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Abstract

What are the effects of strengthening patent protection on income and consumption inequality? To analyze this question, this paper incorporates heterogeneity in the initial wealth of households into a canonical quality-ladder growth model with endogenous labor supply. In this model, I firstly show that the aggregate economy always jumps immediately to a unique and stable balanced-growth path. Given the balanced-growth behavior of the aggregate economy and an exogenous distribution of initial wealth, I then show that the endogenous distribution of assets in subsequent periods is stationary and equal to its initial distribution. The model predicts that strengthening patent protection increases (a) economic growth by stimulating R&D investment and (b) income inequality by raising the return on assets. However, whether it also increases consumption inequality depends on the elasticity of intertemporal substitution. If and only if this elasticity is less (greater) than unity, strengthening patent protection increases (decreases) consumption inequality. For standard parameter values, strengthening patent protection leads to a larger increase in income inequality than consumption inequality.

Keywords: endogenous growth, heterogeneity, income inequality, patent policy

JEL classification: D31, O34, O41

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1. Introduction

What are the effects of strengthening patent protection on income and consumption inequality? To analyze this question, this paper incorporates heterogeneity in the initial wealth of households into a canonical quality-ladder growth model with endogenous labor supply. In this model, I firstly show that the aggregate economy always jumps immediately to a unique and stable balanced-growth path. Given the balanced-growth behavior of the aggregate economy and an exogenous distribution of initial wealth, I then show that the endogenous distribution of assets in subsequent periods is stationary and equal to its initial distribution. The model predicts that strengthening patent protection increases (a) economic growth by stimulating R&D investment and (b) income inequality by raising the return on assets. However, whether it also increases consumption inequality depends on the elasticity of intertemporal substitution. If and only if this elasticity is less (greater) than unity, strengthening patent protection increases (decreases) consumption inequality. For standard parameter values, strengthening patent protection leads to a larger increase in income inequality than consumption inequality.

These theoretical and numerical predictions are consistent with the following stylized facts. Firstly, the level of patent protection in the U.S. is widely believed to have increased since the 80's.[1] Secondly, industrial R&D spending has been increasing since 1980 (see Figure 1). Thirdly, there is well-documented evidence that income inequality has increased significantly over the same period. Interestingly, Krueger and Perri (2006) provide careful empirical evidence based on the Consumer Expenditure Survey from 1980 to 2003 to show that the sharp increase in income inequality was accompanied by a much smaller increase in consumption inequality. Therefore, the findings of the current paper suggest that patent policy may be able to provide a partial explanation on the trends of income and consumption inequality in the U.S. over the past decades.

The effect of patent policy on income inequality can be easily demonstrated as follows. Denote the real income of household h at time t as $y_t(h)$ that equals the sum of a real wage income w_t from

[1] See Jaffe (2000), Gallini (2002) and Jaffe and Lerner (2004) for a comprehensive discussion.

inelastic labor supply[2] and a return from financial assets $r_t v_t(h)$, in which r_t is the real rate of return on asset $v_t(h)$. Denote the share of total assets owned by household *h* as $s_{v,t}(h) \equiv v_t(h)/v_t$, in which v_t is the total and average real value of financial assets in the economy at time *t*. Suppose the distribution of $s_{v,t}(h)$ is stationary and equals to its initial distribution that has a mean of 1 and a standard deviation of σ_v. Let $s_{y,t}(h) \equiv y_t(h)/y_t = [w_t + r_t v_t(h)]/(w_t + r_t v_t)$ denote the share of income earned by household *h*. The first-order condition from the R&D sector is given by $w_t = v_t \varphi$ that equates the marginal cost of hiring an R&D worker (i.e. the wage rate) to the marginal benefit given by the worker's probability φ in obtaining a successful invention that has a value of v_t. In a canonical quality-ladder model, the market value of inventions equals the total value of assets, and there is a unit-continuum of valuable inventions. The share of income earned by household *h* becomes $s_{y,t}(h) = [\varphi + r_t s_{v,t}(h)]/(\varphi + r_t)$, which implies that the standard deviation of $s_{y,t}(h)$ in the steady state is given by $\sigma_y = \sigma_v r/(\varphi + r)$. Because the steady-state return *r* on assets is an increasing function in the equilibrium growth rate, strengthening patent protection increases R&D investment as well as economic growth and hence worsens income inequality.

Related Literature

This paper relates to a number of literatures (a) income inequality and economic growth, (b) endogenous-growth theory and (c) patent policy. The study of inequality and growth has an established and vast literature.[3] Garcia-Penalosa and Turnovsky (2006) incorporate heterogeneity in initial wealth into a canonical AK endogenous-growth model and develop an elegant approach to show that the endogenous distribution of assets is stationary. The current study adopts a similar approach to show that the endogenous distribution of assets is also stationary in a canonical quality-ladder model as in Aghion and Howitt (1992) and Grossman and Helpman (1991). One interesting difference between the two models is

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[2] The model features elastic labor supply to show that this result is robust to labor-supply decisions.
[3] See Aghion et al. (1999) for a recent survey.

that Garcia-Penalosa and Turnovsky's (2006) AK model relies on endogenous labor supply to generate an endogenous income distribution while the quality-ladder model does not.

Upon developing the basic framework, the current study analyzes the effects of patent policy on inequality and growth. This analysis has important policy implications on the problem of R&D underinvestment in the market economy suggested by Jones and Williams (1998, 2000). Given R&D underinvestment, patent policy is an important instrument that can be used to correct for this market failure and increase economic growth. Li (2001) and O'Donoghue and Zweimuller (2004) analyze the effects of patent policy in quality-ladder models that have a representative household. The current paper contributes to this literature by providing a framework that can be applied to investigate the effects of patent policy on income and consumption inequality in addition to growth.

Chou and Talmain (1996), Li (1998), Zweimuller (2000) and Foellmi and Zweimuller (2006) also consider wealth distribution in R&D-growth models, and they focus on the effects of wealth inequality on growth through different channels, such as the concavity/convexity of the labor Engel curve in Chou and Talmain (1996), indivisible consumption of quality goods in Li (1998), hierarchical preferences in Zweimuller (2000) and Foellmi and Zweimuller (2006). The current paper differs from these studies by considering how policy changes affect income and consumption inequality given a certain degree of wealth inequality. Hatipoglu (2008) incorporates finite patent length into Foellmi and Zweimuller's (2006) model to analyze the effects of wealth inequality on growth at different length of patent. In contrast, the current study focuses on patent breadth (i.e. patent protection against imitation) and shows that it may have different qualitative and quantitative effects on income and consumption inequality. As for differences in the policy instruments, patent breadth (length) mainly affects current (future) profits.

2. The Model

I develop a quality-ladder model similar to Aghion and Howitt (1992) and Grossman and Helpman (1991) by adding mainly three features (a) heterogeneity in initial wealth, (b) incomplete patent breadth and (c) endogenous labor supply. Given that quality-ladder models have been well-studied, the model's components are briefly described in Sections 2.1 – 2.3. Section 2.4 defines the decentralized equilibrium and shows that the aggregate economy is always on a unique and stable balanced-growth path. Given the balanced-growth behavior of the aggregate economy and an exogenous distribution of initial wealth, Section 2.5 shows that the endogenous distribution of assets in subsequent periods is stationary. Section 2.6 analyzes the effects of increasing patent breadth on income and consumption inequality. Section 2.7 calibrates the model and numerically evaluates the effects of patent on income/consumption inequality.

2.1. Households

There is a continuum of identical households (except for the initial distribution of wealth) indexed by $h \in [0,1]$. Each household *h* has a standard iso-elastic utility function given by

(1) $$U(h) = \int_0^\infty e^{-\rho t}\left(\frac{[C_t(h) l_t^\phi(h)]^{1-\gamma} - 1}{1-\gamma}\right) dt .$$

$\gamma \in (0,\infty)$ is the inverse of the intertemporal substitution elasticity $\varepsilon \equiv 1/\gamma$. $\gamma = \varepsilon = 1$ corresponds to the case of log utility. $C_t(h)$ is the consumption of final goods. Each household is endowed with one unit of time to allocate between leisure $l_t(h)$ and work $L_t(h)$. $\phi \geq 0$ is a preference parameter on leisure, and setting ϕ to zero corresponds to the case of inelastic labor supply. ρ is the exogenous discount rate. To ensure that lifetime utility is bounded,

(a1) $$\rho > (1-\gamma) g ,$$

where g denotes the balanced-growth rate of consumption.

Each household maximizes utility subject to a sequence of budget constraints given by

(2) $$\dot{V}_t(h) = R_t V_t(h) + W_t L_t(h) - P_t C_t(h) .$$

$V_t(h)$ is the nominal value of financial assets owned by household *h* at time *t*. It is assumed that household *h*'s share of financial assets at time 0 is exogenously given by $s_{v,0}(h) \equiv V_0(h)/V_0$ that has a general distribution function with a mean of $\int_0^1 s_{v,0}(h)dh = 1$ and a standard deviation of σ_v. R_t is the nominal rate of return on financial assets. Household *h* endogenously supplies $L_t(h)$ to earn the nominal wage rate W_t, which is normalized to one. P_t is the price of final goods. From the household's intratemporal optimization, household *h*'s labor supply is determined by

(3) $$1 - L_t(h) = l_t(h) = \phi P_t C_t(h),$$

where $W_t = 1$. From the household's intertemporal optimization, the familiar Euler equation is given by

(4) $$\frac{\dot{C}_t(h)}{C_t(h)} = \frac{1}{\gamma}\left(R_t - \frac{\dot{P}_t}{P_t} - \rho\right) + \phi\left(\frac{1-\gamma}{\gamma}\right)\frac{\dot{l}_t(h)}{l_t(h)}.$$

Lemma 1 shows that the consumption growth rate is the same across households.

Lemma 1: *Aggregate consumption and the consumption for household h evolve according to*

(5) $$\frac{\dot{C}_t(h)}{C_t(h)} = \frac{\dot{C}_t}{C_t} = \frac{R_t - \rho}{\gamma - \phi(1-\gamma)} - \left(\frac{1-\phi(1-\gamma)}{\gamma - \phi(1-\gamma)}\right)\frac{\dot{P}_t}{P_t}$$

for all h. Also, aggregate labor supply is determined by $L_t = 1 - \phi P_t C_t$.

Proof: Differentiate (3) with time and substitute it into (4). As for L_t, integrate (3) with *h*.■

To ensure that the Euler equation has the usual properties, the following parameter condition is assumed.

(a2) $$\gamma - \phi(1-\gamma) > 0.$$

Final goods are produced by a standard Cobb-Douglas aggregator over a continuum of differentiated intermediates goods $i \in [0,1]$ given by

(6) $$C_t = \exp\left(\int_0^1 \ln X_t(i)di\right).$$

I define a new variable $E_t \equiv P_t C_t$ to denote the aggregate nominal expenditure that will be convenient for analyzing the stability of the balanced-growth path. Introducing the price variables in nominal value is also for this purpose.

2.2. Intermediate Goods

There is a continuum of industries indexed by $i \in [0,1]$ producing the differentiated intermediate goods. Each industry i is dominated by a temporary monopolistic leader who holds the patent for the latest technology in the industry. The production function for the leader in industry i is

(7) $$X_t(i) = z^{n_t(i)} L_{x,t}(i) .$$

$L_{x,t}(i)$ is the number of workers in industry *i*. $z > 1$ is the exogenous productivity improvement from each invention, and $n_t(i)$ is the number of inventions that has occurred as of time *t*. Given $z^{n_t(i)}$,

(8) $$MC_t(i) = W_t / z^{n_t(i)} = 1/ z^{n_t(i)}$$

is the nominal marginal cost of production for the leader in industry *i*.

As commonly assumed in the literature, the current and former industry leaders engage in Bertrand competition, and the optimal pricing strategy for the current industry leader is a constant markup over the marginal cost given by

(9) $$P_t(i) = \mu(z,b) MC_t(i) ,$$

where $\mu(z,b) = z^b$ for $b \in (0,1]$ that captures the level of patent breadth. In Aghion and Howitt (1992) and Grossman and Helpman (1991), there is complete patent protection against imitation such that $b = 1$. Li (2001) generalizes the policy environment to capture incomplete patent protection against imitation such that $b \in (0,1)$.[4] Because of incomplete patent protection, the former industry leader's productivity

[4] O'Donoghue and Zweimuller (2004) refer to this form of patent protection as lagging breadth, and they formalize another form of patent protection known as leading breadth (i.e. patent protection against subsequent innovations). For the purpose of the current study, the consideration of lagging patent breadth is sufficient.

increases by a factor of z^{1-b} such that her productivity becomes $z^{1-b}z^{n_t(i)-1} = z^{n_t(i)-b}$. Therefore, the limiting-pricing markup for the current industry leader is given by z^b.

An increase in the level of patent breadth b enables an industry leader to charge a higher markup. The resulting increases in the amount of monopolistic profit and the value of an invention improve the incentives for R&D investment that potentially increase economic growth.

2.3. R&D

Denote the expected nominal value of an invention for industry i as $V_t(i)$. Because of the Cobb-Douglas specification in (6), the amount of monopolistic profit is the same across industries (i.e. $\pi_t(i) = \pi_t$ for $i \in [0,1]$). As a result, $V_t(i) = V_t$ for $i \in [0,1]$. Because patents are the only assets in the economy, V_t equals the total value of assets owned by households. The familiar no-arbitrage condition for V_t is

(10) $$R_t V_t = \pi_t + \dot{V}_t - \lambda_t V_t .$$

The left-hand side of (10) is the nominal return from this asset.[5] The right-hand side of (10) is the sum of (a) the monopolistic profit π_t generated by this asset, (b) the potential capital gain, and (c) the expected capital loss due to creative destruction, in which λ_t is the Poisson arrival rate of the next invention.

There is a continuum of R&D entrepreneurs indexed by $j \in [0,1]$, and they hire workers to create inventions. The expected profit for entrepreneur j is

(11) $$\pi_{r,t}(j) = V_t \lambda_t(j) - W_t L_{r,t}(j) .$$

The Poisson arrival rate of an invention for entrepreneur j is $\lambda_t(j) = \varphi L_{r,t}(j)$, where φ captures the productivity of R&D workers. The first-order condition from the R&D sector is given by

(12) $$V_t \varphi = W_t = 1 .$$

This condition determines the allocation of labor between production and R&D.

[5] As in Grossman and Helpman (1991), the risky asset is valued at the risk-free rate because the idiosyncratic risk for any one leader is fully diversified assuming the existence of a well-functioning stock market.

2.4. Decentralized Equilibrium

The equilibrium is a sequence of prices $\{R_t, W_t, P_t, P_t(i), V_t, V_t(h)\}_{t=0}^{\infty}$ and a sequence of allocations $\{X_t(i), L_{x,t}(i), L_{r,t}(j), L_t(h), C_t(h)\}_{t=0}^{\infty}$ such that in each period,

a. household $h \in [0,1]$ chooses $\{C_t(h), L_t(h)\}$ to maximize (1) subject to (2) taking $\{R_t, W_t, P_t\}$ as given;

b. the monopolistic leader in industry $i \in [0,1]$ chooses $\{P_t(i), L_{x,t}(i)\}$ to maximize profit according to the Bertrand competition and taking $\{W_t\}$ as given;

c. R&D entrepreneur $j \in [0,1]$ chooses $\{L_{r,t}(j)\}$ to maximize profit taking $\{W_t, V_t\}$ as given;

d. the market for final goods clears such that $\int_0^1 C_t(h)dh = C_t = \exp\left(\int_0^1 \ln X_t(i)di\right)$;

e. the labor market clears such that $\int_0^1 L_t(h)dh = L_t = \int_0^1 L_{x,t}(i)di + \int_0^1 L_{r,t}(j)dj$.

The next step is to show that the aggregate economy is always on a unique and stable balanced-growth path. This result can be proven by analyzing the labor-market condition and the law of motion for the aggregate expenditure in a phase diagram of $\{E_t, \lambda_t\}$ to show that the economy always jumps immediately to the intersection of the $\dot{E}_t = 0$ locus and $L_{x,t} + L_{r,t} = L_t$.[6] This exercise is similar to the one in Grossman and Helpman (1991) except with endogenous labor supply and hence is relegated to Appendix A. Another difference is that $\gamma = 1$ in Grossman and Helpman (1991). When $\gamma \in (0, \infty)$, a sufficient condition for the saddle-point stability is

(a3) $$\gamma > 1 - 1/\ln z \equiv \bar{\gamma} \in (-\infty, 1).$$

[6] As in Grossman and Helpman (1991), an implicit assumption behind this result is that at any point in time, each industry has an existing leader with a competitor one step down the quality ladder.

Lemma 2: *The aggregate economy is always on a unique and stable balanced-growth path, in which C_t grows at rate g, P_t decreases at rate g, and both L_t and V_t are constant.*

Proof: See Appendix A.■

Lemma 3: *The balanced-growth equilibrium is characterized by*

(13) $$L^*(\underset{-}{b}) = 1 - \phi\mu(b)\left(\frac{(\gamma-1)\ln z + 1 + \rho/\varphi}{[1+\phi\mu(b)](\gamma-1)\ln z + (1+\phi)\mu(b)}\right),$$

(14) $$L_x^*(\underset{-}{b}) = \frac{1-L^*(b)}{\phi\mu(b)},$$

(15) $$L_r^*(\underset{+}{b}) = 1 - [1+\phi\mu(b)]L_x^*(b),$$

(16) $$g^*(\underset{+}{b}) = \varphi L_r^*(b)\ln z,$$

(17) $$r^*(\underset{+}{b}) \equiv R^* - \dot{P}^*/P^* = \rho + \gamma g^*(b),$$

(18) $$C_t^* = (r^* - g^*)v_t^* + w_t^* L^* = [(r^* - g^*)v_t^* + w_t^*]/(1+\phi).$$

Proof: See Appendix A.■

r^* is the steady-state real interest rate. $w_t \equiv W_t / P_t$ denotes the real wage rate that increases at rate g. $v_t \equiv V_t / P_t$ denotes the real value of assets that also increases at rate g. Note that $C_t^*(1+\phi) > w_t^*$ because $r^* - g^* = \rho + (\gamma-1)g^* > 0$ from (a1). Ensuring $L_r^* > 0$ requires a lower bound on R&D productivity

(a4) $$\varphi > \rho(1+\phi\mu)/(\mu-1).$$

2.5. Distribution of Assets

I adopt a similar approach as in Garcia-Penalosa and Turnovsky (2006) to show that the distribution of assets is stationary. To do this, it is more convenient to rewrite (2) in terms of real variables such that

(19) $$\dot{v}_t(h) = r_t v_t(h) + w_t L_t(h) - C_t(h).$$

The aggregate and average real value of assets evolves according to

(20) $$\dot{v}_t = r_t v_t + w_t L_t - C_t.$$

Combining (19) and (20), the law of motion for $s_{v,t}(h) \equiv v_t(h)/v_t$ is given by

(21) $$\frac{\dot{s}_{v,t}(h)}{s_{v,t}(h)} = \frac{w_t L_t(h) - C_t(h)}{v_t(h)} - \frac{w_t L_t - C_t}{v_t}.$$

Using $w_t[1 - L_t(h)] = \phi C_t(h)$, the equilibrium value of $s^*_{v,t}(h)$ evolves according to a simple linear differential equation given by

(22) $$\dot{s}^*_{v,t}(h) = \frac{C^*_t(1+\phi) - w^*_t}{v^*_t} s^*_{v,t}(h) - \frac{s^*_c(h) C^*_t(1+\phi) - w^*_t}{v^*_t}.$$

(22) describes the potential evolution of $s^*_{v,t}(h)$ given an initial value of $s_{v,0}(h)$. $s^*_c(h) \equiv C^*_t(h)/C^*_t$ is a stationary variable because of Lemma 1. Because C^*_t, w^*_t and v^*_t all increase at rate *g*, the coefficient of $s^*_{v,t}(h)$ and the last term in (22) are constant. Also, the coefficient of $s^*_{v,t}(h)$ is positive because $C^*_t(1+\phi) > w^*_t$ from (18). Therefore, the only solution consistent with long-run stability is $\dot{s}^*_{v,t}(h) = 0$ for all *t*. From (22), $\dot{s}^*_{v,t}(h) = 0$ for all *t* implies that $s^*_{v,t}(h) = s_{v,0}(h)$ and

(23) $$C^*_t(h) = \frac{[C^*_t(1+\phi) - w^*_t] s_{v,0}(h) + w^*_t}{1+\phi} = \frac{(r^* - g^*) v^*_t(h) + w^*_t}{1+\phi}$$

for all *t*. Proposition 1 summarizes the stationarity of the wealth distribution.

Proposition 1: *For every household,* $s^*_{v,t}(h) = s_{v,0}(h)$ *for all t.*

Proof: Proven in the text.■

2.6. Income and Consumption Inequality

Given the previous results that the economy is always on a unique and stable balanced-growth path and the endogenous distribution of assets equals its initial distribution, this section analyzes the effects of increasing patent breadth b on income and consumption inequality. In equilibrium, the real value of income for household *h* is

$$y_t^*(h) = r^* v_t^*(h) + w_t^* L^*(h) = r^* v_t^*(h) + w_t^* - \phi C_t^*(h). \tag{24}$$

From (12), (23) and Proposition 1, the share of income earned by household *h* simplifies to

$$s_{y,t}^*(h) \equiv \frac{y_t^*(h)}{y_t^*} = \frac{(r^* + \phi g^*) s_{v,0}(h) + \varphi}{r^* + \phi g^* + \varphi} \tag{25}$$

for all *t*. (25) implies that the equilibrium standard deviation of real income $\sigma_y^* \equiv \int_0^1 [s_{y,t}^*(h) - 1]^2 dh$ is

$$\sigma_y^* = \left(\frac{r^* + \phi g^*}{r^* + \phi g^* + \varphi} \right) \sigma_v . \tag{26}$$

Following Garcia-Penalosa and Turnovsky (2006) to use the standard deviation of relative income as a measure of income inequality, Proposition 2 summarizes the effect of patent policy on income inequality.

Proposition 2: *An increase in the level of patent breadth increases income inequality.*

Proof: An increase in b raises r^* and g^*, and the resulting increases in r^* and g^* raise σ_y^*.■

Intuitively, the increase in patent protection raises R&D spending and hence the equilibrium growth rate. This higher growth rate drives up the real interest rate, and the resulting higher return on assets increases the relative income of households who have a larger fraction of income coming from assets. Note that increasing patent breadth raises income inequality even in the case of inelastic labor supply (i.e. $\phi = 0$).

The consumption of final goods for household *h* is given by (23). From (12), (18) and Proposition 1, the share of consumption by household *h* is

(27) $$s_{c,t}^{*}(h) \equiv \frac{C_t^{*}(h)}{C_t^{*}} = \frac{(r^{*}-g^{*})s_{v,0}(h)+\varphi}{r^{*}-g^{*}+\varphi}$$

for all *t*. (27) implies that the equilibrium standard deviation of consumption $\sigma_c^{*} \equiv \int_0^1 [s_{c,t}^{*}(h)-1]^2 dh$ is

(28) $$\sigma_c^{*} = \left(\frac{r^{*}-g^{*}}{r^{*}-g^{*}+\varphi}\right)\sigma_v .$$

Proposition 3 summarizes the effect of patent policy on consumption inequality.

Proposition 3: *An increase in the level of patent breadth increases (decreases) consumption inequality if and only if the elasticity of intertemporal substitution* ε *is less (greater) than unity.*

Proof: An increase in b raises r^{*} and g^{*}. (17) shows that the resulting increases in r^{*} and g^{*} lead to a higher (lower) σ_c^{*} if and only if $\gamma = 1/\varepsilon$ is greater (less) than one.■

Intuitively, strengthening patent protection increases economic growth, and this higher growth rate increases the fraction of assets $g^{*}v_t^{*}$ that needs to be saved. At the same time, the higher growth rate also raises the return on assets $r^{*}v_t^{*}$. Whether the increase in return is sufficient to compensate for the increase in saving in order for the fraction of assets consumed to increase depends on the value of $\varepsilon = 1/\gamma$. For a low (high) value of $\varepsilon = 1/\gamma$, the increase in r^{*} is large (small) enough for the fraction of assets consumed to increase (decrease). The larger (smaller) fraction of assets consumed increases (decreases) the relative consumption of households who finance a larger fraction of consumption by asset income. For the benchmark case of log utility, the two effects offset each other leaving the fraction of assets consumed and hence relative consumption as well as consumption inequality unchanged.

Finally, Proposition 4 shows that income inequality is always larger than consumption inequality, and this finding is consistent with the empirical evidence provided by Krueger and Perri (2006).

Proposition 4: $\sigma_y^* > \sigma_c^*$.

Proof: Compare (26) and (28).■

2.7. Numerical Analysis

For illustrative purposes, this section calibrates the model and numerically evaluates the effects of patent on income and consumption inequality. From the model, I express each of the following moments as a function of structural parameters and then use the values of these moments in the data to infer the parameter values. I use standard values for the fraction of time devoted to leisure $l = 0.7$, the real rate of return on assets $r = 0.07$, and total factor productivity growth $g = 0.01$. For the arrival rate of inventions, I set λ to 0.33 such that the average time between arrivals of inventions is 3 years as in Acemoglu and Akcigit (2008).[7] R&D spending as a share of GDP is given by $wL_r/(\pi + wL)$ in the model. Assuming that the increase in R&D spending since the 80's has been driven by patent protection, the hypothetical exercise is to firstly use the time trend of R&D share from 1980 to 2004 to infer a time path for patent breadth b and then examine how the increase in b affects the relative level of income and consumption inequality. Figure 1 plots the time path of R&D share and its trend.

[Insert Figure 1 here]

Given a value for γ, the five moment conditions determine respectively the values of $\{\phi, \rho, z, \varphi, b\}$. As for γ, I use a conservative value of 3 implying an intertemporal substitution elasticity of 0.33 that is within the usual range in the business-cycle literature.[8] In summary, the calibrated parameter values are $\{\gamma = 3, \phi = 2.33, \rho = 0.04, z = 1.03, \varphi = 71.4, b_{1980} = 0.62\}$. The values of the usual parameters seem reasonable, and the large value of φ implies that asset income from patents $r^* v_t^*$ is very small compared

[7] Lanjouw (1998) estimates the obsolescence probability of patents in some industries, and the average estimated value is about 10%. Caballero and Jaffe (2002) estimate the rate of creative destruction to be about 4%. I have also considered these smaller values for $\lambda \in \{0.04, 0.10\}$, and the result that strengthening patent protection has a larger effect on income inequality than on consumption inequality remains robust.

[8] At a lower value of γ (i.e. a larger ε), strengthening patent protection would increase income inequality relative to consumption inequality by even more.

to labor income $w_t^* L^*$, where $w_t^* = \varphi v_t^*$ from (12). This implication also seems reasonable given that labor income and industrial R&D spending are on average about 70% and less than 2% of GDP respectively.

The calibrated value of b gradually increases from 0.62 in 1980 to 0.89 in 2004 implying a substantial increase in the level of patent protection. As a result of the increase in b, the model predicts that the relative standard deviation between income and consumption (i.e. σ_y^* / σ_c^*) would increase from 1.55 to 1.71. This illustrative exercise suggests that for a given degree of wealth inequality, strengthening patent protection leads to a larger increase in income inequality than consumption inequality such that σ_y^* / σ_c^* increases over time.

3. Conclusion

This paper analyzes the effects of patent policy on economic growth, income and consumption inequality. In summary, strengthening patent protection increases growth but worsens income inequality. However, the effect on consumption inequality is ambiguous and depends on the elasticity of intertemporal substitution. To derive these results, this paper incorporates heterogeneity in initial wealth into a canonical quality-ladder model. This class of first-generation endogenous-growth models exhibits scale effects,[9] in which a larger economy experiences faster growth and an economy with growing population experiences an increasing growth rate rather than a balanced-growth path. The current paper avoids these problems by normalizing the population size to one. An interesting extension is to consider the effects of patent policy on growth and inequality in later vintages of R&D-growth models. Given that the canonical quality-ladder model does not feature some important macroeconomic elements, such as capital accumulation, business-cycle shocks and capital-market imperfection, an interesting exercise is to incorporate R&D-driven growth into a computational DSGE model with heterogeneous agents to examine the quantitative importance of patent policy on the distributions of wealth, income and consumption.

[9] See, for example, Jones (1999) for a discussion on scale effects in R&D-growth models.

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Appendix

Proof for Lemma 2: For the stability and uniqueness of the balanced-growth path, I show that the labor-market clearing condition and the law of motion for $E_t = P_t C_t$ can be analyzed in a phase diagram with the same properties as in Grossman and Helpman (1991). The labor-market clearing condition is

(A1) $$L_t = L_{x,t} + L_{r,t}.$$

From aggregate labor supply, $L_t = 1 - \phi E_t$. From the labor share of aggregate expenditure, $L_{x,t} = E_t / \mu$. From the R&D production function, $L_{r,t} = \lambda_t / \varphi$. Therefore, (A1) becomes

(A2) $$E_t = \frac{\mu}{1+\mu\phi}\left(1 - \frac{\lambda_t}{\varphi}\right).$$

From (5), the law of motion for E_t is

(A3) $$\frac{\dot{E}_t}{E_t} = \frac{\dot{P}_t}{P_t} + \frac{\dot{C}_t}{C_t} = \left(\frac{\gamma - 1}{\gamma - \phi(1-\gamma)}\right)\frac{\dot{P}_t}{P_t} + \frac{R_t - \rho}{\gamma - \phi(1-\gamma)}.$$

The price index is $P_t = \exp\left(\int_0^1 \ln P_t(i)di\right) = \mu / Z_t$, where $Z_t \equiv \exp\left(\int_0^1 n_t(i)di \ln z\right) = \exp\left(\int_0^t \lambda_\tau d\tau \ln z\right)$ denotes aggregate technology. Therefore, $\dot{P}_t / P_t = -\dot{Z}_t / Z_t = -\lambda_t \ln z$. As for R_t, using the law of motion for V_t and the first-order condition from the R&D sector $V_t \varphi = 1$ yields

(A4) $$R_t = \frac{\pi_t + \dot{V}_t - \lambda_t V_t}{V_t} = \frac{\pi_t - \lambda_t / \varphi}{1/\varphi}.$$

Profit share is $\pi_t = E_t(\mu - 1)/\mu$. Then, substituting (A4) into (A3) yields

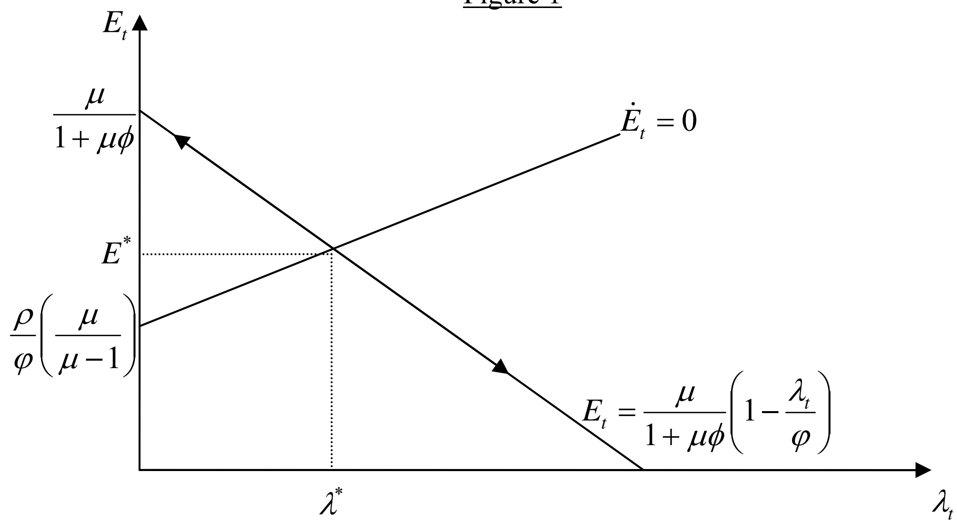
(A5) $$\frac{\dot{E}_t}{E_t} = \frac{\varphi}{\gamma - \phi(1-\gamma)}\left(\frac{\mu - 1}{\mu}\right)E_t - \left(\frac{(\gamma-1)\ln z + 1}{\gamma - \phi(1-\gamma)}\right)\lambda_t - \frac{\rho}{\gamma - \phi(1-\gamma)}.$$

The $\dot{E}_t = 0$ locus is given by

(A6) $$E_t = \left(\left(\frac{(\gamma-1)\ln z + 1}{\varphi}\right)\lambda_t + \frac{\rho}{\varphi}\right)\left(\frac{\mu}{\mu - 1}\right).$$

Figure 1 plots (A2) and (A6) in a phase diagram.

Figure 1



The $\dot{E}_t = 0$ locus is upward sloping from (a3). The coefficient of E_t in (A5) is positive from (a2). The coefficient of λ_t in (A5) is negative from (a2) and (a3). Therefore, the property of the dynamic system is the same as in Grossman and Helpman (1991), in which the economy jumps immediately to $\{\lambda^*, E^*\}$.■

Proof for Lemma 3: Equating (A2) and (A6) yields

(A7) $$\lambda^* = \frac{\varphi(\mu-1) - \rho(1+\mu\phi)}{(1+\mu\phi)(\gamma-1)\ln z + \mu(1+\phi)}.$$

From the R&D production function,

(A8) $$L_r^* = \frac{\lambda^*}{\varphi} = \frac{(\mu-1) - \rho(1+\mu\phi)/\varphi}{(1+\mu\phi)(\gamma-1)\ln z + \mu(1+\phi)} = 1 - \frac{(\gamma-1)\ln z + 1 + \rho/\varphi}{(\gamma-1)\ln z + \mu(1+\phi)/(1+\phi\mu)}.$$

L_r^* is increasing in μ because $\mu(1+\phi)/(1+\phi\mu)$ is increasing in μ. Combining aggregate labor supply $L_t = 1 - \phi P_t C_t$, the price index $P_t = \mu / Z_t$ and the aggregate production function $C_t = Z_t L_{x,t}$ yields

(A9) $$L_t = 1 - \phi P_t C_t = 1 - \phi \mu L_{x,t}.$$

Substituting (A8) and (A9) into the labor-market clearing condition $L_t = L_{x,t} + L_{r,t}$ yields

(A10) $$L_x^* = \frac{(\gamma-1)\ln z + 1 + \rho/\varphi}{(1+\phi\mu)(\gamma-1)\ln z + (1+\phi)\mu}.$$

Differentiating the log of aggregate technology $Z_t = \exp\left(\int_0^t \lambda_\tau d\tau \ln z\right)$ with respect to time yields

(A11) $$g_t \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z.$$

From the aggregate production $C_t = Z_t L_{x,t}$,

(A12) $$\frac{\dot{C}_t^*}{C_t^*} = \frac{\dot{Z}_t^*}{Z_t^*}.$$

From the price index $P_t = \mu / Z_t$,

(A13) $$\frac{\dot{P}_t^*}{P_t^*} = -\frac{\dot{Z}_t^*}{Z_t^*}.$$

Substituting (A12) and (A13) into the Euler equation in (5) yields

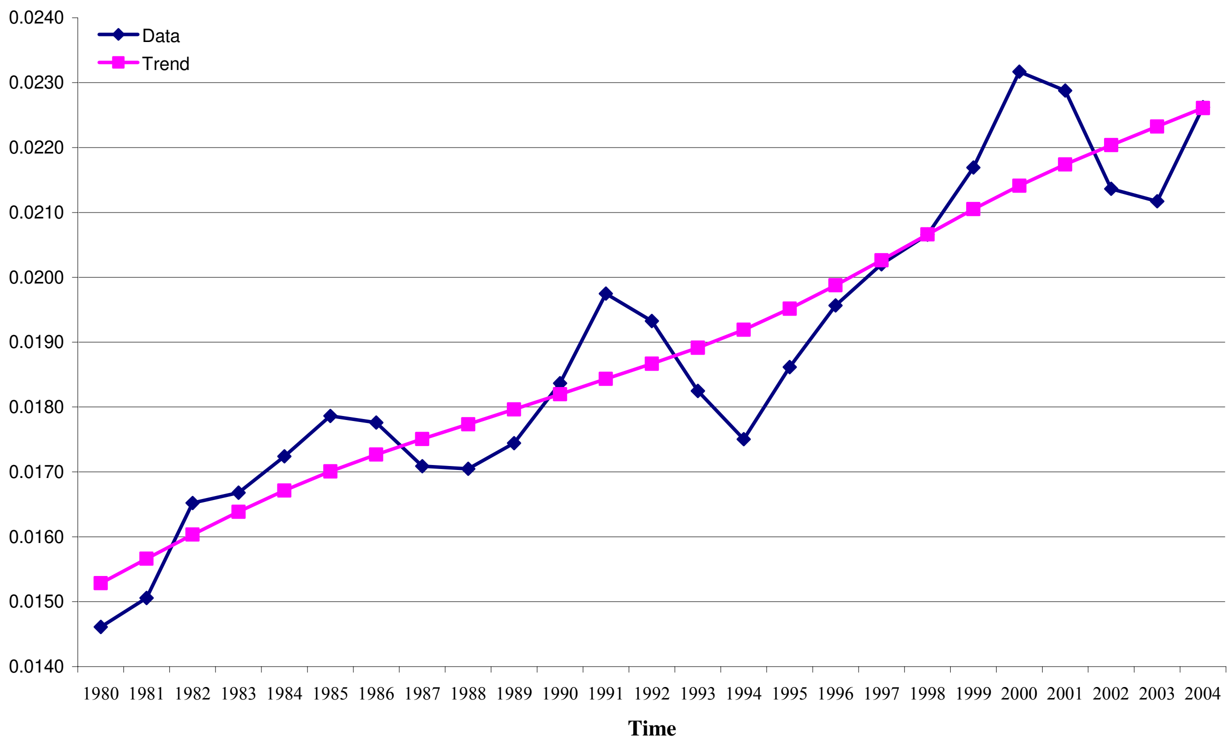
(A14) $$r^* \equiv R^* - \dot{P}^* / P^* = \rho + \gamma g^*.$$

Finally, from (20) and aggregate labor supply $\phi C_t = w_t(1-L_t)$,

(A15) $$C_t^* = (r^* - g^*)v_t^* + w_t^* L^* = (r^* - g^*)v_t^* + w_t^* - \phi C_t^*$$

because $v_t^* = V_t^* / P_t^*$ grows at rate g^*.■

<u>Figure 1: Industrial R&D as a Share of Non-Farm Business-Sector Output in the U.S.</u>



Data sources: (a) National Science Foundation: Division of Science Resources Statistics; and (b) Bureau of Economic Analysis: National Income and Product Accounts.

Footnote: R&D is net of federal spending, and non-farm business-sector output is calculated as GDP net of government spending and farm-sector output. The trend from the data is extracted using a standard HP-filter with a smoothing parameter of 100 for the annual frequency.