Information Design and Sensitivity to Market Fundamentals

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Abstract

I study the problem of firms that disclose verifiable information to each other publicly before engaging in strategic interactions. In a pre-action stage of an Oligopolistic interaction firms must design public signals that can be either interpreted as statistical reports or as slices of physical quantities, i.e. market segments. Before the state of the world is realized, firms choose a signal policy, an estimation technique, about a private individual payoff state and then are forced to publicize the results of the investigations to all other firms before engaging in price or quantity competition. Because signals are made public, when a firm tries to assess the firm's individual payoff, it also ends up revealing the same information to her opponents. Full Disclosure enables companies to adapt to local market fundamentals at the expense of releasing crucial information to the competitors. On the other hand, Partial Revelation makes companies loose optimality of the decisions with regards to the true state of the world but enable them to commit to an aggressive policy of preclusion that increases the frequency of a favorable distribution of players actions. Whereas Partial Revelation acts as a commitment device and preclude entry in otherwise competitive markets, inducing insensitivity of the decisions with respect to local fundamentals, decentralized decision making is a dominant strategy when the profile of competitors is constant across markets or when a company cannot influence the extensive margin entry decision of the competitor with more or less disclosure of information. Since decentralization acts as a way to correlate decisions with local market fundamentals, and running one single policy in multiple states of the world acts as a commitment device to avoid competitors, I describe a trade off between commitment over a distribution of actions versus correlation with states of the world.

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1 Introduction

In this paper I extend a Bayesian Persuasion environment to a model of information exchange. Firms disclose public statistics over their own demand and production cost features before engaging in a game of price or quantity competition. Most models of Persuasion focus on a fixed role between senders and receivers of information. However, in many economically relevant situations of market competition, firms engage in exchange of information, for example when firms choose market segmentations for prices quality or quantity policies. If a firm decides to operate with a smaller capacity in some specific region, this firm is actually revealing information over it's own technological features to it's opponent. In this paper, I study what are the plausible information structures that would arise in sound economic settings à la Cournot and Bertrand with Differentiated Products when all parties are in charge of disclosing publicly information over private individual payoffs. In particular, firms do not have the capacity to induce the same distributions of states of the world as one another.

Assume that N firms share an aggregate market and must decide on prices or quantities. The features of the demand and the production costs, the market fundamentals, are denoted $\theta = (\theta_i)_{i \in N}$ and are jointly distributed according to a common prior μ. States of the world are only observed by firms via Blackwell Experiments that all firms can perform in order to make inference over individual states, θ_i. All firms observe the signal quality of their opponents and, after the signal realization $s = (s_i)_{i \in N}$ is made public, all players update their beliefs to a common posterior $p^s \in \Delta(\Theta)$ and simultaneously choose an action, setting up an equilibrium policy $a_i(p^s)$. Each signal realization, s, and associated distribution over fundamentals, p^s, defines a subgame, G^{p^s}, of complete information and, thus, a Nash Equilibrium, $a(p^s)$, results after $p^s \in \Delta(\Theta)$ is realized. Each signal realization, s, is viewed as a market, because the realization s defines an equilibrium profile of actions. The signal, the informational strategy that firms perform in this paper, can be taken in two possible ways, either as statistical estimations or as nationwide market segmentations.

For example, firms can hire more than one audit company before issuing debt or equity (or a firm may not audit at all). Pharmaceutical companies must release drug trials results and choose how many tests they perform before issuing a new product. In both situations, when the firms choose to perform investigations over a private payoff relevant state of the world they must release information over the quality of their product to the public, and thus to their competitors, before setting up prices (in the case of the medicine example) or quantities (in the case of the debt/equity example). This is the statistical view of Blackwell Experiments, i.e. that firms control the methods of investigations, or estimation, of an uncertain and payoff relevant state of the world. Instead of interpreting a signal as the outcome of an estimation exercise, with the interpretation of outcomes of those experiments being outcomes of a report, as in the previous example, one can also interpret Blackwell Experiments as an automated assignment rule of physical quantities, and, with that, the model can be used to understand market segmentation strategies, in the same way as in Bergemann et al. (2015).

Blackwell experiments can also be used to model market segmentations that are designed as automated assignment rules based on socio-economic and technological factors that determine demand and supply features of a firms structure. Since $\theta_i \in \Theta_i$ can be taken as socio-economic and technological variables, or market fundamentals - wealth, consumption, tax incidence, real estate value etc so that, in the measure theoretic approach, Blackwell Experiments are assignment rules or market segmentations. Multiple industry specific cases arise when using this interpretation, in particular, whenever a firm designs a regional strategy, assigning fundamentals, θ_i, to multiple policy segments, s_i. In the analogy of market segmentations, a Full Disclosure policy is analogous to a Firm choosing one segment s_i, for each state of the world, or market category, θ_i. This is analogous to perfect discrimination and it entails that firms take decisions on a local level. A firm might choose, however, to ignore the variation in it's own fundamentals, a Partial Revelation strategy that is analogous to assigning many states θ_i to one single policy segment, s_i. In this case, the policy adopted will be more insensitive to local market fundamentals because the management decisions are taken on the aggregate level. Thinking from the standpoint of a Cournot game, by running one policy for quantities (per capita), $s_i \in S_i$, over more than one category, $\theta_i \in \Theta_i$, and taking decisions that reflect averages instead of states point by point, a firm might be able to induce more aggressive decisions in regions where that quantity would not be justified by local market fundamentals, taking the opponent out of operation in more regions with this insensitive policy.

Industry examples range from airline routes, car dealers locations, retail and coffeehouse stores etc. For example, Delta Airlines can forcefully choose one policy of flights for places with different local market fundamentals. Delta's policies, in this case, are insensitive to local market conditions as opposed to when Delta assigns one policy of quantities of flights per airport it operates. The two crucial assumptions here are the publicity of the signals, portfolios of policy segments are visible, and the experiments determining the sensitivity of decisions with regards to local market fundamentals so that the model is a good approximation for any automated decision making process that is based on ex-ante and publicly known levels of information.

Let the extensive margin profile of companies to be whether the firms are in or out of operation. In the main result of the paper, I show that Full Disclosure, or Full Decentralization, arises when payoffs are linear in opponents actions and states of the world and when a firm cannot alter the distribution of extensive margin outcomes. The result should be seen as preference for correlation of actions with realized states of the world when the extensive margin realizations are insensitive to information policies. For example, if the model is linear in best responses and the interior solution is valid for all primitive states of the world, θ, then all firms have Full Disclosure of their private types, θ_i, as a weakly dominant information design strategy. If firms must share a market no matter what will be the opponents policies, then connecting decisions with states of the world will be prioritized over a strategy that induces a better distribution of actions via a persuasive opaque strategy. If, on the other hand, running an opaque and insensitive policy is effective in *taking opponents out of operation*, or altering the realizations of the extensive margin, and if the resulting insensitivity with respect to local market conditions does not affect local optimality, then a firm may engage in a policy that is purposefully insensitive to local market conditions in order to increase the frequency of states where the opponents are put out of operation. This means that a firm might choose to ignore information in order to pursue a persuasive policy of deterrence. In the language of market segmentations, the firm engages in policies that are insensitive to local market fundamentals for deterrence reasons.

I also contribute to the technical part of the Persuasion literature by introducing a method for solving Persuasion models with continuous actions under the presence of multiple senders and receivers. Piece-wise affine equilibrium actions yields piece wise convex functions of the senders with respect to the posterior distribution sent publicly. I categorize equilibria associated to posteriors according to classes of equilibrium actions - I categorize an equilibrium profile of actions into the index profile of those who are playing interior or corner solutions - and I show that, in the piece-wise linear equilibrium actions case, the categorization exercise I make fall into the U-Cover method of Lipnowski and Mathevet (2018) - i.e. I cover the domain of posteriors with a finite collection of compact and convex regions where a senders payoff is convex. The U-Cover is obtained by keeping fixed the classes of equilibria, where players are labelled according to the type of action they perform after posterior p, i.e. whether players are in the interior solution or corner solution level. Since actions are affine in those regions, senders utilities are (locally) convex in the posterior, p, because of local linearity in opponents actions. Piece-wise convex value functions, then, yield finitely many extreme points that can be in an optimal policy for a sender, $\tau_i^\star$, as shown in Lipnowski and Mathevet (2018) and Lipnowski and Mathevet (2018). Not only that, these finitely many extreme points are continuously altered by opponents choice of posterior distributions and, thus, an equilibrium exists, from the multiple senders point of view, via standard Maximum Theorems and Kakutani arguments applied to the optimal information best responses $\tau_i^\star(\tau_{-i})$.

This paper endogenizes the information structure of a canonical class of games that embeds generic cases of Cournot and Bertrand with Differentiated products. The model is applied to market segmentations and can explain flooding behavior in the implementation of stores across the nation for preclusion motives on the Cournot side. Flooding, or high levels of quantities in places which *per-se* do not justify with market conditions that decision, arises when the losses of making decisions not connected to local markets are not pronounced across markets. If taking an opponent out of circulation is made with no loss to optimality of decisions with respect to local market conditions, i.e. if local market conditions are homogeneous, firms may engage in a single policy across markets provided that the firm engaging in an insensitive policy can force the opponent out of operation, or that the extensive margin decision can be explored. On the other hand, if some firm cannot force any other firm out of operation, then I show that this firm wishes to connect decisions with states of the world by releasing all information it has. I describe, thus, a trade-off between coordination of actions with states of the world versus the commitment to run aggressive policies and induce desired distribution of actions of the opponents.

2 Literature Review

The Information Design literature has evolved into two somewhat different approaches, namely the Bayes Correlated Equilibria, represented here by Bergemann and Morris (2013), and the Sender - Receiver frameworks, from the work of Kamenica and Gentzkow (2011). While in the Bayesian Persuasion and Cheap Talk literatures the interest lies in the interaction between senders and receivers of information, with a clear focus on the mechanisms, or channels, of communication, in the B.C.E. approach the goal is to explain the effects of information structures on the distribution of outcomes, in a reduced form mentality that enables more generic results. In the Bayes Correlated Equilibria, a somewhat exogenous signal parametric structure gives rise to a distribution of actions and states of nature. Signals are either exogenously given or recovered from a desired "optimal" distribution of actions and states that is attainable and respects prior distributions assumptions. In most models of this branch, there is no presence of competition or market interaction in the design of signals. These papers do not ask why some optimal signals can be seen in market interactions. The question answered in these set of papers is usually of the form: "Given a class of signals and objective functions, what is the optimal signal given a metric of success?". Importantly, these papers abstract from the intricacies of the information and economic environment at hand, optimizing over a parametric class of signals that are simple distributions of actions and states of the world that are attainable.

Bayes Correlated Equilibria, thus, is the proper environment to compare many information structures within a class of signals since the goal is to make computational exercises feasible for arbitrary information structures. B.C.E. is a good form of comparing many information structures. The B.C.E. approach, however, does not answer the question of whether those information structures are plausible or would be observed in natural economic environments. In many situations information design outcomes obey market rules or are transmitted within and across economic institutions in a very specific, industry dependent, way. The presence of private or public signals, for example, is not arbitrary and depends on economic modelling.

The Bayes Persuasion literature, started with Aumann and Maschler (1995) but revived by Kamenica and Gentzkow (2011), is concerned with making predictions of the provision of information performed by economically relevant senders trying to Persuade a receiver to perform a sender optimal action. Models of Persuasion are trying to generate market outcomes in the sense of plausible information structures given a timing and a profile of interactions - one or many senders and one or more receivers, etc. In other words, the Persuasion literature is concerned with explaining the channels in which information is transmitted via plausible strategic interactions between senders and receivers. The focus here is on the economic channels of information transmission with verifiable information.

One of the main challenges in the Bayesian Persuasion literature, nevertheless, has been to find reasonable applications of senders with both reasonable investigative roles and at the same time that carry commitment to disclose reports publicly and in a verifiable manner. My claim in this paper is that it is possible to bring Persuasion models one step further into sound economic models of, for example, Cournot and Bertrand, where nationwide firms *exchange* information in a verifiable manner. A regional strategy coming from the headquarters, for example, is an automated decision that is based on existent, ex-ante, levels of information and that information is visible to all parties.

In the technical aspect, this paper uses methods of covering the domain of a continuous function of posteriors, p, with finitely many compact and convex sets where the sender's utility function is convex, just as in Lipnowski and Mathevet (2019) U-Covers method. Here I am studying continuous actions models with the presence of a benevolent receiver and piece-wise affine equilibrium actions as a function of the posterior p. I obtain a U-Cover in the continuous actions case because equilibrium actions are piece-wise affine in the posteriors for finitely many *classes of equilibria*. Therefore, I show that the stability needed to induce a U-cover in my paper is not in the profile of actions generated by posteriors, but in the *classes of equilibria* that can arise - specifically, if a pair of posteriors generate the same profile of receivers playing interior and corner solutions as equilibrium actions after posterior p, p', then p, p' belong to the same class of equilibria. Linear setups yields convex functions of the parameter in convex regions that translate into the U-Cover - i.e. a finite collections of compact and convex sets in which the sender's utility function is convex in the posterior - just as in Lipnowski and Mathevet (2019).

3 Informational Entry Deterrence

3.1 Cournot with Interior Solutions Lead to Full Revelation

Consider a Cournot Duopoly with homogeneous goods and uncertainty over the marginal costs. Assume that two firms, Firm 1 and Firm 2 compete in a market with a homogeneous good and with linear demand and linear marginal costs. In this market, the only parameter that is uncertain is the firm specific marginal cost, θ_i. Firm 1 has marginal cost denoted by θ_1 and Firm 2 by θ_2.

Thus we can write demand as

$$P^d(Q_i, Q_j) = A - b(Q_j + Q_i)$$

and profits, being revenue minus costs, can then be written as

$$u_i(Q_i, Q_j, \theta_i) = (A - b(Q_j + Q_i) - \theta_i)Q_i$$

with best responses being equal to

$$BR_i(Q_j, \theta_i) = \max\{\frac{A - bQ_j - \theta_i}{2b}, 0\}$$

Instead of assuming a perfect information environment, where the vector θ is known to both players, assume instead that both players are equally uninformed over the true state of the world, the vector of marginal costs $\theta = (\theta_1, \theta_2)$, distributed according to a common prior distribution $\mu \in \Delta(\Theta)$ with marginal $\mu_i = marg_{\Theta_i}\mu(\Theta) \in \Delta(\Theta_i)$. Even though both firms are uninformed over the true state of the world, before actions take place they can assess their individual marginal costs, θ_i, through a Blackwell Experiment in a pre-stage of the interaction.

Blackwell Experiments, or signals, are defined by a pair $X_i = (S_i, \pi_i)$ formed by the support of messages S_i and the function, π_i that take from the relevant state of the world to a distribution over messages, or the realizations of a signal,

$$\pi_i : \Theta_i \to \Delta(S_i)$$

Each Firm chooses its Blackwell Experiments based only on its own parameter of uncertainty, namely the marginal costs. Experiments chosen can be either fully informative, by making $S_i = \Theta_i$ and $\pi_i(\theta_i|\theta_i) = 1, \forall \theta_i \in \Theta_i$, or completely non-informative, in which case a Firm would only emit one single message. In fact, Blackwell Experiments allow the firm to choose between all information structures of a signal. In this sense firms can now choose how much to know about their own cost structure before choosing a quantity strategy. However, I assume that both firms can observe the outcomes of the experiment, making $S = (S_i)_{i=1,2}$ be publicly revealed to both firms and, thus, if a firm decides to know more about their marginal costs at the stage of interaction, they must reveal crucial information to the opponents as well.

Given the profile of experiments $(\pi_i)_{i=1,2}$ made in the first period, the joint signal realization of signals, S, follows a distribution given by

$$\tau(s) = \sum_{\Theta} \pi_1(s_1|\Theta_1)\pi_1(s_2|\Theta_2), \quad s \in S$$

and for each realization $s \in S$, a posterior distribution results

$$p^s(\Theta) = \frac{\mu(\Theta)\pi_1(s_1|\Theta_1)\pi_2(s_1|\Theta_1)}{\sum_{\Theta} \mu(\Theta)\pi_1(s_1|\Theta_1)\pi_1(s_2|\Theta_2)}, \quad s \in S$$

.

After observing the realization of both experiments, i.e. after observing the signal realization $s \in S$, each Firm must then choose a quantity level $Q_i, i = 1, 2$. The public observation of the realization of the random variable $S = (S_i)_{i=1,2}$ effectively gives rise to a Symmetric Information game where payoffs for each player are defined by the random vector $\theta^S = (\theta_i^S)_{i=1,2} \equiv \mathbb{E}[\theta|S]$ distributed according to τ. As a result, for each realization $s \in S$, we can then define a game of symmetric information given by $\{(u_i(.,.,\theta^s), Q_i)_{i=1,2}\}$ with resulting Nash Equilibrium given by $Q_i(\theta^s), s \in S$.

As a result, given the distributions over signals τ with realizations S, a resulting distribution of equilibrium actions as a function of the resulting posterior p^S, $Q(p^S)$, is obtained. Note, however, that because best responses are linear in the marginal costs, θ_i, best responses and consequently equilibrium actions will be dependent only on the posterior first moment. We can then simply write $\tilde{Q}(\theta^S)$ for the equilibrium actions that result from posterior p^S. In other words, the equilibrium actions depend solely on the first moments.

Given a realization of a signal S, with resulting posterior p^S and posterior first moment θ^S, take the equilibrium actions $Q(\theta^S)$. Assume that the interior solution condition is weakly valid. I.E., assume that, given the posterior first moment, θ^S, both players are playing positive amounts as reactions to the opponents actions so that

$$BR_i(Q_j(\theta^S), \theta^S) \geq 0 \Leftrightarrow \frac{A - bQ_j(\theta^S) - \theta_i^S}{2b} \geq 0, i = 1, 2$$

Plugging in the best response of Firm j in the best response of Firm i we reach to the condition

$$Q_i(\theta^S) = \frac{A + \theta_j^S - 2\theta_i^S}{3b} \geq 0 \Leftrightarrow \frac{A + \theta_j^S}{2} \geq \theta_i^S, \quad i = 1, 2$$

.

which is the condition on the sufficient statistic, the first moment, θ^S, that guarantee that best responses are weakly positive. If the interior solution condition is weakly valid, i.e. if $\theta_i^S \geq \dfrac{A + \theta_j^S}{2}$, $i = 1, 2$, then the value function obtained by both players evaluated in equilibrium is

$$v_i(\theta^S) = u_i(Q(\theta^S), \theta_i^S) = \frac{(A + \theta_j^S - 2\theta_i^S)^2}{9b}$$

Note that v_i is convex in the posterior statistic, θ^s for a given realization in the support of signals $s \in S$. Moreover, if v_i is convex in the first moment θ^s, it is also convex in the posterior distribution p^s because the map from posteriors to the firm moment associated with it, $\mathbb{E}_{p^s}\Theta$, is affine p^s. Assume p^s is split into multiple posteriors $p^{S'}$ distributed according to a distribution τ such that $\mathbb{E}_\tau p^{S'} = p^s$. I.E. treat the posterior p^s as a degenerate random variable and consider a mean preserving spread of the distribution p^s into a distribution of posteriors τ' with support $p^{S'}$ such that $\mathbb{E}_{\tau'} p^{S'} = p^s$. Then, by the law of iterated expectations, $\mathbb{E}_{\tau'}\mathbb{E}_{p^{S'}}\Theta = \mathbb{E}_{p^s}\Theta \equiv \theta^s$ and, since the value function is strictly convex in the statistic, $\theta^{S'}$, all players would weakly prefer the mean preserving spread τ' over having a single signal realization s with posterior p^s. Thus if the interior solution condition holds, for any possible $p^{S'} \in \Delta(\Theta)$, both

Firms would weakly prefer more information in the form of mean preserving spreads of p^s.

Remark 1. *Let $L' \in \Delta(\Delta(\Theta))$ be a mean preserving spread of the posterior distribution p^s. In words L' is a distribution of distributions with the property that the average under L' is p^s, or $\mathbb{E}_{L'} p^{L'} = p^s$. Then $L \succeq^{MPS} L' \Leftrightarrow \mathbb{E}_{L'} v_i(\theta^{l'}) > \mathbb{E}_L v_i(\theta^l)$ for v_i convex in θ.*

Any mean preserving spread of a profile of posterior moments, θ^s, when θ^s is seen as a degenerate random variable - or analogously any mean preserving spread of the posterior $p^s \in \Delta(\Theta)$ - would increase the indirect utility function of all players. As the mean preserving spread order over posteriors distributions is essentially a measure of informativeness, we can say that agents prefer more information over less information if we assume that Firms will play interior solutions.

The condition on the primitives that guarantee that the interior solution holds for any potential posterior is that $\theta_i \leq \dfrac{A + \theta_j}{2}, \forall \theta \in \Theta, \forall i = 1, 2$, i.e. that for any potential realization in the support of μ, both players would be playing interior solutions, in the weak sense, if both players knew with certainty that the realized state of the world was $\theta \in \Theta$ and for any $\theta \in \Theta$, where Θ is the support of states of the world.

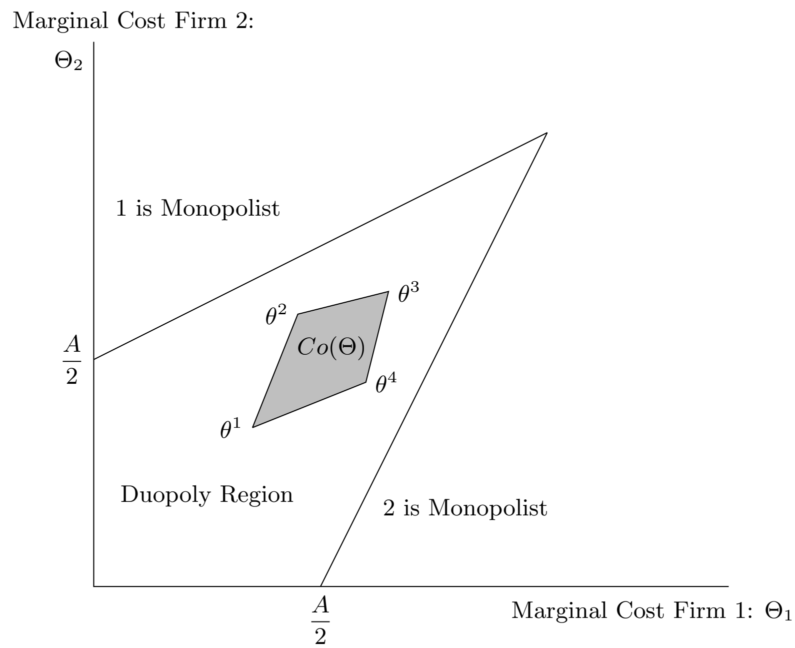


Figure 1: *Duopoly with uncertainty over the marginal cost*

(a) The shaded region represents the attainable posterior moments resulting from Blackwell Experiments so that $\theta^{p^s} \in Co(\Theta)$. The presence of the Interior Solution Condition guarantees Full Disclosure by each firm in equilibrium.

I will show more formally, and on a broader class of utility functions, - the linear-in-best-responses quadratic utility function also used in Bergemann and Morris (2013) - that convexity in fundamentals is driven by: i) the presence of a linear demand and cost functions that yield a linear best response and ii) the assumption that all players are playing interior solutions. Under the presence of a linear best response, more informative, decentralized, posteriors are weakly preferred by all firms and would be chosen in an Information Design market equilibrium.

Blackwell Experiments can assume two interpretations: On one hand Blackwell Experiments may be an estimation procedure that gives a signal S_i and posterior $p_i^{s_i}$. In this sense, experiments are the quality of statistical estimation procedures. On another potential interpretation, on the other hand, Blackwell Experiments can be taken as automated assignment rules. For example, create categories of customers $\theta_i \in \Theta_i$. Assign customers to market segments s_i according to $\pi_i : \theta_i \to \Delta(S_i)$. Then a realization s_i, carries $p_i^{s_i} \in \Delta(\Theta_i)$ in proportions of each type θ_i. This is very similar to the **?** only that, here I ask that each joint realization $s = (s_i)_{i \in N}$ determines a market $p^s \in \Delta(\Theta)$.

In the market segmentation interpretation of Blackwell Experiments, firms would like to decentralize operations and make strategic decisions locally, opting for more informative signals even if this decentralized and informative outcome results in the provision of more information to the other firms as well[1]. Unequivocal preference for more information can be read as preference for coordination of actions with states of the world, θ, in the absence of the capacity of deterrence by Firms. As a result the endogenous information structure that would arise entails high levels of sensitivity of Firms decisions with respect to local market conditions. I.E. Market Segmentations are made thin in order to connect decisions with realized states of the world.

Indeed the Cournot example shown above clarifies the main force of this paper. If firms have to share the market no matter what is the aggregate posterior, $p^s \in \Delta(\Theta)$, for any possible profile of experiments chosen in the pre-stage, π_i, then these firms have a clear incentive to disclose all information and the symmetric information environment becomes effectively a complete information setup as the resulting posteriors are degenerate distributions with Full Revelation side. In other words, firms choose to coordinate actions with states of the world rather than use mixtures of fundamentals to induce desired actions - preclusion for example.

3.2 The General Model

Assume N firms interact in a market and each firm has profits that depend on it's actions, the actions of the opponents and states of the world, $\theta = (\theta_i)_{i\in N} \in (\Theta_i)_{i\in N}$ that are jointly distributed according to a common prior distribution μ. I let $\Theta \equiv \{\theta = (\theta_i)_{i\in N} \in \times_{i\in N}\Theta_i : \mu(\theta) > 0\}$ to denote the support of μ. I assume that the support $\Theta_i \subset \mathbb{R}$ is finite for every firm $i \in N$[2]. Firms simultaneously choose an action $a \in A_i \equiv [0, \overline{a}] \subset \mathbb{R}, \forall i \in N$. Given a profile of states of the world $\theta = (\theta_1, ..., \theta_N) \in \Theta$, payoff functions are such that, for every player i, for every individual state of the world θ_i and action profile $a \equiv (a_i, a_{-i})$,

$$u_i(a_i, a_{-i}, \theta_i) = -\frac{a_i^2}{2c_i} + a_i \sum_{j\neq i} c_{ij} a_j + \theta_i a_i \tag{1}$$

where $c_i > 0$.

Strategic complements happen when $c_{ij} > 0\ \forall i, j \leq N$ and the reverse for strategic substitutes. This functional form (particularly the linearity and separability in actions states of the world) is the standard in the networks literature. In terms of Oligopolistic Competition, the strategic substitutes case leads to a Cournot game whereas the strategic complements case result in a Bertrand with differentiated products case[3]. Note that u_i depends on the state of the world θ only through it's i'th component, the individual state θ_i. Throughout the paper I will use a specific, benchmark, model, namely the classical Cournot with Homogeneous Goods.

[1] Since the experiments are publicly revealed.

[2] States that are unidimensional are not limiting the results as it will be clear in the market segmentation exercise.

[3] See Bergemann and Morris (2013) for more applications including Global Games.

Example 1. *(Cournot Duopoly with Homogeneous Goods) Assume the inverse demand is given by* $P^d = A - b(Q_i + Q_j)$ *and marginal costs are given by* θ_i*. Profits can be written as* $u_i(Q_i, Q_j, \theta_i) = (A - b(Q_i + Q_j) - \theta_i)Q_i$

Instead of a typical private values setting that leads to a Bayesian Game, here the information structure is endogenously chosen.

In the first stage, when firms are equally uninformed over the state of the world, θ, that is jointly distributed according to a common knowledge prior distribution, $\mu \in \Delta(\Theta)$, firms choose a Blackwell Experiment over their own payoff component, θ_i. I let $\Theta \equiv supp(\mu)$ to be the support of the distribution μ. Blackwell Experiments, x_i can be defined as a pair $x_i = (S_i, \pi_i)$ s.t. S_i is a finite signal space and as usual $\pi_i : \Theta_i \to \Delta(S_i)$. I denote the universe of experiments for firm i as $X_i = \{x_i = (S_i, \pi_i) : |S_i| < \infty\}$ which is also a standard notation. Analogously, we can define the space of information policies

$$T_i(\mu_i) \in \Delta(\Delta(\Theta_i)) = \{\tau_i : S_i \to \mathbb{R}_+, s_i \in \Delta(\Theta_i) \forall s_i \in S_i, |S_i| < \infty,$$

$$\sum_{s_i} \tau_i(s_i) = 1, \mathbb{E}_{\tau_i} p_i^{S_i}(\theta_i) = \mu_i(\theta_i), \forall \theta_i \in \Theta_i\}$$

and I note here that the two spaces are connected through a Homeomorphism and so can be treated as identical objects.[4] The choices of experiments $x = (x_i)_{i \in N}$ are made public at the end of period 1.

In period 2, signals are drawn from the corresponding experiments $(x_i)_{i \in N}$, chosen in the previous period, and the profile $s = (s_i)_{i \in N}$ is publicly observed. After observing s and x, firms simultaneously choose actions, $a = (a_i)_{i \in N}$. Note that information is complete and symmetric here since all players must take an action upon observing the same amount of information s and X. At this point all players share a common posterior given by $p^s \in \Delta(\Theta)$[5].

Players update their common posterior to $p^s \in \Delta(\Theta)$ and are then asked to run one single strategy, i.e. choose a strategy $y_i \in \Delta(A_i) \forall i \in N$. Note that in this case a complete information game with payoff uncertainty given by the random variable θ that is now distributed according to $p^s \in \Delta(\Theta)$ unfolds. The realization of a signal $s \in S = (S_i)_{i \in N}$ then determines a subgame. Formally, let

$$U_i^{p^s}(a) \equiv \sum_{\theta \in \Theta} p^s(\theta_i, \theta_{-i}) u_i(a, \theta_i)$$

We can then define the Nash Equilibria associated to each p^s. Let $N(p^s)$ denote the set of Nash equilibria of the simultaneous move game with complete information $G^{p^s} = ((A_i, U_i^{p^s})_{i \in N})$.

[4] Note that $\pi_i(s_i|\theta_i) = \tau_i(s_i) p(\theta_i|s_i)/\mu_0(\theta_i), \forall \theta_i \in \Theta_i$ so that for each τ_i there exists a unique x_i) that represents it and vice-versa. I use both X_i and T_i in the analysis - π_i is used to compute transition probabilities, τ_i is a more familiar object when analyzing choices, since π_i is a function and τ_i is a probability distribution measure.

[5] Specifically the frequencies of the signals are $\tau(s) = \sum_\theta \mu(\theta) \Pi_i \pi_i(s_i|\theta_i)$ and the posterior associated is obtained by $p^s(\theta) = \frac{\mu(\theta) \Pi_i \pi_i(s_i|\theta_i)}{\tau(s)}$, both numbers between 0 and 1.

The objective of the paper is to analyze the information structures, $x = (x_i)_{i\in N}$, that would arise in equilibrium if each company could perform Blackwell Experiments over their own payoff relevant component of the state of the world, i.e. in the first stage each firm chooses a signal $x_i = (S_i, \pi_i)$ such that $\pi_i : \theta_i \to \Delta(S_i)$. In the first stage the strategies are summarized by $x = (X_i)_{i\in N}$ and in the second stage, players anticipate that for any resulting signal s and for any resulting $p^s \in \Delta(\Theta)$ actions $a = (a_i(p^s))_{i\in N}$ are a Nash equilibrium of the game G^{p^s}. Then I want to understand how the information structures, $\tau^\star(\mu, c)$, that arise in equilibrium are affected by (μ, c), i.e. how equilibrium information provision is affected by the assumptions on the prior distribution and the utility function.

When signal s is realized players best respond according to game G^{p^s} and beliefs $\eta^i \in \times_{j\neq i}\Delta(A_j)$. Then the best response of player i at subgame determined by posterior p^s, G^{p^s} is

$$BR_i(\eta^i, p^s) = \underset{a_i\in[0,\overline{a}]}{argmax} \mathbb{E}_{\eta^i} U^{p^s}(a_{-i}, a_i) =$$

$$\min\{\max\{c_i[\mathbb{E}_{\eta^i} \sum_{j\neq i} c_{ij}a_j + \theta_i^{p^s}], 0\}, \overline{a}\} = \min\{\max\{w_i(\eta^i, \theta_i^{p^s}), 0\}, \overline{a}\}$$

with $\theta_i^{p^s} \equiv \mathbb{E}_{\theta_i|p^s}\theta_i$ and $w_i(q^{-i}, \theta_i^{p^s}) = c_i[\mathbb{E}_{\eta^i} \sum_{j\neq i} c_{ij}a_j + \theta_i^{p^s}]$ linear and separable in actions and types. The function w_i is the best response policy of firm i in the case where this firm would play an interior solution, in the weak sense, i.e. when $w \in [0, \overline{a}]$. As this setup yields single valued best responses then only Pure Strategies Nash Equilibria are present and thus only Pure Strategies Nash Equilibria are considered.

In order to describe behavior with this posterior, p^s, we only need to pay attention to the posterior first moments defined to be the first moments after signal realization s is obtained, $\theta^p \equiv \mathbb{E}_p\theta \in Co(\Theta)$. [6] I now make several abuses of notations in order to spare the reader - and the author - from burdensome notation : i) First I may write θ^s to be the resulting statistic θ^{p^s} after signal realization s, ii) I write $w_i(\theta^P)$ to be the resulting function evaluated at the "correct" beliefs where players choose their corresponding equilibrium actions, i.e. $w_i(\theta^p) \equiv w_i(a_{-i}(\theta^p), \theta_i^p)$. In the same manner I write $v_i(\theta^p) = u_i(a(\theta^p), \theta_i^p)$ to be the value function evaluated at the equilibrium actions when the posterior is p. iii) Because equilibrium actions are equivalently defined by it's posterior, p, or it's first moment, θ^p, I may use $a(p)$ or $a(\theta^p)$, and analogously $w_i(p)$ and $v_i(p)$ interchangeably with $w_i(\theta^p)$ and $v_i(\theta^p)$ respectively.

Definition 1. *The Overall Interior Solution condition is satisfied if for every*

[6] Also, for the problem to be clearly specified, I make a small strategic interaction assumption that guarantee that $N(p^s)$ is unique and, thus, indeed a function of p^s and not a correspondence, namely that $|c_i c_{ij}| < 1/\sqrt{N-1}$, $\forall i, j \leq N$ which guarantees that the Nash Equilibrium is unique. For example, the condition holds for a standard homogeneous goods Cournot example.

$\theta \in \Theta : \mu(\theta) > 0$, $w_i(a_{-i}(\theta), \theta_i) \in [0, \overline{a}], \forall i \leq N$, *where* $a_{-i}(\theta)$ *is the prescribed action in a Nash Equilibrium with common knowledge of the state* θ*.*

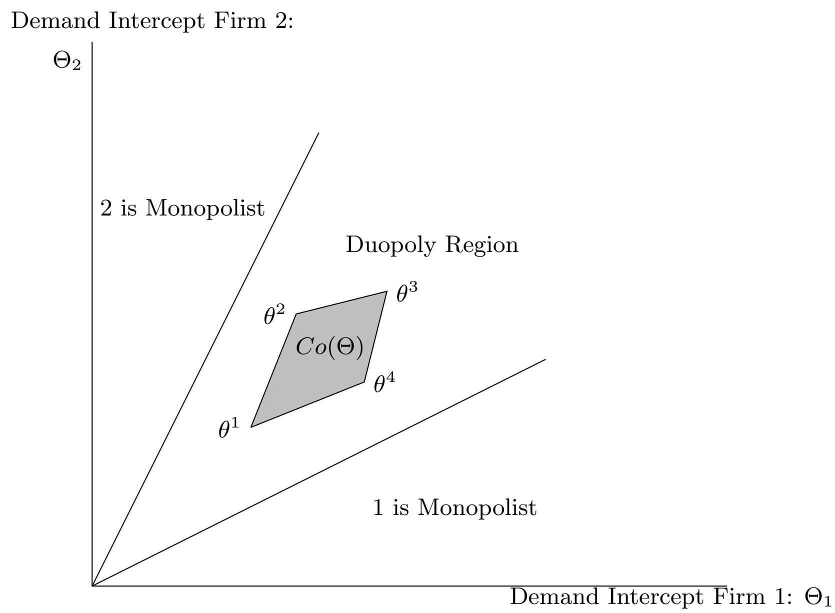


Figure 3: *Duopoly with uncertainty over the Demand Intercept and the presence of the Overall Interior Solution condition - The shaded region represents the space of attainable posterior moments after signal realization s,* $\theta^{p^s} \in Co(\Theta)$*. The presence of the Interior Solution Condition guarantees Full Disclosure by each firm in equilibrium.*

Lemma 1. *Assume that the interior solution condition is satisfied. Then* $v_i(\theta^p)$ *is convex in* $\theta^p \in Co(\Theta)$*. Also,* $\forall i \leq N$, $\forall p \in \Delta(\Theta)$ *and resulting* $\theta^p \in Co(\Theta)$ *such that* $\theta^p = \mathbb{E}_p\theta$*, the equilibrium actions are linear in the posterior statistic or the posterior distribution, i.e.* $a_i(\theta^p) = \mathbb{E}_p[a_i(\theta)]$*. We can, then, write* $a_i(\theta^p) = \beta_i\theta^p \in [0, \overline{a}]$ *,* $\forall i \in N, \forall \theta^p \in Co(\Theta)$ *.*

The proof follows by a guess and verify assumption on linearity applied to a_{-i} and is left to the appendix.

It turns out that the interior solution condition, together with linearity of equilibrium actions, guarantee Full Disclosure by each player as a weakly dominant strategy and, in fact, except for a lower dimensional "slice" of the space of distributions, Full Revelation is a Strictly Dominant Strategy.

Definition 2. *The Non Orthogonality Condition is met for player i and vector* $\beta_i \in \mathbb{R}^N$ *if* $\forall \theta_i, \theta_i' \in \Theta_i$, $\beta_i\mathbb{E}[\theta|\theta_i] \neq \beta_i\mathbb{E}[\theta|\theta_i']$.

Theorem 1. *Assume that the overall interior solution condition is guaranteed and let* $a_i(\theta^p) = \beta_i\theta^p$ *be the equilibrium actions for player i after posterior* *statistic* $\theta^p \in Co(\Theta)$ *as in Lemma* (1). *Then Full Disclosure by each player is a Weakly Dominant Information Strategy, i.e.* $x_i^\star = \{\Theta_i, \theta \to 1_{s=\theta}\}$ *is a best response for any* $x_{-i} \in X_{-i} = (X_j)_{j \neq i}$. *As a result, Full Revelation is always an equilibrium in* $t = 0$. *If the Non-Orthogonality condition is met for player* i*, then Full Disclosure is a strictly dominant action for player* i *and her unique best response.*

Note that $v_i(\theta^{p^s}) = u_i(a_i(\theta^{p^s}), a_{-i}(\theta^{p^s}), \theta_i^{p^s})$ is linear in a_{-i} and a_{-i}, on the other hand, is linear in θ^{p^s}. Thus if player i itself were to not make any change in her actions we would have $v_i(\theta^{p^s}) = u_i(a_i, a_{-i}(\theta^{p^s}), \theta_i^{p^s}) = \mathbb{E}_{\theta|s} u_i(a_i, a_{-i}(\theta), \theta_i)$ - i.e. player i is indifferent between providing more or less information to the other players if just the distribution of opponents actions is taken into account. More revelation, however can only help i make a more informed decision, $a_i()$ and must then be weakly preferred.

Full Revelation thus can be seen as a result of preference for *coordination* of actions with states of the world in detriment of a persuasion attempt to force a desired action by the opponents. Theorem 1 shows that Full Disclosure by each firm is intimately connected to correlation of policies with market fundamentals, as the main objective of the firms under the Overall Interior Solution Condition is to reveal information. In the market segmentation analogy, firms choose one policy for each market category θ_i.

The result of Full Revelation is robust to any prior distribution provided that the Interior Solution condition is met. Thus, in order to check if Full Disclosure is weakly dominant for all players and so that Full Revelation is an equilibrium outcome one must only check for the validity of the Interior Solution Condition at the support Θ, ignoring the weights given by probabilities, μ.

If the model is linear and the Interior Solution Condition is met then Full Revelation, or Full Decentralization in the market segmentation analogy, is a weakly dominant strategy. In the next sections I explore the reverse statement, i.e. whether lack of one of the two conditions can generate lack of Full Revelation.

Remark 2. *If there exists Opacity, then either the model is not linear or the interior solution is failing - i.e.* $\exists \theta$ *and* $i \in N$ *such that* $a_i(\theta) = \{0\} \vee \{A\}$.

Before proceeding to Partial Revelation, I introduce another instance in which Full Revelation is an informational equilibrium, when there is perfect correlation between types among firms.

Proposition 1. *Assume that types are perfectly correlated. I.E. assume that* $\forall i, j \in N$, $\mu_j(\theta_j|\theta_i) = \mathbb{1}_{\theta_j}$ *for some* $\theta_j \in \Theta_j$, $\forall \theta_i \in \Theta_i, \forall i$. *Then Full Revelation is an equilibrium outcome.*

pf: If player i is Fully disclosing, for every other player, $j \neq i$, the value of any experiment is $V_j(\tau_j, \tau_i, \tau_{-i-j}) = \mathbb{E}_s v_j(\theta^s) = \mathbb{E}_{\theta_i} v_j(\theta)$ is independent of the experiments of the opponents. In particular Full Disclosure is a best response for player j. ■

Correlation translates into the power of agents to force an equilibrium distribution of actions and states of the world. A firm can always react to disclosing more information in case of a non degenerate posterior in the support. In the case of one player performing a fully revealing experiment, the other players cannot react by adding more information, since information at this point is already complete, so that these players are indifferent between providing more or less information. Correlation is not the same as homogeneity and thus it is not related to the Interior Solution condition per-se. As a matter of fact, Theorem (1) holds independently of the prior distribution μ, as long as the Interior Solution Condition is valid.

In this section I have shown that either when the Interior Solution Condition holds or when types are perfectly correlated, Full Revelation is an equilibrium and thus an equilibrium is guaranteed to exist. It turns out that an Information Design Equilibrium $\tau^\star$ is always guaranteed to exist. In the appendix I show that standard concavification methods apply and the application of Caratheodory Theorem guarantee that an optimal policy is obtained with at most $|\Theta_i|$ extreme points. The extreme points will be defined by regions where the extensive margin decision of corner solutions is explored to it's maximum and some player is made indifferent between playing interior or one of the corner solutions.

The posteriors in the support of an information policy of player i, $p_i^{s_i}$, are classified into the classes of equilibrium actions that $p_i^{s_i}$ generates, when message s_i is composed with the other signal realizations of the opponents, s_{-i}, yielding joint posterior p^s. For each resulting joint posterior in the support of posterior distributions, p^s, I separate players into bins, each according to the class of equilibrium actions, $a(p^s)$, in which they fall into - whether interior or either of the corner solutions. The Categories for any player i can be threefold; whether i is in the (weak) interior solution region, when $w_i(p^s) \in [0, \overline{a}]$, or i is playing either of the corner solutions, when $w_i(p^s) \in (-\infty, 0]$, the weak lower corner solution regions, or when $w_i(p^s) \in [\overline{a}, \infty)$ the weak upper corner solution region, $\forall i \in N, \forall s \in S$. For each resulting posterior, p, three classes of index profile results and, thus, I let $I(p) = \{i \in N : w_i(p) \in [0, \overline{a}]\}$, $H(p) = \{i \in N : w_i(p) \in [\overline{a}, \infty)\}$ and $L(p) = \{i \in N : w_i(p) \in (-\infty, 0]\}$ to be the resulting three categories of index profiles. For a given posterior p, an equilibrium index profile is thus $\{L, I, H\}$, for $L, I, H \subset N$ and I denote its universe to be $Y = \{\{L, I, H\} : L, I, H \subset N\}$.

Consider now the set of posteriors that induce a certain index profile $y^{-1}(L, I, H) = p \in \Delta(\Theta) : y(p) = \{L, I, H\}$. In the appendix, I show that equilibrium actions are piece-wise affine with respect to the posterior first moment θ^p, or its posterior distribution p, for $p \in y^{-1}(L, I, H)$. To see the reason why actions are piece wise affine in p, or θ^p, assume that $w_i(p)$ is interior in y for all players $i \in N$. Then, for all players $i \in N$, their actions are far from changing category, i.e. $w_i(p) \in (-\infty, 0) \cup (0, \overline{a}) \cup (\overline{a}, \infty)$. Because w_i is linear and separabale in actions of the opponents, a_{-i}, and the first moment, θ_i^p, if we guess that equilibrium actions of the opponents a_{-i} are affine in p for $p \in y^{-1}(L, I, H)$ then for a small enough mean preserving spread of posteriors, $t \in \Delta(\Delta(\Theta))$ with support $\{p^L\} \in \Delta(\Theta)$ that still preserves the index (L, I, H), i.e. such that

$p^L \in y^{-1}(L, I, H)$, $w_i(p^L) = w_i(a_{-i}(p^L), \theta_i^{p^L})$ will be linearly affected by the mean preserving spread of p^L and so the guess of affinity is verified. Thus, for a small enough mean preserving spread, if $w_i \in (-\infty, 0) \cup (0, \overline{a}) \cup (\overline{a}, \infty)$ then $w_i(\mathbb{E}_t p^L) = \mathbb{E}_t w_i(p^L)$ and indeed actions are affine[7].

In this case, $a_i(p)$, the equilibrium actions are piece-wise affine in p, or θ^p, $a(\alpha p + (1-\alpha)p') = \alpha a(p) + (1-\alpha)a(p')$ for p, p' close enough. But then, if player i is sending a non-degenerate posterior $p_i^{s_i^\star}$, then a "small" enough mean preserving spread of $p_i^{s_i^\star}$ would still keep all players in the same index profile so that the same conclusions of Theorem 1 apply. In other words, in the interior regions of w_i, there is preference for more information by all players since then $v_i(\theta^p) = u_i(a_i(\theta^p), a_{-i}(\theta^p), \theta_i^p) = \mathbb{E}_t u_i(a_i(\theta^p), a_{-i}(\theta^{p^L}), \theta_i^{p^L}) \leq \mathbb{E}_t u_i(a_i(\theta^{p^L}), a_{-i}(\theta^{p^L}), \theta_i^{p^L})$ for a small enough mean preserving spread $t \in \Delta(\Delta(\Theta))$. If this interior levels of $w_j(p^{s_i^\star, s_{-i}})$ are true for every $s_{-i} \in S_{-i}$, i.e. if $w_j(ps_i^\star, s_{-i}) \in (-\infty, 0) \cup (0, \overline{a}) \cup (\overline{a}, \infty)$, $\forall s_{-i} \in S_{-i}$, then player i with a non degenerate posterior $p_i^{s_i^\star, s_{-i}}$ would then prefer to increase the levels of information generating a small enough mean preserving spread of $p_i^{s_i^\star, s_{-i}}$. Thus in any optimal non degenerate posterior $p_i^{s_i^\star, s_{-i}}$ of player i, it must be that $w_j(p^{s_i^\star, s_{-i}}) = 0 \vee \overline{a}$ for some $j \in N$ and $s_{-i} \in S_{-i}$.

Since actions are piece-wise linear, each index profile generated by a policy is the result of finitely many intersections of half spaces and thus there exists finitely many extreme points for each s_{-i}, that define the candidates of posteriors $p_i^{s_i^\star}$ that can be in the support of an optimal policy. Moreover, as equilibrium actions, $a_i(p)$, are continuous functions, the extreme points are continuously altered by a change in the opponents information policy τ_{-i} in the Wasserstein Metric (also known as the Optimal Transport Metric or the Kantorovich Distance). [8] The continuity and piece wise affine features of actions, thus, guarantee the application of Maxmimum Theorems and ultimately of a Kakutani Fixed Point in order to establish the existence of an Information Design Equilibrium, $\tau^\star$ in the first stage.

3.3 Cournot Duopoly Continued: Partial Revelation Dominates Full Revelation

In the previous section, I showed that linearity and Interior Solutions lead to Full Revelation. Now I reverse the statement and provide a simple example where the possibility of expelling the opponent out of operation leads to Partial Revelation as an optimal strategy. The intuition for the result is that Partial Revelation increases the frequency of states in which a monopoly happens even

[7] In the appendix I also show that the pre-image of index profiles $y^{-1}(L, I, H)$ form compact and convex subsets of posteriors that are the result of finitely many half spaces, thus, forming the conditions for the search of faces and extreme points.

[8] The Wasserstein metric metrizes the notion of weak convergence and is a distance function specially designed to compute distances of distributions with distinct supports but same metric space.

though it makes firms know less about the realized state of the world, and thus necessarily leads to a loss of optimality. By running a single policy across many θ_i, firm i loses local optimality but engages in a preclusion policy that increases the frequency of a desired state when j is out of operation.

Continuing with the Cournot duopoly case, i.e. $N = 1, 2$. Assume there exists two probable states of the world for each player, i.e., $\Theta_i = \{l_i, h_i\}$ and assume $h_i > l_i > 0$.

Payoff functions are

$$u_i(Q_1, Q_2, \theta_i) = (A - b(Q_i + Q_j) - \theta_i)Q_i$$

I parameterize the prior as $\mu_i(l_i) = \lambda_i; \quad \mu(l_j|l_i) = \lambda_{ij}; \quad \mu(h_j|h_i) = \rho_{ij}$.

A private posterior distribution in this binary state environment is fully characterized by a real number $p_i^{s_i} = p(l_i|s_i) \in (0, 1)$.

After some computation and assuming that $j \neq i$ and that Firm 2 is put out of operation $\theta_j^{s_i^\star, s_j} = \dfrac{A + \theta_i^{s_i^\star, s_j}}{2}$. We have

$$\theta_j^{s_i^\star, s_j} = \frac{A + \theta_i^{s_i^\star, s_j}}{2} \Leftrightarrow$$

$$\frac{p_i^{s_i^\star}}{1 - p_i^{s_i^\star}} = \frac{\rho_{ij}\pi_j^{h_j}(A + h_i - 2h_j) + (1 - \rho_{ij})\pi_j^{l_j}(A + h_i - 2l_j)}{\lambda_{ij}\pi_j^{l_j}(2l_j - (A + l_i)) + (1 - \lambda_{ij})\pi_j^{h_j}(2h_j - (A + l_i))} \tag{2}$$

A sufficient condition for the RHS to be strictly positive is that $l_j > \dfrac{A + l_i}{2}$, $h_j > \dfrac{A + l_i}{2}$, $l_j < \dfrac{A + h_i}{2}$ and $h_j < \dfrac{A + h_i}{2}$. In words, these parametric regions dictate that Firm i is a monopolist whenever state l_i is realized, no matter what is the realization of Firm j , $\theta_j = \{l_j, h_j\}$ and that firm j is playing strictly positive quantities if h_i is realized.

Definition 3. *Agent i is pivotal if $\dfrac{A + l_i}{2} < l_j$ and $\dfrac{A + h_i}{2} > h_j$. In words, in case h_i is realized, player j produces positive quantities in equilibrium, either sharing the market or being a monopolist, and when l_i is realized player i is a monopolist and j is out of operation.*

If i is pivotal, inspection of (2) shows that all terms are positive and the ratio is finite and strictly positive for any policy of Firm j, π_j[9]. From now on I assume that Firm 1 is pivotal.

[9]The pivotal assumption can be graphed in the line below.

$\dfrac{A + l_i}{2}$ ——— l_2 ——— h_2 ——— $\dfrac{A + h_i}{2}$

Remark 3. *Assume that Firm 1 is pivotal. Then there exists a non-degenerate posterior that can be in the support of an optimal information policy for Firm 1.*

I now analyze a case where Firm 1 is pivotal and, by running an opaque informational policy, Firm 1 will have a payoff function that dominates the Full Disclosure strategy for any potential policy of Firm 2, x_2. Let

$$H_1 = \rho_{12}\pi_2^{h_2}(A + h_1 - 2h_2) + (1 - \rho_{12})\pi_2^{l_2}(A + h_1 - 2l_2)$$

denote the gain from the non pivotal player, 2, in the form of the expected quantity produced in equilibrium when the pivotal player, 1, is a high marginal cost type h_1 and cannot deter entry[10]. In the same manner, let

$$L_1 = \lambda_{12}\pi_2^{l_2}(2l_2 - (A + l_1)) + (1 - \lambda_{12})\pi_2^{h_2}(2h_2 - (A + l_1))$$

denote the regret from non pivotal player, Firm 2, from stopping operations when the pivotal 1 is of a strong type, or low marginal cost of operation l_i, and is able to put 2 out of operation. In this case, the non-pivotal Firm 2 would want to be on the consumer side, setting negative quantities, $Q_2 < 0$, but then needs to stop at 0 and thus L_1 denotes its regret.

Observe that H_1 is proportional to the average quantity provided by player 2 when there is high cost, h_1, is realized by firm 1 (see the rule in the figure below.). On the other hand, L_1 measures the average consumption firm 2 would have if it could consume player 1's product, i.e. play negative quantities, in case l_1 is realized and thus I call it the regret quantity of firm 2 of being a producer.

Then (2) can be written as

$$\frac{p_1^{s_1^\star}}{1 - p_1^{s_1^\star}} = \frac{H_1((\pi_2(s_2|h_2), \pi_2(s_2|l_2))}{L_1((\pi_2(s_2|h_2), \pi_2(s_2|l_2))}$$

Thus H_1/L_1 measures a gain-regret ratio for Firm 2 of being a competitor of Firm 1 and completely defines the ratio $\dfrac{p_1^{s_1^\star}}{1 - p_1^{s_1^\star}}$. As a result, the gain-regret ratio of Firm 2 also completely defines the posterior probability $p_1^{s_1^\star}$. The higher is the gain-regret ratio of Firm 2 form being a competitor of Firm 1, the more revealing of the strong state, low marginal cost, Firm 1 must make its signal realization, $s_1^\star$, to be, therefore increasing $p_1^{s_1^\star}$.

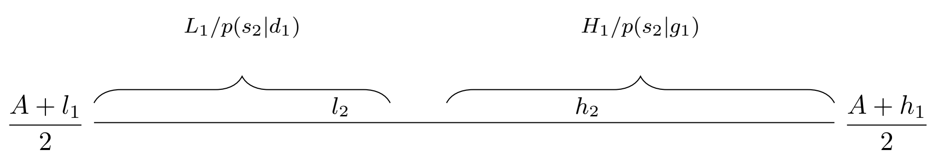
The closer to the Interior solution level of entry h_2 gets, i.e. as $h_2 \searrow \dfrac{A + l_1}{2}$, the lower is the regret from being out of operation, L_1, and the higher the gain H_1 of firm 2 in being a producer independently of Firm 1's type. In this case Firm 1 must make her experiments more revealing towards the pivotal state of deterrence, l_1.

[10] In fact a proportional quantity, $H_1/p(s_2|h_1)$, actually measures the expected quantity provided.

Assume now that Firm 2 is becoming more efficient on both states of the world and this improved efficiency approaches levels of Interior Solution when Firm 1's marginal cost is low, i.e. when $\theta_2 = \frac{A+l_1}{2}$. This is achieved by making the higher marginal cost approach interior solution levels, i.e. $h_2 \searrow \frac{A+l_1}{2}$. As the marginal costs of Firm 2 decrease and approach interior solution levels, the signal from Firm 1 must then be more and more revealing of a high state, low cost, i.e. $p_1^{s_1^\star} \nearrow 1$. Thus, the model presents some form of continuity of the paramaters (l, h), since approximating fundamentals to the Interior Solution leads to more revelation even when Firm 1 emits the opaque signal realization $s_1^\star$.[11].

Remark 4. *(2 Gets Good Too.) If* $h_2 \searrow \frac{A+l_1}{2} \Rightarrow p_1^{s_1^\star} \nearrow 1$, $\forall s_2 \in S_2$, $\forall x_2 = (S_2, \pi_2)$. *In words, if the fundamentals of firm 2 approximate interior solution levels, then any partially revealing strategy from firm 1 approximates full disclosure.*

[11]The gain-regret quantities H_1, L_1 can be visualized in the rule, since a convex combination between $(\frac{A+h_1}{2} - \theta_2)$ yields $\frac{H_1}{p(s_2|h_1)}$ for $\theta_2 = h_2, l_2$ and a convex combination of $(\theta_2 - \frac{A+l_1}{2})$ on $\theta_2 = h_2, l_2$ determine $\frac{L_1}{p(s_2|l_1)}$.



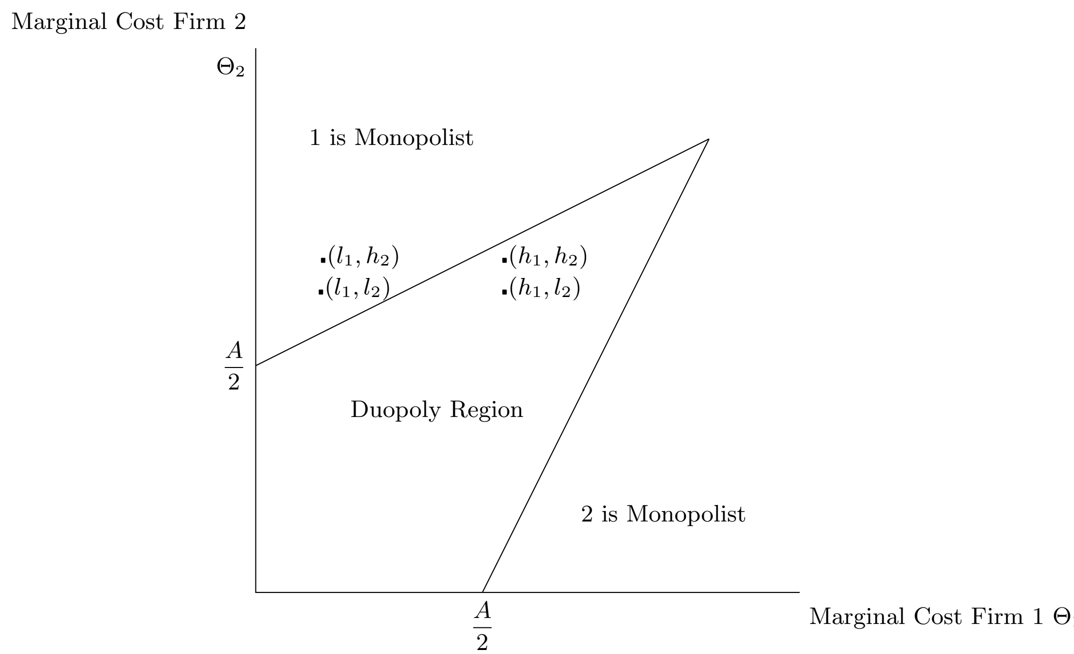


Figure 4: *Duopoly with the Presence of a Pivotal Player (Player 1)*

Consider now a binary signal strategy by player 1. Indeed, by Caratheodory, an optimal strategy is attained with at most 2 extreme points so an optimal strategy is either a binary set of posteriors or a singleton. If player 1 has only one non degenerate posterior, $p_1^\star$, in the support of an optimal best response, $supp(\tau_1^\star(\tau_2))$, and if this posterior is given by $p_1^{s_1^\star} > \lambda_1$ then, by Bayes Plausibility given by the restriction of $\tau_1 \in T_1(\lambda_1)$, we have $p_1' = \frac{\lambda_1 - \tau_1(s_i^\star)p_1^{s_1^\star}}{1-\tau_1(s_1)}$. If only $s_1^\star$ is non degenerate, then $p_1' = 0$ so that $\tau_1(p_1^{s_1^\star}) = \frac{\lambda_1}{p_1^{s_1^\star}} \in (\lambda_1, 1)$ and in case $p_1^{s_1^\star} < \lambda_1$, then $p_1' = 1$ and so $\tau_1(p_1^{s_1^\star}) = \frac{1-\lambda_i}{1-p_1^{s_1^\star}} \in (1-\lambda_1, 1)$. I write the respective value functions in both cases as

$$V_1(\tau_1, \tau_2) = \tau\nu_1(p_1^{s_1^\star}) + (1-\tau)\nu_1(0)$$

if $p_1^{s_1^\star} > \lambda_1$ and

$$V_1(\tau_1, \tau_2) = \tau\nu_1(p_1^{s_1^\star}) + (1-\tau)\nu_1(1)$$

if $p_1^{s_1^\star} < \lambda_1$.

Where $\nu_1(p_1)$ is the expected value of providing $p_1 \in \Delta(\Theta_1)$ by Firm 1 integrated over the opponents signal. To compute the expected value function I label the posteriors p^{s_1,s_2} that are dependent on the opponents signal realization s_2. Let v_1^m and v_1^d to denote equilibrium payoffs with posterior p^s for the cases of monopoly and duopoly for Firm 1 respectively. Integrating over s_2 posterior p_1 will then generate $p^{s_2 p_1}$ as final posterior after signal realization s_2, where I abuse notation and write the label of the signal form Firm 1 as it's own posterior, i.e. I set $s_1 = p_1$[12]. I categorize the equilibrium actions after posterior $p^{s_2 p_1}$ into index regions. In this case it suffices to label which players are playing interior solution levels of quantities and thus I let $I(p) = \{i \in \{1,2\} : w_i \in [0, \overline{a}])\}$ the value of having posterior p_1 in the support will be

$$\nu_1(p_1) = \sum_{s_2} p(s_2|p_1)[\mathbb{1}_{I(p^{s_2 p_1})=\{1,2\}} v_1^d(\theta^s) + \mathbb{1}_{I(p^{s_2 p_1})=\{1\}} v_1^m(\theta_1^s) + \mathbb{1}_{I(p^{s_2 p_1})=\{2\}} 0]$$

$$= \sum_{s_2} p(s_2 \cap \{1,2\}|p_1)(\frac{(A + \theta_2^{s_2,p_1} - 2\theta_1)^2}{9b}) + p(s_2 \cap \{1\}|p_1)(\frac{(\theta_1^s)^2}{4b})$$

Consider then the strategy of player 1 that uses one single non-degenerate posterior, $p_1^{s_1^\star}$, in which $\forall s_2 : w_2(a(\theta^{s_2,s_1^\star}), \theta_2^{s_2,\star_1}) \leq 0$ with indifference condition at s_2 such that $w_2(a(\ \theta^{s_2,s_1^\star}), \theta_2^{s_2,s_1^\star}) = 0$. Note that this strategy entails making player 2 out of operation no matter what is the state of Firm 2. With these assumptions, three types of posteriors can be part of an optimal informational best response and so $supp(\tau_1^\star(\tau_2)) = \{0, p_1^{s_1^\star}, 1\}$ where $p_1^{s_1^\star}$ performs the distribution of actions described in the beginning of the paragraph. I compute the value functions associated.

In the appendix I show that $\nu_1(p_1^{s_1^\star}) > \nu_1(0)$ and that $\nu_1(1) > \nu_1(p_1^{s_1^\star})$. If $h_1 \searrow l_1$ then market fundamentals across individual states for Firm 1 do not vary much and, by running an opaque strategy, Firm 1 is able to increase the frequency of a more favorable distribution of actions without sacrificing local optimality.

The profit for an unequivocal monopoly is $\mathbb{E}_{s_2|p_1} v^m(\theta_1^{s_2,p_1})$. As convergence in fundamentals $h_1 \searrow l_1$ imply that monopoly profits become less uncertain so that $\theta^{s_2,p_1} \to l_1$, then as $h_1 \searrow l_1$ it must be that $\nu_1(p_1^{s_1^\star}) \to \nu_1(1)$, an Opaque strategy will be preferred by Firm 1. In words, making a signal realization be more opaque entails some loss of optimality that comes from Firm 1 not making quantity decisions that are compatible with local market conditions but rather based in average fundamentals. However, as fundamentals converge this loss in adherence to local market fundamentals is less pronounced whereas the benefits of increasing the frequency of monopoly states become more important for Firm 1. The convergence in fundamentals, create then the proper conditions for opacity to arise. We then have

Lemma 2. *(1 is Homogeneous and 2 Does Not Get Too Good) Opacity Arises: Assume player 1 is pivotal. Take a sequence of fundamentals* $(h_1^t, h_2^t, l_2^t)_{t\in\mathbb{N}}$

[12] $p^{s_2 p_1}(\Theta) = \frac{p_1(\Theta_1)\mu(\theta_2|\theta_1)\pi_2(s_2|\Theta_2)}{\sum_\Theta p_1(\Theta_1)\mu(\theta_2|\theta_1)\pi_2(s_2|\Theta_2)}$

such that $h_1^t \searrow l_1$ *and* $h_2^t \to h_2$, $l_2^t \to l_2$ *so that* $\lim p_{1t}^{s_1^\star} = p_1^{s_1^\star}$ *and* $\lim \frac{p_{1t}^{s_1^\star}}{1-p_{1t}^{s_1^\star}} = \lim \frac{H_1^t}{L_1^t} < \infty$*. Then for* t *large enough, Full Disclosure by Firm 1 is strictly dominated by a binary signal strategy with a non-degenerate posterior* $p_1^{s_1^\star}$*.*

In case $p_1^{s_1^\star} > \lambda_1$*,*

$$V(\tau^{F.R.}, \tau_2) = \lambda_1\nu_1(1) + (1-\lambda_1)\nu_1(0) < \tau_1\nu_1(p_1^{s_1^\star}) + (1-\tau_1)\nu_1(0)$$

$$\forall \tau_2 \in T_2(l_2)$$

for l_1 *sufficiently close to* h_1*.*

In case $p_1^{s_1^\star} < \lambda_1$*,*

$$V(\tau^{F.R.}, \tau_2) = \lambda_1\nu_1(1) + (1-\lambda_1)\nu_1(0) < \tau_1\nu_1(p_1^{s_1^\star}) + (1-\tau_1)\nu_1(1)$$

$$\forall \tau_2 \in T_2(g_2)$$

for l_1 *sufficiently close to* h_1*.*

The proof is in the appendix. As market fundamentals for Firm 1 converge, i.e. $h_1 \searrow l_1$, expected profits from the opaque strategy also converge to the profit when firm 1 has complete information over state l_1, i.e. $\nu_1(p_1^{s_1^\star t})$ converges to $\nu_1(1)$. In case $p_1^{s_1^\star} > \lambda_1$, firm has a benefit from the opaque strategy that converges to a convex combination of $\nu_1(0)$ and $\nu_1(1)$ but that puts more probability on $\nu_1(1)$ than the Fully Disclosing experiment because, in the limit, $\tau_1 = \lambda_1/p_1^{s_1^\star} > \lambda_1$. In case $p_1^{s_1^\star} < \lambda_1$ then profits are simply converging to $\nu_1(1) > \lambda_1\nu_1(1) + (1-\lambda_1)\nu_1(0)$. Thus we see that the loss of coordination of policies with market conditions becomes irrelevant and the benefit of running an opaque strategy, or a more uniform approach in the market segmentation analogy of Blackwell Experiments, can become an important tool to take an opponent out of operation. By running one single policy across both states, l_1, h_1, Firm 1 can take Firm 2 in more states, states other than only the ones induced by l_1 but also for those states h_1 bundled into the opaque policy. Thus insensitivity to market fundamentals arise as a policy of deterrence.

I explain more carefully the intuition behind $\lim H_1^t/L_1^t < \infty$. As $h_1 \searrow l_1$ all market fundamentals are converging including player 2's. Moreover, Firm 2 fundamentals are unequivocally getting closer to interior solution level in absolute terms - see the rule graphed below. The condition $\lim H_1^t/L_1^t < \infty$ guarantees that, as fundamentals for Firm 1 converge, there is not enough approximation of the fundamentals of Firm 2 towards the Interior Solution levels. Thus, this condition is making sure that Firm 2 ”does not get good enough”, as market fundamentals converge, so as to force Firm 1 into more and more revealing posteriors, $p_1^{s_1^\star}$.

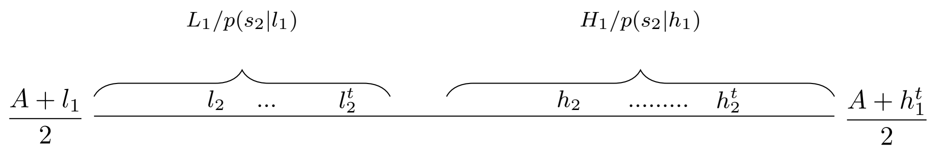


Figure 5: The interval $(\frac{A+l_1}{2}, \frac{A+h_1}{2})$ as $t \to \infty$ and $h_1^t \searrow l_1$

(a) I keep the proportions H_i/L_i relatively fixed as the interval collapses. This yields $\lim p_1^{ts_1^\star} \in (0,1)$. $\lim H^t/L^t < \infty$ should be read as "2 does not get too good" .

Proposition 2. *Assume player 1 is pivotal. Then, if $h_1^t \searrow h_1$ and $\lim \frac{H_1^t}{L_1^t} \in (0,\infty)$, Full disclosure is strictly dominated by a Partial Revelation strategy $\tau_1^\star$, where $supp(\tau_1^\star) = \{p_1^{s_1^\star}, 0\} \cup \{p_1^{s_1^\star}, 1\} \cup \{p_1^{s_1^\star}, p_2^{s_2^\star}\}$.*

If types for each firm were perfectly correlated, Partial Revelation by the pivotal firm would be broken or become irrelevant since Firm 2, the non pivotal Firm, would always have the option to reveal more information in order to understand when the low state of 1, h_1, is realized and thus whether it could benefit from operation. This would destroy any Obfuscation attempt by Firm 1. Here perfect correlation not only induces Full Revelation as a possible equilibrium but it makes all type of Partial Revelation be challenged against a Fully Revealing outcome. Therefore a Full Disclosure strategy serves as a threat point with strong correlation levels. In other words, it is only with the presence of non perfect correlation that Obfuscation can arise.

3.3.1 Application: Market Segmentation in the Rural Areas

I now apply the interpretation of Blackwell Experiments as market segmentations to the competition between two large retailers. Assume that two firms, WM and T are competing and the only source of uncertainty in payoffs is whether there is or not the presence of a tax incentive affecting marginal costs of operation. I assume that demand is given by $P_i^d = A - \kappa_i Q_j - \frac{1}{2c_i} Q_i$ and that marginal costs are given by θ_i.

As I assume a binary state of the world for each firm, I let $\theta_i = \{h_i, l_i\}$ so that in case Firm i received an incentive the state of the world for i is l_i and in case Firm i did not receive a tax incentive, the state is h_i. With this specification we fall back into the setup of the previous section, where the presence of a pivotal firm and the condition of homogeneous market fundamentals made Obfuscation be present in the form of thick market segmentations. In particular, if we assume WM is pivotal, then WM has an incentive to run one single policy across all neighborhoods that are sufficiently close to each other, i.e. the policy should be uniform for sufficiently homogeneous fundamentals, l_{WM}, h_{WM} that have small enough distances $d(l_{WM}, h_{WM}) = ||l_{WM} - h_{WM}||$ is sufficiently small. If a small enough change in taxes is able to make WM take T out of operation , an Opaque strategy from WM dominates Full Disclosure and thus a decentralized, and sensitive to fundamentals, approach, is deferred by a strategy that is the same for multiple states $\theta_{WM} = \{l_{WM}, h_{WM}\}$ in which $d(l_{WM}, h_{WM})$ is sufficiently small. A potential empirical question that emerges then is to whether Walmart's operation is sensitive to tax changes in rural towns or whether a more uniform tax system can induce less competition. [13].

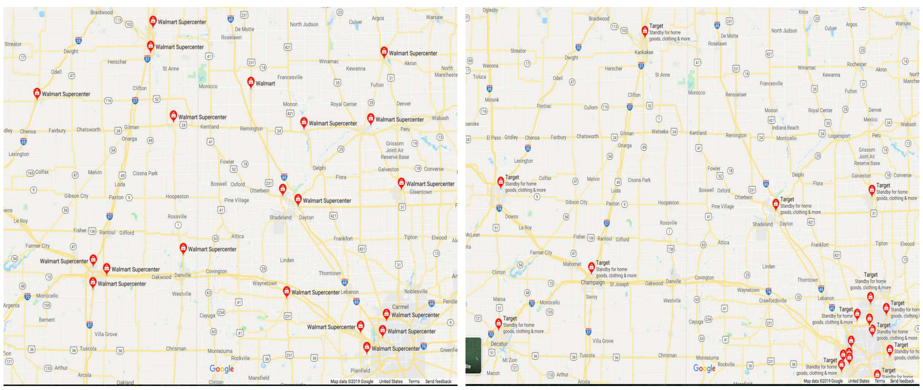


Figure 7: *The distribution of Walmart's and Targets northwest of Indianapolis.*

(a) Note that store policies differ between chains Target, T, and Walmart, WM. Target has a clear presence in the Urban areas of Indianapolis and it's suburbs. As we move away from the urban areas and approach the rural portions, Walmart dominates the map with a uniform strategy. This paper shows that WM policies on the northern portions of this map, the rural areas, may be insensitive to local market conditions such as local taxes because many stores in those regions are there for a preclusion effect and not a market driven motive. Source: Google Maps

[13] In the figure for the surroundings of Indianapolis, look at the northwest region of Indianapolis and the dominance of Walmart over Target. Note that this dominance is true for a small population rural area - I have scanned pretty much the Midwest and South; the google maps images seems to exhibit the same pattern, with Target competing, and, more rarely, dominating, small and large urban areas, whereas Walmart completely dominates the rural and deep suburban areas. In the appendix I provide a satellite image to show that the region depicted is indeed rural.

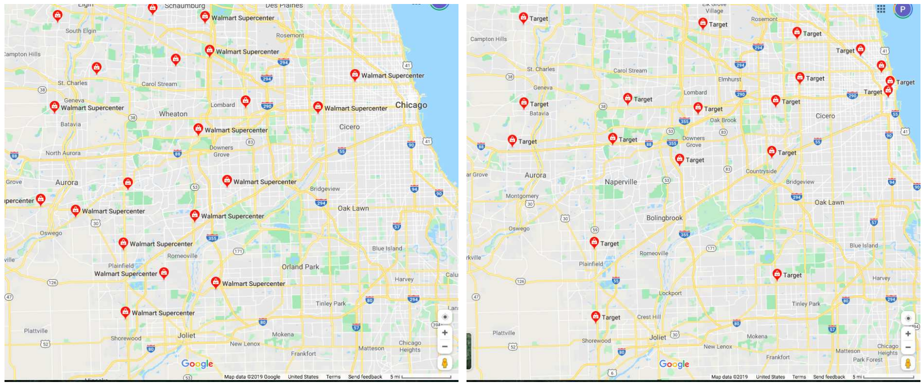


Figure 9: *The distribution of Walmarts and Targets in the Chicago region.*

(a) Note that here both players are playing interior solutions and policies are homogeneous. Source: Google Maps

I see this contrast between rural and subsurban areas and the differences in the large retail chains policies as indication that the model can explain interesting retail market segmentations phenomena, as in Walmarts vs. Targets distributions policies.

4 Conclusion

I have shown that Persuasion models are important tools to endogenize information structures in standard economic models like Cournot or Bertrand. If firms can disclose Blackwell Experiments Publicly, a trade-off between more or less revealing experiments arise. On one hand, Fully Revealing Experiments enable firms to adapt to local market conditions in a nationwide industry, or to adapt to the true state of the world under uncertainty. The cost of more revelation by firms is that, by revealing information publicly, competitors know more about the overall state of the world. In being more informative to themselves, firms end up also revealing information to their opponents. Opaque Signal Structures, on the other hand, hide information from the opponents and from the own firm in the action stage. Firms that know less about their own types can commit better to aggressive policies, increasing the frequency in which the opponent is out of operation - i.e. a deterrence strategy. The cost of being less informative is that companies loose the capacity to coordinate their actions with the realized states of the world.

Therefore, I describe a trade-off between coordination of actions with states of the world and the commitment to run aggressive policies and induce desired distribution of actions by the opponents. Here information in the action stage is symmetric and the setup studied is of continuous static games. Similar models can be used to study models with asymmetric information in the action stage or that have a dynamic setup of negotiation - a bargaining model of information exchange, for example.

In this paper I endogenize the Information Structure of canonical economic setup. In *market* economic setups it may not be reasonable to assume an exogenous signal structure when firms, or economic agents, themselves can engage in informative policies. Therefore a more detailed, industry-based, modelling of the information setups can be of help in understanding the Information Structures that could arise in a market outcome.

In the Cournot case, I show that firms might engage in store implementation policies that insensitive to local market arose as a commitment device to pursuing an aggressive store quantities policy, a "Flooding" behavior, that can be optimal when the flooding firm is indifferent in terms of fundamentals across different markets. In this case controlling the distribution of opponents via insensitivity of the number of stores across markets will be desired because fundamentals are similar across markets. Thus even though the firm policies ignore local market conditions, the sub-optimal assignment of policies is obfuscated by the effect of increasing the frequency in which the firm drives the opponent out of operation.

5 Appendix 1: U-Cover Analogues for Affine Actions

5.1 Local Preference for Information

I start with a reduced form approach that puts in evidence the main technical contribution of this paper. The economic models that are behind the equilibrium assumptions will be made clearer in the next section and should be a motivation for the techniques developed here. Assume N senders are disclosing public signals over a finite set of states of the world, $(\theta_i)_{i\in N}$, that are jointly distributed according to a common prior, $\mu \in \Delta(\Theta)$, with support $\Theta = \{\theta = (\theta_i)_{i\in N} : \mu(\theta) > 0\}$. Importantly, senders can only make Blackwell Experiments over the i'th dimension of the state $\theta = (\theta_i)_{i\in}$, i.e. the Blackwell experiments take the form $(S_i, \pi_i : \Theta_i \to \Delta(S_i))$ for $|S_i < \infty|$, or a Bayes Plausible distribution $(\tau_i, S_i) \in \Delta(\Delta(\Theta_i))$.[14] Note that this environment is not Blackwell Connected, as in Gentzkow and Kamenica (2017), as senders can only make experiments over θ_i but the (payoff relevant) state of the world is the joint random variable $\theta = (\theta_i)_{i\in N}$. Both, Sender i and receiver i, derive utility over (θ_i, a) given by $u_i(a_i, a_{-i,\theta_i})$, assumed linear and separable in a_{-i} and θ_i and I assume that u_i depend on the uncertain variable only through it's ith component, θ_i.

Assumption 1. *$u_i(a_i, a_{-i}, \theta_i)$ is linear and separable in a_{-i}, θ_i.*

After the senders design experiments in period 1, all is made common knowledge, i.e. $(\tau_i)_i$ is observed, and, in period 2, a joint signal s, distributed according to τ and yielding posterior p^s [15], is realized and a set of N receivers with preferences given by $U_i^{p^s} = \mathbb{E}_{p^s} u_i(a_i, a_{-i}, \theta_i), \forall i \in N$ play a simultaneous move complete information game with action space $a_i \in [0, \overline{a}]$ - i.e. $G^{p^s} = \{(U_i^{p^s}, [0, \overline{a}])_{i\in N}\}$. As utility is linear in θ_i, preferences for player i, after posterior p^s, are given by the utility function $U^{p^s}(a_i, a_{-i}) = u_i(a_i, a_{-i}, \theta_i^{p^s})$, for $\theta_i^{p^s} \equiv \mathbb{E}_{p^s}\theta_i$ such that $a_i(\theta^{p^s}) = argmax_{a_i\in[0,\overline{a}]}\{U^{p^s}(a_i, a_{-i}(\theta_i^{p^s}))\}$ is the equilibrium action for player i after posterior p^s. It is a natural consequence of linearity and the environment of public signals then to assume that equilibrium actions depend on p through it's first moment, θ^p, so that $a_i : Co(\Theta) \to A$, $\forall i \in N$. I.E. given a pair $p, p' \in \Delta(\Theta)$ such that $\theta^p = \theta^{p'} \Rightarrow a(\theta^p) = a(\theta^{p'})$.

We can then write the sender's the value from a realized posterior p in period 2 for sender i is $v_i(\theta^p) = u_i(a_i(\theta^p), a_{-i}(\theta^p), \theta_i^p)$ and the value from Blackwell Experiment τ_i is $V_i(\tau_i, \tau_{-i}) = \mathbb{E}_s v_i(\theta^{p^s})$. I am interested in the S.P.E. of the senders of information in the first period, $\tau^\star$, where $\tau_i^\star(\tau_{-i})$ is a signal best response for sender i after belief τ_{-i} of the opponents, when player i anticipates that, when $p^s \in \Delta(\Theta)$ is realized, $(a_i(\theta^{p^s}))_{i\in N}$ will be played by all players .

[14]Letting $mu_i = marg_{\theta_i}\mu$ make (τ_i, S_i) be such that $\tau_i \in \Delta(\Delta(\Theta_i))$ and $\mathbb{E}_\tau p^s = \mu_i$ we have an Homeomorphism between τ and π. For more see Gentzkow and Kamenica (2011).

[15]Specifically the frequencies of the signals are $\tau(s) = \sum_\theta \mu(\theta)\Pi_i\pi_i(s_i|\theta_i)$ and the posterior associated is obtained by $p^s(\theta) = \dfrac{\mu(\theta)\Pi_i\pi_i(s_i|\theta_i)}{\tau(s)}$, both numbers between 0 and 1.

Now, assume that there exists a finite cover Y such that $Co(\Theta) = \underset{y \in Y}{\cup} y$ for y a compact and convex set of $Co(\Theta)$ and Y finite. Thus the map $f : \Delta(\Theta) \rightrightarrows y$ defines a compact cover for $\Delta(\Theta)$ because $g : \Delta(\Theta) \to Co(\Theta)$ is continuous.

Assumption 2. *For each player, $a_i : Co(\Theta) \to [0, \overline{a}]$ is continuous, and $a_i(\theta^p)$ is piece-wise affine in $y \in Y$, $\forall i \in N$, $\forall \theta^p \in f^{-1}(y) \subset Co(\Theta)$. Thus, $\forall \theta^p \in y$, $a_i(\theta^p) = \beta_i^y \theta^p + H_i^y$, if $a_i(\theta^p) \in [0, \overline{a}]$.*

It turns out that under linearity of the actions with respect to posterior moments and under linearity of the payoff function over opponents actions, player i has information revelation as a weakly dominant strategy.

Theorem 2. *$v_i(\theta^p)$ is weakly convex in θ^p in y, $\forall y \in Y$. In other words, given $\tau, \tau' \in \Delta(\Delta(\Theta))$, distributions over posterior distributions s.t. $\tau \succeq_{M.P.S.} \tau'$ and such that $\forall p \in supp(\tau) \cup supp(\tau')$, $f(p) \in y$, we have $\mathbb{E}_\tau v_i(\theta^p) \geq \mathbb{E}_{\tau'} v_i(\theta^{p'})$.*

pf: Given θ^p make $\tau \in \Delta(Co(\Theta))$ be such that $\mathbb{E}_\tau \theta^s = \theta^p$ and that $p^s \in f^1(y), \forall s \in S = (S_i)_{i \in N}$. Then, just taking into consideration the opponents reactions $a_{-i}()$ to the m.p.s. of θ^p, we get $v_i(\theta^p) = u_i(a_i(\theta^p), a_{-i}(\theta^p), \theta_i^p)\mathbb{E}_p) = \mathbb{E}_\tau u_i(a_i(\theta^p), a_{-i}(\theta^{p^s}), \theta_i^{p^s}) \leq \mathbb{E}_\tau v_i(\theta^{p^s})$ since $v_i(\theta^{p^s}) = u_i(a_i(\theta^{p^s}), a_{-i}(\theta^{p^s}), \theta_i^{p^s}) = argmax_{a_i} u_i(a_i, a_{-i}(\theta^{p^s}), \theta_i^{p^s})$. ■

The second equality says that player i is indifferent after revealing more information to the opponents because the opponents reaction is linear, but then a more local point optimization given by $a_i(\theta_i^{p^s})$ is a weak improvement to player i. The theorem should be read as local preference for revelation for designers of information. Thus, as long as the categories of equilibrium actions, y, are fixed, all players have weak preference for more information on the Blackwell order for distributions over posterior distributions, $p \in \Delta(\Theta)$, τ. In particular, we will see that senders will have local preference for more revelation. The importance of this theorem is not only on it's effect on the the strategic forces but on existence of best responses $\tau_i^\star(\tau_{-i})$ with a finite set of candidates $p_i \in supp(\tau_i^\star(\tau_{-i}))$.

Note that for each player i, actions, $a_i()$, is a continuous in $\theta^p \in Co(\Theta)$ and so $v_i()$ is continuous in $\theta^p \in Co(\Theta)$. By Caratheodory, the optimal solution lies in the set of extreme points of the graph of $\nu_i(p_i^{s_i}, \tau_{-i})$ where

$$\nu_i(p_i^{s_i}, \tau_{-i}) \equiv \mathbb{E}_{s_{-i}|s_i} v_i(\theta^{p^s})$$

as a function of $p_i^{s_i}$ - the problem of sender i can be written as $V_i(\tau^\star_{\tau_{-i}, \tau_{-i}}) = max(z : ((z, \mu_i) \in co(p_i, \nu_i(p_i))))$ for $\mu_i = marg_{\theta_i}\mu \in \Delta(\Theta_i)$ and $p_i \in \Delta(\Theta_i)$. Note that $\nu_i()$ is a continuous function of $p_i \in \Delta(\Theta_i)$ and thus it's convex hull is a compact space. However $Graph(\nu_i)$ might have an infinite amount of extreme points so that characterization of optimal best responses $\tau_i^\star()$ might be troublesome. I show now that in the problem I work with there are a finite set of extreme points if actions are affine and value functions are linear and separable in states and actions of the opponents, a_{-i}, θ_i^p.

Suppose that $a_i(\theta^p) = \min\{\max\{w_i(a_{-i}(\theta^p), \theta_i^p), 0\}, \overline{a}\}$, for $w_i(.,.)$ linear and separable in a_{-i} and θ_i^p. Note that w_i being linear in a_{-i}, and a_{-i} being affine in $\theta^p \in Co(\Theta)$ makes $w_i(a_{-i}(\theta^p), \theta_i^p)$ be affine in θ^p [16]. For shortness, I abuse notation and write $w_i(\theta^p) = w_i(a_{-i}(\theta^p), \theta_i^p)$. I let $H(p) = \{i \in N : w_i(\theta^p) \geq \overline{a}\}$ and, analogously, $I(p) = \{i \in N : w_i(\theta^p) \in [0, \overline{a}]\}$, $L() = \{i \in N : w_i(\theta^p) \leq 0\}$ to be the categories , the classes of the equilibrium actions of the receivers after observing posterior, p. Note that $a_i(\theta^p) = \overline{a} \Leftrightarrow i \in H(p)$ and analogously for I and L so that, using the fact that $w_i(\theta^p)$ is affine, these regions define half spaces in $\theta^p \in Co(\Theta)$ and being a finite set of half spaces, we have finitely many extreme points. Now to navigate to $p \in \Delta(\Theta)$, I define $f : \Delta(\Theta) \to (L(), I(), H())$ [17]. Letting $Y \equiv \{y = (L, I, H) : y \in range(f()\}$ note that Y is finite. Moreover, the set $f^{-1}(y)$ is convex because y is convex and f is affine in θ^p inside a specific y by assumption, so that if we complete the *index profile* y with it's closure, $\overline{y} = \{y' : y' \supset y\}$, we get a compact and convex finite dimensional set $\overline{y}$. Moreover, since w_i is affine in θ^p because a_{-i} is affine in θ^p given $\theta^p \in \overline{y}$, then the sets $\overline{y}$ are finite intersections of half spaces and thus have finitely many extreme points.

Now let $d_i(p_i^{s_i}) = (\overline{f(p^s)})_{s_{-i}}$ with range $Z_i = \{z_i \in range(d_i(p_i^{s_i}))\}$ and not that $|Z_i| < \infty$ it's pre-image $d^{-1}(z_i)$ is a compact convex finite dimensional set that is a finite intersection of closed half spaces because the intensive margin decision, $w_i(\theta^{p^s}) = u_i(a_{-i}(\theta^{p^s}), \theta_i^{p^s})$ is a composition of a linear function of opponents actions, $a - i$, with a linear (or affine) function in θ^{p^s} given by $a_{-i}()$ by assumption. Thus I conclude that $d_i^{-1}(z_i)$ has finitely many faces and thus extreme points. Intuitively, d_i gives the distribution of index profiles obtained by i with policy $p_i^{s_i}$ when opponents play τ_{-i}. Following closely Lipnowski and Mathevet (2019), we get

Proposition 3. *$\nu_i()$ is piece-wise convex in $d^{-1}(z_i)$.*

pf: Given $p_i \in d_i^{-1}(z_i)$, pick any mean preserving spread of p_i, (τ_i, S_i) such that $p_i^{s_i} \in supp(\tau_i) : p_i^{s_i} \in d_i^{-1}(z_i), \forall s_i \in S_i$. By the Law of Iterated Expectations, we have

$$\nu_i(p_i) = \mathbb{E}_{s_{-i}|p_i} v_i(\theta^{p_i s_{-i}})$$

$$= \mathbb{E}_{S_{-i}|p_i} v_i(\mathbb{E}_{S_i|s_{-i}} \theta^{s_i, s_{-i}}) \leq \mathbb{E}_{S_{-i}|p_i} \mathbb{E}_{S_i|s_{-i}p_i} v_i(\theta^{s_i s_{-i}})$$

$$= \mathbb{E}_{S_i|p_i} \mathbb{E}_{S_{-i}|p_i S_i} v_i(\theta^{p_i s_i s_{-i}}) = \mathbb{E}_{S_i|p_i} \nu_i(p_i^{s_i})$$

■

A mean preserving spread of p_i induces a mean preserving spread of the resulting posterior p^s and the resulting market fundamental θ^{p^s} for every signal realization of the opponents, s_{-i}. Now we have $d^{-1}(Z_i)$ forms a ν_i-cover to use the term coined in Lipnowski and Mathevet (2019). Now I can apply a similar approach as theirs to prove

Proposition 4. *Given 1 and 2, the following holds:*

[16]Later we will see that w_i is the intensive margin decision of player i, i.e. receiver i's decision after observing a_{-i} and ignoring the constraints given by $a_i \in [0, \overline{a}]$.

[17]After performing the continuous map $\Delta(\Theta) \to Co(\Theta)$ such that $p \to < p, \Theta >$

1. *The problem* $\max\{z : (\mu_i, z) \in Co((p_i^{s_i}, \nu_i(p_i^{s_i})))\}$ *is well defined and obtained with at most* $|\Theta_i|$ *extreme points of* $(p_i^{s_i}\nu_i(p_i))$.

2. $\tau_i^\star(\tau_{-i}) \in \Delta(Ext(d^{-1}(Z_i)))$. *There are finitely many candidates* $|Ext(d^{-1}(Z_i))| < \infty$.

pf: Existence of an information best response is a consequence of continuity of $\nu_i()$. That any non degenerate point $p_i^\star$ must be extremal in whichever z_i it generates is a consequence of convexity of $\nu_i()$ in $d_i^{-1}(z_i)$ given in proposition (3). Finiteness of $Ext(d^{-1}(Z_i))$ comes from $|Z_i| < \infty$ and that $\forall z_i \in Z_i, d_i^{-1}(z_i)$ is a finite intersection of half spaces and thus have finitely many extreme points. ■

Proposition 5. $Ext(d_i^{-1})(z_i) \subset \{p_i^{s_i} \in d_i^{-1}(z_i) : w_j = 0 \vee \overline{a}, j \in N\}$, $\forall z_i \in Z_i$

To see the reason that this proposition is true and in particular the reason that $w_j = 0 \vee A$ for some player j contains the extreme points of the graph of $\nu_i()$ for player i, I show that an analogous rationality to that of Theorem 2 applies in a compact and convex subset of the posterior distributions, $\Delta(\Theta_i)$. Suppose that player i held a non-degenerate posterior $p_i^{s^\star i}$ under τ_i and assume that $a_j(p^{s^\star i s_{-i}}) \in (-\infty, 0) \vee (0, \overline{a}) \vee (\overline{a}, \infty), \forall s_{-i}, \forall j \in N$. Note that a m.p.s. of $p_i^{s^\star i}$ reflects into a m.p.s. of p^s, for each $s_{-i} \in S_{-i}$. There exists, thus, a new strategy that makes a m.p.s. of $p_i^{s^\star i}$ but makes the action profile y^s the same for each s_{-i} because actions are continuous and m.p.s.'s posteriors can be made arbitrarily close to the average distribution $p_i^{s_i}$. Since actions are piece-wise affine inside a region that yields the same index $y = (H, L, I)$, for each s_{-i}, the index $y^{s_{-i}}$ is kept constant and so Theorem (2) applies. A mean preserving spread of $p_i^{s_i}$ induces a mean preserving spread of p^s, $\forall s_{-i} \in S_{-i}$, that is an improvement for Firm i, at least weakly. As in Theorem 1, Linear reactions and the stability of the index of players in each action category, y, translates into preference for more information locally.

The indifference condition, $w_j(a(\theta^{s_i^\star, s_{-i}}), \theta_j^{s_i^\star, s_{-i}}) = 0 \vee \overline{a}$, means that some Firm j is made on the verge of entering the Interior Solution Condition, given by region I. The extreme points of these sets y, then, are points where the corner solution regions H, L are explored at it's maximum level in the mean preserving spread order.

In terms of existence of an information equilibrium, $\tau^\star$, I apply a standard Maximum Theorem leading to a Kakutani's Fixed Point theorem after I introduce a metric for policies τ_j, namely the Wasserstein Metric for discrete probability measures that is widely used in categorization exercises and $k-$nearest neighbours methods in computer science[18]. The Wasserstein metric metrizes the weak convergence notion[19]. Given τ_{-i}, the optimal policy of sender i, $\tau_i^\star(\tau_{-i})$, is obtained as a convex combination of a finite set of candidates $\tau_i^\star \in Co(ext(d^{-1}(Z_i)))$ and thus can be summarized in a finite dimensional

[18]In Computer the Wasserstein is known as the Earth Moving Distance.

[19]I provide a formal treatment of the Wasserstein Metric in the appendix.

fixed vector - by assigning zero probabilities to some extreme point candidates perhaps - and by Caratheodory the optimal $\tau_i^\star$ is obtained with at most $|\Theta_i|$ elements. Now, if we start with the assumption that the opponents strategies τ_{-i} have at most $|\Theta_j|$ for each $j \neq i$ then, without loss, we can pay strict attention to strategies that are $|\Theta_i|$ vectors. Then I can apply the maximum theorem, using the notion of Wasserstein convergence that is analogous to weak convergence, to say that $\tau_i^\star(\tau_{-i})$ is an Upper Hemicontinuous correspondence as a function of τ_{-i} and, with $\tau_i^\star(\tau_{-i})$ being an Upper Hemicontinuous object for each i, existence of an equilibrium, $\tau^\star$, is guaranteed.

Proposition 6. *There exists a best response $\tau_i^\star(\tau_{-i})$. Moreover, the best response is Upper Hemicontinuous w.r.t. $\tau_{-i} \in T_{-i} = (T_j)_{j\neq i}$ in the topology generated by the Wasserstein-1st order Product Metric for τ_{-i}.*

5.2 U-covers and Characterization of Generic Best Responses

In this convex geometry section I develop techniques that allow me to prove proposition (4).

Create categories for the types of linear portion of the best response, the w_i functions, let the index of *active players* be

$$\tilde{I}(\theta^p) = \{i \in N : A \geq w_i(\tilde{a}_{-i}(\theta^p), \theta_i^p) \geq 0\} \subset N$$

where $w_i(a_{-i}, \theta_i^p) = c_i[\sum_{j\neq i} c_{ij} a_j + \theta_i^p]$. This class of sets depict the agents that have some sensitivity of best responses after small perturbations in the equilibrium actions from the opponents. Similarly we can define $\tilde{I} = I \circ \mathbb{E}$

$$I(p) = \{\tilde{I}(\theta^p) : \mathbb{E}_p[\theta] = \theta^p\}$$

Define similar objects for the players that are playing corner solutions, either high or low actions, forming the set of *inactive* players

$$\tilde{L}(\theta^p) = \{i \in N : w_i(\tilde{a}_{-i}(\theta^p), \theta_i^p) \leq 0\} = L(p)$$

letting the range attained by some posterior statistic $\theta^p \in Co(\Theta)$ be

$$\mathcal{L} = \cup_{\theta^p \in Co(\Theta)} \tilde{L}(\theta^p) = \cup_{p\in\Delta(\Theta)} L(p) \subset N$$

and same for the players that are producing a fixed upper bound quantity

$$\tilde{H}(\theta^p) = \{i \in N : w_i(\tilde{a}_{-i}(\theta^p), \theta_i^p) \geq A\} = H(p) \subset N$$

and

$$\mathcal{H} = \cup_{\theta^p \in Co(\Theta)} \tilde{H}(\theta^p) = \cup_{p\in\Delta(\Theta)} H(p)$$

I define, then, the function that takes from market fundamentals to an equilibrium *profile of actions*

$$\tilde{f} : Co(\Theta) \to \mathcal{I} \times \mathcal{H} \times \mathcal{L}$$

The function $\tilde{f}$ indicate the *action profile* resulting from market fundamental $\theta^p \in Co(\Theta)$ associated to posterior $p \in \Delta(\Theta)$. I let $f(p) = \tilde{f} \circ \mathbb{E}_p$ where the operator $\mathbb{E}_p$ takes from $p \in \Delta(\Theta) \to E_p(\theta)$ and defines

$$f : \Delta(\Theta) \to \mathcal{L} \times \mathcal{I} \times \mathcal{H}$$

for $H(p^s) = \{j : w_j(a(\theta^{p^s}), \theta_j^{p^s}) \geq \overline{a}\}$, $L(p^s) = \{j : w_j(a(\theta^{p^s}), \theta_j^{p^s}) \leq 0\}$ and $I(p^s) = \{j : w_j(a(\theta^{p^s}), \theta_j^{p^s}) \in [0, \overline{a}]\}$ and I let

$$f(\Delta(\Theta)) = \tilde{f}(Co(\Theta)) \equiv Y$$

Proposition 7. *$\tilde{a}(\theta^p)$ is piece-wise affine in $\theta^p \in Co(\Theta) \cap \tilde{f}^{-1}(y)$ and, thus, on $\Delta(\Theta) \cap \tilde{f}^{-1}(y)$, for every $p \in f^{-1}(y)$, $\forall y = (I, L, H) \in Y$. Thus for all $i \in I$, $\tilde{a}(\theta^p) = \beta_I^i \theta^p + C_H^i$, $\forall \theta^p \in Co(\Theta) \cap \tilde{f}^{-1}(y)$, $\forall p \in \Delta(\Theta) \cap f^{-1}(y)$.*

pf: Assume $I(\theta^p) = I$. Assume that $a_i(\theta^p) = \beta_{iI}\theta^p$,$\forall i \in I$ and $\beta_{jI} = 0, \forall j \in L$. Then

$$BR_i = c_i[\theta_i^p + \sum_{j \in I} c_{ij}[\sum_{j'} \beta_{jI}^{j'} \theta_{j'}^p] + A \sum_{j \in H} c_{ij}]$$

$$= \underbrace{\theta_i^p (1 + \sum_{j \in I/i} c_{ij} \beta_{jI}^i)}_{\beta_{iI}^i} + \sum_{j \in I/i} \theta_j^p \underbrace{(\underbrace{c_{ij}\beta_{jI}^j}_{D.E.} + \underbrace{\sum_{j' \neq j} \beta_{j'I}^j c_{ij'})}_{I.E.}}_{\beta_{iI}^j} + A \sum_{j \in H} c_{ij}$$

I let $C_{iH} = A \sum_{j \in H} c_{ij}$. It remains to solve for $\beta_{iI}^j, j \in I$ which counts to $|(\beta_{iJ}^j)_{j \in I}| = I^2$. Now for each $i \in I$ we have $|I|$ equations and this solves for $\beta_{iI} = ((\beta_{iI}^j)_{j \in I})_{i \in I}$. to get the result. ■.

Proposition 8. *$f^{-1}(y)$ is a convex set.*

pf: Note that actions are affine, for each $i \in N$, and for each $p \in f^{-1}(y)$. Let $y = (L, I, H)$. Affine actions, as functions of the fundamentals, θ^p, show that the regions $H^{-1}(H)$, $I^{-1}(I)$ and $L^{-1}(L)$ are all half spaces in terms of fundamentals, θ^p. Now note that $f^{-1}(y) = L^{-1}(L) \cap I^{-1}(I) \cap H^{-1}(H)$ which is a finite intersection of half spaces and, as so, it is a convex region. ■

However since the function f may not be continuous in y, f^{-1} is not closed and thus not compact. In order to compactify the object $f^{-1}()$, I will cover a specific y with it's closure. The goal is to compactify y with it's closure and from there, obtain faces of compact convex finite dimensional sets that lead the search for extreme, optimal, points. To this end, and given a specific $y \in Y$, define the collection

$$\overline{y} = \cup\{y' \in Y : y' \supseteq y\}$$

to be the closure of y with universe defined as

$$\overline{Y} = \cup_y \overline{y}$$

Remark 5. *For every $y \in Y$, $f^{-1}(\overline{y})$ is a compact convex finite dimensional set. Moreover, since $f^{-1}(\overline{y})$ is a finite intersection of half spaces, and thus it has finitely many extreme points.*

The closure of y, $\overline{y}$, takes the action profile y and includes in the collection $\overline{y}$ all other action profiles that includes the action profile y but, in addition, the operator adds action profiles to the collection $\overline{y}$, $y' \supset y$, in which additional players are made indifferent between corner and interior solutions. If we take $y = (L^y, I^y, H^y)$, then $i \in K^y$ implies $i \in K^{y'}$ for $y' \in \overline{y}$, for $k = L^y, I^y, H^y \subset N$.

And of course, any market p lives in some $y = f(p)$ and so, for $\overline{y} = \{y' : y' \supseteq y\}$,

$$\Delta(\Theta) = \cup_{y \in Y} f^{-1}(\overline{y})$$

As we have $\Delta(\Theta) = \cup_{y \in Y} f^{-1}(\overline{y})$, Lemma 5.29 from Aliprantis and Border(1999) together with compactness and the cover properties pointed out in the previous two remarks yield

Remark 6.

$$Co(\Delta(\Theta)) = Co\{\cup_{y \in Y} Co(f^{-1}(\overline{y}))\}$$

$$Co(\Delta(\Theta)) = \{\sum_{y \in Y} \lambda_y * p_{\overline{y}} : \lambda \in \Delta(Y), p_{\overline{y}} \in f^{-1}(\overline{y})\}$$

Now the goal is to study the effects of a policy from player i on the distribution of outcomes so I consider the chain of continuous maps that result from fixing τ_{-i} and s_{-i}

$$\Delta(\Theta_i) \times T_{-i} \times S_{-i} \to \Delta(\Theta) \to Co(\Theta) \to a(Co(\Theta)) \to \overline{Y}$$

Therefore, let $\overline{y}(p_i^{s_i}, \tau_{-i}, s_{-i}) = \overline{f(p^s)}$. Then

$$d_i(p_i^{s_i}, \tau_{-i}) = (\overline{y}(p_i^{s_i}, \tau_{-i}, s_{-i}))_{s_{-i} \in S_{-i}}$$

with range

$$Z_i(\tau_{-i}) = \{d_i(p_i, \tau_{-i}) : p_i \in \Delta(\Theta_i)\}$$

and element $z_i \in Z_i$.

Let

$$f|_i : \Delta(\Theta_i) \times (s_j, \tau_j)_{j \neq i} \to Y$$

Note that

$$\Delta(\Theta_i) \equiv d_i^{-1}(Z_i) = \cup_{z_i \in Z_i} d_i^{-1}(z_i) = \\ \cup_{z_i \in Z_i} \cap_{s_{-i}} f|_i^{-1}(\overline{y}_{z_i}^{s_{-i}})$$

Proposition 9. *$d_i^{-1}(z_i)$ is a compact convex set. Moreover, $d_i^{-1}(z_i)$ has finitely many extreme points.*

pf: Compactness and convexity come from the fact that for each y, $f^{-1}(\overline{y})$ is a compact and convex set and $d_i^{-1}(z_i) = \cap_{s_{-i}} f|_i^{-1}(\overline{y}^{s_{-i}})$ which is an intersection of closed and compact spaces. Note that $(a_j(\theta^p))_{j\in N}$ is affine in $\forall p_i \in d_i^{-1}(z_i)$. The inequalities defined in z_i define a set of half spaces in $Co(\Theta) \cap z_i$. As $a_i = \min\{\max\{w_i(a_{-i}(\theta^p), \theta_i^p), 0\}, \overline{a}\}$ determines a closed half space of $\theta^p \in y$, it also defines a half space in p_i because, by the law of iterated expectations, $\theta^{s_{-i}p_i} = \mathbb{E}_{\theta_i\sim p_i}\mathbb{E}_{\theta_{-i}|\theta_i}\theta$ is affine in p_i. I.E. $f|_i^{-1}(\overline{y}^{s_{-i}})$ is a finite intersection of closed half spaces. This means that $d_i^{-1}(z_i)$ has finitely many extreme points. ■

Compactness and Convexity generate the proper conditions for the search over faces and extreme points in those sets using Caratheodory's Theorem. The argument for using z_i as a cover in order to organize the problem comes also from the fact that the policies τ_i can be depicted into distributions over $z_i \in Z_i$

$$Co((\Delta(\Theta_i), \nu_i(\Delta(\Theta_i))) = Co(\cup_{z_i\in Z_i} Co(\Delta(\Theta_i) \cap z_i, \nu_i(\Delta(\Theta_i) \cap z_i))$$

or

$$Co((\Delta(\Theta_i), \nu_i(\Delta(\Theta_i))) = \{(\sum_{z_i\in Z_i} \lambda_{z_i} p_i^{z_i}, \sum_{z_i\in Z_i} \lambda_{z_i}\rho_i(p_i^{z_i} \cap z_i)$$
$$: p_i^{z_i} \in \Delta(\Theta_i) \cap z_i, \forall z_i \in Z_i\}$$

Theorem 3. *Fix τ_{-i} and the resulting $d_i(\Delta(\Theta_i))$. Then*

$$V(\mu_i) = max_{\tau_i\in T_i(\mu_i)}\{v_i : v_i \in Co((\Delta(\Theta_i), \nu_i(\Delta(\Theta_i)))\} = \tag{3}$$

$$\max_{\tau_i\in T_i(\mu_i)}\{v_i : v_i \in Co(\cup_{z_i\in Z_i} \max_{t_i^{z_i}\in T_i(p_i^{z_i}\cap z_i)}\{\tilde{v}_i : \tilde{v}_i \in Co((\Delta(\Theta_i)\cap z_i, \nu_i(\Delta(\Theta_i)\cap z_i))\} \tag{4}$$

pf: From $Co((\Delta(\Theta_i), \nu_i(\Delta(\Theta_i))) = Co(\cup_{z_i\in Z_i} Co((\Delta(\Theta_i) \cap z_i, \nu_i(\Delta(\Theta_i) \cap z_i))\}$ and using Lemma 5.29 of Aliprantis and Border, we write $v_i \in \rho(\mu_{0i})$ as $v_i = \sum_{z_i} \tau_i(z_i)\tilde{v}_i^{z_i}$ for $\tilde{v}_i^{z_i} \in \rho(p_i^{z_i})|_{z_i}$. But $\tilde{v}_i^{z_i} \le V(p_i^{z_i})|_{z_i} = \max_{t_i^{z_i}\in T_i(p_i^{z_i}\cap z_i)}\{\tilde{v}_i : \tilde{v}_i \in Co((\Delta(\Theta_i) \cap z_i, \nu_i(\Delta(\Theta_i) \cap z_i))\}$ and because $T_i(p_i^{z_i} \cap z_i)$ is compact the maximum belongs to the set. Thus $\forall z_i \in Z_i, \exists t_i^{z_i} \in T_i(p_i^{z_i} \cap z_i) \subset \Delta(\Delta(\Theta_i))$: $\mathbb{E}_{t_i^{z_i}} p^{s_{z_i}} = p_i^{z_i}$ and $\mathbb{E}_{t_i^{z_i}} \nu_i(p^{s_{z_i}}) \ge \tilde{v}_i^{z_i}$, i.e. (3) $\le$ (4) .

As for the reverse direction, note again that the domain, $T_i(p_i^{z_i} \cap z_i)$, is a compact set and thus the compact image of the graph of a continuous function is a compact set and thus reaches it's maximum in the set. I.E., $\exists t_i^{z_i} \in \Delta(\Delta(\Theta_i))$ such that $p_i \in supp(t_i^{z_i}) \Rightarrow p_i \in z_i$ and thus $V(p_i^{z_i})|_{z_i}$ is well defined for any $p^{z_i} \in z_i, \forall z_i \in Z_i$ and I let $t_i^{z_i} \in T_i(p_i^{z_i} \cap z_i)$ be the resulting policy $\in \Delta(\Delta(\Theta_i))$. As $v_i \in Co(\cup_{z_i} V(p_i^{z_i})|_{z_i})$ then $v_i = \sum_{z_i} \tau_i(z_i)V(p_i^{z_i})|_{z_i}$ for $\tau_i \in T_i(\mu_{0i})$. Therefore, letting $r_i(s_i^{z_i}) = \tau_i(z_i) * t_i^{z_i}(s_i^{z_i})$ then $\sum_{z_i} \sum_{s_i^{z_i}} r_i(s_i^{z_i}) p_i^{s_i^{z_i}} = \mu_{0i}$ and thus I conclude that $r_i \in T_i(\mu_{0i})$. Finally $v_i = \sum_{s_i^{z_i}} r_i(s_i^{z_i}) \nu_i(p_i^{s_i^{z_i}})$ and thus $v_i \in Co((\Delta(\Theta_i), \nu_i(\Delta(\Theta_i)))$. I.E. (4) $\leq$ (3) ■

The above theorem states that the information design of player i can be separated into two steps. First, agent i assigns a distribution of posterior distributions, $\tau_i \in \Delta(\Delta(\Theta_i))$ with support given by $p_i^{z_i} \in supp(\tau_i)$ whence each $p_i^{z_i} \in supp(\tau_i)$ belongs to a specific z_i, i.e. $p_i^{z_i} \in d^{-1}(z_i)$. From there, player i takes $p_i^{z_i} \in supp(\tau_i^{z_i})$ as the new prior and optimally reallocates, generating signals $q_i^{s_i^{z_i}} \in \Delta(\Theta_i) \cap d^{-1}(z_i)$ under distribution $t_i^{z_i} \in \Delta(\Delta(\Theta_i))$, i.e. for all $q_i^{s_i^{z_i}} \in supp(t_i^{z_i})$, $q_i^{s_i^{z_i}} \in d^{-1}(z_i)$ and, at the same time $\mathbb{E}_{t_i^{z_i}} q_i^{z_i} = p_i^{s_i^{z_i}}$. This constrained reoptimization is only possible because $d_i^{-1}(z_i)$ is a compact, convex, set.

The designers must choose first an allocation of resources to attainable z_i's. Then, managers in charge of local segmentations will shift resources constrained to $p_i \in z_i$ to optimally allocate resources on their own. The headquarters takes this step as given in the value function $V(p_i^{z_i})|_{z_i} = \max_{t_i \in T_i(p_i^{z_i}) \cap z_i} \rho(p_i)$ that is the concavification constrained to z_i. So the objective of the headquarters becomes simple, after we solve the locally concavified problem

$$V(p_i^{z_i})|_{z_i} = \max_{t_i \in T_i(p_i^{z_i}) \cap z_i} \rho(p_i), \forall p_i^{z_i} \in z_i$$

As Caratheodory Theorem indicates, the maximum is attained using (at most $|\Theta_i|$) extreme points and, thus, finding the extreme points of the convex hull of the graph at z_i, $Co((\Delta(\Theta_i \cap z_i), \nu_i(\Delta(\Theta_i \cap z_i)))$ is important. Let me consider the interior of $d_i^{-1}(z_i)$, first.

Linearity of actions restricted to z_i is given by Proposition (7), and thus convexity of the value function, is still present, provided that we restrict the domain of posteriors over θ, p, to the set $z_i = \cap_{s_{-i}} d_i^{-1}(\overline{y}_{s_{-i}})$. Once again mean preserving spreads will be weakly preferred.

Proposition 10. *$\nu_i(p_i) = \mathbb{E}_{s_{-i}|p_i} v_i(\theta^{p_i, s_{-i}})$ is convex in p_i, for $p_i \in d^{-1}(z_i) = \cap_{s_{-i}} f^{-1}|_i(\overline{y}_{s_{-i}})$.*

Linearity once again will guarantee that optimal information policies in $T_i(p_i^{z_i} \cap z_i)$ must be maximal in the mean preserving spread order. An optimal strategy must be such that any mean preserving spread of a posterior $p', p'' : p^s = \alpha p' + (1-\alpha) p''$ has either p' or p'' falling out of $z_i = \cap \overline{y}_{s_{-i}}$. Imagine, on the contrary, that there exists $p' \neq p'' \in d^{-1}(z_i) : \alpha p' + (1-\alpha) p'' = p$. Since player i would have a weak preference for more decentralization, restricted to z_i, $\alpha p'' + (1-\alpha) p''$ would be weakly preferred to p. In fact, we can depict this in terms of convex geometry terms

$$ext(d^{-1}(z_i)) = \cup_{s_{-i}} ext(f^{-1}|_i(\overline{y}_{s_{-i}}) \cap z_i)$$

$$ext(f^{-1}|_i(\overline{y}_{s_{-i}}) \cap z_i) = \{p_i \in \Delta(\Theta_i) \cap z_i : T_i(p_i \cap \overline{y}_{s_{-i}}) = \{p_i\}\}$$

Proposition 11. *There exists an optimal* $t_i \in argmax[V(p_i^{z_i})|_{z_i}]$ *such that* $supp(t_i^{z_i}) \subset ext(d^{-1}(z_i))$. *I.E.* t_i *is maximal in the mean preserving spread order for distributions over* $p_i^{t_i} \in z_i$.

pf: The result follows from Corolary 2.
■

The non-degenerate extreme points in the space of private posteriors are dictated by regions where some player is indifferent between either playing a strictly positive amount and zero, $i \in I \cap L$, or in which players are indifferent between playing a strictly lower than A quantity or A, i.e. $i \in I \cap H$.

Fixing τ_{-i} and a realization s_{-i}, such that $y = y(p_i, s_{-i})$ and $\overline{y} = \{y' : y' \supseteq y\}$, let

$$F(z_i, H) = \bigcup_{y':H_{y'} \cap I_{y'} \neq \emptyset} \{p_i \in \Delta(\Theta_i) \cap z_i \cap y'\}$$

and similarly

$$F(z_i, L) = \bigcup_{y':L_{y'} \cap I_{y'} \neq \emptyset} \{p_i \in \Delta(\Theta_i) \cap z_i \cap y'\}$$

Proposition 12.

$$Bound(f|_i^{-1}(\overline{y}) \cap z_i) = F(z_i, L) \cup F(z_i, H)$$

pf: Note that $\{p_i \in \Delta(\Theta_i) \cap z_i \cap y'\}$ is a proper face of $f|_i^{-1}(\overline{y}) \cap z_i$ for $y' \in q \cap I$, with $q = \{0, \overline{a}\}$, and thus it belongs to the $Bound(f|_i^{-1}(\overline{y}) \cap z_i)$ - since the boundary of any space is the union of it's proper faces.

Now assume $p \notin F(z_i, L) \cup F(z_i, H)$ with the purpose to show that $p \in z_i / Bound(f|_i^{-1}(\overline{y}) \cap z_i)$. Then, $L(p) \cap I(p) \cup H(p) \cap I(p) = \emptyset$. Thus by picking any other element $p' \in f^{-1}(\overline{y}) \cap z_i$, we have $[\alpha p, p'] \subset f^{-1}(\overline{y}) \cap z_i$ for some $\alpha > 1$ and since p' is arbitrary, $p \in Int(f^{-1}(y) \cap z_i)$. ■

As extreme points are faces that are points any extreme posterior distribution for z_i, $p_i \in ext(d^{-1}(z_i))$ must be an extreme element for some $y' \in \overline{y}$ and for some $q = \{0, \overline{a}\}$.

Proposition 13. *There exists an optimal policy,* $\tau^\star(\tau_{-i})$*, such that for all non degenerate* $p_i^{s_i^{z_i}} \in \Delta(\Theta_i) \cap z_i$, $p_i^{s_i^{z_i}} \in ext(F(z_i, q))$ *some* $y' \in \overline{y}$, *for some* $q = \{0, \overline{a}\}$. *In other words, for some* $s = (s_i^\star, s_{-i})$, *for some player* $j \in N$, $w_j(a(\theta^{p^s}), \theta_j^{p^s}) = 0 \vee A$, *if* $p_i^{s_i^\star} \in supp(\tau_i^\star)$ *is non degenerate and for all* $T_i(p_i^{s_i^\star})|_{z_i} = \{p_i^{s_i^\star}\}$

Alternative proof of Proof of Proposition 4**:**

pf: By Theorem 3 (above), there exists an optimal policy $\tau^\star$ s.t. $\forall p_i^{s_i} \in supp(\tau_i^\star), p_i^{s_i} \in Ext(d_i^{-1}(z_i))$ so there exists an optimal policy $\tau^\star \in \Delta(ext(d_i^{-1}(Z_i)))$. By Proposition 9 there are finitely many candidates $|ext(d_i^{-1}(Z_i))| < \infty$. ■

5.3 Uniqueness of Equilibrium Actions w.r.t. Posterior p

In this section I show that, given $u_i(a_i, a_{-i}, \theta_i) = \frac{-a_i^2}{2c_i} + a_i \sum_{j \neq i} c_{ij} a_j + \theta_i a_i$ uniqueness of equilibrium actions for a given posterior distribution p is guaranteed if $|c_i c_{ij}| < \frac{1}{\sqrt{N-1}}$.

Proposition 14. *Assume that* $|c_i c_{ij}| < \frac{1}{\sqrt{N-1}}$. *Then there exists a unique Nash Equilibrium of the game* $G^p = \{(u_i^p, A_i)_{i \leq N}\}$ *for* $u_i^p = \frac{-a_i^2}{2c_i} + a_i \sum_{j \neq i} c_{ij} a_j + \theta_i^p a_i$.

Given two profiles of actions, $a = (a_i)_i, a' = (a'_i)_i$, we want to show that $d(BR(a), BR(a')) < \delta d(a, a')$ where $d(x, y) = [\sum_i (x_i - y_i)^2]^{1/2}$ and $\delta < 1$

$$(d(BR(a), BR(a')))^2 =$$

$$\sum_i (min\{max\{w_i(a_{-i}, \theta_i^p), 0\}, \overline{a}\} - min\{max\{w_i(a'_{-i}, \theta_i^p), 0\}, \overline{a}\})^2$$

$$= \sum_i |min\{max\{w_i(a_{-i}, \theta_i^p), 0\}, \overline{a}\} - min\{max\{w_i(a'_{-i}, \theta_i^p), 0\}, \overline{a}\}|^2$$

$$\leq \sum_i (w_i(a_{-i}, \theta_i^p), 0\} - w_i(a'_{-i}, \theta_i^p))^2$$

$$= \sum_i \sum_{j \neq i} [c_i c_{ij} (a_j - a'_j)])^2$$

$$\leq \sum_i \sum_{j \neq i} [\max_{i,j} |c_i c_{ij}| (a_j - a'_j)]^2$$

$$\leq \sum_i \overline{c}^2 (\sum_{j \neq i} (a_j - a'_j)^2) = (n-1)\overline{c}^2 \sum_i (a_i - a'_i)^2$$

$$= (n-1)\overline{c}^2 d(a, a')^2$$

For $\overline{c} = \max_{i,j} |c_i c_{ij}|$. Thus by making $\overline{c} * \sqrt{N-1} < 1 \Leftrightarrow \overline{c} < \frac{1}{\sqrt{N-1}}$ we are done. ■

5.4 Proof of Lemma 1

pf: Assume that $\forall j \neq i \; a_j(\theta^p) = \mathbb{E}_p a_j(\theta)$. Then

$$BR_i(\tilde{a}_{-i}(\theta^p), \theta_i^p) = \min\{\max\{c_i(\sum_{j \neq i} c_{ij} a_j(\theta^p) + \theta_i^p), 0\}, \overline{a}\} =$$

$$\min\{\max\{c_i(\sum_{j\neq i} c_{ij}\mathbb{E}_p a_j(\theta)+\theta_i^p),0\},\overline{a}\}$$

But then

$$BR_i(\tilde{a}_{-i}(\theta^p),\theta_i^p)=\min\{\max\{\mathbb{E}_p[c_i(\sum_{j\neq i} c_{ij}a_j(\theta)+\theta_i)],0\},\overline{a}\}$$

As we have assumed the Overall Interior Solution conditions is valid, or that $A \geq a_i(\theta) \geq 0$ for every θ, $a_i(\theta) = c_i(\sum_{j\neq i} c_{ij}a_j(\theta)+\theta_i) \in [0,\overline{a}]$, $\forall\theta\in\Theta$. This means that $BR_i(\tilde{a}_{-i}(\theta^p),\theta_i^p) = \min\{\max\{\mathbb{E}_p[a_i(\theta)],0\},\overline{a}\} = \mathbb{E}_p a_i(\theta) = \tilde{a}(\theta^p)$ which is linear in θ. Thus $a_i() \in [0,\overline{a}]$ and is linear in θ. ■

5.5 Proof of Theorem 1

pf: By linearity in a_i given by Lemma (1),

$$V_i(x_i,x_{-i}) \equiv \mathbb{E}_s u_i(\tilde{a}_{-i}(\theta^s),a_i(\theta^s),\theta_i^s) = \mathbb{E}_s u_i(a_i(\theta^s),\mathbb{E}_{\theta_i|s}\tilde{a}_{-i}(\mathbb{E}[\theta_{-i}|s,\theta_i],\theta_i),\mathbb{E}[\theta_i|s])$$

$$= \mathbb{E}_s\mathbb{E}_{\theta_i|s} u_i(a_i(\theta^s),\tilde{a}_{-i}(\mathbb{E}[\theta_{-i}|s,\theta_i],\theta_i),\theta_i) \leq$$

$$\mathbb{E}_s\mathbb{E}_{\theta_i|s} u_i(\tilde{a}_{-i}(\mathbb{E}[\theta_{-i}|\theta_i,s],a_i(\mathbb{E}[\theta_{-i}|\theta_i,s],\theta_i),\theta_i),\theta_i)$$

Note that this result is true for any x_{-i} and thus Full Disclosure is a weakly dominant strategy. As for strict dominance, the result follows iff for any θ_i,θ_i', and for any τ_{-i} and realization s, $a_i(\mathbb{E}[\theta_{-i}|\theta_i,s],\theta_i) \neq a_i(\mathbb{E}[\theta_{-i}|\theta_i',s],\theta_i')$. As $a_i(\mathbb{E}[\theta|\theta_i,s],\theta_i) = \beta_i\mathbb{E}[\theta|\theta_i,s]$. Integrating over s we get $\mathbb{E}_s[a_i(\mathbb{E}[\theta_{-i}|\theta_i,s],\theta_i)] = \beta_i\mathbb{E}[\theta|\theta_i]$ and by assumption $\beta_i\mathbb{E}[\theta|\theta_i] \neq \beta_i\mathbb{E}[\theta|\theta_i']$, for any θ_i,θ_i' so there must be some s such that $\beta_i\mathbb{E}[\theta|\theta_i,s] \neq \beta_i\mathbb{E}[\theta|\theta_i',s] \Leftrightarrow a_i(\mathbb{E}[\theta|\theta_i,s]) \neq a_i(\mathbb{E}[\theta|\theta_i',s])$. This means that player is strictly better off by applying a Full Disclosure policy. ■

5.6 Proof of Lemma 2

We need to prove that given $p_1^{s_1^\star} : w_2(\theta^{s_2 p_1^{s_1^\star}}) = 0, \forall s_2 \in S_2$, the binary signal strategy $\tau_1(p_1^{s_1^\star})\nu_1(p_1^{s_1^\star}) + (1-\tau_1(p_1^{s_1^\star}))\nu_1(p') > \lambda_1(1) + (1-\lambda_1)\nu_1(0)$, for $p'=0$ if $p_1^{s_1^\star} > \lambda$ or $p'=1$ for $p_1^{s_1^\star} < \lambda$. I.E. that a binary signal strategy that is Partially Revealing dominates a Full Disclosure strategy.

pf: Firs note that

$$\nu_1(1) = \frac{(A-l_1)^2}{4b}$$

$$\nu_1(p_1^{s_1^\star}) = \mathbb{E}_{s_2|p_1^{s_1^\star}}[\frac{(A-\theta_1^s)^2}{4b}]$$

Since by the Pivotal assumption when Firm 1 is h_1, Firm 2 is operating with positive quantities - $w_2(\theta^s) > 0$. In case $w_1(\theta^s) < 0$ then $A + \theta_2^s - 2\theta_1^s < 0$

and, in this case, Firm 1 is producing $Q_1 = 0$ when h_1 is realized. Thus $v_1(\theta^{s_2 d_1}) = 0 < \frac{(A+\theta_2^s - 2\theta_1^s)^2}{9b}$.

However, note that since conditional on the realization h_1, Firm 2 would be playing strictly positive quantities - by the Pivotal assumption. But note that Firm 2 is in operation iff $w_2(a(\theta^{s_2,h_1}), \theta_2^{s_2,h_1}) > 0$ iff $\frac{A+h_1-2\theta_2^s}{3b} > 0$ iff $\frac{A-h_1}{2} > \frac{A+\theta_2^s-2h_1}{3}$ iff $\frac{(A-h_1)^2}{4b} = v^m(h_1) > \frac{(A+\theta_2^s-2h_1)^2}{9b} = v^d(\theta_2^s, h_1)$. That means that conditional on being h_1, Firm 1 would prefer to be in a monopoly state.

As $v^m(h_1) = \frac{(A-h_1)^2}{4b} < \frac{(A-\theta_1^s)^2}{4b} = v^m(\theta_1^s)$ we then have $\nu_1(p_1) > \nu_1(0)$. Finally, $\nu_1(p_1) = \frac{(A-\theta_1^s)^2}{4b} < \frac{(A-l_1)^2}{4b} = \nu_1(1)$. This establishes that $\nu_1(0) < \nu_1(p_1) < \nu_1(1)$.

If $1 > p_1^\star > \lambda_1$ then $\tau_1(p_1^{s_1^\star}) = \lim \tau_1(p_1^{\star t}) = \frac{\lambda_1}{p_1^\star} > \lambda$. As $\nu_1(1)\lambda + (1-\lambda)\nu_1(0) < (1-\tau^t)\nu_1(0) + \tau^t \nu_1(p_1^{ts_1^\star}) \Leftrightarrow \nu_1(0) < \frac{\tau^t \nu_1(p_1^{ts_1^\star}) - \lambda\nu_1(1)}{\tau^t - \lambda}$ and, as $\lim \nu_1(p_1^{ts_1^\star}) = \nu_1(1)$ is finite and $\lim \tau^t - \lambda > 0$, then $\lim \frac{\tau^t \nu_1(p_1^{ts_1^\star}) - \lambda\nu_1(1)}{\tau^t - \lambda} = \lim \frac{\tau^t}{\tau^t-\lambda} \lim \nu_1(p_1^{ts_1^\star}) - \lim \frac{\lambda\nu_1(1)}{\tau^t-\lambda} = \nu_1(1) > \nu_1(0)$.

If $p_1^\star < \lambda_1$ then $\nu_1(1)\lambda + (1-\lambda)\nu_1(0) < \lim(1-\tau^t)\nu_1(1) + \tau^t\nu_1(p_1^{ts_1^\star}) = \nu_1(1)$ since $\lim \nu_1(p_1^{ts_1^\star}) = \nu_1(1)$. ∎

5.7 A Generalization of Lemma (2)

In fact, Partial Revelation dominates Full Revelation not only in a Cournot Duopoly with Homogeneous goods, but in a more generic Cournot Duopoly with heterogeneous goods. Let $u_i(a_i, a_{-i}, \theta_i) = \frac{a_i^2}{2} - \kappa_i a_i a_j^2 + \theta_i a_i$ where θ_i here should be read as the net marginal benefit of operation given by the intercept of the demand for firm i minus the marginal cost of firm i, or $\theta_i = A^{\theta_i} - c_{\theta_i}$. Then the same result holds when there exists a binary state for each player $\Theta_i = \{d_i, g_i\}$. Equilibrium quantities of Duopoly as a function of market fundamentals can be written as $\frac{\theta_i^s - \kappa_i\theta_j^s}{1-\kappa_i\kappa_j}$ and Monopoly quantities are simply θ_i^s. Value functions of a Duopoly and Monopoly, respectively, can be written as $(1/2)(\frac{\theta_i^s - \kappa_i\theta_j^s}{1-\kappa_i\kappa_j})^2$ and $(1/2)(\theta_i^s)^2$. In the same manner as before, Firm 2 is in operation iff $w_2(\theta^s) = \theta_2^s - \kappa_1\theta_1^s\theta_2^s - \kappa_2\theta_1^s > 0$ iff $\theta_1^s > \frac{\theta_1^s - \kappa_1\theta_2^s}{1-\kappa_1\kappa_2}$ iff $(1/2)(\theta_1^s)^2 > (1/2)(\frac{\theta_1^s - \kappa_1\theta_2^s}{1-\kappa_1\kappa_2})^2$ and so Firm 1 would prefer to be in a monopoly state whenever Firm 2 is in operation in equilibrium. I.E. $v_1^m(\theta_1^s) > v_1^d(\theta^s)$ whenever $w_2(\theta^s) = \theta_2^s - \kappa_1\theta_1^s > 0$. As a result, once more we will have $\nu_1(p^s) > \nu_1(0)$.

6 Appendix 2:

6.1 Private Information and Public Reports

In the previous setting, a common knowledge game, analogous to a complete information with symmetric information unfolded, since players did not have access to any private information.

I study, now, cases where players receive a perfectly revealing information over her private state θ_i in $t = 1$ according to the market distribution, i.e. that $\theta \sim p^s$, $\forall s \in S$. Here we will see that, once again, linear models entail Decentralization as a weakly dominant strategy in the case where the Overall Interior Solution Condition is assumed binding.

Here goes the timing in more detail:

Timing:

t=0:

$(x_i)_{i \leq I}$ is engineered.

t=1:

s is made public after all agents can access the technologies chosen in $t = 0$, i.e. (x_i, s_i) is known for all i.

In addition, agents receive a perfectly revealing message that tell them what is their private state of nature θ_i, according to the posterior distribution $p(.|s) \in \Delta(\Theta_i)$.

t=2:

The individual information piece is $I_i = \{i_i = (s = (s_i)_{i \leq N}, \theta_i)\}$. A strategy in this stage is a contingency plan $a_i : S \times \Theta_i \to \Delta(A_i)$. A strategy is consistent with beliefs $Y_i : S \times \Theta_i \to \Delta(A_{-i})$ if it is optimal given $p^s \in \Delta(\Theta)$ and Y ,$\forall \theta_i \in \Theta_i$, $\forall s$.

I analyze Bayes Nash Equilibrium on this stage of actions.

Results:

We then have the presence of an incomplete information environment under a common posterior distribution p^s, after realization of signal s. I will denote the Bayes Nash Solution by $BN(p^s)$ to be the profile (for now assumed unique) of best responses analyzed in equilibrium. Specifically, let $p_{-i}^{s,\theta_i} = p(\theta_{-i}|s, \theta_i), \forall \theta_{-i}$ to be the probability of the other-than-i types after realization of signal s and type realization θ_i. Note that p_{-i}^{s,θ_i} can be computed by making $p_{-i}^{s,\theta_i}(\theta_{-i}) = p(s_{-i}|\theta_{-i})p(\theta_{-i}|\theta_i)p(s_i|\theta_i)p(\theta_i)/p(s)p(\theta_i|s)$.

By letting the beliefs be

$$Y_i^{p^s} = \{y_i^{p^s} \in \Delta(\theta_{-i}, a_{-i}) : \int_{a_{-i}} y(da_{-i}, d\theta_{-i}) = p^s(\theta_{-i})\}$$

We then have

$$BR_i(y_i^{p^{s,\theta_i}}, \theta_i) = \underset{a_i \geq 0}{argmax}[\mathbb{E}_{y_i^{p^{s,\theta_i}}} u_i(a_i, y_i^{p^{s,\theta_i}}(a_{-i}), \theta_{-i}), \theta_i)]$$

$$a_i(p^s, \theta_i) = \underset{a_i \geq 0}{argmax}[\mathbb{E}_{p_{-i}^{s,\theta_i}} u_i(a_i, a_{-i}(p^s, \theta_{-i}), \theta_i)]$$

$= \max\{k_i[\mathbb{E}_{p_{-i}^{s_{-i},\theta_i}}[\sum_{j \neq i} c_{ij} a_j(p^s, \theta_j)] + \theta_i], 0\}$.

I will show that, if the Interior Solution Condition is satisfied and the model is linear, equilibrium best responses are unique and linear in the distribution p.

Lemma 3. *If the interior solution condition is satisfied $BR_i(p, \theta_i)$ is linear in p, $\forall i \leq I$.*

pf: $BR_i(p^s, \theta_i) = \max\{k_i[\mathbb{E}_{p_{-i}^{s,\theta_i}} \sum_{j \neq i} c_{ij} a_j(p^s, \theta_j)] + \theta_i], 0\}$. Then, assume that $a_j(p^s, \theta_j) = \mathbb{E}_{p^s} a_j(\theta)$ is linear and strictly positive for every j, θ_j. Then $BR_i(p^s, \theta_i) = max\{\mathbb{E}_{\theta_{-i}|s,\theta_i}[k_i \sum_{j \neq i} c_{ij} \mathbb{E}_{\theta_{-j}|s,\theta_i,\theta_j} a_j(\theta, \theta_j) + \theta_i], 0\} =$
$\mathbb{E}_{p^{s,\theta_i}}[k_i \sum_{j \neq i} c_{ij} a_j(\theta, \theta_j) + \theta_i]$ where the last equality follow from the interior solution condition ■

Proposition 15. *In the case of private values and interior solution, the Bayes Nash Equilibrium strategies generated are linear in p^s. I.E.,*

$$a_i(p^s, \theta_i) = \mathbb{E}_{p^{s\theta_i}} a_i(\theta, \theta_i)$$

Thus Full Disclosure for every i, and Full Revelation as whole, is the unique equilibrium.

In the appendix I provide the computation of the Cournot example with private values which results in a strictly convex function in the posterior $p_i^{s_i}$, thus making the case for full revelation.

$$V(\tau_i, \tau_{-i}) = \mathbb{E}_{s_i} \mathbb{E}_{\theta_i|s_i} \mathbb{E}_{s_{-i}|\theta_i} \mathbb{E}_{\theta_{-i}}[u_i(a_{-i}(\theta_{-i}, s), a(\theta_i, s), \theta_i)]$$

The result is again due to the relationship between linearity and maximization. Intuitively, firms react only linearly to more information coming from you, whereas a better knowledge between firms generate a better coordination in equilibrium.

I have thus concluded that if the interior solution is satisfied there is little room for obfuscation in the case of markets with a linear demand function. It is, thus, a simple consequence of this result that if some degree of obfuscation is present then either the interior solution condition is not satisfied or the linearity and separability assumptions are being challenged.

Corollary 1. *If there is presence of Incomplete Information, then either the interior solution condition is being violated or the model is not linear as defined here.*

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