. Application: Is the Interest Rate Adjustment Dynamics in Malawi Symmetric or Asymmetric?

In this section I apply the concept of multicointegration developed in the preceding sections to practical data. It is known that the influence of monetary policy depends on the effectiveness of the interest rate pass-through, that is the size and the speed to which changes in the central bank policy actions are transmitted to bank retail interest rates. The objective of this section is two-fold: (1) to examine the presence of multicointegration among interest rates in Malawi; (2) to investigate the relationship between the policy-controlled interest rates (Treasury bill rates) and the bank lending rates in Malawi with the view to empirically examine the size and speed of the interest rate pass-through in the long run and short run and determine whether the pass-through process is symmetric or asymmetric, bearing in mind the regime shifts that the financial sector has and continues to experience. Even though my approach to multicointegration assumed I (2) series, I apply it on interest rates, which are more likely I (1), for expository purposes. After all, the methodology developed above can unequivocally be extended to cases where the series are I(1), save only cases where there is a combination of I(1) and I(2) series. I use monthly data from January, 1983 to February, 2019 obtained from the reserve bank of Malawi. Figure 1 below sketches the trends of interest rates in the sample period.

Figure 1; Interest Rate Developments in Malawi



One notices that despite their higher volatility, the deposit, Lending, Treasury bill and policy rates exhibit a regular co-movement suggesting a long-run relationship between these rates; this co-movement is discernible in Figure 1.

As is tradition in time series econometrics, to achieve our objectives I firstly check for seasonality before I carry out stationarity tests. Graphs of seasonal movements of the interest rates are presented in figure 2 below. It is evident that there are no seasonal effects, as expected, in the movements of the rates. In addition, figure 1 shows that there are no deterministic trends in the variables. As such I test the interest rates series directly of unit roots without having to de-seasonalise and/or de-trend them first. I apply four unit root tests, namely the standard augmented Dickey - Fuller (ADF) test, the ERS Dickey-Fuller test with generalized least squares de-trending due to Elliot et al (1996), the Phillips-Perron (PP) test, and the KPSS test due to Kwiatkowski et al. (1992).Table 1 presents the results of applying the four unit root tests. Guided by a graphical inspection of the time series, intercept term is included in all the series. In all the tests but KPSS, test statistics greater than critical values in absolute value terms establish stationarity; the order is reversed in the KPSS test.

Figure 2; Seasonal trends of interest rates in Malawi

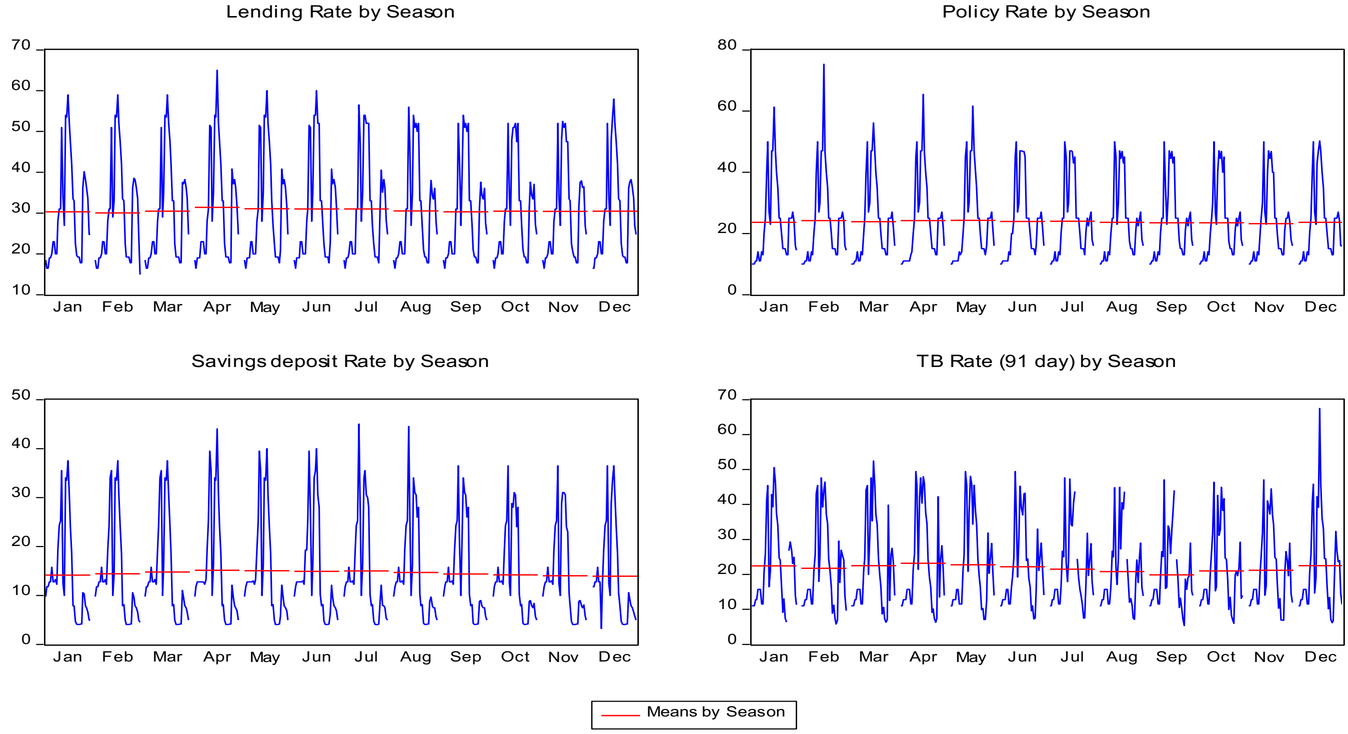


Table 1: Unit root tests of the series

Variable	Test Statistics			
	ADF Test (CV_1 = -3.445) (CV_2 = -2.868) (CV_2 = -2.570)	ERS Test (CV_1 = -2.570) (CV_2 = -1.941) (CV_2 = -1.616)	PP Test (CV_1 = -3.445) (CV_2 = -2.868) (CV_2 = -2.570)	KPSS Test (CV_1 = 0.739) (CV_2 = 0.463) (CV_2 = 0.347)
Lending Rate	Level= -2.217 1st Diff = -18.648	Level= -1.207 1st Diff = -18.589	Level= -1.814 1st Diff = -18.833	Level=0.439 1st Diff =0.169
Treasury Rate	Level= -2.810 1st Diff = -19.320	Level= -2.193 1st Diff = -19.247	Level= -2.752 1st Diff = -19.763	Level= 0.356 1st Diff =0.099
Policy Rate	Level= -2.216 1st Diff = -17.218	Level= -2.212 1st Diff = -12.648	Level=-2.101 1st Diff = -12.334	Level=0.612 1st Diff =0.002
Deposit Rate	Level=-2.200 1st Diff = -18.001	Level= -2.121 1st Diff = -18.234	Level= -2.111 1st Diff = -14.601	Level=0.754 1st Diff =0.012
Note: CV_1 is the 1% critical value for the test with an intercept term only; CV_2 is the 5% critical value for the test with intercepts term only; CV_3 is the 10% critical value for the test with intercept term only.				

From Table 1 above I notice that all the variables are I(1). As such I test for cointegration using the single equation approaches of Engle-Granger and Phillips-Ouliaris tests, the results of which are as presented in table 2 below;

Table 2: Cointegration Test

Equation	Engle-Granger Test	Phillips-Ouliaris Test
Lending Rate	P-value 0.002	P-value 0.000

The results indicate the presence of cointegration of the I(1) interest rate series. I then run the cointegration test equation (1) using the canonical cointegrating regression method and fully modified least squares cointegrating regression method the results of which are presented in table 3.

$$Lending\_Rate_t = \alpha + \beta_1 Policy\_Rate_t + \beta_2 TBrate_t + \delta_1 D1_t + \delta_2 D2_t + \mu_t \qquad (1)$$

Table 3: Usual Cointegrating regression results

Variable Name	Fully Modified	Canonical Regression
Policy Rate	0.749470***	0.741183***
Treasury Bill Rate	0.255362***	0.263168***
D1(Structural break in 1989)	-2.022270***	-1.964723***
D2(Structural break in 2012)	4.970878***	4.937322***
Constant	7.860885***	7.847897***
R-squared	0.951099	0.951039
Adjusted R-squared	0.950637	0.950578
S.E. of regression	2.736215	2.737868
Long-run variance	23.27919	23.27919

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

From these cointegrating regression equations, I derive the residuals and hence compute $s_t = \Delta^{-1}\hat{\mu}_t = \sum_{j=1}^{t}\hat{\mu}_j$. I then run the following model using the fully modified and canonical regression approach;

$$\sum_{j=1}^{t}\hat{\mu}_j = \alpha + \beta_1 \sum_{j=1}^{t} Policy\_rate_j + \beta_2 \sum_{k=1}^{t} TB\_rate_k + \alpha_1 Policy\_rate_t + \alpha_2 TB\_rate_t + \delta SB_t + \phi_1 D1_t + \phi_2 D2_t$$

The results of the multicointegration regression model are presented in table 4 below;

Table 3: Second Level Cointegrating regression results

Variable Name	Fully Modified OLS	Canonical Regression
Policy Rate	-2.660703***	-2.965604***
TB Rate	1.074557*	1.400557
Policy Rate(cumulative)	0.030924	0.033007
TB Rate(cumulative)	-0.034758	-0.036967
D1(Structural break in 1989)	68.64630***	68.19859***
D2(Structural break in 2012)	22.36938**	21.63007*
Constant	-5.157632	-5.201902
R-squared	0.554856	0.550374
Adjusted R-squared	0.548512	0.543966
S.E. of regression	24.83459	25.01001
Long-run variance	3344.149	3367.563

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

From these regression equations, I then derive the residuals. Figure 3 presents the graph depicting the residuals.

Figure 3; Level 2 residuals

I then test the second level residuals for unit root using the conventional ADF, ERS, PP and KPSS tests. The results of these unit root tests are presented in table 5 below;

Table 5: Unit root tests of the residuals in levels

Variable	Test Statistics			
	ADF Test (CV_1 = -3.445) (CV_2 = -2.868) (CV_3 = -2.570)	ERS Test (CV_1 = -2.570) (CV_2 = -1.941) (CV_3 = -1.616)	PP Test (CV_1 = -3.445) (CV_2 = -2.868) (CV_3 = -2.570)	KPSS Test (CV_1 = 0.739) (CV_2 = 0.463) (CV_3 = 0.347)
Canonical residuals	-2.822	-2.051	-3.012	0.074
Fully Modified Residuals	-2.722	-1.919	-2.745	0.078
Note: CV_1 is the 1% critical value for the test with an intercept term only; CV_2 is the 5% critical value for the test with intercepts term only; CV_3 is the 10% critical value for the test with intercept term only.				

From the results in table 5, I reject the null hypothesis of no multi-cointegration implying that there exists level-two cointegration over and above level-one cointegration. This implies that running an error correction model that does not take into account this level two cointegration would to biased estimates due to specification error of the error correction model. Even though this proves the presence of multi-cointegration, the test approaches do not take into account the possibility of asymmetric adjustment. I therefore re-conduct the stationarity test, this time using the following ADF-type regression model that considers asymmetry;

$$\Delta\hat{\mu}_t = D_t \rho_1 \hat{\mu}_{t-1} + (1 - D_t)\rho_2 \hat{\mu}_{t-1} + \sum_{j=1}^{P-1} \gamma \Delta\hat{\mu}_{t-j} + \mu_t$$

I compare the performance of this model against an ADF-type model without taking into account asymmetric adjustment;

$$\Delta\hat{\mu}_t = \rho\hat{\mu}_{t-1} + \sum_{j=1}^{P-1} \gamma_j \Delta\hat{\mu}_{t-j} + \mu_t$$

I run these models using both the fully modified residuals and the canonical residuals, for comparative purposes, the results of which are respectively presented in tables 6 and 7 below.

Table 6: Comparison of ADF tests (with and without asymmetry) Models using FMOLS residuals

Variable Name/Item	Without Asymmetry	With Asymmetry
ρ	0.749470^{***}	-
γ_1	0.255362	-0.040859
γ_2	-2.022270^{***}	-0.137999^{***}
γ_3	4.970878^{***}	0.154083^{***}
ρ_1	-	-0.086791^{***}
ρ_2	-	-0.000369
$\rho_1 = \rho_2$	-	-0.086422^{***}
R-squared	0.071232	0.079522
Adjusted R-squared	0.062280	0.068405
S.E. of regression	6.948221	6.925493
Sum squared resid	20035.28	19856.45
Log likelihood	-1407.603	-1405.720
F-statistic	7.957155	7.153264
Prob(F-statistic)	0.000003	0.000002

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Comparison of ADF tests (with and without asymmetry) Models using Canonical residuals

Variable Name	Without Asymmetry	With Asymmetry
ρ	-0.044245^{***}	-
γ_1	-0.073874	-0.066327
γ_2	-0.176115^{***}	-0.175189^{***}
γ_3	0.143009^{***}	0.139744^{***}
ρ_1	-	-0.095969^{***}
ρ_2	-	-0.004102
$\rho_1 = \rho_2$	-	-0.091867^{***}
R-squared	0.088922	0.096459
Adjusted R-squared	0.080140	0.085547

S.E. of regression	7.757715	7.734883
Sum squared resid	24975.59	24768.97
Log likelihood	-1453.888	-1452.144
F-statistic	10.12604	8.839451
Prob(F-statistic)	0.000000	0.000000

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The results of these models indicate that the asymmetric ADF-type model outperforms the conventional ADF test that disregards the threshold adjustment in terms of R-squared, standard error of the regression and indeed the log-likelihood value. In either case, I fail to reject the null hypothesis of no multi-cointegration. In essence, this proves that there is two-way cointegration among lending rates, policy rate and treasury bill rates in Malawi such that running an error correction model that ignores the second level cointegration leads to biased estimates.

I therefore model the short run dynamics using an error correction model that takes into account both levels of cointegration. In addition I take into account possible asymmetric adjustment and structural breaks. Fuertes, Heffernan and Kalotchou (2006), argue that due to structural shocks, exogenous and endogenous factors, the speed of adjustment may be asymmetric with respect to the magnitude and direction of monetary policy action. Therefore, it is important to relax the symmetric assumption in order to assess the true nature of the adjustment dynamics. To account for the potential asymmetry in the adjustment process I split the residuals (both level one, ECT^{+}_{t-1}, and level two, $MECT^{+}_{t-1}$, obtained from the long-run relationship into two series of positive and negative residuals defined as follows:

$$ECT^{+} = ECT \text{ if } ECT > 0;\ ECT^{+} = 0 \text{ if } ECT < 0$$
$$ECT^{-} = ECT \text{ if } ECT < 0;\ ECT^{-} = 0 \text{ if } ECT > 0$$

$$MECT^{+} = MECT \text{ if } MECT > 0;\ MECT^{+} = 0 \text{ if } MECT < 0$$
$$MECT^{-} = MECT \text{ if } MECT < 0;\ MECT^{-} = 0 \text{ if } MECT > 0$$

The asymmetric residuals specified above are then introduced as separate variables in the error-correction model to obtain the asymmetric short run dynamic model:

$$\Delta I_t = \alpha_i D_{it} + \sum_{j=1}^{J} \beta_j \Delta \Pr ate_{t-j} + \sum_{k=1}^{K} \delta_k \Delta TBrate_{t-k} + \phi_1 ECT^{+}_{t-1} + \phi_2 ECT^{-}_{t-1} + \varphi_1 MECT^{+}_{t-1} + \varphi_2 MECT^{-}_{t-1}$$
$$+ \varepsilon_t$$
$$\forall t = 1,2,\ldots\ldots\ldots\ldots, T$$

where Δ is the first difference operator; I_t represents the interest rate (i.e., the lending rate or the deposit rate) in the current year, t; $\Pr ate$ is the policy rate, $TBrate$ is the Treasury bill rate; D_{it} captures the two dummy variable effects with cut-offs at January 1990 and May 2012 respectively; while ε_t is the white noise error term.

Here ϕ_1 and ϕ_2 are respectively the coefficients of the positive and negative level one error-correction terms while ϕ_1 and ϕ_2 are respectively the coefficients of the positive and negative level two error correction terms. The positive error correction term implies that if the I_t is above its long-run equilibrium value following a decline in the policy rate, it will start falling in the next period; similarly, the negative error correction term suggests that if I_t is below its equilibrium level following an increase in the policy rate, it will start rising in the subsequent period. The coefficients of the error-correction terms provide the information on the speed of adjustment of the bank rates during expansionary and contractionary monetary policy.

The objective of the analysis is to empirically ascertain whether the speed of adjustment of lending rates is different following a positive or negative shock in the policy rates. To measure the potential asymmetric adjustment the null hypothesis has been tested using the Wald test statistic. The null hypothesis posits that the speed of adjustment is the same following a rise or a cut in the policy rate, implying that the coefficients of the error-correction terms in the short run equation are equal, that is $H_0 : \phi_1 = \phi_2$.The results of the error correction terms and Wald test statistics are reported in the table below;

Table 7: The error correction model (using fully modified residuals)

Variable Name	With Level Two Error Terms	Without Level Two Error Terms	Without Asymmetry
$\Delta Prate_{t-1}$	-0.014225***	-0.019912***	-0.020013
$\Delta TBrate_{t-1}$	0.101228***	0.109715***	0.109145***
D1	-0.141428***	-0.065635***	-0.060094
D2	-0.017588***	-0.021349***	-0.024366
$MECT^-_{t-1}$	-0.230666***	-	-
$MECT^+_{t-1}$	-0.178006***	-	-
ECT^-_{t-1}	-0.013838***	-0.222020***	-
ECT^+_{t-1}	-0.006153***	-0.201285***	-
$\phi_1 = \phi_2$	-0.052660***	-0.020735	-
ECT_{t-1}	-	-	-0.214740***
R-squared	0.152866	0.135176	0.135176
Adjusted R-squared	0.136691	0.122851	0.135084
S.E. of regression	1.808664	1.823105	0.124836
Sum squared resid	1370.661	1399.283	1.821041
Log likelihood	-856.3856	-860.8083	1399.432
F-statistic	9.451089	10.96737	-860.8312
Prob(F-statistic)	0.000000	0.000000	13.18168

$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 8: The error correction model (using canonical residuals)

Variable Name	With Level Two Error Terms	Without Level Two Error Terms	Without Asymmetry
$\Delta Prate_{t-1}$	0.015795***	-0.020199***	-0.020329
$\Delta TBrate_{t-1}$	0.097375***	0.108476***	

			0.107895^{***}
D1	-0.145438^{***}	-0.066576^{***}	-0.060749
D2	-0.020599^{***}	-0.019498^{***}	-0.022599
$MECT^{-}_{t-1}$	-0.230753^{***}	-	-
$MECT^{+}_{t-1}$	-0.178736^{***}	-	-
ECT^{-}_{t-1}	-0.015138^{***}	-0.223552^{***}	-
ECT^{+}_{t-1}	-0.005369^{***}	-0.202119^{***}	-
$\phi_1 = \phi_2$	-0.052016^{***}	-0.021433^{***}	-
ECT_{t-1}			-0.216011^{***}
R-squared	0.155798	0.136080	0.135981
Adjusted R-squared	0.139680	0.123768	0.125744
S.E. of regression	1.805531	1.822151	1.820096
Log likelihood	-855.6436	-860.5844	-860.6090
F-statistic	9.665833	11.05231	13.28306
Prob(F-statistic)	0.00000	0.000000	0.00000

$^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

The results of the empirical estimates support the evidence of regime switching adjustment of the lending rates and point to asymmetric adjustment of the lending rates to changes in the policy rates. In addition, the results indicate that the model that considers asymmetric adjustment in both level one and level two error terms outperforms the error correction model that considers asymmetry only in level one error terms which also outperforms a model without regard to asymmetric adjustment.

8. Conclusion

In this paper I extend the I(2) –multicointegration proposed by Berenguer-Rico and Carrion-i-Silvestre (2005), which is itself a generalization of multicointegration approach proposed by Granger and Lee (1989), by relaxing the assumption of symmetric adjustment. To do so I have adopted the Engsted et al. (1997) approach to the concept of multicointegration and have followed Enders and Siklos (2001) to model the multicointegration relation allowing for asymmetric adjustment. I use three theoretical developments: the multivariate invariance principle based on Herrndorf (1984), Phillips and Durlauf (1986), Haldrup (1994) and Gregory and Hansen (1996); the weak convergence to stochastic integrals for dependent heterogeneous processes studied in Hansen (1992) and applied in Gregory and Hansen (1996); and the continuous mapping theorem (CMT) from Billingsley (1968) in order to derive an augmented Dickey-Fuller type of multicointegration test for I(2) series. Our main theoretical result is that the limiting distributions of the estimators and test statistics associated to multicointegration depend on the break fraction parameter, i.e. the cut-off point of the asymmetric response and the break point. On the practical front, I applied the proposed methodology to understanding interest rate pass-through in Malawi. The multicointegration test confirms the presence of two-level cointegration relationship among lending rates, policy rate and Treasury bill rates. It has been established that there are asymmetric adjustments of lending rates, on the first level, to long run steady following a short run adjustment in policy rate.

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