Real Options: Capital Investment Appraisal; Estimating the Market Price of Risk and Application to the Valuation of a New Business

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Abstract

The risk-neutral valuation approach to evaluating an investment avoids the need to estimate risk-adjusted discount rates, but it does require the market price of risk parameters for all stochastic variables. When historical data is available on a particular variable, its market price of risk can be estimated using the capital asset pricing model.

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Setting Up the CAPM

μ = Expected return of the investment asset

σ = Volatility of the return of an investment asset

λ = Market price of risk of the variable

ρ = Instantaneous correlation between the percentage changes in the variable and returns on a broad index of stock market prices.

μ_M = Expected return on broad index of stock market prices

σ_m= Volatility of return on the broad index of stock market prices

r = Short-term risk-free rate.

Because the investment asset is dependent solely on the market variable, the instantaneous correlation between its return on the broad index of stock market prices is also ρ. From the continuous-time version of the capital asset pricing model, we have

$$\mu - r = \frac{\rho\sigma}{\sigma_m}(\mu_m - \mathrm{r})$$

From equation another expression for $\mu - r\ is$:

$$\mu - r = \lambda\sigma$$

It follows that:

$$\lambda = \frac{\rho}{\sigma_m}(\mu_m - r)$$

This equation can be used to estimate λ. This equation is termed as [1] for the purpose of this paper.

A historical analysis of company's sales, quarter by quarter show that percentage changes in sales have correlation of 0.3 with return on the S&P 500 index. The volatility of the S&P 500 is 20% per annum and based, on historical data, the expected excess return of the S&P 500 over the risk-free rate is 5%.

Hence, using the above equation:

$$\frac{0.3}{0.2}\ x\ 0.05 = 0.075$$

When no historical data area available for the particular variable under consideration other similar variables can sometimes be used as proxies. For example, if a plant is being constructed to manufacture a new product, data can be collected on the sales of other similar products. The correlation of the new product with the market index can then be assumed to be the same as that of these other products. In some cases, the estimate of ρ from equation [1] must be based on subjective judgment. If an analyst is convinced that a particular variable is unrelated to the performance of a market index, its market price of risk should be set to zero. Furthermore, for some variables it is not necessary to estimate the market price of risk because the process followed by a variable in a risk-neutral world is the risk-free rate.

Application to the Valuation of a New Business

Traditional methods of business valuation such as applying a price/earnings multiplier to current earning, do not work well for new business. Typically, a company's earnings are negative during its early years as it attempts to gain market share and establish relationships with customers. The company must be valued by estimating future earning and cashflows under different scenarios. The company's future cashflows typically depend on a number of variables such as sales variable costs as a percent of sales, fixed costs and so on. Single estimates should be sufficient as outlined in the previous two section. A Monte-Carlo simulation can then be carried out to generate alternative scenarios for the net cashflow per year in a risk-neutral year. It is like that under some of these scenarios the company does very well and under other it becomes bankrupt and ceases operations (The simulation must have a built-in rule for determining when bankruptcy happens). The value of the company is the present value of the expected cashflow in each year using the risk-free rate for discounting.

Schwartz and Moon (2000) applied this approach to the internet startup Amazon.com at the end of 1999. They assumed that the company's sales revenue, R, and its revenue growth rate, μ are stochastic. Their model is

$$\frac{dR}{R} = \mu \, dt + \sigma(t) dz_1$$

With

$$d\mu = \kappa(\bar{\mu} - \mu)dt + \eta(t)dz_2$$

These equations show that the revenue has an expected growth rate μ, which itself follows a stochastic process. The stochastic process for μ is mean reverting to a long-run average growth rate $\bar{\mu}$ at rate κ.;

The volatility of revenue, $\sigma(t)$, was assumed to deterministic and to decrease exponentially from an initial level of 10% per quarter to a long-run average level of 5% per quarter. The standard deviation of the sales growth rate was also assumed to be deterministic IT decreased exponentially from an initial level of 3% per quarter to zero. The initial sales were $346 million per quarter and the initial growth rate was 11% per quarter. The values of κ and $\bar{\mu}$ were 7% per quarter and 1.5% per quarter respectively. The two Weiner process dz_1 and dz_2were assumed to be uncorrelated. Schwartz and Moon assumed that cost of goods sold would be 75% of sales, other variables expenses would be 19% of sales, and fixed expenses would be $75 million per quarter. The initial tax loss carries forward was $559 million and the tax rate was assumed to be 35%. The market price of risk, λ_R for R was estimated from historical data using the approach described in the previous section. The market price of risk for μ was assumed to be zero. The risk-neutral stochastic process for R was therefore:

$$\frac{dR}{R} = (\mu - \mu_R \sigma)dt + \sigma dz_1$$

While the risk-neutral process for μ was the same as the real-world process given above.

The time horizon for the analysis was 25 years and the terminal value o the company was assumed to be ten-times pretax operating profit. The initial cash positionwas$906 million and the company was assumed to go bankrupt if the cash balance became negative.

Different future scenarios were generated using Monte Carlo simulation. The evaluation of the scenarios involved taking account of the possible exercise of convertible bonds and the possible exercise of employee stock option. The value of the company to the shareholders was calculated the present value of the net cash flows discounted at the risk-free rate.

The market price of Amazon's shares at the end of 1999 was $76.125. The price per share given by the Monte Carlo simulation based on the assumptions we have outlined was $12.42 However, Schwartz and Moon point out that a small change n the assumption about the initial value of η leads to a big change in the price per share. For example, when n(0) was increased from 3% to 6% the share price given by the Monte Carlo simulation increased to about $100.

The CAPM remains an important financial theory that establishes a linear relationship between the required return on an investment and risk. The model is based on the linear relations between an assets beta, the risk-free rate (typically the Treasury bill rate) and the equity risk premium, or the expected return on the market minus the risk-free rate. For the valuation of Internet companies, a simple model to value these companies that is based on fundamentally on assumptions about the expected growth rate of revenue and on expectations about the cost structure of the company.

References

Brealey Myers Allen, Principles of Corporate Finance 9th Edition,

John C Hull, Options, Futures and Other Derivatives, 5th Edition

Peter S Rose Milton H Marquis Money and Capital Markets, 10th Edition

Rashid, Muhammad Mustafa (2019): *A Survey of US and International Financial Regulation Architecture*

Rashid, Muhammad Mustafa (2019): *International Financial Credit Crises; Lessons from Canada.*

Robert J Gordan, Macroeconomics, 11th Edition

Rosen & Gayer, Public Finance, 10th Edition

R. Glenn Hubbard, Money the Financial System and the Economy, 6th Edition

Schwartz and Moon, Rational Pricing of Internet Companies, Financial Analyst Journal, May/June 200, 62-75.

William F. Sharpe (1964), Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, The Journal of Finance, Vol. 19, Issue 3, (Sep 1964), 425-442, 1964