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Abstract

We show the existence of involuntary unemployment based on consumers' utility maximization and firms' profit maximization behavior under monopolistic competition with increasing, decreasing or constant returns to scale technology using a three-period overlapping generations (OLG) model with a childhood period as well as younger and older periods. We also analyze the effects of fiscal policy financed by tax and budget deficit (or seigniorage) to realize full-employment under a situation with involuntary unemployment. We show the following results. 1) In order to maintain the steady state where employment increases at some positive rate, we need a budget deficit (Proposition 1). 2) If the full-employment state is realized, we do not need budget deficit to maintain full-employment (Proposition 2). We also show that involuntary unemployment occurs in the Nash equilibrium.

Keywords: Involuntary unemployment, Three-period overlapping generations model, Monopolistic competition, Nash equilibrium

1. Introduction

In this paper we analyze the effects of fiscal policy to realize full-employment under a situation with involuntary unemployment. Involuntary unemployment in this paper is a situation where workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand. Umada (1997) derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firm behavior is ad-hoc. Otaki (2009) says that there

[1] Lavoie (2001) presented a similar analysis.

exists involuntary unemployment for two reasons: (i) the nominal wage rate is set above the reservation nominal wage rate; and (ii) the employment level and economic welfare never improve by lowering the nominal wage rate. He assume indivisibility (or inelasticity) of individual labor supply. If labor supply is indivisible, it may be 1 or 0. On the other hand, if it is divisible, it takes a real value between 0 and 1. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if the labor supply is divisible and very small, no unemployment exists[2]. However, we show that even if labor supply is divisible, unless it is so small, there may exit involuntary unemployment. We consider consumers' utility maximization and firms' profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2011) and Otaki (2015), and demonstrate the existence of involuntary unemployment without the assumption of wage rigidity.

Also we analyze the effects of fiscal policy financed by tax and budget deficit (or seigniorage). We show the following results.

1. In order to maintain the steady state where employment and output increases at some positive rate, we need a budget deficit. (Proposition 1)
2. If the full-employment state is realized, we do not need budget deficit to maintain full-employment. (Proposition 2)

From these results we can say that in order to realize full-employment from a state with involuntary unemployment we need budget deficit of the government. However, when full-employment is realized, in order to maintain full-employment we need balanced budget. Therefore, additional government expenditure to realize fill-employment should be financed by seigniorage not public debt. If it is financed by public debt, this debt should not be redeemed. It should be bought by the central bank. In this case money supply increases under constant prices of goods.

In the next section we analyze and show the existence of involuntary unemployment under monopolistic competition with increasing or decreasing or constant returns to scale technology using a three-periods OLG model with a childhood period as well as younger (working) and older (retired) periods. Also we consider pay-as-you go pension system for the older generation. In a simple two-periods OLG model falling of the nominal wage rate and the prices of goods may increase consumption and employment by the so-called real balance effect. In such a model consumers have savings for future consumption, but not debt. In a three-periods model with childhood period they consume goods in their childhood period by borrowing

[2]About the indivisible labor supply also please see Hansen (1985).

money from consumers of the previous generation, and must repay their debt in the next period. Real value of the debt is increased by falling of the nominal wage rate and the prices, and consumptions and employment may decrease. In addition to this configuration we consider a pay-as-you go pension system for the older generation which may reduce the savings of consumers. We think our model is more realistic than a simple two-periods OLG model. In Section 3 we examine the effects of a decrease in the nominal wage rate. In our three-period OLG model with pay-as-you-go pension an increase in consumption and employment due to falling of the nominal wage rate and the prices of goods might be small or even negative. In Section 4 we study the fiscal policy financed by tax and budget deficit (or seigniorage) to realize full-employment at a state with involuntary unemployment. We also show that involuntary unemployment occurs in the Nash equilibrium in Section 5.

As we will state in the concluding remarks, the main limitation of this paper is that the good is produced by only labor and there exists no capital and investment of firms. A study of the problem of involuntary unemployment and fiscal policy in such a situation is the theme of future research.

2. Existence of involuntary unemployment

2.1. Consumers

We consider a three-period (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki (2007), Otaki (2009), and Otaki (2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0, 1]$. Good z is monopolistically produced by firm z with increasing or decreasing or constant returns to scale technology.
2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from the younger generation and the government scholarship. They must repay these debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.
3. During Period 1, consumers supply l unit of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.

4. During Period 2, consumers consume the goods using their savings carried over from their Period 1 earnings, and receive the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.
5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period $i, \ i = 1, 2$.
C_i^u: consumption basket of an unemployed consumer in Period $i, \ i = 1, 2$.
$c(z)_i^e$: consumption of good z of an employed consumers in Period i.
$c(z)_i^u$: consumption of good z of an unemployed consumers in Period i.
D: consumption basket of an individual in the childhood period, which is constant.
P_i: the price of consumption basket in Period $i, \ i = 1, 2$.
$p(z)_i$: the price of good z in Period $i, \ i = 1, 2$.
$\rho = \frac{P_2}{P_1}$: (expected) inflation rate (plus one).
W: nominal wage rate.
R: unemployment benefit for an unemployed individual. $R = D$.
$\hat{D}$: consumption basket in the childhood period of a next generation consumer.
Q: pay-as-you-go pension for an individual of the older generation.
Θ: tax payment by an employed individual for the unemployment benefit.
$\hat{Q}$: pay-as-you-go pension for consumers of the younger generation when they retire.
Ψ: tax payment by an employed individual for the pay-as-you-go pension.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
$y(Ll)$: labor productivity, which is increasing or decreasing or constant with respect to "employment $\times$ labor supply" (Ll).

We assume that the population L_f is constant.

We consider a two-step method to consider utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods:

2. Then, they maximize their consumption baskets given the expenditure in each period.

We define the elasticity of the labor productivity with respect to "employment × labor supply" as follows,

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

We assume that $-1 < \zeta < 1$, and ζ is constant. Increasing (decreasing or constant) returns to scale means $\zeta > 0$ ($\zeta < 0$ or $\zeta = 0$).

Since the taxes for unemployed consumers' debts are paid by employed consumers of the same generation, D and Θ satisfy the following relationship.

$$D(L_f - L) = L\Theta.$$

This means

$$L(D + \Theta) = L_f D.$$

The price of the consumption basket in Period 0 is assumed to be 1. Thus, D is the real value of the consumption in the childhood period of consumers.

Also, since the taxes for the pay-as-you-go pension system are paid by employed consumers of younger generation, Q and Ψ satisfy the following relationship:

$$L\Psi = L_f Q.$$

The utility function of employed consumers of one generation over the three periods is written as

$$u(C_1^e, C_2^e, D) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic utility function. The budget constraint is

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

Similarly, the utility function of unemployed consumers is

$$u(C_1^u, C_2^u, D).$$

Their budget constraint is

$$P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q}$$

Since $R = D$,

$$P_1 C_1^u + P_2 C_2^u = \Pi + \hat{Q}.$$

The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 \left(c(z)_i^e\right)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1, 2,$$

and

$$C_i^e = \left(\int_0^1 \left(c(z)_i^u\right)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1, 2.$$

σ is the elasticity of substitution among the goods, and $\sigma > 1$.

The price of consumption basket in Period i is

$$P_i = \left(\int_0^1 (p(z)_i)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}},\ i = 1, 2.$$

Let

$$\alpha = \frac{P_1C_1^e}{P_1C_1^e + P_2C_2^e} = \frac{P_1C_1^u}{P_1C_1^u + P_2C_2^u},$$

$$1 - \alpha = \frac{P_2C_2^e}{P_1C_1^e + P_2C_2^e} = \frac{P_2C_2^u}{P_1C_1^u + P_2C_2^u},$$

Since the utility functions $u(C_1^e, C_2^e, D)$ and $u(C_1^u, C_2^u, D)$ are homothetic, α is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\frac{P_1C_1^e}{P_1C_1^e + P_2C_2^e} = \frac{P_1C_1^u}{P_1C_1^u + P_2C_2^u},$$

$$\frac{P_2C_2^e}{P_1C_1^e + P_2C_2^e} = \frac{P_2C_2^u}{P_1C_1^u + P_2C_2^u},$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e = \alpha\frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1},\ C_2^e = (1-\alpha)\frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2},$$

and

$$C_1^u = \alpha\frac{\Pi + \hat{Q}}{P_1},\ C_2^u = (1-\alpha)\frac{\Pi + \hat{Q}}{P_2}.$$

Lagrange functions in the second step for employed and unemployed consumers are

$$\mathscr{L}_1^e = \left(\int_0^1 \left(c(z)_1^e\right)^{\frac{\sigma}{\sigma-1}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_1^e \left[\int_0^1 p(z)_1 c(z)_1^e dz - \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathscr{L}_2^e = \left(\int_0^1 \left(c(z)_2^e\right)^{\frac{\sigma}{\sigma-1}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_2^e \left[\int_0^1 p(z)_2 c(z)_2^e dz - (1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathscr{L}_1^u = \left(\int_0^1 \left(c(z)_1^u\right)^{\frac{\sigma}{\sigma-1}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_1^u \left[\int_0^1 p(z)_1 c(z)_1^u dz - \alpha(\Pi + \hat{Q})\right],$$

and

$$\mathscr{L}_2^u = \left(\int_0^1 \left(c(z)_2^u\right)^{\frac{\sigma}{\sigma-1}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_2^u \left[\int_0^1 p(z)_2 c(z)_2^u dz - \alpha(\Pi + \hat{Q})\right].$$

λ_1^e, λ_2^e, λ_1^u and λ_2^u are Lagrange multipliers. Solving these maximization problem, the following demand functions of employed and unemployed consumers are derived.

$$c(z)_1^e = \left(\frac{p(z)_1}{P_1}\right)^{-\sigma} \frac{\alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_1}$$

$$c(z)_2^e = \left(\frac{p(z)_2}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_2}$$

$$c(z)_1^u = \left(\frac{p(z)_1}{P_1}\right)^{-\sigma} \frac{\alpha(\Pi + \hat{Q})}{P_1}$$

and

$$c(z)_2^u = \left(\frac{p(z)_2}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(\Pi + \hat{Q})}{P_2}$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e = u\left(\alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1}, (1-\alpha)\frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2}, D\right) - \Gamma(l),$$

and

$$V^u = u\left(\alpha \frac{\Pi + \hat{Q}}{P_1}, (1-\alpha)\frac{\Pi + \hat{Q}}{P_2}, D\right).$$

Let
$$\omega = \frac{W}{P_1},\ \rho = \frac{P_2}{P_1}.$$
Then, since the real value of D in the childhood period is constant, we can write

$$V^e = \varphi\left(\omega l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}, \rho\right) - \Gamma(l),$$

$$V^u = \varphi\left(\frac{\Pi + \hat{Q}}{P_1}, \rho\right),$$

ω is the real wage rate. Denote

$$I = \omega l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}. \tag{1}$$

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial \varphi}{\partial I}\omega - \Gamma'(l) = 0, \tag{2}$$

where
$$\frac{\partial \varphi}{\partial I} = \alpha \frac{\partial u}{\partial C_1^e} + (1-\alpha)\frac{\partial u}{\partial C_2^e}.$$
Given P_1 and ρ the labor supply is a function of ω. From (2) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial \varphi}{\partial I} + \frac{\partial^2 \varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2 \varphi}{\partial I^2}\omega^2}. \tag{3}$$

If $\frac{dl}{d\omega} > 0$, the labor supply is increasing with respect to the real wage rate ω.

2.2. Firms

Let $d(z)_1$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$\begin{aligned} d(z)_1 &= \left(\frac{p(z)_1}{P_1}\right)^{-\sigma} \frac{\alpha(WLl + L_f\Pi - LD - L\Theta + L_f\hat{Q} - L\Psi)}{P_1} \\ &= \left(\frac{p(z)_1}{P_1}\right)^{-\sigma} \frac{\alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_1}. \end{aligned}$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good z in Period 2 is written as

$$d(z)_2 = \left(\frac{p(z)_2}{P_2}\right)^{-\sigma} \frac{(1-\alpha)\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_2}.$$

Let $\overline{d(z)_2}$ be the demand for good z by the older generation. Then

$$\overline{d(z)_2} = \left(\frac{p(z)_1}{P_1}\right)^{-\sigma} \frac{(1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_fD + L_fQ - L_f\bar{Q}\right)}{P_1},$$

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{l}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. $\bar{\alpha}$ is the value of α for the older generation. Q is the pay-as-you-go pension for consumers of the older generation themselves. Let

$$M = (1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_fD + L_fQ - L_f\bar{Q}\right).$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. Net savings is the difference between M and the pay-as-you-go pensions in their Period 2, as follows:

$$M - L_fQ.$$

Their demand for good z is written as $\left(\frac{p(z)_1}{P_1}\right)^{-\sigma} \frac{M}{P_1}$. Government expenditure constitutes the national income as well as the consumption of the younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p(z)_1}{P_1}\right)^{-\sigma} \frac{Y}{P_1}, \tag{4}$$

where Y is the effective demand defined by

$$Y = \alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M.$$

Note that $\hat{D}$ is consumption in the childhood period of a next generation consumer. G is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits (see Otaki (2007), Otaki (2015) about this demand function). Now, we assume that G is financed by seigniorage similarly to Otaki (2007) and Otaki (2009). In a later section, we will consider the government's budget constraint with respect to taxes.

Let L and Ll be employment and the "employment $\times$ labor supply" of firm z. The total employment and the total "employment $\times$ labor supply" are also

$$\int_0^1 Ldz = L, \int_0^1 Lldz = Ll.$$

The output of firm z is $Lly(Ll)$. At the equilibrium $Lly(Ll) = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial p(z)_1} = (y(Ll) + Lly')\frac{\partial (Ll)}{\partial p(z)_1}.$$

From (4)

$$\frac{\partial d(z)}{\partial p(z)_1} = -\sigma\frac{d(z)}{p(z)_1}.$$

The profit of firm z is

$$\pi(z) = p(z)_1 d(z) - \frac{d(z)}{y(Ll)}W.$$

The condition for profit maximization is

$$\begin{aligned}
\frac{\partial \pi(z)}{\partial p(z)_1} =& d(z) + \left(p(z)_1 - \frac{W}{y(Ll)} + \frac{\frac{y'd(z)}{y(Ll)+Lly'}}{y(Ll)^2}W\right)\frac{\partial d(z)}{\partial p(z)_1}\\
=& d(z) + \left(p(z)_1 - \frac{W}{y(Ll)} + \frac{\frac{Lly'}{y(Ll)+Lly'}}{y(Ll)}W\right)\frac{\partial d(z)}{\partial p(z)_1}\\
=& d(z) - \sigma\left(p(z)_1 - \frac{W}{y(Ll)+Lly'}\right)\frac{d(z)}{p(z)_1} = 0
\end{aligned}$$

Therefore, we obtain

$$p(z)_1 = -\frac{\sigma}{(1-\sigma)(1+\zeta)y(Ll)}W.$$

Let $\mu = \frac{1}{\sigma}$. Then,

$$p(z)_1 = \frac{1}{(1-\mu)(1+\zeta)y(Ll)}W.$$

This means that the real wage rate is

$$\omega = (1-\mu)(1+\zeta)y(Ll). \tag{5}$$

With increasing (decreasing or constant) returns to scale, ω is increasing (decreasing or constant) with respect to "employment × labor supply" Ll.

From (1), (2) and (5), we have

$$\frac{\partial\varphi}{\partial I}(1-\mu)(1+\zeta)y(Ll) - \Gamma'(l) = 0,$$

with

$$I = (1-\mu)(1+\zeta)y(Ll)l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}.$$

Then, from (3)

$$\frac{dl}{d(Ll)} = \frac{dl}{d\omega}\frac{d\omega}{d(Ll)} = \frac{\left[\frac{\partial\varphi}{\partial I} + \frac{\partial^2\varphi}{\partial I^2}(1-\mu)(1+\zeta)y(Ll)l\right](1-\mu)(1+\zeta)y'}{\Gamma''(l) - \frac{\partial^2\varphi}{\partial I^2}[(1-\mu)(1+\zeta)y']^2}.$$

Assuming $\frac{dl}{d\omega} > 0$, with increasing (decreasing) returns to scale $y' > 0$ ($y < 0$), this is positive (negative). Since

$$\frac{d(Ll)}{dL} = l + L\frac{dl}{dL}, \tag{6}$$

$$\frac{dl}{dL} = \frac{dl}{d(Ll)}\frac{d(Ll)}{dL} = \left(l + L\frac{dl}{dL}\right)\frac{d(l)}{d(Ll)}.$$

Thus,

$$\frac{dl}{dL} = \frac{l}{1 - L\frac{d(l)}{d(Ll)}}\frac{d(l)}{d(Ll)}.$$

Usually $\frac{dl}{dL}$ and $\frac{d(l)}{d(Ll)}$ have the same sign, and we assume $\frac{d(Ll)}{dL} > 0$ in (6). Also we assume

$$\frac{d(Lly(Ll))}{Ll} = y(Ll) + Lly' > 0. \tag{7}$$

Then, the output $Lly(Ll)$ increases by an increase in L.

Since all firms are symmetric,

$$P_1 = p(z)_1 = \frac{1}{(1-\mu)(1+\zeta)y(Ll)}W. \tag{8}$$

2.3. Involuntary unemployment

Aggregate supply of the good is equal to

$$WL + L_f\Pi = P_1 Lly(Ll).$$

Aggregate demand is

$$\begin{aligned}&\alpha\left(WL + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M\\ =&\alpha\left[P_1Lly(Ll) - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M.\end{aligned}$$

Since they are equal,

$$P_1Lly(Ll) = \alpha\left[P_1Lly(Ll) - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M,$$

or

$$P_1Lly(Ll) = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M}{1-\alpha}.$$

In real terms[3]

$$Lly(Ll) = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M}{(1-\alpha)P_1}, \tag{9}$$

or

$$Ll = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M}{(1-\alpha)P_1y(Ll)}.$$

We define a function

$$\psi(Ll) = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M}{(1-\alpha)P_1y(Ll)}.$$

Since $0 \leq L \leq L_f$ and $0 \leq l \leq 1$, we have $0 \leq Ll \leq L_fl$. Thus, the equilibrium value of Ll is obtained as a fixed point of $\psi(Ll)$.

From (2) and (3) the individual labor supply l is a (usually increasing) function of ω. From (5) ω is a function of Ll. With increasing (decreasing or constant) returns to scale technology it is increasing (decreasing or constant) with respect to Ll or with

[3] $\frac{1}{1-\alpha}$ is a multiplier.

respect to L given l. The individual labor supply l may be increasing or decreasing in L or Ll. However, we assume that Ll is increasing in L. This requires

$$\frac{dLl}{dL} = l + \frac{dl}{dL} > 0.$$

It means $Ll < L_f l$ for $L < L_f$. The equilibrium value of Ll cannot be larger than $L_f l$. However, it may be strictly smaller than $L_f l$. Then, we have $L < L_f$ and involuntary umemployment exists.

If we consider the following budget constraint for the government with a lump-sum tax T on the younger generation consumers[4],

$$G = T,$$

aggregate demand is

$$\begin{aligned}&\alpha\left(WL + L_f\Pi - G - L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\hat{D} + M\\ =&\alpha\left[P_1 Lly(Ll) - G - L_f D + L_f\hat{Q} - L_f Q\right] + G + L_f\hat{D} + M.\end{aligned}$$

Then, we obtain[5]

$$Lly(Ll) = \frac{\alpha\left(-L_f D + L_f\hat{Q} - L_f Q\right) + (1-\alpha)G + L_f\hat{D} + M}{(1-\alpha)P_1}.$$

2.4. Discussion summary

The real wage rate depends on the employment elasticity of the labor productivity and the employment level. But the employment level does not depend on the real wage rate. The real aggregate demand and the employment level are determined by the value of

$$\frac{\alpha\left(-L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\hat{D} + M}{P_1}. \tag{10}$$

If employment is smaller than the labor population, then involuntary unemployment exists.

[4] Of course, only employed consumers pay the taxes.

[5] This equation means that the balanced budget multiplier is 1.

2.5. The case of full-employment

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If $Ll = L_f l$, full-employment is realized. Then, (9) is re-written as

$$L_f l y(L_f l) = \frac{\alpha\left(-L_f D + L_f \hat{Q} - L_f Q\right) + G + L_f \hat{D} + M}{(1-\alpha)P_1}. \tag{11}$$

Since L_f and $L_f l$ are constant (if $L = L_f$, ω is constant) for one generation, this is an identity not an equation. On the other hand, (9) is an equation not an identity. (11) should be re-written as

$$\frac{\alpha\left(-L_f D + L_f \hat{Q} - L_f Q\right) + G + L_f \hat{D} + M}{(1-\alpha)P_1} \equiv L_f l y(L_f l).$$

This yields:

$$P_1 = \frac{1}{(1-\alpha)L_f l y(L_f l)}[\alpha\left(-L_f D + L_f \hat{Q} - L_f Q\right) + G + L_f \hat{D} + M].$$

Then, the nominal wage rate is determined by:

$$W = (1-\mu)(1+\zeta)y(L_f l)P_1.$$

3. Effects of a decrease in the nominal wage rate

In this paper's model, no mechanism determines the nominal wage rate except at the full-employment state. For example, when the nominal value of G increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the prices also rise. If, when G increases, the prices rise considerably, then the outputs might not increase and involuntary unemployment might not decrease. If the prices do not rise or rise only slightly, involuntary unemployment decreases.

Let us examine the effects on employment of a decrease in the nominal wage rate. A decrease in the nominal wage rate induces a decrease in the prices of the goods (see (8)), and it does not directly rescue involuntary unemployment. Proposition 2.1 in Otaki (2016) says

> Suppose that the nominal wage sags. Then, as far as its indirect effects on the aggregate demand are negligible, this only results in causing a proportionate fall in the price level. In other words, a fall in the nominal wage never rescues workers who are involuntarily unemployed.

However, *indirect effects* on aggregate demand due to a fall in the nominal wage rate may exist. We assume that falling of the nominal wage rate and the prices are not predicted by consumers. If the prices of the goods fall, the real value of the older generation's savings increases. But, at the same time, a decrease in the prices of the goods increase the real value of the younger generation consumers' debts.

The real values of the following variables will be maintained even when both the nominal wage rate and the prices fall.

G/P_1: the government expenditure.

$\hat{D}/P_1$: consumption in the childhood period of a next generation consumer.

Q/P_1: pay-as-you-go pension for an older generation consumer.

$\hat{Q}/P_1$: pay-as-you-go pension for a younger generation consumer when he retires.

On the other hand, the nominal value of D and that of $M - L_f Q$, which is the older generation's net savings, does not change. Therefore, from (10), whether the fall in the nominal wage rate increases or decreases the effective demand depends on whether

$$M - L_f Q - \alpha L_f D \tag{12}$$

is positive or negative. This is the so-called real balance effect. If D or Q is large, (12) is negative, and the fall in the nominal wage rate increases involuntary unemployment[6].

4. Several steady states

4.1. Steady state with constant employment under constant prices

First consider a steady state where the employment is constant. If $\eta L < L_f$, involuntary unemployment exists even at the steady state. With constant employment the real wage rate and labor supply are not changed, thus the output is also not changed. We assume also $\rho = 1$, that is, the constant prices of the goods. Consumers correctly predict that the prices are constant. Let T be the tax revenue which is not necessarily equal to G. Then,

$$P_1 Lly(Ll) = \alpha \left[P_1 Lly(Ll) - T - L_f D + L_f \hat{Q} - L_f Q \right] + G + L_f \hat{D} + M. \tag{13}$$

At the steady sate it must be that $\hat{D} = D$ and $\hat{Q} = Q$. Thus,

$$P_1 Lly(Ll) = \alpha \left[P_1 Lly(Ll) - T - L_f D \right] + G + L_f D + M. \tag{14}$$

[6]The discussion in this section is from the different perspectives of the real balance effect for which the argument was fought by Pigou (1943) and Kalecki (1944).

The savings of the younger generation including the pay-as-you-go pension is equal to M. Therefore,

$$(1-\alpha)\left[P_1 Lly(Ll) - T - L_f D\right] = G - T + M = M.$$

This means that:

$$G - T = 0.$$

Therefore, to maintain a sate with constant employment and prices we need balanced budget.

4.2. Steady state with an increase in output by fiscal policy under constant price

Next, consider a steady state where the employment L and the output $Lly(Ll)$ increase by fiscal policy. We assume constant prices of the goods again. Consumers correctly predict that the prices are constant.

If the employment L increases, labor supply l, the real wage rate ω and the labor productivity $y(Ll)$ increase in the case of increasing returns to scale. However, in the case of decreasing returns to scale labor supply, the real wage rate and the labor productivity may decrease. In the former (latter) case the rate of increase in the output is probably larger (smaller) than that the rate of increase in the employment. But we assume that both are positive (see (7)). Let $\eta - 1 > 0$ be the rate of increase in the output.

In this case (14) holds, too. The savings of the younger generation including the pay-as-you-go pension must be equal to ηM. Therefore,

$$(1-\alpha)[P_1 Lly(Ll) - T - L_f D] = G - T + M = \eta M.$$

This means that:

$$G - T = (\eta - 1)M.$$

From this we obtain the following proposition.

Proposition 1. *In order to maintain the steady state where employment and output increase at some positive rate ($\eta - 1 > 0$), a budget deficit is required.*

Let G' and T' be the government expenditure and tax revenue in the next period, (13) is written as

$$P_1 \eta Lly(Ll) = \alpha[P_1 \eta Lly(Ll) - T' - L_f D] + G' + L_f D + \eta M.$$

Suppose that the savings of the younger generation including the pay-as-you-go pension in the next period is equal to $\eta^2 M$. Then,

$$(1-\alpha)[P_1\eta L l y(Ll) - T' - L_f D] = G' - T' + \eta M = \eta^2 M,$$

and we obtain

$$G' - T' = \eta(\eta - 1)M.$$

This is the budget deficit which is necessary to realized an increase in employment in the next period.

On the other hand, if we suppose that the savings of the younger generation including the pay-as-you-go pension is equal to ηM, we have

$$(1-\alpha)[P_1\eta L l y(Ll) - T' - L_f D] = G' - T' + \eta M = \eta M.$$

Then,

$$G' - T' = 0.$$

From this we obtain the following proposition.

Proposition 2. *If $\eta L = L_f$, that is, the full-employment state is realized in the next period, we do not need budget deficit to maintain full-employment.*

Demand and supply of money

The demand for money is the sum of

1. savings of the younger generation,
2. tax payment for government expenditure,
3. tax payment for pay-as-you-go pension,
4. repayment of scholarship,
5. repayment of other debt.

The supply of money is the sum of

1. consumption of the older generation,
2. government expenditure,
3. pay-as-you-go pension,
4. scholarship
5. lending by the younger generation,

At the steady state where the prices of the goods are constant, we have

repayment of debt other than scholarship=lending by the younger generation,
repayment of scholarship=supply of scholarship,

However, if the employment and output increases at the rate $\eta - 1 > 0$, we have

$$\text{savings of the younger generation} = \eta \times \text{consumption of the older generation.}$$

Moreover, the argument above implies

$$\text{tax payment for government expenditure} - \text{government expenditure}$$
$$=(1-\eta) \times \text{consumption of the older generation.}$$

Therefore, the demand for money is equal to the supply of money. Money supply increases by "$(\eta - 1) \times$ consumption of the older generation", which is equal to the budget deficit, under constant prices of the goods.

4.3. Steady state with an increase in employment under inflation or deflation

We consider a steady state where the output $Lly(Ll)$ increase at the rate $\eta - 1$, and the prices of the goods rise or fall at the rate $\rho - 1$. If $\rho > 1 (< 1)$, consumers correctly predict that the prices rise (fall). Let T be the tax revenue which is not necessarily equal to G. Then,

$$P_1 Lly(Ll) = \alpha[P_1 Lly(Ll) - T - L_f D + L_f \hat{Q} - L_f Q] + G + L_f \hat{D} + M.$$

At the steady sate, $\hat{D} = \rho D$ and $\hat{Q} = \rho Q$. Thus,

$$P_1 Lly(Ll) = \alpha[P_1 Lly(Ll) - T - L_f D + (\rho - 1) L_f Q] + G + \rho L_f D + M.$$

The savings of the younger generation including the pay-as-you-go pension must be equal to $\rho\eta M$. Therefore,

$$(1-\alpha)[P_1 Lly(Ll) - T - L_f D + (\rho - 1) L_f Q] = G - T + (\rho - 1)(L_f D + L_f Q) + M = \rho\eta M.$$

This means that:

$$G - T = (\rho\eta - 1)M - (\rho - 1)(L_f D + L_f Q).$$

We approximate $\rho\eta$ by $\rho + \eta - 1$. Then,

$$G - T = (\eta - 1)M + (\rho - 1)(M - L_f D - L_f Q)$$

Without an increase in output ($\eta = 1$), if $M > L_f D + L_f Q$, in order to maintain the steady state with falling prices ($\rho < 1$) (rising prices ($\rho > 1$)) a budget surplus (deficit) is required. If $M < L_f D + L_f Q$, we obtain the inverse results. Similarly to the previous case we need a budget deficit $(\eta - 1)M$ to realize an increase in employment.

4.4. Discussion

From Propositions 1 and 2 we can say that in order to realize full-employment from a state with involuntary unemployment we need budget deficit of the government. However, when full-employment is realized, in order to maintain full-employment we need balanced budget. Therefore, additional government expenditure to realize fill-employment should be financed by seigniorage not public debt. If it is financed by public debt, this debt should not be redeemed. It should be bought by the central bank.

5. Is involuntary unemployment a Nash equilibrium?

We simplify the model up to the previous section to see if involuntary unemployment occurs in the Nash equilibrium. We consider a model with two generations. We assume the following economy. There is one good, one firm, and two younger consumers. Each consumer supplies one unit of labor and the firm produces one unit of the good with one unit of labor. There is only one firm, but it acts competitively, and the price of the good is equal to the nominal wage rate.

1. Consider the following pair of strategies for the firm and consumers. Let $0 \leq t \leq 2.5$.

    Firm: employs one younger consumer and produces one unit of the good.

    Employed younger consumer: supplies one unit of labor, buys and consumes 0.5 units of the good, pays $2t$ units of tax for pay-as-you-go pension for the older generation, and keeps the rest of his income.

    Unemployed younger consumer: consumption is zero.

   Also, there are older consumers who buy 0.5 units of the good in total. $2t$ units of the good are purchased by pension.
   Is the above pair of strategies in a Nash equilibrium? 0.5 units of employed consumers' consumption is due to utility maximization.

    Firm: given the actions of the older consumers and the actions of employed and unemployed consumers it is optimal to employ one consumer and produce one unit of the good.

Employed younger consumer: assuming he is employed, one unit of labor supply and consumption of 0.5 units of the good are the optimal strategies.

Unemployed younger consumer: assuming he is not employed, zero consumption is optimal.

The above confirms the existence of involuntary unemployment in the Nash equilibrium.
In this case even if unemployed consumer consumes 0.5 units of the good, the firm does not produce two units of the good and full-employment is not achieved because the older consumers consume 0.5 units of the good in total.

2. If the government purchases 0.5 units of the good (financed by seigniorage), the following pair of strategies will be in the Nash equilibrium.

   Firm: employs two younger consumers and produces two units of the good.

   Employed younger consumer 1: supplies one unit of labor, buys and consumes 0.5 units of the good, pays t units of tax for pay-as-you-go pension for the older generation, and keeps the rest of his income.

   Employed younger consumer 2: supplies one unit of labor, buys and consumes 0.5 units of the good, pays t units of tax for pay-as-you-go pension for the older generation, and keeps the rest of his income.

   Also, the older consumers buy 0.5 units of the good in total. $2t$ units of the good are purchased by pension.
3. In the next period, above pair of strategies is in the Nash equilibrium with one unit of consumption by the older generation in total and zero government expenditure.

Three generations model without pension

Consider a case where there are childhood period consumers, and no pay-as-you-go pension. Then, the following pair of strategies is in the Nash equilibrium.

Firm: employs one younger consumer and produces one unit of the good.

Employed younger consumer: supplies one unit of labor, buys and consumes 0.3 units of the good, repays 0.2 units of debt, pays 0.2 units of tax for unemployment benefit, and keeps the rest of his income.

Unemployed younger consumer: consumption is zero.

Also, there are 0.4 units of consumption by childhood consumers, and 0.3 units of consumption by the older generation in total.

If the government purchases 0.5 units of the good (financed by seigniorage), the following pair of strategies will be in the Nash equilibrium.

Firm: employs two younger consumers and produces two units of the good.

Employed younger consumer 1: supplies one unit of labor, buys and consumes 0.4 units of the good, repays 0.2 units of debt, and keeps the rest of his income.

Employed younger consumer 2: supplies one unit of labor, buys and consumes 0.4 units of the good, repays 0.2 units of debt, and keeps the rest of his income.

Also, there are 0.4 units of consumption by childhood consumers, and 0.3 units of consumption by the older generation in total.

In the next period, above pair of strategies is in the Nash equilibrium with 0.8 units of consumption by the older generation in total and zero government expenditure.

6. Concluding Remarks

We have examined the existence of involuntary umemployment and the effects of fiscal policy using a three-generation OLG model under monopolistic competition with increasing, decreasing or constant returns to scale. We considered the case of an indivisible labor supply, and we assumed that the good is produced only by labor.

In future research, we want to analyze involuntary unemployment and fiscal policy in a situation where goods are produced by capital and labor, and there exist investment of firms.

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