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Abstract

This paper extends the conventional literature on strategic trade policy in reciprocal dumping model to the context that involves market demand uncertainty and incomplete information. In order to highlight the role of uncertainty and incomplete information, a simple scenario with linear asymmetric demand, additive stochastic market shock, homogeneous products, and identical constant marginal costs are considered. It is shown that incomplete information at industrial level redistributes the option value associated with better information to the country with the better informed firm. As a result, both governments tend to choose tariffs over export subsidies in the Nash equilibrium of the simultaneous strategic trade policy games under complete and incomplete information. This yields a second best outcome. Moreover, we show that Nash equilibrium outcome is inferior to free-trade outcome.

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Keywords: Intra-industry Trade, Incomplete Information, Uncertainty, Bayesian Nash equilibrium, Tariff, Export subsidy.

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1 Introduction

Strategic trade policy has been one of the most intensively researched areas of international trade over the last three decades following the path-breaking seminal work by Brander and Spencer (1985). In their work, both governments adopt trade polices to confer strategic advantage to their respective domestic firm when firms are imperfectly competing with each other. The essence of this literature explains how these strategic trade policies are beneficial at the national level. Further research along this line examines a wide variety of alternative scenarios which involves (but not limited to): the nature of oligopolistic competition in terms of conjectural variation and conduct parameters, available trade policy instruments, information structure, timing and leadership structure, etc.[1]

Most early papers along this line of research in strategic trade policy adopt the third-market setting: the entire output of the rival oligopolistic firms are exported to a market other than the one where firms themselves are located. These papers exclusively focus on how governments can shift oligopoly rents by using export subsidy in favor of their respective domestic firms. (e.g., see Cooper and Riezman (1989), Shivakumar (1993) and Caglayan (2000)). On the other hand, in the so-called reciprocal market model, first analyzed by Brander and Krugman (1983), domestic and foreign firms are assumed to compete in each other's market. The typical case, where firms engage in Cournot competition in homogeneous product gives rise to intra-industry trade of reciprocal dumping variety. An implication of this setting for strategic trade policy is that consumer's surplus becomes significantly important in determining social welfare for governments. Brander and Spencer (1985) and Dixit (1984) are the pioneering papers that analyze equilibrium in strategic trade policy in the deterministic reciprocal market.In the reciprocal dumping model, import restriction becomes an additional viable policy regime. Nash equilibrium levels of subsidy game and tariff game usually involves positive subsidy and tariff rate. However, the welfare ranking of the policy regimes is generally ambiguous.

Although the so-called third market model is a useful simplification for isolating rent-shifting motive for strategic trade policy, reciprocal dumping model is apparently the more realistic scenario. Important policy issues such as export subsidies and countervailing duties can only be analyzed in the reciprocal dumping model. However, a limitation of the literature to date is that policy making in reciprocal market is analyzed with no uncertainty. Most recently, Anam and Chiang (2018) extend Cooper and Riezman (1989) strategic trade policy in reciprocal markets that are interdependent, and they replace the constant marginal cost assumption with quadratic cost functions. They show that

[1] For a complete survey of these early literature incorporating extensions in strategic trade policy, see Brander (1995).

quadratic cost functional form has significant impact on the sign of export subsidy, and the market correlation plays an important role in determining Nash equilibrium in choosing optimal policy regime.

Unlike Anam and Chiang (2018), this paper retains basic assumption of constant marginal cost and segmented markets, but the added value of this paper is to introduce incomplete information at industrial level.[2] In order to highlight the impact of uncertainty, the simplest framework for the reciprocal market is assumed. There is one firm in the home and foreign countries producing a homogeneous good at constant marginal cost. The firms compete in both markets as Cournot duopolists, similar to Brander (1981). The two markets are segmented, and in one market demand is deterministic while in the other demand is stochastic .In stage one, both governments are assumed to simultaneously commit to either a tariff or an export subsidy policy before the resolution of uncertainty. In stage two, the level of each policy instrument is set again before random variable is revealed. In order to highlight the role of incomplete information at industrial level, we discuss two possible information partition: (i) both firms are able to observe the realized demand and (ii) only the domestic firm can observe true state of nature. The true market demand is revealed according to different information partition at the beginning of stage three. Finally, in stage three, the firms set their profit-maximizing output in both markets taking the present strategic trade policy level as given. As in the conventional analysis, both governments arrive at the Nash strategic trade policy equilibrium by backward induction. An important feature of this setup (in both information partition cases) is that governments and the firms stand to capture option value associated with the ability to make strategic decisions after the resolution of uncertainty.

We construct a game in which each government chooses the policy regime between tariffs and export subsidies in stage one. This gives rise to four possible scenarios: subsidy v.s. subsidy (S, S), tariff v.s. tariff (T, T), tariff v.s. subsidy (T, S) and subsidy v.s. tariff (S, T). The Bayesian Nash equilibrium levels of each policy is determined in stage two. The firms (one domestic and the other foreign) then compete against each other. The multi-stage game is then solved by backward induction. Two different information partitions (complete information and incomplete information at industrial level) are considered for each pair of strategies. National welfare associated with each policy combination is then used to characterize the choice of policy regimes.

[2] Ning (2020a) extends Cooper and Riezman (1989) by incorporating incomplete information at industrial level in a third market model. He shows that flexibility is no longer desirable when one firm has more information about the third market than the other firm, and thus export quota becomes a strictly dominant strategy for the country with less informed firm. On the other hand, Ning (2020b) introduces incomplete information at industrial level in examining social welfare equivalence issue of tariff and quota from the importing country's perspective. He shows that a tariff is always superior to a quota as long as incomplete information persists at industrial level.

Several interesting results emerge from our analysis. We show that Nash equilibrium export subsidy dominates the corresponding tariff equilibrium for both information partitions. Our analysis demonstrates that incomplete information at industrial level redistributes the option value associated with better information.Incomplete information is shown to shift option value to the country with the more informed firm. As in the optimal strategic choice of trade policy games, we show that (T, T) is the unique Nash equilibrium policy combination for both trade games with complete and incomplete information. Moreover, the trade games with complete and incomplete information have a prisoner dilemma property since (S, S) is a Pareto the outcome. Furthermore, we also show that the Bayesian Nash equilibrium outcome is inferior to the outcome without any government interventions.

The paper is organized as follows. Section 2 presents the basic reciprocal dumping model. Section 3 derives the sub-game equilibrium for the trade game with incomplete information and that under complete information. Section 4 characterizes and analyzes the optimal choice of policy regimes. Section 5 relates our results to free-trade outcome. Finally, section 6 concludes the paper.

2 The Model

Consider a two-country (home and foreign) model in which each country has only one firm producing a homogeneous good. These firms are identical except for their country of operation if there is no government intervention. For computational as well as expositional ease, we assume that the demand function in each country is linear with constant slope b. In order to highlight the effect of incomplete information, we assume the demand in country 1 is uncertain, but the demand in country 2 is deterministic. Thus, the inverse demand functions for countries 1 and 2 are given by

$$p_1 = a - b\left(q_1 + x_2\right) + \varepsilon,$$

and

$$p_2 = a - b\left(q_2 + x_1\right),$$

where p_i is the commodity price in country i, q_i is the delivery of the local firm to the local market, x_j is the export of firm j to market i, and ε is a random disturbance term in the domestic market demand for all $i, j = 1, 2$ and $i \neq j$. Following the convention, country 1 is the home country and country 2 the foreign country.

For simplicity, we assume the random variable ε is binary. Specifically,

$$\varepsilon \in \Omega \equiv \{-V, V\},$$

where $V \in \mathbb{R}$. The subjective common prior probability measure over Ω is assumed to be $(\theta, 1-\theta)$. In other words, we assume that the bad state $-V$ occurs with probability θ and the good state V occurs with probability $1-\theta$, respectively.

Assume each firm produces final goods at a constant marginal cost of $c > 0$. The cost function for firm i is therefore assumed to be

$$C_i = c\left(q_i + x_i\right),$$

for $i = 1, 2$. Homogeneous product with identical cost structure implies that the trade is entirely of the intra-industry type.

Following Anam and Chiang (2018), our trade game consists of three stages. In stage one, each government commits to a policy instrument to be either export subsidy or tariff. The corresponding levels of the policy regimes are then set in stage two once the specific trade policy is prescribed in the previous stage. At the beginning of stage three, the random variable ε is either revealed to both firms (trade game of complete information) or only to the domestic firm (trade game of incomplete information). Both firms then play a Cournot game by setting outputs (both for local and foreign delivery) to maximize their profits, given the optimal policies chosen by the governments in previous stages. To ease presentation, the equilibrium analysis for stage sub-game under complete information is relegated to the Appendix. In the main text, we exclusively focus our analysis on the stage game under incomplete information.

Figure 1 shows the timing of uncertainty resolution for the trade game under incomplete information. Note that the random variable ε is realized at the beginning of stage three. The random variable, once realized, is only observed by firm 1. Given that the better informed firm would stand to capture option values at the expense of the less informed firm, the implication that comes out of the game under incomplete information is expected to be substantially different from that under perfect information. Since the random variable becomes known at the beginning of stage three, both governments make their policy choices without having any knowledge of true demand in the home market.

Our model differs from Anam and Chiang (2018) in three ways: Firstly, in their model, the cost function for each firm is assumed to be quadratic. The increasing marginal cost in their model has

Figure 1: Timing of Three-Stage Trade Game with Incomplete Information

Stage 1		Stage 2		Beginning of Stage 3		Stage 3
Both governments select policy modes	$\rightarrow$	Both governments select policy levels	$\rightarrow$	Only firm 1 observes ε	$\rightarrow$	Firms play Cournot game

significant impact on the levels of trade policies.[3] In our model, however, the market equilibrium is separable given segmented markets and constant marginal costs in order to highlight the role of information. Secondly, they assume identical market demand in both countries while we assume asymmetric market demand. Lastly, they only consider the complete information trade game while we also conduct equilibrium analysis for incomplete information at industrial level.

Without loss of generality, we assume $\theta = \frac{1}{2}$ in the subsequent analysis. Given this, the expected value of random variable is $E(\varepsilon) = 0$, and its variance is $Var(\varepsilon) = \sigma^2 = V^2$. This simplified assumption allows us to see clearly the effect of market volatility on the choice of policy regime.

In what follows, we derive the perfect Bayesian equilibrium for various policy regimes by backward induction, beginning with stage three.

3 Subgame Equilibrium

In this section, we derive equilibrium output levels for both firms in stage three for trade game with incomplete information under various trade policies. The expected social welfare level for both governments is then calculated by reverting back to previous stages given that both governments anticipate equilibrium output levels in the last stage. We use Bayesian Nash solution concept in solving equilibrium output levels for trade game with incomplete information. The complete derivation of subgame equilibrium for trade game with complete information at industrial level can be found in the Appendix. We use superscript CI to denote variable choices for trade game with complete information, and we use superscript II to denote variable choices for trade game with incomplete information. We also use (a_1, a_2) to denote pair of strategies each government chooses in the first stage, where $a_i \in A_i \equiv \{S, T\}$ for $i = 1, 2$.[4]

Next, we examine the Bayesian-Nash equilibrium for our three-stage game for four policy combinations: (S, S), (T, T), (T, S), and (S, T).

[3] Specifically, quadratic cost function can make optimal export subsidy level negative.

[4] For example, (S, T) represents that government 1 chooses export subsidy and government 2 imposes tariff.

3.1 Subsidy Game (S,S)

We start with the subsidy game in which both governments choose to the amount of export subsidy for their firms in the first stage. The domestic firm, being able to observe the true demand in market 1, solves the following maximization problem:

$$\begin{aligned}\max_{q_{1L}^{II}(S,S),x_1^{II}(S,S)} \pi_{1L}^{II}(S,S) &= \left(a - b\left(q_{1L}^{II}(S,S) + x_2^{II}(S,S)\right) - V\right) q_{1L}^{II}(S,S) \\ &+ \left(a - b\left(q_2^{II}(S,S) + x_1^{II}(S,S)\right)\right) x_1^{II}(S,S) \\ &- c\left(q_{1L}^{II}(S,S) + x_1^{II}(S,S)\right) + s_1 x_1^{II}(S,S), \qquad (1)\end{aligned}$$

if $\varepsilon = -V$ and

$$\begin{aligned}\max_{q_{1H}^{II}(S,S),x_1^{II}(S,S)} \pi_{1H}^{II}(S,S) &= \left(a - b\left(q_{1H}^{II}(S,S) + x_2^{II}(S,S)\right) + V\right) q_{1H}^{II}(S,S) \\ &+ \left(a - b\left(q_2^{II}(S,S) + x_1^{II}(S,S)\right)\right) x_1^{II}(S,S) \\ &- c\left(q_{1H}^{II}(S,S) + x_1^{II}(S,S)\right) + s_1 x_1^{II}(S,S), \qquad (2)\end{aligned}$$

if $\varepsilon = V$, where s_1 is the subsidy granted by government 1. Nevertheless, the foreign firm, being unable to observe the true state of nature in market 1, solves the following maximization problem:

$$\begin{aligned}\max_{q_2^{II}(S,S),x_2^{II}(S,S)} E\left(\pi_2^{II}(S,S)\right) &= \left(a - b\left(q_2^{II}(S,S) + x_1^{II}(S,S)\right)\right) q_2^{II}(S,S) \\ &+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a - b\left(q_1^{II}(S,S) + x_2^{II}(S,S)\right) + \varepsilon\right) x_2^{II}(S,S) \\ &- c\left(q_2^{II}(S,S) + x_2^{II}(S,S)\right) + s_2 x_2^{II}(S,S), \qquad (3)\end{aligned}$$

where $q_1^{II}(S,S) \in \left\{q_{1L}^{II}(S,S), q_{1H}^{II}(S,S)\right\}$ depends on the realization of ε and s_2 is the subsidy given by the foreign government. The best response functions for above optimization problems are

$$\begin{aligned}BR_{1L} &= q_{1L}^{II}(S,S) \in \arg\max \pi_{1L}^{II}(S,S), \\ BR_{1H} &= q_{1H}^{II}(S,S) \in \arg\max \pi_{1H}^{II}(S,S), \\ BR_{1x} &= x_1^{II}(S,S) \in \arg\max \pi_{1L}^{II}(S,S), \\ BR_2 &= q_2^{II}(S,S) \in \arg\max \pi_2^{II}(S,S), \\ BR_{2x} &= x_2^{II}(S,S) \in \arg\max \pi_2^{II}(S,S).\end{aligned}$$

Solving these equations simultaneously yields Bayesian Nash equilibrium points for stage three game given any (s_1, s_2, V):

$$\begin{aligned}
q_{1L}^{II}(S,S) &= \frac{1}{6b}\left(2(a-c) - 3V - 2s_2\right), \\
q_{1H}^{II}(S,S) &= \frac{1}{6b}\left(2(a-c) + 3V - 2s_2\right), \\
x_{1}^{II}(S,S) &= \frac{1}{3b}\left(a - c + 2s_1\right), \\
q_{2}^{II}(S,S) &= \frac{1}{3b}\left(a - c - s_1\right), \\
x_{2}^{II}(S,S) &= \frac{1}{3b}\left(a - c + 2s_2\right).
\end{aligned}$$

It is evident that a higher export subsidy provided by the domestic government increases the export of the domestic firm to the foreign market, but a higher export subsidy given by the foreign government to its own firm lowers the local delivery of the domestic firm to the domestic market. This is due to profit shifting as noted in Brander and Spencer (1985).

Folding back to the second stage, both governments then set subsidy rates to maximize their respective social welfare function, given that they anticipate the Bayesian Nash equilibrium output level in the subsequent stage. The social welfare functions for both governments under the subsidy game are the sum of producer's surplus and consumer's surplus net of total subsidy, given by

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{II}(S,S)\right) &= PS_1^{II}(S,S) + CS_1^{II}(S,S) - S_1^{II}(S,S) \\
&= \frac{1}{2}\left(\pi_{1L}^{II}(S,S) + \pi_{1H}^{II}(S,S)\right) \\
&+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{II}(S,S) + x_2^{II}(S,S)\right)^2 + \frac{b}{2}\left(q_{1H}^{II}(S,S) + x_2^{II}(S,S)\right)^2\right) \\
&- s_1 x_1^{II}(S,S),
\end{aligned} \tag{4}$$

$$\begin{aligned}
\max_{s_2} E\left(SW_2^{II}(S,S)\right) &= PS_2^{II}(S,S) + CS_2^{II}(S,S) - S_2^{II}(S,S) \\
&= \pi_2^{II}(S,S) + \frac{b}{2}(q_2^{II}(S,S) + x_1^{II}(S,S))^2 - s_2 x_2^{II}(S,S).
\end{aligned} \tag{5}$$

The best response functions of stage-two game are

$$\begin{aligned} BR_1(s_2) &= s_1 \in \arg\max E\left(SW_1^{II}(S,S)\right), \\ BR_2(s_1) &= s_2 \in \arg\max E\left(SW_2^{II}(S,S)\right). \end{aligned}$$

Solving yields Bayesian Nash equilibrium level of export subsidies

$$s_1^* = s_2^* = \frac{1}{4}(a-c). \tag{6}$$

Clearly, the expected level of Bayesian Nash equilibrium subsidy is the same for both countries mainly because both governments make their policy choices on the basis of the distribution of ε. With no surprise, these optimal subsidy rates are identical to the solution under perfect information (see equation (45) in Appendix A).

By substitution, we can obtain the expected output levels as follows:

$$\begin{aligned} E\left(q_{1L}^{II}(S,S)\right) &= \frac{1}{4b}(a-c) - \frac{1}{2b}V, \\ E\left(q_{1H}^{II}(S,S)\right) &= \frac{1}{4b}(a-c) + \frac{1}{2b}V, \\ E\left(x_1^{II}(S,S)\right) &= \frac{1}{2b}(a-c), \\ E\left(q_2^{II}(S,S)\right) &= \frac{1}{4b}(a-c), \\ E\left(x_2^{II}(S,S)\right) &= \frac{1}{2b}(a-c). \end{aligned}$$

Now we are able to compare these output levels for the subsidy game under incomplete information with the output levels under complete information. Note that $E\left(x_2^{II}(S,S)\right)$ is the weighted average of $E\left(x_{2L}^{CI}(S,S)\right)$ and $E\left(x_{2H}^{CI}(S,S)\right)$[5], given our assumption that each state occurs with equal probability. Without being able to observe the true market demand in the domestic country, the foreign firm now can only supply a state-independent export to the domestic market. As a consequence, the domestic firm tends to deliver lower (higher) local output in low (high) market demand compare to the complete information case under subsidy game. This can be seen as $E\left(q_{1L}^{II}(S,S)\right) < E\left(q_{1L}^{CI}(S,S)\right)$ and $E\left(q_{1H}^{II}(S,S)\right) > E\left(q_{1H}^{CI}(S,S)\right)$. Since the foreign market involves no uncertainty, there is no change in the foreign firm's local delivery and the domestic firm's export to the foreign market.

[5] See Appendix A for expected output levels under subsidy game with complete information.

Given optimal subsidy levels with incomplete information, the expected social welfare for both countries under subsidy game with incomplete information are

$$E\left(SW_1^{II}(S,S)\right) = \frac{15}{32b}(a-c)^2 + \frac{3}{8b}\sigma^2, \tag{7}$$
$$E\left(SW_2^{II}(S,S)\right) = \frac{15}{32b}(a-c)^2. \tag{8}$$

It is worth noting that now the expected social welfare for the foreign country contains no term associated with option value. The incomplete information at industrial level shifts the entire option value to the domestic country. As the domestic firm is able to make output decisions after it observes the random variable, incomplete information for the foreign firm enables the domestic firm to fully capture the option value. As a result, the domestic country ends up with higher expected social welfare relative to the foreign country due to option value effects. This can be seen from $E\left(SW_1^{II}(S,S)\right) - E\left(SW_2^{II}(S,S)\right) = \frac{3}{8b}\sigma^2$. Moreover, it can be easily verified that $\frac{\partial E\left(SW_1^{II}(S,S)\right)}{\partial \sigma^2} > 0$.[6] This gives us the following proposition.

Proposition 1. *For export subsidy game with incomplete information at industrial level,*

i. the entire option value shifts to the domestic country due to incomplete information against the foreign firm;

ii. social welfare for the domestic country increases as variance increases;

iii. the domestic country ends up with higher social welfare due to option value effects.

Next, we turn to examine sub game equilibrium with incomplete information under tariffs.

3.2 Tariff Game (T,T)

Under tariffs, the domestic firm solves the following maximization problem:

$$\begin{aligned}\max_{q_{1L}^{II}(T,T),\, x_1^{II}(T,T)} \pi_{1L}^{II}(T,T) &= \left(a - b\left(q_{1L}^{II}(T,T) + x_2^{II}(T,T)\right) - V\right) q_{1L}^{II}(T,T) \\ &+ \left(a - b\left(q_2^{II}(T,T) + x_1^{II}(T,T)\right)\right) x_1^{II}(T,T) \\ &- c\left(q_{1L}^{II}(T,T) + x_1^{II}(T,T)\right) - t_2 x_1^{II}(T,T),\end{aligned} \tag{9}$$

[6] A similar result obtained by Cooper and Riezman (1989), Chen and Hwang (2006) and Anam and Chiang (2018).

if $\varepsilon = -V$ and

$$\begin{aligned}
\max_{q_{1H}^{II}(T,T),x_1^{II}(T,T)} \pi_{1H}^{II}(T,T) &= \left(a - b\left(q_{1H}^{II}(T,T) + x_2^{II}(T,T)\right) + V\right) q_{1H}^{II}(T,T) \\
&+ \left(a - b\left(q_2^{II}(T,T) + x_1^{II}(T,T)\right)\right) x_1^{II}(T,T) \\
&- c\left(q_{1H}^{II}(T,T) + x_1^{II}(T,T)\right) - t_2 x_1^{II}(T,T), \qquad (10)
\end{aligned}$$

if $\varepsilon = V$, where t_2 is the tariff imposed by the foreign government on exports from the domestic firm.

On the contrary, the foreign firm maximizes the expected profit function given a common prior:

$$\begin{aligned}
\max_{q_2^{II}(T,T),x_2^{II}(T,T)} E\left(\pi_2^{II}(T,T)\right) &= \left(a - b\left(q_2^{II}(T,T) + x_1^{II}(T,T)\right)\right) q_2^{II}(T,T) \\
&+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a - b\left(q_1^{II}(T,T) + x_2^{II}(T,T)\right) + \varepsilon\right) x_2^{II}(T,T) \\
&- c\left(q_2^{II}(T,T) + x_2^{II}(T,T)\right) - t_1 x_2^{II}(T,T), \qquad (11)
\end{aligned}$$

where $q_1^{II}(T,T) \in \left\{q_{1L}^{II}(T,T), q_{1H}^{II}(T,T)\right\}$ depend on the realization of ε, and t_1 is the tariff imposed by the domestic government on exports from the foreign firm. The corresponding best response functions for above maximization problems are

$$\begin{aligned}
BR_{1L} &= q_{1L}^{II}(T,T) \in \arg\max \pi_{1L}^{II}(T,T), \\
BR_{1H} &= q_{1H}^{II}(T,T), \in \arg\max \pi_{1H}^{II}(T,T), \\
BR_{1x} &= x_1^{II}(T,T) \in \arg\max \pi_{1L}^{II}(T,T), \\
BR_2 &= q_2^{II}(T,T) \in \arg\max \pi_2^{II}(T,T), \\
BR_{2x} &= x_2^{II}(T,T) \in \arg\max \pi_2^{II}(T,T).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium output levels given any (t_1, t_2, V):

$$\begin{aligned}
q_{1L}^{II}(T,T) &= \frac{1}{6b}\left(2(a-c) - 3V + 2t_1\right), \\
q_{1H}^{II}(T,T) &= \frac{1}{6b}\left(2(a-c) + 3V + 2t_1\right), \\
x_1^{II}(T,T) &= \frac{1}{3b}\left(a - c - 2t_2\right), \\
q_2^{II}(T,T) &= \frac{1}{3b}\left(a - c + t_2\right), \\
x_2^{II}(T,T) &= \frac{1}{3b}\left(a - c - 2t_1\right).
\end{aligned}$$

Going backward to the second stage, each government chooses a tariff rate t_i to maximize its expected social welfare given that each government anticipates firms' strategic behavior in the last stage. The expected social welfare for each government is specified as the sum of producer's surplus, consumer's surplus, and tariff revenue. Given the common prior assumption, we can write each government's problem as

$$\begin{aligned}\max_{t_1} E\left(SW_1^{II}(T,T)\right) &= PS_1^{II}(T,T) + CS_1^{II}(T,T) + TR_1^{II}(T,T) \\ &= \frac{1}{2}\left(\pi_{1L}^{II}(T,T) + \pi_{1H}^{II}(T,T)\right) \\ &+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{II}(T,T) + x_2^{II}(T,T)\right)^2 + \frac{b}{2}\left(q_{1H}^{II}(T,T) + x_2^{II}(T,T)\right)^2\right) \\ &+ t_1 x_2^{II}(T,T),\end{aligned} \tag{12}$$

$$\begin{aligned}\max_{t_2} E\left(SW_2^{II}(T,T)\right) &= PS_2^{II}(T,T) + CS_2^{II}(T,T) + TR_2^{II}(T,T) \\ &= \pi_2^{II}(T,T) + \frac{b}{2}(q_2^{II}(T,T) + x_1^{II}(T,T))^2 + t_2 x_1^{II}(T,T).\end{aligned} \tag{13}$$

The best response functions for stage two game are

$$\begin{aligned}BR_1(t_2) &= t_1 \in \arg\max E\left(SW_1^{II}(T,T)\right), \\ BR_2(t_1) &= t_2 \in \arg\max E\left(SW_2^{II}(T,T)\right).\end{aligned}$$

Hence the Bayesian Nash equilibrium policy rates are

$$t_1^* = t_2^* = \frac{1}{3}(a-c). \tag{14}$$

The optimal tariff rates set by the governments are the same under incomplete information. They are again identical to that under complete information (see Appendix B) due to no change in information partition at the national level. Given these optimal tariff rates, we can get the expected output levels in stage three as follows:

$$
\begin{aligned}
E\left(q_{1L}^{II}\left(T,T\right)\right) &= \frac{4}{9b}\left(a-c\right)-\frac{1}{2b}V,\\
E\left(q_{1H}^{II}\left(T,T\right)\right) &= \frac{4}{9b}\left(a-c\right)+\frac{1}{2b}V,\\
E\left(x_{1}^{II}\left(T,T\right)\right) &= \frac{1}{9b}\left(a-c\right),\\
E\left(q_{2}^{II}\left(T,T\right)\right) &= \frac{4}{9b}\left(a-c\right),\\
E\left(x_{2}^{II}\left(T,T\right)\right) &= \frac{1}{9b}\left(a-c\right).
\end{aligned}
$$

Similar to the argument we made for subsidy game under incomplete information, the well-informed firm is more aggressive (conservative) in the good (bad) market 1, and the ill-informed firm tends to deliver a state-independent export to the market 1. This is due to the effect of incomplete information at the industrial level. Given the optimal tariff rates, we can calculate the expected social welfare for each country as

$$
\begin{aligned}
E\left(SW_{1}^{II}\left(T,T\right)\right) &= \frac{65}{162b}\left(a-c\right)^{2}+\frac{3}{8b}\sigma^{2}, && (15)\\
E\left(SW_{2}^{II}\left(T,T\right)\right) &= \frac{65}{162b}\left(a-c\right)^{2}. && (16)
\end{aligned}
$$

As we can see, the option value associated with better information now completely shifts to the domestic country under tariffs when the domestic firm is a information monopolist. One can also easily verify that $\frac{\partial E\left(SW_{1}^{II}(T,T)\right)}{\partial\sigma^{2}}>0$. The above can be summarized in the following proposition.

Proposition 2. *For tariff game under incomplete information,*

i. the entire option value shifts to the domestic country due to the information deficiency of the foreign firm;

ii. social welfare for the domestic country increases as variance increases;

iii. the domestic country ends up with higher social welfare due to option value effects.

Next, we examine equilibrium points under mixed games with incomplete information.

3.3 Mixed Game (T,S)

For the mixed game, we first analyze the sub-game equilibrium when the domestic government imposes tariff on the goods imported to market 1 and the foreign government subsidizes firm 2's export.

In stage three, the maximization problems for the domestic firm are

$$\begin{aligned}
\max_{q_{1L}^{II}(T,S),x_1^{II}(T,S)} \pi_{1L}^{II}(T,S) &= \left(a-b\left(q_{1L}^{II}(T,S)+x_2^{II}(T,S)\right)-V\right)q_{1L}^{II}(T,S) \\
&+ \left(a-b\left(q_2^{II}(T,S)+x_1^{II}(T,S)\right)\right)x_1^{II}(T,S) \\
&- c\left(q_{1L}^{II}(T,S)+x_1^{II}(T,S)\right), \qquad (17)
\end{aligned}$$

if $\varepsilon=-V$ and

$$\begin{aligned}
\max_{q_{1H}^{II}(T,S),x_1^{II}(T,S)} \pi_{1H}^{II}(T,S) &= \left(a-b\left(q_{1H}^{II}(T,S)+x_2^{II}(T,S)\right)+V\right)q_{1H}^{II}(T,S) \\
&+ \left(a-b\left(q_2^{II}(T,S)+x_1^{II}(T,S)\right)\right)x_1^{II}(T,S) \\
&- c\left(q_{1H}^{II}(T,S)+x_1^{II}(T,S)\right), \qquad (18)
\end{aligned}$$

if $\varepsilon=V$. The ill-informed foreign firm solves the following maximization problem, given a common prior:

$$\begin{aligned}
\max_{q_2^{II}(T,S),x_2^{II}(T,S)} E\left(\pi_2^{II}(T,S)\right) &= \left(a-b\left(q_2^{II}(T,S)+x_1^{II}(T,S)\right)\right)q_2^{II}(T,S) \\
&+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a-b\left(q_1^{II}(T,S)+x_2^{II}(T,S)\right)+\varepsilon\right)x_2^{II}(T,S) \\
&- c\left(q_2^{II}(T,S)+x_2^{II}(T,S)\right) \\
&- t_1x_2^{II}(T,S)+s_2x_2^{II}(T,S), \qquad (19)
\end{aligned}$$

where $q_1^{II}(T,S)\in\left\{q_{1L}^{II}(T,S),q_{1H}^{II}(T,S)\right\}$ depend on the realization of ε, t_1 is the tariff imposed by government 1 and s_2 is the export subsidy provided by government 2.

The corresponding best response functions for the stage three game are

$$\begin{aligned}
BR_{1L} &= q_{1L}^{II}(T,S)\in\arg\max\pi_{1L}^{II}(T,S), \\
BR_{1H} &= q_{1H}^{II}(T,S)\in\arg\max\pi_{1H}^{II}(T,S), \\
BR_{1x} &= x_1^{II}(T,S)\in\arg\max\pi_{1L}^{II}(T,S), \\
BR_2 &= q_2^{II}(T,S)\in\arg\max\pi_2^{II}(T,S), \\
BR_{2x} &= x_2^{II}(T,S)\in\arg\max\pi_2^{II}(T,S).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium points for stage three game given any (t_1, s_2, V):

$$\begin{aligned}
q_{1L}^{II}(T,S) &= \frac{1}{6b}\left(2(a-c) - 3V - 2s_2 + 2t_1\right), \\
q_{1H}^{II}(T,S) &= \frac{1}{6b}\left(2(a-c) + 3V - 2s_2 + 2t_1\right), \\
x_1^{II}(T,S) &= \frac{1}{3b}(a-c), \\
q_2^{II}(T,S) &= \frac{1}{3b}(a-c), \\
x_2^{II}(T,S) &= \frac{1}{3b}(a-c+2s_2-2t_1).
\end{aligned}$$

In stage two, the domestic government chooses tariff rate and the foreign government chooses subsidy rate in order to maximize their expected social welfare given that both governments anticipate that firms behave strategically. The expected social welfare for the domestic country is the sum of producer's surplus, consumer's surplus, and tariff revenue, while the expected social welfare for the foreign country is the sum of producer's surplus and consumer's surplus net of total subsidy. Formally, we have

$$\begin{aligned}
\max_{t_1} E\left(SW_1^{II}(T,S)\right) &= PS_1^{II}(T,S) + CS_1^{II}(T,S) + TR_1^{II}(T,S) \\
&= \frac{1}{2}\left(\pi_{1L}^{II}(T,S) + \pi_{1H}^{II}(T,S)\right) \\
&+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{II}(T,S) + x_2^{II}(T,S)\right)^2 + \frac{b}{2}\left(q_{1H}^{II}(T,S) + x_2^{II}(T,S)\right)^2\right) \\
&+ t_1 x_2^{II}(T,S),
\end{aligned} \tag{20}$$

$$\begin{aligned}
\max_{s_2} E\left(SW_2^{II}(T,S)\right) &= PS_2^{II}(T,S) + CS_2^{II}(T,S) - S_2^{II}(T,S) \\
&= \pi_2^{II}(T,S) + \frac{b}{2}(q_2^{II}(T,S) + x_1^{II}(T,S))^2 - s_2 x_2^{II}(T,S).
\end{aligned} \tag{21}$$

The corresponding best response functions are

$$\begin{aligned}
BR_1(s_2) &= t_1 \in \arg\max E\left(SW_1^{II}(T,S)\right), \\
BR_2(t_1) &= s_2 \in \arg\max E\left(SW_2^{II}(T,S)\right).
\end{aligned}$$

Solving stage two game gives us the Bayesian-Nash equilibrium policy rates:

$$t_1^* = \frac{5}{14}(a-c), \tag{22}$$
$$s_2^* = \frac{1}{14}(a-c). \tag{23}$$

Note that these optimal policy rates are identical to the mixed game with complete information.[7] Given these, the expected output in stage three can be summarized as follows:

$$\begin{aligned}
E\left(q_{1L}^{II}(T,S)\right) &= \frac{3}{7b}(a-c) - \frac{1}{2b}V, \\
E\left(q_{1H}^{II}(T,S)\right) &= \frac{3}{7b}(a-c) + \frac{1}{2b}V, \\
E\left(x_1^{II}(T,S)\right) &= \frac{1}{3b}(a-c), \\
E\left(q_2^{II}(T,S)\right) &= \frac{1}{3b}(a-c), \\
E\left(x_2^{II}(T,S)\right) &= \frac{1}{7b}(a-c).
\end{aligned}$$

The corresponding expected social welfare functions are

$$E\left(SW_1^{II}(T,S)\right) = \frac{449}{882b}(a-c)^2 + \frac{3}{8b}\sigma^2, \tag{24}$$
$$E\left(SW_2^{II}(T,S)\right) = \frac{101}{294b}(a-c)^2. \tag{25}$$

Obviously, that the expected social welfare for the domestic country increases with market volatility.

3.4 Mixed Game (S,T)

In this case, the domestic government subsidizes firm 1's exports to the foreign market, while the foreign government imposes tariff on the imports from firm 1.

[7] Optimal rates for complete information can be found in Appendix C. The same reasoning in previous sections is also applied here.

As above, the problems for the domestic firm is

$$\begin{aligned}\max_{q_{1L}^{II}(S,T),x_1^{II}(S,T)} \pi_{1L}^{II}(S,T) &= \left(a - b\left(q_{1L}^{II}(S,T) + x_2^{II}(S,T)\right) - V\right) q_{1L}^{II}(S,T) \\ &+ \left(a - b\left(q_2^{II}(S,T) + x_1^{II}(S,T)\right)\right) x_1^{II}(S,T) \\ &- c\left(q_{1L}^{II}(S,T) + x_1^{II}(S,T)\right) \\ &- t_2 x_1^{II}(S,T) + s_1 x_1^{II}(S,T), \end{aligned} \tag{26}$$

if $\varepsilon = -V$ and

$$\begin{aligned}\max_{q_{1H}^{II}(S,T),x_1^{II}(S,T)} \pi_{1H}^{II}(S,T) &= \left(a - b\left(q_{1H}^{II}(S,T) + x_2^{II}(S,T)\right) + V\right) q_{1H}^{II}(S,T) \\ &+ \left(a - b\left(q_2^{II}(S,T) + x_1^{II}(S,T)\right)\right) x_1^{II}(S,T) \\ &- c\left(q_{1H}^{II}(S,T) + x_1^{II}(S,T)\right) \\ &- t_2 x_1^{II}(S,T) + s_1 x_1^{II}(S,T), \end{aligned}$$

if $\varepsilon = V$, where t_2 is the tariff imposed by the foreign government and s_1 is the export subsidy given by the domestic government.

The foreign firm solves the following maximization problem, given a common prior

$$\begin{aligned}\max_{q_2^{II}(S,T),x_2^{II}(S,T)} E\left(\pi_2^{II}(S,T)\right) &= \left(a - b\left(q_2^{II}(S,T) + x_1^{II}(S,T)\right)\right) q_2^{II}(S,T) \\ &+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a - b\left(q_1^{II}(S,T) + x_2^{II}(S,T)\right) + \varepsilon\right) x_2^{II}(S,T) \\ &- c\left(q_2^{II}(S,T) + x_2^{II}(S,T)\right), \end{aligned} \tag{27}$$

where $q_1^{II}(S,T) \in \left\{q_{1L}^{II}(S,T), q_{1H}^{II}(S,T)\right\}$ depend on the realization of ε.

The best response functions for stage three game are

$$\begin{aligned} BR_{1L} &= q_{1L}^{II}(S,T) \in \arg\max \pi_{1L}^{II}(S,T), \\ BR_{1H} &= q_{1H}^{II}(S,T) \in \arg\max \pi_{1H}^{II}(S,T), \\ BR_{1x} &= x_1^{II}(S,T) \in \arg\max \pi_{1L}^{II}(S,T), \\ BR_2 &= q_2^{II}(S,T) \in \arg\max \pi_2^{II}(S,T), \\ BR_{2x} &= x_2^{II}(S,T) \in \arg\max \pi_2^{II}(S,T). \end{aligned}$$

Solving yields the Bayesian-Nash equilibrium points for stage three game given any (s_1, t_2, V):

$$\begin{aligned}
q_{1L}^{II}(S,T) &= \frac{1}{6b}\left(2(a-c)-3V\right), \\
q_{1H}^{II}(S,T) &= \frac{1}{6b}\left(2(a-c)+3V\right), \\
x_{1}^{II}(S,T) &= \frac{1}{3b}\left(a-c+2s_1-2t_2\right), \\
q_{2}^{II}(S,T) &= \frac{1}{3b}\left(a-c-s_1+t_2\right), \\
x_{2}^{II}(S,T) &= \frac{1}{3b}\left(a-c\right).
\end{aligned}$$

In stage two, the domestic government chooses subsidy rate and the foreign government chooses tariff rate to maximize their expected social welfare:

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{II}(S,T)\right) &= PS_1^{II}(S,T) + CS_1^{II}(S,T) - S_1^{II}(S,T) \\
&= \frac{1}{2}\left(\pi_{1L}^{II}(S,T) + \pi_{1H}^{II}(S,T)\right) \\
&+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{II}(S,T) + x_2^{II}(S,T)\right)^2 + \frac{b}{2}\left(q_{1H}^{II}(S,T) + x_2^{II}(S,T)\right)^2\right) \\
&- s_1 x_1^{II}(S,T),
\end{aligned} \tag{28}$$

$$\begin{aligned}
\max_{t_2} E\left(SW_2^{II}(S,T)\right) &= PS_2^{II}(S,T) + CS_2^{II}(S,T) + TR_2^{II}(S,T) \\
&= \pi_2^{II}(S,T) + \frac{b}{2}(q_2^{II}(S,T) + x_1^{II}(S,T))^2 + t_2 x_1^{II}(S,T).
\end{aligned} \tag{29}$$

The corresponding best response functions for stage two game are

$$\begin{aligned}
BR_1(t_2) &= s_1 \in \arg\max E\left(SW_1^{II}(S,T)\right), \\
BR_2(s_1) &= t_2 \in \arg\max E\left(SW_2^{II}(S,T)\right).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium level of policy rates:

$$s_1^* = \frac{1}{14}(a-c), \tag{30}$$

$$t_2^* = \frac{5}{14}(a-c). \tag{31}$$

Given these optimal policy rates, we can then calculate the expected output decisions by firms in the last stage:

$$\begin{aligned}
E\left(q_{1L}^{II}(S,T)\right) &= \frac{1}{3b}(a-c) - \frac{1}{2b}V, \\
E\left(q_{1H}^{II}(S,T)\right) &= \frac{1}{3b}(a-c) + \frac{1}{2b}V, \\
E\left(x_{1}^{II}(S,T)\right) &= \frac{1}{7b}(a-c), \\
E\left(q_{2}^{II}(S,T)\right) &= \frac{3}{7b}(a-c), \\
E\left(x_{2}^{II}(S,T)\right) &= \frac{1}{3b}(a-c).
\end{aligned}$$

The corresponding expected social welfare functions are

$$E\left(SW_{1}^{II}(S,T)\right) = \frac{101}{294b}(a-c)^2 + \frac{3}{8b}\sigma^2, \tag{32}$$

$$E\left(SW_{2}^{II}(S,T)\right) = \frac{449}{882b}(a-c)^2. \tag{33}$$

One can easily verify that the expected social welfare for the domestic country increases with market volatility. This together with the findings for (T,S) yield the following proposition:

Proposition 3. *For mixed game (either (T,S) or (S,T)) with incomplete information,*

i. *the entire option value shifts to the domestic country due to incomplete information;*

ii. *social welfare for the domestic country increases as variance increases.*

4 Choice of Regimes

Given that we have derived sub-game equilibrium points for trade game with incomplete information and trade game with complete information (see Appendices A to D), we are now ready to investigate what governments would do in order maximize their social welfare in stage one. This can be modeled as two strategic games.

To this end, we construct two strategic games that involve choosing policy regimes by two governments simultaneously in the first stage. We denote G_1 and G_2 as the choice of policy games with expected payoffs derived under incomplete information and under complete information (see Appendices), respectively. For both games:

i. the set of players are government 1 and 2;

ii. the set of actions are $\{S, T\}$;

iii. the payoffs are in terms of expected social welfare;

iv. the preference relations of governments are over expected social welfare.

Table 1 and 2 illustrate the normal form representation of game G_1 and G_2, respectively. Government 1 is the row player and government 2 is the column player in both games.

Table 1: Normal form representation of game G_1

Government 1 \ Government 2	Subsidy	Tariff
Subsidy	$E\left(SW_1^{II}(S,S)\right)$, $E\left(SW_2^{II}(S,S)\right)$	$E\left(SW_1^{II}(S,T)\right)$, $E\left(SW_2^{II}(S,T)\right)$
Tariff	$E\left(SW_1^{II}(T,S)\right)$, $E\left(SW_2^{II}(T,S)\right)$	$E\left(SW_1^{II}(T,T)\right)$, $E\left(SW_2^{II}(T,T)\right)$

Table 2: Normal form representation of game G_2

Government 1 \ Government 2	Subsidy	Tariff
Subsidy	$E\left(SW_1^{CI}(S,S)\right)$, $E\left(SW_2^{CI}(S,S)\right)$	$E\left(SW_1^{CI}(S,T)\right)$, $E\left(SW_2^{CI}(S,T)\right)$
Tariff	$E\left(SW_1^{CI}(T,S)\right)$, $E\left(SW_2^{CI}(T,S)\right)$	$E\left(SW_1^{CI}(T,T)\right)$, $E\left(SW_2^{CI}(T,T)\right)$

We are now ready to conduct equilibrium analysis of these two games using conventional Nash solution concept. Our presentation here follows rationalizability concept proposed by Pearce (1984). Let us first consider the game G_1. Assuming that government 2 chooses subsidy to begin with, the best response for government 1 is to choose tariff since $E\left(SW_1^{II}(T,S)\right) - E\left(SW_1^{II}(S,S)\right) > 0$. On the other hand, given that government 2 chooses tariff, the best response for government 1 is to choose tariff as well since $E\left(SW_1^{II}(T,T)\right) - E\left(SW_1^{II}(S,T)\right) > 0$. Both best responses from government 1 is independent of market volatility, and government 1 strictly prefers to impose tariff on imports from firm 2 no matter which trade policy government 2 prescribed. Therefore, subsidy is strictly dominated by tariff from government 1's perspective. After strategy subsidy is eliminated by government 1, tariff is also a best response from government 2's point of view. As a result, (T, T) is the only pair of strategy that survives the iterated elimination of strictly dominant strategy. Hence (T, T) is the unique Nash equilibrium of G_1. This gives us

Proposition 4. *The unique Nash equilibrium of game G_1 is (T, T), regardless of market volatility.*

The similar argument and reasoning from rationalizability can also be applied to G_2, yielding

Proposition 5. *The unique Nash equilibrium of game G_2 is (T, T), regardless of market volatility.*

This is not surprising since game G_1 and G_2 are actually variants of classical *prisoner's dilemma* game. Although the pair of strategy (S, S) gives better payoffs to both governments than the payoffs from (T, T), (S, S) is never achievable as long as we do not allow the governments to cooperate. Our result is consistent with the conventional wisdom that a Cournot-Nash game played by governments and firms will lead them to choose a tariff in equilibrium with certainty.

It is worth noting that the unique Nash equilibrium is independent of variance. This is because the option value associated with better information is the same under all pairs of strategies chosen by governments. For example, the option value to the domestic country is $\frac{3}{8b}\sigma^2$ for all pair of strategies with incomplete information. Therefore, the market volatility plays no role in determining optimal strategy in choosing between subsidy and tariff. This results is in sharp contrast to Anam and Chiang (2018) in that the choice of trade policy game is shown to depend on the correlation between two markets. This is not the case in the current paper because we assume that two markets are stochastically independent. Moreover, our results are also different from Cooper and Riezman (1989) in that the choice is shown to depend on the degree of uncertainty. While their model assumes that the information about market demand is symmetrically revealed to both firms, we here focus on the effect of information asymmetry on the choice of trade policies.

Notice that although incomplete information for the foreign firm does not affect the optimal strategic choice between tariff and subsidy, it, nevertheless, redistributes the option value associated with better information. As shown in trade game with incomplete information, the well-informed domestic firm is able to capture entire option value, since it is able to make output decision after the resolution of uncertainty.

Because Nash equilibrium (T, T) implies a prisoner's dilemma outcome, we shall consider a special case which both governments agree on free-trade agreement. Then we compare these social welfare levels from free-trade to our Nash equilibrium output to determine which trade policy (tariff or free-trade agreement) is optimal.

5 Free-Trade with Incomplete Information

In this section, we show that no intervention is superior to the Bayesian-Nash equilibrium (T, T). To see this, first consider the domestic firm's maximization problem:

$$\max_{q_{1L}^{II}, x_1^{II}} \pi_{1L}^{II} = \left(a - b\left(q_{1L}^{II} + x_2^{II}\right) - V\right) q_{1L}^{II} + \left(a - b\left(q_2^{II} + x_1^{II}\right)\right) x_1^{II} - c\left(q_{1L}^{II} + x_1^{II}\right), \tag{34}$$

if $\varepsilon = -V$;

$$\max_{q_{1H}^{II}, x_1^{II}} \pi_{1H}^{II} = \left(a - b\left(q_{1H}^{II} + x_2^{II}\right) + V\right) q_{1H}^{II} + \left(a - b\left(q_2^{II} + x_1^{II}\right)\right) x_1^{II} - c\left(q_{1H}^{II} + x_1^{II}\right), \tag{35}$$

if $\varepsilon = V$. The foreign firm, being ill-informed, solves the following maximization problem given common prior:

$$\max_{q_2^{II}, x_2^{II}} E\left(\pi_2^{II}\right) = \left(a - b\left(q_2^{II} + x_1^{II}\right)\right) q_2^{II} + \frac{1}{2}\sum_{\varepsilon \in \Omega}\left(a - b\left(q_1^{II} + x_2^{II}\right) + \varepsilon\right) x_2^{II} - c\left(q_2^{II} + x_2^{II}\right), \tag{36}$$

where $q_1^{II} \in \left\{q_{1L}^{II}, q_{1H}^{II}\right\}$ depend on the realization of ε.

The best response functions for above problems are

$$\begin{aligned}
BR_{1L} &= q_{1L}^{II} \in \arg\max \pi_{1L}^{II}, \\
BR_{1H} &= q_{1H}^{II} \in \arg\max \pi_{1H}^{II}, \\
BR_{1x} &= x_1^{II} \in \arg\max \pi_{1L}^{II}, \\
BR_2 &= q_2^{II} \in \arg\max \pi_2^{II}, \\
BR_{2x} &= x_2^{II} \in \arg\max \pi_2^{II}.
\end{aligned}$$

Solving yields

$$
\begin{aligned}
q_{1L}^{II} &= \frac{1}{3b}(a-c) - \frac{1}{2b}V, \\
q_{1H}^{II} &= \frac{1}{3b}(a-c) + \frac{1}{2b}V, \\
x_{1}^{II} &= \frac{1}{3b}(a-c), \\
q_{2}^{II} &= \frac{1}{3b}(a-c), \\
x_{2}^{II} &= \frac{1}{3b}(a-c).
\end{aligned}
$$

By substitution, one obtains the expected social welfare functions for home and foreign countries, respectively.

$$
\begin{aligned}
E\left(SW_1^{II}\right) &= PS_1^{II} + CS_1^{II} \\
&= \frac{1}{2}\left(\pi_{1L}^{II} + \pi_{1H}^{II}\right) + \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{II} + x_2^{II}\right)^2 + \frac{b}{2}\left(q_{1H}^{CI} + x_2^{II}\right)^2\right) \\
&= \frac{4}{9b}(a-c)^2 + \frac{3}{8b}\sigma^2,
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
E\left(SW_2^{II}\right) &= PS_2^{II} + CS_2^{II} \\
&= \pi_2^{II} + \frac{b}{2}(q_2^{II} + x_1^{II})^2 \\
&= \frac{4}{9b}(a-c)^2.
\end{aligned}
\tag{38}
$$

It follows that

$$
\begin{aligned}
E\left(SW_1^{II}\right) - E\left(SW_1^{II}(T,T)\right) &> 0, \\
E\left(SW_2^{II}\right) - E\left(SW_2^{II}(T,T)\right) &> 0.
\end{aligned}
$$

The result is summarized in

Proposition 6. *With incomplete information at industrial level, Nash equilibrium outcome for game G_1, (T,T), is inferior to free-trade outcome for both countries.*

6 Conclusion

The main focus of this paper is to reexamine the strategic trade policy in the context of reciprocal markets under incomplete information, an issue that has been ignored in the literature. Several departures from conventional strategic trade policy wisdom are observed. First, an export subsidy is superior to a tariff if one country is active and can unilaterally choose a trade policy against its rival. But interestingly, tariffs turn out to be a non-cooperative equilibrium outcome when both governments are active and set trade policies against each other. The result holds regardless of market volatility and information partition. Incomplete information for the foreign firm redistributes the option value, associated with ability to make decision after the resolution of uncertainty, to the country with more information. Second, we show that the Nash equilibrium outcome under incomplete information (i.e., (T, T)) is inferior to the equilibrium outcome absent of any form of government intervention (i.e., *Laissez-faire*).

The paper adopts some simplified assumptions to highlight our assertions. These include a trade game being simultaneous-move, policy modes being restricted to tariffs and export subsidies, marginal costs being constant, and the random disturbance being additive in form. A natural extension of current model would be to expand policy set to include quota as well as a sequential game structure. Therefore, the countervailing policy model can be revisited to take into account the possibility that one country proactively intervenes by adopting a strategic trade policy while the other responds optimally by retaliating with appropriate instruments in response to the leader's choice. The equilibrium choice of the leader government would then be determined by backward induction as usual. It is possible that equilibrium response to a subsidy may be something other than a countervailing duty. But we leave it for future research.

Appendices

In this Appendix, we derive the equilibrium outputs, subsidies/tariffs, and policy regimes in stage three for trade game with complete information under various trade policies. As in the main text, the actions taken by various decision makers are summarized in the following figure:

Figure 2: Timing of Three-Stage Trade Game with Complete Information

Stage 1		Stage 2		Beginning of Stage 3		Stage 3
Both governments select policy modes	$\rightarrow$	Both governments select policy levels	$\rightarrow$	Both firms observe ε	$\rightarrow$	Firms play Cournot game

Four policy combinations, (S,S), (T,T), (T,S) and (S,T), under complete information are examined in sequence.

A Subsidy Game (S,S)

We first consider the game, where export subsidy is committed to by both governments, as the policy instrument in stage one. Under complete information assumption, the solution through backward induction starts in stage three when the random variable ε is known to both domestic and foreign firms.

Starting from the last stage, if the true market condition is $-V$ in country 1, each firm's problem is

$$\begin{aligned}\max_{q_{1L}^{CI}(S,S),x_{1}^{CI}(S,S)} \pi_{1L}^{CI}(S,S) &= \left(a-b\left(q_{1L}^{CI}(S,S)+x_{2L}^{CI}(S,S)\right)-V\right)q_{1L}^{CI}(S,S)\\ &+ \left(a-b\left(q_{1}^{CI}(S,S)+x_{1}^{CI}(S,S)\right)\right)x_{1}^{CI}(S,S)\\ &- c\left(q_{1L}^{CI}(S,S)+x_{1}^{CI}(S,S)\right)+s_1x_{1}^{CI}(S,S), \end{aligned} \tag{39}$$

$$\begin{aligned}\max_{q_{2}^{CI}(S,S),x_{2L}^{CI}(S,S)} \pi_{2L}^{CI}(S,S) &= \left(a-b\left(q_{2}^{CI}(S,S)+x_{1}^{CI}(S,S)\right)\right)q_{2}^{CI}(S,S)\\ &+ \left(a-b\left(q_{1L}^{CI}(S,S)+x_{2L}^{CI}(S,S)\right)-V\right)x_{2L}^{CI}(S,S)\\ &- c\left(q_{2}^{CI}(S,S)+x_{2L}^{CI}(S,S)\right)+s_2x_{2L}^{CI}(S,S). \end{aligned} \tag{40}$$

Similarly, if the true market demand is high in country 1, each firm's problem can be written as

$$\begin{aligned}\max_{q_{1H}^{CI}(S,S),x_1^{CI}(S,S)} \pi_{1H}^{CI}(S,S) &= \left(a-b\left(q_{1H}^{CI}(S,S)+x_{2H}^{CI}(S,S)\right)+V\right)q_{1H}^{CI}(S,S)\\ &+ \left(a-b\left(q_2^{CI}(S,S)+x_1^{CI}(S,S)\right)\right)x_1^{CI}(S,S)\\ &- c\left(q_{1H}^{CI}(S,S)+x_1^{CI}(S,S)\right)+s_1x_1^{CI}(S,S),\end{aligned} \tag{41}$$

$$\begin{aligned}\max_{q_2^{CI}(S,S),x_{2H}^{CI}(S,S)} \pi_{2H}^{CI}(S,S) &= \left(a-b\left(q_2^{CI}(S,S)+x_1^{CI}(S,S)\right)\right)q_2^{CI}(S,S)\\ &+ \left(a-b\left(q_{1H}^{CI}(S,S)+x_{2H}^{CI}(S,S)\right)+V\right)x_{2H}^{CI}(S,S)\\ &- c\left(q_2^{CI}(S,S)+x_{2H}^{CI}(S,S)\right)+s_2x_{2H}^{CI}(S,S),\end{aligned} \tag{42}$$

where s_i is the subsidy per unit of export for $i = 1, 2$, and the subscript L and H stands for the variable choice for each firm in the respective states[8]. The best response functions for the above maximization problems are

$$\begin{aligned}BR_{1L} &= q_{1L}^{CI}(S,S)\in\arg\max\pi_{1L}^{CI}(S,S),\\ BR_{1H} &= q_{1H}^{CI}(S,S)\in\arg\max\pi_{1H}^{CI}(S,S),\\ BR_{1x} &= x_1^{CI}(S,S)\in\arg\max\pi_{1L}^{CI}(S,S),\\ BR_2 &= q_2^{CI}(S,S)\in\arg\max\pi_{2L}^{CI}(S,S),\\ BR_{2xL} &= x_{2L}^{CI}(S,S)\in\arg\max\pi_{2L}^{CI}(S,S),\\ BR_{2xH} &= x_{2H}^{CI}(S,S)\in\arg\max\pi_{2H}^{CI}(S,S).\end{aligned}$$

Given these best response functions, we can obtain the Nash equilibrium points in each state for

[8]Henceforth, we use subscript L to represent variable choice when the random variable is $-V$, and we use subscript H to represent variable choice when the random variable is V.

any given level of s_1 and s_2 as follows:

$$\begin{aligned}
q_{1L}^{CI}(S,S) &= \frac{1}{3b}(a-c-V-s_2), \\
q_{1H}^{CI}(S,S) &= \frac{1}{3b}(a-c+V-s_2), \\
x_{1}^{CI}(S,S) &= \frac{1}{3b}(a-c+2s_1), \\
q_{2}^{CI}(S,S) &= \frac{1}{3b}(a-c-s_1), \\
x_{2L}^{CI}(S,S) &= \frac{1}{3b}(a-c-V+2s_2), \\
x_{2H}^{CI}(S,S) &= \frac{1}{3b}(a-c+V+2s_2).
\end{aligned}$$

The solutions characterize the Nash equilibrium in each market given (s_1, s_2, V).

Folding back to the second stage, both governments set subsidy rates to maximize respective social welfare given that they anticipate the Nash equilibrium output levels in stage three across different states of nature. The social welfare functions for both governments under export subsidy are specified as the sum of producer's surplus and consumer's surplus net of total subsidy. Hence, the problem for each government can be written as

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{CI}(S,S)\right) &= PS_1^{CI}(S,S) + CS_1^{CI}(S,S) - S_1^{CI}(S,S) \\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(S,S) + \pi_{1H}^{CI}(S,S)\right) \\
&+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{CI}(S,S) + x_{2L}^{CI}(S,S)\right)^2 + \frac{b}{2}\left(q_{1H}^{CI}(S,S) + x_{2H}^{CI}(S,S)\right)^2\right) \\
&- s_1 x_1^{CI}(S,S),
\end{aligned} \tag{43}$$

$$\begin{aligned}
\max_{s_2} E\left(SW_2^{CI}(S,S)\right) &= PS_2^{CI}(S,S) + CS_2^{CI}(S,S) - S_2^{CI}(S,S) \\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(S,S) + \pi_{2H}^{CI}(S,S)\right) \\
&+ \frac{b}{2}(q_2^{CI}(S,S) + x_1^{CI}(S,S))^2 \\
&- \frac{1}{2}\left(x_{2L}^{CI}(S,S) + x_{2H}^{CI}(S,S)\right) s_2.
\end{aligned} \tag{44}$$

The best response functions of stage two game are

$$\begin{aligned} BR_1(s_2) &= s_1 \in \arg\max E\left(SW_1^{CI}(S,S)\right), \\ BR_2(s_1) &= s_2 \in \arg\max E\left(SW_2^{CI}(S,S)\right). \end{aligned}$$

Solving yields Bayesian Nash equilibrium level of export subsidies

$$s_1^* = s_2^* = \frac{1}{4}(a-c). \tag{45}$$

By Substitution, one can obtain the expected output levels for each firm in stage three as follows:

$$\begin{aligned} E\left(q_{1L}^{CI}(S,S)\right) &= \frac{1}{4b}(a-c) - \frac{1}{3b}V, \\ E\left(q_{1H}^{CI}(S,S)\right) &= \frac{1}{4b}(a-c) + \frac{1}{3b}V, \\ E\left(x_1^{CI}(S,S)\right) &= \frac{1}{2b}(a-c), \\ E\left(q_2^{CI}(S,S)\right) &= \frac{1}{4b}(a-c), \\ E\left(x_{2L}^{CI}(S,S)\right) &= \frac{1}{2b}(a-c) - \frac{1}{3b}V, \\ E\left(x_{2H}^{CI}(S,S)\right) &= \frac{1}{2b}(a-c) + \frac{1}{3b}V. \end{aligned}$$

Given equation (45), the expected social welfare for both countries are

$$E\left(SW_1^{CI}(S,S)\right) = \frac{15}{32b}(a-c)^2 + \frac{1}{3b}\sigma^2, \tag{46}$$

$$E\left(SW_2^{CI}(S,S)\right) = \frac{15}{32b}(a-c)^2 + \frac{1}{9b}\sigma^2. \tag{47}$$

It is straightforward to show that $\frac{\partial E\left(SW_i^{CI}(S,S)\right)}{\partial \sigma^2} > 0$ for $i = 1,2$. That is, the expected social welfare increases with market volatility. Notice that the second term in expected social welfare functions is the option value associated with better information. Apparently, $E\left(SW_1^{CI}(S,S)\right) - E\left(SW_2^{CI}(S,S)\right) > 0$, indicating that the domestic country enjoys higher option values than the foreign country.

Next, we turn to examine the sub game equilibrium under tariff game.

B Tariff Game (T,T)

We now turn to the tariff game between the two countries. Similar to the equilibrium analysis in the previous section, we start from the last stage in which both firms observe the true market condition in country 1.

If the true market demand in country 1 is low, we can write firms' problems as

$$\begin{aligned}
\max_{q_{1L}^{CI}(T,T),x_1^{CI}(T,T)} \pi_{1L}^{CI}(T,T) &= \left(a - b\left(q_{1L}^{CI}(T,T) + x_{2L}^{CI}(T,T)\right) - V\right) q_{1L}^{CI}(T,T) \\
&+ \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) x_1^{CI}(T,T) \\
&- c\left(q_{1L}^{CI}(T,T) + x_1^{CI}(T,T)\right) - t_2 x_1^{CI}(T,T), \qquad (48)
\end{aligned}$$

$$\begin{aligned}
\max_{q_2^{CI}(T,T),x_{2L}^{CI}(T,T)} \pi_{2L}^{CI}(T,T) &= \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) q_2^{CI}(T,T) \\
&+ \left(a - b\left(q_{1L}^{CI}(T,T) + x_{2L}^{CI}(T,T)\right) - V\right) x_{2L}^{CI}(T,T) \\
&- c\left(q_2^{CI}(T,T) + x_{2L}^{CI}(T,T)\right) - t_1 x_{2L}^{CI}(T,T). \qquad (49)
\end{aligned}$$

On the other hand, if the true market demand in country 1 is high, the profit functions for both firms are

$$\begin{aligned}
\max_{q_{1H}^{CI}(T,T),x_1^{CI}(T,T)} \pi_{1H}^{CI}(T,T) &= \left(a - b\left(q_{1H}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right) + V\right) q_{1H}^{CI}(T,T) \\
&+ \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) x_1^{CI}(T,T) \\
&- c\left(q_{1H}^{CI}(T,T) + x_1^{CI}(T,T)\right) - t_2 x_1^{CI}(T,T), \qquad (50)
\end{aligned}$$

$$\begin{aligned}
\max_{q_2^{CI}(T,T),x_{2H}^{CI}(T,T)} \pi_{2H}^{CI}(T,T) &= \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) q_2^{CI}(T,T) \\
&+ \left(a - b\left(q_{1H}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right) + V\right) x_{2H}^{CI}(T,T) \\
&- c\left(q_2^{CI}(T,T) + x_{2H}^{CI}(T,T)\right) - t_1 x_{2H}^{CI}(T,T), \qquad (51)
\end{aligned}$$

where t_i is the tariff imposed by the home government on foreign imports for $i = 1, 2$. The best response functions for above problems are

$$
\begin{aligned}
BR_{1L} &= q_{1L}^{CI}(T,T) \in \arg\max \pi_{1L}^{CI}(T,T), \\
BR_{1H} &= q_{1H}^{CI}(T,T) \in \arg\max \pi_{1H}^{CI}(T,T), \\
BR_{1x} &= x_{1}^{CI}(T,T) \in \arg\max \pi_{1L}^{CI}(T,T), \\
BR_{2} &= q_{2}^{CI}(T,T) \in \arg\max \pi_{2L}^{CI}(T,T), \\
BR_{2xL} &= x_{2L}^{CI}(T,T) \in \arg\max \pi_{2L}^{CI}(T,T), \\
BR_{2xH} &= x_{2H}^{CI}(T,T) \in \arg\max \pi_{2H}^{CI}(T,T).
\end{aligned}
$$

Given these best response functions, the Nash equilibrium output levels given any (t_1, t_2, V) for stage three game are

$$
\begin{aligned}
q_{1L}^{CI}(T,T) &= \frac{1}{3b}(a-c-V+t_1), \\
q_{1H}^{CI}(T,T) &= \frac{1}{3b}(a-c+V+t_1), \\
x_{1}^{CI}(T,T) &= \frac{1}{3b}(a-c-2t_2), \\
q_{2}^{CI}(T,T) &= \frac{1}{3b}(a-c+t_2), \\
x_{2L}^{CI}(T,T) &= \frac{1}{3b}(a-c-V-2t_1), \\
x_{2H}^{CI}(T,T) &= \frac{1}{3b}(a-c+V-2t_1).
\end{aligned}
$$

Moving backward to the second stage, each government selects the tariff rate to maximize its social welfare given that each government anticipates Nash equilibrium outputs from both firms in stage three. The maximization problem facing each government under tariffs is the sum of producer's surplus, consumer's surplus and tariff revenue, given by

$$
\begin{aligned}
\max_{t_1} E\left(SW_1^{CI}(T,T)\right) &= PS_1^{CI}(T,T) + CS_1^{CI}(T,T) + TR_1^{CI}(T,T) \\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(T,T) + \pi_{1H}^{CI}(T,T)\right) \\
&+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{CI}(T,T) + x_{2L}^{CI}(T,T)\right)^2 + \frac{b}{2}\left(q_{1H}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right)^2\right) \\
&+ \frac{1}{2}t_1\left(x_{2L}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right), \qquad (52)
\end{aligned}
$$

$$\begin{aligned}
\max_{t_2} E\left(SW_2^{CI}(T,T)\right) &= PS_2^{CI}(T,T) + CS_2^{CI}(T,T) - TR_2^{CI}(T,T) \\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(T,T) + \pi_{2H}^{CI}(T,T)\right) \\
&+ \frac{b}{2}(q_2^{CI}(T,T) + x_1^{CI}(T,T))^2 \\
&+ t_2 x_1^{CI}(T,T). \qquad (53)
\end{aligned}$$

The best response functions of stage two game are

$$\begin{aligned}
BR_1(t_2) &= t_1 \in \arg\max E\left(SW_1^{CI}(T,T)\right), \\
BR_2(t_1) &= t_2 \in \arg\max E\left(SW_2^{CI}(T,T)\right).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium level of tariff

$$t_1^* = t_2^* = \frac{1}{3}(a-c). \qquad (54)$$

It is worth noting that both governments impose same amount of tariff rate since there is no information advantage of one country over another country at national level. Substituting the optimal tariff rate in equation (54) into best response functions of both firms, we can get the expected output in stage three as

$$\begin{aligned}
E\left(q_{1L}^{CI}(T,T)\right) &= \frac{4}{9b}(a-c) - \frac{1}{3b}V, \\
E\left(q_{1H}^{CI}(T,T)\right) &= \frac{4}{9b}(a-c) + \frac{1}{3b}V, \\
E\left(x_1^{CI}(T,T)\right) &= \frac{1}{9b}(a-c), \\
E\left(q_2^{CI}(T,T)\right) &= \frac{4}{9b}(a-c), \\
E\left(x_{2L}^{CI}(T,T)\right) &= \frac{1}{9b}(a-c) - \frac{1}{3b}V, \\
E\left(x_{2H}^{CI}(T,T)\right) &= \frac{1}{9b}(a-c) + \frac{1}{3b}V.
\end{aligned}$$

Given these, the expected social welfare for both countries can then be written as

$$\begin{aligned}
E\left(SW_1^{CI}(T,T)\right) &= \frac{65}{162b}(a-c)^2 + \frac{1}{3b}\sigma^2, \qquad (55) \\
E\left(SW_2^{CI}(T,T)\right) &= \frac{65}{162b}(a-c)^2 + \frac{1}{9b}\sigma^2. \qquad (56)
\end{aligned}$$

Hence, $\frac{\partial E\left(SW_i^{CI}(T,T)\right)}{\partial\sigma^2} > 0$ for $i = 1, 2$. This implies that higher market volatility, both at home and abroad, enhances the expected social welfare. The stochastic term represents the gain in social welfare associated with the option value accruing to firms from being able to wait for the resolution of uncertainty. It is also worth noting that the domestic country ends up with higher expected social welfare due to higher option value effects.

Next, we turn to examine the mixed game, either (T, S) or (S, T), under complete information in the next two sub-sections.

C Mixed Game (T,S)

We now turn to a scenario where country 1 imposes a tariff on firm 2's export, while country 2 subsidizes its exports. As above, the three-stage game is solved by the backward induction, beginning with the last stage.

In stage three, both firms observe the random variable ε before making their output decisions. If $\varepsilon = -V$, their problems can be written as

$$\begin{aligned}\max_{q_{1L}^{CI}(T,S),x_1^{CI}(T,S)} \pi_{1L}^{CI}(T,S) &= \left(a - b\left(q_{1L}^{CI}(T,S) + x_{2L}^{CI}(T,S)\right) - V\right) q_{1L}^{CI}(T,S) \\ &+ \left(a - b\left(q_2^{CI}(T,S) + x_1^{CI}(T,S)\right)\right) x_1^{CI}(T,S) \\ &- c\left(q_{1L}^{CI}(T,S) + x_1^{CI}(T,S)\right), \end{aligned} \tag{57}$$

$$\begin{aligned}\max_{q_2^{CI}(T,S),x_{2L}^{CI}(T,S)} \pi_{2L}^{CI}(T,S) &= \left(a - b\left(q_2^{CI}(T,S) + x_1^{CI}(T,S)\right)\right) q_2^{CI}(T,S) \\ &+ \left(a - b\left(q_{1L}^{CI}(T,S) + x_{2L}^{CI}(T,S)\right) - V\right) x_{2L}^{CI}(T,S) \\ &- c\left(q_2^{CI}(T,S) + x_{2L}^{CI}(T,S)\right) \\ &+ s_2 x_{2L}^{CI}(T,S) - t_1 x_{2L}^{CI}(T,S). \end{aligned} \tag{58}$$

Conversely, if $\varepsilon = V$, firms make

$$\begin{aligned}\max_{q_{1H}^{CI}(T,S),x_1^{CI}(T,S)} \pi_{1H}^{CI}(T,S) &= \left(a - b\left(q_{1H}^{CI}(T,S) + x_{2H}^{CI}(T,S)\right) + V\right) q_{1H}^{CI}(T,S) \\ &+ \left(a - b\left(q_2^{CI}(T,S) + x_1^{CI}(T,S)\right)\right) x_1^{CI}(T,S) \\ &- c\left(q_{1H}^{CI}(T,S) + x_1^{CI}(T,S)\right), \end{aligned} \tag{59}$$

$$\begin{aligned}
\max_{q_2^{CI}(T,S),x_{2H}^{CI}(T,S)} \pi_{2H}^{CI}(T,S) &= \left(a-b\left(q_2^{CI}(T,S)+x_1^{CI}(T,S)\right)\right)q_2^{CI}(T,S) \\
&+ \left(a-b\left(q_{1H}^{CI}(T,S)+x_{2H}^{CI}(T,S)\right)+V\right)x_{2H}^{CI}(T,S) \\
&- c\left(q_2^{CI}(T,S)+x_{2H}^{CI}(T,S)\right) \\
&+ s_2 x_{2H}^{CI}(T,S) - t_1 x_{2H}^{CI}(T,S), \qquad (60)
\end{aligned}$$

where t_1 is the tariff imposed by the domestic government on the imports from the foreign firm, and s_2 is the export subsidy granted by the foreign government to its own firm's export to market 1. The corresponding best response functions for the above maximization problems are

$$\begin{aligned}
BR_{1L} &= q_{1L}^{CI}(T,S) \in \arg\max \pi_{1L}^{CI}(T,S), \\
BR_{1H} &= q_{1H}^{CI}(T,S) \in \arg\max \pi_{1H}^{CI}(T,S), \\
BR_{1x} &= x_1^{CI}(T,S) \in \arg\max \pi_{1L}^{CI}(T,S), \\
BR_{2} &= q_2^{CI}(T,S) \in \arg\max \pi_{2L}^{CI}(T,S), \\
BR_{2xL} &= x_{2L}^{CI}(T,S) \in \arg\max \pi_{2L}^{CI}(T,S), \\
BR_{2xH} &= x_{2H}^{CI}(T,S) \in \arg\max \pi_{2H}^{CI}(T,S).
\end{aligned}$$

Given these best response functions, the Nash equilibrium outputs given (t_1, s_2, V) for stage three game are

$$\begin{aligned}
q_{1L}^{CI}(T,S) &= \frac{1}{3b}(a-c-V-s_2+t_1), \\
q_{1H}^{CI}(T,S) &= \frac{1}{3b}(a-c+V-s_2+t_1), \\
x_1^{CI}(T,S) &= \frac{1}{3b}(a-c), \\
q_2^{CI}(T,S) &= \frac{1}{3b}(a-c), \\
x_{2L}^{CI}(T,S) &= \frac{1}{3b}(a-c-V+2s_2-2t_1), \\
x_{2H}^{CI}(T,S) &= \frac{1}{3b}(a-c+V+2s_2-2t_1).
\end{aligned}$$

In stage two, the expected social welfare for the domestic country is the sum of producer's surplus, consumer's surplus and tariff revenue, while the expected social welfare for the foreign country is the sum of producer's surplus and consumer's surplus net of total subsidy. Government 1 chooses t_1 and government 2 chooses s_2 to maximize their expected social welfare:

$$\begin{aligned}
\max_{t_1} E\left(SW_2^{CI}(T,S)\right) &= PS_1^{CI}(T,S) + CS_1^{CI}(T,S) + TR_1^{CI}(T,S) \\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(T,S) + \pi_{1H}^{CI}(T,S)\right) \\
&+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{CI}(T,S) + x_{2L}^{CI}(T,S)\right)^2 + \frac{b}{2}\left(q_{1H}^{CI}(T,S) + x_{2H}^{CI}(T,S)\right)^2\right) \\
&+ \frac{1}{2}t_1\left(x_{2L}^{CI}(T,S) + x_{2H}^{CI}(T,S)\right), \qquad (61)
\end{aligned}$$

$$\begin{aligned}
\max_{s_2} E\left(SW_2^{CI}(T,S)\right) &= PS_2^{CI}(T,S) + CS_2^{CI}(T,S) - S_2^{CI}(T,S) \\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(T,S) + \pi_{2H}^{CI}(T,S)\right) \\
&+ \frac{b}{2}(q_2^{CI}(T,S) + x_2^{CI}(T,S))^2 \\
&- \frac{1}{2}\left(x_{2L}^{CI}(T,S) + x_{2H}^{CI}(T,S)\right)s_2. \qquad (62)
\end{aligned}$$

The best response functions of stage two game are

$$\begin{aligned}
BR_1(s_2) &= t_1 \in \arg\max E\left(SW_1^{CI}(T,S)\right), \\
BR_2(t_1) &= s_2 \in \arg\max E\left(SW_1^{CI}(T,S)\right).
\end{aligned}$$

Solving yields

$$t_1^* = \frac{5}{14}(a-c), \qquad (63)$$

$$s_2^* = \frac{1}{14}(a-c). \qquad (64)$$

This implies that the optimal tariff imposed by government 1 and the optimal export subsidy set by government 2 are positive. Moreover, the tariff imposed by government 1 is higher than the export subsidy granted by government 2. With these optimal policy levels, we can obtain the expected Nash equilibrium outputs for both firms:

$$\begin{aligned}
E\left(q_{1L}^{CI}(T,S)\right) &= \frac{3}{7b}(a-c) - \frac{1}{3b}V, \\
E\left(q_{1H}^{CI}(T,S)\right) &= \frac{3}{7b}(a-c) + \frac{1}{3b}V, \\
E\left(x_{1}^{CI}(T,S)\right) &= \frac{1}{3b}(a-c), \\
E\left(q_{2}^{CI}(T,S)\right) &= \frac{1}{3b}(a-c), \\
E\left(x_{2L}^{CI}(T,S)\right) &= \frac{1}{7b}(a-c) - \frac{1}{3b}V, \\
E\left(x_{2H}^{CI}(T,S)\right) &= \frac{1}{7b}(a-c) + \frac{1}{3b}V.
\end{aligned}$$

Moreover, the expected social welfares for home and foreign countries are given by

$$E\left(SW_{1}^{CI}(T,S)\right) = \frac{449}{882b}(a-c)^{2} + \frac{1}{3b}\sigma^{2}, \tag{65}$$

$$E\left(SW_{2}^{CI}(T,S)\right) = \frac{101}{294b}(a-c)^{2} + \frac{1}{9b}\sigma^{2}. \tag{66}$$

It is straightforward to verify that $\frac{\partial E\left(SW_{i}^{CI}(T,S)\right)}{\partial \sigma^{2}} > 0$ for $i = 1, 2$. That is, social welfare under mixed policies (T, S) increases with market volatility.

D Mixed Game (S,T)

Finally, we examine the last possible pair of strategies chosen by the governments in stage one under complete information. That is, country 1 subsidizes its exports to market 2, while country 2 imposes a tariff on imported goods from firm 1.

If the true state in market 1 is $-V$, the profit functions for each firm are

$$\begin{aligned}
\max_{q_{1L}^{CI}(S,T),\, x_{1}^{CI}(S,T)} \pi_{1L}^{CI}(S,T) &= \left(a - b\left(q_{1L}^{CI}(S,T) + x_{2L}^{CI}(S,T)\right) - V\right) q_{1L}^{CI}(S,T) \\
&+ \left(a - b\left(q_{2}^{CI}(S,T) + x_{1}^{CI}(S,T)\right)\right) x_{1}^{CI}(S,T) \\
&- c\left(q_{1L}^{CI}(S,T) + x_{1}^{CI}(S,T)\right) \\
&+ s_{1} x_{1}^{CI}(S,T) - t_{2} x_{1}^{CI}(S,T), \qquad (67)
\end{aligned}$$

$$\begin{aligned}\max_{q_2^{CI}(S,T),x_{2L}^{CI}(S,T)} \pi_{2L}^{CI}(S,T) &= \left(a-b\left(q_2^{CI}(S,T)+x_1^{CI}(S,T)\right)\right)q_2^{CI}(S,T)\\ &+ \left(a-b\left(q_{1L}^{CI}(S,T)+x_{2L}^{CI}(S,T)\right)-V\right)x_{2L}^{CI}(S,T)\\ &- c\left(q_2^{CI}(S,T)+x_{2L}^{CI}(S,T)\right). \qquad (68)\end{aligned}$$

On the other hand, if the true state in market 1 is V, the profit functions are

$$\begin{aligned}\max_{q_{1H}^{CI}(S,T),x_1^{CI}(S,T)} \pi_{1H}^{CI}(S,T) &= \left(a-b\left(q_{1H}^{CI}(S,T)+x_{2H}^{CI}(S,T)\right)+V\right)q_{1H}^{CI}(S,T)\\ &+ \left(a-b\left(q_2^{CI}(S,T)+x_1^{CI}(S,T)\right)\right)x_1^{CI}(S,T)\\ &- c\left(q_{1H}^{CI}(S,T)+x_1^{CI}(S,T)\right)\\ &+ s_1x_1^{CI}(S,T)-t_2x_1^{CI}(S,T), \qquad (69)\end{aligned}$$

$$\begin{aligned}\max_{q_2^{CI}(S,T),x_{2H}^{CI}(S,T)} \pi_{2H}^{CI}(S,T) &= \left(a-b\left(q_2^{CI}(S,T)+x_1^{CI}(S,T)\right)\right)q_2^{CI}(S,T)\\ &+ \left(a-b\left(q_{1H}^{CI}(S,T)+x_{2H}^{CI}(S,T)\right)+V\right)x_{2H}^{CI}(S,T)\\ &- c\left(q_2^{CI}(S,T)+x_{2H}^{CI}(S,T)\right), \qquad (70)\end{aligned}$$

where s_1 is the export subsidy granted by government 1 on firm 1's export to market 2, and t_2 is the tariff rate imposed by government 2 on firm 1's export to market 2. The best response functions for above optimization problems are

$$\begin{aligned}BR_{1L} &= q_{1L}^{CI}(S,T)\in\arg\max\pi_{1L}^{CI}(S,T),\\ BR_{1H} &= q_{1H}^{CI}(S,T)\in\arg\max\pi_{1H}^{CI}(S,T),\\ BR_{1x} &= x_1^{CI}(S,T)\in\arg\max\pi_{1L}^{CI}(S,T),\\ BR_2 &= q_2^{CI}(S,T)\in\arg\max\pi_{2L}^{CI}(S,T),\\ BR_{2xL} &= x_{2L}^{CI}(S,T)\in\arg\max\pi_{2L}^{CI}(S,T),\\ BR_{2xH} &= x_{2H}^{CI}(S,T)\in\arg\max\pi_{2H}^{CI}(S,T).\end{aligned}$$

This gives

$$\begin{aligned}
q_{1L}^{CI}(S,T) &= \frac{1}{3b}(a-c-V), \\
q_{1H}^{CI}(S,T) &= \frac{1}{3b}(a-c+V), \\
x_{1}^{CI}(S,T) &= \frac{1}{3b}(a-c+2s_1-2t_2), \\
q_{2}^{CI}(S,T) &= \frac{1}{3b}(a-c-s_1+t_2), \\
x_{2L}^{CI}(S,T) &= \frac{1}{3b}(a-c-V), \\
x_{2H}^{CI}(S,T) &= \frac{1}{3b}(a-c+V).
\end{aligned}$$

These are Nash equilibrium outputs across states.

In stage 2, government 1 chooses s_1 and government 2 chooses t_2 to maximize their expected social welfare given by

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{CI}(S,T)\right) &= PS_1^{CI}(S,T) + CS_1^{CI}(S,T) - S_1^{CI}(S,T) \\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(S,T) + \pi_{1H}^{CI}(S,T)\right) \\
&+ \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{CI}(S,T) + x_{2L}^{CI}(S,T)\right)^2 + \frac{b}{2}\left(q_{1H}^{CI}(S,T) + x_{2H}^{CI}(S,T)\right)^2\right) \\
&- s_1 x_1^{CI}(S,T), \qquad (71)
\end{aligned}$$

$$\begin{aligned}
\max_{t_2} E\left(SW_2^{CI}(S,T)\right) &= PS_2^{CI}(S,T) + CS_2^{CI}(S,T) + TR_2^{CI}(S,T) \\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(S,T) + \pi_{2H}^{CI}(S,T)\right) \\
&+ \frac{b}{2}(q_2^{CI}(S,T) + x_1^{CI}(S,T))^2 \\
&+ t_2 x_1^{CI}(S,T). \qquad (72)
\end{aligned}$$

The best response functions for stage two game are

$$\begin{aligned}
BR_1(t_2) &= s_1 \in \arg\max E\left(SW_1^{CI}(S,T)\right), \\
BR_2(s_1) &= t_2 \in \arg\max E\left(SW_2^{CI}(S,T)\right).
\end{aligned}$$

Solving gives us Bayesian Nash equilibrium level of policy rate

$$s_1^* = \frac{1}{14}(a-c), \tag{73}$$

$$t_2^* = \frac{5}{14}(a-c). \tag{74}$$

With no surprise, the optimal policy rates for export subsidy and tariff is identical that under (T,S). Given s_1^* and t_2^*, the expected outputs for both firms can be obtained as follows:

$$\begin{aligned}
E\left(q_{1L}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c-V), \\
E\left(q_{1H}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c+V), \\
E\left(x_{1}^{CI}(S,T)\right) &= \frac{1}{7b}(a-c), \\
E\left(q_{2}^{CI}(S,T)\right) &= \frac{3}{7b}(a-c), \\
E\left(x_{2L}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c-V), \\
E\left(x_{2H}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c+V).
\end{aligned}$$

The corresponding social welfares are

$$E\left(SW_1^{CI}(S,T)\right) = \frac{101}{294b}(a-c)^2 + \frac{1}{3b}\sigma^2, \tag{75}$$

$$E\left(SW_2^{CI}(S,T)\right) = \frac{449}{882b}(a-c)^2 + \frac{1}{9b}\sigma^2. \tag{76}$$

Again, we have $\frac{\partial E\left(SW_i^{CI}(S,T)\right)}{\partial \sigma^2} > 0$ for $i = 1, 2$. In other words, social welfare under mixed policies (S,T) is also enhanced with higher variance. This together with the sub-game equilibrium analysis under (T,S), we can conclude that the social welfare increases with market volatility for mixed games under complete information.

These equilibrium social welfare levels in different sub-games under complete information are used to construct choice of policy game in section 4.

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