Liquidity requirement and banks' lending

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20 January 2020

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Abstract

This study proposes a model that describes banks' decisions about how much liquidity they hold and analyzes how liquidity regulations affect the amount of their lending. In literature, it is pointed out that banks are likely to hold ex-post excess liquidity under a liquidity regulation when some depositors make decisions based on the banks' soundness. This result implies that the regulation forces banks to suffer an unnecessary decrease of their lending, and thus, they would try to mitigate the loss by adjusting their portfolio. The aim of this study is to investigate whether banks' lending decreases or not when there exist multiple sets of assets that satisfy a liquidity regulation. In addition, we analyze two types of liquidity regulation; one focuses on banks' survivability, and the other focuses on continuity of their liquidity holding. The model shows that, even when there exist other ways to satisfy the regulations besides holding only reserves, banks still hold an ex-post excess amount of liquidity under either type of liquidity regulation. However, the model also shows that the amount of banks' lending varies according to how they satisfy the liquidity regulation and the probability that a severe reduction of lending happens depends partly on the regulation's type. These results implies that banks' decisions for mitigating losses caused by liquidity regulations lead to an undesired outcome, and thus, we consider more carefully banks' decisions under liquidity regulations.

Key words: Bank, Liquidity regulation, Excess liquidity

JEL classification: E02, G21, G28

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1 Introduction

After the financial crisis of 2007–2008, the danger of negative externalities that highly indebted financial institutions face gained attention and the Basel Committee developed a new regulatory framework on banks, Basel Ⅲ , to make the financial system stable. This framework introduces new rules governing banks' debt structures and requirements for holding certain types of liquid asset. There is, however, a remarkable asymmetry between the economic analysis of the capital and liquidity regulations.

As to analyses of capital regulations, the pioneering work of Modigliani and Miller (1958) provides a theoretical framework. After the introduction of the international regulations on banks' capital in 1988, there are some models that analyze banks' capital such as Keeley (1990). Then, in 2000s, especially after the financial crisis, there are a large number of studies on the banks' capital and capital regulations. Although there is little agreement on the optimal level of requirements and the regulations' costs, there exist some common settings for analyzing capital regulations.

However, the amount of discussion about regulating liquidity is much less than that on capital regulations, and moreover, there is no benchmark theory regarding regulating liquidity provision by intermediaries. Before the financial crisis, there were studies of liquidity provision by financial intermediaries, but liquidity regulations got not so much attentions.

Nevertheless, Basel Ⅲ introduces two new concepts, the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR), and the deadline by which banks will be compelled to meet requirements for these ratios is 2019. Thus, we need to consider what the optimal way to regulate bank's liquidity is. To investigate this issue, Diamond and Kashyap (2016) provide a baseline model.

Diamond and Kashyap (2016) analyze two types of liquidity regulation that represent NSFR and LCR, and show that two important results are obtained. First, banks must hold an excess amount of safe assets and reduce their lending regardless of the regulation's type when some depositors determine whether or not they should withdraw their deposits early based on the banks' soundness and when the regulation restricts the banks' decisions. Second, which type of regulation is optimal depends on the banks' heterogeneity. If the bank's heterogeneity is sufficiently large, LCR-type regulation can lead to a smaller reduction of lending than NSFR-type one does, otherwise NSFR-type one leads to a smaller reduction of the banks' lending.

The result that banks suffer an unnecessary reduction of lending under liquidity regulations implies that the banks can suffer losses, and thus, they would try to mitigate the losses in some way. Considering that some of their safe assets are needed just to satisfy the regulation's requirement, they would be able to replace the assets with some other assets that can work as sources of both liquidity and profits. In the situation where banks are reluctant to hold excess safe assets under a liquidity regulation, introducing such assets raises a new question of how banks satisfy the liquidity regulation. When there exists another asset that can be held as less effective liquidity but yield larger return than the safe asset, there is a probability that the banks can substitute these assets for some of the unnecessary safe assets and reduce losses by the regulation. Thus, we need to analyze how the regulation affects banks' decisions and the reduction of their lending taking their choices of assets into account.

To address these issues in more depth, we suppose that there are "liquid assets" in addition to safe assets and lending (long-term assets). Then, we investigate how a bank satisfies a requirement of a liquidity regulation and how its decision and the amount of its lending vary according to the regulation's type.

In the model presented in this study, there exist one bank and its depositors. The bank raises funds by offering demand deposits to the depositors and by issuing shares, and then, it uses some of the funds to invest in assets and lends the remainder. Because the deposits are demand deposits, the depositors can withdraw their deposits as they want, and thus, some of withdrawals happen before the bank receives the return of tits lending. Then, in order to repay to these depositors, it needs to hold some liquidity and/or liquidate its assets and/or loans. When there exists a liquidity regulation, the bank need to satisfy its requirement.

In this study, we analyze two types of liquidity regulation, one focuses on banks' survivability, and the other requires them holding liquidity at any time. The model shows that, under either type of regulation, the bank still holds an ex-post excess amount of liquidity as Diamond and Kashyap (2016) show, even when it can use the liquid assets as sources of liquidity. This result derives as follows. A requirement of a liquidity regulation must be determined based on the most dangerous case, that is, the largest amount of early repayment. Then, when some depositors determine whether or not to withdraw their deposits early based on the bank's soundness, a regulator requires that it holds enough liquidity to repay to these depositors. However, when it holds enough liquidity under the regulation and satisfies the requirement, the depositors choose not to withdraw their deposits early because they think the bank is sound enough. Thus, it holds an ex-post excess amount of liquidity under the liquidity regulation, and this result does not depend on what assets it holds as liquidity.

Although introducing the liquid assets cannot solve the problem of excess liquidity, the existence of the liquid assets yields two new results. First, under a liquidity regulation, the model shows that the bank can choose to hold both the safe assets and the liquid assets as well as to rely only on the safe assets and that the former is chosen when the discount rate for the liquidated liquid asset is sufficiently large. In other words, when the liquid assets work efficiently as liquidity, the bank substitutes them for the safe assets. Second, the model shows that, although holding only the safe assets and holding both the safe assets and the liquid assets are indifferent regarding a liquidity regulation, the latter causes a larger reduction of the bank's lending than the former does, and, at the same time, the bank obtains larger profits from the former portfolio. In other words, the bank chooses to decrease its lending as the privately optimal response to the regulation. Moreover, the probability that it chooses to hold both the safe assets and the liquid assets depends partly on the regulation's type. These results imply that banks' decisions for mitigating losses caused by a liquidity regulation lead to an undesired outcome with respect to the amount of their lending, and thus, we consider more carefully banks' decisions under liquidity regulations.

The remainder of this paper is organized as follows. Section 2 reviews literature. Section 3 presents the model and explains its settings, and Section 4 examines the bank's decision. Section 5 presents the comparison of two types of liquidity regulation, and Section 6 presents our conclusion.

2 Review of literature

As it is mentioned above, there has been little literature on liquidity regulations, particularly before the crisis. The early contributes are Rochet (2004, 2008) and Allen and Gale (2004). These papers focus on some market failures and consider which problems we need liquidity regulations to solve. Rochet argues that simple liquidity ratios can potentially deal with problems in payment systems and moral hazard problems at the individual bank level due to opaqueness of assets. Allen and Gale (2004) study regulations of the financial system using a welfare analysis, and argue that there may be a role for liquidity regulations when markets for aggregate risks are incomplete.

After the crisis and implementation of new liquidity requirements for banks such as the Liquidity Coverage Ratio (LCR) and the Net Stability Funding Ratio (NSFR) [1], some researchers investigate the effectiveness of liquidity regulations by comparing them with other regulations, such as capital regulations.

Vives (2014) supposes that banks can become either insolvent or illiquid and analyzes efficient combinations of capital and liquidity regulations by using a global game analysis. He finds that the two regulations are not simple substitutes, and their effectiveness vary according to whether banks' problems are insolvency or illiquidity. In particular, if depositors are more likely to run, the liquidity regulation can enhance the banks' stability. Calomiris et al. (2018) point out that a liquidity regulation improves banks' incentives to manage risk more easily than a capital regulation because banks' liquidity is more observable and verifiable for outsiders than their capital. Perotti and Suarez (2011) develop a model in which banks use too much amount of short term funding and compare a liquidity regulation with Pigovian taxes. They point out that whether or not the liquidity regulation solves the problem depends on what heterogeneity dominates. When the banks differ in credit opportunities, Pigovian taxes are best, whereas the liquidity regulation is best when they differ in their risk taking incentives. Walther (2016) also compares a liquidity regulation with Pigovian taxes and argues that a constraint-based liquidity regulation works efficiently and it does not require knowledge of banks' private information such as funding costs and average returns to investment, whereas efficient Pigovian taxes need these information.

Some studies show that a liquidity regulation does not only improve banks' stability with respect to illiquidity but also it changes status of aggregate economy, and in turn, affects banks' decisions. Farhi et al. (2009) investigate how a liquidity regulation affects banks' risk taking and point out that, in the financial system under the liquidity regulation, there exists a sufficient amount of aggregate

[1] Cecchetti and Kashyap (2018) explain the objects of the two tools. LCR urges banks to hold enough amounts of liquidity, whereas NSFR guarantees their stable maturity transformations.

investment in short-term assets to achieve the efficient risk sharing amount the banks.

However, some negative effects of liquidity regulations are also pointed out. One of them is that a liquidity regulation can reduce banks' lending and liquidity creation. Roberts et al. (2018) empirically investigate the effects of liquidity regulation and find that banks subject to LCR create less liquidity per dollar of assets. Thus, they argue that LCR has a negative effect on banks' lending and at least some of this effect is unlikely to be due to capital regulations. Carletti et al. (2018) theoretically analyze how banks choose their portfolio under liquidity and/or capital regulations and point that, because holding so much liquidity reduces their profits and increases probability of their insolvency, the liquidity regulation does not always improve their stability.

One of the other negative externality of liquidity regulations is that it can deteriorate adverse selection in interbank markets and lead to collapse of them because fewer sales of banks' assets reflect cash needs under a liquidity regulation. Malherbe (2014) considers interbank markets with elastic demands for banks' assets and points out that the adverse selection impairs liquidity provision between banks and then leads them to hold more liquidity. He concludes that this negative feedback effect may result in hoarding behavior and a market breakdown. Heider et al. (2015) investigate a similar problem at interbank markets with inelastic demands and argue that the market breakdown can be prevented when the liquidity regulation is implemented at the appropriate level.

Diamond and Kashyap (2016) consider banks' excess liquidity as a negative externality of a liquidity regulation. The first study of banks' excess liquidity is Cooper and Ross (1998). Based on Diamond and Dybvig (1983), they investigate banks' portfolio choices and point out that they hold excess liquidity when liquidation cost of an insolvent bank is sufficiently high. Although Cooper and Ross (1998) argue that there exists a probability that banks hold excess liquidity when they become insolvent, Ennis and Keister (2006) extend the results of Cooper and Ross (1998) and point out that the reason why banks hold excess liquidity is only to prevent their depositors' massive withdraw. Then, based on these results, Diamond and Kashyap (2016) find that banks hold an ex-post excess amount of liquidity when some of depositors' withdrawals depend on the banks' soundness under a liquidity regulation. In addition, they argue that the amount of excess liquidity varies according to the type of liquidity regulation and that which type of regulation is effective depends on the heterogeneity of the banks.

As mentioned above, however, these researches does not consider banks' portfolio choices with respect to their assets that work as liquidity. Thus, banks' response to a liquidity regulation is not investigated fully. Recently, some researchers analyze the effects of a liquidity regulation with respect to banks' choices of liabilities such as insecure debt and secured debt (Matta and Perotti, 2015; Körding and Scheubel, 2018). However, researches of changes of banks' safe assets under a liquidity regulation are still scarce, and thus, this study provides some insights regarding this question.

3 Model

In this section, based on Diamond and Kashyap (2016), we develop a model in which a bank makes decision about its liquidity holding.

3.1 Baseline settings

There are two types of actor in the economy: a bank (banker), its depositors, and there are three dates: $t = 0, 1, 2$. In addition, there exists a liquidity regulation. In this study, we consider two types of regulation, and both require that the bank does not go bankrupt at $t = 1$. We assume that there is no uncertainty in the economy, and thus, both the banks and its depositors can estimate the other's decisions precisely.

At $t = 0$, the bank raises funds, invests some of the funds in some assets and lends the remainder. On the other hand, the bank's depositors obtain demand deposits. At $t = 1$, some of the depositors decide to withdraw their deposit, and the bank repays to them by using its asset returns and/or liquidating its assets and/or its claim of the lending. And then, at $t = 2$, it receives the returns of its assets and lending if it still holds them. Then, it repays to the remaining depositors and the banker obtains the residual profit if it exists. Therefore, in the model, the bank tries to maximize its remaining profit by choosing the optimal portfolio at $t = 0$ under a constraint of the liquidity regulation.

3.2 Settings: depositors

Suppose that there exist many depositors and their total size is normalized as 1. At $t = 0$, each of them obtains D units of demand deposits. The gross rate of return of one-period deposit is assumed r_d, and thus, the gross rate of return is r_d^2 when a deposit is withdrawn at $t = 2$.

The depositors can withdraw their deposits either $t = 1$ or $t = 2$. We assume their decisions as follows.

Assumption 1. *There exist three types of depositor: impatient, nervous, and patient, and the ratios of each type to total depositors are η, δ, $1 - \eta - \delta$, respectively. The impatient depositors always withdraw their deposits at $t = 1$, and the patient depositors always withdraw their deposits at $t = 2$. The nervous depositors withdraw their deposits at $t = 1$ unless the bank satisfies a constraint by the liquidity regulation.*

Then, the bank's repayment at $t = 1$ is $\eta D r_d$ or $(\eta + \Delta) D r_d$ based on its decision (soundness).

3.3 Settings: bank

Suppose that the bank raises funds B by offering demand deposits to the depositors and by issuing shares to itself. Because it raises funds D by receiving deposit, $B - D$ is raised by issuing shares.

At $t = 0$, it decides how to use B so that its residual profit at $t = 2$ is maximized. With respect to assets that it can invest at $t = 0$ and lending, we have the following assumption.

Assumption 2. *There exist two types of asset that the bank can invest at $t = 0$: safe assets and liquid assets. For simplification, we assume that there exists no asset to invest at $t = 1$. One unit of invest at $t = 0$ in the safe asset returns r_s at $t = 1$ On the other hand, one unit of invest at $t = 0$ in the liquid asset returns R_1 at $t = 2$.*

Moreover, the bank can lend its funds at $t = 0$. One unit of lending returns R_2 at $t = 2$.

In addition, we assume about the liquidation of the bank's assets and loans as follows.

Assumption 3. *At $t = 1$, the bank can liquidate its liquid assets and lending (loans) to raise funds. Liquidating one unit of liquid asset returns βR_1, and liquidating one unit of loan returns αR_2, where $0 \leq \alpha < \beta < 1$ is satisfied.*

Then, we have the following assumption about the returns of the assets.

Assumption 4. *The returns of the safe asset, the liquid asset and the lending satisfy*

$$\alpha R_2 < \beta R_1 < r_s < R_1 < R_2.$$

Therefore, in order to obtain some amount of funds at $t = 1$, investing in the safe asset is the most efficient way. On the other hand, in order to obtain some amount of funds at $t = 2$, lending is the most efficient way.

Denote the ratios of the bank's investment in the liquid assets and the safe assets as $s_1,\ s_2$, respectively, and $s_1,\ s_2$ are nonnegative and satisfy and $s_1 + s_2 \leq 1$. Then, the maximum amount of funds that the bank can raise at $t = 1$ without liquidating its loan is $s_2 B r_s + \beta s_1 B R_1$. We denote this amount as **the bank's liquidity** and the pair (s_1, s_2) as **the bank's liquidity decision**.

4 Analysis

4.1 No liquidity regulation

As a first benchmark, we consider the bank's liquidity decision when there exists no liquidity regulation. Because there are no uncertainty and no constraint on the bank's decision by liquidity regulation, the nervous depositors does not withdraw their deposits at $t = 1$, and thus, the amount that the bank needs to repay is $\eta r_d D$.

Define **the efficient ratio of safe assets** as follows.

Definition 1. *Define* ***the efficient ratio of safe asset*** *as $\ddot{s}_2$, that is, the ratio with that the bank can obtain funds just equal to the repayment to the impatient depositors at $t = 1$. This ratio is expressed as*

$$\ddot{s}_2 \equiv \frac{\eta D r_d}{B r_s}.$$

Then, because the bank knows that it needs funds to repay $\eta D r_D$ and that investing in safe assets is the most efficient way to obtain funds at $t=1$, its liquidity decision (s_1, s_2) satisfies $s_2 \leq \ddot{s}_2$. In addition, because holding excess liquidity between $t=1$ and $t=2$ is less efficient than holding loans, the optimal liquidity decision is $(0, \ddot{s}_2)$. The following proposition summarizes the bank's optimal liquidity decision.

Proposition 1. *When the nervous depositors does not withdraw their deposits at $t=1$, that is, $\Delta = 0$, the bank's optimal liquidity decision satisfies $(s_1, s_2) = (0, \ddot{s}_2)$.*

In other words, when only the impatient deposits withdraw at $t=1$, the bank's optimal liquidity decision is investing in only the safe assets and the ratio is the efficient ratio of safe assets.

4.2 Analysis 2: with liquidity regulation

4.2.1 Liquidity under a liquidity regulation

In this subsection, we consider the bank's liquidity decision when there exists a liquidity regulation. Denote the bank's residual profit at $t=2$ when its liquidity decision is (s_1, s_2) as $\mathcal{R}(s_1, s_2)$. Suppose that only the impatient depositors withdraw their deposits at $t=1$, that is, $\Delta = 0$ is satisfied. Then, the outcome of the bank's liquidity decision can be classified into four cases based on its behavior at $t=1$. In the first case, it has enough return of the safe assets to repay $\eta D r_d$ to the impatient depositors at $t=1$. In the second case, return of its safe assets is not enough to repay $\eta D r_d$ but it hold a sufficient amount of the liquid assets, and thus, it liquidates (some part of) its liquid assets. In the third case, both of its assets are not enough to repay $\eta D r_d$, and thus, it liquidate (some part of) its lending. In the fourth case, its assets and lending are not enough to raise funds to repay $\eta D r_d$, and thus, it goes bankrupt. Based on our assumption, however, it must not go bankrupt at $t=1$ under liquidity regulations, and thus, we do not consider the fourth case.

Denote the ratio of the liquidated liquid assets in the second case as μ, and the ratio of the liquidated lending in the third case as ν. In addition, denote the bank's residual profits in the former three cases as $\mathcal{R}^A(s_1, s_2)$, $\mathcal{R}^B(s_1, s_2)$ and $\mathcal{R}^C(s_1, s_2)$, respectively. Then, they are expressed as

$$\begin{aligned}
\mathcal{R}^A(s_1, s_2) &\equiv (1 - s_1 - s_2)BR_2 + s_1 BR_1 + s_2 B r_s - \eta D r_d - (1-\eta) D r_d^2, \\
&= BR_2 - B(R_2 - R_1)s_1 - B(R_2 - r_s)s_2 - \eta D r_d - (1-\eta) D r_d^2,
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}^B(s_1, s_2) &\equiv (1 - s_1 - s_2)BR_2 + (s_1 - \mu)BR_1 + \left[\mu\beta BR_1 + s_2 B r_s - \eta D r_d\right] - (1-\eta) D r_d^2, \\
&= BR_2 - B(R_2 - R_1)s_1 - B\left(R_2 - \frac{r_s}{\beta}\right)s_2 - \frac{\eta D r_d}{\beta} - (1-\eta) D r_d^2,
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}^C(s_1, s_2) &\equiv (1 - s_1 - s_2 - \nu)BR_2 + \left[\nu\alpha BR_2 + \beta s_1 BR_1 + s_2 B r_s - \eta D r_d\right] - (1-\eta) D r_d^2, \\
&= BR_2 + B\left(\frac{\beta R_1}{\alpha} - R_2\right)s_1 + B\left(\frac{r_s}{\alpha} - R_2\right)s_2 - \frac{\eta D r_d}{\alpha} - (1-\eta) D r_d^2,
\end{aligned}$$

where $\mu \equiv (\eta D r_d - s_2 B r_s)/\beta B R_1$ and $\nu \equiv (\eta D r_d - \beta s_1 B R_1 - s_2 B r_s)/\alpha B R_2$.

Suppose that there exists a liquidity regulation that adds a constraint on the bank's decision. We consider two types of liquidity regulation: NSFR-type and LCR-type.

NSFR-type regulation requires that the bank's residual profit at $t = 2$ is nonnegative even when it liquidates (some part of) its loans. In other words, it must not go bankrupt under NSFR-type regulation. On the other hands, LCR-type regulation requires that it holds liquidity more than some fraction of remaining repayment between $t = 1$ and $t = 2$. In other words, it always hold some liquidity under LCR-type regulation.

First, consider NSFR-type regulation. Although the requirement of the regulation seems to mean that $\mathcal{R}^C(s_1, s_2) \geq 0$ is satisfied, it is not correct. Because $\mathcal{R}^C(s_1, s_2)$ is defined based on $\Delta = 0$, $\mathcal{R}^C(s_1, s_2) \geq 0$ does not always guarantee that the bank's residual profit is non-negative even when $\Delta \neq 0$ is satisfied. In other words, satisfying $\mathcal{R}^C(s_1, s_2) \geq 0$ is not enough to satisfy the requirement of NSFR-type regulation and it is needed that the bank's residual profit must be non-negative even when the repayment at $t = 1$ is $(\eta + \Delta) D r_d$. This residual profit is easily calculated by replacing η in $\mathcal{R}^C(s_1, s_2)$ with $\eta + \Delta$. Then, the constraint is expressed as

$$BR_2 + B\left(\frac{\beta R_1}{\alpha} - R_2\right) s_1 + B\left(\frac{r_s}{\alpha} - R_2\right) s_2 - \frac{(\eta + \Delta) D r_d}{\alpha} - (1 - \eta - \Delta) D r_d^2 \geq 0,$$

$$\Leftrightarrow s_1 \geq -\frac{r_s - \alpha R_2}{\beta R_1 - \alpha R_2} s_2 + \frac{(\eta + \Delta) D r_d + \alpha(1 - \eta - \Delta) D r_d^2 - \alpha B R_2}{B(\beta R_1 - \alpha R_2)} \equiv \underline{s}_1^N(s_2).$$

Second, consider LCR-type regulation. As it is in the case of NSFR-type regulation, the constraint must be defined based on the case where the repayment at $t = 1$ is $(\eta + \Delta) D r_d$. LCR-type regulation requires the bank holding liquidity more than some fraction of remaining repayment between $t = 1$ and $t = 2$ and we denote this fraction as ρ. Then, the constraint is expressed as

$$s_2 B r_s + \beta s_1 B R_1 \geq (\eta + \Delta) r_d D + \rho(1 - \eta - \Delta) r_d^2 D,$$

$$\Leftrightarrow s_1 \geq -\frac{r_s}{\beta R_1} s_2 + \frac{(\eta + \Delta) r_d D + \rho(1 - \eta - \Delta) r_d^2 D}{\beta B R_1} \equiv \underline{s}_1^L(s_2).$$

When there exists i-type regulation, the bank's liquidity decision $(\bar{s}_2, \bar{s}_2)$ must satisfy at least $\bar{s}_1 \geq \underline{s}_1^i(\bar{s}_2)$ where $i = N$ under NSFR-type regulation and $i = L$ under LCR-type one. Then, consider whether or not $(s_1, s_2) = (0, \ddot{s}_2)$ satisfies the conditions. Because $s_2 B r_s + \beta s_1 B R_1$ with $(s_1, s_2) = (0, \ddot{s}_2)$ is $\eta D r_d$, it is clear that $(0, \ddot{s}_2)$ does not satisfy the constraint of LCR-type regulation. With regard to NSFR-type one, $0 \geq \underline{s}_1^N(\ddot{s}_2)$ is not satisfied with some parameters. In addition, when $0 \geq \underline{s}_1^N(\ddot{s}_2)$, the bank's optimal liquidity decision under no regulation is also optimal under the regulation, and thus, there is no need to regulate the bank. In other words, when liquidity regulations affect the bank's decision, it cannot choose $(0, \ddot{s}_2)$ under the regulations.

However, when it changes its decision and satisfies the regulations, the nervous depositors regard it as sound and does not withdraw their deposits at $t = 1$. Then, it implies that the actual repayment at $t = 1$ is $\eta D r_d$ and the ex-post optimal liquidity decision is $(0, \ddot{s}_2)$ that the bank cannot choose. The result is summarized as follows.

Proposition 2. *Suppose that the bank can hold the safe assets and/or the liquid assets as liquidity. In addition, suppose that there exists NSFR-type regulation or LCR-type one and that the bank cannot choose $(s_1, s_2) = (0, \ddot{s}_2)$ under either of the regulations. Then, regardless of the type of regulation, the bank holds an ex-post excess amount of liquidity, and thus, the bank's lending decrease under the liquidity regulations.*

This result is also obtained in Diamond and Kashyap (2016) where the liquid assets do not exists. Thus, we have the following corollary.

Corollary 1. *Introducing another assets that can be used to obtain liquidity in addition to the safe assets does not prevent unnecessary decrease of banks' lending happening.*

4.2.2 Bank's decision under a liquidity regulation

Suppose that there exists i-type regulation and $0 \leq \underline{s}_1^i(\ddot{s}_2)(i = NorL)$ is not satisfied, in other words, the bank cannot chose $(s_1, s_2) = (0, \ddot{s}_2)$ under the regulations. In addition, assume that $\underline{s}_1^N(s_2) < \underline{s}_1^L(s_2)$ is satisfied $\forall s_2 \in [0, 1]$. This assumption is just to keep the figures simple and does not affect the results described here. Because the actual repayment at $t = 1$ is $\eta D r_d$, the bank's residual profit is defined as $\mathcal{R}^A(s_1, s_2), \mathcal{R}^B(s_1, s_2)$ or $\mathcal{R}^C(s_1, s_2)$ in the previous subsection.

First, consider the bank's liquidity decision when its residual profit is $\mathcal{R}^C(s_1, s_2)$. In this case, the bank's liquidity is not enough to repay $\eta D r_d$ at $t = 1$. This implies that $s_2 B r_s + \beta s_1 B R_1 < \eta D r_d$ is satisfied. Then, by rewriting the inequality, we have

$$s_1 < -\frac{r_s}{\beta R_1} s_2 + \frac{\eta D r_d}{\beta B R_1} = -\frac{r_s}{\beta R_1}(s_2 - \ddot{s}_2) \equiv s_1^c(s_2).$$

The bank's liquidity decision (s_1, s_2) that satisfies the above inequality exists the lower part of the line $s_1 = s_1^c(s_2)$ in Figure 1.

Because at least $s_2 B r_s + \beta s_1 B R_1 \geq (\eta + \Delta) D r_d$ is satisfied under LCR-type regulation, this case does not happen under LCR-type one. Then, suppose that there exists NSFR-type regulation. The constraint of NSFR-type regulation is expressed as

$$s_1 \geq -\frac{r_s - \alpha R_2}{\beta R_1 - \alpha R_2} s_2 + \frac{(\eta + \Delta) D r_d + \alpha(1 - \eta - \Delta) D r_d^2 - \alpha B R_2}{B(\beta R_1 - \alpha R_2)}.$$

As it is mentioned above, we suppose that $(s_1, s_2) = (0, \ddot{s}_2)$ does not satisfy the constraint of NSFR-type regulation, and thus, the line $s_1 = \underline{s}_1^N(s_2)$ intersects with x axis at the left part of the point $(0, \ddot{s}_2)$. In addition, because $r_s > \beta R_1$ is satisfied from the assumption, we have

$$-\frac{r_s}{\beta R_1} > -\frac{r_s - \alpha R_2}{\beta R_1 - \alpha R_2}.$$

Then, as it is shown in Figure 1, there is no liquidity decision (s_1, s_2) that satisfies both $s_1 < s_1^c(s_2)$ and $s_1 \geq \underline{s}_1^N(s_2)$ at the same time. It implies that the bank's residual profit does not expressed as

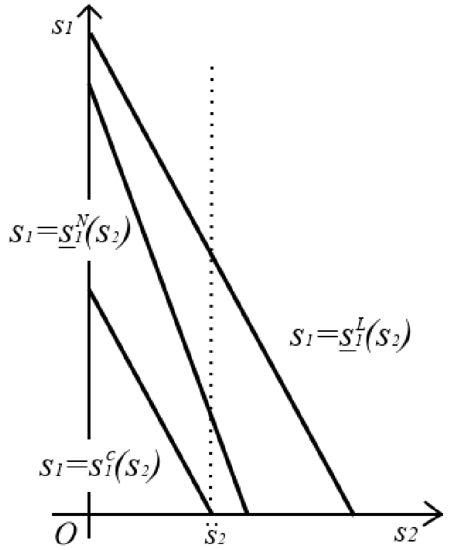

Figure 1: the constraints regarding $\mathcal{R}^C(s_1, s_2)$

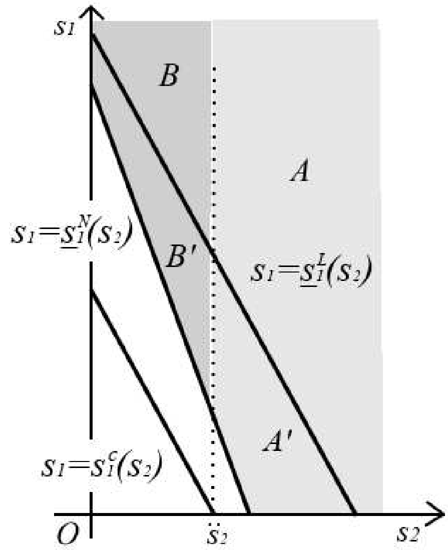

Figure 2: the regions separated by $s_2 = \ddot{s}_2$

$\mathcal{R}^C(s_1, s_2)$ under either NSFR-type regulation or LCR-type one. In other words, the liquidation of the bank's loans does not happen under the regulations.

Second, consider the bank's liquidity decision when its residual profit is $\mathcal{R}^B(s_1, s_2)$. In this case, its safe assets in not enough to repay $\eta D r_d$ at $t = 1$ but it holds a sufficient amount of the liquid assets. It implies that $s < \ddot{s}_2$ and $\eta D r_d \leq s_2 B r_s + \beta s_1 B R_1$ are satisfied. As it is shown in Figure 1, $\eta D r_d \leq s_2 B r_s + \beta s_1 B R_1$, that is, $s_1^c(s_2) \leq s_1$ is always satisfied when the bank's liquidity decision satisfies one of the constraints of the regulations. Then, when its residual profit is $\mathcal{R}^B(s_1, s_2)$, the bank's liquidity decision (s_1, s_2) exists on the region B under LCR-type regulation and on the region $B + B'$ under LCR-type one in Figure 2.

Suppose that there exists NSFR-type regulation. Then, the bank's problem is expressed as

$$\max_{s_1,s_2} \mathcal{R}^B(s_1, s_2) \equiv BR_2 - B(R_2 - R_1)s_1 - B\left(R_2 - \frac{r_s}{\beta}\right)s_2 - \frac{\eta D r_d}{\beta} - (1-\eta)Dr_d^2$$

$$s.t. \quad 0 \leq s_1 \leq 1, \quad 0 \leq s_2 < \ddot{s}_2, \quad 0 \leq s_1 + s_2 \leq 1,$$

$$\underline{s}_1^N(s_2) \leq s_1.$$

With fixed value $\bar{s}_2$, it is clear that $\mathcal{R}^B(s_1, \bar{s}_2)$ is decreasing in s_1, and thus, $\underline{s}_1^N(\bar{s}_2)$ is the optimal ratio of the liquid assets. Then, by substituting $s_1 = \underline{s}_1^N(\bar{s}_2)$ into $-B(R_2 - R_1)s_1 - B\left(R_2 - \frac{r_s}{\beta}\right)\bar{s}_2$, we have

$$\begin{aligned}
&- B(R_2 - R_1)\bar{F} + B(R_2 - R_1)\frac{r_s - \alpha R_2}{\beta R_1 - \alpha R_2}\bar{s}_2 - B\left(R_2 - \frac{r_s}{\beta}\right)\bar{s}_2, \\
&= -B(R_2 - R_1)\bar{F} + \frac{BR_2}{\beta(\beta R_1 - \alpha R_2)}(\beta - \alpha)(r_s - \beta R_1)\bar{s}_2, \\
&\text{where} \quad \bar{F} \equiv \frac{(\eta + \Delta)Dr_d + \alpha(1 - \eta - \Delta)Dr_d^2 - \alpha BR_2}{B(\beta R_1 - \alpha R_2)}.
\end{aligned}$$

Thus, $\mathcal{R}^B\big(\underline{s}_1^N(s_2), s_2\big)$ is increasing in s_2. It implies that the optimal liquidity decision under NSFR-type regulation (the region $B + B'$) is $(\underline{s}_1^N(\check{s}_2), \check{s}_2)$ where $\check{s}_2$ is sufficiently close to $\ddot{s}_2$ but not equal to $\ddot{s}_2$.

Next, suppose that there exists LCR-type regulation. With fixed value $\bar{s}_2$, $\mathcal{R}^B(s_1, \bar{s}_2)$ is decreasing in s_1. Thus, $\underline{s}_1^L(\bar{s}_2)$ is the optimal ratio of the liquid assets. Then, by substituting $s_1 = \underline{s}_1^L(\bar{s}_2)$ into $-B(R_2 - R_1)s_1 - B\left(R_2 - \frac{r_s}{\beta}\right)\bar{s}_2$, we have

$$\begin{aligned} & -B(R_2 - R_1)\bar{L} + B(R_2 - R_1)\frac{r_S}{\beta R_1}\bar{s}_2 - B\left(R_2 - \frac{r_s}{\beta}\right)\bar{s}_2, \\ & = -B(R_2 - R_1)\bar{L} + \frac{BR_2}{\beta R_1}(r_s - \beta R_1)\bar{s}_2, \\ & \text{where} \quad \bar{L} \equiv \frac{(\eta + \Delta)Dr_d + \rho(1 - \eta - \Delta)Dr_d^2}{\beta R_1}. \end{aligned}$$

Thus, $\mathcal{R}^B\big(\underline{s}_1^N(s_2), s_2\big)$ is increasing in s_2 under LCR-type regulation. Therefore, the optimal liquidity decision under LCR-type regulation (the region B) is $(\underline{s}_1^L(\check{s}_2), \check{s}_2)$.

Finally, consider the bank's liquidity decision when its residual profit is $\mathcal{R}^A(s_1, s_2)$. In this case, it holds a sufficient amount of assets to repay ηDr_d at $t = 1$. It implies that $\ddot{s}_2 \leq s_2$ and $\eta Dr_d \leq s_2 Br_s + \beta s_1 BR_1$ are satisfied, and the latter is satisfied when the bank's liquidity decision satisfies either the constraint of NSFR-type regulation or LCR-type one. Then, when the bank's residual profit is $\mathcal{R}^A(s_1, s_2)$, its liquidity decision (s_1, s_2) exists on the region A under LCR-type regulation and on the region $A + A'$ under LCR-type one in Figure 2.

Suppose that there exists NSFR-type regulation. Then, the bank's problem is expressed as

$$\begin{aligned} \max_{s_1, s_2} \mathcal{R}^A(s_1, s_2) \equiv & BR_2 - B(R_2 - R_1)s_1 - B(R_2 - r_s)s_2 - \eta Dr_d - (1 - \eta)Dr_d^2, \\ & s.t. \quad 0 \leq s_1 \leq 1, \quad \ddot{s}_2 \leq s_2 \leq 1, \quad s_1 + s_2 \leq 1, \\ & \underline{s}_1^N(s_2) \leq s_1. \end{aligned}$$

With fixed value $\bar{s}_2$, it is clear that $\mathcal{R}^A(s_1, \bar{s}_2)$ is decreasing in s_1. Thus, $\underline{s}_1^N(\bar{s}_2)$ is the optimal ratio of liquid assets. Then, by substituting $s_1 = \underline{s}_1^N(\bar{s}_2)$ into $-B(R_2 - R_1)s_1 - B(R_2 - r_s)\bar{s}_2$, we have

$$\begin{aligned} & -B(R_2 - R_1)\bar{F} + B(R_2 - R_1)\frac{r_s \alpha R_2}{\beta R_1 - \alpha R_2}\bar{s}_2 - B(R_2 - r_s)\bar{s}_2, \\ & = -B(R_2 - R_1)\bar{F} + B\left[(R_2 - R_1)\frac{r_s - \alpha R_2}{\beta R_1 - \alpha R_2} - (R_2 - r_s)\right]\bar{s}_2, \end{aligned}$$

where the sign of the coefficient of $\bar{s}_2$ depends on the parameters. Thus, we have

$$\frac{\partial \mathcal{R}^A\big(\underline{s}_1^N(s_2), s_2\big)}{\partial s_2} \geq 0 \quad \Leftrightarrow \quad \frac{(R_2 - R_1)r_s + (R_1 - r_s)\alpha R_2}{R_1(R_2 - r_s)} \equiv \tilde{\beta}^N \geq \beta.$$

Denote s_2 that satisfied $\underline{s}_1^N(s_2) = 0$ as $\hat{s}_2^N$, that is, the point where the line $s_1 = \underline{s}_1^N(s_2)$ intersects with s_2 axis. Then, when $\tilde{\beta}^N \geq \beta$, $\mathcal{R}^A\big(\underline{s}_1^N(s_2), s_2\big)$ is increasing in s_2 and the optimal liquidity decision is $(s_1, s_2) = (0, \hat{s}_2^N)$. On the other hand, when $\tilde{\beta}^N < \beta$, $\mathcal{R}^A\big(\underline{s}_1^N(s_2), s_2\big)$ is decreasing in s_2 and the optimal liquidity decision is $(s_1, s_2) = \big(\underline{s}_1^N(\ddot{s}_2), \ddot{s}_2\big)$.

Next, suppose that there exists LCR-type regulation.With fixed value $\bar{s}_2$, $\mathcal{R}^A(s_1, \bar{s}_2)$ is decreasing in s_1. Thus, $\underline{s}_1^L(\bar{s}_2)$ is the optimal ratio of the liquid assets. Then, by substituting $s_1 = \underline{s}_1^L(\bar{s}_2)$ into $-B(R_2 - R_1)s_1 - B(R_2 - r_s)\bar{s}_2$, we have

$$\begin{aligned}
&-B(R_2 - R_1)\bar{L} + B(R_2 - R_1)\frac{r_s}{\beta R_1}\bar{s}_2 - B(R_2 - r_s)\bar{s}_2, \\
&= -B(R_2 - R_1)\bar{L} + B\left[(R_2 - R_1)\frac{r_s}{\beta R_1} - (R_2 - r_s)\right]\bar{s}_2,
\end{aligned}$$

where the sign of the coefficient of $\bar{s}_2$ depends on the parameters. Thus, we have

$$\frac{\partial \mathcal{R}^A\big(\underline{s}_1^N(s_2), s_2\big)}{\partial s_2} \geq 0 \quad \Leftrightarrow \quad \frac{r_s(R_2 - R_1)}{R_1(R_2 - r_s)} \equiv \tilde{\beta}^L \geq \beta.$$

Denote s_2 that satisfied $\underline{s}_1^L(s_2) = 0$ as $\hat{s}_2^L$, that is, the point where the line $s_1 = \underline{s}_1^L(s_2)$ intersects with s_2 axis. Then, when $\tilde{\beta}^L \geq \beta$, the optimal liquidity decision is $(s_1, s_2) = (0, \hat{s}_2^L)$. On the other hand, when $\tilde{\beta}^L < \beta$, the optimal liquidity decision is $(s_1, s_2) = \big(\underline{s}_1^L(\ddot{s}_2), \ddot{s}_2\big)$.

Then, compare $\mathcal{R}^A(s_1, s_2)$ with $\mathcal{R}^B(s_1, s_2)$ with some liquidity decision $(\bar{s}_1, \bar{s}_2)$. By calculating $\mathcal{R}^A(\bar{s}_1, \bar{s}_2) - \mathcal{R}^B(\bar{s}_1, \bar{s}_2)$, we have

$$\mathcal{R}^A(\bar{s}_1, \bar{s}_2) - \mathcal{R}^B(\bar{s}_1, \bar{s}_2) = \frac{1-\beta}{\beta}(\eta D r_d - s_2 B r_s).$$

Thus, $\mathcal{R}^A\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big) = \mathcal{R}^B\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big)$ $(i = N, L)$ is satisfied. However, $s_2 < \ddot{s}_2$ is satisfied when the bank's residual profit is defined as $\mathcal{R}^B(s_1, s_2)$. Moreover, as it is explained above, $\mathcal{R}^B\big(\underline{s}_1^i(s_2), s_2\big)$ $(i = N, L)$ is increasing in s_2. Thus, with any liquidity decision (s_1, s_2) in the region $B + B'$, we have

$$\mathcal{R}^B(s_1, s_2) \leq \mathcal{R}^B\big(\underline{s}_1^i(\check{s}_2), \check{s}_2\big) < \mathcal{R}^A\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big) \qquad (i = N, L).$$

In addition, when the bank' residual profit is defined as $\mathcal{R}^A(s_1, s_2)$, $\mathcal{R}^A\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big) < \mathcal{R}^A(0, \hat{s}_2^i)$ $(i = N, L)$ is satisfied, because $(s_1, s_2) = (0, \hat{s}_2^i)$ $(i = L, N)$ is optimal. Then, we have the following result.

Proposition 3. *Suppose that the bank can hold the safe assets and/or the liquid assets as liquidity. In addition, suppose that there exists NSFR-type regulation or LCR-type one and the bank cannot choose $(s_1, s_2) = (0, \ddot{s}_2)$ under either of the regulations. Then, the bank's optimal liquidity decision (s_1^*, s_2^*) satisfies following properties, regardless which type of regulations exists.*

1. *The maximum value of $\mathcal{R}^B(s_1, s_2)$ is always smaller than the maximum value of $\mathcal{R}^A(s_1, s_2)$, and thus, $\ddot{s}_2 \leq s_2^*$ is satisfied.*

2. *When β is sufficiently large, the bank's optimal liquidity decision is $(s_1^*, s_2^*) = \big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big)$, otherwise, $(s_1^*, s_2^*) = (0, \hat{s}_2^i)$ is satisfied, where $i = N$ under NSFR-type regulation and $i = L$ under LCR-type one.*

In addition, because it is clear that $\tilde{\beta}^N > \tilde{\beta}^L$ is satisfied, we have the following corollary regarding how likely the bank is to choose $s_1 > 0$ as the optimal liquidity decision.

Corollary 2. *With the same parameters, the bank is more likely to choose $\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big)$ rather than $(0, \hat{s}_2^i)$ as the optimal liquidity decision under LCR-type regulation than it is under NSFR-type regulation.*

In other words, LCR-type regulation is more likely to lead to the bank's investment in the liquid assets than NSFR-type regulation is.

5 Comparing two types of liquidity regulation

5.1 Two types of the liquidity regulation

As it is explained in the previous subsection, the bank's liquidity decision (s_1, s_2) under i-type liquidity regulations is $\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big)$ or $(0, \hat{s}_2^i)$, and actual decision depends on how large β is and which type of regulation exists. Then, because $\mathcal{R}^A(s_1, s_2)$ is decreasing in both of s_1 and s_2, which type of regulation is more optimal depends on which of the lines $s_1 = \underline{s}_1^N(s_2)$ and $s_1 = \underline{s}_1^L(s_2)$ exists more left in the region $\ddot{s}_2 \leq s_2$. Although the exact positions of the two lines depend on many parameters and it is difficult to obtain clear results, there are some implications.

When the line $s_1 = \underline{s}_1^L(s_2)$ exists on the left part of the line $s_1 = \underline{s}_1^N(s_2)$ in the region $\ddot{s}_2 \leq s_2$, at least $\underline{s}_1^N(s_2) > \underline{s}_1^L(s_2)$ is satisfied. The inequality is expressed as

$$\frac{(\eta+\Delta)Dr_d + \alpha(1-\eta-\Delta)Dr_d^2 - \alpha BR_2}{\beta R_1 - \alpha R_2} > \frac{(\eta+\Delta)Dr_d + \rho(1-\eta-\Delta)Dr_d^2}{\beta BR_1}.$$

Define $\gamma \equiv \beta BR_1/[\beta BR_1 - \alpha R_2](< 1)$, and then, the inequality can be rewritten as

$$\frac{\gamma(\eta+\Delta)Dr_d + \gamma\alpha(1-\eta-\Delta)Dr_d^2}{\beta BR_1} - \frac{\alpha BR_2}{\beta R_1 - \alpha R_2} > \frac{(\eta+\Delta)Dr_d + \rho(1-\eta-\Delta)Dr_d^2}{\beta BR_1},$$

$$\frac{Dr_d}{\beta BR_1}\big[-(1-\gamma)(\eta+\Delta)Dr_d + (\gamma\alpha-\rho)(1-\eta-\Delta)r_d\big] - \frac{\alpha BR_2}{\beta R_1 - \alpha R_2} > 0.$$

Thus, in order that $\underline{s}_1^L(s_2) < \underline{s}_1^N(s_2)$ is satisfied, at least $\gamma\alpha - \rho > 0$ must be satisfied. Because both α and γ are smaller than 1, $\gamma\alpha - \rho > 0$ implies that ρ is sufficiently close to 0. Therefore, when LCR-type regulation is sufficiently strict, ρ is so large that $\underline{s}_1^L(s_2) < \underline{s}_1^N(s_2)$ is not satisfied. Then, the line $s_1 = \underline{s}_1^L(s_2)$ exists on the right part of the line $s_1 = \underline{s}_1^N(s_2)$ in the region $\ddot{s}_2 \leq s_2$ as in the Figure 1.

This implication can be derived in another way. The constraint of LCR-type regulation is expressed as

$$\begin{aligned} & s_2Br_s + \beta s_1BR_1 \geq (\eta+\Delta)r_dD + \rho(1-\eta-\Delta)r_d^2D, \\ \Leftrightarrow \quad & \big[s_2Br_s + \beta s_1BR_1 - (\eta+\Delta)r_dD\big] \geq \rho(1-\eta-\Delta)r_d^2D. \end{aligned}$$

The constraint of NSFR-type one is expressed as

$$
\begin{aligned}
& BR_2 + B\left(\frac{\beta R_1}{\alpha} - R_2\right)s_1 + B\left(\frac{r_s}{\alpha} - R_2\right)s_2 - \frac{(\eta+\Delta)Dr_d}{\alpha} - (1-\eta-\Delta)Dr_d^2 \geq 0, \\
\Leftrightarrow\quad & \alpha(1-s_1-s_2)BR_2 + s_2Br_s + \beta s_1 BR_1 \geq (\eta+\Delta)Dr_d + \alpha(1-\eta-\Delta)Dr_d^2, \\
\Leftrightarrow\quad & \left[s_2Br_s + \beta s_1 BR_1 - (\eta+\Delta)Dr_d\right] \geq -\alpha\left[(1-s_1-s_2)BR_2 - (1-\eta-\Delta)Dr_d^2\right].
\end{aligned}
$$

Thus, LCR-type regulation requires the bank holding more liquidity than the amount of maximum repayment at $t=1$ because it pays more attention to holding liquidity between $t=1$ and $t=2$. On the other hand, NSFR-type regulation permits the bank to hold less liquidity than $(\eta+\Delta)Dr_d$ if it can survive at $t=2$. Requiring the bank to hold more liquidity means that it must choose large s_1 and s_2, and thus, the constraint of LCR-type regulation is likely to exist the left part of the constraint of NSFR-type one.

The above result implies that $\left(\underline{s}_1^N(\ddot{s}_2), \ddot{s}_2\right)$ and $(0, \hat{s}_2^N)$ are likely to be more optimal for the bank than $\left(\underline{s}_1^L(\ddot{s}_2), \ddot{s}_2\right)$ and $(0, \hat{s}_2^L)$. Moreover, it implies that the amount of lending $(1-s_1^*-s_2^*)B$ can be larger under NSFR-type regulation than it is under LCR-type regulation. Thus, one of the implication of the model is that NSFR-type regulation is more likely to be optimal than LCR-type regulation. However, as Diamond and Kashyap (2016) points out, this result depends on the model's setting that $\eta+\Delta$ is fixed.

Suppose that there are many banks in the economy and each bank's depositors have different values regarding η and Δ. Then, first, consider how to regulate these banks by one NSFR-type regulation. When a regulator sets the variable $\eta+\Delta$ at some level to make the banks stable, he or she must set the level so that even the bank with maximum repayment at $t=1$ can survive. Then, because all of the other banks must obey the constraint that is based on the largest value of $\eta+\Delta$, they need to hold more liquidity than the amount that is calculated by using their own $\eta+\Delta$. Thus, the total amount of excess liquidity in the economy can be so large. Next, consider how to regulate these banks by one LCR-type regulation. In this case, the regulator can use not $\eta+\Delta$ but ρ as the tool to make them stable, and thus, they can decide the amount of liquidity based on their own $\eta+\Delta$. As a result, although each bank has some amount of excess liquidity, the total amount of excess liquidity in the economy can be kept relatively moderate, especially when ρ is small. Therefore, when we consider heterogeneous banks, LCR-type regulation can be optimal than NSFR-type regulation.

5.2 Bank's lending and liquidity regulations

As it is explained above, when the bank can hold the liquid assets as liquidity, it can choose not only to hold only the safe assets but also to hold both the safe assets and the liquid assets. Then, either liquidity decision satisfies the regulation's requirement regardless of the regulation's type. It implies that these two liquidity decisions are indifferent with respect of the bank's stability. However, it does not always guarantee that these two decision are indifferent with regard to the amounts of the bank's lending.

Suppose that there exists NSFR-type regulation. Then, the bank's liquidity decision (s_1, s_2) is $\big(\underline{s}_1^N(\ddot{s}_2), \ddot{s}_2\big)$ when $\tilde{\beta}^N \le \beta$ is satisfied, whereas it is $(0, \hat{s}_2^N)$ when $\beta \le \tilde{\beta}^N$ is satisfied. Denote the amount of its lending when its liquidity decision is $(0, \hat{s}_2^N)$ as $\bar{M}^N B$. It implies that $\bar{M}^N B \equiv (1-0-\hat{s}_2^N)B$ is satisfied and we have $\bar{M}^N \equiv 1-\hat{s}_2^N$. Then, suppose that, with some liquidity decision $(\bar{s}_1, \bar{s}_2)$, the amount of its lending is larger than or equal to $\bar{M}^N B$, that is, $\bar{M}^N B \le (1-\bar{s}_1-\bar{s}_2)B$ is satisfied. Then, by rewriting this inequality, we have

$$\begin{aligned} \bar{M}^N B \le (1-\bar{s}_1-\bar{s}_2)B \quad &\Leftrightarrow \quad \bar{s}_1 \le -\bar{s}_2 + 1 - \bar{M}^N, \\ &\Leftrightarrow \quad \bar{s}_1 \le -\bar{s}_2 + \hat{s}_2^N. \end{aligned}$$

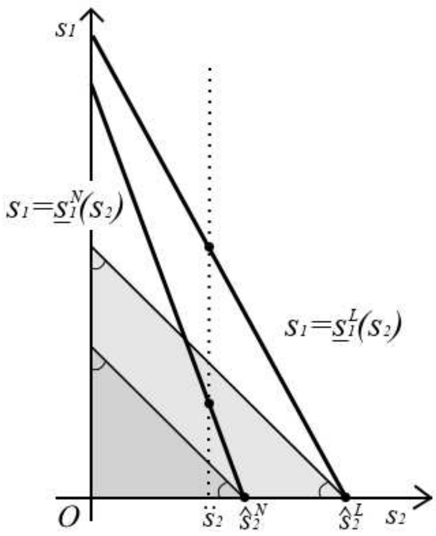

Figure 3: the region where the amount of bank's lending increases compared with $(0, \hat{s}_2^i)$

Thus, if the liquidity decision $\big(\underline{s}_1^N(\ddot{s}_2), \ddot{s}_2\big)$ leads to at least the same amount of its lending as $(0, \hat{s}_2^N)$ does, the point $\big(\underline{s}_1^N(\ddot{s}_2), \ddot{s}_2\big)$ must not exist on the upper part of the line $s_1 = -\bar{s}_2 + \hat{s}_2^N$. Then, because the slope of the line $s_1 = -\bar{s}_2 + \hat{s}_2^N$ is -1 and the slope of the line $s_1 = \underline{s}_1^N(s_2)$ is $-(r_s - \alpha R_2)/(\beta R_1 - \alpha R_2) < -1$, point $\big(\underline{s}_1^N(\ddot{s}_2), \ddot{s}_2\big)$ exists on the upper part of the line $s_1 = -\bar{s}_2 + \hat{s}_2^N$, as it is in Figure 3. In other words, liquidity decision $\big(\underline{s}_1^N(\ddot{s}_2), \ddot{s}_2\big)$ always leads to smaller amount of the bank's lending than $(0, \hat{s}_2^N)$ does. When there exists LCR-type regulation, we still obtain a similar result, that is, the liquidity decision $\big(\underline{s}_1^L(\ddot{s}_2), \ddot{s}_2\big)$ always leads to smaller amount of the bank's lending than $(0, \hat{s}_2^L)$ does.

Then, the results are summarized as follows.

Proposition 4. *Suppose that the bank's optimal liquidity decision is* $\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big)$ *or* $(0, \hat{s}_2^i)$ *where* $i = N$ *under NSFR-type regulation and* $i = L$ *under LCR-type one. Then, regardless which type regulations exists and which of the liquidity decision is optimal for the bank, the amount of the bank's lending with* $\big(\underline{s}_1^i(\ddot{s}_2), \ddot{s}_2\big)$ *is always smaller than that with* $(0, \hat{s}_2^i)$.

In other words, holding liquid assets as liquidity involves the bank's lending decreasing as the outcome of its profit maximization. In addition, as it is mentioned above, holding both the liquid assets and the safe assets in more likely to be chosen under LCR-type regulation than it is under NSFR-type one. Then, we have the following corollary.

Corollary 3. *With same parameters, the amount of the bank's lending is more likely to decrease under LCR-type regulation than it is under NSFR-type regulation.*

6 Conclusion

In this study, we analyze how a bank chooses its portfolio under liquidity regulations in the economy there exist safe assets, liquid assets and lending. The results are summarized as follows.

First, the analysis shows that, regardless type of the liquidity regulation, the bank still holds an ex-post excess amount of liquidity even when it can use the liquidity assets as sources of liquidity besides the safe assets (reserves). This result mainly depends on the model's setting that the bank needs to prepare the nervous depositors' withdrawals that are not actually taken place under the regulations, and thus, introducing another assets cannot solve the problem.

Second, the analysis shows that relying only on the safe assetsis not the only choice of the bank under liquidity regulations and that holding both the safe assets and the liquid assets can be chosen when the return of liquidating the liquid asset is sufficient large. In other words, if there exist assets that are slightly inefficient as sources of liquidity but yield more return than the safe assets, banks have an incentive to substitute these assets for the safe assets. In addition, in this case, the model shows that the amount of the safe assets is equal to the amount that is actually needed. In other words, with respect to the safe assets, banks hold ex-post efficient amount of the assets.

Third, the analysis shows that, although holding both the safe assets and the liquid assets and holding only the safe assets are indifferent with regard to the liquidity regulations, the former causes larger reduction of the bank's lending than the latter does, even when the former is optimal for its profit. Taking the second result into account, this result implies that it is optimal for banks to reduce their lending in order to keep the safe assets at the ex-post efficient amount. Thus, if a regulator wants to make banks stable but does not want to decrease their lending, the liquidity regulation causes the incompatibility between the regulator's and the banks' objects. In addition, the model shows that the probability that banks choose to hold both the safe assets and the liquid assets depends partly on the type of liquidity regulation.

These results imply that banks' decisions for mitigating losses caused by liquidity regulations lead to an undesired outcome with respect of the amount of their lending, and thus, we consider more carefully banks' decisions under liquidity regulations.

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