Vector autoregressive-based Granger causality test in the presence of instabilities

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VAR-based Granger-causality Test in the Presence of Instabilities

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Abstract

In this article, we review Granger-causality tests robust to the presence of instabilities in a Vector Autoregressive framework. We also introduce the gcrobustvar command, which illustrates the procedure in Stata. In the presence of instabilities, the Granger-causality robust test is more powerful than the traditional Granger-causality test.

Keywords: gcrobustvar, Granger-causality, VAR, instability, structural breaks, local projections.

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1 Introduction

Vector Autoregressive (VAR) models have played an important role in macroeconomic analysis since Sims (1980). A VAR is a multi-equation, multi-variable linear model where each variable is in turn explained by its own lagged values, as well as current and past values of the remaining variables. Compared with a univariate autoregression, VARs provide both a systematic way to capture the rich dynamics in multiple time series as well as a coherent and credible approach to forecasting.

Granger (1969) causality is a useful tool for characterizing the dependence among time series in reduced-form VARs, and Granger-causality test statistics are widely used to examine whether lagged values of one variable help to predict another variable – see Stock and Watson (2001).

However, VAR analyses in macroeconomic data face important practical challenges: economic time-series data are prone to instabilities (see Stock and Watson (1996, 1999, 2003, 2006), Rossi (2013), Clark and McCracken (2006b)) and VARs estimates may be prone to instabilities as well (see Boivin and Giannoni (2006), Kozicki and Tinsley (2001), and Cogley and Sargent (2001, 2005)).

Thus, given the widespread use of VARs and the evidence of instabilities, it is potentially important to allow for changes over time where doing VAR-based statistical inference. As demonstrated in Rossi (2005), statistical tests that are based on stationarity assumptions are invalid in the presence of instabilities. Since the traditional Granger-causality test assumes stationarity, it is not reliable in the presence of instabilities and may lead to incorrect inference.

In this article, we present the **gcrobustvar** command, which illustrates how to test Granger-causality in a way that is robust to the presence of instabilities. The test is based on methodologies developed by Rossi (2005) and includes the robust versions of the mean and exponential Wald tests (Andrews and Ploberger (1994)), the Nyblom (1989) test, and the Quandt (1960) and Andrews (1993) quasi-likelihood-ratio test, jointly testing for both parameter instability and Granger-causality. In the presence of instabilities, the Granger-causality robust tests are more powerful than the traditional Granger-causality test. The tests can also be used to find the point in time in which Granger-causality either appears or breaks down in the data. Besides, the test is valid for reduced-form VAR models as well as VAR-based direct multistep (VAR-LP) forecasting models. The former assume homoskedastic idiosyncratic shocks, while the latter are estimated via Local Projections (see Jordà (2005)), and, hence, assume heteroskedastic and serially correlated idiosyncratic shocks.

We first introduce the tests, then present the Stata commands that implement them. Then, we illustrate the empirical implementation of the robust Granger-causality tests using a three-variable (inflation, unemployment and interest rate) VAR model with four lags as in Stock and Watson (2001), as well as a direct multistep VAR-LP forecasting model. Finally, we compare the results with those based on a traditional Granger-causality test.

The remainder of this paper is organized as follows. Section 2 describes the theoretical framework and the Granger-causality robust tests. Section 3 introduces the **gcrobustvar** command, which implements the Granger-causality robust tests in Stata. Section 4 applies the Granger-causality robust tests in the three-variable VAR and compares the results with the traditional Granger-causality test. Section 5 applies the Granger-causality robust tests in the direct multistep VAR-LP forecasting model.

2 VAR-based Granger-Causality Test in the Presence of Instabilities

2.1 Motivation

In the presence of instabilities, as is shown in Rossi (2005), traditional Granger-causality tests may have no power. Consider one of the equations in a two-variable VAR with one lag and fixed prediction horizon h for example: $y_{t+h} = \beta_t x_{t-1} + \rho y_{t-1} + \varepsilon_{t+h}$, $t = 2, 3, ..., T$. For simplicity, we assume that $x_{t-1}, \varepsilon_{t+h} \overset{i.i.d.}{\sim} N(0,1)$, and x_{t-1}, y_{t-1}, ε_{t+h} are independent of each other. Suppose the parameter β_t changes through time in the following way:

$$\beta_t = 2/3\,(t \leq T/3) - 1/3(t > T/3) \tag{1}$$

In this example, a traditional Granger-causality test would be a t-test applied on the full-sample Ordinary Least Squares (OLS) parameter estimator $\hat{\beta}^{OLS}$:

$$\begin{aligned}
\hat{\beta}^{OLS} &= \left(\sum_{t=2}^{T} x_{t-1}^2\right)^{-1} \sum_{t=2}^{T} x_{t-1} y_{t+h} \\
&= \left(T^{-1}\sum_{t=2}^{T} x_{t-1}^2\right)^{-1} T^{-1}\left[\sum_{t=2}^{T/3} x_{t-1}^2\,(2/3) + \sum_{t=T/3+1}^{T} x_{t-1}^2\,(-1/3)\right] \\
&\quad + \left(T^{-1}\sum_{t=2}^{T} x_{t-1}^2\right)^{-1} T^{-1}\sum_{t=2}^{T} x_{t-1} y_{t-1}\rho + \left(T^{-1}\sum_{t=2}^{T} x_{t-1}^2\right)^{-1} T^{-1}\sum_{t=2}^{T} x_{t-1}\varepsilon_{t+h} \quad \underset{p}{\rightarrow} 0
\end{aligned} \tag{2}$$

since $T^{-1}\sum_{t=2}^{T} x_{t-1}^2 \underset{p}{\rightarrow} E\left(x_t^2\right)=1$, $T^{-1}\sum_{t=2}^{T} x_{t-1}y_{t-1} \underset{p}{\rightarrow} 0$, and $T^{-1}\sum_{t=2}^{T} x_{t-1}\varepsilon_{t+h} \underset{p}{\rightarrow} 0$.

Eq. (2) implies that we don't reject the null hypothesis even if x_{t-1} does Granger-cause y_{t+h} in reality. This failure to reject results from the violation of the stationarity assumption underlying traditional Granger-causality tests, as the predictive ability is unstable across time. Thus, traditional Granger-causality tests can be inconsistent if there are instabilities in the parameters. Without losing generality, this conclusion can be generalized to instabilities other than eq. (1) by varying the time as well as the magnitude of the break. Note that this conclusion is empirically relevant as evidence shows that parameter estimates change substantially in sign and magnitude across time, see for example Goyal and Welch (2008) and Rossi (2005).

Considering the possibility of parameter instabilities, Rossi (2005) proposes tests to evaluate the predictive ability in the situation where the parameter might be time-varying by testing jointly the significance of the predictors and their stability over time. Consider a simple Granger-causality regression: $y_{t+h}=\beta_t x_t+\varepsilon_{t+h}$, $t=1,2,...,T$, where β_t changes at some unknown point in time, τ: $\beta_t=\beta_1\cdot 1\left(t\leq\tau\right)+\beta_2\cdot 1(t>\tau)$.[1] Let $\widehat{\beta}_{1\tau}$ and $\widehat{\beta}_{2\tau}$ denote the OLS estimators before and after the break.[2] In respect of the null hypothesis of no Granger-causality at any point in time, i.e., $H_0:\beta_t=\beta=0$, the robust test builds on two components: $\frac{\tau}{T}\widehat{\beta}_{1\tau}+\left(1-\frac{\tau}{T}\right)\widehat{\beta}_{2\tau}$ and $\widehat{\beta}_{1\tau}-\widehat{\beta}_{2\tau}$. A test on whether the first component (the full-sample estimate of the parameter)[3] is zero detects situations in which the parameter β_t is constant and different from zero. A test on whether the second component (the difference between the parameters estimated in the two sub-samples) is zero detects situations in which the parameter changes, which is a complement to detecting situations in which the regressor Granger-causes the dependent variable in such a way that the parameter changes but the average of the estimates equals zero as in eq. (1). Rossi (2005) proposes several test statistics, including QLR_T^*, $Mean-W_T^*$ and $Exp-W_T^*$[4]. The corresponding critical values of the asymptotic distributions under the null are tabulated in Rossi's (2005) Table B1.

Note that a test for structural breaks would not necessarily be the correct approach either. In fact,

[1] Rossi (2005) considered various forms of instabilities, more general case of testing possibly nonlinear restrictions in models estimated with Generalized Method of Moments (GMM), and tests on subsets of parameters.

[2] $\widehat{\beta}_{1\tau}=\left(\frac{1}{\tau}\sum_{t=1}^{\tau}x_t x_t'\right)^{-1}\left(\frac{1}{\tau}\sum_{t=1}^{\tau}x_t y_{t+h}\right)$, $\widehat{\beta}_{2\tau}=\left(\frac{1}{T-\tau}\sum_{t=\tau+1}^{T}x_t x_t'\right)^{-1}\left(\frac{1}{T-\tau}\sum_{t=\tau+1}^{T}x_t y_{t+h}\right)$

[3] The first component is the full-sample estimate of the parameter: $\frac{\tau}{T}\widehat{\beta}_{1\tau}+\left(1-\frac{\tau}{T}\right)\widehat{\beta}_{2\tau}=\left(\frac{1}{T}\sum_{t=1}^{T}x_t x_t'\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T}x_t y_{t+h}\right)$

[4] Please refer to Rossi (2005) for detailed expressions of these statistics

while in the previous example the researcher would identify a break, a structural break test is not sufficient nor necessary for the existence of Granger-causality. In fact, immagine that a variable has predictive content for another variable and the predictive ability is constant over time, that is $\beta_t = \beta$. A structural break test is not necessary nor sufficient to detect predictive ability. The approach taken in this paper is to jointly test $\beta_t = \beta = 0$, which also avoids issues of multiple testing that one would incur into in case he/she separately tests instability and Granger-causality.

Note also that the way the possible presence of instabilities is modeled here is via a one-time break; such an approach has been proved to be more powerful than CUSUM tests – see Andrews, Lee and Ploberger (1996). Andrews, Lee and Ploberger (1996) derived the optimal tests (the exponential averages of the Wald test statistics) for one or more changepoints at unknown times in a multiple linear regression model. They compare the power of the optimal exponential tests with that of other tests in the literature such as the likelihood ratio or supF test, the CUSUM test in Brown, Durbin and Evans (1975), and the midpoint F test considering a one-time break in parameter, and they find that the optimal tests perform quite well in finite samples compared to the other tests considered and that the CUSUM test performs very poorly.

2.2 Framework

We consider two types of VAR specifications. The first is a reduced-form VAR with time-varying parameters:

$$\begin{aligned} A_t(L)y_t &= u_t \\ A_t(L) &= I - A_{1,t}L - A_{2,t}L^2 - \cdots - A_{p,t}L^p \\ u_t &\overset{i.i.d}{\sim} (O, \Sigma) \end{aligned} \tag{3}$$

where $y_t = [y_{1,t}, y_{2,t}, \ldots, y_{n,t}]'$ is an $(n \times 1)$ vector, and $A_{j,t}, j = 1, \ldots, p$, are $(n \times n)$ time-varying coefficient matrices.

The second is a direct multistep VAR-LP forecasting model with time-varying parameters. By iterating eq (3), y_{t+h} can be projected onto the linear space generated by $(y_{t-1}, y_{t-2}, ..., y_{t-p})'$, specifically

$$y_{t+h} = \Phi_{1,t}y_{t-1} + \Phi_{2,t}y_{t-2} + \cdots + \Phi_{p,t}y_{t-p} + \epsilon_{t+h} \tag{4}$$

where $\Phi_{j,t}, j = 1, \ldots, p$ are functions of $A_{j,t}, j = 1, \ldots, p$ in eq (3), and ϵ_{t+h} is a moving average of the errors u from time t to $t+h$ in eq (3) and therefore uncorrelated with the regressors but serially correlated itself[5]. Note that $h=0$ is a special case where eq (4) degenerates to eq (3), thus we focus on eq (4) from now onwards.

Let θ_t be an appropriate subset of $vec(\Phi_{1,t}, \Phi_{2,t}, \ldots, \Phi_{p,t})$. The null hypothesis of the Granger-causality robust test is:

$$H_0: \quad \theta_t = 0 \qquad \forall t = 1, 2 \ldots T \tag{5}$$

The statistics to test H_0 in eq (5), following from Rossi (2005), are $ExpW^*$ (the exponential Wald tests), $MeanW^*$ (the mean Wald tests), $Nyblom^*$ (the Nyblom test), and QLR^* (the Quandt likelihood ratio tests).[6]

The optimal exponential Wald test statistic ($ExpW^*$) and the optimal mean Wald test statistic ($MeanW^*$) are based on the exponential test statistics proposed in Andrews and Ploberger (1994). The optimal mean Wald test statistic is designed for alternatives that are very close to the null hypothesis; while the optimal exponential Wald test statistic is designed for testing against more distant alternatives. The optimal Nyblom test statistic ($Nyblom^*$) is based on the Nyblom (1989) test, which is the locally most powerful invariant test for the constancy of the parameter process against the alternative that the parameters follow a random walk process. The optimal Quandt likelihood ratio test statistic (QLR^*) is based on Andrews (1993) Sup-LR test (or the Quandt likelihood ratio (QLR) test), which considers the supremum of the statistics over all possible break dates of the Chow statistic designed for the alternatives for a fixed break date.

2.3 A special case: Granger-causality test

The traditional Granger-causality test is a special case where the parameters in eq (4) are time invariant, i.e. for $j = 1, \ldots, p$, we replace $\Phi_{j,t}$ with $\Phi_j, t = 1, \ldots, T$. Thus, eq (4) becomes:

$$y_{t+h} = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \cdots + \Phi_p y_{t-p} + \epsilon_{t+h} \tag{6}$$

To consider a more concrete example, Stock and Watson (2001) study a three-variable VAR with four lags and $h=0$. The variables included are inflation (π_t), unemployment (u_t) and interest rate (R_t). Their

[5]See Jorda (2005) for more details of Local Projections.
[6]See Rossi (2005) for more details of constructing the statistics.

reduced-form VAR is:

$$\begin{bmatrix} \pi_t \\ u_t \\ R_t \end{bmatrix} = \Phi_1 \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \\ R_{t-1} \end{bmatrix} + \Phi_2 \begin{bmatrix} \pi_{t-2} \\ u_{t-2} \\ R_{t-2} \end{bmatrix} + \Phi_3 \begin{bmatrix} \pi_{t-3} \\ u_{t-3} \\ R_{t-3} \end{bmatrix} + \Phi_4 \begin{bmatrix} \pi_{t-4} \\ u_{t-4} \\ R_{t-4} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{\pi} \\ \epsilon_t^{u} \\ \epsilon_t^{R} \end{bmatrix}$$
$$\Phi_j = \begin{bmatrix} \phi_j^{\pi,\pi} & \phi_j^{\pi,u} & \phi_j^{\pi,R} \\ \phi_j^{u,\pi} & \phi_j^{u,u} & \phi_j^{u,R} \\ \phi_j^{R,\pi} & \phi_j^{R,u} & \phi_j^{R,R} \end{bmatrix}, \qquad j = 1, \ldots, 4 \tag{7}$$

Thus, in Stock and Watson (2001), the reduced-form VAR involves three equations: current unemployment as a function of past values of unemployment, inflation and the interest rate; current inflation as a function of past values of inflation, unemployment and the interest rate; and current interest rate as a function of past values of inflation, unemployment and the interest rate. Stock and Watson (2001) consider traditional Granger-causality tests in each equation where the null hypothesis is: $H_0^* : \theta = 0$, where θ is the appropriate subset of $vec(\Phi_1, \Phi_2, \ldots, \Phi_p)$. For example, unemployment doesn't Granger-cause inflation if:

$$\phi_1^{\pi,u} = \phi_2^{\pi,u} = \phi_3^{\pi,u} = \phi_4^{\pi,u} = 0 \tag{8}$$

If unemployment does not Granger-cause inflation, then lagged values of unemployment are not useful for predicting inflation.

3 The gcrobustvar command

3.1 The gcrobustvar command

Syntax

The **gcrobustvar** command is the Stata command that implements the VAR-based Granger-causality robust test. The general syntax of the **gcrobustvar** command is

gcrobustvar *depvarlist, pos(#,#) [nocons horizon(#) lags(numlist) trimming(level)]*

depvarlist is a list of dependent variables, that is, all the variables in y_t in the notation in eq (6).

pos(#,#) is a numeric list (i.e. "numlist" in Stata) including two integers indicating the positions of the targeted dependent variable and restricted regressor respectively. For example, if we are testing whether the second variable $y_{2,t}$ Granger-causes the first variable $y_{1,t}$ in the presence of instabilities, then we assign the numeric list as *pos(1,2)*, where the first integer 1 refers to the position of the targeted dependent variable in the VAR (i.e. $y_{1,t}$ in this example) and the second integer 2 refers to the position of the targeted restricted regressor in the VAR (i.e. $y_{2,t}$ in this example).

Options

nocons suppresses the constant term. The default regression includes the constant term.

horizon(#) specifies the targeted horizon, i.e. h in the notation in eq (3). The default, i.e. not specifying *horizon(#)*, refers to a reduced-form VAR assuming homoskedastic idiosyncratic shocks. When *horizon(h)* ($h \geq 0$) is specified, the command assumes heteroskedastic and serially correlated idiosyncratic shocks, and chooses the truncation lag used in the estimation of the long run variance. The truncation lag is automatically determined using Newey and West (1994) optimal lag-selection algorithm. Note that *horizon(0)* refers to a reduced-form VAR assuming heteroskedastic and serially correlated idiosyncratic shocks, and *horizon(h)* ($h > 0$) refers to the $(h+1)$-step-ahead forecasting model, see eq (3). For example, in a one-year-ahead VAR-LP forecasting model with quarterly data, *horizon(3)* should be specified.

lags(numlist) is a numeric list that specifies the lags included in the VAR. The default is *lags(1 2)*. This option takes a numlist and not simply an integer for the maximum lag. For example, *lags(2)* would include only the second lag in the model, whereas *lags(1 2)* would include both the first and second lags in the model. The shorthand to indicate the range follows ”numlist” in Stata.

trimming(level) is the trimming parameter. As is standard in the structural break literature, the possible break dates are usually trimmed to exclude the beginning and end of the sample period. If we specify *trimming(μ)*, the range where we search for instabilities is set to be $[\mu T, (1-\mu)T]$, where T is the number of total periods. The default is *trimming(0.15)*, which is recommended in the structural break literature and commonly used in practice.

Stored results

gcrobustvar stores the following macros and matrices in **r()**:

Macros

r(cmd) gcrobustvar

r(cmdline) command as typed

Matrices

r(result_stat) A 4-by-1 matrix containing four statistics: $ExpW^*$, $MeanW^*$, $Nyblom^*$, $SupLR^*$.

r(result_pv) A 4-by-1 matrix containing four p-values, corresponding respectively to $ExpW^*$, $MeanW^*$, $Nyblom^*$, $SupLR^*$.

r(result_wald) A column vector containing wald statistics across time, the supremum of which is the optimal Quandt likelihood ratio test statistic (QLR^*).

3.2 Empirical Example of Practical Implementation in Stata

In what follows, we illustrate how to use the **gcrobustvar** command to implement the Granger-causality robust test in Stata. The data (GCdata.xlsx, provided with the article files) include quarterly U.S. data on the rate of price inflation (π_t), the unemployment rate (u_t), the interest rate (R_t, specifically, the federal funds rate) from 1959:I - 2000:IV. These are the same variables used in Stock and Watson (2001). Inflation is computed as $\pi_t = 400 \times ln(P_t/P_{t-1})$, where P_t is the chain-weighted GDP price index. Quarterly data on u_t and R_t are quarterly averages of their monthly values.

Consider the inflation equation in (3):

$$\begin{aligned} &\pi_t = c_t^\pi + \Phi_t^{\pi,\pi}(L)\pi_t + \Phi_t^{\pi,u}(L)u_t + \Phi_t^{\pi,R}(L)R_t + \epsilon_t^\pi \\ \text{where} \quad &\Phi_t^{\cdot,\cdot}(L) = \phi_{1,t}^{\cdot,\cdot}L + \phi_{2,t}^{\cdot,\cdot}L^2 + \phi_{3,t}^{\cdot,\cdot}L^3 + \phi_{4,t}^{\cdot,\cdot}L^4 \end{aligned} \tag{9}$$

Suppose we are interested in testing whether unemployment (u) Granger-causes inflation (π) and we want the test to be robust to instabilities over time. That is, we want to test whether the coefficients of lagged values of unemployment (u) are zero across time:

$$H_0: \quad \phi_{j,t}^{\pi,u} = 0 \qquad \forall j = 1,2,3,4 \quad \forall t = 1,2\ldots T$$

Implementing the Granger-causality Tests in the Presence of Instabilities

The following scripts implement the Granger-causality robust test. We first import the data, claim the data to be time series, and import the pvalue tables needed for the tests:

```
.
. * import data
. import excel GCdata.xlsx, sheet(SW2001) firstrow clear

.
. * time-series settings
. generate year = int(pdate)

. generate quarter = (pdate - int(pdate))*4 + 1

. generate tq = yq(year, quarter)

. format tq %tq

. tsset tq
        time variable:  tq, 1959q1 to 2000q4
                delta:  1 quarter

.
. * import p-value table
. mata:
------------------------------------------------- mata (type end to exit) ------------------
: mata clear

:
: mata matuse pvtable,replace
(loading pvap0opt[34,21], pvapiopt[34,21], pvnybopt[34,21], pvqlropt[34,21])

: st_matrix("r(pvap0opt)",pvap0opt)

: st_matrix("r(pvapiopt)",pvapiopt)

: st_matrix("r(pvnybopt)",pvnybopt)

: st_matrix("r(pvqlropt)",pvqlropt)

:
: end
---------------------------------------------------------------------------------------------
.
. mat pvap0opt = r(pvap0opt)

. mat pvapiopt = r(pvapiopt)

. mat pvnybopt = r(pvnybopt)

. mat pvqlropt = r(pvqlropt)
```

Then we run the Granger-causality robust test using the **gcrobustvar** command. When we run the **gcrobustvar** command, important information (variables, lags, etc) will be displayed:

```
.
. * run gcrobust test for a VAR
. gcrobustvar pi u R, pos(1,2) lags(1/4)
Running the Granger Causality Robust Test...
Setting:
 Variables in VAR: pi u R
 Lags in VAR:1 2 3 4
 h is 0 (reduced-form VAR).
 Trimming parameter is .15
 Constant is included.
 Assuming homoskedasticity in idiosyncratic shocks.
```

The results are dispayed in the following script. The **gcrobustvar** command provides the four optimal test statistics ($ExpW^*$, $MeanW^*$, $Nyblom^*$, QLR^*) and their corresponding p-values.

```
Results of Granger Causality Robust Test: Lags of u Granger cause pi
ExpW*,MeanW*,Nyblom*,QLR* -- and their p-values below

                        ExpW      MeanW     Nyblom      SupLR
statistics(pi:u)   9.1974592  17.047538   4.689049  23.187923
   p-value(pi:u)   .07383773  .05866136  .07939046  .07069996
```

Here is how we get all the inputs of the **gcrobustvar** command. $depvarlist$ lists the variables included in the VAR, i.e. π, u, R in this order. Since we are testing whether lags of the second variable u_t Granger-cause the first variable π_t in the presence of instabilities, we assign the following positions $pos(1,2)$. As for the options, we include the constant term and include four lags, i.e.$lags(1/4)$, as Stock and Watson (2001). Besides, we assume homoskedasticity and choose the standard trimming parameter 0.15.

Here is how to interpret the results. Let's take the exponential Wald tests statistics, denoted as $ExpW^*$, as an example. The value of the test statistic $ExpW^*$ is 9.20, and the p-value is 0.07. Thus, the test rejects the null hypothesis that unemployment (u) doesn't Granger-cause inflation (π) for all t at the 10% significance level.

4 Comparison with the Traditional Granger-Causality Test

In this section, we compare the robust Granger-causality tests with the traditional Granger-causality test in the three-variable VAR model in Stock and Watson (2001). The VAR includes a constant term, four lags and assumes homoskedastic idiosyncratic shocks.

Table 1 reports the p-values of the traditional Granger-causality Wald statistics. The results show that π Granger-causes R, u Granger-causes both π and R, R Granger-causes u at the 5% significance level.

Table 1: Traditional Reduced-form VAR-based Granger-Causality Tests

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.25	0.00
u	0.01	0.00	0.00
R	0.22	0.00	0.00

Note: This table reports p-values of the Wald statistics of the traditional Granger-causality test. $h = 0$ (i.e. the reduced-form VAR model), $lags = (1, 2, 3, 4)$, assuming homoskedastic idiosyncratic shocks.

Table 2 reports the p-values of the robust Granger-causality test statistics (for $ExpW^*$, $MeanW^*$, $Nyblom^*$ and QLR^*, respectively). We are testing whether the restricted regressor Granger-causes the dependent variable in the presence of instabilities. For example, if we consider the dependent variable π and the restricted regressor R, we are testing whether R Granger-causes π in a way robust to instabilities across time, i.e. whether the coefficients of lags of R are constant and equal to zero over time. The p-value of the $ExpW^*$ statistics in Panel A in Table 2 is 0.01, so the test does reject the null at the 5% significance level. Hence, R does Granger-cause π.

Table 2: Robust Granger-Causality Tests in the Reduced-form VAR

Panel A ExpW*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.20	0.00
u	0.07	0.00	0.00
R	0.01	0.00	0.00

Panel B MeanW*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.44	0.00
u	0.06	0.00	0.00
R	0.20	0.01	0.00

Panel C Nyblom*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.22	0.00
u	0.08	0.00	0.00
R	0.03	0.02	0.00

Panel D QLR*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.08	0.00
u	0.07	0.00	0.00
R	0.00	0.00	0.00

Note: This table reports p-values of the statistics of the Granger-causality robust test. $h = 0$ (i.e. the reduced-form VAR model), $lags = (1, 2, 3, 4)$, $pistart = 0.15$, assuming homoskedastic idiosyncratic shocks.

Comparing Table 1 and Table 2, the empirical conclusions differ if a researcher uses the Granger-causality robust test instead of the traditional Granger-causality test. In fact, R doesn't Granger-cause π at the 5% significance level in the traditional Granger-causality test, but R does Granger-cause π at the 5% significance level in the Granger-causality robust test according to the $ExpW^*$, $Nyblom^*$, $SupLR^*$ test statistics. Hence, there is empirical evidence that lagged values of R can predict π but the predictive ability only shows up sporadically over time, which is the reason why the traditional Granger-causality test doesn't detect it.

This command also returns a graph showing the whole sequence of the Wald statistics across time[7], which gives more information on when the Granger-causality occurs. Take the test of whether unemployment (u) Granger-causes inflation (π) as an example, figure 1 documents the whole sequence of the Wald statistics testing whether unemployment (u) Granger-causes inflation (π). The sequence of the Wald statistic over time t (depicted by a continuous line in Figure 1) is above the 10% critical value line (depicted by the orange dashed lines) around 1970q1 and 1980q1. The figure is saved as gcrobustvar_pi_u.

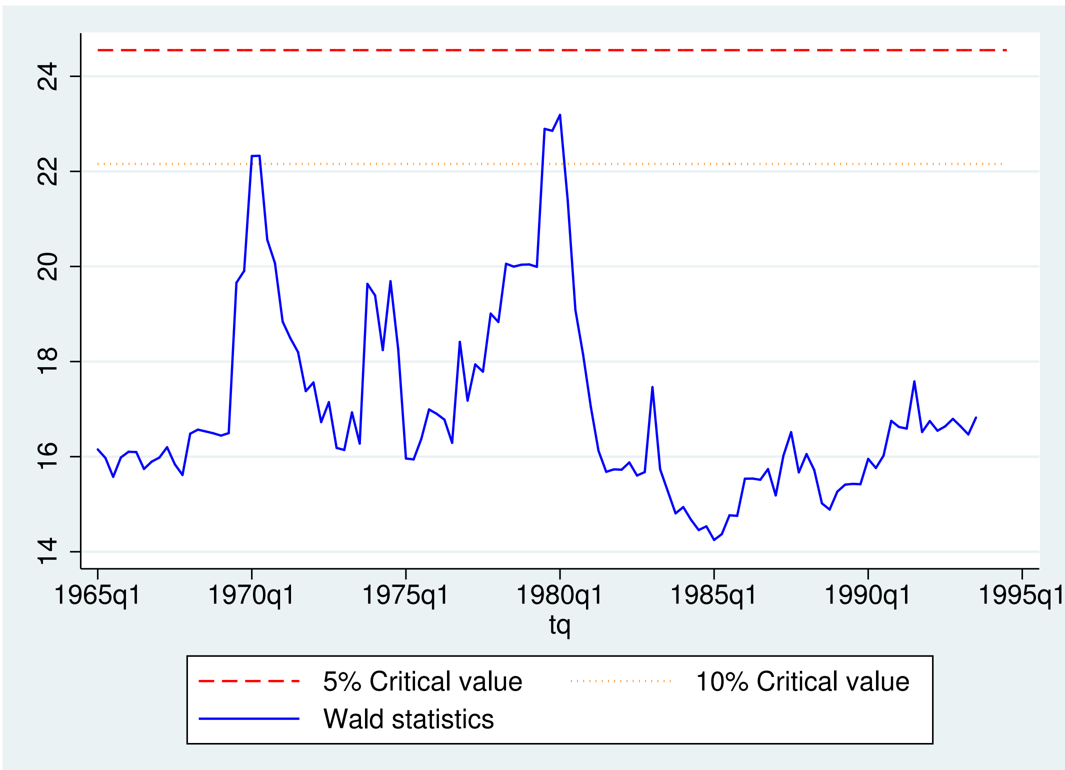

Figure 1: Wald statistics, testing whether unemployment (u) Granger-causes inflation (π)

[7]The optimal Quandt likelihood ratio test statistic (QLR^*) is the supremum of the statistics over all possible break dates of the Chow statistic designed for the alternatives for a fixed break date.

5 Robust Granger-Causality Tests in Local Projections

Section 4 considers the reduced-form VAR assuming homoskedastic idiosyncratic shocks. In this section, we extend the VAR analysis to Jorda's (2005) Local Projections by implementing the direct multistep VAR-LP forecasting model in eq (6) and assuming heteroskedastic and serially correlated idiosyncratic shocks. Allowing for heteroskedasticity and serial correlation in idiosyncratic shocks is important when the researcher extends the VAR analysis to Local Projections, where the error terms in eq (6) can be both heteroskedastic and serially correlated.

We consider the one-year-ahead VAR-LP forecasting model with a constant term, four lags and assuming heteroskedastic and serially correlated idiosyncratic shocks. The setting is similar to Section 4 except that we specify $h = 3$ and relax the homoskedasticity assumption.

The following is the command to implement the Granger-causality robust test to investigate whether the coefficients on $R_{t-1}, R_{t-2}, R_{t-3}, R_{t-4}$ in the equation where the dependent variable is π_{t+3} are zero across time in the one-year-ahead VAR-LP forecasting model. To test other coefficients, the command is similarly implemented, except for adjusting the input of $pos(\#,\#)$.

```
. gcrobustvar pi u R, pos(1,3) lags(1/4) horizon(3)
Running the Granger Causality Robust Test...
Setting:
 Variables in VAR: pi u R
 Lags in VAR:1 2 3 4
 h is 3 (4-step-ahead VAR-LP forecasting model).
 Trimming parameter is .15
 Constant is included.
 Assuming heteroskedasticity and serial correlation in idiosyncratic shocks.
```

Table 3 reports the p-values of the robust Granger-causality test statistics (the $ExpW^*$, $MeanW^*$, $Nyblom^*$ and QLR^* statistics respectively). The results show that lags of inflation (π) can significantly forecast the one-year-ahead unemployment (u) and interest rate (R), lags of unemployment can significantly forecast the one-year-ahead inflation and interest rate, and lags of interest rate can significantly forecast the one-year-ahead inflation and unemployment.

Table 3: Robust Granger-causality Tests in the Direct Multistep VAR-LP Forecasting Model

Panel A ExpW*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.00	0.00
u	0.00	0.00	0.00
R	0.00	0.00	0.00

Panel B MeanW*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.00	0.00
u	0.00	0.00	0.00
R	0.00	0.00	0.00

Panel C Nyblom*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.00	0.00
u	0.00	0.00	0.00
R	0.00	0.00	0.00

Panel D QLR*

	Dependent Variable		
Restricted Regressors	π	u	R
π	0.00	0.00	0.00
u	0.00	0.00	0.00
R	0.00	0.00	0.00

Note: This table reports p-values of the statistics of the Granger-causality robust test. $h = 3$ (i.e. the one-year-ahead VAR-LP forecasting model), $lags = (1, 2, 3, 4)$, $pistart = 0.15$, assuming heteroskedastic and serially correlated idiosyncratic shocks.

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