Choice of Trade Policy with Incomplete Information

Ning, Haokai

Department of Economics, York University

1 February 2020

Choice of Trade Policy with Incomplete Information*

Haokai Ning†

Abstract

This paper studies the design of trade policies in an uncertain third market with incomplete information. Governments in each of the two countries select either direct quantity controls or subsidies in an attempt to shift profits in favour of their own firms in an oligopolistic setting. It is shown that the country with firms having information disadvantage tends to choose the direct quantity control, while the country with well-informed firms would use export subsidy (export quota) when the degree of uncertainty is sufficiently high (low).

Journal of Economic Literature classification number: C72, D82, F13, L13.

Key words: Uncertainty, incomplete information, Bayesian Nash equilibrium, strategic trade policy, Cournot competition.

*I thank professors Mahmudul Anam, Shin-Hwan Chiang and Kin Chung Lo for their encouragement and valuable comments. All remaining errors are my own responsibility.

†Department of Economics, York University. E-mail: neoning@yorku.ca

1 Introduction

Strategic trade policy, which combines international trade theory and industrial organization, has been brought to our attention due the seminal work by Brander and Spencer (1985). Through these policies, governments influence the behavior of their domestic firms in their subsequent strategic interaction with foreign firms. Strategic trade policy is designed to shift profits towards domestic firms when market in the destination country is imperfectly competitive. Brander and Spencer (1985) show that an export subsidy can shift rents from the foreign to the domestic firm, hence provides a new explanation for the use of export subsidies. Moreover, Krishna (1989) shows that direct quantity constraints on exports can perform a similar role in oligopolistic markets under certain conditions.

A weakness of these results is that these theoretical models constrain the policy instrument selected by governments. For instance, Brander and Spencer (1985) restrict their attention to export subsidy or tax. Another shortcoming of conventional strategic trade policy models is that they assume that governments have complete information about export markets. However, real policymakers are unlikely to meet the information requirements assumed by theorists. Cooper and Riezman (1989) is the first paper to introduce uncertainty and incomplete information in the context of strategic trade policy.[1] In Cooper-Riezman three-country trade model à la Brander and Spencer (1985), policymakers can use either price incentives (export subsidies) or direct quantity controls (export quotas), but they are not fully informed about demand in the export market. Cooper and Riezman (1989) argue that governments will subsidize domestic firms when the degree of export market uncertainty is high and export quotas when uncertainty is low. In addition, they show that the number of firms plays a critical role in equilibrium. Countries with a large number of firms will tax, while countries with fewer firms will subsidize exports in a bilateral subsidy game.

The role of uncertainty in strategic trade policy models have been examined in other contexts as well. Shivakumar (1993) analyzes the importance of demand uncertainty for the optimal choice of trade policy instrument as inCooper and Riezman (1989). However, Shivakumar (1993)'s focus is the timing of policy implementation, the choice being whether to do it before or after the resolution of uncertainty. Grant and Quiggin (1997) show that whether the equilibrium modes of trade intervention in the presence of uncertainty is a specific, ad valorem or quadratic trade tax, emerges endogenously

[1] Ning (2020a) introduces incomplete information at industrial level in examining social welfare equivalence issue of tariff and quota from the importing country's perspective. He shows that a tariff is always superior to a quota as long as incomplete information persists at industrial level. Moreover, Ning (2020b) also introduces incomplete information into reciprocal dumping model, and he shows that the equilibrium outcome from choosing between tariff and subsidy results a prisoner dilemma outcome.

from the parameters of the model. He uses an equilibrium concept using supply functions proposed by Klemperer and Meyer (1989). Caglayan (2000) extends Cooper and Riezman (1989) framework to the to the context where firms from both countries have imperfect information about uncertain demand in the third country but receive a signal about the stochastic term. Although, in this case, firms have imperfect information about demand in the destination country, the signal firms received equips them with better information than policymakers.

In addition to the choice of trade policy instruments under uncertainty, which is the main focus of the previous papers, specific policy instrument and information asymmetry have also been considered in the context of strategic trade policy. Qui (1994) compares the impact of complete information and incomplete information on export subsidies in a model where only one government is active, and the cost of a firm is unknown to both government and the rival firm. For Cournot competition, he finds the government prefers to be informed and thus offers complete-information subsidies. Maggi (1999) examines equilibrium trade policies when firms have better information than governments about the profitability of the industry. The main result is that, governments offer non-linear export subsidies inducing firms to reveal information. The firms respond by expanding outputs, thereby adding to the Brander–Spencer prisoner's dilemma problem. Similarly, Creane and Miyagiwa (2008) study an information acquisition model in which firms have incentive to disclose information to the governments in a three-country trade model. They prove that firms disclose demand and cost information to governments in a Cournot competition, and governments are caught in an informational prisoner's dilemma. Anam and Chiang (2000) extend Brander and Spencer (1985) single export market framework with demand uncertainty to the context of multiple correlated export markets. They demonstrate that when firms engage in quantity competition in two stochastic and positively correlated markets, it may be optimal to tax exports to the more volatile market while subsidizing it in the relatively stable market when firms are risk-averse.

The current paper investigates the effect of incomplete information at industrial level on choice of trade policies. We model a three-country international trade model à la Brander and Spencer (1985) with demand uncertainty in the third country, which is the destination of the other two country's exports. Similar to Cooper and Riezman (1989), we focus on the choice of trade policy, assuming governments from exporting countries have no information about demand condition in the destination country. Unlike Cooper and Riezman (1989), we assume that one of the exporting firms has an information advantage over the other. Specifically, we let firm from one of the exporting country have complete information about demand in the third country, while firm from the other country is assumed to be incompletely informed. Our equilibrium results are significantly different from Cooper and Riezman (1989) and Caglayan (2000). We show that direct quantity control becomes the dominant strategy for country with the incompletely-informed firm. By contrast, the equilibrium choice between export subsidy and export quota for the country with the fully informed firm depends on the degree of uncertainty in the third market. We demonstrate that what drives our results is the option value associated with the ability to make decisions armed with more information about the uncertainty parameters. This paper is organized as follows: section 2 outlines the basic third-market model and information partition. Section 3 derives sub game equilibrium for various pairs of strategies. Section 4 characterizes and analyzes the optimal choice of policy regimes. Finally, section 5 concludes the paper.

2 The Model

Following Brander and Spencer (1985), we assume that there are two firms, one domestic (denote as firm 1) and one foreign (denote as firm 2), producing a homogeneous product exclusively for exports.[2] The export competition takes place in a neutral third country where the demand is subject to some random disturbances. For simplicity, assume that the inverse demand function in this export market is

$$p = a - b\left(q_1 + q_2\right) + \varepsilon,$$

where q_1 and q_2 represent the export of the domestic and foreign firms respectively. The parameters a and b are both positive and ε is a random variable, defined over a finite set Ω, which reflects stochastic demand conditions. For tractability, it is assumed that the disturbance term ε is binary, which takes only two possible values: $\varepsilon \in \Omega = \{\varepsilon_l, \varepsilon_h\}$, where $\varepsilon_h > \varepsilon_l$. The subjective common prior for ε_l and ε_h to occur are θ and $1-\theta$, respectively. Hence the expected value of random variable is $E(\varepsilon) = \theta\varepsilon_l + (1-\theta)\varepsilon_h$, and the variance of random variable is $var\left(\varepsilon\right) = E\left(\varepsilon^2\right) - E\left(\varepsilon\right)^2 = \sigma^2$.

The cost of production for firm i $(i = 1, 2)$ is assumed to be linear in output, i.e.,

$$C_i = cq_i.$$

Following Cooper and Riezman (1989), our model consists of three stages. In stage one, each

[2] Our model is readily to be extended to have N_1 number of firms in the domestic country and N_2 number of firms in the foreign country as in Cooper and Riezman (1989).

government commits to a policy instrument before the realization of random variable ε. The levels of the policy instruments are then set in the second stage, again before the realization of ε. The values of ε becomes known at the beginning of stage three. Unlike the Cooper-Riezman's model, we assume that ε is fully observable to the foreign firm, but such information is not available to the home firm. However, the home firm does know the distribution of ε.[3] That is, the home firm faces information disadvantage in competing with the foreign firm in export competition. This permits us to gain some insights into the effect of incomplete information on policy choices by each country. Both firms play a Cournot-Nash game and set outputs to maximize profits under incomplete information in stage three, given optimal policies chosen by the governments in previous stages. Since the foreign firm sets output after observing realized market demand, it stands to capture the option value associated with being able to wait for the resolution of uncertainty. Figure 1 shows the time of moves and the (partial) resolution of uncertainty.

Figure 1: Three-stage trade game with incomplete information

Stage 1		Stage 2		Beginning of Stage 3		Stage 3
Governments select policy forms	$\rightarrow$	Governments select policy levels	$\rightarrow$	Trade policies and levels are revealed to firms, and firm 2 realizes ε.	$\rightarrow$	Firms select quantities

Before getting into the heart of present model, it may be useful to consider the case where there is no government interventions and firms can fully observe the market condition when ε is realized. Under this limiting case, firm i's profit is therefore

$$\pi_i = pq_i - cq_i,$$

for $i = 1, 2$. One can easily verify that the Cournot-Nash equilibrium of output is

$$q_i^* = \frac{a - c + \varepsilon}{3b},$$

for $i = 1, 2$. For the output to be non-negative, it requires $a - c + \varepsilon \geq 0$. This benchmark case serves as a basis for comparison when the governments seek to shift profits in favor of their native firms by providing an export subsidy (Brander and Spencer (1985)).

[3]This is reflected in our common prior assumption.

In what follows, we construct a three-stage perfect Bayesian equilibria for various policy regimes using backward induction. Subsidy and quota games are examined in sequence and their welfare implications are compared. By using this approach, we can guarantee that the equilibrium is a perfect Bayesian equilibrium.

3 Equilibrium of Subgames

In this section, we derive equilibrium levels of output produced by firms and levels of government intervention using Bayesian Nash solution concept in each subgame (stage two and stage three). In other words, we derived the equilibrium points when two governments choose the following four combinations of forms of intervention:

i. (S, S): both governments grant export subsidy;

ii. (Q, Q): both governments impose export quota;

iii. (Q, S): government 1 imposes export quota while government 2 provides export subsidies;

iv. (S, Q): government 1 grants export subsidy while government 2 imposes export quotas.

Denote $q_i(a_1, a_2)$ as firm i's decision on output level as a reaction to a pair of strategy in forms of intervention chosen by both governments, where $a_1 \in A_1 = \{S, Q\}$ and $a_2 \in A_2 = \{S, Q\}$.[4] For each of the following subsections, expected social welfare for both countries are also derived.

3.1 Bilateral Subsidy Game: (S,S)

We first consider the game, where subsidy is committed to by governments as the instrument of protection, in stage one. The solution through backward induction starts in stage three when ε becomes known to firm 2.

The game involves two firms and has two states of nature (ε_l and ε_h) that are asymmetrically revealed to the firms involved. The possible actions of each player are the amount of outputs (or exports) which is defined over $[0, \infty)$. Technically, it is an incomplete information (Bayesian) game. In what follows, we characterize the Bayesian Nash equilibrium. Specifically, each firm maximize its (expected) profit by setting quantities given conjectures on the quantities chosen by its rival. In a Bayesian Nash equilibrium, these conjectures will be confirmed.

[4] A_i is the set of actions for government i in the stage one game. We use S stands for export subsidy and Q stands for export quota.

Let s_1 and s_2 be the amount of subsidy given by governments 1 and 2, respectively, to their own firm. Being unable to observe the true state of nature, firm 1 thus chooses $q_1(S, S)$ to maximize the expected profits, given by

$$\begin{aligned} E(\pi_1(S,S)) &= \theta\left(\left(a - b\left(q_1(S,S) + q_2^l(S,S)\right) + \varepsilon_l\right) q_1(S,S) - cq_1(S,S) + s_1 q_1(S,S)\right) \\ &+ (1-\theta)\left(\left(a - b\left(q_1(S,S) + q_2^h(S,S)\right) + \varepsilon_h\right) q_1(S,S) - cq_1(S,S) + s_1 q_1(S,S)\right), \end{aligned} \tag{1}$$

where θ and $1 - \theta$ are the subjective probabilities associated with ε_l and ε_h.

On the other hand, firm 2 is able to observe the true state of nature once it is realized. Hence, it can make its production decision according to the demand condition. If $\varepsilon = \varepsilon_l$, firm 2 chooses $q_2^l(S, S)$ to maximize its profit:

$$\pi_2^l(S,S) = \left(a - b\left(q_1(S,S) + q_2^l(S,S)\right) + \varepsilon_l\right) q_2^l(S,S) - cq_2^l(S,S) + s_2 q_2^l(S,S). \tag{2}$$

Conversely, if $\varepsilon = \varepsilon_h$, firm 2 sets $q_2^h(S, S)$ so as to maximize

$$\pi_2^h(S,S) = \left(a - b\left(q_1(S,S) + q_2^h(S,S)\right) + \varepsilon_l\right) q_2^h(S,S) - cq_2^h(S,S) + s_2 q_2^h(S,S). \tag{3}$$

The best response functions for the above maximization problems are

$$\begin{aligned} BR_1(q_2^l, q_2^h) &= q_1 \in \arg\max E(\pi_1(S,S)), \\ BR_{2l}(q_1) &= q_2^l \in \arg\max \pi_2^l(S,S), \\ BR_{2h}(q_1) &= q_2^h \in \arg\max \pi_2^h(S,S). \end{aligned}$$

Given these, we obtain the Bayesian Nash equilibrium points as follows:

$$\begin{aligned} q_1(S,S) &= \frac{a - c + 2s_1 - s_2}{3b} + \frac{E(\varepsilon)}{3b}, \\ q_2^l(S,S) &= \frac{a - c - s_1 + 2s_2}{3b} + \frac{\varepsilon_l}{2b} - \frac{E(\varepsilon)}{6b}, \\ q_2^h(S,S) &= \frac{a - c - s_1 + 2s_2}{3b} + \frac{\varepsilon_h}{2b} - \frac{E(\varepsilon)}{6b}. \end{aligned} \tag{4}$$

These equations characterize the Bayesian Nash equilibrium points in market 3 for given values of $(s_1, s_2, \varepsilon_l, \varepsilon_h, \theta)$. Notice that higher values of s_1 leads increasing output by country 1's firm, while higher values of s_2 from its rival country causes firm 1's output to fall. This is due to the fact that governments recognize increasing export subsidy levels lead to output expansions by their firms and output reductions by rival firms[5].

In stage two, both governments maximize their expected social welfare by choosing export subsidy levels, and they take into account the responses from firms (incompletely informed) in stage three. That is, a government's objective is to choose a value for the ex post subsidy that maximizes the expected value of its firm's profit net of subsidies since we can ignore consumer's surplus due to absence of domestic consumers. This implies that income distribution is not an important determinant of social welfare for each country.

Information partition for both countries indicates both countries have no information about true demand in country 3, and country 1 only anticipates its firm has one reaction function of output level in the following stage (due to lack of information). We can write expected social welfare for country 1 as

$$E(SW_1(S,S)) = E(\pi_1(S,S)) - s_1 q_1(S,S), \tag{5}$$

where

$$E(\pi_1(S,S)) = \frac{(a - c + 2s_1 - s_2 + E(\varepsilon))^2}{9b}.$$

On the other hand, country 2 knows its firm will have complete information about true demand in market 3, then country 2's expected social welfare given the common prior $(\theta, 1-\theta)$ on Ω can be written as

$$E(SW_2(S,S)) = \theta\left(\pi_2^l(S,S) - s_2 q_2^l(S,S)\right) + (1-\theta)\left(\pi_2^h(S,S) - s_2 q_2^h(S,S)\right), \tag{6}$$

where

$$\begin{aligned}
\pi_2^l(S,S) &= \frac{(2a - 2c - 2s_1 + 4s_2 + 3\varepsilon_l - E(\varepsilon))^2}{36b}, \\
\pi_2^h(S,S) &= \frac{(2a - 2c - 2s_1 + 4s_2 + 3\varepsilon_h - E(\varepsilon))^2}{36b}.
\end{aligned}$$

Similar to solving Bayesian Nash equilibrium points in the third stage, each country maximize its expected social welfare by setting export subsidy levels given conjectures on the levels chosen by the other government in the second stage.

[5] E.g., traditional view on profit shifting motivation as in Brander and Spencer (1985).

The best response functions for stage two game are

$$\begin{aligned}
BR_1(s_2) &= s_1 \in \arg\max E(SW_1(S,S)), \\
BR_2(s_1) &= s_2 \in \arg\max E(SW_2(S,S)).
\end{aligned}$$

Solving yields the Bayesian Nash equilibrium level of subsidy rate as

$$s_1^* = s_2^* = \frac{a-c}{5} + \frac{E(\varepsilon)}{5}. \tag{7}$$

Note that $s_i > 0$ for $i = 1, 2$. That is, both governments choose to subsidize their own firms. It is worth noting that the equilibrium export subsidy levels are symmetric for both governments even with incomplete information at industrial level. This is because governments 1 and 2 hold the same prior beliefs about random variable ε and $q_1(S,S) = \theta q_2^l(S,S) + (1-\theta)(q_2^h(S,S))$. Also note that when $E(\varepsilon) = 0$, the equilibrium export subsidy is reduced the export subsidy obtained by Cooper and Riezman (1989) even though information symmetry is assumed in their paper.

Substituting equation (7) into the expected social welfare functions for both governments, we get:

$$E(SW_1(S,S)) = \frac{2((a-c)+E(\varepsilon))^2}{25b}, \tag{8}$$

$$E(SW_2(S,S)) = \frac{2((a-c)+E(\varepsilon))^2}{25b} + \frac{\sigma^2}{4b}. \tag{9}$$

Notice that $E(SW_2(S,S))$ is the sum of firm's profits net of export subsidy and the variance term (which is absent in $E(SW_1(S,S))$). This indicates that higher variance of ε benefits the country with better information (i.e., country 2 in our model). This can be seen by calculating $\frac{\partial E(SW_2(S,S))}{\partial \sigma^2} = \frac{1}{4b} > 0$ and $E(SW_2(S,S)) - E(SW_1(S,S)) = \frac{1}{4b}(\sigma^2) > 0$. Obviously, if there is no uncertainty in demand (hence no information problem), we obtain the classical result of Brander and Spencer (1985). This gives us

Proposition 1. *Under a bilateral subsidy game, the difference in expected social welfare between two countries is the option value effect enjoyed by the country with well-informed firm. Moreover, the expected social welfare for country with better informed firm increases with market volatility.*

Having more information at industrial level does not always benefit the foreign firm. This occurs especially when $\varepsilon = \varepsilon_l$. Specifically, firm 2's expected profit is lower (higher) than firm 1's expected profit if the true market demand in country 3 is low (high). To see this, calculate

$$
\begin{aligned}
E(\pi_1(S,S)) &= \frac{4(a-c+E(\varepsilon))^2}{25b}, \\
E(\pi_2^l(S,S)) &= \frac{(4a-4c+5\varepsilon_l-E(\varepsilon))^2}{100b}, \\
E(\pi_2^h(S,S)) &= \frac{(4a-4c+5\varepsilon_h-E(\varepsilon))^2}{100b}.
\end{aligned} \tag{10}
$$

It can be easily verified that $E\left(\pi_2^l\left(S,S\right)\right) < E\left(\pi_1\left(S,S\right)\right) < E\left(\pi_2^h\left(S,S\right)\right)$. This is because firm 1 with incomplete information always produces the moderate level of output (i.e., a weighted average output level given $(\theta, 1-\theta)$ over Ω). Nevertheless, when the true demand is low, firm 2 will be forced to produce less than expected, given that firm 1 is incapable of reacting to it for lack of information. Conversely, when the true demand is high, firm 2 will respond by producing much more to take advantage of good market conditions, given that it anticipates no action from firm 1.

It is also worth noting the differences among $E(\pi_2^l(S,S))$, $E(\pi_1(S,S))$ and $E(\pi_2^h(S,S))$ depend on the common prior $(\theta, 1-\theta)$ over Ω. For any fixed parameters, if firm 1 is more pessimistic (i.e., higher θ), it will lose more profit relative to firm 2 in high demand. But if firm 1 is more optimistic (i.e., lower θ), the opposite is true.

3.2 Bilateral Quota Game: (Q,Q)

The equilibrium analysis under bilateral export quota subgame is easy to analyze. In this case, governments impose export level restrictions for their own firms in the second stage. For simplicity, assume that export quota levels are binding for both firms in both countries. That is, both firms will produce positive outputs in stage three, given export restrictions set by their home governments in stage two.

Each government sets an export ceiling to the third market for their respective firms in stage two. Since the actions taken by both governments occur before the resolution of random variable, each government chooses $\overline{q}_i\left(Q,Q\right)$ for $i=1,2$ so as to maximize their expected social welfare given by

$$
\begin{aligned}
E(\pi_1(Q,Q)) &= \theta\left(\left(a-b\left(\overline{q}_1(Q,Q)+\overline{q}_2(Q,Q)\right)+\varepsilon_l\right)\overline{q}_1(Q,Q)-c\overline{q}_1(Q,Q)\right) \\
&+ (1-\theta)\left(\left(a-b\left(\overline{q}_1(Q,Q)+\overline{q}_2(Q,Q)\right)+\varepsilon_h\right)\overline{q}_1(Q,Q)-c\overline{q}_1(Q,Q)\right),
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
E\left(\pi_2\left(Q,Q\right)\right) &= \theta\left(\left(a-b\left(\overline{q}_2\left(Q,Q\right)+\overline{q}_1\left(Q,Q\right)\right)+\varepsilon_l\right)\overline{q}_2\left(Q,Q\right)-c\overline{q}_2\left(Q,Q\right)\right) \\
&+ \left(1-\theta\right)\left(\left(a-b\left(\overline{q}_2\left(Q,Q\right)+\overline{q}_1\left(Q,Q\right)\right)+\varepsilon_h\right)\overline{q}_2\left(Q,Q\right)-c\overline{q}_2\left(Q,Q\right)\right), \qquad (12)
\end{aligned}
$$

respectively. The best response functions for above optimization problems are

$$
\begin{aligned}
BR_1(\overline{q}_2\left(Q,Q\right)) &= \overline{q}_1\left(Q,Q\right) \in \arg\max E(\pi_1(Q,Q)), \\
BR_2(\overline{q}_1\left(Q,Q\right)) &= \overline{q}_2\left(Q,Q\right) \in \arg\max E(\pi_2(Q,Q)).
\end{aligned}
$$

Solving yields the Bayesian Nash equilibrium level of quota:

$$
\overline{q}_1^*\left(Q,Q\right)=\overline{q}_2^*\left(Q,Q\right)=\frac{a-c}{3b}+\frac{E\left(\varepsilon\right)}{3b}. \qquad (13)
$$

Substituting equation (13) into each firm's expected profit functions, we obtain the expected social welfare for both countries as follows:

$$
E\left(SW_1\left(Q,Q\right)\right)=E\left(SW_2\left(Q,Q\right)\right)=\frac{\left(\left(a-c\right)+E\left(\varepsilon\right)\right)^2}{9b}. \qquad (14)
$$

It is worth noting that both government end up with same level of social welfare under export quota game. In this case, there is no option value to be had for firm 2 even though it has more information about the market condition than firm 1 does. With output being state-independent under quota, the well-informed firm looses the option of setting output according to the market conditions and therefore, there is no option value effect associated with information advantage for firm 2.

Next, we turn to examine equilibrium for mixed games.

3.3 Export Quota v.s. Export Subsidy: (Q,S)

So far we have derived subgame equilibrium when both governments choose symmetric strategies. We now consider the scenarios that governments 1 and 2 choose to intervene with asymmetric policy instruments. Suppose that government 1 chooses direct quantity control and government 2 decides to subsidize its own firm. Under this circumstance, government 1 imposes an output constraint on firm 1, while government 2 selects the amount of subsidy for firm 2 in stage 1. Given our assumption that a quota is binding, this limits what firm 1 can sell to the third market. Letting $\overline{q}_1(Q,S)$ be the quota imposed on firm 1 by government 1, firm 2's profits for $\varepsilon = \varepsilon_h$ and $\varepsilon = \varepsilon_l$ are

$$\begin{aligned} \pi_2^l(Q,S) &= \left(a - b\left(\overline{q}_1(Q,S) + q_2^l(Q,S)\right) + \varepsilon_l\right) q_2^l(Q,S) - cq_2^l(Q,S) + s_2 q_2^l(Q,S), \quad (15) \\ \pi_2^h(Q,S) &= \left(a - b\left(\overline{q}_1(Q,S) + q_2^h(Q,S)\right) + \varepsilon_l\right) q_2^h(Q,S) - cq_2^h(Q,S) + s_2 q_2^h(Q,S). \quad (16) \end{aligned}$$

Therefore, in stage three, firm 2 chooses $q_2^l(Q,S)$ to maximize $\pi_2^l(Q,S)$ and $q_2^h(Q,S)$ to maximize $\pi_2^h(Q,S)$.

The best response functions for firm 2's problems are

$$\begin{aligned} BR_{2l}(\overline{q}_1(Q,S)) &= q_2^l(Q,S) \in \arg\max \pi_2^l(Q,S), \\ BR_{2h}(\overline{q}_1(Q,S)) &= q_2^h(Q,S) \in \arg\max \pi_2^h(Q,S). \end{aligned}$$

Solving yields

$$\begin{aligned} q_2^l(Q,S) &= \frac{a - c + s_2 - b\overline{q}_1(Q,S) + \varepsilon_l}{2b}, \quad (17) \\ q_2^h(Q,S) &= \frac{a - c + s_2 - b\overline{q}_1(Q,S) + \varepsilon_h}{2b}, \quad (18) \end{aligned}$$

given any level of s_2.

Given these, government 1 then chooses the export quota level while government 2 selects optimal subsidy level in stage two. For the third market model, the expected social welfare for country 1 can therefore be written as

$$\begin{aligned} E\left(SW_1(Q,S)\right) &= \theta\left(\left(a - b\left(\overline{q}_1(Q,S) + q_2^l(Q,S)\right) + \varepsilon_l\right)\overline{q}_1(Q,S) - c\overline{q}_1(Q,S)\right) \\ &+ (1-\theta)\left(\left(a - b\left(\overline{q}_1(Q,S) + q_2^h(Q,S)\right) + \varepsilon_h\right)\overline{q}_1(Q,S) - c\overline{q}_1(Q,S)\right). \quad (19) \end{aligned}$$

Similarly, the expected social welfare for country 2 is specified as producer's surplus net of subsidy, given by

$$\begin{aligned} E\left(SW_2(Q,S)\right) &= \theta\left(\pi_2^l(Q,S) - s_2 q_2^l(Q,S)\right) \\ &+ (1-\theta)\left(\pi_2^h(Q,S) - s_2 q_2^l(Q,S)\right), \quad (20) \end{aligned}$$

where

$$\begin{aligned}
\pi_2^l(Q,S) &= \frac{(a-c+s_2-b\overline{q}_1(Q,S)+\varepsilon_l)^2}{4b}, \\
\pi_2^h(Q,S) &= \frac{(a-c+s_2-b\overline{q}_1(Q,S)+\varepsilon_h)^2}{4b}.
\end{aligned}$$

Government 1 chooses $\overline{q}_1(Q,S)$ to maximize its expected social welfare, while government 2 chooses s_2 to maximize its expected social welfare simultaneously. The best response functions are

$$\begin{aligned}
BR_1(s_2) &= \overline{q}_1(Q,S) \in \arg\max E(SW_1(Q,S)), \\
BR_2(\overline{q}_1(Q,S)) &= s_2 \in \arg\max E(SW_2(Q,S)).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium policy level:

$$\overline{q}_1^*(Q,S) = \frac{a-c}{2b} + \frac{E(\varepsilon)}{2b}, \tag{21}$$

$$s_2^* = 0. \tag{22}$$

Hence, the optimal policy for government 2 is not to intervene at all. The reason behind this result is that firm 2 knows what firm 1 will produce given the quota is binding and it can also fully observe the true market demand and act accordingly. Profit maximization by firm 1 ensures social welfare maximization for the entire country. Therefore, there is no role for government 1 to play. This explains $s_2^* = 0$.

Substituting $s_2^* = 0$ into firm 2's best response function, we get the expected output for firm 2 in equilibrium:

$$\begin{aligned}
E(q_2^l(Q,S)) &= \frac{a-c}{4b} + \frac{\varepsilon_l}{2b} - \frac{E(\varepsilon)}{4b}, \\
E(q_2^h(Q,S)) &= \frac{a-c}{4b} + \frac{\varepsilon_h}{2b} - \frac{E(\varepsilon)}{4b}.
\end{aligned}$$

The corresponding social welfare for each country under (Q,S) are

$$E(SW_1(Q,S)) = \frac{((a-c)+E(\varepsilon))^2}{8b}, \tag{23}$$

$$E(SW_2(Q,S)) = \frac{((a-c)+E(\varepsilon))^2}{16b} + \frac{var(\varepsilon)}{4b}. \tag{24}$$

Note that the second term in country 2's social welfare is the option value associated with better information. It is easy to verify that the expected social welfare for country 2 increases as market volatility increases. Moreover, it is straightforward to obtain

$$E\left(SW_2\left(Q,S\right)\right)-E\left(SW_1\left(Q,S\right)\right)>(<)\ 0,$$

if and only if $var\left(\varepsilon\right)>(<)\frac{\left((a-c)+E(\varepsilon)\right)^2}{4}$. This implies that country 2 is better off (worse off) relative to country 1 when the degree of uncertainty is sufficiently high (low).

3.4 Export Subsidy VS Export Quota: (S,Q)

Here, we consider the last pair of strategies (S,Q) that are selected by two governments in the first stage: with government 1 choosing subsidy and government 2 imposing direct quantity control. As usual, our analysis starts from the last stage. Firm 1 does not know the true market demand in the third market, but it can however observe the export quota level $\overline{q}_2\left(S,Q\right)$ set by government 2. Firm 1's expected profit function is

$$\begin{aligned} E\left(\pi_1\left(S,Q\right)\right) &= \theta\left(\left(a-b\left(q_1\left(S,Q\right)+\overline{q}_2\left(S,Q\right)\right)+\varepsilon_l\right)q_1\left(S,Q\right)\right) \\ &+ \left(1-\theta\right)\left(\left(a-b\left(q_1\left(S,Q\right)+\overline{q}_2\left(S,Q\right)\right)+\varepsilon_h\right)q_1\left(S,Q\right)\right) \\ &- cq_1\left(S,Q\right)+s_1q_1\left(S,Q\right). \end{aligned} \tag{25}$$

Being unable to observe the realized demand, firm 1 produces a fixed state-independent output for the third market, given $\overline{q}_2\left(S,Q\right)$. Thus, it chooses $q_1(S,Q)$ to maximize its profit and the best response function is

$$BR_1(\overline{q}_2\left(S,Q\right))=q_1(S,Q)\in\arg\max E\left(\pi_1\left(S,Q\right)\right).$$

Solving yields

$$q_1\left(S,Q\right)=\frac{a-c+s_1-b\overline{q}_2\left(S,Q\right)}{2b}+\frac{E\left(\varepsilon\right)}{2b},$$

given any level of s_1.

In stage two, government 1 sets the subsidy level s_1 to maximize its expected social welfare (specified as producer's surplus net of subsidy) given by

$$E(SW_1\left(S,Q\right))=E\left(\pi_1\left(S,Q\right)\right)-s_1q_1(S,Q), \tag{26}$$

where

$$E\left(\pi_1\left(S,Q\right)\right)=\frac{\left(a-c+s_1+E\left(\varepsilon\right)-b\overline{q}_2\left(S,Q\right)\right)^2}{4b}.$$

At the same time, government 2 selects the export quota $\overline{q}_2\left(S,Q\right)$ to maximize its social welfare (specified as producer's surplus):

$$\begin{aligned}
E(SW_2\left(S,Q\right)) &= \theta\left(\left(a-b\left(q_1(S,Q)+\overline{q}_2\left(S,Q\right)\right)+\varepsilon_l\right)\overline{q}_2\left(S,Q\right)\right) \\
&+ (1-\theta)\left(\left(a-b\left(q_1(S,Q)+\overline{q}_2\left(S,Q\right)\right)+\varepsilon_h\right)\overline{q}_2\left(S,Q\right)\right) \\
&- c\overline{q}_2\left(S,Q\right). \qquad (27)
\end{aligned}$$

The best response functions for the above optimization problems are

$$\begin{aligned}
BR_1(\overline{q}_2\left(S,Q\right)) &= s_1\in\arg\max E(SW_1\left(S,Q\right)), \\
BR_2(s_1) &= \overline{q}_2\left(S,Q\right)\in\arg\max E(SW_2\left(S,Q\right)).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium policy rate:

$$\begin{aligned}
s_1^* &= 0, \qquad (28) \\
\overline{q}_2^*\left(S,Q\right) &= \frac{(a-c)}{2b}+\frac{E\left(\varepsilon\right)}{2b}. \qquad (29)
\end{aligned}$$

Surprisingly, the optimal policy regime is symmetric as in (Q,S). Here, government 1 now grants no subsidy to firm 1 if government 2 imposes an export quota on firm 2. This can be explained as follows. Government 1 makes its choice based on its conjecture on government 2's best response; if there is no deviation from government 2's actual choice, then the Bayesian Nash equilibrium holds in stage two. Since government 1 and firm 1 have same information partition over Ω and firm 1 is well informed about the export quota imposed by government 2 at the beginning of stage three, firm 1 actually knows what firm 2 will produce. In this case, firm 2 loses the option of responding to market conditions since its output is fixed by the quota and is therefore no longer state contingent. Therefore, government 1 would leave full decision to its own firm (i.e., firm 1) since profit maximization ensures social welfare maximization in the third market setting. In short, government 1 would choose no subsidy.

Substituting stage two equilibrium points into firm 1's best response function, we get the expected output by firm 1 in stage three:

$$E(q_1(S,Q)) = \frac{a-c}{4b} + \frac{E(\varepsilon)}{4b}.$$

The corresponding expected social welfare for both couturiers under (S,Q) are

$$E(SW_1(S,Q)) = \frac{((a-c)+E(\varepsilon))^2}{16b}, \tag{30}$$

$$E(SW_2(S,Q)) = \frac{((a-c)+E(\varepsilon))^2}{8b}. \tag{31}$$

This together with the finding under (Q,Q) imply that the export quota essentially eliminates firm's ability to react to market conditions after the resolution of uncertainty. Hence no option value available for the country with the better-informed firm. This can be seen in $E(SW_2(Q,Q))$ and $E(SW_2(S,Q))$ that have no σ^2 term. This gives us

Proposition 2. *The export quota eliminates option values for the country with the better-informed firm.*

We are now ready to endogenize the choice of policy regimes in stage one. To this end, a normal form game is constructed next.

4 Choice of Trade Policy

The equilibrium analysis for each sub-game in the previous section indicates that the form of intervention does not lead to the same welfare outcome. We can now pose the obvious question: what will the governments choose in stage one when the foreign firm has better information about the market condition over the domestic firm? We answer this question with the help of normal form game outlined below.

Let a n-tuple $G \equiv < N, (A_i), (SW_i), f, (\succeq_i) >$ be a strategic game of choice of trade policy. The specification of the game G is as follows:

i. the finite set of players N consist of two players: government 1 and 2;

ii. for each government $i \in N$, the set of actions $A_i \equiv \{Subsidy, Quota\}$;

iii. the set SW_i represents the set of expected social welfare for government i;

iv. a function $f : A_i \to SW_i$ associates consequences with action profiles;

v. for each government $i \in N$, the preference relation of government i is $\succeq_i$ over SW_i.[6]

We introduce a set of expected social welfare SW here because each government cares about its social welfare, but not about the profiles of export subsidy or export quota level that generate that social welfare level.

Normal form representation of game G is illustrated in Table 1. Government 1 is the row player and government 2 is the column player. The first expression in each cell is the expected social welfare level for country 1 and the second expression in each cell is the expected social welfare level for country 2.

Table 1: Normal form representation of game G

Government 1	Government 2: Subsidy	Government 2: Quota
Subsidy	$\frac{2((a-c)+E(\varepsilon))^2}{25b}, \frac{2((a-c)+E(\varepsilon))^2}{25b} + \frac{\sigma^2}{4b}$	$\frac{((a-c)+E(\varepsilon))^2}{16b}, \frac{((a-c)+E(\varepsilon))^2}{8b}$
Quota	$\frac{((a-c)+E(\varepsilon))^2}{8b}, \frac{((a-c)+E(\varepsilon))^2}{16b} + \frac{\sigma^2}{4b}$	$\frac{((a-c)+E(\varepsilon))^2}{9b}, \frac{((a-c)+E(\varepsilon))^2}{9b}$

In order to illustrate Nash equilibrium of game G, we adopt the rationalizability concept by Pearce (1984) for presentation purpose. It follows that the game G is dominant solvable. We begin by proving the following lemma.

Lemma 1. *Subsidy is a strictly dominated strategy for government 1.*

Proof. Let $B_1(a_2)$ be the set of government 1's best actions

$$B_1(a_2) \equiv \{a_1 \in A_1 | (a_1, a_2) \succeq_1 (a_1', a_2) \, \forall a_1' \in A_1\}.$$

Given government 2 chooses *Subsidy*, we have

$$B_1(Subsidy) = \{Quota\},$$

[6] For example, an action $a_i \succeq a_i'$ if and only if $f(a_i, a_{-i}) \succeq f(a_i', a_{-i}) \; \forall a_i, a_i' \in A_i$.

since $f\left(Quota, Subsidy\right) \succ_1 f\left(Subsidy, Subsidy\right)$ or $\frac{((a-c)+E(\varepsilon))^2}{8b} > \frac{2((a-c)+E(\varepsilon))^2}{25b}$;

Similarly, given government 2 chooses $Quota$, we have

$$B_1\left(Quota\right) = \{Quota\},$$

since $f\left(Quota, Quota\right) \succ_1 f\left(Subsidy, Quota\right)$ or $\frac{((a-c)+E(\varepsilon))^2}{9b} > \frac{((a-c)+E(\varepsilon))^2}{16b}$.

Hence we have

$$B_1\left(a_2\right) = \{Quota\} \quad \forall a_2 \in A_2.$$

$\square$

Therefore, the *subsidy* will never be chosen by government 1. In other words, government 1 will always prefer to choose the *quota* irrespective of the policy choice of government 2. In view of *subsidy* being a strictly dominated strategy for government 1, government 2 will respond by choosing a quota (subsidy) if $\sigma^2 \leq (>) \frac{7}{36}\left((a-c)+E\left(\varepsilon\right)\right)^2$. This is proven in the following lemma.

Lemma 2. *The optimal strategy for government 2 depends on the variance over Ω. Specifically, a quota (subsidy) is chosen if $\sigma^2 \leq (>) \frac{7}{36}\left((a-c)+E\left(\varepsilon\right)\right)^2$.*

Proof. Let $B_2\left(Quota\right)$ be the set of government 2's best responses to government 1's strategy $Quota$

$$B_2\left(Quota\right) \equiv \left\{a_2 \in A_2 \middle|\, \left(Quota, a_2\right) \succeq_2 \left(Quota, a_2'\right)\right\}.$$

If $f\left(Quota, Quota\right) \succ_2 f\left(Subsidy, Quota\right)$, we have $\frac{((a-c)+E(\varepsilon))^2}{9b} \geq \frac{((a-c)+E(\varepsilon))^2}{16b} + \frac{\sigma^2}{4b}$, that is

$$B_2\left(Quota\right) = \{Quota\} \qquad if \quad \sigma^2 \leq \frac{7}{36}\left((a-c)+E\left(\varepsilon\right)\right)^2.$$

On the other hand, if $f\left(Subsidy, Quota\right) \succ_2 f\left(Quota, Quota\right)$, we have $\frac{((a-c)+E(\varepsilon))^2}{16b} + \frac{\sigma^2}{4b} \geq \frac{((a-c)+E(\varepsilon))^2}{9b}$, that is

$$B_2\left(Quota\right) = \{Subsidy\} \qquad if \quad \sigma^2 \geq \frac{7}{36}\left((a-c)+E\left(\varepsilon\right)\right)^2.$$

Thus the best response to government 1's strategy $Quota$ for government 2 depends on σ^2. $\square$

As we can see, for a small variance, setting $Quota$ becomes the dominant strategy for government 2. Since neither governments has information advantage over each other, there is no option value to country 2 and both countries end up having the same expected level of social welfare. On the other hand, for sufficiently large variance, choosing *Subsidy* becomes the dominant strategy for government 2. The option value accounts for a significant part of social welfare for country 2. By lemmas 1 and 2, we have the following proposition.

Proposition 3. *Nash equilibrium of game G is*

i. (Q, Q) *if* $\sigma^2 \leq \frac{7}{36}\left((a-c)+E(\varepsilon)\right)^2$*;*

ii. (Q, S) *if* $\sigma^2 \geq \frac{7}{36}\left((a-c)+E(\varepsilon)\right)^2$.

Our result is in sharp contrast to Cooper and Riezman (1989) and Caglayan (2000). In their studies, both firms have complete information about demand condition in the third market, and any of four pairs of strategies can become Nash equilibrium trade policy choice, depending on the market volatility. Here, we show that *Quota* becomes the dominant strategy for country with the less informed firm. Hence the possibility of getting (S, S) and (S, Q) are eliminated. As the flexibility provided by setting subsidy can only be beneficial to the country with a well-informed firm, incomplete information itself redistributes the option value from the less-informed to the well-informed country. But the option value is accrues to the country with a better informed firm only if the variance is sufficiently large.

5 Conclusion

The literature on the choice of strategic trade policy in oligopolistic industry under uncertainty points to a variety of combinations of modes of intervention that could emerge as an equilibrium outcome in a three-country setting (see Brander and Spencer (1985)). When we consider policymakers choosing between export subsidy/tax and export quota, the optimal choice of trade policy depends on the degree of uncertainty in the third market, and all four combinations of subsidy and quota could be equilibrium under demand uncertainty. The novelty of this paper is that it analyzes the optimal choice of strategic trade policy in a third market model when information is incomplete across the duopoly exporters to the third market.

Our principle results can be summarized as follows. In a choice of trade policy game,,export subsidy is a dominated strategy for country with incompletely informed firms. In other words, imposing direct quantity control is always optimal for country with ill-informed firms irrespective of what form of interventions is chosen by the other country. This holds for any degree of market uncertainty. For country with completely informed firms, the optimal choice of trade policy depends on the volatility of market demand in the third market. If the market in the third country is relatively stable, imposing export quota is optimal. Conversely, if market demand in the third country is relatively volatile, subsidy turns out to be the optimal choice. This is driven by the option value effect associated with better information.

Our analysis rests heavily on a number of simplifying assumptions. Firstly, the specification of the demand structure is linear with an additive shock. A more general setting with respect to the demand function could be introduced. Secondly, the source of uncertainty is unique in our theoretical framework. We only consider the single source of uncertainty in market demand. In addition to country specific shocks, one may introduce a richer environment of uncertainty by taking firm specific shocks into consideration. Thirdly, we impose a common prior assumption on beliefs for all players involved. Further research research could consider a more general setting by allowing different prior beliefs about the states of nature. Lastly, we have assumed that consumers only reside in the third country. This assumption is only appropriate if the good is produced solely for export or if the domestic market can be isolated from trade policies by the use of consumption subsidies/taxes. This assumption simplifies our analysis since consumer's surplus is excluded from social welfare calculation. A more realistic setting may include the domestic consumption of goods produced by the domestic firm.

References

Anam, M. and Chiang, S.-H. (2000) Export market correlation and strategic trade policy, *Canadian Journal of Economics*, **33**, 41–52.

Brander, J. A. and Spencer, B. J. (1985) Export subsidies and international market share rivalry, *Journal of International Economics*, **18**, 83–100.

Caglayan, M. (2000) Uncertainty resolution and strategic trade policy in oligopolistic industries, *Review of International Economics*, **8**, 311–318.

Cooper, R. and Riezman, R. (1989) Uncertainty and the choice of trade policy in oligopolistic industries, *The Review of Economic Studies*, **56**, 129–140.

Creane, A. and Miyagiwa, K. (2008) Information and disclosure in strategic trade policy, *Journal of International Economics*, **75**, 229–244.

Grant, S. and Quiggin, J. (1997) Strategic trade policy under uncertainty: Sufficient conditions for the optimality of ad valorem, specific and quadratic trade taxes, *International Economic Review*, **38**, 187–203.

Klemperer, P. D. and Meyer, M. A. (1989) Supply function equilibria in oligopoly under uncertainty, *Econometrica*, **57**, 1243–1277.

Krishna, K. (1989) Trade restrictions as facilitating practices, *Journal of International Economics*, **26**, 251–270.

Maggi, G. (1999) Strategic trade policy under incomplete information, *International Economic Review*, **40**, 571–594.

Ning, H. (2020a) On the equivalence of tariffs and quotas with incomplete information, manuscript, York University.

Ning, H. (2020b) Strategic trade policy in reciprocal dumping model with incomplete information, manuscript, York University.

Pearce, D. G. (1984) Rationalizable strategic behavior and the problem of perfection, *Econometrica*, **52**, 1029–1050.

Qui, L. D. (1994) Optimal strategic trade policy under asymmetric information, *Journal of International Economics*, **36**, 333–354.

Shivakumar, R. (1993) Strategic trade policy: Choosing between export subsidies and export quotas under uncertainty, *Journal of International Economics*, **35**, 169–183.